

Homework 8

Physics 112A

Problem 5.12 Use the result of Ex. 5.6 to calculate the magnetic field at the center of a uniformly charged spherical shell, of radius R and total charge Q , spinning at constant angular velocity ω

$$B(z) = \frac{\mu_0 I}{4\pi} \frac{\cos\theta}{r^2} 2\pi R = \frac{\mu_0 I}{2} \frac{R^2}{(R^2 + z^2)^{\frac{3}{2}}}$$

θ is from the center of the sphere instead of from the ring, so $\cos\theta \rightarrow \sin\theta$.

$$\begin{aligned} R &\rightarrow R\sin\theta \\ dI &= KRd\theta \\ &= \sigma v R d\theta \\ &= \frac{Q}{4\pi R^2} R\sin\theta \omega R d\theta \\ &= \frac{Q\omega}{4\pi} \sin\theta d\theta \\ dB &= \frac{2\pi\mu_0}{4\pi} \frac{R\sin^2\theta}{R^2} dI \\ &= \frac{\mu_0}{2R} \sin^2\theta \frac{Q\omega}{4\pi} \sin\theta d\theta \\ &= \frac{Q\omega\mu_0}{8\pi R} \int_0^\pi \sin^3\theta d\theta \\ &= \frac{Q\omega\mu_0}{8\pi R} \left[\frac{1}{3} \cos^3\theta - \cos\theta \right]_0^\pi \\ &= \boxed{\frac{Q\omega\mu_0}{6\pi R}} \end{aligned}$$

Problem 5.13 Suppose you have two infinite straight-line charges λ , a distance d apart, moving along at a constant speed v . How great would v have to be in order for the magnetic attraction to balance the electrical repulsion? Work out the actual number. Is this a reasonable sort of speed?

$$\begin{aligned}
E &= \frac{\lambda L}{2\pi\epsilon_0 d^2} \\
B &= \frac{\mu_0 I L}{4\pi d^2} \\
F_C &= -F_L \\
\frac{1}{4\pi\epsilon_0} \frac{(\lambda L)^2}{d^2} &= -\lambda L \left(\frac{\lambda L}{2\pi\epsilon_0 d^2} + v \frac{\mu_0 I L}{4\pi d^2} \right) \\
\frac{1}{4\epsilon_0} &= -\frac{1}{2\epsilon_0} - \frac{\mu_0 v^2}{4} \\
1 &= -2 - \mu_0 \epsilon_0 v^2 \\
v^2 &= \frac{1}{\mu_0 \epsilon_0} \\
v &= \boxed{\frac{1}{\sqrt{\mu_0 \epsilon_0}}}
\end{aligned}$$

The speed of light is also $\frac{1}{\sqrt{\mu_0 \epsilon_0}}$, so this is not possible.

Problem 5.14 A steady current I flows down a long cylindrical wire of radius a . Find the magnetic field, both inside and outside the wire if:

(a) the current is uniformly distributed over the surface of the wire.

When $s < a$:

$$\begin{aligned}
\int B \cdot dl &= \mu_0 I \\
B 2\pi s &= 0 \\
B &= \boxed{0}
\end{aligned}$$

When $s > a$:

$$\begin{aligned}
\int B \cdot dl &= \mu_0 I \\
B 2\pi s &= \mu_0 I \\
B &= \boxed{\frac{\mu_0 I}{2\pi s} \hat{\phi}}
\end{aligned}$$

(b) the current is distributed in such a way that J is proportional to s , the distance from the axis.

$$\begin{aligned}
J(s) &= ks \\
I &= \int_0^a ks\pi s ds \\
&= \frac{1}{3}k\pi[s^3]_0^a \\
&= \frac{1}{3}k\pi a^3 \\
k &= \frac{3I}{a^3\pi}
\end{aligned}$$

When $s < a$:

$$\begin{aligned}
I_{\text{enc}} &= \int_0^s \frac{3I}{a^3\pi} s\pi s ds \\
&= \frac{3I}{a^3} \frac{1}{3} [s^3]_0^s \\
&= I \frac{s^3}{a^3} \\
\int B \cdot dl &= \mu_0 I \\
B2\pi s &= \mu_0 I \frac{s^3}{a^3} \\
B &= \boxed{\frac{\mu_0 I}{2\pi} \frac{s^2}{a^3} \hat{\phi}}
\end{aligned}$$

When $s > a$:

$$\begin{aligned}
\int B \cdot dl &= \mu_0 I_{\text{enc}} \\
B2\pi s &= \mu_0 I \\
B &= \boxed{\frac{\mu_0 I}{2\pi s} \hat{\phi}}
\end{aligned}$$

Problem 5.25 Find the magnetic vector potential of a finite segment of straight wire carrying a current I . [Put the wire on the z -axis, from z_1 to z_2 , and use Eq. 5.66.] Check that your answer is consistent with Eq. 5.37.

Eq. 5.66:

$$A = \frac{\mu_0 I}{4\pi} \int \frac{dl}{r}$$

Using cylindrical coordinates:

$$\begin{aligned}
 r^2 &= z^2 + s^2 \\
 A &= \frac{\mu_0 I}{4\pi} \int_{z_1}^{z_2} \frac{dz}{\sqrt{z^2 + s^2}} \hat{z} \\
 &= \frac{\mu_0 I}{4\pi} \int_{z_1}^{z_2} [\ln(z + \sqrt{s^2 + z^2})]_{z_1}^{z_2} \\
 &= \boxed{\frac{\mu_0 I}{4\pi} \ln \left[\frac{z_2 + \sqrt{s^2 + z_2^2}}{z_1 + \sqrt{s^2 + z_1^2}} \right] \hat{z}}
 \end{aligned}$$

Checking consistency: