12

STATIC EQUILIBRIUM

EXERCISES

Section 12.1 Conditions for Equilibrium

10. **INTERPRET** This problem asks us to find the net force and torque on an object, given all the forces acting on the object and the positions at which they are applied. The net force is simply the sum of all the forces acting on the object, and the net torque is the sum of all the torques acting on the object.

DEVELOP Sum the *x*- and *y*-components of the given forces individually to find the *x*- and *y*-components of the net force. For part (b), apply Equation 12.2 to find the net torque, where $\vec{r}_i$ is the position vector indicating where each force is applied.

EVALUATE (a) The sum of the *x*-components of the forces is $F_x = 2\text{ N} + (-5\text{ N}) + 3\text{ N} = 0\text{ N}.$ The sum of the *y*-components of the forces is $F_y = 3\text{ N} + (-7\text{ N}) + 4\text{ N} = 0\text{ N}.$ Thus, the net force on the object is $\vec{F}_{\text{net}} = F_x\hat{i} + F_y\hat{j} = \vec{0}.$

(b) Evaluating Equation 12.2 to find the torques about the origin due to each force, we have

$$\vec{\tau}_1 = \vec{r}_1 \times \vec{F}_1 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3\text{ m} & 0\text{ m} & 0\text{ m} \\ 2\text{ N} & 3\text{ N} & 0\text{ N} \end{vmatrix} = (9\text{ N}\cdot\text{m})\hat{k}$$

$$\vec{\tau}_2 = \vec{r}_2 \times \vec{F}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1\text{ m} & 2\text{ m} & 0\text{ m} \\ -5\text{ N} & -7\text{ N} & 0\text{ N} \end{vmatrix} = (17\text{ N}\cdot\text{m})\hat{k}$$

$$\vec{\tau}_3 = \vec{r}_3 \times \vec{F}_3 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2\text{ m} & 6\text{ m} & 0\text{ m} \\ 3\text{ N} & 4\text{ N} & 0\text{ N} \end{vmatrix} = (-26\text{ N}\cdot\text{m})\hat{k}$$

The net torque is then $\vec{\tau}_{\text{net}} = \vec{\tau}_1 + \vec{\tau}_2 + \vec{\tau}_3 = (9\text{ N}\cdot\text{m} + 17\text{ N}\cdot\text{m} - 26\text{ N}\cdot\text{m})\hat{k} = \vec{0}.$

ASSESS Notice that each torque is perpendicular to both the force and the position vector from which it is composed.

11. **INTERPRET** We have been told that, for calculating the net torque, the choice of pivot point does not matter if the sum of the forces is zero (i.e., the object is in static equilibrium). For this problem, we will test this claim by calculating the torques from the previous problem about two new points

DEVELOP The three forces are $\vec{F}_1 = 2\hat{i} + 3\hat{j}\text{ N},$ applied at point $(x, y) = (3\text{ m}, 0\text{ m});$ $\vec{F}_2 = -5\hat{i} - 7\hat{j}\text{ N},$ applied at $(x, y) = (-1\text{ m}, 2\text{ m});$ and $\vec{F}_3 = 3\hat{i} + 4\hat{j}\text{ N},$ applied at $(x, y) = (-2\text{ m}, 6\text{ m}).$ Use Equation 11.2, $\vec{\tau} = \vec{r} \times \vec{F},$ to find the torques due to these three forces around the points $(3\text{ m}, 4\text{ m})$ and $(1\text{ m}, -3\text{ m}).$ To find the value of $\vec{r}$ for an arbitrary point, take the vector difference between the point where the force is applied and the point used as the pivot: $\vec{r} = (\vec{r}_{\text{applied}} - \vec{r}_{\text{pivot}}).$

EVALUATE If applying the first force about the point $(3\text{ m}, 4\text{ m}),$ the position vector is $\vec{r}_1 = (3\text{ m} - 3\text{ m})\hat{i} + (0\text{ m} - 4\text{ m})\hat{j} = (0\text{ m})\hat{i} + (-4\text{ m})\hat{j},$ so the torque due to $\vec{F}_1$ is

$$\vec{\tau}_1 = \vec{r}_1 \times \vec{F}_1 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0\text{ m} & -4\text{ m} & 0\text{ m} \\ 2\text{ N} & 3\text{ N} & 0\text{ N} \end{vmatrix} = \left[(0+8)\text{ N}\cdot\text{m})\right]\hat{k} = (8\text{ N}\cdot\text{m})\hat{k}$$

For the second force, the position vector is $\vec{r}_2 = (-1\text{ m} - 3\text{ m})\hat{i} + (2\text{ m} - 4\text{ m})\hat{j} = (-4\text{ m})\hat{i} + (-2\text{ m})\hat{j}$, so the torque due to $\vec{F}_2$ is

$$\vec{\tau}_2 = \vec{r}_2 \times \vec{F}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -4\text{ m} & -2\text{ m} & 0\text{ m} \\ -5\text{ N} & -7\text{ N} & 0\text{ N} \end{vmatrix} = \left[(28-10)\text{ N}\cdot\text{m})\right]\hat{k} = (18\text{ N}\cdot\text{m})\hat{k}$$

For the third force, the position vector is $\vec{r}_3 = (-2\text{ m} - 3\text{ m})\hat{i} + (6\text{ m} - 4\text{ m})\hat{j} = (-5\text{ m})\hat{i} + (2\text{ m})\hat{j}$, so the torque due to $\vec{F}_3$ is

$$\vec{\tau}_3 = \vec{r}_3 \times \vec{F}_3 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -5\text{ m} & 2\text{ m} & 0\text{ m} \\ 3\text{ N} & 4\text{ N} & 0\text{ N} \end{vmatrix} = \left[(-20-6)\text{ N}\cdot\text{m})\right]\hat{k} = (-26\text{ N}\cdot\text{m})\hat{k}$$

Summing these torques to find the net torque gives $\vec{\tau}_{\text{net}} = (8\text{ N}\cdot\text{m} + 18\text{ N}\cdot\text{m} - 26\text{ N}\cdot\text{m})\hat{k} = \vec{0}$. If applying the first force about the point $(1\text{ m}, -3\text{ m})$, the position vector is $\vec{r}_1 = (3\text{ m} - 1\text{ m})\hat{i} + (0\text{ m} + 3\text{ m})\hat{j} = (2\text{ m})\hat{i} + (3\text{ m})\hat{j}$, so the torque due to $\vec{F}_1$ is

$$\vec{\tau}_1 = \vec{r}_1 \times \vec{F}_1 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2\text{ m} & 3\text{ m} & 0\text{ m} \\ 2\text{ N} & 3\text{ N} & 0\text{ N} \end{vmatrix} = \left[(6-6)\text{ N}\cdot\text{m})\right]\hat{k} = (0\text{ N}\cdot\text{m})\hat{k}$$

For the second force, the position vector is $\vec{r}_2 = (-1\text{ m} - 1\text{ m})\hat{i} + (2\text{ m} + 3\text{ m})\hat{j} = (-2\text{ m})\hat{i} + (5\text{ m})\hat{j}$, so the torque due to $\vec{F}_2$ is

$$\vec{\tau}_2 = \vec{r}_2 \times \vec{F}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2\text{ m} & 5\text{ m} & 0\text{ m} \\ -5\text{ N} & -7\text{ N} & 0\text{ N} \end{vmatrix} = \left[(14+25)\text{ N}\cdot\text{m})\right]\hat{k} = (39\text{ N}\cdot\text{m})\hat{k}$$

For the third force, the position vector is $\vec{r}_3 = (-2\text{ m} - 1\text{ m})\hat{i} + (6\text{ m} + 3\text{ m})\hat{j} = (-3\text{ m})\hat{i} + (9\text{ m})\hat{j}$, so the torque due to $\vec{F}_3$ is

$$\vec{\tau}_3 = \vec{r}_3 \times \vec{F}_3 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3\text{ m} & 9\text{ m} & 0\text{ m} \\ 3\text{ N} & 4\text{ N} & 0\text{ N} \end{vmatrix} = \left[(-12-27)\text{ N}\cdot\text{m})\right]\hat{k} = (-39\text{ N}\cdot\text{m})\hat{k}$$

Summing these torques to find the net torque gives $\vec{\tau}_{\text{net}} = (0\text{ N}\cdot\text{m} + 39\text{ N}\cdot\text{m} - 39\text{ N}\cdot\text{m})\hat{k} = \vec{0}$.

ASSESS Indeed, for calculating the net torque, if the sum of the forces is zero, the choice of pivot point does not matter.

12. **INTERPRET** This problem involves finding a force and the point at which to apply it so that static equilibrium is obtained when this force is joined with the other given forces. We must thus find the force needed so that the net force and the net torque about the origin is zero.

DEVELOP Apply Equations 12.1 and 12.2. From the graph, we see that for case (a),

$$\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = F\hat{j} + F\hat{i} + \vec{F}_3$$

where $\vec{F}_3$ is the unknown force. The net torque is

$$\vec{\tau}_{\text{net}} = \vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2 + \vec{r}_3 \times \vec{F}_3 = \vec{r}_1 \times F\hat{j} + \vec{r}_2 \times F\hat{i} + \vec{r}_3 \times \vec{F}_3$$

where $\vec{r}_1 = (2\text{ m})\hat{j}$, $\vec{r}_2 = (1\text{ m})\hat{j}$, and $\vec{r}_3 = (x_3\text{ m})\hat{i} + (y_3\text{ m})\hat{j}$ is unknown. Solve the unknowns by imposing the condition for equilibrium, $\vec{F}_{\text{net}} = 0$ and $\vec{\tau}_{\text{net}} = 0$. For case (b),

$$\vec{F}_{\text{net}} = -F\hat{i} + F\hat{i} + \vec{F}_3$$

where $\vec{F}_3$ is the unknown force. The net torque has the same form as for (a) with the same position vectors, but the forces are $\vec{F}_1 = -F\hat{i}$ and $\vec{F}_2 = F\hat{i}$.

EVALUATE (a) Applying the condition of zero net force gives $\vec{F}_3 = -F\left(\hat{i} + \hat{j}\right)$. Applying the condition of zero net torque gives

$$\vec{\tau} = 0 = \overbrace{\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0\text{ m} & 2\text{ m} & 0\text{ m} \\ 0 & F & 0 \end{vmatrix}}^{\vec{r}_1\times\vec{F}_1} + \overbrace{\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0\text{ m} & 1\text{ m} & 0\text{ m} \\ F & 0 & 0 \end{vmatrix}}^{\vec{r}_2\times\vec{F}_2} + \overbrace{\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x_3 & y_3 & 0\text{ m} \\ -F & -F & 0 \end{vmatrix}}^{\vec{r}_3\times\vec{F}_3} = -F\hat{k} + \left(-Fx_3 + Fy_3\right)\hat{k}$$

$$1 = -x_3 + y_3$$

where we have used the result just derived for F_3. We have thus found that applying a force $\vec{F}_3 = -F\left(\hat{i} + \hat{j}\right)$ at any point (x_3, y_3) that satisfies the equation $1 = -x_3 + y_3$ will result in static equilibrium.

(b) Applying the condition of zero net force gives $\vec{F}_3 = 0$. Applying the condition of zero net torque gives

$$\vec{\tau} = 0 = \overbrace{\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0\text{ m} & 2\text{ m} & 0\text{ m} \\ -F & 0 & 0 \end{vmatrix}}^{\vec{r}_1\times\vec{F}_1} + \overbrace{\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0\text{ m} & 1\text{ m} & 0\text{ m} \\ F & 0 & 0 \end{vmatrix}}^{\vec{r}_2\times\vec{F}_2} + \overbrace{\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x_3 & y_3 & 0\text{ m} \\ 0 & 0 & 0 \end{vmatrix}}^{\vec{r}_3\times\vec{F}_3}$$

$$= 2F\hat{k} + \left(-F\right)\hat{k} = F\hat{k}$$

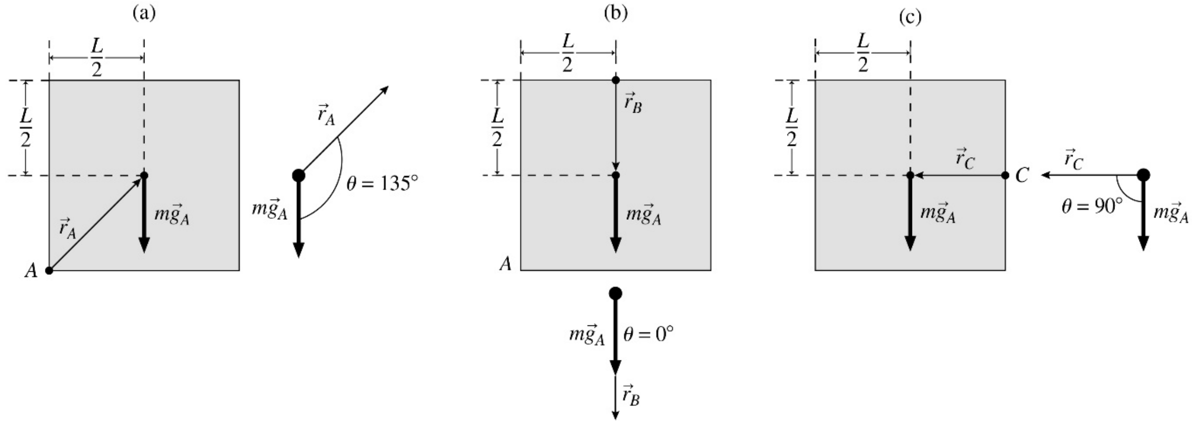
where we have used the result just derived for F_3. Thus, no single nonzero force can simultaneously satisfy both conditions for static equilibrium, so this situation is not static.

ASSESS Notice that both net force and net torque must be zero to attain static equilibrium.

Section 12.2 Center of Gravity

13. **INTERPRET** This problem involves finding the center of gravity of an object and calculating the torque that gravity applies on an object about several different points. We can treat gravity as if it acts only at the center of gravity of the object, and so calculate the position vectors from this point.

DEVELOP Make a drawing of the plate that shows the center of gravity and the position vectors from each point to the center of gravity (see figure below). By symmetry, the center of gravity is located at the center of the uniform plate. The force due to gravity acts at the center of gravity in the downward direction. Apply Equation 11.2, or its scalar analog, Equation 10.10, to find the torque due to gravity about the three points given.



EVALUATE (a) For point A, the angle θ between the position vector and the force vector is 135°, so the magnitude of the torque is

$$\tau = rF\sin\theta = \frac{L}{\sqrt{2}}mg\sin\left(135^\circ\right) = \frac{mgL}{2}$$

(b) For point B, the angle θ between the position vector and the force vector is 0°, so the torque is zero because sin(0°) = 0.
(c) For point C, the angle θ between the position vector and the force vector is 90°, so the torque is

$$\tau = rF\sin\theta = \frac{L}{2}mg\sin\left(90^\circ\right) = \frac{mgL}{2}$$

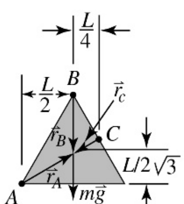
and the magnitude of the torque is $mgL/2$.
ASSESS The torque has the same magnitude in (a) and (b), but acts in opposite directions. You can confirm this by the right-hand rule.

14. **INTERPRET** In this problem we are asked to find the gravitational torque about various pivot points.
DEVELOP The torque about a point is given by Equation 10.10, $\tau = rF\sin\theta = r_\perp F$, where $r_\perp = r\sin\theta$ is the lever arm. In our problem, the center of gravity (CG) is at the center of the triangle, which is at a perpendicular distance of $\frac{L}{2\sqrt{3}}$ from any side. We regard CG as the point at which all the mass is concentrated.
EVALUATE **(a)** The lever arm of the weight about point *A* is $r_{\perp,A} = L/2$. Therefore, the gravitational torque about *A* is

$$\tau_A = r_{\perp,A}F_g = \frac{L}{2}mg = \frac{1}{2}mgL$$

(b) The lever arm about point *B* is zero (the line of action passes through point B). Therefore, we have $\tau_B = 0$.
(c) The lever arm about point *C* is $r_{\perp,C} = L/4$ (*C* is halfway up from the base) so

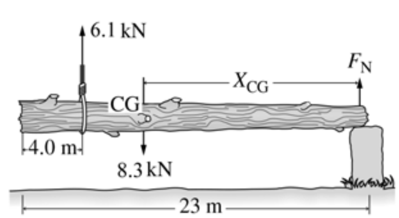
$$\tau_C = r_{\perp,C}F_g = \frac{L}{4}mg = \frac{1}{4}mgL$$



(b)

ASSESS The torques $\vec{\tau}_A$ and $\vec{\tau}_C$ are in opposite directions. If pivoted at A, the gravitational torque $\vec{\tau}_A$ tends to rotate the triangle CW. On the other hand, when pivoted at C, $\vec{\tau}_C$ gives rise to a CCW rotation.

