34 PARTICLES AND WAVES

EXERCISES

Section 34.2 Blackbody Radiation

11. **INTERPRET** This is a problem about blackbody radiation. We want to explore the connection between temperature and the radiated power.

DEVELOP From the Stefan–Boltzmann law (Equation 34.1), $P = \sigma A T^4$, we see that the total radiated power, or luminosity, of a blackbody is proportional to T^4.

EVALUATE Doubling the absolute temperature increases the luminosity by a factor of $2^4 = 16$.

ASSESS A blackbody is a perfect absorber of electromagnetic radiation. As the temperature of the blackbody increases, its radiated power also goes up.

12. **INTERPRET** This problem explores blackbody characteristics of a star. We are to find the power radiated, the peak emission wavelength, and the median emission wavelength of the star Rigel.

DEVELOP To a good approximation, the surface of Rigel radiates like a blackbody, so the power radiated per unit area may be found from the Stefan–Boltzmann law (Equation 34.1), dividing both sides by the area A:

$$\frac{P}{A} = \sigma T^4$$

The peak and median wavelengths may be found from Equations 34.2a and 34.2b.

EVALUATE **(a)** The power radiated per unit area is

$$\frac{P}{A} = \sigma T^4 = \left[5.67\times10^{-8}\ \text{W}/\left(\text{m}^2\cdot\text{K}^4\right)\right]\left(1.0\times10^4\,\text{K}\right)^4 = 5.7\times10^8\ \text{W/m}^2.$$

(b) From Equation 34.2a, the peak wavelength is

$$\lambda_{\text{peak}} = \frac{2.898\ \text{mm}\cdot\text{K}}{1.00\times10^4\ \text{K}} = 290\ \text{nm}$$

(c) From Equation 34.2b, the median wavelength is

$$\lambda_{\text{median}} = \frac{4.11\,\text{mm}\cdot\text{K}}{1.00\times10^4\ \text{K}} = 411\,\text{nm}$$

ASSESS The median wavelength is longer than the peak wavelength because of the long-wavelength tail of the temperature distribution (see Fig. 34.2).

13. **INTERPRET** We are given the temperature of a blackbody (i.e., the Earth) and asked to find the wavelengths that correspond to peak radiance and median radiance.

DEVELOP The wavelength at which a blackbody at a given temperature radiates the maximum power is given by Wien's displacement law (Equation 34.2a):

$$\lambda_{\text{peak}} T = 2.898\ \text{mm}\cdot\text{K}$$

Similarly, the median wavelength, below and above which half the power is radiated, is given by Equation 34.2b:

$$\lambda_{\text{median}} T = 4.11\ \text{mm}\cdot\text{K}$$

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EVALUATE Using the above formulae, we obtain

$$\lambda_{\text{peak}} = \frac{2.898 \text{ mm}\cdot\text{K}}{255 \text{ K}} = 11.4 \text{ µm}$$
$$\lambda_{\text{median}} = \frac{4.11 \text{ mm}\cdot\text{K}}{255 \text{ K}} = 16.2 \text{ µm}$$

The wavelengths are in the infrared region of the electromagnetic spectrum.

ASSESS Note that $\lambda_{\text{median}} > \lambda_{\text{peak}}$.

14. **INTERPRET** This problem involves blackbody radiation. We are given the wavelength at which the peak power is emitted, and are asked to find the temperature.

DEVELOP Apply Equation 34.2a

$$\lambda_{\text{peak}} T = 2.898 \text{ mm}\cdot\text{K}$$

with $\lambda_{\text{peak}} = 40$ µm.

EVALUATE The asteroid's temperature is

$$T = \frac{2.898 \text{ mm}\cdot\text{K}}{40 \text{ µm}} = 72 \text{ K}$$

ASSESS This is a cold asteroid. According to NASA, the temperature of a typical asteroid in the asteroid belt is around 200 K.

15. **INTERPRET** We are to find the wavelength for the peak radiance of solar blackbody radiation, and the median wavelength. In both cases, we'll use the per-unit-wavelength basis; Equations 34.2a and 34.2b.

DEVELOP Wien's law (Equation 34.2a) gives us the peak wavelength: $\lambda_{\text{peak}} T = 2.898 \text{ mm}\cdot\text{K}$. The median wavelength is given by Equation 34.2b: $\lambda_{\text{median}} T = 4.11 \text{ mm}\cdot\text{K}$. The temperature of the Sun is $T = 5800$ K, so we can use these equations to solve for the respective wavelengths.

EVALUATE Inserting the temperature gives

(a) $\lambda_{\text{peak}} = \dfrac{2.898 \text{ mm}\cdot\text{K}}{5800 \text{ K}} = 5.000\times10^{-4} \text{ mm} = 500.0 \text{ nm}$

(b) $\lambda_{\text{median}} = \dfrac{4.11 \text{ mm}\cdot\text{K}}{5800 \text{ K}} = 7.086\times10^{-4} \text{ mm} = 708.6 \text{ nm}$

ASSESS The peak wavelength is near the center of the visible spectrum (green) and the median wavelength is just beyond the visible in the near-infrared region.

Section 34.3 Photons

16. **INTERPRET** This problem explores the connection between frequency and energy. We are given the frequency of photons and are asked to find the energy in electron volts (eV).

DEVELOP Apply Equation 34.6, $E = hf$.

EVALUATE **(a)** For $f = 1.0$ MHz, $E = hf = \left(4.136\times10^{-15} \text{ eV}\cdot\text{s}\right)\left(1.0\times10^{-6} \text{ Hz}\right) = 4.1\times10^{-9} \text{ eV}$

(b) For $f = 5.0 \times 10^{14}$ Hz, $E = hf = \left(4.136\times10^{-15} \text{ eV}\cdot\text{s}\right)\left(5.0\times10^{14} \text{ Hz}\right) = 2.1 \text{ eV}$

(c) For $f = 3.0 \times 10^{18}$ Hz, $E = hf = \left(4.136\times10^{-15} \text{ eV}\cdot\text{s}\right)\left(3.0\times10^{18} \text{ Hz}\right) = 12 \text{ keV}$

ASSESS The energy for the photon of part **(a)** corresponds roughly to radio frequencies, that for part **(b)** is visible light, and that for part **(c)** is X-ray radiation.

17. **INTERPRET** We're asked to express the range of human eye sensitivity in terms of photon energies.

DEVELOP The photon energy is given by Equation 34.6: $E = hf$, or in terms of wavelength: $E = hc/\lambda$.

EVALUATE The limits of human eye sensitivity are

$$E_{\text{min}} = \frac{hc}{\lambda_{\text{max}}} = \frac{1240 \text{ eV}\cdot\text{nm}}{700 \text{ nm}} = 1.8 \text{ eV}$$
$$E_{\text{max}} = \frac{hc}{\lambda_{\text{min}}} = \frac{1240 \text{ eV}\cdot\text{nm}}{400 \text{ nm}} = 3.1 \text{ eV}$$

ASSESS We've used the common shorthand of $hc = 1240 \text{ eV}\cdot\text{nm}$.

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18. **INTERPRET** We are to find the rate at which photons are produced by a cellphone with a power of 0.600 W.

DEVELOP Apply Equation 34.6, $E = hf$ to find the photon energy E. To find the photon production rate, divide the oven power by the photon energy (in joules).

EVALUATE For f = 787 MHz, $E = hf = \left(4.136\times10^{-15}\ \text{eV}\cdot\text{s}\right)\left(787\times10^{6}\ \text{Hz}\right) = 3.26\times10^{-6}\ \text{eV}$.

The photon production rate is

$$P/E = \frac{0.600\ \text{J/s}}{3.26\times10^{-6}\ \text{eV/photon}}\overbrace{\left(\frac{1\,\text{eV}}{1.6\times10^{-19}\ \text{J}}\right)}^{=1} = 1.15\times10^{24}\ \text{s}^{-1}$$

ASSESS The production rate is quite large because the photon energy is quite small.

19. **INTERPRET** The problem asks for a comparison of the power output by a red laser and a blue laser. The lasers emit photons at the same rate, but the photon energy of each laser is different.

DEVELOP Using $\lambda = c/f$ and Equation 34.6, $E = hf$, the ratio of the photon energies is

$$\frac{E_{\text{blue}}}{E_{\text{red}}} = \frac{f_{\text{blue}}}{f_{\text{red}}} = \frac{\lambda_{\text{red}}}{\lambda_{\text{blue}}}$$

EVALUATE Using the above equation, the ratio of the energies is

$$\frac{E_{\text{blue}}}{E_{\text{red}}} = \frac{\lambda_{\text{red}}}{\lambda_{\text{blue}}} = \frac{650\ \text{nm}}{450\ \text{nm}} = 1.44$$

Since the lasers emit photons at the same rate, this is also the ratio of their power outputs. Thus the power of the blue laser is 1.44 times that of the red laser.

ASSESS Blue lasers, with shorter wavelength (higher frequency), are more energetic than red lasers.

20. **INTERPRET** We are to find the minimum work function that would allow photons to be ejected by 990-nm light.

DEVELOP Equation 34.7, $k_{\max} = hf - \phi$, gives the maximum kinetic energy for electrons ejected by light at a frequency f from a material with a work function ϕ. Use $c = \lambda f$ to convert this to a function of wavelength. The result is

$$\phi = \frac{hc}{\lambda} - K_{\max}$$

The minimum work function occurs for $K_{\max}$ 0.

EVALUATE For $K_{\max} = 0,$ the work function is $\phi = hc/\lambda = (1240\ \text{eV}\cdot\text{nm})/(990\ \text{nm}) = 1.25\ \text{eV}.$

ASSESS The work function for many materials is greater than this, so 990-nm light would not be able to eject electrons from these materials.

Section 34.4 Atomic Spectra and the Bohr Atom

21. **INTERPRET** This problem is about the energy levels of a hydrogen atom using the Bohr model. We are interested in the wavelengths of the first three lines in the Lyman series.

DEVELOP The wavelength can be calculated using Equation 34.9:

$$\frac{1}{\lambda} = R_{\text{H}}\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right)$$

where $R_{\text{H}} = 1.097\times10^{7}\,\text{m}^{-1}$ is the Rydberg constant and $n_2 = 1$ for the Lyman series.

EVALUATE The first three lines correspond to $n_1 = 2, 3,$ and 4, and the wavelengths are, respectively,

$$\lambda = \frac{1}{R_{\text{H}}}\frac{n_1^2 n_2^2}{n_1^2 - n_2^2} = \frac{1}{R_{\text{H}}}\frac{n_1^2}{n_1^2 - 1} = 122\ \text{nm, } 103\ \text{nm, and } 97.2\ \text{nm}$$

Note that $R_{\text{H}}^{-1} = (0.01097)^{-1}\ \text{nm} = 91.2\ \text{nm},$ which is the Lyman series limit.

ASSESS The wavelengths are less than 400 nm. Therefore, the Lyman spectral lines are in the ultraviolet regime.

22. **INTERPRET** This problem involves finding the spectral line in the Paschen series that corresponds to the wavelength 954.7 nm.

