15

FLUID MOTION

EXERCISES

Section 15.1 Density and Pressure

11. **INTERPRET** This problem requires us to calculate the total mass of a substance, given its density and volume.
DEVELOP The density ρ is defined as the mass per unit volume, or $\rho = m/V$. Given the volume and the density, the mass can be calculated by solving this equation for *m*: $m = \rho V$. Because we are given the mass in units of $\mathrm{kg/m^3}$ and the volume in terms of L, we will convert L to $\mathrm{m^3}$ using the conversion factor

$$(1\ \mathrm{L})\left(\frac{10^3\ \mathrm{cm}^3}{\mathrm{L}}\right)\left(\frac{1\ \mathrm{m}}{10^2\ \mathrm{cm}}\right)^3 = 10^{-3}\ \mathrm{m}^3$$

$$1 = 10^{-3}\ \mathrm{m}^3/\mathrm{L}$$

EVALUATE Inserting the given quantities $\rho = 1600\ \mathrm{kg/m^3}$ and $V = (0.95\ \mathrm{L})(10^{-3}\ \mathrm{m^3/L}) = 0.95\times10^{-3}\ \mathrm{m}^3$, we find

$$m = \rho V = (1600\ \mathrm{kg/m^3})(0.95\times10^{-3}\ \mathrm{m^3}) = 1.5\ \mathrm{kg}$$

ASSESS The mass is linearly proportional to the volume and to the density.

12. **INTERPRET** This problem is about the volume fraction of water that consists of atomic nuclei, which for this problem are considered to be solid. Given the density of water and atomic nuclei, we are to find the fraction of space (i.e., volume) in water that is *not* empty space (i.e., the fraction of the volume occupied by the nuclei).
DEVELOP The volume occupied by the nuclei is $V_\mathrm{n} = m_\mathrm{n}/\rho_\mathrm{n}$, and the volume occupied by water is $V_\mathrm{w} = m_\mathrm{w}/\rho_\mathrm{w} \approx m_\mathrm{n}/\rho_\mathrm{w}$ if we ignore the mass of the electrons (which is much, much less than the mass of the nuclei so $m_\mathrm{w} = m_\mathrm{n} + m_\mathrm{e} \approx m_\mathrm{n}$). The fraction of the water volume occupied by the nuclei is $V_\mathrm{w}/V_\mathrm{n}$.
EVALUATE The fraction of volume occupied by nuclei in water is

$$\frac{V_w}{V_\mathrm{n}} \approx \left(\frac{m_\mathrm{n}}{\rho_\mathrm{w}}\right)\left(\frac{\rho_\mathrm{n}}{m_\mathrm{n}}\right) = \frac{\rho_\mathrm{n}}{\rho_\mathrm{w}} = \frac{10^3}{10^{17}} = 10^{-14}$$

ASSESS The result agrees with the fact that almost the entire mass of an atom is concentrated in its nucleus, which is made up of protons and neutrons that are much more massive than the electrons.

13. **INTERPRET** This problem involves calculating the density, given the mass and the volume of a substance, and calculating the volume were this same mass to have a different density.
DEVELOP Use the definition of density $\rho = m/V$ to find the density of the air in the cylinder, which we will call ρ_1. For part (b), let ρ_1 go to $\rho_2 = 1.2\ \mathrm{kg/m^3}$ and keep the mass the same ($m = 9.8$ kg) to calculate the new volume V_2 occupied by the air.
EVALUATE **(a)** The density of the air in the cylinder is

$$\rho_1 = \frac{m}{V_1} = \frac{9.8\ \mathrm{kg}}{0.041\ \mathrm{m}^3} = 240\ \mathrm{kg/m^3}$$

(b) The volume occupied by this mass of air at atmospheric density is

$$V_2 = \frac{m}{\rho_2} = \frac{9.8\ \mathrm{kg}}{1.2\ \mathrm{kg/m^3}} = 8.2\ \mathrm{m}^3$$

Assess These volumes are small enough that any variation in the density of the air due to gravity may be ignored.

14. **Interpret** This problem is about computing the weight of a column of air that extends from the Earth's surface to the top of the atmosphere.

Develop As shown in Equation 15.1, pressure measures the normal force per unit area exerted by a fluid, or $p = F/A$. Thus, knowing the pressure and the area, the force is given by their product. At the surface of the Earth, the atmospheric pressure is p_{atm} = 101.3 kPa, so we can find the force exerted by this pressure using $F = p_{atm}A$.

Evaluate The weight of a 1 m^2 column of air is

$$w_{air} = p_{atm}A = (101.3 \text{ kPa})\left(1 \text{ m}^2\right) = 101.3 \text{ kN}$$

Assess This is the force that's pushing down on the 1-m^2 cross-sectional area. It is enormous! For this problem, you may be tempted to use Equation 15.2, because you consider (rightly) that the pressure changes as a function of height in the atmosphere. However, because air is compressible, its density also changes as a function of height, as does the force due to gravity. Thus, Equation 15.2 would give

$$dp = \rho(h)g(h)dh$$

Given $\rho(h)$ and $g(h)$, we could integrate this expression, but it is much simpler to use the fact that the atmospheric pressure at the surface of the Earth is 101.3 kPa.

15. **Interpret** This problem involves calculating the pressure given the force and the area over which the force is applied.

Develop Both the force exerted on the diamonds and the size of the surface upon which the force is exerted are given. We can insert these into Equation 15.1 to find the pressure.

Evaluate The pressure resulting from the force on the diamonds is

$$p = \frac{F}{A} = \frac{6000 \text{ N}}{\pi\left(1.0\times10^{-4} \text{ m}\right)^2} = 200 \text{ GPa}$$

Assess This pressure is equivalent to approximately 2 million atmospheres.

16. **Interpret** This problem is about finding the force that must be exerted over a given area to result in a pressure of 135 atm.

Develop As shown in Equation 15.1, pressure measures the normal force per unit area exerted by a fluid, $p = F/A$. The cross-sectional area of the paper clip is $A = \pi(d/2)^2 = \pi d^2/4$, where d is the diameter. Assuming the force is applied normal to the paper clip cross section, the requisite force is given by $F = pA$.

Evaluate An average pressure of 135 atm over the area results in a force of

$$F = pA = \frac{(135\times101.3 \text{ kPa})\pi(0.8\times10^{-3} \text{ m})^2}{4} = 6.874 \text{ N} \approx 6.9 \text{ N}$$

Assess Pressure is inversely proportional to the area over which the force is exerted. The smallness of the cross section of the paper clip is what makes the pressure so large.

Section 15.2 Hydrostatic Equilibrium

17. **Interpret** This problem involves calculating the density of an incompressible fluid, given the pressure increase as a function of depth.

Develop For an incompressible fluid, the increase in pressure with depth is given by Equation 15.3, $p = p_0 + \rho gh$. Applying this equation to the two depths h_1 and h_2, we can write

$$p_1 = p_0 + \rho g h_1$$
$$p_2 = p_0 + \rho g h_2$$
$$p_2 - p_1 = \rho g\left(h_2 - h_1\right)$$

Given that $p_2 - p_1 = 100$ kPa for $h_2 - h_1 = 8$ m, we can calculate the density ρ.

Evaluate Solving for the density, we find

$$\rho = \frac{p_2 - p_1}{g(h_2 - h_1)} = \frac{100\times 10^3 \text{ Pa}}{(9.8 \text{ m/s}^2)(8.0 \text{ m})} = 1.3\times 10^3 \text{ kg/m}^3$$

ASSESS Checking the units of this expression, we find

$$\frac{\text{Pa}}{(\text{m/s}^2)(\text{m})} = \frac{\text{N}\cdot\text{m}^{-2}}{(\text{m/s}^2)(\text{m})} = \frac{\text{kg}\cdot\text{m}\cdot\text{s}^{-2}\cdot\text{m}^{-2}}{(\text{m/s}^2)(\text{m})} = \frac{\text{kg}}{\text{m}^3}$$

as expected for density.

18. **INTERPRET** This problem is about hydrostatic equilibrium. Given the maximum pressure the submarine can withstand, we want to find its maximum-allowable depth below water.

DEVELOP The external pressure p at a depth h below the surface of water is given by Equation 15.1:

$$p = p_0 + \rho g h$$

where the pressure $p_0 = 101.3$ kPa is atmospheric pressure pushing down on the surface of the water. A typical density for open ocean seawater (which varies with salinity) is $\rho = 1027 \text{ kg/m}^3$.

EVALUATE The depth corresponding to $p = 62$ MPa is

$$h = \frac{p - p_0}{\rho g} = \frac{57.5 \text{ MPa} - 101.3 \text{ kPa}}{(1027 \text{ kg/m}^3)(9.8 \text{ m/s}^2)} = 5.7 \text{ km}$$

ASSESS The pressure the submarine can withstand is quite high (more than $560\times p_{\text{atm}}$). Note that our result is not exact because water at this depth is slightly compressible.

19. **INTERPRET** This problem involves calculating the depth at which water pressure is 1 MPa.

DEVELOP Equation 15.3 $p = p_0 + \rho g h$ gives the pressure as a function of depth. For this problem, we can assume that p_0 = 1 atm = 101.3 kPa and that we are dealing with fresh water with a density of $\rho = 10^3$ kg/m^3. Thus, we can solve for the depth h at which the pressure $p = 1 \times 10^6$ Pa.

EVALUATE Solving for the depth h, we find

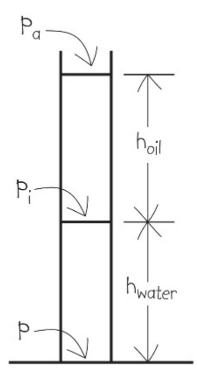
$$h = \frac{p - p_0}{\rho g} = \frac{(1 - 0.1013)\times 10^6 \text{ Pa}}{(1.0\times 10^3 \text{ kg/m}^3)(9.8 \text{ m/s}^2)} = 92 \text{ m}$$

ASSESS The depth is a little less in salt water because its density is slightly greater.

20. **INTERPRET** We have an open tube filled with water on the bottom and oil on the top of water. The two fluids do not mix. We want to find the gauge pressure at the bottom of the tube.

DEVELOP The pressure pushing down on the oil at the top of the tube is the atmospheric pressure, p_{atm} (see figure below). Using Equation 15.3, the absolute pressure at the oil-water interface is $p_{\text{i}} = p_{\text{atm}} + \rho_{\text{oil}} g h_{\text{oil}}$ and the pressure at the bottom of the water is,

$$p = p_{\text{i}} + \rho_{\text{water}} g h_{\text{water}} = p_{\text{atm}} + \rho_{\text{oil}} g h_{\text{oil}} + \rho_{\text{water}} g h_{\text{water}}$$



EVALUATE Therefore, the gauge pressure at the bottom is

$$\begin{aligned}\Delta p = p - p_{\text{atm}} &= (\rho_{\text{oil}} h_{\text{oil}} + \rho_{\text{water}} h_{\text{water}}) g \\ &= \left[\left(0.82\times10^3\,\text{kg/m}^3\right)(0.05\text{ m}) + \left(1.00\times10^3\,\text{kg/m}^3\right)(0.05\text{ m})\right]\left(9.8\text{ m/s}^2\right) \\ &= 890\text{ Pa}\end{aligned}$$

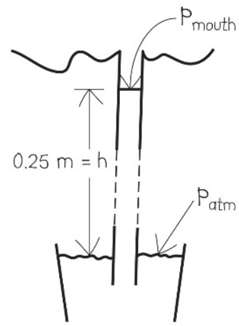
ASSESS Oil is less dense than water. Therefore it flows on top of the water. The gauge pressure at the bottom of the tube is due to the weight of both the oil and the water.

21. **INTERPRET** This problem involves calculating the pressure difference needed between that pushing down on the water in the cup and that pushing down on the water in the straw to generate the force needed to lift the water up the given amount.

DEVELOP Make a sketch of the situation (see figure below). We applying Equation 15.3, which for this situation takes the form

$$p_{\text{atm}} = p_{\text{mouth}} + \rho g h$$

where $h = 0.25$ m and $p_{\text{atm}} = 101.3$ kPa.



EVALUATE Solving the expression above for $p_{\text{atm}} - p_{\text{mouth}}$ gives

$$p_{\text{atm}} - p_{\text{mouth}} = \rho g h = \left(1.0\times10^3\text{ kg/m}^3\right)\left(9.8\times10^3\text{ N/m}^3\right)(0.25\text{ m}) = 2.45\text{ kPa}$$

ASSESS This corresponds to a reduction of

$$\frac{2.45\text{ kPa}}{101.3\text{ kPa}} = 2.4\%$$

22. **INTERPRET** In this problem, we are given two barometric pressures, one for the eye of a hurricane, another for a fair-weather region. We want to compare the level of the ocean surface between these two regions.

DEVELOP The pressure difference between two points leads to a difference h in sea level given by Equation 15.3: $p - p_0 = \rho g h$.