15. **INTERPRET** The log is in equilibrium under the torques exerted by the cable, gravity, and the wall.
DEVELOP Because the tree is in equilibrium, we know that the sum of the torques is zero. Since we don't know the force exerted by the wall at the right end of the tree, we choose the contact point with the wall as our pivot point so that the torque from this force is zero. As for the given forces, gravity acts at the center of gravity, producing a torque of $\tau_g = x_{CG}F_g$, where x_{CG} is the unknown distance between the wall and the center of gravity.
See the figure below. The cable's tension produces a torque in the opposite direction: $\tau_c = -x_C T$, where $x_c = 23\text{ m} - 4.0\text{ m} = 19\text{ m}$.



EVALUATE Setting the sum of the torques to zero gives $\tau_g = -\tau_c$, from which we can find the center of gravity:

$$x_{CG} = \frac{x_c T}{F_g} = \frac{(19\text{ m})(6.1\text{ kN})}{(8.3\text{ kN})} = 13.96\text{ m}$$

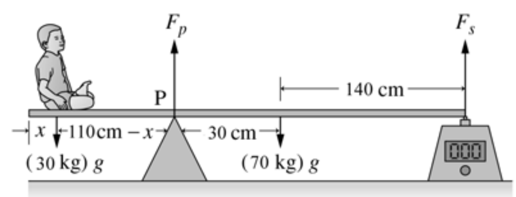
where this distance is relative to the wall.

ASSESS This seems reasonable. As expected, the center of gravity is between the cable and the wall; otherwise, the tree couldn't possibly be in equilibrium.

Section 12.3 Examples of Static Equilibrium

16. **INTERPRET** In this problem, we are asked to find where the child should sit on the pivot-supporting board so that the scale at the right end will read zero. To solve this problem, consider the center of gravity of the board, and apply the conditions for static equilibrium

DEVELOP Draw a diagram of the situation, with the force of gravity acting at the center of gravity of the board (which is at the middle of the uniform board; see figure below). The force applied by the supporting pivot is F_p, and the force applied by the scale is F_s. Apply the condition for static equilibrium (Equation 12.2). If we consider torques about the pivot point (so that the force exerted by the pivot does not contribute), then Equation 12.2, $\Sigma\vec{\tau}_i = \Sigma(\vec{r}_i \times \vec{F}_i) = \vec{0}$, is sufficient to determine the position of the child. Note that this choice of pivot point means that we must measure distances from this point, so we will need to calculate the distance at which the child sits from the left end of the board using $x = 1.10\text{ m} - x_c$, where x_c is the child's position relative to the pivot point.



EVALUATE As shown in the figure above, the weight F_b of the board (acting at its center of gravity), the weight F_c of the child (acting at a distance x from the left end), and the scale force F_s acting at 170 cm from the pivot produce zero torque about the pivot:

$$\left(\sum\tau\right)_P = 0 = F_c x_c - F_b x_{cm} + F_s x_s$$

$$x_c = \frac{F_b x_{cm} - F_s x_s}{F_c} = \frac{m_b g x_{cm} - F_s x_s}{m_c g}$$

If the scale reads zero, $F_s = 0$, and the child's position is

$$x_c = \frac{m_b g x_{cm}}{m_c g} = \frac{m_b}{m_c} x_{cm} = \frac{70\text{ kg}}{30\text{ kg}}(0.30\text{ m}) = 0.70\text{ m}$$

to the left of the pivot, or $x = 1.10\text{ m} - 0.70\text{ m} = 0.40\text{ m}$ from the left end of the board.

ASSESS If the child moves closer to the pivot (increasing x), then F_s will increase, so the reading of the scale will increase. One can also show that, without the child, the reading would be $F_s = 120\text{ N}$ (to two significant figures).

17. **INTERPRET** The interpretation of this problem is the same as that for the preceding problem, except that the position of the child must be adjusted so that the scale at the right end of the board will indicate the given quantities

DEVELOP The development of this problem is the same as that for Problem 16.

EVALUATE (a) Starting with the same relationship as that for Problem 16, we now insert $F_s = 90\text{ N}$ to find the position of the child. The result is

$$x_c = \frac{m_b g x_{cm} - F_s x_s}{m_c g} = \frac{(70\text{ kg})(9.8\text{ m/s}^2)(0.30\text{ m}) - (90\text{ N})(1.7\text{ m})}{(30\text{ kg})(9.8\text{ m/s}^2)} = 0.18\text{ m}$$

from the pivot point, or $x = 1.10\text{ m} - 0.18\text{ m} = 0.92\text{ m}$ from the left end of the board.

(b) With $F_s = 220\text{ N}$, we have

$$x_c = \frac{(70\text{ kg})(9.8\text{ m/s}^2)(0.30\text{ m}) - (220\text{ N})(1.7\text{ m})}{(30\text{ kg})(9.8\text{ m/s}^2)} = -0.57\text{ m}$$

from the pivot point (i.e., 0.57 m to the right of the pivot point), or $x = 1.10\text{ m} - (-0.57\text{ m}) = 1.67\text{ m}$ from the left end of the board.

ASSESS The answer to part (b) is reasonable because if there were no child present, the weight of the board alone could only supply a force

$$F_s = m_b g\left(\frac{x_{cm}}{x_s}\right) = (70\text{ kg})(9.8\text{ m/s}^2)\left(\frac{0.30\text{ m}}{1.7\text{ m}}\right) = 120\text{ N}$$

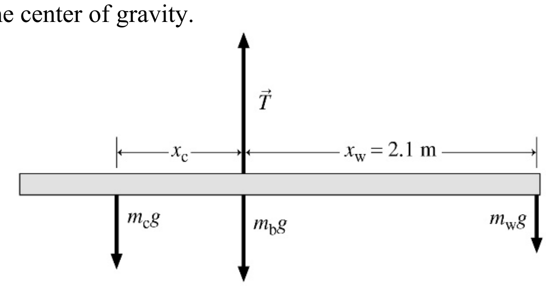
to two significant figures. Thus, with the child to the right of the pivot point, we expect the scale reading to be greater than 120 N, which corresponds to what we find in part (b).

18. **INTERPRET** This problem is about static equilibrium. We want to know where the concrete should be placed so that the long beam suspended by a cable and with a steelworker standing at one end will be in equilibrium.

DEVELOP Make a sketch of the situation, showing all the forces and the positions at which they are applied (see figure below). Because the beam is uniform, its center of mass is at its geometric center, which is where the cable is attached and where we can consider gravity to act (i.e., its center of gravity). At equilibrium, the sum of the torques on the beam about any point will be zero. We choose to calculate the torque about the center of gravity (which is the same as the center of gravity for a uniform gravitational field), so that the tension of the cable and the force of gravity on the beam itself do not enter (because they act at a distance zero from the chosen pivot point). Therefore,

$$\left(\sum \tau\right)_{cm} = 0 = -m_w g x_w + m_s g x_s$$

where the negative sign enters because the worker is to the left of the center of gravity, which we choose to be the negative direction. The quantities m_c and m_w are the masses of the concrete and the worker, and $x_s = 2.1$ m is the distance from the worker to the center of gravity.



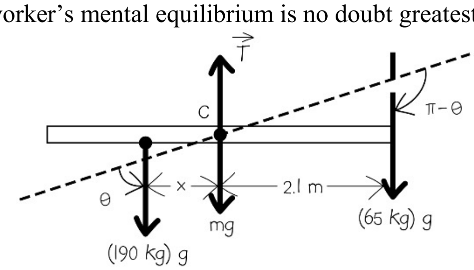
EVALUATE Using the values given in the problem statement, we have

$$x = \left(\frac{m_w}{m_c}\right)x_s = \left(\frac{65\text{ kg}}{190\text{ kg}}\right)(2.1\text{ m}) = 0.72\text{ m}$$

to the right of the center of gravity.

ASSESS The lighter the concrete, the longer the lever arm (x) must be for the system to remain in static equilibrium.

Note also that because $\sin\theta = \sin(\pi - \theta)$ will cancel from the torque equation, the beam need not be horizontal to be in equilibrium. However the worker's mental equilibrium is no doubt greatest when the beam is horizontal.

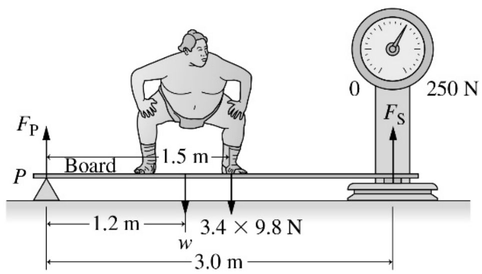


19. **INTERPRET** This is a problem of static equilibrium, so we know that the sum of the forces must be zero and that the sum of the torques about any point must be zero. We can use these concepts to find the weight of the sumo wrestler standing at the given position.

DEVELOP Make a sketch of the situation that shows the pivot point, the forces, and where the forces are applied (see figure below). We chose to calculate the torque about the pivot point P, so this will serve as the origin for measuring distances. Applying the condition (Equation 12.2) for equilibrium gives

$$\left(\sum\tau\right)_{\mathrm{P}} = 0 = \overbrace{\vec{r}_{\mathrm{P}}}^{=0}\times\vec{F}_{\mathrm{P}} + \vec{r}_{\mathrm{w}}\times\vec{F}_{w} + \vec{r}_{\mathrm{cm}}\times\vec{F}_{\mathrm{b}} + \vec{r}_{\mathrm{S}}\times\vec{F}_{\mathrm{S}} = x_{\mathrm{w}}w_{\mathrm{w}} + x_{\mathrm{cm}}m_{\mathrm{b}}g + x_{\mathrm{S}}F_{\mathrm{S}}$$

where the subscript w refers to the wrestler, b to the board, and s to the scale. We can solve for equation for the upward force F_s exerted by the scale.



EVALUATE Inserting the given quantities into the expression for net torque gives

$$x_{\mathrm{w}}w_{\mathrm{w}} = -x_{\mathrm{cm}}m_{b}g + x_{\mathrm{S}}F_{\mathrm{S}}$$

$$w_{\mathrm{w}} = \frac{-x_{\mathrm{cm}}m_{b}g + x_{\mathrm{S}}F}{-x_{\mathrm{w}}} = \frac{-(1.5\ \mathrm{m})(3.4\ \mathrm{kg})\left(9.8\ \mathrm{m/s^2}\right)+(3.0\ \mathrm{m})(210\ \mathrm{N})}{-1.2\ \mathrm{m}} = 480\ \mathrm{N}$$

to two significant figures.

ASSESS In pounds, the wrestler weighs about 110 pounds.

Section 12.4 Stability

20. **INTERPRET** The problem is about the stability of the roller coaster as it moves along the track described by a height function. We want to identify the equilibrium point and classify its stability.

DEVELOP The potential energy of the roller coaster car is

$$U(x) = mgh(x) = mg\left(0.65x - 0.013x^2\right)$$

The equilibrium condition is given by Equation 12.3, $dU/dx = 0$. In addition, the equilibrium condition may be classified according to its second derivative:

$$\frac{d^2U}{dx^2}\begin{cases} >0, & \text{stable} \\ <0, & \text{unstable} \\ =0, & \text{neutral}\end{cases}$$

EVALUATE **(a)** Applying Equation 12.3, the condition for equilibrium, gives

$$0 = \frac{dU}{dx} = mg(0.65 - 0.026x) \quad\Rightarrow\quad x = 25\ \mathrm{m}$$

Thus, at 25 m from the origin, the roller coaster will be in an equilibrium position.

(b) Using Equation 12.5, the condition for stable equilibrium, gives

$$\frac{d^2U}{dx^2} = -0.026mg < 0$$

so this is an unstable equilibrium (i.e., a peak, not a valley).

ASSESS The point $x = 25$ m with $U(25\ \mathrm{m}) = 8.1mg$ corresponds to a local maximum where the potential-energy curve is concave downward. Therefore, the point is unstable.

21. **INTERPRET** We are given a potential function, and are asked to find the positions of any stable and unstable equilibria.

DEVELOP The condition for equilibrium is that $dU/dx = 0$, and the equilibrium point is stable if $d^2U/dx^2 > 0$ and unstable if $d^2U/dx^2 < 0$. The function we are given is $U(x) = 2x^3 - 2x^2 - 7x + 10$.

EVALUATE We take the derivative and equate it to zero to find the locations of any equilibrium points.

$$\frac{dU}{dx} = \frac{d}{dx}\left(2x^3 - 2x^2 - 7x + 10\right) = 6x^2 - 4x - 7 = 0$$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(6)(-7)}}{2(6)} = \left\{\frac{2-\sqrt{46}}{6}, \quad \frac{2+\sqrt{46}}{6}\right\}$$

There are equilibrium points at $x_1 = \left(2-\sqrt{46}\right)/6 \approx -0.797\text{ m}$ and $x_2 = \left(2+\sqrt{46}\right)/6 \approx 1.46\text{ m}$.
Next, we find the second derivative of U, and evaluate it at these two points to ascertain their stability.

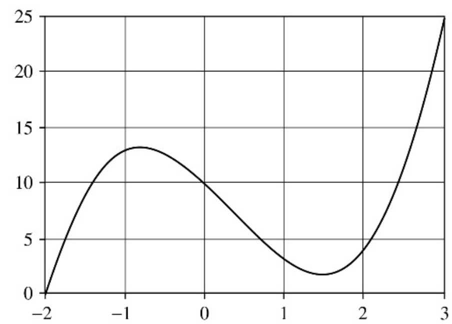
$$\frac{d^2U}{dx^2} = \frac{d^2}{dx^2}\left(2x^3 - 2x^2 - 7x + 10\right) = \frac{d}{dx}\left(6x^2 - 4x - 7\right) = 12x - 4$$

$$\text{for } x = \frac{2-\sqrt{46}}{6} \quad \Rightarrow \quad \frac{d^2U}{dx^2} = -13.6 < 0$$

$$\text{for } x = \frac{2+\sqrt{46}}{6} \quad \Rightarrow \quad \frac{d^2U}{dx^2} = 13.6 > 0$$

Thus, the first solution is unstable, and the second is stable.

ASSESS The potential energy function is plotted in the figure below, where it can be seen that the points we found are indeed equilibria. The point at –0.797 is unstable, and the point at 1.46 is metastable.



EXAMPLE VARIATIONS

22. **INTERPRET** In this problem we want to revisit the static equilibrium of a ladder lying up against a wall that is discussed in Example 12.2, but with a different coefficient of friction.

DEVELOP We treat this problem as is done in Example 12.2, with the same coordinate system and choosing the bottom of the ladder as the pivot point. This results in the equations:

$$\text{Force}, x: \ \mu n_1 - n_2 = 0$$

$$\text{Force}, y: \ n_1 - mg = 0$$

$$\text{Torque}: \ Ln_2 \sin\varphi - \frac{1}{2}Lmg\cos\varphi = 0$$

EVALUATE Like in Example 12.2, we use the force equations to rewrite the expression for the torque, and obtain the minimum angle for which the ladder won't slip is given by

$$L\mu mg \sin\varphi = \frac{1}{2}Lmg\cos\varphi$$

$$\tan\varphi = \frac{1}{2\mu} \rightarrow \varphi = \text{atan}\frac{1}{2\mu} = 46.0°$$

ASSESS The larger the frictional coefficient, the more horizontal force holding the ladder in place, and the smaller the angle at which it can safely lean.

23. **INTERPRET** In this problem we want to revisit the static equilibrium of a ladder lying up against a wall that is discussed in Example 12.2, but with a person standing on the ladder.