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DEVELOP The wavelengths in the Paschen series for hydrogen are given by Equation 34.9, with $n_2 = 3$ and $n_1 = 4, 5, 6....,$ which gives

$$\lambda = \frac{9n_1^2}{R_H^{-1}\left(n_1^2 - 9\right)}$$

EVALUATE With $\lambda = 954.7$ nm and $R_H = 0.01097\ \text{nm}^{-1}$, one finds

$$9n_1^2 = (954.7 \times 0.01097)\left(n_1^2 - 9\right)$$

or

$$n_1 = \sqrt{\frac{9 \times 10.5}{10.5 - 9}} = 8$$

which corresponds to the fifth line in this series.

ASSESS The wavelength 954.7 nm is in the infrared portion of the spectrum.

23. **INTERPRET** This problem is about the ionization energy of a hydrogen atom in its ground state. We want to find the wavelength that corresponds to a photon carrying this much energy.

DEVELOP The energy of the ground state of hydrogen is given by Equation 34.12b (with $n = 1$): $E_1 = -13.6$ eV. Therefore, the ionization energy is $E_I = |E_1| = 13.6$ eV (the subscript "I" is for ionization). For a photon whose wavelength is λ, the energy it carries is (Equation 34.6) $E = hf = hc/\lambda$.

EVALUATE A photon with energy $E_I = 13.6$ eV has wavelength

$$\lambda = \frac{hc}{E_I} = \frac{1240\ \text{eV} \cdot \text{nm}}{13.6\ \text{eV}} = 91.2\ \text{nm}$$

ASSESS This is the same as the Lyman series limit (Equation 34.9 with $n_2 = 1$ and $n_1 = \infty$) $R_H^{-1} = hc/13.6$ eV, and lies in the ultraviolet.

24. **INTERPRET** We are to find the energy level of a Bohr hydrogen atom that has a diameter of 8.57 nm.

DEVELOP The diameter of a hydrogen atom in the Bohr model is found in Equation 34.13, from which

$$2r_n = 2n^2 a_0, \text{ so } n^2 = \frac{8.57\ \text{nm}}{2(0.0529\ \text{nm})} = 81\ \text{nm}$$

or $n = 9$.

EVALUATE This is the eighth excited state.

ASSESS The radius of this state is two orders of magnitude larger than that of the ground state.

Section 34.5 Matter Waves

25. **INTERPRET** In this problem, we are asked to find the de Broglie wavelength of Earth orbiting the Sun and an electron moving at the given speed.

DEVELOP For nonrelativistic momentum, Equation 34.14 becomes

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

EVALUATE **(a)** Using the orbital speed given and Earth's mass from Appendix E gives

$$\lambda = \frac{h}{mv} = \frac{6.626 \times 10^{-34}\ \text{J} \cdot \text{s}}{\left(5.97 \times 10^{24}\ \text{kg}\right)\left(30 \times 10^3\ \text{m/s}\right)} = 3.7 \times 10^{-63}\ \text{m}$$

(b) For the given electron,

$$\lambda = \frac{h}{mv} = \frac{6.626 \times 10^{-34}\ \text{J} \cdot \text{s}}{\left(9.11 \times 10^{-31}\ \text{kg}\right)\left(20 \times 10^3\ \text{m/s}\right)} = 36.4\ \text{nm}$$

ASSESS Earth's de Broglie wavelength is much smaller than the smallest physically meaningful distance.

26. **INTERPRET** This problem involves finding the momentum of an electron with the given de Broglie wavelength.

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DEVELOP For a nonrelativistic electron $(v \ll c,)$ we can use the classical expression $p = mv$ for momentum. The de Broglie wavelength is then $\lambda = h/(mv)$.

EVALUATE Solving for velocity gives

$$v = \frac{h}{m\lambda} = \frac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{\left(9.11\times10^{-31}\ \text{kg}\right)\left(10\times10^{-3}\ \text{m}\right)} = 7.3\ \text{cm/s}$$

ASSESS This velocity is far less than the speed of light, so we were justified in using the classical expression for momentum.

27. **INTERPRET** The problem asks what relative speed must an electron have in order to have the same de Broglie wavelength as a proton.

DEVELOP Since $\lambda = h/p$ (Equation 34.14), the same de Broglie wavelength means the same momentum.

EVALUATE At nonrelativistic speeds, the equal momenta implies $m_p v_p = m_e v_e$, or

$$\frac{v_e}{v_p} = \frac{m_p}{m_e} = \frac{1.672\times10^{-27}\text{kg}}{9.109\times10^{-31}\text{kg}} = 1836$$

This says that the electron will need to be moving 1836 times faster than the proton.

ASSESS The problem would be more complicated if the momenta were relativistic: $p = \gamma mv$ (Equation 33.7).

28. **INTERPRET** We are to find the de Broglie wavelength of electrons with various kinetic energies.

DEVELOP We shall use Equation 34.14, $\lambda = h/p$, for the de Broglie wavelength. Because the largest kinetic energy, 50 keV, is small compared to the electron's rest energy, $mc^2 = 511$ keV, we can use the nonrelativistic expression $K = p^2/(2m)$; so, the expression for the de Broglie wavelength becomes

$$\lambda = \frac{h}{\sqrt{2mK}} = \frac{hc}{\sqrt{2mc^2K}}.$$

EVALUATE **(a)** For $K = 50$ eV,

$$\lambda = \frac{\left(4.136\times10^{-15}\ \text{eV}\cdot\text{s}\right)\left(3.00\times10^{8}\ \text{m/s}\right)}{\sqrt{2(511\ \text{MeV})(50\ \text{eV})}} = 5.5\ \text{pm}$$

(b) For $K = 3.0$ keV,

$$\lambda = \frac{\left(4.136\times10^{-15}\ \text{eV}\cdot\text{s}\right)\left(3.00\times10^{8}\ \text{m/s}\right)}{\sqrt{2(511\ \text{MeV})(3.0\ \text{keV})}} = 7.1\ \text{pm}$$

(c) For $K = 50$ keV,

$$\lambda = \frac{\left(4.136\times10^{-15}\ \text{eV}\cdot\text{s}\right)\left(3.00\times10^{8}\ \text{m/s}\right)}{\sqrt{2(511\ \text{MeV})(50\ \text{keV})}} = 5.49\ \text{pm}$$

ASSESS For part (c), the relativistic relation $p = \sqrt{K^2 + 2mc^2K/c}$ leads to a result that differs by about 0.0005%.

Section 34.6 The Uncertainty Principle

29. **INTERPRET** We want to find the minimum uncertainty in the velocity of a proton, given the uncertainty in its position.

DEVELOP To find Δv, use the uncertainty principle, $\Delta x \Delta p \geq \hbar$ (Equation 34.15) with $\Delta p = m\Delta v$ and $\Delta x = 1$ fm.

EVALUATE The above equation gives

$$\Delta v = \frac{\Delta p}{m} \geq \frac{\hbar}{m\Delta x} = \frac{(197.3\ \text{MeV}\cdot\text{fm}/c)}{(938\ \text{MeV})(1\ \text{fm})} = 0.21c = 6\times10^{7}\ \text{m/s}$$

ASSESS The quantity $\Delta p = \hbar/\Delta x = 197.3\ \text{MeV}/c$ is barely small enough compared to $mc = 938\ \text{MeV}/c$ to justify using the nonrelativistic relation $p = mv$, but this is good enough for the purpose of approximation.

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30. **INTERPRET** This problem involves the uncertainty principle, which we shall use to determine the precision with which we can measure an electron's velocity and position.

DEVELOP The uncertainty principle (Equation 34.15) is $\Delta x \Delta p \geq \hbar$. For the speeds considered in this problem, we can use the nonrelativistic expression $p = mv$ for momentum, which leads to

$$\Delta x(m\Delta v) \geq \hbar$$

$$m \geq \frac{\hbar}{\Delta x\ \Delta v} = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{(2\pi)\left(2\times10^{-6}\ \text{m}\right)(2\ \text{m/s})} = 3\times10^{-29}\ \text{kg}$$

where we have used $\Delta x = 2\ \mu\text{m}$ and $\Delta v = 2$ m/s. Insert the mass of the electron and proton to see if the inequality is satisfied.

EVALUATE For an electron, $m = 9.11 \times 10^{-31}$ kg, so the inequality is not satisfied. The mass of a proton is $m = 1.67 \times 10^{-27}$ kg, so the inequality is satisfied.

ASSESS Thus, we cannot determine both the velocity and position to the desired precision for an electron, but we can do so for a proton.

31. **INTERPRET** In this problem, we want to find the uncertainty in the position of a proton given the uncertainty in its velocity.

DEVELOP To find Δx, we use the uncertainty principle, $\Delta x \Delta p \geq \hbar$ (Equation 34.15), where $\Delta p = m\Delta v.$ We take the uncertainty in velocity to be the full range of variation given; that is, $\Delta v = 0.25\ \text{m/s} - (-0.25\ \text{m/s}) = 0.50\ \text{m/s}$.

EVALUATE The position uncertainty of the proton is

$$\Delta x \geq \frac{\hbar}{m\ \Delta v} = \frac{1.055\times10^{-34}\ \text{J}\cdot\text{s}}{\left(1.67\times10^{-27}\ \text{kg}\right)(0.50\ \text{m/s})} = 130\ \text{nm}$$

ASSESS The smaller the uncertainty Δv in velocity, the greater the uncertainty Δx in position.

32. **INTERPRET** We are given the uncertainty in the speed of an electron and are asked to find the uncertainty in its position.

DEVELOP The electron is moving at

$$v = \frac{(50\times10^{6}\ \text{m/s})c}{3.00\times10^{8}\ \text{m/s}} = 0.17c$$

so we can use the classical approximation for the momentum, $p = mv$. Thus, the uncertainty in momentum is

$$\Delta p = m\Delta v$$

where $\Delta v = (0.20)(50 \times 10^6\ \text{m/s})$. Insert this into the expression for the uncertainty principle (Equation 34.15) to find the minimum uncertainty in position.

EVALUATE This minimum uncertainty in position Δx is

$$\Delta x \geq \frac{\hbar}{\Delta p} = \frac{\hbar}{m\Delta v} = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{(2\pi)\left(9.11\times10^{-31}\ \text{kg}\right)(0.20)\left(50\times10^{6}\ \text{m/s}\right)} = 12\ \text{pm}$$

ASSESS AS a rule of thumb, relativistic formulae should be used for $v \geq 0.3c$.

33. **INTERPRET** The alpha particle is confined in the oganesson nucleus with Δx equal to the diameter of the nucleus. We are to find the minimum energy of the alpha particle using the uncertainty principle.

DEVELOP Using the same reasoning as given in Example 34.6, for a helium-4 nucleus $\left(m = 6.64\times10^{-27}\,\text{kg}\right)$ confined to a oganesson nucleus $(\Delta x \approx 15\ \text{fm}),$ the uncertainty principle requires that

$$K = \frac{p^2}{2m} \geq \frac{1}{2m}\left(\frac{\hbar}{2\Delta x}\right)^2$$

EVALUATE From the above equation, we find the minimum kinetic energy to be

$$K_{\text{min}} = \frac{1}{2m}\left(\frac{\hbar}{2\Delta x}\right)^2 = \frac{1}{2mc^2}\left(\frac{\hbar c}{2\Delta x}\right)^2 = \frac{1}{2(3.74\ \text{GeV})}\left(\frac{197.3\ \text{MeV}\cdot\text{fm}}{2(15\ \text{fm})}\right)^2 = 5.8\ \text{keV}$$

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Where we have used

$$mc^2 = \left(6.64\times10^{-27}\,\text{kg}\right)\left(3.0\times10^{8}\,\text{m/s}\right)^2\left(\frac{1\,\text{eV}}{1.6\times10^{-19}\,\text{J}}\right) = 3.74\text{ GeV}$$

ASSESS This is smaller than the 5 MeV estimated for the nucleon in Example 34.6 by about a factor of 10^3 because Δx is 15 times larger, but the weight of the alpha particle is about 4 times greater. Most estimates of nuclear energies for single-particle states, based on the uncertainty principle, give values of the order of 1 MeV, consistent with experimental measurements.