EVALUATE With $p = 1.0$ atm and $p_0 = 0.94$ atm, we obtain

$$\text{h} = \frac{p - p_0}{\rho g} = \frac{1.0\text{ atm} - 0.94\text{ atm}}{(1000\text{ kg/m}^3)(9.8\text{ m/s}^2)}\overbrace{\left(\frac{1.013\times10^5\text{ Pa}}{1\text{ atm}}\right)}^{=1} = 0.62\text{ m}$$

ASSESS The fair-weather region is 0.62 m *below* the sea level at the eye of the hurricane.

Section 15.3 Archimedes' Principle and Buoyancy

23. **INTERPRET** This problem involves the buoyancy force, which will help us to carry a concrete block if it is submerged in water. We can use Archimedes' principle and Newton's second law to calculate the most massive concrete block we could lift underwater. We are given the mass of the largest block we can carry on land and the density of the concrete.

DEVELOP Make a free-body diagram of the situation (see Fig. 15.8). Applying Newton's second law to the concrete block gives

$$\vec{F}_{\text{net}} = m_{\text{w}}\vec{a}$$
$$F_{\text{b}} + F_{\text{app}} - m_{\text{w}}g = 0$$
$$F_{\text{app}} = m_{\text{w}}g - F_{\text{b}}$$

where the subscript w indicates the mass we can carry under water. The maximum force we can apply is $F_{\text{app}} = m_{\text{L}}g$, where $m_{\text{L}} = 25$ kg is the maximum mass we can carry on land. The buoyancy force on a block is $F_{\text{b}} = \rho_{\text{w}}gV_{\text{c}}$, where $\rho_{\text{w}} = 1.0 \times 10^3\ \text{kg/m}^3$ is the density of water and V_{c} is the volume of the concrete block, which is given by $V_{\text{c}} = m_{\text{w}}/\rho_{\text{c}}$ where $\rho_{\text{c}} = 2200\ \text{kg/m}^3$ is the density of the concrete. We can solve this expression for the maximum mass m_{w} that we can carry in water.

EVALUATE Inserting the given quantities in the expression for F_{app} and solving for m_{w} gives

$$m_{\text{L}}g = m_{\text{w}}g - \rho_{\text{w}}gV_{\text{c}} = m_{\text{w}}g - \rho_{\text{w}}g\left(\frac{m_{\text{w}}}{\rho_{\text{c}}}\right) = m_{\text{w}}g\left(1 - \frac{\rho_{\text{w}}}{\rho_{\text{c}}}\right)$$
$$m_{\text{w}} = m_{\text{L}}\left(\frac{\rho_{\text{c}}}{\rho_{\text{c}} - \rho_{\text{w}}}\right) = 46\ \text{kg}$$

ASSESS We can check this solution by looking at what happens if $\rho_{\text{c}} = \rho_{\text{w}}$. In this case, the "block" would have neutral buoyancy and we would be able to lift any size.

24. **INTERPRET** Given the apparent weight of a jewel when submerged in water, we want to know whether the jewel is a sapphire or not. To answer this question, we need to calculate the density of the jewel using Archimedes' principle.

DEVELOP According to Archimedes' principle, the buoyancy force on an object is equal to the weight of the fluid displaced by the object. Mathematically, this is expressed as

$$F_{\text{b}} = m_{\text{w}}g = \rho_{\text{w}}V_{\text{j}}g = \rho_{\text{w}}\frac{m_{\text{j}}}{\rho_{\text{j}}}g = \left(\frac{\rho_{\text{w}}}{\rho_{\text{j}}}\right)w_{\text{j}}$$

where $w_{\text{j}} = m_{\text{j}}g$ is the true weight of the jewel. The apparent weight w'_{j} of the jewel is

$$w'_{\text{j}} = w_{\text{j}} - F_{\text{b}} = w_{\text{j}} - \left(\frac{\rho_{\text{w}}}{\rho_{\text{j}}}\right)w_{\text{j}} = \left(1 - \frac{\rho_{\text{w}}}{\rho_{\text{j}}}\right)w_{\text{j}}$$

EVALUATE Solving the expression above for the density of the jewel gives

$$\rho_{\text{j}} = \frac{\rho_{\text{w}}}{1 - w'_{\text{j}}/w_{\text{j}}} = \frac{1.0\times10^3\ \text{kg/m}^3}{1.0 - (0.056\ \text{N})/(0.0083\ \text{kg}\times9.8\ \text{m/s}^2)} = 3.2\times10^3\ \text{kg/m}^3$$

The value is too small for the density of sapphire $(\rho_{\text{sapphire}} = 3.98\times10^3\ \text{kg/m}^3)$.

ASSESS Knowing the apparent weight of a submerged object allows us to determine the density of the object.

25. **INTERPRET** This problem involves the buoyancy force (Archimedes' principle), which applies to objects in air just as it does to objects in water. We can use this to find the fractional error that occurs when weighing styrofoam in air as opposed to in a vacuum.

DEVELOP In a vacuum, the weight of a given volume V of styrofoam is

$$w = mg = \rho_{\text{s}}Vg$$

In air, the buoyancy force F_{b} acts against gravity, so that the apparent weight w' of the same volume of styrofoam is

$$w' = w - F_{\text{b}} = \rho_{\text{s}}Vg - \rho_{\text{w}}Vg$$

The fractional error Δ is

$$\Delta = \frac{w - w'}{w}$$

EVALUATE Inserting the expression for the weights gives

$$\Delta = \frac{w - w'}{w} = \frac{\rho_s Vg - (\rho_s Vg - \rho_a Vg)}{\rho_s Vg} = \frac{\rho_a}{\rho_s} = \frac{1.2\ \text{kg/m}^3}{160\ \text{kg/m}^3} = 0.75\%$$

ASSESS This error of almost 1% may not be negligible in all situations

26. **INTERPRET** In this problem we are given the volume and mass of a steel drum and asked to determine whether or not it will float in water when filled with water or with gasoline.

DEVELOP An object will float in water if its average density is less than the density of water. This follows from Archimedes' principle, since the volume of water displaced by an object floating on the surface is less than its total volume, that is, $V_{\text{dis}} < V$. Since the buoyant force equals the weight of a floating object,

$$F_b = \rho_{H_2O} g V_{\text{dis}} = W = \rho_{\text{av}} g V$$

and this implies $\rho_{\text{av}} < \rho_{H_2O}$.

EVALUATE **(a)** When the drum is filled with water, the combination of steel $(\rho_{\text{steel}} > \rho_{H_2O})$ and water will have a higher density than pure water, so the drum will sink.

(b) When the drum is filled with gasoline, its average density is

$$\rho_{\text{av}} = \frac{M_{\text{steel}} + M_{\text{gas}}}{V_{\text{steel}} + V_{\text{gas}}}$$

We'll assume that the volume of the steel shell is negligible compared to the volume of the gasoline $(V_{\text{steel}} \ll V_{\text{gas}})$. Since $M_{\text{gas}} = \rho_{\text{gas}} V_{\text{gas}}$,

$$\rho_{\text{av}} \approx \frac{M_{\text{steel}} + M_{\text{gas}}}{V_{\text{gas}}} = \frac{M_{\text{steel}}}{V_{\text{gas}}} + \rho_{\text{gas}} = \frac{16\ \text{kg}}{0.23\ \text{m}^3} + 860\ \text{kg/m}^3 = 930\ \text{kg/m}^3$$

which is less than $\rho_{H_2O} = 1000\ \text{kg/m}^3$. Therefore, the drum will float in this case.

ASSESS Even though steel has greater density than water, the effective volume of the steel drum includes all of the liquid inside it. Thus, when filled with gasoline, its average density is then less than that of water, and hence it will float.

Sections 15.4 and 15.5 Fluid Dynamics and Applications

27. **INTERPRET** This problem involves calculating the flow rate of an incompressible fluid in a pipe with a varying cross section. We use the principle of conservation of mass to find the speed of fluid flow in the narrow section, given the speed of flow in the wide section and the diameters of each section.

DEVELOP The continuity Equation 15.5 for a liquid is $vA = \text{constant}$. Applying this equation to the pipe in question gives $v_1 A_1 = v_2 A_2$, where $A_1 = \pi (d_1/2)^2$ and $A_2 = \pi (d_2/2)^2$, with $d_1 = 2.6$ cm and $d_2 = 1.6$ cm. Given that the speed in the wide section is $v_1 = 1.7$ m/s, we can solve for v_2.

EVALUATE Inserting the given quantities into the expression derived using the continuity equation gives

$$v_1 A_1 = v_2 A_2$$

$$v_2 = v_1 \left(\frac{A_1}{A_2}\right) = v_1 \left(\frac{\pi d_1^2}{\pi d_2^2}\right) = v_1 \left(\frac{d_1}{d_2}\right)^2 = (1.7\ \text{m/s}) \left(\frac{2.6\ \text{cm}}{1.6\ \text{cm}}\right)^2 = 4.5\ \text{m/s}$$

ASSESS As expected, the speed increases in the narrow section.

28. **INTERPRET** This problem is an exercise in dimensional analysis. We are asked to show that pressure has units of energy density.

DEVELOP Analogous to mass density, energy density has units of energy per unit volume. In SI units, this is J/m^3. The units of pressure are force per unit area, which in SI units is N/m^2. Use the fact that $\text{J} = \text{N} \cdot \text{m}$ to show that N/m^2 = J/m^3.

EVALUATE Multiplying both numerator and denominator by units of length gives

$$\underbrace{\left(\frac{\text{N}}{\text{m}^2}\right)}_{\text{pressure}}\underbrace{\left(\frac{\text{m}}{\text{m}}\right)}_{1}=\frac{\text{N}\cdot\text{m}}{\text{m}^3}=\frac{\text{J}}{\text{m}^3}$$

which is energy density.

ASSESS Thus, we find that the pressure can be considered an expression of the internal energy density in a fluid, as implied by Bernoulli's Equation 15.6.

29. **INTERPRET** This problem deals with fluid flow rate. The key concepts involved are conservation of mass and the continuity equation.

DEVELOP The mass flow rate is given by Equation 15.4, $R_\text{m} = \rho vA,$ and the volume flow rate is given by Equation 15.5, $R_\text{V} = vA.$ The speed v of the flow can be determined once the flow rates R_m and R_V and the corresponding cross-sectional area A is known.

EVALUATE **(a)** Inserting the given quantities, the volume flow rate for the Nile River is

$$R_\text{V} = vA = \frac{R_\text{m}}{\rho} = \frac{2.8\times10^7 \text{ kg/s}}{1.0\times10^3 \text{ kg/m}^3} = 2.8\times10^4 \text{ m}^3/\text{s}$$

(b) At a point in the river where the cross-sectional area is given, the average speed of flow is

$$v = \frac{R_\text{V}}{A} = \frac{2.8\times10^4 \text{ m}^3/\text{s}}{(1.8\times10^3 \text{ m})(8.2 \text{ m})} = 1.9 \text{ m/s}$$

ASSESS The flow speed we find is reasonable. Note that the actual flow rate of any river varies with the season, local weather, vegetation conditions, human water consumption, and so on.

30. **INTERPRET** This problem involves conservation of mass, as expressed by the continuity Equations 15.4 and 15.5. We can use this to find the flow speeds in the hose and in the nozzle**.**

DEVELOP Applied to the hose, Equation 15.4 gives

$$R_\text{m} = \rho v_1 A_1 = 15 \text{ kg/s}$$

where v_1 is the flow speed in the hose, $A_1 = \pi(d_1/2)^2,$ with $d_1 = 10$ cm, and $\rho = 1.0\times10^3 \text{ kg/m}^3$ is the density of fresh water. Applying Equation 15.5 to the hose and to the nozzle gives

$$v_1 A_1 = v_2 A_2$$

which allows us to solve for the flow speed v_2 at the nozzle.

EVALUATE **(a)** In the hose, the flow speed is

$$v_1 = \frac{R_\text{m}}{\rho A_1} = \frac{R_\text{m}}{\rho\pi(d_1/2)^2} = \frac{22 \text{ kg/s}}{\pi\left(1.0\times10^3 \text{ kg/m}^3\right)}\left(\frac{2}{0.10 \text{ m}}\right)^2 = 2.8 \text{ m/s}$$

(b) In the nozzle, the flow speed is

$$v_2 = v_1\frac{A_1}{A_2} = v_1\left(\frac{d_1}{d_2}\right)^2 = (2.8 \text{ m/s})\left(\frac{10 \text{ cm}}{2.1 \text{ cm}}\right)^2 = 64 \text{ m/s}$$

ASSESS By narrowing the nozzle, the exit speed of the fluid is increased by over an order of magnitude, which gives the exiting water greater linear momentum so that it travels farther through the air.

31. **INTERPRET** This problem deals with flow speed of a fluid, which in this case is the blood in the artery. The key concepts involved are mass conservation and the continuity equation.

DEVELOP Apply the continuity Equation 15.5 vA = constant. Without the clot, we have $v_1 A_1$ = constant. With the clot, we have $v_2 A_2$ = constant, where $A_2 = 0.20A_1$. Because the constant is the same, we can equate these two expressions for the volume flow rate and solve for v_2.