DEVELOP We treat this problem as is done in Example 12.2, with the same coordinate system and choosing the bottom of the ladder as the pivot point. Adding the person of given mass m_p a distance x up the ladder, we obtain the equations:

$$\text{Force}, x: \ \mu n_1 - n_2 = 0$$

$$\text{Force}, y: \ n_1 - \left(m_1 + m_\mathrm{p}\right) g = 0$$

$$\text{Torque}: \ L n_2 \sin\varphi - \frac{1}{2} L m_1 g \cos\varphi - x m_\mathrm{p} g \cos\varphi = 0$$

EVALUATE Like in Example 12.2, we use the force equations to rewrite the expression for the torque, and obtain the distance x is given by

$$L\mu\left(m_1 + m_\mathrm{p}\right) g \sin\varphi = \frac{1}{2} L m_1 g \cos\varphi + x m_\mathrm{p} g \cos\varphi$$

$$x = \frac{\left(L\mu\left(m_1 + m_\mathrm{p}\right) g \sin\varphi - \frac{1}{2} L m_1 g \cos\varphi\right)}{m_\mathrm{p} g \cos\varphi} = \frac{L}{m_\mathrm{p}}\left(\mu\left(m_1 + m_\mathrm{p}\right)\tan\varphi - \frac{m_1}{2}\right) = 3.00\,\mathrm{m}$$

ASSESS A person of the given weight, for the given parameters, can make it ¾ of the way up the ladder before it begins to slip.

24. **INTERPRET** In this problem we want to revisit the static equilibrium case discussed in Example 12.2, but instead with a log having a different mass distribution and a person standing on top.

DEVELOP We treat this problem as is done in Example 12.2, with the same coordinate system and choosing the bottom of the log as the pivot point, but noting the center of mass of the log is now a distance $L/3$ up the log. Adding the person of given mass m_p a distance $L/2$ up the log, we obtain the equations:

$$\text{Force}, x: \ \mu n_1 - n_2 = 0$$

$$\text{Force}, y: \ n_1 - \left(m_1 + m_\mathrm{p}\right) g = 0$$

$$\text{Torque}: \ L n_2 \sin\varphi - \frac{1}{3} L m_1 g \cos\varphi - \frac{1}{2} L m_\mathrm{p} g \cos\varphi = 0$$

EVALUATE Like in Example 12.2, we use the force equations to rewrite the expression for the torque, and obtain the minimum angle for which the log won't slip is given by

$$L\mu\left(m_1 + m_\mathrm{p}\right) g \sin\varphi = \frac{1}{3} L m_1 g \cos\varphi + \frac{1}{2} L m_\mathrm{p} g \cos\varphi$$

$$\tan\varphi = \frac{\left(\frac{1}{3} m_1 + \frac{1}{2} m_\mathrm{p}\right)}{\mu\left(m_1 + m_\mathrm{p}\right)} \rightarrow \varphi = \operatorname{atan}\frac{\left(\frac{1}{3} m_1 + \frac{1}{2} m_\mathrm{p}\right)}{\mu\left(m_1 + m_\mathrm{p}\right)} = 21.0^\circ$$

ASSESS The lower down the log (closer to the pivot point chosen) the center of mass is located, the smaller the minimum angle for which the log won't slip.

25. **INTERPRET** In this problem we want to revisit the static equilibrium case discussed in the previous problem, but we want to consider a new angle and determine how far up the log the person can go before it slips.

DEVELOP We treat this problem as is done in Example 12.2, with the same coordinate system and choosing the bottom of the log as the pivot point, but noting the center of mass of the log is now a distance $L/3$ up the log. Adding the person of given mass m_p a distance cL up the log, we obtain the equations:

$$\text{Force}, x: \ \mu n_1 - n_2 = 0$$

$$\text{Force}, y: n_1 - \left(m_l + m_p\right) g = 0$$

$$\text{Torque}: Ln_2 \sin\varphi - \frac{1}{3} Lm_l g \cos\varphi - cLm_p g \cos\varphi = 0$$

Here c is the fraction of L which the person is able to traverse before the log slips.

Evaluate Like in Example 12.2, we use the force equations to rewrite the expression for the torque, and obtain the fraction of the log's length c is given by

$$L\mu\left(m_l + m_p\right) g \sin\varphi = \frac{1}{3} Lm_l g \cos\varphi + cLm_p g \cos\varphi$$

$$c = \frac{\left(L\mu\left(m_l + m_p\right) g \sin\varphi - \frac{1}{3} Lm_l g \cos\varphi\right)}{Lm_p g \cos\varphi} = \frac{1}{m_p}\left(\mu\left(m_l + m_p\right)\tan\varphi - \frac{m_l}{3}\right)$$

Considering the same log mass, frictional coefficient and climber in the preceding problem, we find that c is equal to 0.90. This means the climber won't be able to get to the right-hand end of the log without it slipping, and can only get 90% of the way across.

Assess A less massive climber should be able to make it across the entire length of the log. This mass can be determine by setting $c = 1$ and solving for m_p leaving all other parameters the same.

26. **Interpret** In this problem we want to revisit the stability analysis discussed in Example 12.4, but considering different values for the given coefficients.

Develop We treat this problem as is done in Example 12.4, where we first want to find the equilibrium positions using Equation 12.3, which we then examine to determine their stability.

Evaluate Equation 12.3 states that equilibria occur where the potential energy has a maximum or minimum—that is, where its derivative is zero. Taking the derivative of U and setting it to zero gives

$$0 = \frac{dU}{dx} = 2ax - 4bx^3 = 2x\left(a - 2bx^2\right)$$

This equation has solutions when $x = 0$ and when $x = \pm\sqrt{a/2b}$. However since the given value for a is negative, there is no real solution for the latter set of equilibrium points. To evaluate the stability of the $x = 0$ equilibrium point we take the second derivative and evaluate it at $x = 0$, giving

$$\frac{d^2U}{dx^2} = 2a - 12bx^2 \rightarrow \frac{d^2U(0)}{dx^2} = 2a$$

Since this quantity is less than zero for the given value of a, we find that there is an unstable equilibrium located at $x = 0$, and no stable equlibria.

Assess Plotting this potential we see that it resembles Fig. 12.7b, clearly depicting the unstable nature of the equilibrium found at $x = 0$.

27. **Interpret** In this problem we want to revisit the stability analysis discussed in Example 12.4, but considering different values for the given coefficients.

Develop We treat this problem as is done in Example 12.4, where we first want to find the equilibrium positions using Equation 12.3, which we then examine to determine their value for the desired stability.

Evaluate Equation 12.3 states that equilibria occur where the potential energy has a maximum or minimum—that is, where its derivative is zero. Taking the derivative of U and setting it to zero gives

$$0 = \frac{dU}{dx} = 2ax - 4bx^3 = 2x\left(a - 2bx^2\right)$$

This equation has solutions when $x = 0$ and when $x = \pm\sqrt{a/2b}$. We find that the value of a for which we have an unstable equilibrium at $x = \pm 3\,\text{nm}$, is given by $a = \left(9\,\text{nm}^2\right)(2)\left(1\,\text{aJ/nm}^4\right) = 18\,\text{aJ/nm}^2$. We can also determine whether there is still a stable equilibrium at $x = 0$ by the taking the second derivative of the potential and evaluate it at $x = 0$.

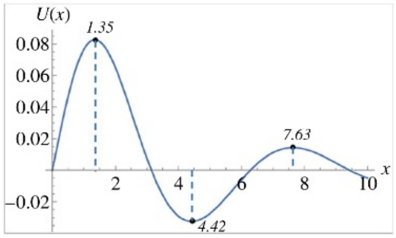
$$\frac{d^2U}{dx^2} = 2a - 12bx^2 \rightarrow \frac{d^2U(0)}{dx^2} = 2a$$

Since this quantity is greater than zero for the given value of a, we find that there is still a stable equilibrium located at $x = 0$.

ASSESS Since none the signs of the coefficients in the expression for the potential energy do not differ from the original example, the quantitative properties of its stability remain the same as in the original example.

28. **INTERPRET** In this problem we want to determine the equilibrium points of a given potential as well as their respective stability.

DEVELOP To determine the location of the equilibrium points for the given potential, $U(x) = \sin x / (x^2 + 10)$, we plot it in the region $0 < x < 10$ and take a look.



EVALUATE From the figure we can see that there is one stable equilibrium located at $x = 4.42$ and two unstable equilibria located at $x = 1.35$ and $x = 7.63$. These were found by locating the maxima and minima within the region of interest. We are able to determine their stability from the behavior depicted in figs. 12.7a and 12.7b for stable and unstable equilibria, respectively.

ASSESS We could have also determine the locations and stability of the equilibrium points by numerically solving the equations we would have obtained by taking the first and second derivatives of the given potential.

29. **INTERPRET** In this problem we want to determine the equilibrium points of a given two-dimensional potential as well as their respective stability.

DEVELOP We treat this problem as is done in Example 12.4, where we first want to find the equilibrium positions using Equation 12.3, which we then examine to determine their value for the desired stability.

EVALUATE Equation 12.3 states that equilibria occur where the potential energy has a maximum or minimum—that is, where its derivative is zero. Taking the derivative of U and setting it to zero along each dimension gives

$$0 = \frac{dU}{dx} = 2ax$$

$$0 = \frac{dU}{dy} = -2ay$$

These equations have solutions when $x = 0$ and when $y = 0$. Thus, this potential has an equilibrium point located at $x = 0$, $y = 0$. To determine whether it is stable to small displacements along either direction we take the second derivative of the potential along each direction and evaluate them at the equilibrium coordinate.

$$\frac{d^2U(0)}{dx^2} = 2a > 0$$

$$\frac{d^2U(0)}{dy^2} = -2a < 0$$

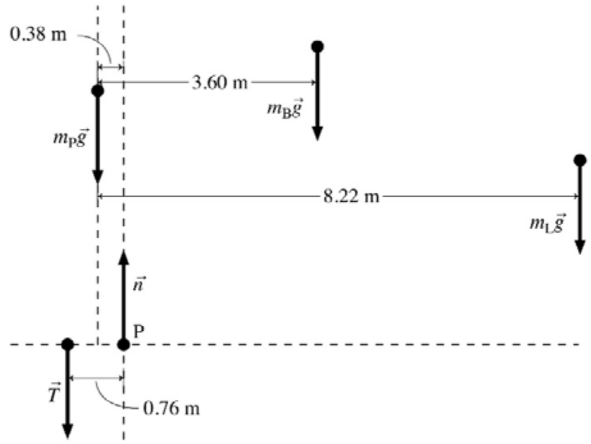
Since the value of a is positive, the equilibrium is stable in x but unstable in y.

ASSESS The equilibrium point we found is referred to as a saddle point, since the potential surface resembles a horse's saddle. It has concave shape along one dimension and a convex shape along the other.

PROBLEMS

30. **INTERPRET** This problem is about static equilibrium. We want to find that tension force the left-hand bolt must withstand to keep the traffic signal system upright.

DEVELOP The forces on the traffic signal structure, and their lever arms about the pivot point P, are shown in the figure. We have chosen point P at the right edge of the base plate because, if the structure is just in equilibrium, the normal force will be applied at this point and the bolt on this point will supply zero tensile force.



Note that each lever arm is just the perpendicular distance between the line on which the force is applied and the point of rotation (this is given by the factor $\sin\theta$ in the cross product for torque, see Equation 10.10). The two conditions for equilibrium are (1) that the net force be zero and (2) that the net torque be zero. The former condition gives

$$0=\sum\vec{F}=n-T-m_Pg-m_Bg-m_{SL}g$$

where the subscripts P, B, and SL refer to the pole, bar, and stoplight, respectively. The second condition leads to

$$0=\left(\sum\tau\right)_P=r_TT+r_Pm_Pg-r_Bm_Bg-r_{SL}m_{SL}g$$

where we have defined the out-of-page torque as positive, and r_T = 0.76 m, r_B = 3.60 m − 0.76/2 m = 3.22 m, and r_{SL} = 8.22 m − 0.76/2 m = 7.84 m. The second condition is sufficient to solve for T, which is the tension that the left-hand bolt must withstand to keep the structure in equilibrium.

EVALUATE Solving condition 2 for T and inserting the given quantities gives

$$\begin{aligned}T&=\frac{-r_Pm_Pg+r_Bm_Bg+r_{SL}m_{SL}g}{r_T}\\&=\frac{-(0.38\text{ m})(321\text{ kg})+(3.22\text{ m})(175\text{ kg})+(7.84\text{ m})(64.7\text{ kg})}{0.76\text{ m}}\left(9.8\text{ m/s}^2\right)=12.2\text{ kN}\end{aligned}$$

to three significant figures.

ASSESS From condition 1 for equilibrium, we can find the normal force. The result is

$$n=T+m_Pg+m_Bg+m_{SL}g=12234\text{ N}+(321\text{ kg}+175\text{ kg}+64.7\text{ kg})\left(9.8\text{ m/s}^2\right)=17.7\text{ kN}$$

Thus, the normal force is greater than the weight of the structure because it must not only prevent the structure from falling in the gravitational field, but it must also prevent it from rotating.

31. **INTERPRET** This problem involves finding the torque about a pivot point given the forces and the positions at which they are applied. This is also a problem concerning equilibrium, in which the net force and torque on an object must be zero. We can use this to find the force required of the deltoid muscle to keep the arm in equilibrium.

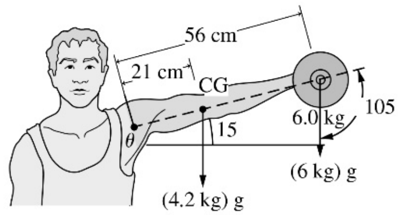
DEVELOP The figure below shows the forces involved and where they are applied. To find the torque about the shoulder due to the arm and the 6-kg mass, we sum the torques. This gives

$$\tau_a=r_{cm}m_{arm}g\sin\theta+r_{arm}mg\sin\theta$$

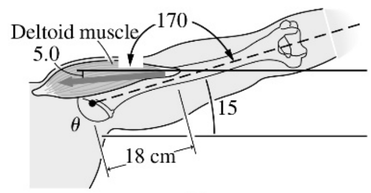
where r_{cm} = 21 cm, r_{arm} = 56 cm , m_{arm} = 4.2 kg, m = 6.0 kg, and θ = 105°. To find the tensile force required of the deltoid muscle, we require that all the torques (including now the torque due to the deltoid muscle) sum to zero (this is the condition for equilibrium, see Equation 12.2). This gives

$$\left(\sum \tau\right)_{\text{shoulder}} = 0 = r_{\text{d}} T \sin\theta_{\text{d}} + r_{\text{cm}} m_{\text{arm}} g \sin\theta + r_{\text{arm}} mg \sin\theta = r_{\text{d}} T \sin\theta_{\text{d}} + \tau_{\text{a}}$$

where θ_d = 170° and r_d = 18 cm. We can solve this for the tensile force T of the deltoid muscle.



(a)



(b)

EVALUATE (a) The torque due to the arm and the mass is

$\tau_{\text{a}} = r_{\text{cm}} m_{\text{arm}} g \sin\theta + r_{\text{arm}} mg \sin\theta = \left[(0.21\text{ m/s})(4.2\text{ kg}) + (0.56\text{ m})(6.0\text{ kg})\right]\left(9.8\text{ m/s}^2\right)\sin\left(255^\circ\right) = 4.0\times10^1\text{ N}\cdot\text{m}$ to two significant figures.

(b) The magnitude of the tensile force supplied by the deltoid is thus

$$T = \frac{\tau_{\text{a}}}{r_{\text{d}} \sin\theta_{\text{d}}} = \frac{40.2\text{ N}\cdot\text{m}}{(0.18\text{ m})\sin\left(170^\circ\right)} = 1.3\text{ kN}$$

ASSESS By the right-hand rule, we see that the torque supplied by the deltoid muscle is out of the page, whereas the torque due to the arm and the mass is into the page. Thus, the deltoid muscle must supply a force that is (1.3 kN)/[(9.8 m/s2)(6.0 kg)] = 22 times weight in order to lift it. This is not very efficient, and it underscores the comment at the end of Example 12.3 that the musculoskeletal structure of the human extremities evolved for speed and range of motion, not mechanical advantage.

32. **INTERPRET** The problem is about static equilibrium. The sphere is supported by a rope attached to the wall. The friction between the sphere and the wall helps keep it in equilibrium. We want to find the smallest possible value for the coefficient of friction.

DEVELOP In equilibrium, there is no net force acting on the sphere. In addition, the sum of the torques about the center of the sphere must be zero, so the frictional force is up, as shown. Using Equations 12.1 and 12.2, the equilibrium conditions can be written as

$$\sum F_x = n - T\sin\theta = 0$$
$$\sum F_y = T\cos\theta + f - Mg = 0$$
$$\left(\sum \tau\right)_c = Rf - \left(\frac{R}{2}\right)T\cos\theta = 0$$

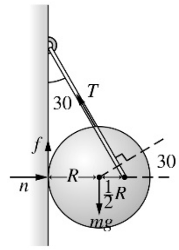
The above equations can be used to solve for the frictional force f. The coefficient of friction can then be obtained from $f \le \mu_s n$.