EXAMPLE VARIATIONS

34. **INTERPRET** This is a problem about electron transitions in hydrogen atoms, albeit of unusual size. The Bohr model applies.

DEVELOP Using the same reasoning as given in Example 34.2, we will use the given energy level to obtain the atomic diameter with Equation 34.13, and apply Equation 34.9 to find the transition wavelength.

EVALUATE **(a)** Using Equation 34.13 we find a diameter of

$$d = 2n^2a_0 = 2(36)^2(0.0529\,\text{nm}) = 137\,\text{nm}$$

(b) Using Equation 34.9 we find the wavelength of light emitted after transitioning is equal to

$$\lambda = \left[R_\text{H}\left(\frac{1}{35^2} - \frac{1}{36^2}\right)\right]^{-1} = 2.04\,\text{mm}$$

ASSESS Our atomic diameter is some 1400 times that of ground-state hydrogen, with a transition wavelength of 2.04 mm, corresponding to a frequency of about 147 GHz.

35. **INTERPRET** This is a problem about electron transitions in hydrogen atoms, albeit of unusual size. The Bohr model applies.

DEVELOP Using the same reasoning as given in Example 34.2, we will use the given starting level (n_1 = 21) and the transition wavelength (λ = 76.44 μm), along with Equation 34.9, to determine the state after the photon emission.

EVALUATE Using Equation 34.9 we find the energy level of atom after transitioning is equal to

$$n_2 = \left[\frac{1}{\lambda R_\text{H}} + \frac{1}{n_1^{\,2}}\right]^{-1/2} = 17$$

ASSESS The atomic diameter of the final state is approximately 31 nm, some 300 times greater than that of ground-state hydrogen.

36. **INTERPRET** This is a problem about electron transitions in doubly ionized lithium atoms, which are hydrogen-like in that there is one active electron. The Bohr model applies.

DEVELOP Using the same reasoning as given in Example 34.2, we will use the given energy level and modified Rydberg constant, along with Equation 34.9, to find the transition wavelength. We are told the modified Rydberg constant is greater by a factor of the square of the ratio of the nuclear charges, that is: $\left(Z_\text{Li}/Z_\text{H}\right)^2 = 3^2$.

EVALUATE Using Equation 34.9 we find the wavelength of light emitted after transitioning is equal to

$$\lambda = \left[3^2R_\text{H}\left(\frac{1}{2^2} - \frac{1}{3^2}\right)\right]^{-1} = 72.9\,\text{nm}$$

ASSESS This transition lies in the ultraviolet region of the electromagnetic spectrum.

37. **INTERPRET** This is a problem about electron transitions in doubly ionized lithium atoms, which are hydrogen-like in that there is one active electron. The Bohr model applies..

DEVELOP Using the same reasoning as given in Example 34.2, we will determine which transitions in Li will result in emission of light in the visible region of the electromagnetic spectrum (400–700 nm). We will solve for the value of n in Equation 34.9, when considering a transition from n to n – 1. We will also use the modified Rydberg constant $R_\text{Li} = 3^2R_\text{H}$.

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EVALUATE Using Equation 34.9 we will find the initial energy level of the Li atom, for a transition to a level below it (i.e., $n_2 = n - 1$), that results in a wavelength in the visible region of the spectrum. We first express the wavelength in terms of the energy levels.

$$\lambda = \left[R_{\text{Li}}\left(\frac{1}{(n-1)^2} - \frac{1}{n^2}\right)\right]^{-1} = \frac{n^2(n-1)^2}{R_{\text{Li}}(2n-1)}$$

Setting the limits on the wavelengths we are considering: $\lambda_{\text{min}} = 400\,\text{nm}$, and $\lambda_{\text{max}} = 700\,\text{nm}$, we arrive at:

$$\lambda_{\text{min}} R_{\text{Li}} < \frac{n^2(n-1)^2}{(2n-1)} < \lambda_{\text{max}} R_{\text{Li}}$$

$$39.5 < \frac{n^2(n-1)^2}{(2n-1)} < 69.1$$

The only value which results in emission within this region is the $n = 5$ energy level.

ASSESS The wavelength of light emitted in the transition from the $n = 5$ level to the $n = 4$ level in doubly ionized lithium is approximately 450 nm.

38. INTERPRET The proton is confined in the uranium-238 nucleus with Δx equal to the diameter of the nucleus. We are to find the minimum energy of the proton using the uncertainty principle.

DEVELOP Using the same reasoning as given in Example 34.6, for a proton $\left(m = 938\,\text{MeV/c}^2\right)$ confined to a uranium-238 nucleus $(\Delta x \approx 15\text{ fm})$, the uncertainty principle requires that

$$K = \frac{p^2}{2m} \geq \frac{1}{2m}\left(\frac{\hbar}{2\Delta x}\right)^2$$

EVALUATE From the above equation, we find the minimum kinetic energy to be

$$K_{\text{min}} = \frac{1}{2m}\left(\frac{\hbar}{2\Delta x}\right)^2 = \frac{1}{2mc^2}\left(\frac{\hbar c}{2\Delta x}\right)^2 = \frac{1}{2(938\text{ MeV})}\left(\frac{197.3\text{ MeV}\cdot\text{fm}}{2(15\text{ fm})}\right)^2 = 23\text{ keV}$$

ASSESS This is smaller than the 5 MeV estimated for the nucleon in Example 34.6 by about a factor of 15^2 because Δx is 15 times larger.

39. INTERPRET Given the kinetic energy of the outermost electrons in boron we are to find an estimate for the diameter of the boron atom using the uncertainty principle.

DEVELOP Using the same reasoning as given in Example 34.6, for an electron $\left(m = 0.511\,\text{MeV/c}^2\right)$ confined to a boron atom with a given energy $(K \approx 8\text{ eV})$, the uncertainty principle requires that

$$K = \frac{p^2}{2m} \geq \frac{1}{2m}\left(\frac{\hbar}{2\Delta x}\right)^2$$

EVALUATE From the above equation, we find the boron atom diameter to be

$$\Delta x = \frac{\hbar c}{2\sqrt{K\left(2mc^2\right)}} = \frac{197.3\text{ MeV}\cdot\text{fm}}{\sqrt{(8\text{eV})(2\times 0.511\text{MeV})}} = 35\,\text{pm}$$

ASSESS Our answer is of the same order of magnitude as is more rigorously calculated for the atomic radius of boron, making it a valid approximation.

40. INTERPRET An electron is confined in a transistor with Δx equal to 1.45 nm. We are to find the minimum energy of the electron using the uncertainty principle.

DEVELOP Using the same reasoning as given in Example 34.6, for an electron $\left(m = 0.511\,\text{MeV/c}^2\right)$ confined to a transistor of size $(\Delta x \approx 1.45\text{ nm})$, the uncertainty principle requires that

$$K = \frac{p^2}{2m} \geq \frac{1}{2m}\left(\frac{\hbar}{2\Delta x}\right)^2$$

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EVALUATE From the above equation, we find the minimum kinetic energy to be

$$K_{\min} = \frac{1}{2m}\left(\frac{\hbar}{2\Delta x}\right)^2 = \frac{1}{2mc^2}\left(\frac{\hbar c}{2\Delta x}\right)^2 = \frac{1}{2(0.511\text{ MeV})}\left(\frac{197.3\text{ MeV}\cdot\text{fm}}{2(1.45\text{ nm})}\right)^2 = 4.5\text{ meV}$$

ASSESS The size of the space occupied by the electron is much larger than what it would be confined to where it bound to a nucleus, making its minimum energy much lower.

41. **INTERPRET** Considering the energy at room temperature as the highest kinetic energy one would like electrons inside an electronic device to have, we are to find the minimum width of the device which could operate under these conditions using the uncertainty principle.

DEVELOP Using the same reasoning as given in Example 34.6, for an electron $\left(m = 0.511\text{MeV/c}^2\right)$ confined to an electronic device with a given energy $(K \approx 25\text{ meV})$, the uncertainty principle requires that

$$K = \frac{p^2}{2m} \geq \frac{1}{2m}\left(\frac{\hbar}{2\Delta x}\right)^2$$

EVALUATE From the above equation, we find the width of the device to be

$$\Delta x = \frac{\hbar c}{2\sqrt{K\left(2mc^2\right)}} = \frac{197.3\text{ MeV}\cdot\text{fm}}{2\sqrt{(25\text{ meV})(2\times 0.511\text{MeV})}} = 0.62\,\text{nm}$$

ASSESS The size of the space occupied by the electron is much larger than what it would be confined to where it bound to a nucleus, making its minimum energy much lower.

PROBLEMS

42. **INTERPRET** This problem involves blackbody radiation, which we can use to find the power emitted per unit area by the lamp within the given wavelength range.

DEVELOP The power emitted is $dP = R(\lambda,T)\; d\lambda$ (Equation 34.3) where

$$R(\lambda,T) = \frac{2\pi hc^2}{\lambda^5\left(e^{hc/\lambda kT} - 1\right)}$$

Since $d\lambda = 0.100$ nm is such a small interval around 645 nm, integration is not necessary.

EVALUATE With $2\pi hc^2 = 3.74\times 10^{-16}\text{ W}\cdot\text{m}^2$ and $hc/k_B = 1.44\times 10^{-2}\text{ K}\cdot\text{m}$, and with $\lambda = 645$ nm and $T = 2780$ K, we find

$$dP = \left(3.74\times 10^{-16}\text{ W}\cdot\text{m}^2\right)(0.100\text{ nm})\left(6.45\times 10^{-7}\text{ m}\right)^{-5}\left(e^{8.03} - 1\right)^{-1} = 108\text{ W/m}^2$$

ASSESS If we knew how this power compared to the total power emitted by the filament, we could estimate its surface area using the 60 W at which it operates.

$$s \approx \frac{60\text{ W}}{0.8\text{ kW/m}^2} = 7.5\text{ cm}^2$$

which seems like a reasonable order-of-magnitude estimate.

43. **INTERPRET** We are given the temperature of the Sun, which we shall treat as a blackbody, and asked to compare its radiance at two different wavelengths.

DEVELOP The radiance of a blackbody is given by Equation 34.4:

$$R(\lambda,T) = \frac{2\pi hc^2}{\lambda^5\left(e^{hc/\lambda kT} - 1\right)}$$

This equation allows us to compare the radiance at two different wavelengths.

