# Homework 10

Physics 112A

**Problem 6.8** A very long circular cylinder of radius $R$ carries a magnetization $M = ks^2\hat{\phi}$, where $k$ is a constant. Find the magnetic field due to $M$, for points inside and outside the cylinder (and far from the ends).

$$\begin{aligned}
J_b &= \nabla \times M\hat{\phi} \\
&= \frac{1}{s}\frac{\partial}{\partial s}[sks^2]\hat{z} \\
&= 3ks\hat{z} \\
K_b &= M\hat{\phi} \times \hat{n} \\
&= -ks^2\hat{z} \\
&= -kR^2\hat{z}
\end{aligned}$$

When $s < R$:

$$\begin{aligned}
\int B \cdot dl &= \mu_0 \int_0^s J_b da \\
B2\pi s &= \mu_0 \int_0^s 3ks2\pi s ds \\
B &= \frac{3\mu_0 k}{s}[\frac{1}{3}s^3]_0^s \\
&= \boxed{\mu_0 ks^2\hat{\phi}}
\end{aligned}$$

When $s > R$:

$$\begin{aligned}
\int B \cdot dl &= \mu_0(\int_0^R J_b da + \int_0^R K_b dl) \\
B2\pi s &= \mu_0(\int_0^R 3ks2\pi s ds + \int_0^R -kR^2 2\pi ds) \\
&= \mu_0(2\pi kR^3 - 2\pi kR^2[s]_0^R) \\
B &= \boxed{0}
\end{aligned}$$

**Problem 6.10** An iron rod of length $L$ and square cross section (side length $a$) is given a uniform longitudinal magnetization $M$, and then bent around into a circle with a narrow gap (width $w$). Find the magnetic field at the center of the gap, assuming $w << a << L$. [Hint: Treat it as the superposition of a complete torus plus a square loop with revered current.]

**RETURN LATER**

**Problem 6.12** An infinitely long cylinder, of radius $R$, carries a "frozen-in" magnetization, parallel to the axis,

$$M = ks\hat{z}$$

where $k$ is a constant and $s$ is the distance from the axis; there is no free current anywhere. Find the magnetic field inside and outside the cylinder by two different methods:

**(a)** Locate all the bound currents, and calculate the field they produce.

$$\begin{aligned} J_b &= \nabla \times M\hat{z} \\ &= -\frac{\partial}{\partial s} ks\hat{\phi} \\ &= -k\hat{\phi} \\ K_b &= M\hat{z} \times \hat{n} \end{aligned}$$

**(b)** Use Ampere's law to find $H$, and then get $B$.