EVALUATE The x-component of the force equation gives $T = n/\sin\theta$. Substituting this into the torque equation gives

$$f = \frac{1}{2}T\cos\theta = \frac{n}{2}\frac{\cos\theta}{\sin\theta} = \frac{n}{2}\cot\theta$$

Since $f \le \mu_s n$, the minimum coefficient of friction is

$$\mu_s = \frac{1}{2}\cot\theta = \frac{1}{2}\cot 30° = \frac{\sqrt{3}}{2} = 0.866$$



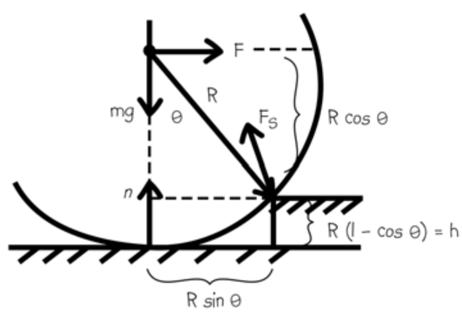
ASSESS If the angle θ is increased, then the minimum coefficient of friction would decrease. Note that μ_s is independent of the mass and radius of the sphere; it depends only on the angle θ.

33. **INTERPRET** You need to specify the minimum horizontal force needed to push the cart over the step. The cart will be in static equilibrium until the force is sufficient to overcome the obstacle.

DEVELOP The forces that you need to account for are: the force, F, exerted by the person; the weight of the cart, mg; the normal force, n, between the wheel and the ground; and the force from the step, F_s. You can assume that the person is only pushing and not lifting the cart in any way. In any case, the person's force and the cart's weight are both exerted on the wheel at the wheel's axle. See the figure below. Since you don't know the magnitude or the direction of the step force, you should choose the point where the wheel meets the step as the pivot point. Before the cart starts moving, the wheel is in static equilibrium, so the sum of the torques around this pivot point should be zero:

$$\left(\sum \tau\right)_{\text{step}} = mgR\sin\theta - nR\sin\theta - FR\cos\theta = 0$$

where we have used the trig identity: $= \cos\theta$.



As the person pushes harder, more of the cart's weight shifts from the ground (supported by n) to the step (supported by F_s). Eventually, the normal force will go to zero, and the wheel will rotate around the point where it meets the step. Therefore, the minimum force needed to push the cart over the step is that which makes $n = 0$:

$$mgR\sin\theta - FR\cos\theta = 0 \rightarrow F = mg\tan\theta$$

EVALUATE To find the minimum force, we need to find the angle θ. We know that the step height is equal to: $h = R(1-\cos\theta)$. So

$$\theta = \cos^{-1}\left[1 - \frac{h}{R}\right] = \cos^{-1}\left[1 - \frac{8\text{ cm}}{\frac{1}{2}\cdot 60\text{ cm}}\right] = 42.83^{\circ}$$

Plugging this into the force equation from above:

$$F = mg\tan\theta = (65\text{ kg})\left(9.8\text{ m/s}^2\right)\tan\left(42.83^{\circ}\right) = 590\text{ N}$$

ASSESS The person is essentially using the wheel as a lever arm to lift the loaded cart up onto the step. The wider the wheel, the longer the lever arm. This is reflected in the fact that a larger R results in a smaller θ and a smaller F.

34. **INTERPRET** In this problem, we want to find the tension in the Achilles tendon and the contact force at the ankle joint when the foot is in static equilibrium

DEVELOP If we approximate the bones in the foot as a massless, planar, rigid body, the equilibrium conditions for the situation depicted in Fig. 12.20 are:

$$0 = \sum F_x = T\sin\theta - F_{C,x}$$
$$0 = \sum F_y = T\cos\theta + n - F_{C,y}$$
$$0 = \left(\sum \tau\right)_{\text{ankle joint}} = n(12.5\text{ cm}) - (T\cos\theta)(5.25\text{ cm})$$

where $\theta = 25^\circ$.

EVALUATE (a) We first note that the normal force is simply equal to the weight of the person, $n = 735$ N. Substituting this into the torque equation, the tension in the Achilles tendon is

$$T = \frac{n(12.5\text{ cm})}{(5.25\text{ cm})\cos\theta} = \frac{(735\text{ N})(12.5\text{ cm})}{(5.25\text{ cm})\cos 25^\circ} = 1.93\times 10^3\text{ N}$$

(b) Substituting the value of T into the force equations, we find

$$F_{C,x} = T\sin\theta = (1.93\times 10^3\text{ N})\sin 25^\circ = 816\text{ N}$$
$$F_{C,y} = T\cos\theta + n = (1.93\times 10^3\text{ N})\cos 25^\circ + 735\text{ N} = 2.49\times 10^3\text{ N}$$

Therefore, the contact force at the ankle joint is

$$F_C = \sqrt{F_{C,x}^2 + F_{C,y}^2} = \sqrt{(816\text{ N})^2 + (2.49\times 10^3\text{ N})^2} = 2.62\times 10^3\text{ N}$$

ASSESS The tension in the Achilles tendon is about twice the weight of the person. This is because the Achilles tendon is very close to the ankle joint, leading to a small lever arm. The contact force at the ankle is roughly three times the weight of the person. The problem demonstrates that in order to maintain a static equilibrium, many parts of our body often experience forces that are greater than our own weight.

35. **INTERPRET** This problem involves calculating the force required to maintain a leaning ladder in equilibrium. The ladder experiences forces due to gravity (acting on the ladder and on the person climbing the ladder), a normal force due to the frictionless wall against which it is leaning, and forces due to the floor (friction and normal force). Using the conditions for equilibrium (i.e., sum of the forces and torques must be zero), we are asked to find how high a person of a given mass may climb the ladder before the ladder slips.

DEVELOP Draw a sketch of the ladder that includes all the forces acting on it (see figure below). For the ladder to remain in equilibrium, the forces on it must sum to zero. Summing the forces in the *x*- and *y*-directions, this condition leads to

$$\sum F_x = 0 = f_s - F_{\text{wall}} = \mu_s n - F_{\text{wall}}$$
$$\sum F_y = 0 = n - m_L g - mg$$

where *m* is the mass of the person climbing the ladder, and we have expressed the force due to static friction as $f_s = m_s n$. To remain in equilibrium, the torques on the ladder must also sum to zero, which gives

$$\left(\sum\tau\right)_A = m_L g\frac{L}{2}\overbrace{\sin(\pi-\theta)}^{\sin(\theta)} + mgr\overbrace{\sin(\pi-\theta)}^{=\sin(\theta)} - F_{\text{wall}}L\overbrace{\sin\left(\frac{\pi}{2}+\theta\right)}^{=\cos(\theta)}$$
$$= \frac{m_L g}{2}\sin(\theta) + mg\alpha\sin(\theta) - F_{\text{wall}}\cos(\theta)$$

where $\alpha = L/r$ is the position of the person on the ladder, expressed in units of ladder lengths. For the ladder to not slip, the force applied by the wall must be less than or equal to the force due to friction:

$$F_{wall} \le f_s$$

Using the equations we obtained by imposing the conditions for equilibrium, we obtain

$$\left(\frac{m_L g}{2}+mg\alpha\right)\tan(\theta)\le\mu_s n=\mu_s\left(m_L g+mg\right)$$

$$\alpha\le\frac{\mu_s}{m}\left(m_L+m\right)\cot\theta-\frac{m_L}{2m}$$

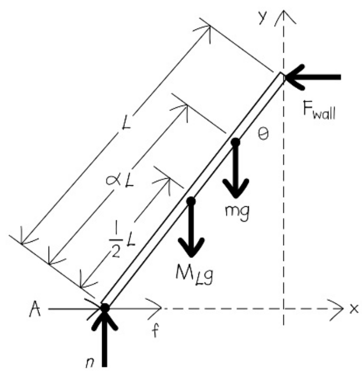
The maximum value of α is 1 (this corresponds to the person climbing to the top of the ladder, so $\alpha = L/L = 1$), so if the right-hand side of the expression above is less than unity, the person can climb to the top of the ladder without it slipping. Explicitly, the condition for the person to be able to climb to the top of the ladder is

$$1\le\frac{\mu_s}{m}\left(m_L+m\right)\cot\theta-\frac{m_L}{2m}$$

The most massive person that can climb to the top of the ladder may be found by setting $\alpha = 1$ and solving for the mass m. This gives

$$1=\frac{\mu_s}{m}\left(m_L+m\right)\cot\theta-\frac{m_L}{2m}$$

$$m=\frac{m_L\left(\mu_s\cot\theta-1/2\right)}{1-\mu_s\cot\theta}$$



EVALUATE Inserting the values given, we find

$$1\le\frac{\mu_s}{m}\left(m_L+m\right)\cot\theta-\frac{m_L}{2m}=\frac{0.26}{65\text{ kg}}\left(5.0\text{ kg}+65\text{ kg}\right)\cot\left(15°\right)-\frac{5.0\text{ kg}}{2\left(65\text{ kg}\right)}=1.0$$

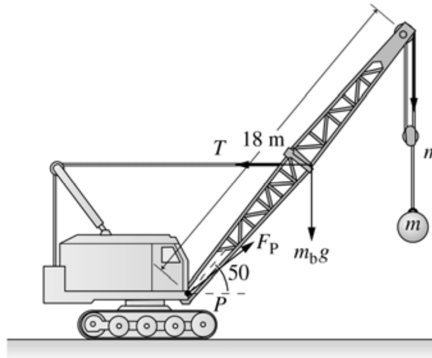
to two significant figures. Thus, the inequality is satisfied, so the 65-kg person can climb to the top of the ladder without it slipping. The most massive person that can climb the ladder is

$$m=\frac{m_L\left(\mu_s\cot\theta-1/2\right)}{1-\mu_s\cot\theta}=\frac{\left(5.0\text{ kg}\right)\left[\left(0.26\right)\cot\left(15°\right)-0.50\right]}{1-\left(0.26\right)\cot\left(15°\right)}=79\text{ kg}$$

ASSESS We see that if the ladder is more massive, a more massive person may climb it. This is because the more massive ladder will generate a greater force due to friction on the floor. If $\mu_s\to 0$, then $m\to 0$, as expected.

36. **INTERPRET** In this problem, we want to find the tension in the cable supporting the boom so that the boom is in static equilibrium. We will thus apply the condition for equilibrium, which is that the sum of the forces and torques on the boom must be zero.

DEVELOP For the boom to remain in static equilibrium, the torques must satisfy Equation 12.2 (i.e., sum to zero). Since we are only asked about the tension, we can focus on the torque about point P. The forces on the boom are shown superposed on the figure below. By assumption, T is horizontal and acts at the center of mass of the boom. To find T, we compute the torques about P.



EVALUATE The condition for equilibrium implies that $(\Sigma\tau)_P = 0$, or

$$T(L/2)\sin\theta - m_b g(L/2)\cos\theta - mgL\cos\theta = 0$$

which can be solved to give

$$T = (2m + m_b)g\cot\theta = (4400\text{ kg} + 1750\text{ kg})(9.8\text{ m/s}^2)\cot(50°) = 5.1\times10^4\text{ N}$$

ASSESS To see that our result makes sense, let's consider the following cases: **(i)** $\theta = 90°$: In this limit, the boom is vertical and the tension in the cable vanishes $[\cot(90°) = 0]$, as expected. **(ii)** $m = 0$: When no mass is hanging from the end of the boom, the tension in the cable reduces to $T = m_b g\cot\theta$. This comes from the force equations at P (with $m = 0$):

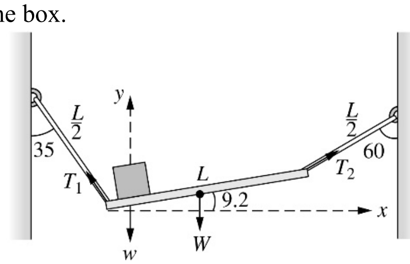
$$0 = \sum F_x = F_P\cos\theta - T\sin\theta$$
$$0 = \sum F_y = F_P\sin\theta - m_b g$$

37. **INTERPRET** The problem involves static equilibrium, so we can apply the condition that the sum of the forces and torques on the board must be zero.

DEVELOP Sketch the board, showing all the forces acting on it and the position at which the forces are applied (see figure below). The conditions for equilibrium (about the origin drawn on the figure) are:

$$\sum F_x = 0 = T_2\sin(60°) - T_1\sin(35°)$$
$$\sum F_y = 0 = T_2\cos(60°) + T_1\cos(35°) - w - W$$
$$\left(\sum\tau\right)_0 = 0 = T_2 L\sin(20.8°) - WL\sin(99.2°)/2$$

which we can solve for the weight w of the box.



EVALUATE From the condition of zero torque, we find that

$$T_2 = \frac{W\sin(99.2°)}{2\sin(20.8°)} = (1.39)W$$

Inserting this result into the x-component of the zero-force condition gives

$$T_1 = \frac{T_2\sin(60°)}{\sin(35°)} = \frac{(1.39)W\sin(60°)}{\sin 35°} = (2.10)W$$

Inserting both these results into the y component of the zero-force condition gives

$$w = T_2\cos(60°) + T_1\cos(35°) - W = W\left[(1.39)\cos(60°) + 2.10\cos(35°) - 1\right] = 1.4\text{ W}$$

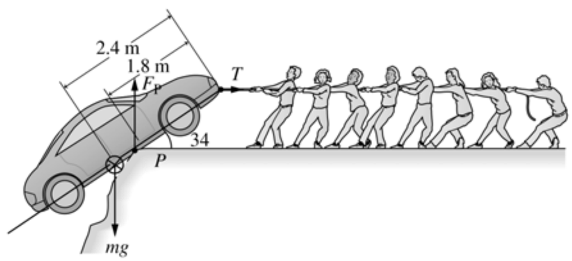
ASSESS Drawing an accurate sketch is helpful for this problem.

38. **INTERPRET** In this problem, we want to find the force that people apply to pull the car so that it is in static equilibrium. The applied force is equal to the tension in the rope. Because the car is in equilibrium, we know that the forces and torques acting on the car sum to zero.

DEVELOP For the car to remain in static equilibrium, the torques must satisfy Equation 12.2. Three forces act on the car: the tensile force of the rope, the force due to gravity, and the force exerted by the embankment (see the figure below). If we evaluate the torques about point P, the unknown force F_p exerted by the edge of the embankment does not contribute, so the tension necessary to keep the car in equilibrium can be found directly (i.e., without needing to invoke the zero-force condition of Equation 12.1). Thus, the zero-torque condition for equilibrium gives

$$\left(\sum \vec{\tau}\right)_{\mathrm{P}} = 0 = \vec{r}_{\mathrm{cm}} \times m_{\mathrm{car}}\vec{g} + \vec{r}_{\mathrm{rope}} \times \vec{T}$$
$$= m_{\mathrm{car}} g(L-l)\sin(\pi/2-\theta) - Tl\sin\theta$$

where $L = 2.4$ m, $l = 1.8$ m, and $\theta = 34°$.



EVALUATE Inserting the given quantities gives

$$T = m_{\mathrm{car}} g\left(\frac{L-l}{l}\right)\cot\theta$$
$$= (1270 \text{ kg})(9.8 \text{ m/s}^2)\left(\frac{2.4 \text{ m} - 1.8 \text{ m}}{1.8 \text{ m}}\right)\cot(34°)$$
$$= 6.2 \text{ kN}$$

ASSESS Note that if $l \geq L$ (when the center of mass lies above the edge of the embankment), then the tension must be negative to maintain equilibrium. Because it is not possible to apply a negative tensile force, the car would not remain in equilibrium but would tip forward onto the embankment, as expected.

39. **INTERPRET** This problem is about maintaining a ladder in static equilibrium. The ladder leans against a wall, as in Example 12.2, but the wall now has friction. Thus, in addition to the forces listed in Example 12.2, we must add the force due to static friction at the top of the ladder. This force will resist motion, so it is directed upward.

