# Homework 9

## Physics 112A

**Problem 5.37** A circular loop of wire, with radius $R$, lies in the $xy$ plane (centered at the origin) and carries a current $I$ running counterclockwise as viewed from the positive $z$-axis.

**(a)** What is its magnetic dipole moment ?

$$\begin{aligned} m &= I \int da' \\ &= \boxed{I\pi R^2 \hat{z}} \end{aligned}$$

**(b)** What is the (approximate) magnetic field at point far from the origin ?

$$\begin{aligned} A &= \frac{\mu_0}{4\pi} \frac{m\hat{z} \times \hat{r}}{r^2} \\ &= \frac{\mu_0}{4\pi} \frac{m[cos\theta\hat{r} - sin\theta\hat{\theta}] \times \hat{r}}{r^2} \\ &= \frac{\mu_0}{4\pi} \frac{msin\theta}{r^2} \hat{\phi} \\ B &= \frac{\mu_0 m}{4\pi} \left(\frac{1}{rsin\theta} \frac{\partial}{\partial\theta}[sin\theta \frac{sin\theta}{r^2}]\hat{r} - \frac{1}{r} \frac{\partial}{\partial r}[r \frac{sin\theta}{r^2}]\hat{\theta}\right) \\ &= \frac{\mu_0 m}{4\pi r^3}[2cos\theta\hat{r} + sin\theta\hat{\theta}] \\ &= \boxed{\frac{\mu_0 I R^2}{4r^3}[2cos\theta\hat{r} + sin\theta\hat{\theta}]} \end{aligned}$$

**(c)** Show that, for points on the $z$-axis, your answer is consistent with the exact field (Ex. 5.6), when $z >> R$.

At the $z$-axis, $\hat{r} = \hat{z}$ and $\theta = 0$, so

$$B = \frac{\mu_0 I R^2}{2z^3} \hat{z}$$

From Ex. 5.6:

$$B(z) = \frac{\mu_0 I}{2} \frac{R^2}{(R^2+z^2)^{\frac{3}{2}}} \hat{z}$$
$$= \frac{\mu_0 I R^2}{2z^3} \hat{z}$$

**Problem 5.39**

**(a)** A phonograph record of radius $R$, carrying uniform surface charge $\sigma$, is rotating at constant angular velocity $\omega$. Find its magnetic dipole moment.

$$I = \int \sigma v dr$$
$$= \sigma\omega \int_0^R r dr$$
$$= \frac{1}{2}\sigma\omega R^2$$
$$m = \frac{1}{2}\sigma\omega R^2 \int_0^R \pi r dr$$
$$= \boxed{\frac{1}{4}\pi\sigma\omega R^4 \hat{z}}$$

**(b)** Find the magnetic dipole moment of the spinning spherical shell in Ex. 5.11. Show that for points $r > R$ the potential is that of a perfect dipole.

$$dI = \frac{dq}{t}$$
$$= \frac{\sigma\omega 2\pi R^2 sin\theta d\theta}{2\pi}$$
$$= \omega\sigma R^2 sin\theta d\theta$$
$$a = \pi(R sin\theta)^2$$
$$m = \pi\sigma\omega R^4 \int_0^\pi sin^3\theta d\theta$$
$$= \pi\sigma\omega R^4 [\frac{1}{3}cos\theta - cos\theta]_0^\pi$$
$$= \boxed{\frac{4}{3}\pi\sigma\omega R^4 \hat{z}}$$

$$A = \frac{\mu_0}{4\pi}\frac{4}{3}\pi\sigma\omega R^4 \hat{\phi}$$
$$= \frac{1}{3r^2}\mu_0\sigma\omega R^4 sin\theta\hat{\phi}$$

**5.44** A current $I$ flows to the right through a rectangular bar of conducting material, in the prescence of a uniform magnetic field $B$ pointing out of the page.

**(a)** The moving charges will be deflected $\boxed{\text{downwards.}}$

**(b)** Find the resulting potential difference (the Hall voltage) between the top and bottom of the bar, in terms of $B$, $v$ (the speed of the charges), and the relevant dimensions of the bar.

$$\begin{aligned} q(E + vB) &= 0 \\ E &= -vB \\ V &= -\int_0^t E \cdot dl \\ &= \boxed{vBt} \end{aligned}$$

**(c)** The potential would be greater at the top plate instead.