EVALUATE From the above equation (also see Example 34.1), the ratio of the blackbody radiances for the two given wavelengths is

$$\frac{R(\lambda_2,T)}{R(\lambda_1,T)} = \left(\frac{\lambda_1}{\lambda_2}\right)^5\left(\frac{e^{hc/\lambda_1 kT} - 1}{e^{hc/\lambda_2 kT} - 1}\right) = \left(\frac{5}{2}\right)^5\left(\frac{146.9}{2.66\times 10^5}\right) = 5.4\times 10^{-2}$$

where $\lambda_1 = 500$ nm, $\lambda_2 = 200$ nm, $T = 5800$ K, and $hc/k = 1.449\times 10^{-2}\text{ m}\cdot\text{K}$.

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ASSESS The characteristic radiance as a function of wavelength is shown in Fig. 34.2. For a given wavelength, the radiance increases with temperature.

44. INTERPRET We are to compare the Rayleigh–Jeans law to the Planck's formula for blackbody radiation at the three wavelengths given.

DEVELOP The ratio of the radiances for the Rayleigh–Jeans and Planck's laws is

$$\frac{R_{\text{R-J}}}{R_{\text{P}}}=\frac{\left(2\pi ckT/\lambda^4\right)\lambda^5\left(e^{hc/\lambda kT}-1\right)}{2\pi hc^2}=\frac{e^{hc/\lambda kT}-1}{hc/(\lambda kT)}$$

Accurate values of $hc/\lambda kT$, for $T=2000$ K and $\lambda=1$ mm, 10 μm, and 1 μm are 7.245×10^{-3}, 0.7245, and 7.245, respectively.

EVALUATE The percent difference is

$$\left(\frac{R_{\text{R-J}}}{R_{\text{P}}}-1\right)\times100\%$$

which equals **(a)** 0.36%, **(b)** 47%, and **(c)** (1.9×10^4)% for the three given wavelengths.

ASSESS The error becomes increasingly large as the wavelengths approach the visible part of the spectrum.

45. INTERPRET This problem is about blackbody radiation. We are given the wavelengths that correspond to peak radiance and asked to find the temperature of the blackbody. We also want to compare the radiance at two different wavelengths.

DEVELOP The wavelength at which a blackbody at a given temperature radiates the maximum power is given by Wien's displacement law (Equation 34.2a): $\lambda_{\text{peak}}T=2.898\text{ mm}\cdot\text{K}$. For part (b), to compare the radiance at two different wavelengths, we use Equation 34.4:

$$R(\lambda,T)=\frac{2\pi hc^2}{\lambda^5\left(e^{hc/\lambda kT}-1\right)}$$

EVALUATE **(a)** Equation 34.2a gives

$$T=\frac{2.898\text{ mm}\cdot\text{K}}{\lambda_{\text{peak}}}=\frac{2.898\times10^6\text{ nm}\cdot\text{K}}{690\text{ nm}}=4200\text{ K}$$

(b) Using the result obtained in **(a)**, we have $hc/(kT)=3.45\text{ μm}$, and the ratio of the radiances is

$$\frac{R(\lambda=400\text{ nm})}{R(\lambda=700\text{ nm})}=\left(\frac{700}{400}\right)^5\left(\frac{137.2}{5568}\right)=0.40$$

ASSESS The wavelengths considered are in the visible spectrum. Note that the characteristic radiance as a function of wavelength is shown in Fig. 34.2. For a given wavelength, the radiance increases with temperature.

46. INTERPRET This problem involves blackbody radiation, which we can use to find the power emitted by the Earth's entire surface area at two different regions of the electromagnetic spectrum.

DEVELOP The power emitted is $dP=R(\lambda,T)\,d\lambda$ (Equation 34.3) where

$$R(\lambda,T)=\frac{2\pi hc^2}{\lambda^5\left(e^{hc/\lambda kT}-1\right)}$$

Since $d\lambda$ = 1.0 nm is such a small interval around both wavelength regions, integration is not necessary.

EVALUATE We calculate the surface area of Earth using the radius found in Appendix E to be

$$S_{\text{E}}=4\pi R_{\text{E}}^2=4\pi\left(6.37\times10^6\text{m}\right)^2=5.10\times10^{14}\text{m}^2$$

With $2\pi hc^2=3.74\times10^{-16}\text{ W}\cdot\text{m}^2$ and $hc/k_B=1.44\times10^{-2}\text{ K}\cdot\text{m}$, and T = 288 K, we find for the two wavelength ranges

$$P_{0.55\,\mu\text{m}}=S_{\text{E}}\times dP=\left(5.10\times10^{14}\text{m}^2\right)\left(3.74\times10^{-16}\text{ W}\cdot\text{m}^2\right)(1\text{ nm})\left(5.5\times10^{-7}\text{ m}\right)^{-5}\left(e^{90.9}-1\right)^{-1}=1.2\times10^{-18}\text{ W}$$

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$$P_{10\,\mu\text{m}} = S_\text{E} \times dP = \left(5.10\times10^{14}\,\text{m}^2\right)\left(3.74\times10^{-16}\ \text{W}\cdot\text{m}^2\right)(1\ \text{nm})\left(1.0\times10^{-5}\ \text{m}\right)^{-5}\left(e^{4.99}-1\right)^{-1} = 1.3\times10^{13}\,\text{W}$$

ASSESS This large difference (over 30 orders of magnitude) shows how Earth emits much more strongly in the mid-infrared region than in the visible. This radiation is preferentially absorbed by greenhouse gases in the atmosphere and contributes to the heat trapping cycle which results in global climate change.

47. **INTERPRET** We are given the power output at various frequencies and asked to find the rate of photon emission.
DEVELOP The rate dN/dt of photon emission is the electromagnetic power output divided by the photon energy:

$$\frac{dN}{dt} = \frac{P}{E} = \frac{P}{hf} = \frac{P\lambda}{hc}$$

where we have used Equation 34.6, $E = hf$.
EVALUATE **(a)** For the antenna, the rate is

$$\frac{dN}{dt} = \frac{P}{hf} = \frac{1.5\ \text{kW}}{\left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)(89.5\ \text{MHz})} = 2.5\times10^{28}\ \text{s}^{-1}$$

(b) For the laser, we have

$$\frac{dN}{dt} = \frac{P\lambda}{hc} = \frac{(1.4\ \text{mW})(633\ \text{nm})}{\left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00\times10^{8}\ \text{m/s}\right)} = 4.5\times10^{15}\ \text{s}^{-1}$$

(c) Similarly, for the X-ray machine, the rate is

$$\frac{dN}{dt} = \frac{P\lambda}{hc} = \frac{(2.6\ \text{kW})(0.1\ \text{nm})}{\left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00\times10^{8}\,\text{m/s}\right)} = 1.3\times10^{18}\ \text{s}^{-1}$$

ASSESS For a general device at a given power output, the rate of photon production decreases with the energy of the photon; the more energetic the photons, the smaller the rate of production because each photon carries more energy.

48. **INTERPRET** This problem involves the photoelectric effect. Given the maximum kinetic energy with which electrons emerge from an aluminum surface, we are to find the wavelength of the illuminating radiation.
DEVELOP Einstein's equation for the photoelectric effect (Equation 34.7) gives

$$K_\text{max} = \frac{hc}{\lambda} - \phi$$
$$\lambda = \frac{hc}{K_\text{max} + \phi}$$

where ϕ = 4.28 eV (see Table 34.1) is the work function of Al.
EVALUATE For K_max = 1.3 eV, the wavelength is

$$\lambda = \frac{hc}{K_\text{max}+\phi} = \frac{\left(4.136\times10^{-15}\ \text{eV}\cdot\text{s}\right)\left(3.00\times10^{8}\,\text{m/s}\right)}{1.3\ \text{eV} + 4.28\ \text{eV}} = 220\ \text{nm}$$

to two significant figures.
ASSESS This wavelength is in the ultraviolet portion of the electromagnetic spectrum.

49. **INTERPRET** This problem involves the photoelectric effect. We want to find the cutoff frequency and the maximum energy of electrons ejected by shining light with the given frequency on copper.
DEVELOP At the cutoff frequency, $K_\text{max} = 0$, and the photon energy equals the work function, $\phi = hf_\text{cutoff}$ (see Equation 34.7), which we can find in Table 34.1. For part (b), apply Equation 34.7 to find the maximum kinetic energy possible for the given frequency f.

$$K_\text{max} = hf - \phi$$

EVALUATE **(a)** The work function of copper is $\phi_\text{Cu} = 4.65\ \text{eV}$. Therefore, the cutoff frequency is

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$$f_{\text{cutoff}} = \frac{\phi_{\text{Cu}}}{h} = \frac{4.65\,\text{eV}}{4.136\times10^{-15}\,\text{eV}\cdot\text{s}} = 1.12\times10^{15}\,\text{Hz}$$

(b) The maximum kinetic energy of the ejected electrons is

$$K_{\text{max}} = hf - \phi = \left(4.136\times10^{-15}\,\text{eV}\cdot\text{s}\right)\left(2.1\times10^{15}\,\text{Hz}\right) - 4.65\,\text{eV} = 4.04\,\text{eV}$$

ASSESS Upon illuminating copper with photons at 8.69 eV, it takes 4.65 eV to overcome the work function of copper, leaving the electrons with 4.04 eV of kinetic energy.

50. **INTERPRET** This problem involves the photoelectric effect. Given the electric potential energy difference needed to stop electrons emitted from a surface by the given radiation, we are to determine the work function of the material.

DEVELOP The electron's maximum kinetic energy is expended in crossing the stopping potential (see text), so

$$eV_{\text{s}} = K_{\text{max}}$$

where V_{s} = 1.8 V. Apply Equation 34.7 to find the work function;

$$K_{\text{max}} = hf - \phi = \frac{hc}{\lambda} - \phi$$

EVALUATE **(a)** for λ = 365 nm, the work function is

$$\begin{aligned}\phi &= \frac{hc}{\lambda} - K_{\text{max}} \\ &= \frac{hc}{\lambda} - eV_{\text{s}} \\ &= \frac{\left(6.626\times10^{-34}\,\text{J}\cdot\text{s}\right)\left(3.00\times10^{8}\,\text{m/s}\right)}{365\,\text{nm}} - 1.8\,\text{eV} = 1.6\,\text{eV}\end{aligned}$$

(b) At the new wavelength,

$$eV_{\text{s}} = \frac{hc}{\lambda} - \phi = \frac{1240\,\text{eV}\cdot\text{nm}}{280\,\text{nm}} - 1.60\,\text{eV} = 2.79\,\text{eV}$$

or

$$V_{\text{s}} = 2.8\,\text{V}$$

to two significant figures.

ASSESS The stopping potential increases because it takes a bigger "hill" to stop the electrons emitted by the 280-nm radiation, which contains photons of higher energy than radiation at 365 nm.

51. **INTERPRET** We are asked to explain why plants are green using the absorption peaks in the chlorophyll molecule.

DEVELOP We can convert the wavelength peaks into energy peaks using $E = hc/\lambda$, and the shorthand $hc = 1240\,\text{eV}\cdot\text{nm}$.