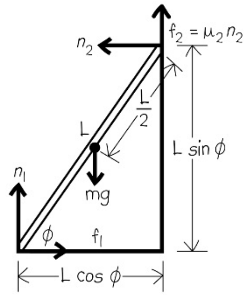
DEVELOP Make a sketch of the situation that shows all the forces acting on the ladder (see figure below). The force f_2 due to static friction at the top of the ladder is given by Equation 5.2, $f_2 \leq \mu_{2,s} n_2$. As per Example 12.2, we apply the two conditions for static equilibrium (i.e., zero net force and zero net torque on the object). The net force in the x-direction has not changed, but the net force in the y-direction has changed due to the addition of f_2. Thus, the net force equilibrium equations read

$$\text{Force, } x \quad \mu_1 n_1 - n_2 = 0$$
$$\text{Force, } y \quad n_1 - mg + f_2 = 0$$

Similarly, the equilibrium equation now reads

$$Ln_2 \sin\phi - \frac{L}{2} mg\cos\phi + Lf_2 \overbrace{\sin(\pi/2-\phi)}^{=\cos\phi} = 0$$
$$Ln_2 \sin\phi - \frac{L}{2} mg\cos\phi + Lf_2 \cos\phi = 0$$

where the last term is due to the torque by the new friction force. Using the inequalities $f_2 \le \mu_2 n_2$ and $f_1 \le \mu_1 n_1$ we can now find the minimum angle ϕ at which the ladder can be positioned without slipping.



EVALUATE We will take the maximum values for the friction forces, which will give the maximum angle ϕ at which the ladder can lean. Solving for ϕ from the torque equation, we find

$$\tan\phi = \frac{mg - 2f_2}{2n_2} = \frac{mg - 2\mu_2 n_2}{2n_2} = \frac{mg}{2n_2} - \mu_2$$

From the horizontal force equation, we have $\mu_1 n_1 = n_2$. Inserting this into the horizontal equation leads to

$$mg = n_1 + f_2 = n_1 + \mu_2 n_2 = n_1 + \mu_2 \mu_1 n_1 = n_1(1 + \mu_2 \mu_1)$$

Using these two results in the expression for $\tan\phi$ gives

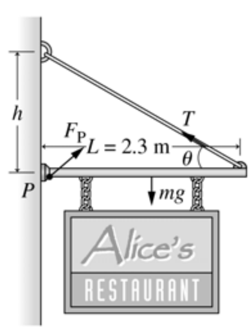
$$\tan\phi = \frac{n_1(1+\mu_2\mu_1)}{2\mu_1 n_1} - \mu_2 = \frac{(1+\mu_1\mu_2)}{2\mu_1} - \frac{2\mu_1\mu_2}{2\mu_1} = \frac{1-\mu_1\mu_2}{2\mu_1}$$

$$\phi = \operatorname{atan}\left[\frac{1-\mu_1\mu_2}{2\mu_1}\right]$$

ASSESS Does this make sense? The units of the argument for atan are dimensionless, as they should be. If we let $\mu_1 \to 0$, then $\phi \to 90^\circ$, meaning the ladder can only remain in equilibrium if it is vertical. Letting $\mu_2 \to 0$, we find the result of Example 12.2, as expected. Note that this problem involves four forces, n_1, n_2, f_1, and f_2, while there are only three equations (involving two forces and one torque). Problems like this where there are more unknowns than equations are called indeterminate. To solve such types of problems, we typically supplement the equilibrium equations with equation involving elasticity.

40. **INTERPRET** This problem is about static equilibrium. The hanging sign is being supported by a rod and a cable. Given the maximum tension in the cable, we want to find the minimum height above the pivot for anchoring the cable to the wall so that the sign remains in static equilibrium.

DEVELOP Suppose that the sign is centered on the rod so that its center of mass lies under the center of the rod. Then the total weight of the rod and the sign, $M = 51\text{ kg} + 8.0\text{ kg} = 59\text{ kg}$, may be considered to act through the center of the rod, as shown below. For the sign to remain in static equilibrium, the forces and the torques must satisfy Equations 12.1 and 12.2. In this situation, since we are only interested in the minimum height h, and not F_p, it is sufficient to focus only on the torques about point P.



EVALUATE In equilibrium, we have $(\Sigma\tau)_P = 0$. Note that the pivot force F_p makes no contribution to the torque about P because it acts along a line through the pivot point. The equilibrium condition leads to

$$0=\left(\sum \tau\right)_P = TL\sin\theta - Mg(L/2)$$

Therefore, the tension is

$$T=\frac{Mg}{2\sin\theta}=\frac{1}{2}Mg\sqrt{1+\frac{L^2}{h^2}}$$

where we have used $\tan\theta h/L$ and the identity $1+\cot^2\theta=\csc^2\theta.$ Solving for *h*, we obtain

$$h=L\left[\left(\frac{2T}{Mg}\right)^2-1\right]^{-\frac{1}{2}}$$

Given the maximum tension T_{max} that could be withstood in the cable, the condition that must be met by *h* for the sign to remain in static equilibrium is

$$h\ge L\left[\left(\frac{2T_{max}}{Mg}\right)^2-1\right]^{-1/2}=(2.3\text{ m})\left[\left(\frac{2(755\text{ N})}{(51\text{ kg}+8\text{ kg})(9.8\text{ m/s}^2)}\right)^2-1\right]^{-1/2}=0.95\text{ m}$$

So the minimum height is $h_{min}=0.95$ m.

ASSESS If the maximum tension were $T_{max}<Mg/2,$ static equilibrium would not be possible!

41. **INTERPRET** In this problem we want to determine the energy needed to bring the pipe from a metastable equilibrium state to an adjacent unstable equilibrium state.

DEVELOP We can identify the energy involved in changing the equilibrium state to be the gravitational potential energy. There will be work done on the pipe to change its position from the metastable to the adjacent unstable equilibrium state, which is generated when the pipe is balanced on the tip of its bottom surface. This can be calculated by finding the change in the gravitational potential of the center of mass of the pipe as it goes from one state to the other. While it is in the metastable state, the center of mass is located halfway up its vertical length L and at the center of its circular cross section of diameter D. Setting the zero of the potential energy to be the surface upon which the pipe originally rests on results in the initial potential energy $U_i = MgL/2$. Once the pipe has been shifted to the unstable state, the center of mass is located halfway along a diagonal that goes from the bottom of one edge to the opposite top edge. The length of this diagonal line is equal to $d=\sqrt{L^2+D^2}$, and thus the final configuration's potential energy is given by $U_f = Mgd/2$.

EVALUATE Taking the difference in potential energies results in the energy needed to bring the pipe into the unstable equilibrium state, which we find is equal to

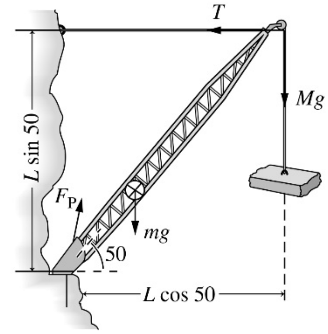
$$E=U_f-U_i=\frac{1}{2}Mg\left(\sqrt{L^2+D^2}-L\right)$$

ASSESS This state is very unstable, and can easily shift to the more stable equilibrium state found when the pipe lies flat on its side.

42. **INTERPRET** This problem involves finding the tension required to maintain the crane in static equilibrium. To find the tension, we can apply the two conditions of static equilibrium, which are that the forces and torques on the crane must sum to zero.

DEVELOP Make a diagram of the situation that shows the forces and their positions at which they act (see figure below). As in Problem 35, the equilibrium condition for torques about the pivot does not contain the unknown pivot force, and thus allows the tension to be directly determined without use of the force equations. Thus, the condition for zero torque gives

$$\begin{aligned}\left(\sum\tau\right)_P &= 0 = TL\overbrace{\sin(\pi-\theta)}^{=\sin\theta} - MgL\overbrace{\sin(\pi/2+\theta)}^{=\cos\theta} - \frac{mgL}{3}\overbrace{\sin(\pi/2+\theta)}^{=\cos\theta}\\ &= TL\sin\theta - MgL\cos\theta - \frac{mgL}{3}\cos\theta\end{aligned}$$



EVALUATE Solving for the tension and inserting the given quantities gives

$$T = \left(M + \frac{m}{3}\right) g \cot(50°) = \left(2500\text{ kg} + \frac{830\text{ kg}}{3}\right)(9.8\text{ m/s}^2)\cot(50°) = 23\text{ kN}$$

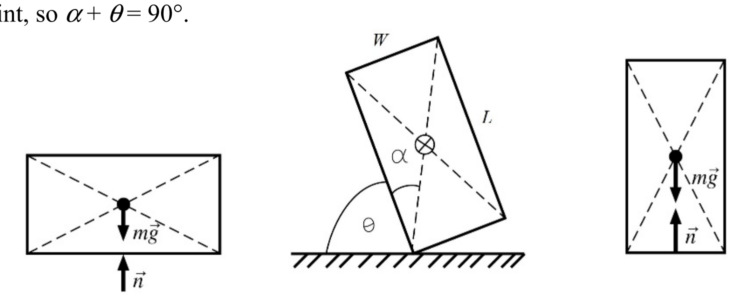
ASSESS Thus the cable must support a force slightly less than that of the hanging mass (= 2500 kg × 9.8 m/s^2 = 24.5 kN).

43. **INTERPRET** In this problem we are given a block that is balancing on its corner so that its long side makes an angle θ with the horizontal, and we want to find the values of θ for which the block is in equilibrium. We are to comment on whether these are stable or unstable equilibria.

DEVELOP Let the block be tilted in a plane perpendicular to its thickness, as shown in the figure below. The two conditions for equilibrium (see Equations 12.1 and 12.2) are that (1) there is zero net force on the block and (2) there is zero net torque on the block:

$$\sum F = 0$$
$$\left(\sum \tau\right)_P = 0$$

If the block is lying on its long side we can consider the normal force from the floor to act directly under the center of gravity, as shown in the sketch (because the block is uniform, so the normal force over the entire surface may be replaced by a sum of the normal forces acting at the geometric center of the block). This force cancels the force due to gravity. Furthermore, there is no torque on the block since the two forces (normal and gravity) are antiparallel. If the block is lying on its short side, the situation is the same, except that it would be easier to destabilize the block because the pivot point is closer to the line along which gravity acts. If the block is balanced on its corner, as in the middle sketch, then the zero-torque condition demands that the force due to gravity act through the pivot point, so $\alpha + \theta = 90°$.



EVALUATE The condition $\alpha + \theta = 90°$ is an unstable equilibrium. The angle θ may be found by noting that $\tan\alpha = W/L$, so

$$\theta = 90° - \tan^{-1}\left(\frac{W}{L}\right)$$

The expression can be rewritten as $\tan^{-1}\left(\frac{W}{L}\right) = 90° - \theta$, or $\frac{W}{L} = \tan(90° - \theta) = \cot\theta$. Thus, the equivalent condition is $\tan\theta = \frac{L}{W}$, or $\theta = \tan^{-1}\left(\frac{L}{W}\right)$.

ASSESS One may also show that the general expression for the torque about the pivot point is given by

$$\tau = r_{\perp}F = \frac{mg}{2}(L\cos\theta - W\sin\theta)$$

Thus, the condition that $\tau = 0$ implies that

$$\tan\theta = \frac{L}{W}$$

44. **INTERPRET** This problem involves finding the equilibrium points of a particle given its potential energy function. From the discussion in Chapter 12 on stability, we know that the equilibria will occur where the potential energy function is flat (i.e., has zero derivative), and that an equilibrium will be stable (unstable) if, at the equilibrium position, the second derivative is positive (negative).

DEVELOP The equilibrium condition $dU/dx = 0$ (Equation 12.3) leads to

$$3(x/x_0)^2 + 2a(x/x_0) + 4 = 0$$
$$\frac{x}{x_0} = \frac{-2a \pm 2\sqrt{a^2 - 12}}{6}$$

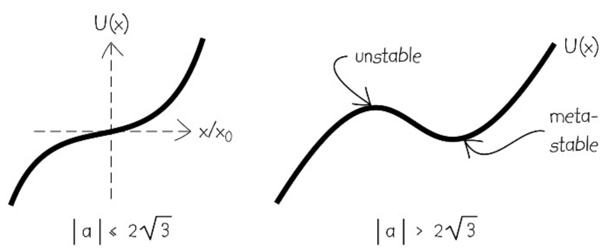
This quadratic has two real roots if the discriminant is positive (i.e., $a^2 - 12 > 0$, or $|a| > 2\sqrt{3}$)

EVALUATE The equilibria are $(x/x_0)_{\pm} = \frac{1}{3}\left(-a \pm \sqrt{a^2 - 12}\right)$. The second derivative of the potential energy, evaluated at these roots, is

$$\left(\frac{d^2U}{dx^2}\right)_{\pm} = \frac{U_0}{x_0^2}\left[6\left(\frac{x}{x_0}\right)_{\pm} + 2a\right] = \pm 2\sqrt{a^2 - 12}\left(\frac{U_0}{x_0^2}\right)$$

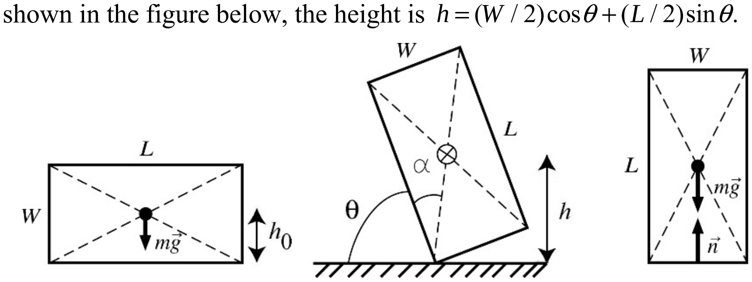
Thus, the positive root is a position of metastable equilibrium (see Equation 12.4), whereas the "minus" root represents unstable equilibrium (Equation 12.5).

ASSESS A plot of the potential energy, which is a cubic, will clarify these remarks. For $|a| > 2\sqrt{3}$ $U(x)$ has no wiggles, as shown. (U passes through the origin, but its position depends on the value of a, and is not shown.)



45. **INTERPRET** This problem involves finding the unstable equilibrium point of a $w\times w\times L$ rectangular block, given its potential energy as a function of tilt angle θ.

DEVELOP From the discussion in Chapter 12 on stability, we know that the equilibria will occur where the potential energy function is flat (i.e., has zero derivative), and that an equilibrium will be unstable if, at the equilibrium position, the second derivative is negative. Initially when the block is resting on its long side, the height of the center of mass (CM) of the block measured from the surface is $h_0 = W/2$. When the block is tiled at an angle θ, as shown in the figure below, the height is $h = (W/2)\cos\theta + (L/2)\sin\theta$.



EVALUATE **(a)** The potential energy of the block is

$$U(\theta) = mg(h - h_0) = mg\left(\frac{L}{2}\sin\theta + \frac{W}{2}\cos\theta - \frac{W}{2}\right) = \frac{1}{2}mg\left[L\sin\theta - W(1-\cos\theta)\right]$$

(b) Differentiating U with respect to θ gives

$$\frac{dU}{d\theta} = \frac{1}{2}mg\left(L\cos\theta - W\sin\theta\right)$$

The equilibrium condition $dU/d\theta = 0$ (Equation 12.3) leads to $\tan\theta = L/W$, which agrees with that obtained in problem 37.

(c) The second derivative of $U(\theta)$ is

$$\frac{d^2U}{d\theta^2} = -\frac{1}{2}mg\left(L\sin\theta + W\cos\theta\right)$$

At $\theta = 90°$, we have

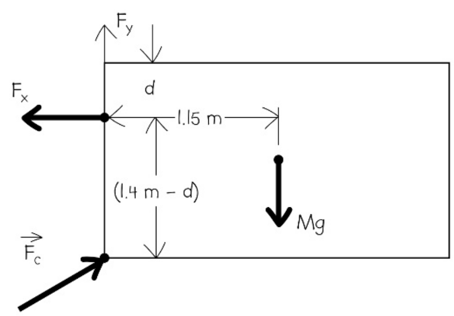
$$\left(\frac{d^2U}{d\theta^2}\right)_{\theta=90°} = -\frac{1}{2}mgL < 0$$

The negative sign implies that the upright position is an unstable equilibrium.

ASSESS When the block is in the upright position (resting on the short side), its potential energy is $U = mgL/2$, which is greater than $U_0 = mgh_0 = mgW/2$.

46. **INTERPRET** We are asked how far down a bolt can be placed on a hanging sign without exerting more than the given amount of force on the bolt. The sign is in static equilibrium.

DEVELOP There are three forces acting on the sign: the force from the bolt, the contact force from the bottom corner, and the weight of the sign. Since the sign has uniform density, the weight acts at the center of the sign, as shown in the figure below. The contact force at the bottom may have a vertical as well as a horizontal component.