EVALUATE Solving for v_2 and inserting the given quantities gives

$$v_2 = v_1\left(\frac{A_1}{A_2}\right) = v_1\left(\frac{A_1}{0.20A_1}\right) = \left(\frac{0.35\,\text{m/s}}{0.20}\right) = 1.75\,\text{m/s} \approx 1.8\,\text{m/s}$$

ASSESS The flow speed of blood increases in the region where the cross-sectional area of the artery has been reduced due to clotting. Notice that the initial diameter of the artery is not needed to solve this problem; it suffices to know the ratio of the arterial cross sections.

EXAMPLE VARIATIONS

32. **INTERPRET** This problem involves the balance of the buoyancy and gravitational forces exerted on of a floating object of a given density.

DEVELOP Archimedes' principle applies here and states that the buoyancy force is equal to the weight of water displaced by the submerged portion of the iceberg. Our plan is to find the gravitational and buoyancy forces, and then equate their magnitudes to get the submerged volume. Since we're looking for volume, we'll write any masses as products of density and volume.

EVALUATE The iceberg's weight is $w_{\text{ice}} = m_{\text{ice}}g = \rho_{\text{ice}}V_{\text{ice}}g$, where V_{ice} is the volume of the entire iceberg. Only the submerged portion displaces water, so the volume of displaced water is V_{sub}, and the weight of the displaced water is therefore $w_{\text{water}} = m_{\text{water}}g = \rho_{\text{water}}V_{\text{sub}}g$. By Archimedes' principle, w_{water} is equal in magnitude to the buoyancy force, which balances gravity when the iceberg is in equilibrium. Equating the two gives $\rho_{\text{water}}V_{\text{sub}}g = \rho_{\text{ice}}V_{\text{ice}}g$, which we solve to get

$$\frac{V_{\text{sub}}}{V_{\text{ice}}} = \frac{\rho_{\text{ice}}}{\rho_{\text{water}}} = \frac{952\,\text{kg/m}^3}{1030\,\text{kg/m}^3} = 0.925$$

ASSESS Our result means that 92.5% of the iceberg's volume is under water, leaving only 7.5% showing.

33. **INTERPRET** This problem involves the balance of the buoyancy and gravitational forces exerted on a floating object of a given density.

DEVELOP Much like in the original example, the ratio of the submerged volume to the total volume is equal to the ratio of the iceberg's density to that of ocean water. We are given the submerged portion of the composite iceberg floating in ocean water, so we can determine its density. We are also given the total mass, and the densities of pure ice and rock embedded within it, so we can determine their relative masses.

EVALUATE Using the results of the original example, where the buoyant force has been equated to the gravitational force, we find the composite iceberg's density is equal to

$$\frac{\rho_{\text{iceberg}}}{\rho_{\text{water}}} = 0.955 \rightarrow \rho_{\text{iceberg}} = 0.955\rho_{\text{water}} = 983.7\,\text{kg/m}^3$$

We know that volume and mass of the iceberg is equal to the sum of those of the pure ice and rock which are contained within it

$$V_{\text{iceberg}} = V_{\text{ice}} + V_{\text{rock}} \rightarrow \frac{m_{\text{iceberg}}}{\rho_{\text{iceberg}}} = \frac{m_{\text{ice}}}{\rho_{\text{ice}}} + \frac{m_{\text{rock}}}{\rho_{\text{rock}}}$$

$$m_{\text{iceberg}} = m_{\text{ice}} + m_{\text{rock}} = 138{,}000\,\text{tons}$$

From these, we can develop an expression for one of the masses (e.g., just the ice) in terms of the known quantities, and find the mass of the other portion (the rock portion) by subtracting from the total mass. We begin by eliminating the mass of the rock, then solving for the mass of the ice. Then we plug in the given and calculated quantities.

$$\frac{m_{\text{iceberg}}}{\rho_{\text{iceberg}}} = \frac{m_{\text{ice}}}{\rho_{\text{ice}}} + \frac{(m_{\text{iceberg}} - m_{\text{ice}})}{\rho_{\text{rock}}}$$

$$m_{\text{ice}}\left(\frac{1}{\rho_{\text{ice}}} - \frac{1}{\rho_{\text{rock}}}\right) = m_{\text{iceberg}}\left(\frac{1}{\rho_{\text{iceberg}}} - \frac{1}{\rho_{\text{rock}}}\right)$$

$$m_{\text{ice}}\left(\frac{\rho_{\text{rock}}-\rho_{\text{ice}}}{\rho_{\text{ice}}\rho_{\text{rock}}}\right)=m_{\text{iceberg}}\left(\frac{\rho_{\text{rock}}-\rho_{\text{iceberg}}}{\rho_{\text{iceberg}}\rho_{\text{rock}}}\right)$$

$$m_{\text{ice}}=m_{\text{iceberg}}\left(\frac{\rho_{\text{ice}}}{\rho_{\text{iceberg}}}\right)\left(\frac{\rho_{\text{rock}}-\rho_{\text{iceberg}}}{\rho_{\text{rock}}-\rho_{\text{ice}}}\right)=124{,}000\text{ tons}$$

$$m_{\text{rock}}=m_{\text{iceberg}}-m_{\text{ice}}=14{,}000\text{ tons}$$

ASSESS The rock makes up about 10% of the total mass and about 4% of the total volume in the iceberg.

34. **INTERPRET** This problem involves the balance of the buoyancy and gravitational forces exerted on a floating object of a given density.

DEVELOP Much like in the original example, the ratio of the submerged volume to the total volume is equal to the ratio of the slab's density to that of mercury. We are given the mass and dimensions of the slab, so we can determine its density. We are also given the density of mercury, so we can find the percentage of the slab's volume which is submerged.

EVALUATE Using the results of the original example, where the buoyant force has been equated to the gravitational force, we find the ratio is equal to

$$\frac{V_{\text{sub}}}{V_{\text{slab}}}=\frac{\rho_{\text{slab}}}{\rho_{\text{mercury}}}=\frac{m_{\text{slab}}/V_{\text{slab}}}{\rho_{\text{mercury}}}=\frac{(1700\,\text{kg})/(1.5\,\text{m})^2(0.30\,\text{m})}{(13.69\times10^3\,\text{kg/m}^3)}=0.18$$

We find the 18% of the slab's volume was below the surface or mercury.

ASSESS Equating the first and third terms in our expression above might seem strange, since it relates the submerged slab volume to the ratio of slab mass and mercury density: $V_{\text{sub}}=m_{\text{slab}}/\rho_{\text{mercury}}$. However, looking at the expressions obtained from the equation of forces that lead us to the ratio of volumes, we see these terms are indeed equivalent.

35. **INTERPRET** This problem involves the balance of the buoyancy and gravitational forces exerted on a floating object whose volume we wish to find.

DEVELOP Much like in the original example, the ratio of the submerged volume to the total volume is equal to the ratio of the probe's density to that of the lake. We are given the mass and some of the slab's dimensions. We are also given the density of the lake, and we are told the percentage of the probe's volume which should be submerged. Thus, we can calculate the missing dimension (L_{probe}) of the probe that would result in such a submersion.

EVALUATE Using the results of the original example, where the buoyant force has been equated to the gravitational force, we find the ratio is equal to

$$\frac{V_{\text{sub}}}{V_{\text{probe}}}=\frac{\rho_{\text{probe}}}{\rho_{\text{lake}}}=\frac{m_{\text{probe}}/V_{\text{probe}}}{\rho_{\text{lake}}}=\frac{1}{2}$$

$$V_{\text{probe}}=\frac{2m_{\text{probe}}}{\rho_{\text{lake}}}\rightarrow L_{\text{probe}}=\frac{2m_{\text{probe}}}{\left(\pi R_{\text{probe}}^2\right)\rho_{\text{lake}}}=2.25\,\text{m}$$

ASSESS From this expression we can see that for the given dimensions of the probe's diameter, the length of the probe is directly proportional to the denominator of the fraction submerged. Had the scientists wanted the probe to be ¼ submerged, the length would need to increase by a factor of 2.

36. **INTERPRET** We're dealing with a flow of water, an incompressible liquid. So, we can apply our problem-solving strategy for fluid dynamics.

DEVELOP We're interested in the water's velocity at the hole, so the hole is one of the points we'll use in the fluid equations. Both the hole and the top of the tube are at atmospheric pressure p_{a}. As it's described in the original example, we can make the approximation $v=0$ at the top—and thus we know both p and v at the top. Lastly, we set the potential zero to be at the location of the hole, so the potential energies at the hole and at the top are, respectively, equal to 0 and ρgh. Knowing this, we see Bernoulli's equation becomes

$$p_a + \rho g h = p_a + \frac{1}{2}\rho v_{\text{hole}}^2$$

Where the terms on the left are the top surface and those on the right are at the hole.

EVALUATE Atmospheric pressure cancels, and we solve for the unknown flow velocity at the hole:

$$v_{\text{hole}} = \sqrt{2gh} = \sqrt{(2)\left(9.8\,\text{m/s}^2\right)(2.68\,\text{m})} = 7.25\,\text{m/s}$$

ASSESS This is the same result we would get by dropping an object from a height h—and for the same reason: conservation of energy.

37. **INTERPRET** We're dealing with a flow of water, an incompressible liquid. So, we can apply our problem-solving strategy for fluid dynamics.

DEVELOP We want to use water's velocity at the hole to determine the size of the tube, so the hole is one of the points we'll use in the fluid equations. Both the hole and the top of the tube are at atmospheric pressure p_a. As it's described in the original example, we can make the approximation $v = 0$ at the top—and thus we know both p and v at the top. Lastly, we set the potential zero to be at the location of the hole, so the potential energies at the hole and at the top are, respectively, equal to 0 and $\rho g h$. Knowing this, we see Bernoulli's equation becomes

$$p_a + \rho g h = p_a + \frac{1}{2}\rho v_{\text{hole}}^2$$

Where the terms on the left are the top surface and those on the right are at the hole.

EVALUATE Atmospheric pressure cancels, and we solve for the unknown depth of the tank:

$$h = \frac{v_{\text{hole}}^2}{2} = \frac{(5.46\,\text{m/s})^2}{2} = 1.52\,\text{m}$$

ASSESS This is the same result we would get by dropping an object from a height h—and for the same reason: conservation of energy.

38. **INTERPRET** We're dealing with a flow of water, an incompressible liquid. So, we can apply our problem-solving strategy for fluid dynamics.

DEVELOP We're interested in the water's velocity at the hole, so the hole is one of the points we'll use in the fluid equations. The hole at the bottom is at atmospheric pressure p_a, while the top of the tank is exposed to air pressurized to p_p 186 kPa. As it's described in the original example, we can make the approximation $v = 0$ at the top—and thus we know both p and v at the top. Lastly, we set the potential zero to be at the location of the hole, so the potential energies at the hole and at the top are, respectively, equal to 0 and $\rho g h$. Knowing this, we see Bernoulli's equation becomes

$$p_p + \rho g h = p_a + \frac{1}{2}\rho v_{\text{hole}}^2$$

Where the terms on the left are the top surface and those on the right are at the hole.

EVALUATE Solving for the speed at which the water will initially emerge we obtain

$$v_{\text{hole}} = \sqrt{\frac{2\left(\left(p_p - p_a\right) + \rho g h\right)}{\rho}} = 14.9\,\text{m/s}$$

Where we have used $\rho = 1000\,\text{kg/m}^3$.

ASSESS This result makes intuitive sense since we would expect the flow speed to increase at the bottom when the top is exposed to a higher pressure.

39. **INTERPRET** We're dealing with a flow of water, an incompressible liquid. So, we can apply our problem-solving strategy for fluid dynamics.

DEVELOP We're interested in using the water's velocity at the end of the hose to determine the water pressure inside the container p_c, so the end of the hose is one of the points we'll use in the fluid equations, which is at atmospheric pressure p_a. As it's described in the original example, we can also make the approximation $v = 0$ at the top/inside the canister. Lastly, we are told to neglect pressure variations with height inside the container, so we can remove the potential-energy terms. Knowing this, we see Bernoulli's equation becomes

$$p_c = p_a + \frac{1}{2}\rho v_{\text{hole}}^2$$

Where the term on the left is inside the canister and those on the right are at the end of the hose.

EVALUATE Evaluating the pressure inside the canister for which the water will initially emerge at the given speed, we obtain

$$p_c = p_a + \frac{1}{2}\rho v_{\text{hole}}^2 = 278\,\text{kPa}$$

ASSESS The pressure inside the container is approximately 2.74 times that of atmospheric pressure.

PROBLEMS

40. **INTERPRET** This problem involves calculating the surface area required for the given force to produce the given pressure. The force in question is the force exerted by gravity on the two people lying on the bed, and the pressure is the pressure the water exerts on the bed lining.

DEVELOP Pressure is the force per unit area, or $p = F / A$ (Equation 15.1). When the two people lie on the bed, they exert a pressure due to gravity, given by $F = mg$, where $m = 150$ kg. Given that this leads to a pressure increase of Δp 4700 Pa, we can solve for the area over which their bodies are in contact with the bed.