EVALUATE **(a)** The energy peaks in chlorophyll's absorption spectrum are at

$$E_1 = \frac{1240\,\text{eV}\cdot\text{nm}}{430\,\text{nm}} = 2.9\,\text{eV}$$
$$E_2 = \frac{1240\,\text{eV}\cdot\text{nm}}{662\,\text{nm}} = 1.9\,\text{eV}$$

(b) These absorption peaks correspond to blue and red wavelengths, near the limits of the human visible range. The light that is not absorbed is reflected, and this is what we humans observe. Since the reflected light is primarily in the green region of the visible spectrum between blue and red, we perceive plants to be green.

ASSESS Plants also reflect a lot of infrared light with wavelengths longer than 700 nm.

52. **INTERPRET** This problem involves Compton scattering, which we can use to find the initial wavelength of photons that scatter at the given angle from electrons.

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Develop Apply Equation 34.8, which describes Compton scattering (i.e., the scattering of photons off electrons). For a photon that loses half its initial energy, we have

$$f = \frac{1}{2} f_0$$

so the wavelength shift $\Delta\lambda$ is

$$\Delta\lambda = \lambda - \lambda_0 = \frac{c}{f} - \frac{c}{f_o} = \frac{2c}{f_0} - \frac{c}{f_0} = \frac{c}{f_0} = \lambda$$

Evaluate Using this result for $\Delta\lambda$ in Equation 34.8 gives

$$\lambda = \frac{h}{c}(1 - \cos\theta) = \frac{h}{c} = 2.43 \text{ pm}$$

Assess The energy of these photons is about 0.5 MeV.

53. **Interpret** This problem involves the photoelectric effect. We are given the maximum speed of electrons ejected from potassium and asked to find the wavelength of the light that ejected the electrons.
Develop The maximum speed of the ejected electrons is related to the wavelength of the light by Einstein's photoelectric effect equation (Equation 34.7):

$$K_{\text{max}} = \frac{1}{2} m v_{\text{max}}^2 = hf - \phi = \frac{hc}{\lambda} - \phi$$

We shall use this equation to find λ.
Evaluate The maximum kinetic energy of the electron is

$$K_{\text{max}} = \frac{1}{2} m v_{\text{max}}^2 = \frac{1}{2}\left(mc^2\right)\left(\frac{v_{\text{max}}}{c}\right)^2 = \frac{1}{2}(511 \text{ keV})\left(\frac{5.00\times10^5 \text{ m/s}}{3.00\times10^8 \text{ m/s}}\right)^2 = 0.71 \text{ eV}$$

From Equation 34.7 and using Table 34.1 to find the work function ϕ, we find the wavelength to be

$$\lambda = \frac{hc}{K_{\text{max}} + \phi} = \frac{1240 \text{ eV}\cdot\text{nm}}{0.71 \text{ eV} + 2.30 \text{ eV}} = 412 \text{ nm}$$

to two significant figures.
Assess Strictly speaking, the result should be reported as 41×10^1 nm to make the significant figures more obvious. The cutoff wavelength of potassium is

$$\lambda_{\text{cutoff}} = hc/\beta = (1240 \text{ eV}\cdot\text{nm})/(2.30 \text{ eV}) = 539 \text{ nm}$$

For the photoelectric effect to take place, we require $\lambda \le \lambda_{\text{cutoff}}$.

54. **Interpret** This problem involves the photoelectric effect. Given the maximum electron energy for two different illuminating wavelengths we are to find the work function of the material.
Develop The photoelectric effect equations (Equation 34.7) for the two experimental runs are

$$K_{\text{max}} = \frac{hc}{\lambda} - \phi = 2.8 \text{ eV}$$

and

$$K'_{\text{max}} = \frac{hc}{\kappa'} - \phi = \frac{hc}{1.5\lambda} - \phi = 1.1 \text{ eV}$$

which we can solve for the work function ϕ and the initial wavelength λ.
Evaluate **(a)** Subtracting the two equations gives

$$\lambda = \frac{hc}{3(1.7 \text{ eV})} = \frac{\left(4.136\times10^{-15} \text{ eV}\cdot\text{s}\right)\left(3.00\times10^8 \text{ m/s}\right)}{3(1.7 \text{ eV})} = 243 \text{ nm}$$

Adding the two equations gives

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$$\phi = \frac{5hc}{6\lambda} - \frac{3.9\ \text{eV}}{2} = \frac{5\left(4.136\times10^{-15}\ \text{eV}\cdot\text{s}\right)\left(3.00\times10^{8}\ \text{m/s}\right)}{6\left(243\ \text{nm}\right)} - 1.85\ \text{eV} = 2.3\ \text{eV}$$

(b) From the calculation above, we find $\lambda = 240$ nm (to two significant figures).

ASSESS This material is probably potassium, which has a work function of $\phi = 2.3$ eV.

55. **INTERPRET** This problem is about Compton scattering of a photon with an electron. We are interested in the wavelength of the scattered photon and the kinetic energy of the electron.

DEVELOP The Compton shift of wavelength is given by Equation 34.8:

$$\Delta\lambda = \frac{h}{mc}(1-\cos\theta) = \lambda_C(1-\cos\theta)$$

where $\lambda_c = h/(mc) = 2.43$ pm is the Compton wavelength of the electron. By conservation of energy, the kinetic energy of the scattered electron is equal to the energy lost by the photon.

EVALUATE **(a)** From Equation 34.8, the wavelength of the scattered photon is

$$\lambda' = \lambda + \Delta\lambda = 160\ \text{pm} + 2.43\ \text{pm}\left[1-\cos(135°)\right] = 160\ \text{pm} + 4.15\ \text{pm} = 164\ \text{pm}$$

(b) The kinetic energy of the scattered electron is

$$K = E - E' = \frac{hc}{\lambda} - \frac{hc}{\lambda'} = \frac{hc\Delta\lambda}{\lambda\lambda'} = \frac{(1240\ \text{eV}\cdot\text{nm})(4.15\ \text{pm})}{(160\ \text{pm})(164\ \text{pm})} = 196\ \text{eV}$$

ASSESS For X-rays, the wavelength is in the range 0.01–10 nm, so the detection of the Compton shift in X-rays is difficult.

56. **INTERPRET** This problem involves Compton scattering of an X-ray at 90° from a stationary electron. We are to find the kinetic energy of the electron after the scattering event.

DEVELOP In Compton scattering, the kinetic energy of the recoil electron equals the energy lost by the photon:

$$K = E - E' = \frac{hc}{\lambda} - \frac{hc}{\lambda'} = \frac{hc(\lambda'-\lambda)}{\lambda\lambda'} = \frac{E\lambda_c(1-\cos\theta)}{\lambda + \lambda_c(1-\cos\theta)}$$

where we used Equation 34.8 for $\Delta\lambda = \lambda' - \lambda$.

EVALUATE For the given data ($\theta = 90°$, $\lambda = 400$ pm, $E = hc/\lambda = 3.1$ keV, and $\lambda_C = 2.43$ pm), we find

$$K = \frac{E\lambda_c}{\lambda + \lambda_c} = (3.1\ \text{keV})\frac{2.43}{402.43} = 18.7\ \text{eV}$$

to two significant figures.

ASSESS This (nonrelativistic) energy corresponds to a speed of

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2Kc^2}{mc^2}} = \left(3.00\times10^{8}\ \text{m/s}\right)\sqrt{\frac{2(18.7\ \text{eV})}{511\ \text{keV}}} = 2.6\times10^{6}\ \text{m/s}$$

57. **INTERPRET** We want to find the minimum uncertainty in the components of the velocity of an electron, given the size of the carbon nanotube.

DEVELOP We consider the z (along the tube) and the radial (perpendicular to z) components of the velocity separately. To find Δv_i, use the uncertainty principle, $\Delta x_i \Delta p_i \ge \hbar$ (Equation 34.15) with $\Delta p_i = m\Delta v_i$, $\Delta r = 1.2$ nm and $\Delta z = 370$ nm.

EVALUATE **(a)** Along the tube, the above equation gives

$$\Delta v_z = \frac{\Delta p_z}{m} \ge \frac{\hbar}{m\Delta z} = \frac{\left(1.055\times10^{-34}\ \text{J}\cdot\text{s}\right)}{\left(9.11\times10^{-31}\ \text{kg}\right)\left(370\times10^{-9}\ \text{m}\right)} = 313\ \text{m/s}$$

(a) Along the radial direction, the uncertainty is

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$$\Delta v_r = \frac{\Delta p_r}{m} \ge \frac{\hbar}{m\Delta r} = \frac{\left(1.055\times10^{-34}\ \text{J}\cdot\text{s}\right)}{\left(9.11\times10^{-31}\ \text{kg}\right)\left(1.2\times10^{-9}\ \text{m}\right)} = 9.6\times10^{4}\ \text{m/s} = 96\ \text{km/s}$$

Assess The uncertainty in Δv_i is inversely proportional to Δx_i. In this case, since $\Delta r << \Delta z$, we have $\Delta v_r >> \Delta v_z$.

58. **Interpret** We are to find the de Broglie wavelength of an electron that has been accelerated through a potential difference.

Develop We shall use Equation 34.14 $\lambda = h/p$ for the de Broglie wavelength. Because the kinetic energy, $K = eV = 3.35$ keV is small compared to the electron's rest energy $mc^2 = 511$ keV, we can use the nonrelativistic expressions $K = eV = p^2/(2m)$, so the expression for the de Broglie wavelength becomes

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2meV}}.$$

Evaluate For $V = 3.35$ kV, the de Broglie wavelength is

$$\lambda = \frac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{\sqrt{2\left(9.11\times10^{-31}\ \text{kg}\right)(1.6\times10^{-19}\ \text{C})(3350\ \text{V})}} = 21.2\ \text{pm}$$

Assess The wavelength varies as $1/\sqrt{V}$. The greater the value of V, the smaller the wavelength.

59. **Interpret** An electron is confined in a channel with width $\Delta x = 6.6$ nm. We are to find the minimum energy of the electron using the uncertainty principle.

Develop Using the same reasoning as given in Example 34.6, for an electron $(mc^2 = 0.511\ \text{MeV})$ confined to a channel of width $\Delta x = 6.6\ \text{nm},$ the uncertainty principle requires that

$$K = \frac{p^2}{2m} \ge \frac{1}{2m}\left(\frac{\hbar}{2\Delta x}\right)^2$$

Evaluate From the above equation, we find the minimum kinetic energy to be

$$K_{\text{min}} = \frac{1}{2m}\left(\frac{\hbar}{2\Delta x}\right)^2 = \frac{1}{2mc^2}\left(\frac{\hbar c}{2\Delta x}\right)^2 = \frac{1}{2(0.511\ \text{MeV})}\left(\frac{197.3\ \text{MeV}\cdot\text{fm}}{2\left(6.6\times10^{6}\ \text{fm}\right)}\right)^2 = 2.2\times10^{-4}\ \text{eV} = 3.5\times10^{-23}\ \text{J}$$

Assess The minimum kinetic energy varies with $1/(\Delta x)^2$. The value of Δx here is 66 times that considered in Example 34.5, and therefore the minimum energy is $1/(66)^2$, or about 4000 times smaller than the 1 eV calculated there.

60. **Interpret** This problem involves the Bohr atom (i.e., a hydrogen atom). We are to find the highest possible energy for a photon emitted by such an atom.