We are only interested in the horizontal tension, F_x, provided by the bolt. Therefore it makes sense to choose the bottom corner as the pivot point (that way we can ignore the force acting there). Since the sign is in static equilibrium, the sum of the torques around this point is zero:

$$\sum\tau = Mg\left(1.15\text{ m}\right) - F_x\left(1.4\text{ m} - d\right) = 0$$

EVALUATE Setting the horizontal tension to the maximum the bolt can handle $\left(F_x = 2.1\text{ kN}\right)$, the maximum value of d is:

$$d = 1.4\text{ m} - \frac{Mg}{F_x}\left(1.15\text{ m}\right) = 1.4\text{ m} - \frac{\left(160\text{ kg}\right)\left(9.8\text{ m/s}^2\right)}{\left(2100\text{ N}\right)}\left(1.15\text{ m}\right) = 0.54\text{ m}$$

ASSESS As we would expect, the maximum distance would be longer (i.e., farther down from the top of the sign) if we had a stronger bolt that could withstand a greater amount of horizontal tension.

47. **INTERPRET** In this problem, we want to revisit the static equilibrium of a ladder lying up against a wall that is discussed in Example 12.2, but with a ladder of slightly different mass distribution

DEVELOP We treat this problem as is done in Example 12.2, with the same coordinate system and choosing the bottom of the ladder as the pivot point. This results in the equations:

$$\begin{aligned}
&\text{Force}, x: \quad \mu n_1 - n_2 = 0\\
&\text{Force}, y: \quad n_1 - mg = 0\\
&\text{Torque}: \quad Ln_2 \sin\phi - \frac{1}{3} Lmg\cos\phi = 0
\end{aligned}$$

where the only thing that has changed is the magnitude of the torque applied by the weight of the ladder due to its newly distributed mass.

EVALUATE Like in Example 12.2, we use the force equations to rewrite the expression for the torque and obtain the minimum angle for which the ladder won't slip, given by

$$L\mu mg\sin\phi = \frac{1}{3} Lmg\cos\phi$$

$$\tan\phi = \frac{1}{3\mu} \quad \to \quad \phi = \text{atan}\frac{1}{3\mu}$$

ASSESS The location of the center of mass will not change the net force acting on the ladder, but it will determine the location about which the torque is applied relative to the chosen pivot point.

48. **INTERPRET** In this problem, we want to revisit the static equilibrium of a human arm holding a pumpkin that is discussed in Example 12.3, but with a larger value for the maximum tension tolerable

DEVELOP We treat this problem as is done in Example 12.3, with the same coordinate system and choosing the elbow as the pivot point. This results in the equations:

$$\begin{aligned}
&\text{Force}, x: \quad F_{cx} - T\cos\theta = 0\\
&\text{Force}, y: \quad T\sin\theta - F_{cy} - mg - Mg = 0\\
&\text{Torque}: \quad x_1 T\sin\theta - x_2 mg - x_3 Mg = 0
\end{aligned}$$

EVALUATE Like in Example 12.3, we rewrite the equation for the torque, but in this case, we want to obtain an expression for the mass M of the pumpkin.

$$M = \frac{x_1 T\sin\theta - x_2 mg}{x_3 g}$$

Using the values given in Example 12.3, along with the new value for the maximum tension of $T = 600\text{ N}$, we find the mass of the pumpkin is $M = 5.6\text{ kg}$.

ASSESS A heavier pumpkin will impart a large torque upon the arm holding it, which means that a greater tension tolerance will be necessary to comfortably maintain the static equilibrium of the system.

49. **INTERPRET** We want to find the energy required to bring a cube of side s from a stable to an unstable equilibrium. The problem is equivalent to finding the increase in potential energy of the system. We can consider that the entire mass of the cube is concentrated at its center of gravity, which is its geometric center for a uniform cube in a uniform gravitational field.

DEVELOP When resting in a stable equilibrium position, the center of mass of a uniform cube of side s is at a distance $y_0 = s/2$ above the tabletop. When balancing on a corner, the center of mass is now a distance

$$y = \sqrt{(s/2)^2 + (s/2)^2 + (s/2)^2} = \frac{\sqrt{3}}{2}s$$

above the corner resting on the tabletop.

EVALUATE From the above, the potential energy difference is

$$\Delta U = mg\ \Delta y_{\text{cm}} = mgs\left(\frac{\sqrt{3}}{2} - \frac{1}{2}\right) = 0.366\ mgs$$

This is the energy required to bring the cube to an unstable equilibrium.

ASSESS Raising the vertical distance of the center of mass increases the potential energy of the cube. In general, the stability of a system decreases as its potential energy is increased.

50. **INTERPRET** This problem is similar to Problem 43, except that this time we are dealing with an isosceles triangular block instead of a cube. Thus, we will use the same approach here (i.e., calculate the change in potential energy considering that all the mass is concentrated at the center of mass).

DEVELOP To compare the position of the center of mass for the isosceles block when it's on its base with its position when the isosceles block is on its apex, we first need to calculate the center of mass of the object. See the figure below for a sketch of the isosceles block, which consists of a pyramid with each face being an isosceles triangle with sides of length *b*. The height of the pyramid is *h*. Because the pyramid is symmetric about a vertical line through its apex, we know that the center of mass will lie on this line—we only need to calculate at what height. Creating a volume element (see side view in sketch) allows us to relate an infinitesimal mass element *dm* to the vertical position of that element. The result is

$$\frac{dm}{M} = \frac{dV}{V} = \frac{1}{V}\left(\frac{b}{h}\right)^2 (h-y)^2$$

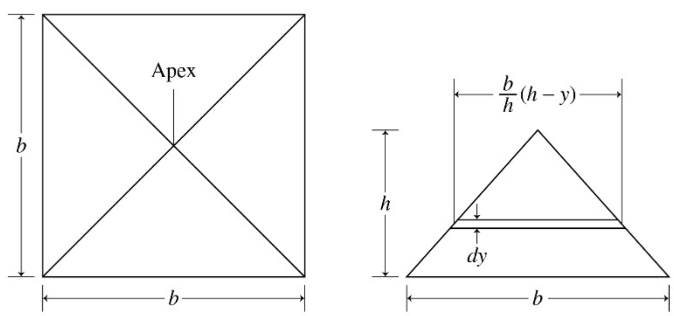
The volume of the isosceles pyramid is

$$V = \int dV = \left(\frac{b}{h}\right)^2 \int_0^h dy(h-y)^2 = \frac{bh^2}{3}$$

so the vertical position of the center of mass is

$$y_{\text{cm}} = \int y dm = \frac{1}{V}\left(\frac{b}{h}\right)^2 \int_0^h dy(h-y)^2 = \frac{h}{4}$$

Using this result, we can calculate the change in potential energy involved when flipping the pyramid from its base onto its apex.



EVALUATE On its base, $y_{\text{cm}} = h/4$, so on its apex, $y_{\text{cm}} = 3h/4$. The difference in potential energy is therefore

$$\Delta U = mg\ \Delta y_{\text{cm}} = mg(3h/4 - h/4) = \frac{mgh}{2}$$

ASSESS For an two-dimensional isosceles triangle, $y_{\text{cm}} = h/2$ (see Example 9.3). Therefore, the energy required to invert it is $\Delta U = mg\left(\frac{2}{3}h - \frac{1}{3}h\right) = \frac{1}{3}mgh.$ Thus, it takes relatively more energy to flip the isosceles block than the two-dimensional isosceles triangle.

51. **INTERPRET** This problem deals with a ladder leaning against a frictionless wall, and we want to verify the condition under which any person (with any mass) can climb to the top, and also the condition in which nobody can climb to the top. The conditions of equilibrium (Equations 12.1 and 12.2) will apply.

DEVELOP The forces on the uniform ladder are shown in the sketch below. The person of mass *m* is positioned on the ladder a fraction α of its total length *L* from the bottom. Equilibrium conditions of zero net force and zero net torque require that

$$0 = \sum F_x = f - F_{\text{wall}}$$
$$0 = \sum F_y = n - (m_L + m)g$$
$$0 = \left(\sum \tau\right)_A = n_w L \overbrace{\sin(\theta + \pi/2)}^{=\cos\theta} - \frac{m_L gL}{2}\overbrace{\sin(\pi - \theta)}^{=\sin\theta} - mg\alpha L\overbrace{\sin(\pi - \theta)}^{=\sin\theta} = n_w L\cos\theta - \frac{m_L gL}{2}\sin\theta - mg\alpha L\sin\infty$$

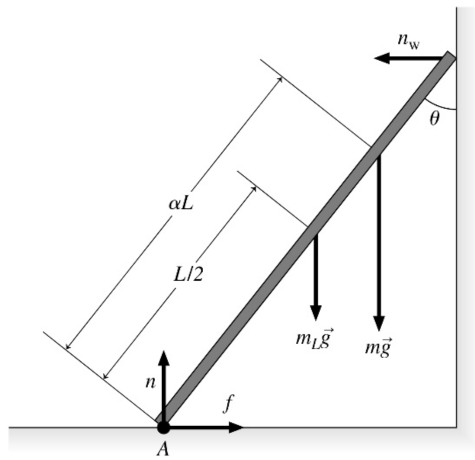
The ladder will not slip if $f \le \mu_s n$. Using the equations above, this condition can be rewritten as

$$f = n_w = \left(\frac{1}{2}m_L + \alpha m\right)g\tan\theta \le \mu_s n = \mu_s(m_L + m)g$$

or

$$\alpha \le \frac{\mu_s(m_L + m)\cot\theta - m_L/2}{m} = \mu_s\cot\theta + \frac{m_L}{m}\left(\mu_s\cot\theta - \frac{1}{2}\right)$$

Here, we used the horizontal force equation to find f, the torque equation to find n_w, and the vertical force equation to find n.



EVALUATE For a person at the top of the ladder, $\alpha = 1$, and the condition for no slipping becomes

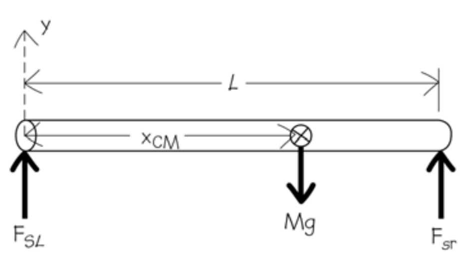
$$m \le m_L\left(\frac{\mu_s\cot\theta - 1/2}{1 - \mu_s\cot\theta}\right) = m_L\left(\frac{\mu_s - \tan\theta/2}{\tan\theta - \mu_s}\right)$$

Since m is positive, this condition cannot be fulfilled if $\mu_s \le \frac{1}{2}\tan\theta$, that is, no one can climb to the top without causing the ladder to slip. However, if $\mu_s = \tan\theta$, the limit is ∞ so anyone can climb to the top.

ASSESS When the coefficient of friction becomes too small, $\mu_s\cot\theta < 1/2$, or $\mu_s < \tan(\theta/2)$ (see Example 12.2), slipping will occur and it is no longer possible for the ladder to remain in static equilibrium. In this situation, nobody can climb up to the top of the ladder without making the ladder slip, regardless of his or her mass.

52. **INTERPRET** The scales in this problem will read what force they must exert on the nonuniform rod to keep it in static equilibrium.

DEVELOP There are three vertical forces acting on the rod: the weight, Mg; the contact force from the scale on the left, F_{sl}; and the contact force from the scale on the right, F_{sr}. The weight acts at the center of mass, x_{cm}, as depicted in the figure below.



As this is a case of static equilibrium, the sum of the vertical forces and the sum of the torques around the left end are both zero:

$$F_{sl} - Mg + F_{sr} = 0$$
$$Mgx_{cm} - F_{sr}L = 0$$

To find the center of mass, we first obtain the total mass of the rod by integrating:

$$M = \int_0^L x\lambda dx = \int_0^L (a + bx)dx = aL + \tfrac{1}{2}bL^2 = \left(1.0\tfrac{\text{kg}}{\text{m}}\right)(2.0\text{ m}) + \tfrac{1}{2}\left(2.0\tfrac{\text{kg}}{\text{m}^2}\right)(2.0\text{ m})^2 = 6\text{ kg}$$

Plugging this into Equation 9.4 for the center of mass gives:

$$x_{cm} = \frac{1}{M}\int_0^L x\lambda dx = \frac{1}{M}\int_0^L \left(ax + bx^2\right)dx = \frac{\frac{1}{2}aL^2 + \frac{1}{3}bL^3}{M} = 0.611L = 1.22\text{ m}$$

EVALUATE From the torque equation, we find the force exerted by the right scale:

$$F_{sr} = \frac{Mgx_{cm}}{L} g\left(\frac{1}{2}aL + \frac{1}{3}bL^2\right) = g\left[\frac{1}{2}\left(1.0\frac{\text{kg}}{\text{m}}\right)(2.0\text{ m}) + \frac{1}{3}\left(2.0\frac{\text{kg}}{\text{m}^2}\right)(2.0\text{ m})^2\right] = 36\text{ N}$$

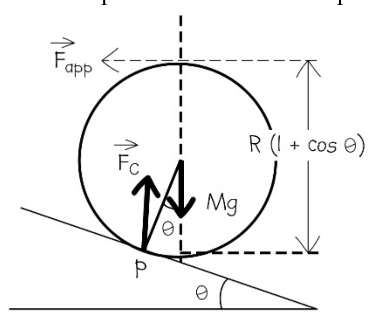
Plugging this into the force equation and solving for the force exerted by the left scale gives:

$$F_{sl} = Mg - F_{sr} = g\left(\frac{1}{2}aL + \frac{1}{6}bL^2\right) = g\left[\frac{1}{2}\left(1.0\frac{\text{kg}}{\text{m}}\right)(2.0\text{ m}) + \frac{1}{6}\left(2.0\frac{\text{kg}}{\text{m}^2}\right)(2.0\text{ m})^2\right] = 23\text{ N}$$

ASSESS The two forces added together give the weight, as they should. The right scale reads a greater weight since the rod gets increasingly denser from left to right. This is also why the center of mass is closer to the right-side end.

53. **INTERPRET** In this problem a wheel has been placed on a slope. We want to apply a horizontal force at its highest point to keep it from rolling down. We will apply the conditions for static equilibrium to solve this problem; namely that the sum of the forces on the wheel must be zero and the sum of the torques on the wheel must be zero.

DEVELOP Consider the conditions for static equilibrium of the wheel, under the action of the forces shown in the sketch below. Here F_{app} is the applied horizontal force, F_c is the contact force of the incline (normal plus friction), and we assume that the center of mass is at the geometric center of the wheel. For the wheel to remain in static equilibrium, the forces must satisfy Equation 12.1. Our plan is to compute the torques about P using Equation 12.2. Note that the contact force, F_c does not create a torque because it acts at the pivot point so it has no lever arm.



EVALUATE The torques about the point of contact sum to zero, or

$$0 = \left(\sum \tau\right)_P = F_{app}R(1 + \cos\theta) - MgR\sin\theta$$

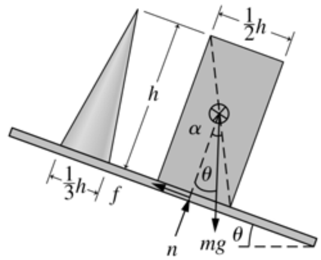
Therefore, the applied force is

$$F_{app} = Mg\frac{\sin\theta}{1 + \cos\theta} = Mg\tan\left(\frac{\theta}{2}\right)$$

ASSESS The applied force vanishes when $\theta = 0$ (flat surface), and becomes maximum when $\theta = 90°$. In this limit, $F_{app} = Mg$ and points vertically upward.

54. **INTERPRET** This problem asks us to find whether a rectangular block (the solid cone is for Problem 56) will tip over or slide as we increase the angle of incline. We are given the coefficient of static friction, so we can calculate all the forces acting on the block and use the conditions that the forces and the torques must sum to zero for an object to remain in equilibrium

DEVELOP Suppose that the block is oriented with two sides parallel to the direction of the incline and that its center of mass is at the center. Make a sketch of the situation showing all the forces acting on the block (see below). From the condition for equilibrium that the forces parallel to the incline must sum to zero, we find the condition for sliding is $mg\sin\theta > f_s^{\max} = \mu_s n = \mu_s mg\cos\theta,$ where we have applied the condition that the forces perpendicular to the incline must sum to zero to find $n = mg\cos\theta.$ This leads to the condition $\tan\theta > \mu_s$ for sliding to occur. The condition for tipping is that the center of mass pass over to the right of the lower corner of the block (see sketch). At this point, $\theta > \alpha,$ where $\alpha = \text{atan}(w/h)$ is the diagonal angle of the block.