EVALUATE The area is

$$A = \frac{F}{\Delta p} = \frac{mg}{\Delta p} = \frac{(150\text{ kg})\left(9.8\text{ m/s}^2\right)}{4700\text{ Pa}} = 0.31\text{ m}^2$$

ASSESS Considering that a typical height for a female is 1.6 m and for a male is 1.8 m, the width over which their bodies are in contact with the bed is estimated to be

$$w = \frac{A}{L} = \frac{0.31\text{ m}^2}{1.6\text{ m} + 1.8\text{ m}} = 9.1\text{ cm}$$

which seems reasonable, given that this is an average width (it is no doubt greater at the shoulders and less at the feet).

41. **INTERPRET** This problem involves calculating the area needed for a given pressure to produce a given force. We are given the mass and the gauge pressure of the tires, and we want to find the total tire area that's in contact with the road.

DEVELOP As shown in Equation 15.1, pressure measures the normal force per unit area exerted by a fluid. For this problem, the fluid is air. The force exerted on the road by the tires is the weight of the car, $F = mg$.

EVALUATE With a gauge pressure of $p = 270$ kPa, the contact area is

$$A = \frac{F_g}{p} = \frac{mg}{p} = \frac{(1950\text{ kg})\left(9.8\text{ m/s}^2\right)}{27\times10^4\text{ Pa}} = 0.0708\text{ m}^2 = 708\text{ cm}^2$$

ASSESS Our result implies that the contact area of each wheel is about 175 cm^2, or the area of a $22 \times 8\text{ cm}^2$ rectangular surface, which seems reasonable.

42. **INTERPRET** You want to know if your seatmate could potentially pull the emergency window inward, given that the pressure inside the plane is higher than the pressure outside.

DEVELOP The force from the outside air pushing in on the window is $F_{\text{out}} = P_{\text{out}}A.$ The force from the inside air pushing out on the window is $F_{\text{in}} = P_{\text{in}}A.$ Since the inside pressure is higher, there will be a net force pushing out on the window, and your seatmate will have to overcome that force to open the window.

EVALUATE The minimum force needed to pull in the window is

$$F \ge (P_{\text{out}} - P_{\text{in}})A = (0.8\text{ atm} - 0.2\text{ atm})\left[\frac{101.3\text{ kPa}}{1\text{ atm}}\right](0.5\text{ m}\times 0.6\text{ m}) = 18\text{ kN}$$

This would be like trying to lift over 1800 kg, roughly the combined weight of 23 adults, so it's not likely that your seatmate will be able to open the window.

43. **INTERPRET** We have an open tube filled with water at the bottom and oil on top of the water. The two fluids do not mix. We want to find the gauge pressures at the oil-water interface as well as at the bottom of the tube

DEVELOP The pressure pushing down on the oil at the top of the tube is atmospheric pressure, p_{atm}. The gauge pressure at the interface of the oil and the water is the difference between the absolute pressure and the atmospheric pressure, or $\Delta p = p_{\text{i}} - p_{\text{atm}} = \rho_{\text{oil}} g h_{\text{oil}}$. To find h_{oil}, note that

$$m_{\text{oil}} = \rho_{\text{oil}} V_{\text{oil}} = \rho_{\text{oil}} A_{\text{tube}} h_{\text{oil}} = 5.6 \text{ g}$$

where V_{oil} is the volume of oil and A_{tube} is the cross-sectional area of the tube. The gauge pressure at the bottom is the total weight of the fluid divided by the cross-sectional area of the tube, which is

$$\Delta p_{\text{bot}} = \frac{m_{\text{w}} g}{A_{\text{tube}}} + \frac{m_{\text{oil}} g}{A_{\text{tube}}}$$

EVALUATE **(a)** The gauge pressure at the interface is

$$\Delta p = p_{\text{i}} - p_{\text{atm}} = \rho_{\text{oil}} g h_{\text{oil}} = \frac{m_{\text{oil}} g}{A_{\text{tube}}} = \frac{(0.0056 \text{ kg})(9.8 \text{ m/s}^2)}{\pi(0.017 \text{ m})^2/4} = 240 \text{ Pa}$$

(b) The gauge pressure at the bottom is the total weight of the fluid divided by the cross-sectional area of the tube,

$$\Delta p_{\text{bot}} = (m_{\text{w}} + m_{\text{oil}}) \frac{g}{A_{\text{tube}}} = (0.0056 \text{ kg} + 0.0056 \text{ kg}) \left(\frac{9.8 \text{ m/s}^2}{\pi(0.017 \text{ m})^2/4} \right) = 480 \text{ kPa}$$

ASSESS Oil floats on top of water because its density is lower than that of water. The gauge pressure at the bottom of the tube is due to the weight of both the oil and the water. The absolute pressure there would be equal to the sum of the gauge pressure and the atmospheric pressure. Note that because $m_{\text{w}} = m_{\text{oil}}$ in this problem, the gauge pressure at the bottom is just twice that at the interface.

44. **INTERPRET** You must determine the total force on a dam from the water in a lake. In this case, the pressure from the water is not constant, but varies with depth.

DEVELOP The water pressure is given by Equation 15.3: $p = p_0 + \rho g h$. You can assume the pressure at the surface is $p_0 = 1$ atm. To find the total force of the water on the dam, divide the dam into thin strips of height dh that stretch along the dam's width L. The force on a given strip is $dF = pdA = pLdh$, and the total force is the integral over the full depth of the lake, H.

EVALUATE The water exerts a force on the dam of

$$F = \int dF = \int_0^H pLdh = p_0 LH + \tfrac{1}{2}\rho g L H^2 = \left(p_0 + \tfrac{1}{2}\rho g H\right) LH$$
$$= \left[(101.3 \text{ kPa}) + \tfrac{1}{2}(1000 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(95 \text{ m})\right](1500\text{m} \times 95\text{m}) = 81 \text{ GN}$$

If the force increased by 50%, it would be 120 GN. Therefore, the dam needs to be reinforced to withstand 20 GN more.

ASSESS The calculated force is equivalent to the pressure at a depth of $\tfrac{1}{2}H$ multiplied by the total area of the dam, LH. In other words, the linear dependence of pressure with depth implies that the average pressure is equal to the pressure at the midpoint.

45. **INTERPRET** The U tube contains two liquids, oil and water, in hydrostatic equilibrium. We want to find their height difference.

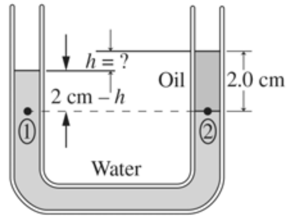
DEVELOP The pressure at point 2 in the figure below, which is the oil-water interface, is

$$p_2 = p_{\text{atm}} + \rho_{\text{oil}} g l$$

where $l = 2.0$ cm. The pressure at point 1, which is at the same height as point 2, is

$$p_1 = p_{\text{atm}} + \rho_{\text{w}} g (l - h)$$

From Equation 15.3, $p = p_0 + \rho g h$, we see that the pressure at points at the same height is the same, so $p_1 = p_2$. Using the information that $\rho_{\text{oil}} = 0.92 \rho_{\text{w}}$ allows us to solve for h.



EVALUATE Equating the two pressures leads to $\rho_{\text{oil}} g l = \rho_{\text{w}} g(l-h)$ or

$$h = \left(1 - \frac{\rho_{\text{oil}}}{\rho_{\text{w}}}\right) l = \left(1 - \frac{0.92\rho_{\text{w}}}{\rho_{\text{w}}}\right) l = (1-0.92)(2.0\text{ cm}) = 1.6\text{ mm}$$

ASSESS Note that the final answer does not depend on the atmospheric pressure, p_{atm}, because this pressure pushes down equally on both the oil and the water. The U tube can be used to measure the density of a fluid if we know the height difference h and the density of the other fluid.

46. **INTERPRET** The hydraulic system you are designing must have tubes that support a certain amount of pressure when the fluid inside them is compressed by the hydraulic cylinder.

DEVELOP Given the force from the cylinder, the pressure in the fluid will be $p = F/\pi r^2$, where *r* is the radius of the cylinder $(r = d/2)$.

EVALUATE The pressure of the fluid in the hydraulic system will be

$$p = \frac{F}{\pi (d/2)^2} = \frac{6.1\text{ kN}}{\pi (6.0\text{ cm}/2)^2} = 2.2\text{ MPa}$$

To be safe, you choose tubing that can withstand at least 50% more pressure than calculated, that is, 3.3 MPa. Since the tubing is sold only in multiples of ½ MPa, you choose 3.5 MPa.

ASSESS The answer seems reasonable. The units work out since $1\text{ Pa} = 1\text{ N/m}^2$.

47. **INTERPRET** This problem involves calculating the pressure given the force and the area over which the force is applied.

DEVELOP The pressure maintained within the hydraulic brake system is the constant, while the magnitude of the force exerted by/on either end will be determined by the size of the piston/cylinder that exert the said force. Knowing the size of the pistons/cylinders on either end, we can use Equation 15.1 to find the relationship between the forces exerted on either end. Knowing these, we can use conservation of energy to relate the relative motion of each piston when a force is exerted on one end.

EVALUATE Since the pressure is the same throughout, we relate the ratio of force to surface area at each end by

$$p = \frac{F_{\text{s}}}{A_{\text{s}}} = \frac{F_{\text{l}}}{A_{\text{l}}} \to \frac{F_{\text{s}}}{r_{\text{s}}^2} = \frac{F_{\text{l}}}{r_{\text{l}}^2}$$

Where the subscripts s and l correspond to the small and large pistons/cylinders, respectively. Thus, the force F_{s} necessary on the small piston to get a force F_{l} on the large piston is equal to

$$F_{\text{s}} = \frac{r_{\text{s}}^2}{r_{\text{l}}^2} F_{\text{l}} = \frac{(0.0052\,\text{m})^2}{(0.0105)^2}(3.25\,\text{kN}) = 798\,\text{N}$$

If the small piston moves by a certain amount x_{s} as a force F_{s} is applied on it, then the work done on it will be given by $W_{\text{s}} = F_{\text{s}} x_{\text{s}}$. This piston will in turn do the same amount of work on the water, and the water will do the same amount of work on the large piston, where it applies a force F_{l} causing it to move a distance x_{l}. Equating these two shows the distance by which the large piston moves is equal to

$$x_{\text{l}} = \frac{F_{\text{s}}}{F_{\text{l}}} x_{\text{s}} = \frac{(798\,\text{N})}{(3.25\times 10^3\text{ N})}(8.80\times 10^{-3}\text{ m}) = 2.16\,\text{mm}$$

We see that amount of work done on both pistons is equal to $F_{\text{s}} x_{\text{s}} = F_{\text{l}} x_{\text{l}} = 7.03\,\text{J}$

ASSESS The factor by which the applied force is magnified is given by the square of the ratio between output and input cylinders.

48. **INTERPRET** This problem involves using Archimedes' principle to calculate the apparent weight of an object in water given its composition.

DEVELOP This problem is similar to Problem 15.25, where we found the following result for the apparent weight of an object in water:

$$w' = w - F_b = w - \left(\frac{\rho_w}{\rho}\right)w = \left(1 - \frac{\rho_w}{\rho}\right)w$$

where w and w' are the true weight and apparent weight, respectively. For the alloy crown, the density is

$$\rho_{\text{alloy}} = \frac{m_G + m_S}{V} = \frac{V_G\rho_G + V_S\rho_S}{V}$$

where the subscripts G and S refer to gold and silver, respectively. Using the information that $V_G = 0.75V$ and $V_S = 0.25V$, this gives

$$\rho_{\text{alloy}} = 0.75\rho_G + 0.25\rho_S$$

which we can use in the expression above for the apparent weight.

EVALUATE **(a)** For the pure gold crown, the apparent weight is

$$w'_G = \left(1 - \frac{\rho_w}{\rho_G}\right)w = \left(1 - \frac{1.00\ \text{g/cm}^3}{19.3\ \text{g/cm}^3}\right)(25.0\ \text{N}) = 23.7\ \text{N}$$

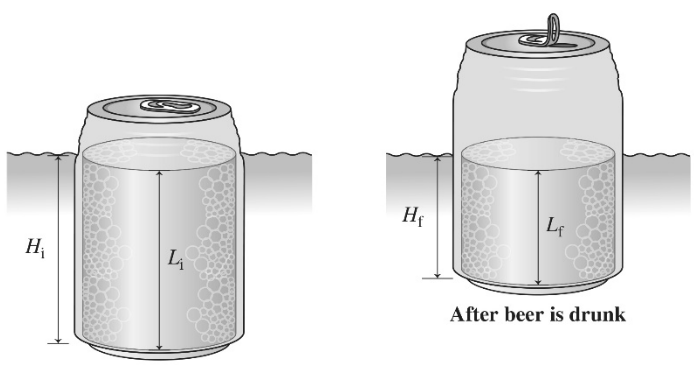
(b) The apparent weight of the alloy crown is

$$w'_{\text{alloy}} = \left(1 - \frac{\rho_w}{\rho_{\text{alloy}}}\right)w = \left(1 - \frac{1.00\ \text{g/cm}^3}{(0.750)(19.3\ \text{g/cm}^3) + (0.250)(10.5\ \text{g/cm}^3)}\right)(25.0\ \text{N}) = 23.5\ \text{N}$$

ASSESS This corresponds to ~0.8% difference.