Develop The energy of the photon emitted in a hydrogen atom transition between adjacent states $(n_1 \to n_2 = n_1 - 1)$ is

$$E = \frac{hc}{\lambda} = hcR_{\text{H}}\left[(n_1-1)^{-2} - n_1^{-2}\right] = hcR_{\text{H}}(2n_1-1)n_1^{-2}(n_1-1)^{-2}$$

(see Equations 34.6 and 34.9 and the discussion of the Bohr atom in the text).

Evaluate **(a)** The maximum allowed energy occurs for $n_1 = 2$, which gives

$$E_{\text{max}} = \frac{3}{4}hcR_{\text{H}} = \frac{3}{4}(13.6\ \text{eV}) = 10.2\ \text{eV}$$

(b) The upper (initial) energy level is $n_1 = 2$ and the lower (final) energy level is $n_2 = 1$.

Assess This energy is in the ultraviolet portion of the electromagnetic spectrum.

61. **Interpret** This problem is about the wavelength and energy of the photon emitted when a Rydberg hydrogen atom undergoes a transition.

Develop The wavelength of the photon can be calculated using Equation 34.9:

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$$\frac{1}{\lambda} = R_{\text{H}}\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right)$$

where $R_{\text{H}} = 1.097\times10^{7}\ \text{m}^{-1}$ is the Rydberg constant. Once we know the wavelength, the energy of the photon may be found using $E = hf = hc/\lambda$.

EVALUATE **(a)** The above equation gives

$$\lambda = \frac{1}{R_{\text{H}}}\frac{n_1^2 n_2^2}{n_1^2 - n_2^2} = \left(\frac{1}{1.097\times10^{7}\ \text{m}^{-1}}\right)\frac{(179)^2(180)^2}{(180)^2 - (179)^2} = 26.4\ \text{cm}$$

(b) The energy of the emitted photon is

$$E = \frac{hc}{\lambda} = \frac{1.24\times10^{-6}\ \text{eV}\cdot\text{m}}{0.264\ \text{m}} = 4.70\ \mu\text{eV}$$

ASSESS The long wavelength corresponds to the radio region of the electromagnetic spectrum.

62. **INTERPRET** We are to find the maximum energy for a photon emitted by a transition in the Lyman series and in the Balmer series in a hydrogen atom.

DEVELOP Apply Equation 13.12b,

$$E_n = \frac{13.6\ \text{eV}}{n_2^2}$$

where n_2 is lowest quantum number for the given series. From Fig. 34.11, we see that $n_2 = 1$ for the Lynam series and $n_2 = 2$ for the Balmer series. To find the corresponding wavelength, use Equation 34.6 $E = hf = hc/\lambda$.

EVALUATE **(a)** For the Lyman series, the highest energy photon is

$$E_1 = \frac{13.6\ \text{eV}}{1^2} = 13.6\ \text{eV}$$

The wavelength of this is

$$\lambda = \frac{hc}{E} = \frac{1240\ \text{eV}\cdot\mu\text{m}}{13.6\ \text{eV}} = 91.2\ \text{nm}$$

(b) For the Balmer series, the highest energy photon is

$$E_2 = \frac{13.6\ \text{eV}}{2^2} = 3.40\ \text{eV}$$

so the wavelength is

$$\lambda = \frac{hc}{E} = \frac{1240\ \text{eV}\cdot\mu\text{m}}{3.40\ \text{eV}} = 365\ \text{nm}$$

ASSESS The Lyman series limit is in the ultraviolet. The Balmer series limit is at the very high-energy end of the visible spectrum.

63. **INTERPRET** The hydrogen atom undergoing a downward transition emits a photon. We are interested in the original state of the atom, given the energy of the photon and the quantum number of the final state of the atom.

DEVELOP The wavelength of the photon can be calculated using Equation 34.9:

$$\frac{1}{\lambda} = R_{\text{H}}\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right)$$

where $R_{\text{H}} = 1.097\times10^{7}\ \text{m}^{-1}$ is the Rydberg constant.Once we know the wavelength, the energy of the photon may be found using Equation 34.6:

$$E = hf = \frac{hc}{\lambda} = hcR_{\text{H}}\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right)$$

EVALUATE Solving for n_1 gives

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$$n_1 = \left[n_2^{-2} - E/\left(hcR_{\text{H}}\right)\right]^{-1/2} = \left[(225)^{-2} - \left(9.32\ \mu\text{eV}/(13.6\ \text{eV})\right)\right]^{-1/2} = 229$$

Note that $hcR_{\text{H}} = 13.6$ eV is the ionization energy.

ASSESS The wavelength of the emitted photon is 0.133 m, which falls into the radio wave spectrum.

64. **INTERPRET** This problem involves conservation of energy and the Bohr model of the atom. We can use these two concepts to find the energy of an electron ejected from the ground state of a hydrogen atom by a 50-eV photon.

DEVELOP Energy must be conserved, so the final kinetic energy is the original (ground state) energy plus the absorbed photon energy.

EVALUATE The final kinetic energy is thus

$$K = E_0 + E_\gamma = -13.6\ \text{eV} + 50\ \text{eV} = 36.4\ \text{eV}$$

ASSESS This energy is much less than the electron's rest mass, so relativistic effects do need to be considered.

65. **INTERPRET** This problem involves the Bohr model of the atom. We are to find the ionization energy of a hydrogen atom in its first excited state.

DEVELOP Using Equation 34.12b, the energy of the first excited state (n = 2) is

$$E_2 = \frac{-13.6\ \text{eV}}{2^2} = -3.40\ \text{eV}$$

whereas an ionized atom (with zero electron kinetic energy) has energy zero.

EVALUATE Thus, we must supply an energy E_{I} such that

$$0 = E_{\text{I}} + (-3.40\ \text{eV})$$
$$E_{\text{I}} = 3.40\ \text{eV}$$

ASSESS The ionization energy here is only ¼ of the case where the atom is in the ground state. The higher the value of n, the smaller the ionization energy because the electron is less tightly bound to the nucleus.

66. **INTERPRET** This problem involves the uncertainty principle applied to the variable energy and time. We shall use this to find the uncertainty in energy in a cosmic ray particle passing through a detector of a given thickness at near the speed of light.

DEVELOP The uncertainty principle (Equation 34.16) for energy and time is

$$\Delta t \Delta E \ge \hbar$$

(note that the units of $\hbar$ are time-energy). Apply this to find the minimum energy given the uncertainty in time of

$$\Delta t = \frac{\Delta x}{v} \simeq \frac{0.01\text{m}}{3.0\times10^8\ \text{m/s}} = 3.33\times10^{-11}\text{s}\ .$$

EVALUATE The uncertainty in energy is

$$\Delta E \ge \frac{\hbar}{\Delta t} = \frac{1.055\times10^{-34}\ \text{J}\cdot\text{s}}{3.33\times10^{-11}\ \text{s}} = 3\times10^{-24}\ \text{J}$$

ASSESS This energy uncertainty is called the natural linewidth.

67. **INTERPRET** The resolution of a microscope depends on the wavelength used. A smaller de Broglie wavelength will improve the resolution of an electron microscope. We are to find the minimum electron speed that will make its de Broglie wavelength less than 460 nm.

DEVELOP The resolution of the electron microscope is better than that of the optical microscope with 460-nm light if the de Broglie wavelength λ of the electrons is less than 460 nm. Thus,

$$\lambda = \frac{h}{p} < 460\ \text{nm}$$

Since $p = mv$ (for nonrelativistic electrons), the above condition allows us to obtain the minimum electron speed.

EVALUATE The above inequality gives

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$$v > \frac{h}{m(460\ \text{nm})} = \frac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{(9.11\times10^{-31}\ \text{kg})(460\ \text{nm})} = 1.58\ \text{km/s}$$

So the minimum speed is $1.58\ \text{km/s}$.

ASSESS Since $460\ \text{nm} \gg \lambda_c = h/mc = 2.42\ \text{pm}$ (Compton wavelength of the electron), our use of the nonrelativistic momentum was justified. The electron microscope can provide resolutions down to about 1 nm and magnifications of 10^6.

68. **INTERPRET** You want to see what energy of electrons is needed to resolve microtubules. You'll need to consider the de Broglie wavelength of the particles.

DEVELOP To resolve the microtubules, the electron microscope that you buy will need electrons with de Broglie wavelength less than or equal to the size of the microtubules (25 nm). This corresponds to a momentum of $p = h/\lambda$, and a kinetic energy of $K = p^2/2m$. With these equations, you can calculate the minimum energy you need for an electron microscope.

EVALUATE The minimum electron kinetic energy needed is

$$K = \frac{h^2}{2m\lambda^2} = \frac{(hc)^2}{2mc^2\lambda^2} = \frac{(1240\ \text{eV}\cdot\text{nm})^2}{2(511\ \text{keV})(25\ \text{nm})^2} = 2.4\ \text{meV}$$

where we have used the rest energy of the electron, $mc^2 = 511$ keV. The minimum kinetic energy is far below 40 keV, so you don't need to buy the more expensive microscope, since the less expensive microscope will work.

ASSESS By the above arguments, the 40 keV electron microscope should have resolution of around 6 pm, since that is the corresponding de Broglie wavelength of the electrons. However, there are complications involved in focusing a beam of electrons, so most electron microscopes have resolutions around a nanometer.

69. **INTERPRET** This problem involves the uncertainty principle. We want to find the minimum velocity of an electron based on its uncertainty in position.

DEVELOP Using the uncertainty principle given in Equation 34.15, with $\Delta x = 15\ \text{nm}$ (the width of the well), we have

$$\Delta p \geq \frac{\hbar}{\Delta x} = \frac{197.3\ \text{eV}\cdot\text{nm}/c}{15\ \text{nm}} = 13.15\ \text{eV}/c$$

This is small compared to $mc = 511\ \text{keV}/c$, so nonrelativistic formulas are sufficient (see Example 34.6).

Therefore, $\Delta v = \Delta p/m = 2v$.

EVALUATE The above conditions lead to

$$v = \frac{\Delta p}{2m} \geq \frac{13.15\ \text{eV}/c}{2(511\ \text{keV}/c^2)} = 1.29\times10^{-5}c = 3.86\times10^{3}\ \text{m/s} = 3.86\ \text{km/s}$$

Thus, the minimum speed is $v_{\text{min}} = 3.86\ \text{km/s}$.

ASSESS Quantum wells have important applications in the field of semiconductor fabrication. Note that the result is given to two significant digits, as warranted by the data.

70. **INTERPRET** This problem involves the uncertainty principle applied to the variable energy and time. We shall use this to find the uncertainty in energy in a typical atomic transition given the lifetime of the excited atomic state.

DEVELOP The uncertainty principle (Equation 34.16) for energy and time is

$$\Delta t \Delta E \geq \hbar$$

(note that the units of $\hbar$ are time-energy). Apply this to find the minimum energy given the uncertainty in time of $\Delta t = 10^{-8}$ s.

EVALUATE The uncertainty in energy is

$$\Delta E \geq \frac{\hbar}{\Delta t} = \frac{6.582\times10^{-16}\ \text{eV}\cdot\text{s}}{10^{-8}\ \text{s}} = 7\times10^{-8}\ \text{eV}$$

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ASSESS This energy uncertainty is called the natural linewidth.

71. **INTERPRET** This problem involves energy-time uncertainty. We are interested in the minimum measurement time needed to measure the energy with the desired precision.