EVALUATE For $\mu_s = 0.46,$ the sliding condition is $\theta > \text{atan}(0.46) = 24.7^\circ.$ For tipping, with $h = 2w,$ $\alpha = \text{atan}(1/2) = 26.6^\circ.$ Therefore, this block slides before tipping over.

ASSESS For a block with a wider base, the tipping angle will be greater, as expected.

55. **INTERPRET** In this problem, a rectangular block is placed on an incline. Given the coefficient of friction between the block and the incline, we'd like to find out under what condition the block would tip before sliding

DEVELOP We suppose that the block is oriented with two sides parallel to the direction of the incline and that its center of mass is at the geometric center of the block. The condition for tipping over is that the center of mass lie to the left of the lower corner of the block (see sketch for Problem 12.54). Thus, $\theta > \alpha,$ where

$$\alpha = \text{atan}\left(\frac{w}{h}\right)$$

is the diagonal angle of the block. The condition for sliding is

$$mg\sin\theta > f_s^{\max} = \mu_s n = \mu_s mg\cos\theta$$

or $\tan\theta > \mu_s.$

EVALUATE For the rectangular block with $w = h/2,$ it tips over when $\theta > \alpha = \text{atan}(w/h) = \text{atan}(1/2)$ but will slide when $\theta > \text{atan}(\mu_s).$ Thus, if $\mu_s > \tan\alpha = 1/2,$ the block in Problem 54 will tip before sliding.

ASSESS We find that sliding happens first if $\mu_s < w/h.$ This makes sense because when the coefficient of friction is small, the block has a greater tendency to slide. On the other hand, when the coefficient of friction is large $(\mu_s > w/h),$ we would expect tipping to take place first.

56. **INTERPRET** This problem is similar to the preceding two problems, except that the object involved in this problem is a cone instead of a rectangular block. We can treat it in the same way, but we need to first find the center of mass of the cone, which we will do by integration

DEVELOP Because the cone is symmetric about an axis perpendicular to its base, we know the center of mass will lie on this axis. Thus, we need only find its height above the base. As per Example 9.3, we will slice the cone into infinitesimal elements parallel to the cone's base (similar to the strategy used for the isosceles block in Problem 12.50). Because the cone is uniform, the ratio of infinitesimal mass to total mass can be equated to the ratio of infinitesimal volume to total volume. Thus,

$$\frac{dm}{M} = \frac{dV}{V} = \frac{\pi r(y)^2 dy}{V}$$

where $V = \pi h^3/108$ is the cone volume, and $r(y)$ is the radius of the infinitesimal slice and is a function of the position y of the slice. Specifically, $r(y) = (h/6)(h-y)/h = (h-y)/6.$ Combining these results, we have

$$dm = \left(\frac{108M}{\pi h^3}\right)\frac{\pi(h-y)^2 dy}{36} = \frac{3M(h-y)^2 dy}{h^3}$$

Using this mass element, the height of the center of mass is

$$y_{\rm cm} = \frac{1}{M}\int y\,dm = \frac{3}{h^3}\int_0^h y(h-y)^2\,dy = \frac{h}{4}$$

which is the same result as for the isosceles pyramid of Problem 12.50. From Problem 54, we know that the condition for sliding first is $\mu_s < \tan\alpha$, where α is the angle between the centerline and the line from the edge of the base through the center of mass.

EVALUATE For the cone, the angle α is such that $\tan\alpha = r / y_{\rm cm} = (h/6)/(h/4) = 2/3$. Therefore, if $\mu_s > 2/3$, the cone will tip over before sliding.

ASSESS Because the cone has more mass near its base relative to the rectangle, it is more stable than the rectangle. Thus, for $1/2 < \mu_s < 2/3$, the rectangle will tip, whereas the cone will slide.

57. **INTERPRET** In this problem we want to verify the statement that the choice of pivot point does not matter when applying the conditions for static equilibrium.

DEVELOP With reference to Fig. 12.29, we follow the hint given in the problem statement and write

$$\vec{\tau}_{\rm P} = \sum_i \vec{r}_{{\rm P}i}\times\vec{F}_i = \sum_i\left[\left(\vec{r}_{{\rm O}i} + \vec{R}\right)\times\vec{F}_i\right] = \sum_i \vec{r}_{{\rm O}i}\times\vec{F}_i + \vec{R}\times\sum_i \vec{F}_i = \vec{\tau}_O + \vec{R}\times\vec{F}_{\rm net}$$

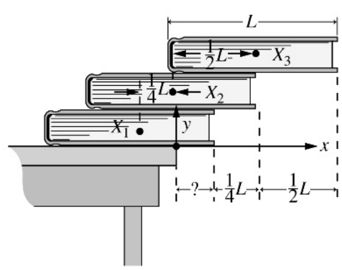
EVALUATE When the system is in static equilibrium, the total force and torque acting on the system vanish; that is $\vec{F}_{\rm net} = 0$ and $\vec{\tau}_P = \vec{0}$. Therefore, we have $\vec{\tau}_{\rm P} = \vec{\tau}_{\rm O} = \vec{0}$ so, the total torque about any two points is the same.

ASSESS If the angle θ is increased, then the corresponding coefficient of friction must also be increased to keep the pole from slipping.

58. **INTERPRET** We want to see how far we can let three books overhang a table without falling. The scales in this problem will read what force they must exert on the nonuniform rod to keep it in static equilibrium.

DEVELOP The top book is situated so that its center of mass is right above the edge of the middle book. And the center of mass of the two top books is right above the edge of the bottom book. This alignment of the center of mass with an edge allows the normal force to cancel out the weight. Otherwise there would be a nonzero torque about the edge, and the books would fall. Therefore, we need to ensure that the center of mass of the three books is above the edge of the table.

EVALUATE Let's define the edge of the table as the origin and x_1 as the location of the bottom book's center of mass. As such, the middle book's CM will be at $x_2 = x_1 + \frac{1}{4}L$, and the top book's CM will be at $x_3 = x_2 + \frac{1}{2}L$. See the figure below.



As explained above we want the center of mass of the three books combined to be above the edge of the desk at $x = 0$:

$$x_{\rm cm} = 0 = \frac{1}{3m}\sum mx_i = \frac{1}{3}\left[x_1 + \left(x_1 + \tfrac{1}{4}L\right) + \left(x_1 + \tfrac{1}{4}L + \tfrac{1}{2}L\right)\right]$$

This implies that $x_1 = -\frac{1}{3}L$, which means that only $\frac{1}{2}L - \frac{1}{3}L = \frac{1}{6}L$ of the bottom book can overhang the desk.

ASSESS The normal forces in this system are all located at the edges, with the books teetering above. This is an unstable equilibrium, since any slight deviation could misalign the downward-pointing weights and upward-pointing normal forces.

59. **INTERPRET** This problem is about static equilibrium. The forces acting on the pole are the tension in the rope, gravity acting at the center of mass of the pole, and the contact force of the incline (perpendicular component n and parallel component f). We want to find the minimum coefficient of friction that will keep the pole from slipping.

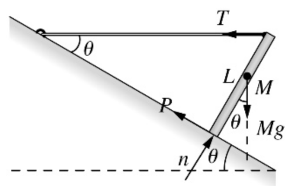
DEVELOP Make a sketch of the situation that shows the forces and the positions at which they are applied (see figure below). Applying the equilibrium condition of zero net torque (Equation 12.2) about the center of mass shows that a frictional force f must act up the plane if the rod is to remain in static equilibrium. Since the weight mg of the rod, and the normal force n contribute no torques about the center of mass, there must be a force to oppose the torque due to the tension T. The equations for static equilibrium (parallel and perpendicular components of Equation 12.1, and CCW-positive component of Equation 12.2) are:

$$0=\sum F_{\parallel} = f + T\cos\theta - mg\sin\theta$$
$$0=\sum F_{\perp} = n - T\sin\theta - mg\cos\theta$$
$$0=\left(\sum\tau\right)_{\rm cm} = T(L/2)\overbrace{\sin(\pi/2+\theta)}^{=\cos\theta} - f\,L/2 = T(L/2)\cos\theta - f\,L/2$$

The solutions for the forces are $f=\frac{1}{2}mg\sin\theta,\ T=\frac{1}{2}mg\tan\theta,$ and

$$n=\frac{1}{2}mg\left(2\cos\theta+\frac{\sin^2\theta}{\cos\theta}\right)$$

subject to the condition that $f\le\mu n$.



EVALUATE Therefore,

$$\sin\theta\le\mu\left(2\cos\theta+\frac{\sin^2\theta}{\cos\theta}\right)$$
$$\mu\ge\frac{\tan\theta}{2+\tan^2\theta}$$

ASSESS If the angle θ is increased, then the corresponding coefficient of friction must also be increased in order to keep the pole from slipping. By use of the identities $\sin 2\theta = 2\sin\theta\cos\theta$, $\cos 2\theta=\cos^2\theta-\sin^2\theta$, and $\sin^2\theta = 1-\cos^2\theta$, this may be rewritten as

$$\mu\ge\frac{\sin(2\theta)}{3+\cos(2\theta)}$$

60. **INTERPRET** This problem involves finding the incline for Problem 12.53 that demands the largest minimum coefficient of static friction. We can apply the same principles as described in Chapter 12 for finding the maximum of a potential energy function; namely, that the first derivative with respect to the independent variable (θ, in this case) must be zero and the second derivative evaluated at this point should be negative.

DEVELOP The largest minimum coefficient of friction found in Problem 53 will occur when $d\mu_{\rm min}/d\theta = 0$, or

$$\mu_{\rm min}=\frac{\tan\theta}{2+\tan^2\theta}$$
$$\frac{d\mu_{\rm min}}{d\theta}=0=\frac{(2+\tan^2\theta)\sec^2\theta-\tan\theta(2\tan\theta\sec^2\theta)}{(2+\tan^2\theta)^2}=\frac{2-\tan^2\theta\sec^2\theta}{(2+\tan^2\theta)^2}$$

The second derivative is

$$\frac{d^2\mu_{\min}}{d\theta^2}=0=\frac{1}{\left(2+\tan^2\theta\right)^4}\left[\left(2+\tan^2\theta\right)^2\left(4\sec^2\theta\tan\theta-2\tan\theta\sec^4\theta-2\tan^3\theta\sec^2\theta\right)\right.$$
$$\left.-4\tan\theta\sec^2\theta\left(2\sec^2\theta-\tan^2\theta\sec^2\theta\right)\left(2+\tan^2\theta\right)\right]$$

EVALUATE The extrema will occur at

$$0=2-\tan^2\theta$$
$$\theta=\operatorname{atan}\left(\pm\sqrt{2}\right)=54.7^\circ$$

where we have chosen the positive sign as the physically meaningful solution. Inserting this result into the expression for the second derivative gives

$$d^2\mu_{\min}/d\theta^2\Big|_{\theta=54.7^\circ}=\frac{(2+2)^2\left(12\sqrt{2}-18\sqrt{2}-12\sqrt{2}\right)-12\sqrt{2}(6-6)(2+2)}{(2+2)^4}=-\frac{9\sqrt{2}}{8}<0$$

so this extremum is a maximum.

ASSESS Calculating the second derivative is facilitated by noting that $\tan^2\theta = 2$ and $\sec^2\theta = 3$. At $\theta = 54.7°$, the minimum coefficient of static friction needed to prevent the pole from slipping is

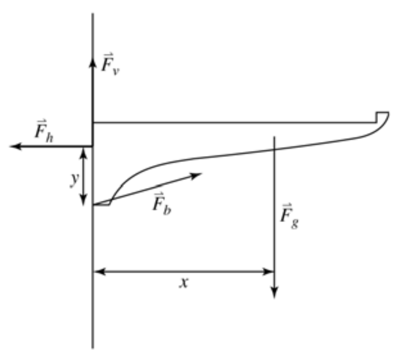
$$\mu_{\min}=\frac{\tan\left(54.7^\circ\right)}{2+\tan^2\left(54.7^\circ\right)}=\frac{\sqrt{2}}{2+2}=0.35$$

61. **INTERPRET** We use equilibrium methods to find the horizontal component of force on a bookshelf bracket tab. The bookshelf is in equilibrium, so the sum of the forces and the sum of the torques are both zero. Since the sum of the forces is zero, we may use any point as the pivot for calculating the torques. We would expect that the horizontal force is much larger than the weight of the books, since the books have more leverage than the bracket.

DEVELOP We start by drawing a diagram showing the forces and their approximate locations, as shown in the figure below. The mass of books is $m = 35$ kg, the distance $y = 4.5$ cm, and the distance $x = 12$ cm. Since we know nothing about the force $\vec{F}_b$ acting on the bottom corner of the bracket, we will use that point as our pivot point. The sum of torques around this point must be zero, which gives

$$\sum\tau=0=F_g x-F_h y.$$

Use this to find F_h.



EVALUATE Solving for F_h and inserting the known values gives

$$F_h=F_g\frac{x}{y}=mg\frac{x}{y}=\frac{(35\text{ kg})\left(9.8\text{ m/s}^2\right)(12\text{ cm})}{4.5\text{ cm}}=915\text{ N}$$

to two significant figures.

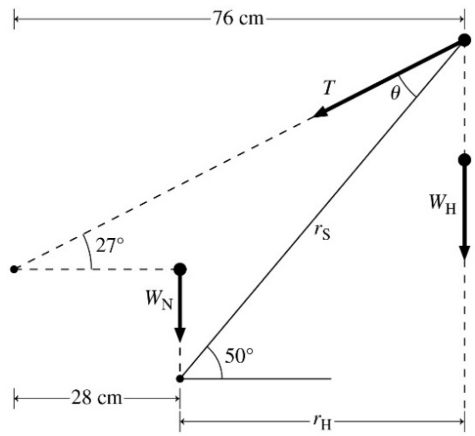
ASSESS The force is much larger than the weight of the books, as we expected.

62. **INTERPRET** In this static equilibrium problem, the tension in a particular neck ligament is estimated.

DEVELOP The spine is assumed to be a straight rod, which presumably exerts some force in an outward direction parallel to its length. If we choose the base of this rod as our pivot point, then there will be no torque from the spine's force around this point. There also won't be any torque from the neck's weight, $\vec{W}_N$, since the neck's center of mass is above the rod's base, according to Fig. 12.32. That leaves only two forces that contribute a torque around the chosen pivot: the tension, $\vec{T}$, from the nuchal ligament and the weight from the head, $\vec{W}_H$. We can write the sum of the torques around the base of the rod as

$$\sum \tau = r_S T \sin\theta - r_H W_H = 0$$

where the new variables are defined in the figure below. The horizontal distance between the rod's base and the head's center of mass is $r_H = 76\text{cm} - 28\text{cm} = 48\text{cm}$. The length of the rod representing the animal's spine is $r_S = r_H / \cos 50^\circ = 75\text{cm}$. And the angle between the spine and the ligament is $\theta = 50^\circ - 27^\circ = 23^\circ$.



EVALUATE From the torque equation, the tension in the nuchal ligament is

$$T = \frac{r_H W_H}{r_S \sin\theta} = \frac{\cos 50^\circ}{\sin 23^\circ}(29\text{ kg})(9.8\text{ m/s}^2) = 470\text{ N}$$

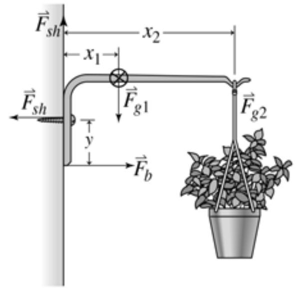
ASSESS From the figure, it is obvious that the spine will have to provide some force outward in order to balance the weight from the head and the tension from the nuchal ligament.

63. **INTERPRET** We use equilibrium methods to find the horizontal component of force on a bracket mounting screw. The bracket is in equilibrium, so the sum of the forces and the sum of the torques are both zero. Since the sum of the forces is zero, we may use any point as the pivot for calculating the torques. We would expect that the horizontal force is much larger than the weight of the plant, since the plant has more leverage than the bracket.

DEVELOP We start by drawing a diagram showing the forces and their approximate locations, as shown in the figure below. The mass of the plant is $m_2 = 3.6$ kg, the mass of the bracket is $m_1 = 0.65$ kg, the distance $y = 7.2$ cm, the distance $x_1 = 9.0$ cm, and the distance $x_2 = 28$ cm. Since we know nothing about the force $\vec{F}_b$ acting on the bottom corner of the bracket, we will use that point as our pivot point. The sum of the torques around this point must be zero, which gives

$$\left(\sum \tau\right)_b = 0 = F_{g1}x_1 + F_{g2}x_2 - F_{sh}y$$

Use this to find F_{sh}.