49. **INTERPRET** You're asked to determine how much the accused person drank, given the change in the buoyant force of a keg of beer.

DEVELOP By Archimedes' principle, the buoyant force on the keg is equal to the weight of the fluid displaced. The keg is probably made of aluminum and is filled with a mixture of beer and air. Let's assume that the level of the beer in the keg was initially L_i, and the keg was submerged in the water to a depth of H_i, as shown in the figure below.



Before beer is drunk

After beer is drunk

As such, the initial volume of beer would be $\pi r^2 L_i$, and the initial volume of water displaced would be $\pi r^2 H_i$ (where we have assumed the aluminum shell is thin enough that the interior and exterior radii are essentially the same). The total weight of the keg and beer (neglecting the weight of the air) is balanced by the buoyant force, which equals the weight of the water displaced.

$$F_g = \left(m_{\text{keg}} + \rho_{\text{beer}}\left(\pi r^2 L_i\right)\right)g = F_b = \rho_{\text{water}}\left(\pi r^2 H_i\right)g$$

A similar equation can be written for the keg at the end of the day, using L_f for the final level of beer and H_f for the final depth. Subtracting these two equations gives

$$L_i - L_f = H_i - H_f$$

where we have used the fact that $\rho_{beer} \approx \rho_{water}$.

EVALUATE You are told that the keg rose by 1.2 cm, so the level of beer fell by that same amount. This corresponds to a volume of beer:

$$\Delta V = \pi r^2 (L_i - L_f) = \pi\left(\tfrac{1}{2} \cdot 40 \text{ cm}\right)^2 (1.2 \text{ cm}) = 1.5 \text{ L}$$

where we've used the conversion $1 \text{ cm}^3 = 1 \text{ mL}$. In terms of English units $(1 \text{ L} = 33.8 \text{ oz})$, the accused drank 51 oz, which would imply that he or she was legally impaired.

ASSESS The defendant may want a more careful analysis, taking into account the thickness of the keg's outer shell, but this would actually increase the estimated beer volume that he or she drank.

50. **INTERPRET** This problem involves Archimedes' principle for floating objects, which we can use to find the mass of the beaker, and, subsequently, the number of 16-g rocks we can put in the beaker before it sinks.

DEVELOP Archimedes' principle for floating objects tells us that the weight of the beaker equals the weight of water displaced by one-third of its volume, so the maximum weight of rocks the beaker can carry and still float is equal to the weight of water displaced by two-thirds of the beaker's volume, or

$$nm_R g \le \frac{2}{3} V \rho_w g$$

where $m_R = 16 \text{ g}$ is the mass of each rock.

EVALUATE Solving the expression above for n gives

$$n \le \frac{2V\rho_w}{m_R} = \frac{2\pi(3.5 \text{ cm})^2 (12 \text{ cm})(1.0 \text{ g/cm}^3)}{3(16 \text{ g})} = 19.2$$

So, 19 rocks is the maximum number the beaker can support before sinking.

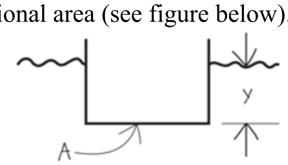
ASSESS Because n is an integer, we do not need to report it to the two significant figures that is warranted by the data.

51. **INTERPRET** This problem involves applying Archimedes' principle to find the minimum water depth for the load-carrying ship to navigate.

DEVELOP Archimedes' principle states that the buoyancy force is equal to the weight of the water displaced by the floating supertanker:

$$F_b = m_w g = \rho_w g V = \rho_w g A y$$

where we have used the fact that the volume V of water displaced is proportional to its draft (depth y in the water); $V = Ay$, where A is the cross-sectional area (see figure below).



EVALUATE Because the total mass of the full supertanker is three times that when empty, three times the buoyancy force is needed to support the ship. From the expression above, we see that the buoyancy force is proportional to the draft y, so y must increase threefold. Thus, $y_{full} = 3y_{empty} = 3(8.0 \text{ m}) = 24 \text{ m}$.

ASSESS Our result is independent of the mass of the supertanker. The heavier the supertanker plus load, the deeper it will submerge.

52. **INTERPRET** This problem is about the buoyancy force provided by a balloon filled with gas.

DEVELOP By Archimedes' principle, the balloon's buoyant force will be equal to the weight of air it displaces. We want this force to be greater than the weight of the mass M plus the mass m of the gas:

$$F_b = \rho_a g V \ge (M + m) g$$

We aren't given the volume of the balloon, but if we neglect the thickness of the balloon's outer shell, then we can assume $V = m/\rho_g$.

EVALUATE Solving for the minimum mass, *m*, gives

$$m \geq M \frac{\rho_{\rm g}}{\rho_{\rm a} - \rho_{\rm g}}$$

ASSESS The less dense the gas, the less of it we need. But note that the balloon cannot lift up if $\rho_{\rm g} \geq \rho_{\rm a}$, as we would imagine.

53. **INTERPRET** This problem is about the buoyancy force provided by the helium balloon and the hot air balloon, which we can use to calculate how much He is needed to lift a given mass.

DEVELOP For the balloon to lift off, the buoyancy force must exceed the weight of the load (mass *M*, including the balloon) plus the gas (mass *m*):

$$F_{\rm b} \geq (M + m)g$$

where the buoyancy force is simply equal to $F_{\rm b} = \rho_{\rm air} g V$. If we neglect the volume of the balloon's skin and others compared to that of the gas it contains, then $V = m/\rho_{\rm gas}$. Therefore,

$$m = \rho_{\rm gas} V = \rho_{\rm gas} \frac{F_{\rm b}}{\rho_{\rm air} g} \geq \frac{\rho_{\rm gas}}{\rho_{\rm air}}(M + m)$$

$$m \geq \left(\frac{\rho_{\rm gas}}{\rho_{\rm air} - \rho_{\rm gas}} \right) M$$

EVALUATE **(a)** When the gas is helium, the density ratio is $\rho_{\rm air}/\rho_{\rm He} = (1.2\ {\rm kg/m^3})/(0.18\ {\rm kg/m^3}) = 6.67,$ and

$$m \geq \left(\frac{\rho_{\rm gas}}{\rho_{\rm air} - \rho_{\rm gas}} \right) M = \left(\frac{1}{6.67 - 1} \right)(280\ {\rm kg}) = 49\ {\rm kg}$$

(b) For hot air, $\rho_{\rm gas} = 0.9\rho_{\rm air},$ and

$$m \geq \left(\frac{\rho_{\rm gas}}{\rho_{\rm air} - \rho_{\rm gas}} \right) M = \left(\frac{0.9}{1 - 0.9} \right)(280\ {\rm kg}) = 2500\ {\rm kg}$$

ASSESS These masses correspond to gas volumes of 275 m^3 for helium and 2330 m^3 for hot air, which are reasonable for a helium-filled balloon and a hot air balloon.

54. **INTERPRET** This problem involves calculating the change in pressure exerted on a floating surface by an increasing force, and determining the amount needed to lower its depth by a given amount.

DEVELOP Equation 15.2, $dp/dh = \rho g$, relates the rate of change in pressure p with depth h to the fluid density ρ and the gravitational acceleration g . We are told the dimensions of the float, along with its mass M and that of the children who will be on top of it, m . Thus, we can relate this change in pressure to the maximum desired change in depth (the thickness of the float). We want to find the maximum number n of children who can stay afloat before the depth changes by an amount equal to the thickness of the float.

EVALUATE We express the change in pressure as a function of the number n of people who lie on top, plug it into Equation 15.2, solve for the number n

$$dp = \frac{dF}{A} = \frac{(M + nm)g}{(l \times w)} = \rho g\, dh$$

$$n = \frac{[(\rho\, dh)(l \times w) - M]}{m} = \frac{[(1000\,{\rm kg/m^3})(0.1\,{\rm m})(1.8\,{\rm m} \times 2.4\,{\rm m}) - (20\,{\rm kg})]}{(50\,{\rm kg})} = 8$$

The maximum number of children the float accommodates before the water comes over its top surface is 8.

ASSESS The float can hold approximately 410 kg on top before sinking a depth equal to its thickness.

55. **INTERPRET** This problem deals with flow speed of a fluid, which in this case is the blood in the artery. The key point involved here is Bernoulli's equation.

DEVELOP The continuity equation, $vA = \text{constant},$ as given in Equation 15.5, is a reasonable approximation for blood circulation in an artery. Neglecting any pressure differences due to height, we find, from Bernoulli's equation, that

$$p + \frac{1}{2}\rho v^2 = p' + \frac{1}{2}\rho v'^2$$

EVALUATE We're told that the clot reduces the cross-sectional area by 80%, so $A' = 0.20A$, and

$$v' = v\left(\frac{A}{A'}\right) = (0.35 \text{ m/s})\frac{A}{0.20A} = 1.75 \text{ m/s}$$

From Bernoulli's equation, the gauge pressure at the clot is

$$p' = p + \tfrac{1}{2}\rho\left(v^2 - v'^2\right) = 16 \text{ kPa} + \tfrac{1}{2}\left(1.06\frac{\text{g}}{\text{cm}^3}\right)\left[(0.35\text{m/s})^2 - (1.75\text{m/s})^2\right] = 14 \text{ kPa}$$

ASSESS The flow speed of blood increases in the region where the cross-sectional area of the artery has been reduced due to clotting. Since $p + \frac{1}{2}\rho v^2 = \text{constant},$ the gauge pressure must decrease.

56. **INTERPRET** You're consulting a maple syrup company. Their system has a thin vertical tube that acts as a kind of barometer. They've asked you to derive a formula relating the height in the tube to the volume flow rate.
DEVELOP The Bernoulli equation relates the syrup flow through the pipe at the height h_1, to the syrup flow at the base of the thin glass tube:

$$p + \frac{1}{2}\rho v^2 + \rho g h_1 = p' + \frac{1}{2}\rho v'^2$$

The pressure at height h_1 is equal to atmospheric pressure: $p = p_0$, while the pressure at the base of the thin glass tube satisfies Equation 15.3: $p = p_0 + \rho g h_2$. Because the syrup is essentially incompressible, the volume flow rate, Av, is constant through the pipe, which means the speeds are related by:

$$v' = \frac{A}{A'}v = 2v$$

EVALUATE Putting together the information above, the Bernoulli equation simplifies to:

$$\frac{1}{2}\left[(2v)^2 - v^2\right] = g\left(h_1 - h_2\right)$$

The volume flow rate is therefore:

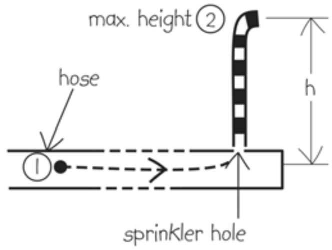
$$Av = A\sqrt{\frac{2}{3}g\left(h_1 - h_2\right)}$$

ASSESS If the height in the vertical tube were equal to h_1, then the volume flow rate would be zero. The maximum volume flow rate would correspond to $h_2 = 0$.

57. **INTERPRET** This problem involves the flow of water, which we can consider to be an incompressible fluid. Bernoulli's equation allows us to find the maximum height reached by the water coming out from the hose.
DEVELOP Make a sketch of the situation (see figure below). The flow of water in the hose can be described by Bernoulli's equation (Equation 15.6):

$$p + \frac{1}{2}\rho v^2 + \rho g y = \text{constant}$$

The pressure, velocity, and height of the water in the hose (point 1) are $p_1 = p_{\text{atm}} + \Delta p_1 = p_{\text{atm}} + 120 \text{ kPa},$ $v_1 \approx 0,$ and $y_1 = 0,$ respectively. At the highest point attained by a jet of water emerging from a hole (point 2), $p_2 = p_{\text{atm}},$ $v_2 \approx 0,$ and $y_2 = h.$ We can equate the result of Bernoulli's equation at points 1 and 2 to find h.



EVALUATE Using Bernoulli's equation, we have

$$p_1 = p_2 + \rho gh$$
$$p_{\rm atm} + \Delta p_1 = p_{\rm atm} + \rho gh$$
$$h = \frac{\Delta p_1}{\rho g} = \frac{120\ \text{kPa}}{\left(1.0\times 10^3\ \text{kg/m}^3\right)\left(9.8\ \text{m/s}^2\right)} = 12\ \text{m}$$

ASSESS At the maximum height, all the work done by pressure has been converted to potential energy of the fluid. Energy is conserved in the process (ignoring dissipative forces such as air resistance).

58. **INTERPRET** This problem involves the flow of an (essentially) incompressible fluid, so we can apply Bernoulli's equation to find the volume flow rate of water through the solar collector pipe.