DEVELOP The electron is nonrelativistic $(v/c = 1/300 \ll 1)$, so we can use the nonrelativistic expression $K = mv^2/2$ for kinetic energy. The desired uncertainty in the kinetic energy is

$$\Delta E = 2(0.01\%)\times\left(\frac{1}{2}mv^2\right) = 10^{-4}mc^2\left(\frac{v}{c}\right)^2 = 10^{-4}(511\text{ keV})\left(\frac{1}{300}\right)^2 = 5.68\times10^{-4}\text{ eV}$$

The minimum time can then be calculated using Equation 34.16, $\Delta E\Delta t \geq \hbar$.

EVALUATE An energy measurement of this precision requires a time

$$\Delta t \geq \frac{\hbar}{\Delta E} = \frac{6.582\times10^{-16}\text{ eV}\cdot\text{s}}{5.68\times10^{-4}\text{ eV}} = 1\text{ ps}$$

to a single significant figure.

ASSESS The energy-time uncertainty principle implies that the minimum measurement time must necessarily go up in order to achieve a greater accuracy in energy measurement.

72. **INTERPRET** We are to derive the classical Rayleigh–Jeans law (Equation 34.5) for blackbody radiation from Planck's law (Equation 34.3).

DEVELOP For $\lambda \gg hc/kT$, the exponent in Planck's law (Equation 34.3) is much, less than unity, so we can express the exponential function as

$$\exp\left(\frac{hc}{\lambda kT}\right) \approx 1 - \frac{hc}{\lambda kT}$$

Insert this result into Planck's law.

EVALUATE With the above approximation, Planck's law takes the form

$$R(\lambda,T) \approx \frac{2\pi hc^2}{\lambda^5\left[1 - hc/(\lambda kT) - 1\right]} = \frac{2\pi hc^2}{\lambda^5\, hc/(\lambda kT)} = \frac{2\pi ckT}{\lambda^4}$$

which is the Rayleigh–Jeans law.

ASSESS Thus, the Rayleigh–Jeans law is an approximation of the more accurate Planck's law.

73. **INTERPRET** In this problem, we want to show that if a photon's wavelength is equal to a particle's Compton wavelength, then the photon's energy is equal to the particle's rest energy.

DEVELOP From Equation 34.8, we see that the Compton wavelength of a particle is $\lambda_C = h/mc$. The rest energy of a particle is $E = mc^2$ (see discussion preceding Equation 33.9). From Equation 34.6, we see that the energy of a photon is $E = hf = hc/\lambda$.

EVALUATE When the wavelength of the photon is $\lambda = \lambda_C$, its energy is

$$E = \frac{hc}{\lambda_C} = \frac{hc}{h/(mc)} = mc^2$$

which is the same as the particle's rest energy.

ASSESS This result is not surprising because photons have zero rest mass, so their energy is completely kinetic. Have you ever seen a photon that is not moving at the speed of light?

74. **INTERPRET** This problem involves the frequencies (i.e., energies) of the electronic transitions in hydrogen. We are to show that transitions to a level n from all higher levels cover the given frequency range.

DEVELOP From Equation 34.9, we see that the frequency of a transition in which the electron is initially in state n_1 and ends up in state $n_2 = n$ is

$$f = \frac{c}{\lambda} = cR_H\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right) = cR_H\left(\frac{1}{n^2} - \frac{1}{n_1^2}\right)$$

The possible values for n_1 are $n + 1$, $n + 2$, …, so the lowest possible frequency occurs for $n_1 = n + 1$ and the highest possible frequency occurs for $n_1 = \infty$.

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EVALUATE The frequency range is

$$\Delta f = f\left(n_1 = \infty\right) - f\left(n_1 = n+1\right) = cR_{\mathrm{H}}\left(\frac{1}{n^2} - 0\right) - cR_{\mathrm{H}}\left(\frac{1}{n^2} - \frac{1}{\left(n+1\right)^2}\right) = cR_{\mathrm{H}}\frac{1}{\left(n+1\right)^2}$$

ASSESS This agrees with the formula given in the problem statement.

75. **INTERPRET** This problem involves Compton scattering of a photon off an electron that is initially at rest (zero kinetic energy). We are to find an expression for the initial photon energy given the final total energy (kinetic energy plus rest energy) of the electron.

DEVELOP For Compton scattering at 90°, Equation 34.8 reduces to $\lambda = \lambda_0 + \lambda_{\mathrm{C}}$. In terms of the photon energy (Equation 34.6) $E = hf = hc/\lambda$ and the electron's Compton wavelength [Equation 34.8, $\lambda_{\mathrm{C}} = hc\big/\left(m_e c^2\right)$], this can be written as

$$\frac{1}{E} = \frac{1}{E_0} + \frac{1}{m_e c^2}$$

or

$$E = \frac{E_0 m_e c^2}{E_0 + m_e c^2}$$

The recoil electron's kinetic energy is

$$K_e = \left(\gamma - 1\right) m_e c^2 = E_0 - E = E_0 - \frac{E_0 m_e c^2}{E_0 + m_e c^2} = \frac{E_0^2}{E_0 + m_e c^2}$$

This is a quadratic equation in E_0, namely $E_0^2 - \left(\gamma - 1\right) mc^2 \left(E_0 + m_e c^2\right) = 0.$ The positive solution corresponds to the initial photon energy that we seek.

EVALUATE The positive solution for E_0 is

$$E_0 = \frac{1}{2}\left[\left(\gamma-1\right) m_e c^2 + \sqrt{\left(\gamma-1\right)^2 m_e^2 c^4 + 4\left(\gamma-1\right) m^2 c^4}\right] = \frac{1}{2} m_e c^2 \left[\left(\gamma-1\right) + \sqrt{\left(\gamma-1\right)\left(\gamma+3\right)}\right]$$

ASSESS With some algebra, the kinetic energy of the recoiled electron can be written as

$$K_e = \left(\gamma-1\right) m_e c^2 = \frac{E_0^2}{E_0 + m_e c^2} = \frac{1}{2} m_e c^2 \frac{\left[\left(\gamma-1\right) + \sqrt{\left(\gamma-1\right)\left(\gamma+3\right)}\right]^2}{\left(\gamma+1\right) + \sqrt{\left(\gamma-1\right)\left(\gamma+3\right)}}$$

In the nonrelativistic limit where $\gamma \approx 1 + \frac{1}{2} v^2 \big/ c^2$, the above expression reduces to the expected result $K_e \approx \frac{1}{2} m_e v^2$.

76. **INTERPRET** We are to derive Wien's law (see Section 34.2 on blackbody radiation) from Planck's law.

DEVELOP If we introduce the dimensionless variable $x = hc/(\lambda kT)$ into Planck's law, $R(\lambda, T)$ is proportional to $x^5\big/\left(e^x - 1\right)$. This can be a maximum when its derivative with respect to x is zero, which leads to the equation $e^x = 5\big/\left(5 - x\right)$.

EVALUATE For a maximum, this condition is satisfied by a value of x nearly equal to 5 (since $x = 0$ corresponds to a minimum radiance). The value $x_{\max}$ can be found numerically to be about 4.965, so

$$\lambda_{\max} T = \frac{hc}{4.965k} = 2.898\ \mathrm{mm} \cdot \mathrm{K}$$

which is Equation 34.2.

ASSESS We have shown the desired relationship. Notice that Wien's law is not a classical approximation, as is the case for the Rayleigh–Jeans law.

77. **INTERPRET** We shall use conservation of energy and conservation of momentum (with relativistic expressions for both energy and momentum) to derive equations related to Compton's equation, and then derive the equation for the Compton shift.

DEVELOP The energy of a photon is $E = hf = hc/\lambda$ (Equation 34.6) and for a particle it is $E = \gamma mc^2$ (Equation 33.9). The relativistic momentum is $\vec{p} = \gamma m \vec{u}$ for the electron and $\vec{p} = \hat{i}\, h/\lambda_0$ for the photon, where $\hat{i}$

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is the direction of the photon's motion. We will use conservation of energy to obtain one of the desired equations, and conservation of momentum in two dimensions to obtain the other two equations.

EVALUATE The initial energy is the energy of the photon plus the rest energy of the electron:

$$E_\mathrm{i} = hc/\lambda_0 + mc^2.$$

The final energy is the energy of the new photon plus the relativistic energy of the moving electron:

$$E_\mathrm{f} = hc/\lambda + \gamma mc^2$$

Equating these two energies (by conservation of energy) gives us the first of the three desired equations:

$$\frac{hc}{\lambda_0} + mc^2 = \frac{hc}{\lambda} + \lambda mc^2.$$

The next two equations come from the initial momentum, $\vec{p}_\mathrm{i} = \hat{i}\, h/\lambda_0$ and the components of final momentum $p_{\mathrm{f}x} = \frac{h}{\lambda}\cos\theta + \gamma mu\cos\phi$ and $p_{\mathrm{f}y} = 0 = \gamma mu\sin\phi - \frac{h}{\lambda}\sin\theta$. By conservation of momentum, we can equate the initial and final momentum in each direction, which leads to

$$p_{\mathrm{i}x} = p_{\mathrm{f}x} \quad \Rightarrow \quad \frac{h}{\lambda_0} = \frac{h}{\lambda}\cos\theta + \gamma mu\cos\phi$$

$$p_{\mathrm{i}y} = p_{\mathrm{f}y} = 0 \quad \Rightarrow \quad 0 = \frac{h}{\lambda}\sin\theta - \gamma mu\sin\phi$$

These are the second two of the desired relationships we were to derive. Solving these three equations for $\lambda_0 - \lambda$ directly is a lengthy algebraic process. An easier approach is to start with the momentum in vector form and use the law of cosines:

$$\vec{p}_\gamma = \vec{p}_{\gamma'} + \vec{p}_{e'} \to p_{e'}^2 = p_\gamma^2 + p_{\gamma'}^2 - 2p_\gamma p_{\gamma'}\cos\theta$$

$$p_{e'}^2 = \left(\frac{h}{\lambda_0}\right)^2 + \left(\frac{h}{\lambda}\right)^2 - 2\frac{h}{\lambda_0}\frac{h}{\lambda}\cos\theta$$

We now use conservation of energy in the form $\frac{hc}{\lambda_0} + mc^2 = \frac{hc}{\lambda} + \sqrt{m^2c^4 + p_{e'}^2c^2}$, and solve for $p_{e'}^2$ to obtain

$$p_{e'}^2 = \frac{\left(\frac{hc}{\lambda_0} - \frac{hc}{\lambda} + mc^2\right)^2 - m^2c^4}{c^2}$$

We equate the two equations for $p_{e'}^2$ to obtain

$$\frac{\left(\frac{hc}{\lambda_0} - \frac{hc}{\lambda} + mc^2\right)^2 - m^2c^4}{c^2} = \left(\frac{h}{\lambda_0}\right)^2 + \left(\frac{h}{\lambda}\right)^2 - 2\frac{h}{\lambda_0}\frac{h}{\lambda}\cos\theta$$

$$\cancel{\left(\frac{hc}{\lambda_0}\right)^2 + \left(\frac{hc}{\lambda}\right)^2} - \frac{2c^3hm}{\lambda} + \frac{2c^3hm}{\lambda_0} - \frac{2c^2h^2}{\lambda\lambda_0} = \cancel{\left(\frac{hc}{\lambda_0}\right)^2 + \left(\frac{hc}{\lambda}\right)^2} - 2\frac{hc}{\lambda_0}\frac{hc}{\lambda}\cos\theta$$

$$\frac{mc}{\lambda_0} - \frac{mc}{\lambda} = \frac{h}{\lambda\lambda_0}(1 - \cos\theta)$$

$$\lambda - \lambda_0 = \frac{h}{mc}(1 - \cos\theta)$$

ASSESS We have derived the equation for the Compton shift, using conservation of energy and momentum.

