Problem 5.12. Use the result of Ex. 5.6 to calculate the magnetic field at the center of a uniformly charged spherical shell, of radius R and total charge Q, spinning at constant angular velocity ω.

Ex 5.6, current ring:

$$B(z) = \frac{\mu_0 I}{4\pi}\left(\frac{\cos\theta}{\imath^2}\right)2\pi R = \frac{\mu_0 I}{2}\frac{R^2}{(R^2+z^2)^{3/2}}. \tag{5.41}$$

Problem 5.13. Suppose you have two infinite straight-line charges λ, a distance d apart, moving along at a constant speed v (Fig. 5.25). How great would v have to be in order for the magnetic attraction to balance the electrical repulsion? Work out the actual number. Is this a reasonable sort of speed?[14]

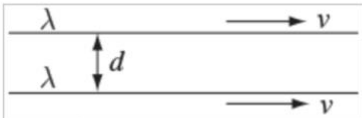

Fig. 5.25

Fig. 5.25

Problem 5.14. A steady current I flows down a long cylindrical wire of radius a (Fig. 5.39). Find the magnetic field, both inside and outside the wire if:

Fig. 5.39

Fig. 5.39

(a) the current is uniformly distributed over the surface of the wire;

(b) the current is distributed in such a way that J is proportional to s, the distance from the axis.

Problem 5.25. Find the magnetic vector potential of a finite segment of straight wire carrying a current I. [Put the wire on the z-axis, from z_1 to z_2, and use Eq. 5.66.] Check that your answer is consistent with Eq. 5.37.

Problem 5.26.

(a) What current density would produce the vector potential $\mathbf{A} = k\hat{\boldsymbol{\phi}}$ (where k is a constant), in cylindrical coordinates?
(b) Consider an azimuthally symmetric magnetic field; it points in the z direction, and its magnitude is a function only of s. Check that

$$\mathbf{A} = A(s)\hat{\boldsymbol{\phi}}, \quad \text{where } A(s) = \frac{1}{s}\int_0^s B(s')s'\,ds',$$

by calculating its divergence and curl. (This generalizes Ex. 5.12.)

Problem 5.27. If $\mathbf{B}$ is *uniform*, show that $\mathbf{A}(\mathbf{r}) = -\frac{1}{2}(\mathbf{r} \times \mathbf{B})$ works. That is, check that $\nabla \cdot \mathbf{A} = 0$ and $\nabla \times \mathbf{A} = \mathbf{B}$. Is this result unique, or are there other functions with the same divergence and curl?

Problem 5.28.

(a) By whatever means you can think of (short of looking it up), find the vector potential a distance s from an infinite straight wire carrying a current I. Check that $\nabla \cdot \mathbf{A} = 0$ and $\nabla \times \mathbf{A} = \mathbf{B}$.

(b) Find the magnetic potential *inside* the wire, if it has radius R and the current is uniformly distributed.