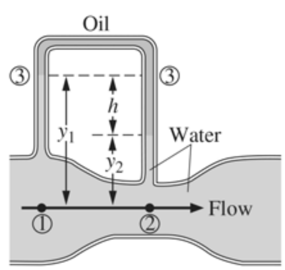
DEVELOP Make a sketch of the situation (see figure below). Applying Bernoulli's equation (Equation 15.6) and the continuity equation (Equation 15.5) to points 1 and 2 in the flowmeter, we can calculate the volume rate of flow:

$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2 \text{ and } v_1 A_1 = v_2 A_2$$
$$p_1 - p_2 = \frac{1}{2}\rho\left(v_2^2 - v_1^2\right) = \frac{1}{2}\rho v_1^2 A_1^2\left(\frac{1}{A_2^2} - \frac{1}{A_1^2}\right)$$
$$\text{or } R_{\rm v} = v_1 A_1 = \pm\sqrt{\frac{2(p_1 - p_2)}{\rho\left(A_2^{-2} - A_1^{-2}\right)}}$$

where the positive or negative sign indicates that the flow rate can go in either direction. This is the same calculation as in Example 15.7. Note that pressure variation with height in the flowmeter is assumed to be negligible. The pressure difference $p_1 - p_2$ is related to the difference in height h and the density of oil $\rho_{\rm oil}$ in the manometer (where the fluid is assumed to be stagnant). In terms of the pressure at point 3 (on the left-hand side), the pressure at point 1 may be written as $p_1 = p_3 + \rho g y_1.$ Likewise, the pressure at point 2 may be written as $p_2 = p_3 + \rho g y_2 + \rho_{\rm oil} gh.$ Subtracting these two expressions gives

$$p_1 - p_2 = \left(\rho - \rho_{\rm oil}\right)gh$$

where we have used the fact that $y_1 - y_2 = h.$



EVALUATE Using $A = \pi d^2/4,$ we find the volume flow rate to be

$$R_{\rm v} = v_1 A_1 = \sqrt{\frac{2(p_1 - p_2)}{\rho\left(A_2^{-2} - A_1^{-2}\right)}} = \frac{\pi}{4}\sqrt{\frac{2\left(\rho - \rho_{\rm oil}\right)gh}{\rho\left(d_2^{-4} - d_1^{-4}\right)}}$$

$$= \frac{\pi}{4}\sqrt{\frac{2\left(1.0\text{ g/cm}^3 - 0.82\text{ g/cm}^3\right)\left(980\text{ cm/s}^2\right)\left(1.9\text{ cm}\right)}{\left(1.0\text{ g/cm}^3\right)\left[\left(0.64\text{ cm}\right)^{-4} - \left(1.9\text{ cm}\right)^{-4}\right]}} = 8.4\text{ cm}^3/\text{s}$$

ASSESS Can you convince yourself that the units of this expression are correct?

59. **INTERPRET** A narrower section is placed in a pipe carrying an incompressible fluid. We are to find the flow speed in the pipe and the volume flow rate, given the pressure difference between the fluid in the pipe and the fluid in the narrow section. We will assume that the flow is nonturbulent and use Bernoulli's equation. The velocity is related to the cross-sectional area by the continuity equation.

DEVELOP The pressure difference between the venturi and the unrestricted pipe is $\Delta P = 15$ kPa. The radius of the unconstricted pipe is $r_1 = 0.0115$ m, and the radius of the constricted region is $r_2 = 0.0065$ m. We will assume that any height changes are negligible and will take the density of water to be $\rho = 1.0\times10^3\text{ kg/m}^3$. Bernoulli's equation is $p + \rho gh + \frac{1}{2}\rho v^2 = \text{constant}$, and the continuity equation for incompressible fluids such as water is $v_1A_1 = v_2A_2$. We equate the result of Bernoulli's equation for the unrestricted pipe equal to that for the restricted pipe, with $h_1 = h_2$.

$$p_1 + \cancel{\rho g h_1} + \frac{1}{2}\rho v_1^2 = p_2 + \cancel{\rho g h_2} + \frac{1}{2}\rho v_2^2$$

Use the continuity Equation 15.5 (for liquid), $v_2 = v_1A_1/A_2$, and solve for v_1:

EVALUATE **(a)** The flow speed v_1 of the water is

$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\left(\frac{v_1A_1}{A_2}\right)^2 \Rightarrow p_1 - p_2 = \frac{1}{2}\rho v_1^2\left(\frac{A_1^2}{A_2^2} - 1\right)$$

$$v_1 = \pm\sqrt{\frac{2\left(p_1 - p_2\right)}{\rho\left[\left(r_1/r_2\right)^4 - 1\right]}} = \pm\sqrt{\frac{2\left(15\text{ kPa}\right)}{\left(1.0\times10^3\text{ kg/m}^3\right)\left(1.8^4 - 1\right)}} = \pm1.8\text{ m/s}$$

where the positive and negative signs indicate that the flow may be in either direction. Without loss of generality, we can use the positive value.

(b) To find the volume flow rate in m^3/s, we multiply the flow speed by the area of the pipe, which gives

$$R_V = v_1A_1 = v_1\pi\frac{d_1^2}{4} = \frac{\pi\left(1.8\text{ m/s}\right)\left(0.023\text{ m}\right)^2}{4} = 7.5\times10^{-4}\text{ m}^3/\text{s} = 0.75\text{ L/s}$$

ASSESS Both the flow speed and volume flow rate seem reasonable for a small pipe such as this one.

60. **INTERPRET** This problem involves Archimedes' principle, which we can apply to the balloon that is submerged in air to find the maximum weight that it can support.

DEVELOP For an object submerged in a liquid (we can consider air as a liquid for this problem because there is no motion nearing the speed of sound in air in the problem), the buoyancy force is given by mass of the liquid displaced by the object's volume, which is

$$F_b = \rho_{air}V_{balloon}g$$

The force due to gravity pulling the balloon toward the Earth is

$$F_g = \left(m_{He} + m_{balloon} + nm_{clip}\right)g = \left(V_{balloon}\rho_{He} + m_{balloon} + nm_{clip}\right)g$$

where n is the number of paper clips hanging from the balloon. For the balloon to remain airborne, $F_b > F_g$, which we can solve for n.

EVALUATE Solving the inequality for n and inserting the given values gives

$$\rho_{\rm air}V_{\rm balloon}g > \left(V_{\rm balloon}\rho_{\rm He} + m_{\rm balloon} + nm_{\rm clip}\right)g$$
$$n < \frac{\rho_{\rm air}V_{\rm balloon} - V_{\rm balloon}\rho_{\rm He} - m_{\rm balloon}}{m_{\rm clip}}$$

Given that the density of air under standard conditions is $\rho_{\rm air}$ = 1.2 kg/m^3 and that the $V_{\rm balloon} = 4\pi r^3/3$, we have

$$n < \frac{\left(4\pi r^3/3\right)\left(\rho_{\rm air} - \rho_{\rm He}\right) - m_{\rm balloon}}{m_{\rm clip}} = \frac{(4\pi/3)\left(0.14\ {\rm m}\right)^3\left(1.2\ {\rm kg/m^3} - 0.18\ {\rm kg/m^3}\right) - 0.0016\ {\rm kg}}{0.63\times10^{-3}\ {\rm kg}} = 16.1$$

Thus, the maximum number is 16, and 17 paper clips will cause the balloon to lose its buoyancy.

ASSESS With 16 paper clips attached, the balloon will rise until its average density equals the density of the surrounding air. Note that we have neglected the volume of the paper clips and of the rubber that makes up the balloon, both of which experience a small buoyancy force of their own.

61. **INTERPRET** This problem deals with blood flow through an artery that is obstructed by a plaque.

DEVELOP We'll assume there's no appreciable change in the gravitational potential, so Bernoulli's equation can be written as:

$$p + \frac{1}{2}\rho v^2 = p' + \frac{1}{2}\rho v'^2$$

We're told that the pressure drops by 6% $(p' = 0.94p)$. The blood flow can be approximated by the continuity equation $vA = v'A'$. We are given p, ρ, and v, and we want to find the fraction of the area that is obstructed: $\left(A - A'\right)/A$.

EVALUATE Putting together the information that we have:

$$p - p' = 0.06p = \frac{1}{2}\rho\left[\left(\frac{A}{A'}v\right)^2 - v^2\right]$$

Rearranging the terms, the fraction of the area that is obstructed is

$$\frac{\Delta A}{A} = 1 - \left(1 + \frac{0.12p}{\rho v^2}\right)^{-1/2} = 1 - \left(1 + \frac{0.12\left(10\ {\rm kPa}\right)}{\left(1.06\ {\rm g/cm^3}\right)\left(31\ {\rm cm/s}\right)^2}\right)^{-1/2} = 0.72$$

ASSESS This is a rather large blockage, but surprisingly, the pressure drops by only 6%.

62. **INTERPRET** This problem involves the flow of an incompressible fluid through a pipe of varying diameter, so we can apply Bernoulli's equation to find the density of the fluid.

DEVELOP This problem is identical to Problem 15.59, except that we are given the pressure difference $p_1 - p_2$ and are asked to find the density of the fluid in the pipe. In that problem, we derived the relationship

$$p_1 - p_2 = \frac{1}{2}\rho v_1^2\left(\frac{A_1^2}{A_2^2} - 1\right)$$

which we can solve for the density ρ of the oil. For this problem, the subscripts 1 and 2 refer to the unconstricted pipeline and the venturi, respectively.

EVALUATE The density ρ of the oil is

$$\rho = \frac{2\left(p_1 - p_2\right)}{v_1^2\left(A_1^2/A_2^2 - 1\right)} = \frac{2\left(16\ {\rm kPa}\right)}{\left(1.9\ {\rm m/s}\right)^2\left(16 - 1\right)} = 590\ {\rm kg/m^3}$$

ASSESS This density is about half that of water, which seems reasonable.

63. **INTERPRET** This problem involves calculating the change in height of water that a pump induces when a given change in pressure is achieved.

DEVELOP Equation 15.2, $dp/dh = \rho g$, relates the rate of change in pressure p with depth h to the fluid density ρ and the gravitational acceleration g. We are told amount by which the pump changes the pressure dp above ground, and we want to determine the amount by which the height of the water dh in the well will change by.

EVALUATE Solving for the height change dh from Equation 15.2, we obtain

$$dh = \frac{dp}{\rho g} = \frac{(1-0.333)(1.01\times 10^5\ \text{kPa})}{(1000\ \text{kg/m}^3)(9.8\ \text{m/s}^2)} = 6.89\ \text{m}$$

Meaning the maximum well depth at which this pump will function is 6.89 m.

ASSESS The depth at which shallow well pumps operate depends on the density of the water being pumped and the achievable change in pressure at ground level.

64. **INTERPRET** This problem is about finding the total pressure force exerted on a spherical submersible at the bottom of a trench.

DEVELOP The external pressure p at a depth h below the surface of water is given by Equation 15.1:

$$p = p_0 + \rho g h$$

where the pressure p_0 = 101.3 kPa is atmospheric pressure pushing down on the surface of the water. A typical density for open ocean seawater (which varies with salinity) is $\rho = 1027\ \text{kg/m}^3$. The pressure force is $F = pA$, where $A = 4\pi r^2$ is the surface area of a sphere of radius r. The net force on the spherical vessel is equal to the difference between the outward force (pushing inward) and the inward force (pushing out).

EVALUATE The pressure at the depth of 11 km is

$$p_{\text{out}} = p_0 + \rho g h = 101.3\ \text{kPa} + (1027\ \text{kg/m}^3)(9.8\ \text{m/s}^2)(1.1\times 10^4\ \text{km}) = 1.11\times 10^8\ \text{Pa}$$

The inner and outer radii of the vessel are $r_{\text{in}} = 0.545$ m and $r_{\text{out}} = 0.545\ \text{m} + 0.064\ \text{m} = 0.609$ m, respectively. Thus, the net force on the sphere is

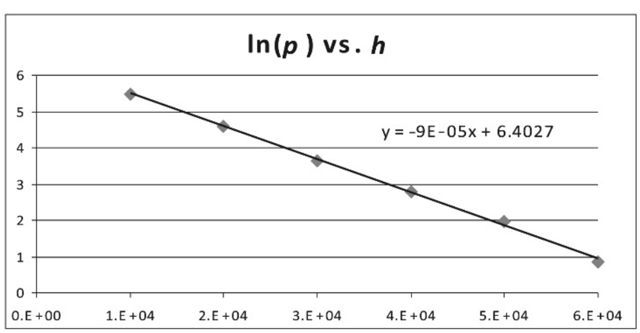
$$F_{\text{net}} = p_{\text{out}}A_{\text{out}} - p_{\text{in}}A_{\text{in}} = (1.11\times 10^8\ \text{Pa})4\pi(0.609\ \text{m})^2 - (1.01\times 10^5\ \text{Pa})4\pi(0.545\ \text{m})^2 = 5.2\times 10^8\ \text{N}$$

ASSESS The pressure the submersible must withstand is quite high (more than 1000 × p_{atm}). Note that our result is not exact because water at this depth is slightly compressible. Note also that the answer is obtained if you just use the inward force from the water, and not subtract the effect of the 1 atm pressure inside.