78. **INTERPRET** We are to integrate the radiance equation (Equation 34.3, Planck's law) over all wavelengths and show that the resulting total power radiated per unit area is equivalent to the Stefan–Boltzmann law (Equation 34.1).

DEVELOP The Stefan–Boltzmann law gives the power per area as $P/A = \sigma T^4$, and the radiance equation is

$$R(\lambda, T) = \frac{2\pi hc^2}{\lambda^5\left(e^{hc/(\lambda kT)} - 1\right)}$$

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Substituting $hc/(\lambda kT)$ by the integration variable x gives

$$R(x,T)=\frac{2\pi k^5T^5}{h^4c^3}\left(\frac{x^5}{e^x-1}\right)$$

The differentials dx and $d\lambda$ are related by

$$d\lambda=-\frac{hc}{x^2kT}dx$$

We will integrate $R(x,T)d\lambda$ over all λ.

EVALUATE Performing the integration gives

$$\frac{P}{A}=\int_0^\infty R(\lambda,T)\,d\lambda=\frac{2\pi k^5T^5}{h^4c^3}\int_0^\infty\left(\frac{x^5}{e^x-1}\right)\left(-\frac{hc}{x^2kT}\right)dx=-\frac{2\pi k^4T^4}{h^3c^2}\int_0^\infty\left(\frac{x^3}{e^x-1}\right)dx$$
$$=\frac{2k^4\pi^5}{15c^2h^3}T^4$$

ASSESS This is equivalent to the Stefan–Boltzmann law, with $\sigma=2k^4\pi^5/\left(15h^3c^2\right)$.

79. **INTERPRET** We are to numerically verify the median wavelength as given by Equation 34.2b by numerically integrating the radiance equation from zero to the value given by 34.2b—the result should be 1/2.

DEVELOP We shall numerically integrate Equation 34.3

$$R(\lambda,T)=\frac{2\pi hc^2}{\lambda^5\left(e^{hc/(\lambda kT)}-1\right)}$$

from $\lambda=0$ to $\lambda_{\text{median}}=0.00411/T$. If the value we get is half the value obtained when numerically integrating to "infinity," then we will have verified that λ_{median} is indeed the median wavelength.

EVALUATE With most numeric software packages or simple integration routines, it is easiest to substitute some value of T. For example, for $T=1000$ K, the integral to λ_{median} is 28,379 and the integral to some approximately "infinite" number is 56,704. Other values of T give different values, but in each case the integral to λ_{median} is approximately half the integral over all values of λ.

ASSESS The median wavelength is given to only three significant figures. The numeric value of the integral taken to λ_{median} is half the numeric value of the entire integral to more than three significant figures, so we have verified that the median wavelength given is correct.

80. **INTERPRET** We are to show that the correspondence principle (see Section 34.7) holds for the Bohr model in that the frequency of a photon emitted in a $n+1\to n$ transition for large n equals the orbital frequency of the electron. We shall do this by taking the limit of both the transition energy and the orbital frequency as $n\to\infty$.

DEVELOP The energy of the photon emitted in a transition from n_1 to n_2 is $\Delta E=ke^2\left[2a_0\left(1/n_2^2-1/n_1^2\right)\right]$, where $n_2=n$ and $n_1=n+1$. The frequency of the photon is $\Delta E/h$. The orbital frequency can be calculated from the orbital radius $r=-ke^2/(2E)=ke^2/\left(mv^2\right)$ and the orbital velocity $v=n\hbar/(mr)$ using $f_\text{O}=v/(2\pi r)$.

We will use the binomial expansion to approximate f_γ and compare the result with the orbital frequency f_O.

EVALUATE From $v=n\hbar/(mr)$ and $r=ke^2/\left(mv^2\right)$ we obtain the orbital radius

$$r=\frac{n^2\hbar^2}{e^2mk}$$

The orbital frequency is

$$f_\text{O}=\frac{v}{2\pi r}=\frac{e^4mk^2}{2\pi\hbar^3n^3}=\frac{e^2k}{ha_0n^3},\ \text{where}\ a_0=\frac{\hbar^2}{e^2mk}$$

The frequency of the photon emitted is

$$f_\gamma=\frac{ke^2}{2ha_0}\left(\frac{1}{n^2}-\frac{1}{(n+1)^2}\right)=\frac{ke^2}{2ha_0}\frac{1}{n^2}\left(1-\frac{1}{(1+1/n)^2}\right)$$

so for $1/n=1$

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$$f_\gamma \approx \frac{ke^2}{2ha_0n^2}\left[1-\left(1-\frac{2}{n}\right)\right] = \frac{ke^2}{2ha_0n^2}\left(\frac{2}{n}\right) = \frac{e^2k}{ha_0n^3}$$

ASSESS For large values of n, the optical frequency f_γ is the same as the orbital frequency f_O.

81. **INTERPRET** This problem involves a photoelectric effect experiment. Given the data of the stopping potential as a function of wavelength, we are to determine the Planck's constant, work function, and the material.

DEVELOP The electron's maximum kinetic energy is expended in crossing the stopping potential, so

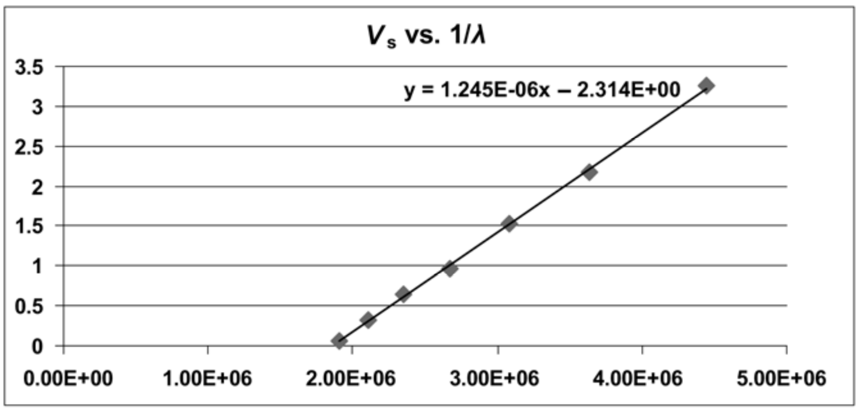
$$eV_s = K_{max}$$

Using Equation 34.7, we obtain

$$eV_s = hf - \phi = \frac{hc}{\lambda} - \phi \quad \Rightarrow \quad V_s = \frac{hc}{e\lambda} - \frac{\phi}{e}$$

Thus, plotting V_s vs. $1/\lambda$ will give a straight line with slope equal to hc/e, which allows us to experimentally determine h. Similarly, the y-intercept corresponds to $-\phi/e$, from which we deduce the work function ϕ.

EVALUATE **(a)** The plot is shown below.



The slope of the best-fit line is 1.245×10^{-6} V·m which allows us to deduce the value of h:

$$h = \frac{e(1.245\times10^{-6}\ \text{V}\cdot\text{m})}{c} = \frac{(1.6\times10^{-19}\ \text{C})(1.245\times10^{-6}\ \text{V}\cdot\text{m})}{3.0\times10^{8}\ \text{m/s}} = 6.65\times10^{-34}\ \text{J}\cdot\text{s}$$

(b) The y-intercept is $-\phi/e = -2.314$ V, #which implies a work function of $\phi \approx 2.31$ eV.

(c) From Table 34.1, we determine the material to be potassium.

ASSESS Work function ϕ is the minimum energy required to eject an electron, and the value is typically a few eV for most metals.

82. **INTERPRET** We investigate the relation between particle lifetimes and uncertainty in their rest energy measurements.

DEVELOP From Equation 34.16, the uncertainties in the energy and time are constrained by $\Delta E \Delta t \ge \hbar$. Therefore, the lifetime of a given particle will be inversely proportional to the uncertainty in its rest energy: $\tau \propto 1/\Delta E$.

EVALUATE The shortest lifetime will correspond to the curve with the largest uncertainty in its rest energy. In the graph, this is particle C.

The answer is **(c)**.

ASSESS The distribution width shown in the graph is called the natural linewidth and is denoted by Γ. It is called "natural" to signify that this uncertainty is inherent to the particle and does not, like other uncertainties, come simply from the imperfect instruments used to collect the data.

83. **INTERPRET** We investigate the relation between particle lifetimes and uncertainty in their rest energy measurements.

DEVELOP As argued above, the lifetime is related to the uncertainty in its rest energy by $\tau \approx \hbar/\Delta E$.

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EVALUATE For an uncertainty of 1 MeV, the lifetime must be roughly

$$\tau \approx \frac{\hbar}{\Delta E} = \frac{h}{2\pi \cdot \Delta E} = \frac{6.63\times10^{-34}\,\text{J}\cdot\text{s}}{2\pi(1\text{ MeV})}\left[\frac{1\text{ eV}}{1.6\times10^{-19}\,\text{J}}\right] = 7\times10^{-22}\,\text{s}$$

The answer is **(b)**.

ASSESS Although this seems like an incredibly short amount of time, there are many particles that have lifetimes in this range.

84. **INTERPRET** We investigate the relation between particle lifetimes and uncertainty in their rest energy measurements.

DEVELOP The inverse relation between energy uncertainty and lifetime is $\Delta E \approx \hbar / \tau$.

EVALUATE A longer lifetime leads to a narrower range in the energy measurement. By Einstein's mass-energy equivalence, this corresponds to a narrower range in the mass, as well.

The answer is **(d)**.

ASSESS Some particles, like the proton and the electron, appear to have infinite lifetimes, so we'd expect the uncertainty in their mass to be near to zero.

85. **INTERPRET** We investigate the relation between particle lifetimes and uncertainty in their rest energy measurements.

DEVELOP Using Einstein's mass-energy equation $(E = mc^2)$, we can write the time-energy uncertainty inequality as $\Delta m \approx \hbar / c^2\tau$.

EVALUATE Plugging in the lifetime, the mass range is

$$\Delta m \approx \frac{\hbar}{c^2\tau} = \frac{6.63\times10^{-34}\,\text{J}\cdot\text{s}}{2\pi\left(3\times10^{8}\,\text{m/s}\right)^2\left(10^{-7}\,\text{s}\right)}\left[\frac{1\text{ u}}{1.661\times10^{-27}\,\text{kg}}\right] = 7\times10^{-18}\,\text{u}$$

The answer is **(c)**.

ASSESS In particle physics, $10^{-7}\,\text{s}$ is a relatively long lifetime. A particle with roughly this long of a lifetime is the charged pion with $\tau = 2.6\times10^{-8}\,\text{s}$ and a mass of 0.15 u.

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