Homework 6

Physics 112A

Problem 4.8 Show that the interaction energy of two ideal dipoles separated by a displacement **r** is

$$U = \frac{1}{4\pi\epsilon_0}\frac{1}{r^3}[\vec{p}_1 \cdot \vec{p}_2 - 3(\vec{p}_1 \cdot \hat{r})(\vec{p}_2 \cdot \hat{r})]$$

Eq. 3.104:

$$\vec{E}_{dip}(r) = \frac{1}{4\pi\epsilon_0}\frac{1}{r^3}[3(\vec{p} \cdot \hat{r})\hat{r} - \vec{p}]$$

Prob. 4.7:

$$U = -\vec{p} \cdot \vec{E}$$

The interaction energy is the energy from the second dipole $\vec{p}_2$ in the electric field $\vec{E}$ from the first dipole.

$$\begin{aligned}
U &= -\vec{p}_2 \cdot \vec{E}_1 \\
&= -\vec{p}_2 \cdot \frac{1}{4\pi\epsilon_0}\frac{1}{r^3}[3(\vec{p}_1 \cdot \hat{r})\hat{r} - \vec{p}_1] \\
&= -\frac{1}{4\pi\epsilon_0}\frac{1}{r^3}\vec{p}_2 \cdot [3(\vec{p}_1 \cdot \hat{r})\hat{r} - \vec{p}_1] \\
&= \boxed{\frac{1}{4\pi\epsilon_0}\frac{1}{r^3}[\vec{p}_2 \cdot \vec{p}_1 - 3(\vec{p}_1 \cdot \hat{r})(\vec{p}_2 \cdot \hat{r})]}
\end{aligned}$$

Problem 4.10 A sphere of radius R carries a polarization

$$\vec{P}(r) = k\vec{r}$$

where k is a constant and **r** is the vector from the center.
(a) Calculate the bound charges σ_b and ρ_b

$$\begin{aligned}
\sigma_b &= \vec{P} \cdot \hat{n} \\
&= k\vec{r} \cdot \hat{r} \\
&= \boxed{kR}
\end{aligned}$$

$$
\begin{aligned}
\rho_b &= -\nabla \cdot \vec{p} \\
&= -(\frac{1}{r^2}\frac{\partial}{\partial r}[r^2 kr]) \\
&= -\frac{1}{r^2}(3r^2 k) \\
&= \boxed{-3k}
\end{aligned}
$$

(b) Find the field inside and outside the sphere

$$
\begin{aligned}
E_{in}(4\pi r^2) &= \frac{Q}{\epsilon} \\
&= \frac{1}{\epsilon}\rho_b V \\
&= \frac{1}{\epsilon}(-3k)(\frac{4}{3}\pi r^3) \\
E_{in} &= \frac{1}{\epsilon}(-4k\pi r^3)(\frac{1}{4\pi r^2}) \\
&= \boxed{-\frac{kr}{\epsilon}\hat{r}}
\end{aligned}
$$

$$
\begin{aligned}
Q &= \rho_b V + \sigma_b A \\
&= 0 \\
\therefore E_{out} &= \boxed{0}
\end{aligned}
$$