65. **INTERPRET** In this problem you are asked to analyze the data of Mars's atmospheric pressure as a function of altitude, and deduce Mars's surface pressure.

DEVELOP We follow the hint given in the problem statement and plot the natural logarithm versus altitude h. A pressure of the form $p = p_0 e^{-h/h_0}$ would give $\ln p = \ln p_0 - h/h_0$, where p_0 is the surface pressure at scale height h_0.

EVALUATE The plot of ln$p$ vs. h is shown below.



(a) With $\ln p_0 = 6.4027$ obtained from the plot, we find the surface pressure to be $p_0 = e^{6.4027} = 603$ Pa.

(b) Similarly, $-1/h_0 = -9.0\times 10^{-5}$, the scale height is $h_0 = 1/(9.0\times 10^{-5}) = 1.1\times 10^4$ m, or about 11 km.

ASSESS The variation of pressure with height in Mars's atmosphere, like that of Earth, follows from Equation 15.2: $dp = -\rho g\ dh.$ Since pressure and density are proportional, we have $dp = -pdh/h_0$, which can be integrated to give $p = p_0 e^{-h/h_0}$.

66. **INTERPRET** This problem involves the continuity equation for liquid flow (i.e., Equation 15.5), which we can use, combined with kinematics, to find the expression for the diameter of the falling water column.

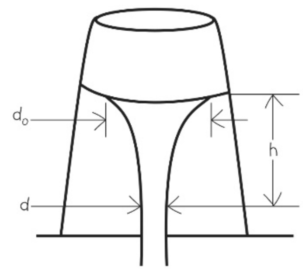
DEVELOP The water experiences constant acceleration (ignoring air resistance) due to the force of gravity, and it falls in a straight line, so we can apply Equation 2.11 to describe its speed. For this problem, the acceleration is $a = g$ (taking the downward direction to be positive) and the distance traveled is $x - x_0 = h$, so the Equation 2.11 takes the form

$$v^2 = v_0^2 + 2gh$$

The continuity Equation 15.5 gives

$$v_0 A_0 = vA$$
$$v_0\left(\frac{\pi d_0^2}{4}\right) = v\left(\frac{\pi d^2}{4}\right)$$
$$v_0 d_0^2 = vd^2$$

where the diameters are as shown in the sketch below. Combine these expressions and solve for d.



EVALUATE The diameter d of the water column is

$$v^2 = v_0^2 + 2gh \quad \Rightarrow \quad v_0^2\left(\frac{d_0^2}{d^2}\right)^2 = v_0^2 + 2gh$$
$$d = d_0 \sqrt[4]{\frac{v_0^2}{v_0^2 + 2gh}}$$

ASSESS If $v_0 = 0$, this expression reduces to $d = 0$, which is nonphysical. The reason for this is that the water cannot have zero velocity and still pour out of a faucet.

67. **INTERPRET** This problem involves finding the force on a suction cup due to atmospheric pressure. Given this force, we can find the force due to friction that allows the suction cup to support objects.

DEVELOP The normal force on the suction cup is the result of atmospheric pressure. We assume a perfect vacuum inside the cup. The force f_s due to static friction that supports the object of mass m must satisfy

$$mg = f_s \le \mu_s n = \mu_s p_{atm} A = \mu_s p_{atm}(\pi d^2/4)$$

EVALUATE The maximum mass that can be supported by the suction cup is, therefore,

$$m_{max} = \frac{\mu_s p_{atm} \pi d^2}{4g} = \frac{(0.68)(101.3\ \text{kPa})\pi(0.040\ \text{m})^2}{4(9.8\ \text{m/s}^2)} = 9\ \text{kg}$$

ASSESS The maximum value of 9 kg is about the mass of a baby. The force on the cup due to the atmospheric pressure is quite large.

68. **INTERPRET** This problem involves the flow of air at a speed much below the speed of sound in air, so we can apply Bernoulli's equation to find the speed of the air flowing past point A.

DEVELOP We can assume that any difference in height between A and B is negligible, and we know that $v_B = 0$, so applying Bernoulli's Equation 15.6 to points A and B and equating the results gives $p_B = p_A + \frac{1}{2}\rho v_A^2$, which we can solve for v_A.

EVALUATE The air speed at point A is thus $v_A = \sqrt{2(p_B - p_A)/\rho}$

ASSESS Even though Equation 15.6 applies strictly to incompressible steady fluid flow, density variations in a gas are generally insignificant when the flow speed is much less than the speed of sound.

69. **INTERPRET** You want to verify the power output of a proposed wind farm.

DEVELOP From the text, you know that the theoretical maximum power per unit area that can be extracted from the wind is $\frac{8}{27}\rho v^3$, where the air density is given by $\rho = 1.2\ \text{kg/m}^3$. The plan is to build 900 turbines, each with blade diameter of 110 m, in an area where the average wind speed is $10\ \text{m/s}$.

EVALUATE If you assume that the turbines, on average, generate 30% of the theoretical maximum power, the total power that the wind farm could produce is

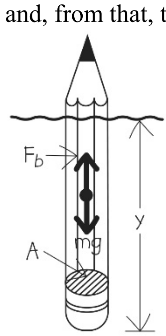
$$P_{\text{tot}} = N(0.3)\left(\frac{8}{27}\rho v^3\right)(\pi r^2) = (900)(0.3)\left[\frac{8}{27}\left(1.2\frac{\text{kg}}{\text{m}^3}\right)\left(10\frac{\text{m}}{\text{s}}\right)^3\right]\pi\left(\frac{1}{2}110\ \text{m}\right)^2 = 0.9\ \text{GW}$$

No, the wind farm cannot conceivably replace a 1-GW nuclear power plant.

ASSESS Even if the wind farm can, on average, generate the desired power, there will be fluctuations in the output due to changing weather conditions. Utility companies are still reluctant to entirely abandon coal and nuclear power plants, which supply a more stable baseline of power.

70. **INTERPRET** This problem involves the buoyancy force and simple harmonic motion (see Chapter 13). The pencil is never completely submerged, so we can apply Archimedes' principle for floating objects to calculate the force as a function of submersion depth, then apply the concepts of simple harmonic motion to find the period of the pencil's oscillation.

DEVELOP Make a sketch of the situation (see figure below). The two vertical forces (taking the downward direction to be positive) on the pencil are its weight, mg, and the buoyancy force $F_b = -\rho g A y$, where A is the cross-sectional area of the pencil and y is it submersion depth. Thus, the vertical component of Newton's second law (for constant mass) is $m\, d^2y/dt^2 = mg - \rho g A y.$ At equilibrium, $mg = \rho g A L$, so the equation of motion, written in terms of the displacement from equilibrium, is $d^2(y-L)/dt^2 = -(\rho g A/m)(y-L)$. Compare this formula to Equation 13.3 to find the effective force constant and, from that, the period.



EVALUATE Equation 1.3.3, which is Newton's second law for simple harmonic motion, reads

$$m\frac{d^2x}{dt^2} = -kx$$

Comparing this to the expression derived for the pencil shows that $x = y - L$ and $k = \rho g A = gm/L$ because $\rho = m/(AL)$, so from Equations 13.7a and 13.5, we find

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{L}{g}}$$

ASSESS This formula says that the period does not depend on the mass of the pencil, nor on its diameter, but only on its length (and the acceleration due to gravity, but that is essentially constant on the surface of the Earth).

71. **INTERPRET** In this problem we want to find the time it takes for the can to drain out all its water through its hole. An integral is needed since the water level is continuous, from 0 to height h.

DEVELOP Let y be the height of the water above the bottom of the can, then $-dy/dt$ is the magnitude of the flow speed of the top surface of the water draining out (y decreases as a function of time). The continuity equation gives

$$-\frac{dy}{dt}A_0 = v_1 A_1 \quad \rightarrow \quad dt = -\left(\frac{A_0}{A_1}\right)\frac{dy}{v_1}$$

where subscript 1 refers to the small hole in the bottom. For most of the time, $v_1 \approx \sqrt{2gy}$ (see Example 15.6) and we assume the top of the can is open.

EVALUATE Carrying out the integration, we find the total time required to be

$$t = \int dt \approx -\int_h^0 \left(\frac{A_0}{A_1}\right)\frac{dy}{\sqrt{2gy}} = \frac{A_0}{A_1\sqrt{2g}}2\sqrt{y}\Big|_0^h = \frac{A_0}{A_1}\sqrt{\frac{2h}{g}}$$

ASSESS This result is approximate since dy/dt cannot be neglected compared to v_1 when y is small. If we use Bernoulli's equation without this approximation, then

$$\frac{1}{2}\rho\left(\frac{dy}{dt}\right)^2 + \rho g y = \frac{1}{2}\rho v_1^2$$

since the pressure is atmospheric pressure at both the top of the can and the hole. Combining with the continuity equation gives

$$v_1 = \sqrt{2gy + (dy/dt)^2} = -\left(\frac{A_0}{A_1}\right)\frac{dy}{dt} \quad \rightarrow \quad \frac{dy}{dt} = -\sqrt{\frac{2gy}{(A_0/A_1)^2 - 1}}$$

Integration of this yields a more exact outflow time of $t = \sqrt{\frac{2h}{g}[(A_0/A_1)^2 - 1]}$.

72. **INTERPRET** Using the fact that the density and pressure in Earth's atmosphere are proportional: $\rho = p/h_0 g$, where h_0 is a constant, we want to find the atmospheric density as a function of height.

DEVELOP The variation of pressure with height in the Earth's atmosphere follows from Equation 15.2 (with h replaced by $-h$, since height is positive upward whereas depth is positive downward). Thus, $dp = -\rho g\ dh$. If pressure and density are proportional, then

$$dp = -\frac{p dh}{h_0}$$

Integrating this expression yields pressure as a function of height h.

EVALUATE **(a)** The above equation can be integrated from the surface values, $h = 0$ and $p = p_a$ (atmospheric pressure) to yield

$$\int_{p_a}^{p} \frac{dp}{p} = \ln\left(\frac{p}{p_0}\right) = -\int_0^h \frac{dh}{h_0} = -\frac{h}{h_0}$$

$$p(h) = p_0 e^{-h/h_0}$$

This is called the law of atmospheres; it applies exactly if the temperature is constant.

(b) The pressure will drop to half its surface value when $p(h) = p_0/2$, or

$$\frac{p_0}{2} = p_0 e^{-h/h_0}$$

$$h = h_0 \ln(2) = (8.2\ \text{km})\ln(2) = 5.68\ \text{km} \approx 5.7\ \text{km}$$

ASSESS Given that three quarters of the Earth's atmosphere is within 11 km of the surface of the Earth, this result seems reasonable.

73. **INTERPRET** Using the fact that the density and pressure in Earth's atmosphere are proportional, we are to use the result of the previous problem to express the atmospheric density as a function of height and to find the height below which half the Earth's atmospheric mass lies.

DEVELOP From Problem 15.68, we have the atmospheric pressure is $p = p_0 e^{-h/h_0}$. Combining this with the given information that $\rho = p/(h_0 g)$, we can express the density as a function of height h.

EVALUATE **(a)** The atmospheric density as a function of height is

$$\rho(h) = \frac{p}{h_0 g} = \frac{p_0}{h_0 g} e^{-h/h_0} = \rho_0 e^{-h/h_0}$$

(b) The mass of atmosphere contained in a thin spherical shell of thickness dh, at height h, is

$$dm = \rho dV = \left(\rho_0 e^{-h/h_0}\right) 4\pi \left(R_E + h\right)^2 dh = 4\pi \rho_0 \left(R_E + h\right)^2 e^{-h/h_0} dh$$

where R_E is the radius of the Earth and $R_E + h$ is the radius of the shell. The mass of atmosphere below height h_1 is

$$M(h_1) = \int_0^{h_1} dm = 4\pi \rho_0 R_E^2 \int_0^{h_1} \left(1 + 2\frac{h}{R_E} + \frac{h^2}{R_E^2}\right) e^{-h/h_0} dh$$

The integrals can be evaluated easily enough with the use of the table of integrals in Appendix B. However, if $h_1/R_E \ll 1$, only the first term is important. (Even if h_1 is large, the exponential term is negligibly small for $h_1 \ll h_0$ and none of the terms contribute significantly for large h.) To a good approximation, therefore

$$M(h_1) \approx 4\pi \rho_0 R_E^2 \int_0^{h_1} e^{-h/h_0} dh = 4\pi \rho_0 R_E^2 h_0 \left(1 - e^{-h_1/h_0}\right)$$

The total mass of the atmosphere is approximately $M(\infty) = 4\pi \rho_0 R_E^2 h_0$, so the height bounding half the total mass is given by the equation $M/2 = M\left(1 - e^{-h_1/h_0}\right)$ or

$$h_1 = h_0 \ln(2) = (8.2\ \text{km})\ln(2) = 5.8\ \text{km}$$

ASSESS This is the same result as we obtained for Problem 15.72.