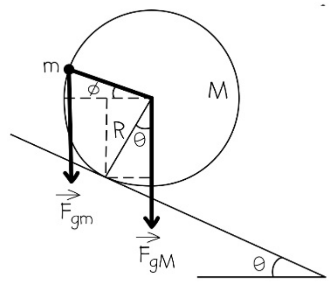
EVALUATE Solving the expression above for F_{sh} and inserting the given quantities gives

$$F_{sh} = \frac{F_{g1}x_1 + F_{g2}x_2}{y} = \frac{(m_1x_1 + m_2x_2)g}{y} = \frac{(0.65\text{ kg})(9.0\text{ cm}) + (3.6\text{ kg})(28\text{ cm})}{7.2\text{ cm}}(9.8\text{ m/s}^2) = 145.2\text{ N}.$$

ASSESS The force is much larger than the weight of the plant, as we expected.

64. **INTERPRET** We return to the problem of the disk with the off-center mass, this time finding the minimum off-center mass required to balance the disk. Again, we find the torques on the disk, and require that the sum of torques be zero.

DEVELOP We will use the point of contact between disk and ramp as the pivot for calculating torques. As we see in the figure, the torque around the contact point due to the smaller mass is a maximum when $\phi = 0$. When this is the case, the counterclockwise torque is $\tau_2 = F_{gm}R(1-\sin\theta)$. The clockwise torque is $\tau_1 = F_{gM}R\sin\theta$. The magnitudes of these two torques must be the same for the disk to be in equilibrium.



EVALUATE Equating the two torques and solving for the mass give

$$\tau_1 = \tau_2$$

$$M\cancel{gR}\sin\theta = m\cancel{gR}(1-\sin\theta) \quad \Rightarrow \quad m = \frac{M\sin\theta}{1-\sin\theta}$$

Because this mass corresponds to the ϕ=0, which gives the largest torque, this mass will have to be larger if it is positioned at an angle $\phi \neq 0$. Thus we have $m \ge M\sin\theta/(1-\sin\theta)$.

ASSESS If we solve this problem for a general position f of the mass m, we find

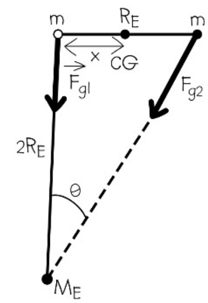
$$0 = F_{gm}(R\cos\phi - R\sin\theta) - F_{gM}\sin\theta$$

$$m = \frac{M\sin\theta}{\cos\phi - \sin\theta} \ge \frac{M\sin\theta}{1-\sin\theta}$$

where the inequality follows from the fact that $\cos\phi < 1$ for $0 < \phi < 180°$.

65. **INTERPRET** This problem involves two masses, separated by a massless rod, hovering far above the Earth. We need to find the net gravitational force on these masses, the net torque around the center of mass, and the center of gravity. We will use the equation for gravitational force, as well as the equation for torque. The center of gravity is not the same as the center of mass, in this case, since there is a change in gravitational field between the two masses.

DEVELOP We start with a diagram, as shown in the figure below. We use Newton's law of universal gravity $\vec{F}_g = GM_Em/r^2$ (Equation 8.1) to find the force on each end of the spacecraft, then find the vector sum of the forces to obtain the net gravitational force. Use the forces on each end, and the angle found from the figure, to find the torque on each end and thus the net torque around the center of mass. The center of gravity is the point on the spacecraft at which the torques will be zero. We have labeled the distance from the left end of the spacecraft to the center of gravity x.



EVALUATE **(a)** The gravitational force on the left mass is $\vec{F}_1 = -GM_{\text{E}}m/(2R_{\text{E}})^2\,\hat{j}$. The gravitational force on the right mass is $F_2 = -GM_{\text{E}}m\left(4R_{\text{E}}^2 + R_{\text{E}}^2\right)^{-2}$, directed at an angle $\theta = \text{atan}\left[R_{\text{E}}/(2R_{\text{E}})\right] = \text{atan}(1/2) = 26.7°$. We note in passing that $\sin\theta = 1/\sqrt{5}$ and $\cos\theta = 2/\sqrt{5}$. We break F_2 into x- and y-components:

$$\vec{F}_2 = -G\frac{M_E m}{4R_E^2 + R_E^2}\left(\sin\theta\hat{i} + \cos\theta\hat{j}\right) = -G\frac{M_E m}{5R_E^2}\left(\frac{1}{\sqrt{5}}\hat{i} + \frac{2}{\sqrt{5}}\hat{j}\right).$$

Now we add the vector components to find the total force: $\vec{F} = -G\frac{M_{\text{E}}m}{R_{\text{E}}^2}\left[\left(\frac{1}{4} + \frac{2}{\sqrt{5}}\right)\hat{j} + \left(\frac{1}{\sqrt{5}}\right)\hat{i}\right]$. The magnitude of this force is $F = G\frac{M_{\text{E}}m}{R_{\text{E}}^2}(1.229)$, and the direction of the force is $\text{atan}\left[\left(1/\sqrt{5}\right)/\left(1/4 + 2/\sqrt{5}\right)\right] = 21.3°$ left of the negative y-axis.

(b) The net torque around the center of mass is

$$\tau = -F_1\frac{R_E}{2} + F_{2\hat{j}}\frac{R_E}{2} = -G\frac{M_E m}{4R_E^{\cancel{2}}}\left(\frac{\cancel{R_E}}{2}\right) + G\frac{M_E m}{5R_E^{\cancel{2}}}\frac{\cancel{2}}{\sqrt{5}}\left(\frac{\cancel{R_E}}{\cancel{2}}\right)$$
$$= G\frac{M_E m}{R_E}\left(-\frac{1}{8} + \frac{1}{5\sqrt{5}}\right) = G\frac{M_E m}{R_E}(-0.0356)$$

(c) To find the center of gravity, repeat the calculation for (b) but use x for the left-hand distance and $(R_{\text{E}} - x)$ for the right-hand distance, setting $\tau = 0$. Solving this for x will give the distance of the center of gravity from the left end, which we can compare to $R_{\text{E}}/2$.

$$0 = -F_1 x + F_{2\hat{j}}(R_{\text{E}} - x) = -G\frac{M_E m}{4R_E{}^2}(x) + G\frac{M_E m}{5R_E{}^2}\frac{2}{\sqrt{5}}(R_E - x)$$
$$= -\frac{1}{4}x + \frac{2}{5\sqrt{5}}(R_{\text{E}} - x)$$
$$x\left(\frac{1}{4} + \frac{2}{5\sqrt{5}}\right) = R_{\text{E}}\frac{2}{5\sqrt{5}}$$
$$x = 0.417R_{\text{E}}$$
$$\frac{R_{\text{E}}}{2} - x = 0.083$$

ASSESS The torque is negative, which in this case means counterclockwise as we would expect. The center of gravity in this case is not at the center of mass, due to the decrease in gravitational force with altitude.

66. **INTERPRET** In this problem, we examine the stability of an SUV as it negotiates a turn on a flat road

DEVELOP As discussed in the textbook, the static stability factor (SSF) is given by $t/2h$, where t is the width between the wheels, and h is the height of the center of gravity above the road. The rollover condition is $v^2/rg = t/2h$. In addition, we are told that an SUV without ECS (electronic stability control system) has $\text{SSF} = 1.06$. To successfully negotiate the turn, we require that $v^2/rg < \text{SSF}$.

EVALUATE **(a)** With a speed $v = 90\ \text{km/h} = 25\ \text{m/s}$, we obtain

$$\frac{v^2}{rg} = \frac{(25\ \mathrm{m/s})^2}{(70\ \mathrm{m})(9.8\ \mathrm{m/s^2})} = 0.91$$

Since this value is less than $\mathrm{SSF} = 1.06$, we conclude that the SUV can successfully negotiate the turn.

(b) The initial height of the CM before the cargo goes on the roof is $h_0 = t/2(\mathrm{SSF}) = (1.63\ \mathrm{m})/(2\cdot 1.06) = 0.7689\ \mathrm{m}$. In the presence of a cargo, the new CM is

$$h = \frac{m_{\mathrm{SUV}}h_0 + m_{\mathrm{cargo}}h_c}{m_{\mathrm{SUV}} + m_{\mathrm{cargo}}} = \frac{(2100\ \mathrm{kg})(0.7689\ \mathrm{m}) + (300\ \mathrm{kg})(2.1\ \mathrm{m})}{2100\ \mathrm{kg} + 300\ \mathrm{kg}} = 0.94\ \mathrm{m}$$

Thus, the new SSF is $t/2h = (1.63\ \mathrm{m})/(2\cdot 0.94\ \mathrm{m}) = 0.87$. In this case, the maximum speed becomes

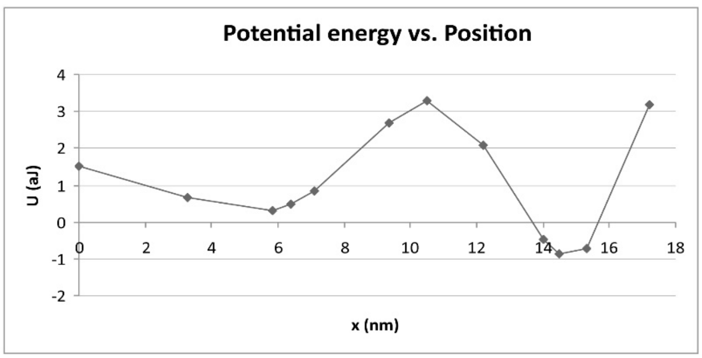
$$v_{\max} = \sqrt{(\mathrm{SSF})rg} = \sqrt{(0.87)(70\ \mathrm{m})(9.8\ \mathrm{m/s^2})} = 24.3\ \mathrm{m/s} = 88\ \mathrm{km/h}$$

ASSESS As expected, raising the CM of the combined cargo-SUV system lowers its maximum safe speed.

67. **INTERPRET** In this problem we're given the data of an electron's potential energy as a function of position. We would like to determine the equilibrium positions.

Develop We know that the equilibria will occur where the potential energy function is flat (i.e., has zero derivative), and that an equilibrium will be stable (unstable) if, at the equilibrium position, the second derivative is positive (negative).

EVALUATE The plot is shown below.



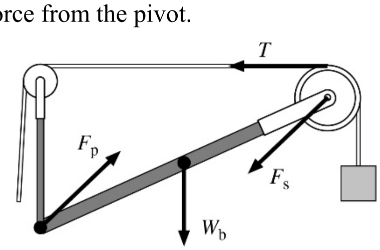
From the plot, we identify three equilibrium positions where $dU/dx = 0$. They are located approximately at $x = 6$ nm, 11 nm, and 14 nm. The equilibria at $x = 6$ and 14 nm are stable since $d^2U/dx^2 > 0$, but the one at $x =$ 11 nm is unstable since $d^2U/dx^2 < 0$.

ASSESS A stable equilibrium corresponds to a local minimum (concaved up), while an unstable equilibrium corresponds to a local maximum (concaved down).

68. **INTERPRET** We're asked to analyze a boom and pulley system.

Develop There are four forces on the boom: the weight of the boom, W_b, the tension in the boom rope, T, the force on the pulley from the sampling rope, F_s, and the force exerted by the pivot F_p.

EVALUATE When the boom rope is horizontal, the tension is horizontal. There will be downward forces from the boom's weight and from the force on the pulley, as shown in the figure below. However, these downward forces will be balanced by the upward force from the pivot.



The answer is (c).

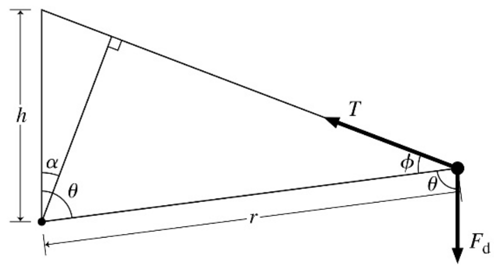
ASSESS The force from the pivot is like the normal force from a surface except that it can usually point in any direction.

69. **INTERPRET** We're asked to analyze a boom and pulley system.

DEVELOP To find the maximum tension, we need to parameterize the tension by a singular variable. We can simplify the problem by combining the downward forces from the boom's weight and the sample rope into one variable, F_d, which we assume acts on the end of the boom. Therefore, the sum of the torques around the pivot is:

$$\sum \tau = rT\sin\phi - rF_d\sin\theta = 0$$

where r is the length of the boom and the two angles are defined in the figure below.



We have defined in the figure another angle, α, as well as the height, h, of the vertical support. The parameters h, r, and F_d are constant, whereas the three angles change as the boom rotates. The angles are related to each other by: $\theta + \phi - \alpha = 90^\circ$, and $h\cos\alpha = r\sin\phi$. We plug these two relations into the torque equation and use some of the trig identities in Appendix A to arrive at the tension as a function of the angle θ between the boom and the vertical:

$$T = F_d\sqrt{1 + \frac{r^2}{h^2} - 2\frac{r}{h}\cos\theta}$$

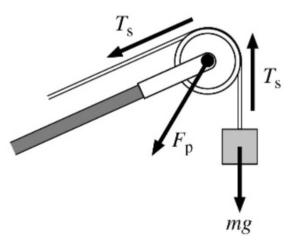
EVALUATE As θ increases from zero (straight up), the tension increases as well, reaching a maximum at $\theta = 90^\circ$ when the boom is horizontal.

The answer is (a).

ASSESS The tension keeps increasing as θ continues to 180°, but we assume that the boom cannot rotate any lower than a horizontal position, otherwise it would drop into the river.

70. **INTERPRET** We're asked to analyze a boom and pulley system.

DEVELOP In the static case, the weight of the apparatus, mg, is balanced by the tension in the sample rope, T_s. If we assume the sample rope is massless and its pulley is frictionless, the tension will be the same on both sides of the pulley. This results in a force on the pulley, as shown in the figure below. Because the pulley is attached to the boom, F_p acts on the boom as well.



EVALUATE If the apparatus is descending at constant speed, the sum of the forces acting on it must still be zero. Therefore, the force on the pulley remains the same as in the static case. This means the forces and torques on the boom are unchanged.

The answer is (c).

ASSESS If the apparatus were lowered at an accelerating rate, the tension in the sample rope would be $T_s = m(g - a)$. This would reduce the force on the pulley and would thus mean less tension would be needed in the boom rope.

71. **INTERPRET** We're asked to analyze a boom and pulley system.

DEVELOP Let y be the length of boom rope between the end of the boom and the pulley on the vertical support. From the figure in Problem 12.65 and the law of cosines in Appendix A:

$$y^2 = r^2 + h^2 - 2rh\cos\theta$$

As the rope is pulled at constant speed v_0, the length y will be getting shorter: $dy/dt = \dot{y} = -v_0$, where we use dots to indicate time derivatives. We want to find how the boom's angle changes with respect to the horizontal. Since the angle θ is with respect to the vertical, we switch to its complement: $\beta = 90^\text{o} - \theta$, such that

$$\sin\beta = \frac{r^2 + h^2 - y^2}{2rh}$$

We'll take the derivative of this equation with respect to time.

EVALUATE Taking the first derivative with respect to time and using the chain rule (see Appendix A) gives:

$$(\cos\beta)\dot{\beta} = \frac{-2y\dot{y}}{2rh} \quad \rightarrow \quad \dot{\beta} = \frac{v_0}{rh}\frac{y}{\cos\beta} > 0$$

The derivative $\dot{\beta}$ is positive because all of the other terms are positive (as long as $\beta \geq 0$). This implies that the angle is increasing, as we would expect. To find the rate at which it is increasing, we need to take the second derivative with respect to time:

$$\frac{d}{dt}\dot{\beta} = \ddot{\beta} = \frac{v_0}{rh}\left[\frac{\dot{y}}{\cos\beta} - \frac{y}{\cos^2\beta}(-\sin\beta)\dot{\beta}\right] = \frac{v_0^2}{rh\cos\beta}\left[1 + \frac{y^2}{rh}\frac{\tan\beta}{\cos\beta}\right]$$

All of the terms are again positive, so the angle increases at an increasing rate.

The answer is (b).

ASSESS The result implies that the boom's angle is changing faster as it points more and more vertical. This makes sense since the fastest rate at which the angle could be changing is $\dot{\beta} = v_0 / r$, that is, when the velocity is perpendicular to the radius. This occurs at the top, when the boom nears the vertical.