74. **INTERPRET** This problem involves a container full of water that is rotating at a given angular speed. We are to find an expression for the pressure on the bottom of the container as a function of the distance from the axis of rotation and an expression for the depth of the water as a function of radial distance. Solving this problem involves Newton's second law and circular motion.

DEVELOP When the water is in equilibrium at constant angular velocity, the vertical change in pressure balances the weight of the water, the radial change in pressure supplies the centripetal acceleration, and there is no change in pressure in the direction tangent to the rotation (i.e., the pressure is constant along a horizontal circle). Introduce vertical, radial, and tangential coordinates, y, r, and ϕ, respectively, with the origin at the bottom center of the pan and the y-axis positive upward (these are cylindrical coordinates). Consider a fluid element $dm = \rho\ dV = \rho\ dr(rd\varphi)\,dy$ as shown below, where ρ is the density and dV is the volume element. Applying Newton's second law ($F = ma$) to mass element tells us that the vertical pressure difference must balance the gravitational force, as per Equation 15.2, which gives

$$\begin{aligned} 0 &= dF_\text{p} + dF_g \\ &= -dp_y A_y - \rho g\ dV \\ \partial p/\partial y &= -\rho g \end{aligned}$$

Here, $A_y = rdrd\phi$ is the area of the faces perpendicular to the y-direction, and we use the partial derivative because the pressure varies with both y and r. Note that $\partial p/\partial y$ is negative because the pressure increases with depth (decreasing y).

Similarly, applying Newton's second law in the radial direction tells us that the pressure force in the radial direction equals the mass element times the centripetal acceleration, or

$$\begin{aligned} dF_\text{p} &= ma_\text{c} \\ dp_r A_r &= dm\omega^2 r = \rho dV \omega^2 r \end{aligned}$$

(Recall that $a_\text{c} = v^2/r = -\omega^2 r$.) In this equation, $A_r = rd\phi dy$ is the area of the faces perpendicular to the radial direction. Since $dV = A_r dr$, we find

$$\begin{aligned} dp_r &= \rho\omega^2 r dr \\ \partial p/\partial r &= \rho\omega^2 r \end{aligned}$$

after canceling $-A_y$. Here, $\partial p/\partial r$ is positive because the pressure increases with r. Because the pressure varies with r and y, we have

$$\frac{\partial^2 p(r,y)}{\partial r \partial y} = \frac{\partial}{\partial r} p(r,y) + \frac{\partial}{\partial y} p(r,y) = \rho\omega^2 r - \rho g$$

For an incompressible fluid, ρ is a constant (not a function of r and y), so

$$\partial p/\partial y = -\rho g \quad \text{and} \quad \partial p/\partial r = \rho\omega^2 r$$

which we can integrate with respect to r and y to obtain

$$p(r,y) = -\rho g y + \rho\omega^2 r^2/2 + C$$

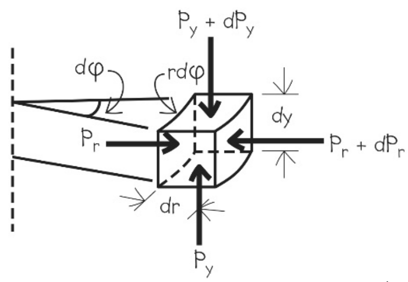
The constant term C can be evaluated, since at the surface above the center the pressure is atmospheric pressure, so

$$p(0, h_0) = p_a = -\rho g h_0 + \mathrm{C}$$
$$C = p_a + \rho g h_0$$

Inserting this constant back into the expression for pressure gives

$$p(r,y) = p_a - \rho g(y - h_0) + \rho\omega^2 r^2/2$$

which we can use to express the pressure at the bottom of the container and to find the height of the liquid in the container.



EVALUATE **(a)** Along the bottom of the pan $p(r,0) = p_a + \rho g h_0 + \rho\omega^2 r^2/2$.
(b) The pressure at the water's surface is the atmospheric pressure p_a for all values of r, so the height of the surface, $y = h(r)$ is given by the equation $p[r, h(r)] = p_a$, or $-\rho g[h(r) - h_0] + \rho\omega^2 r^2/2 = 0$. Solving this expression for $h(r)$ gives

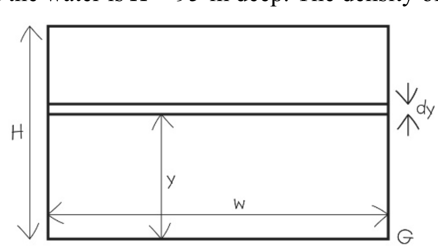
$$h(r) = h_0 + \omega^2 r^2 = 2g$$

which is a parabola.

ASSESS Such a technique is used to shape large mirrors for astronomical telescopes by a process called spin casting.

75. **INTERPRET** In Problem 15.44, we determined the force on a dam due to the water behind the dam. In this problem, we find the torque around the bottom edge of the same dam. We will use $F = pA$ and $\tau = yF$, as shown in the figure below.

DEVELOP The pressure varies with depth, according to $p(y) = \rho g(H - y)$. We will find the force $dF = pdA$, and thus the torque $d\tau = ydF$ from each horizontal strip across the dam. Integrating $d\tau$ gives us the total torque. The dam has width $w = 1500$ m and the water is $H = 95$-m deep. The density of water is $\rho = 1000\ \text{kg/m}^3$.



EVALUATE The pressure $d\tau = ydF = y[p(y)]dA = y[\rho g(H - y)](wdy)$. Integrate this from $y = 0$ to $y = H$.

$$\tau = \rho g w \int_0^H y(H-y)\,dy = \rho g w\left(\frac{H}{2}y^2 - \frac{y^3}{3}\right)\Bigg|_0^H = \rho g w\left(\frac{H^3}{2} - \frac{H^3}{3}\right) = \frac{1}{6}\rho g w H^3$$

$$= \frac{1}{6}\left(1.0\times10^3\ \text{kg/m}^3\right)\left(9.8\ \text{m/s}^2\right)(1500\ \text{m})(95\ \text{m})^3 = 2.1\times10^{12}\ \text{N}\cdot\text{m}$$

ASSESS The units in our final equation are

$$\tau = \left(\frac{\text{kg}}{\text{m}^3}\right)\left(\frac{\text{m}}{\text{s}^2}\right)\cdot\text{m}\cdot\text{m}^3 = \overbrace{\text{kg m/s}^2}^{\text{N}}\cdot\text{m} = \text{N}\cdot\text{m}$$

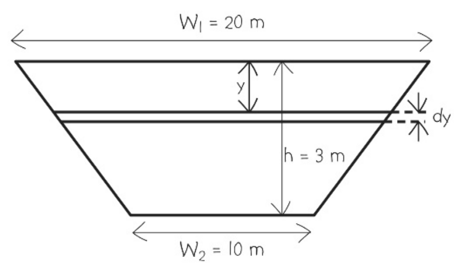
which work out correctly.

76. **INTERPRET** To find the net force on one vertical wall of a swimming pool, we integrate the pressure times the area of each horizontal strip of wall, from the top to the bottom of the pool. The pressure varies with depth.

DEVELOP First, we draw a picture of the situation, as shown in the figure below. The width varies linearly from 22 m at the top to 15 m at the bottom, so

$$w(y) = w_1 - \frac{(w_1 - w_2)y}{h} = 22 - \frac{7y}{3.3}$$

The pressure at depth y is $p = \rho g y$, and the force on the strip shown is $dF = p\,dA$, where $dA = w(y)\,dy$. The density of the water is $\rho = 1000\ \text{kg/m}^3$. Integrate dF from $y = 0$ to $y = h$.



EVALUATE The force on the side of the pool is

$$F = \int_0^{3.3}\rho g y\left(22 - \frac{7}{3.3}y\right)dy = \left(1.0\times10^3\ \text{kg}\right)\left(9.8\ \text{m/s}^2\right)(94.38\ \text{m}) = 9.25\times10^5\ \text{N} = 925\ \text{kN}$$

ASSESS Note that this force does not depend on the size of the pool in the other horizontal direction. In other words, if the pool had this cross section and were a centimeter wide, the force on this wall would be the same!

77. **INTERPRET** We are to find the expected density of a mix of immiscible liquids and compare it with a measured density to see if the mix is what it should be. The density of a mix of liquids should be the total mass divided by the total volume.

DEVELOP The "official" dressing is 1 part Reggio Emilia balsamic vinegar to 3 parts oil, measured by volume. So the dressing should have volume $4V$ and mass $m_{\text{vinegar}} + m_{\text{oil}} = \rho_{\text{vinegar}}V + 3\rho_{\text{oil}}V$. Calculate the density of this mix, and compare it with the measured density $\rho' = 0.99\ \text{g/cm}^3$. If the density of the sample is lower than it should be, then ordinary balsamic vinegar has probably been used. The density of oil is $\rho_{\text{oil}} = 0.92\ \text{g/cm}^3$, and the density of Reggio Emilia balsamic vinegar $\rho_{\text{vinegar}} = 1.20\ \text{g/cm}^3$.

EVALUATE

$$\rho = \frac{\rho_{\text{vinegar}}V + 3\rho_{\text{oil}}V}{4V} = \frac{\rho_{\text{vinegar}} + 3\rho_{\text{oil}}}{4} = 0.99\ \text{g/cm}^3$$

ASSESS The dressing has not been altered.

78. **INTERPRET** The question here is really "Is the water pressure sufficient to get the water to the top floor?" We'll look at it the other way: if there was a pipe full of water from the top to the bottom of the building, what would be the pressure at the bottom? If the pressure at the bottom of this hypothetical pipe is less than the measured pressure at the water heater, then there is enough pressure at the water heater to get hot water to the top floor.

DEVELOP To find the hypothetical pressure in a pipe of height $h = 33$ ft, we will use $p = \rho h$, where here ρ is the *weight* density of water in the English system:

$$\rho = \frac{mg}{V} = \frac{1000\ \text{kg}}{\text{m}^3}\left(\frac{1\ \text{lb}}{0.454\ \text{kg}}\right)\left(\frac{0.3048\ \text{m}}{1\ \text{ft}}\right)^3 = 62.4\ \text{lb/ft}^3$$

We will compare this hypothetical pressure to the measured pressure:

$$p_m = 18\ \text{lb/in}^2\left(\frac{12\ \text{in}}{1\ \text{ft}}\right)^2 = 2600\ \text{lb/ft}^2$$

EVALUATE The hypothetical pressure, or minimum pressure needed to reach the top of the building is:

$$p = \rho h = \left(62.4\ \text{lb/ft}^3\right)\left(33\ \text{ft}\right) = 2059\ \text{lb/ft}^3\left(\frac{1\ \text{ft}}{12\ \text{in}}\right)^2 = 14\ \text{psi}$$

Therefore, the measured pressure is 4 psi more than the minimum pressure needed.

ASSESS A quicker way to solve this is to remember that atmospheric pressure (about 15 psi) can support a column of water about 32 feet high. This pressure (18 psi) is higher, so it can push water even higher.

79. **INTERPRET** You are asked to find the maximum load that a ship can hold, given the size and shape of the hull, and the initial waterline of the ship.

DEVELOP By Archimedes' principle, the buoyant force is equal to the weight of the water displaced by the ship. To find the volume of water displaced, we'll need the formula for the area of an isosceles triangle with apex angle θ and height h: $A = h^2 \tan\theta / 2$.

EVALUATE When the ship is empty, the buoyant force only needs to support the weight of the ship:

$$m_{\text{ship}} g = \rho_{\text{H}_2\text{O}} V g = \rho_{\text{H}_2\text{O}}\left(L h_1^2 \tan\tfrac{\theta}{2}\right) g$$

When the maximum cargo load, m_{max}, is placed on the ship, the entire hull is submerged:

$$\left(m_{\text{ship}} + m_{\text{max}}\right) g = \rho_{\text{H}_2\text{O}}\left(L h_0^2 \tan\tfrac{\theta}{2}\right) g$$

We subtract these two equations to find the maximum load:

$$m_{\text{max}} = \rho_{\text{H}_2\text{O}} L \tan\tfrac{\theta}{2}\left(h_0^2 - h_1^2\right)$$

ASSESS A bigger concern for your design might be whether this shape of hull is stable. It is not immediately obvious that the center of gravity is below the center of buoyancy, see Fig. 15.10.

80. **INTERPRET** We're asked to consider some of the physics of arterial stenosis.

DEVELOP The volume flow rate is the cross-sectional area multiplied by the flow speed, Av.

EVALUATE Like for most fluids, we can assume that blood is incompressible. Therefore, to conserve mass, the volume flow rate must be constant.

The answer is **(b)**.

ASSESS The only time the volume rate is not constant is when the fluid's density changes in response to the change in the cross-sectional area of the flow.

81. **INTERPRET** We're asked to consider some of the physics of arterial stenosis.

DEVELOP The flow speed has to change in order to keep the volume flow rate constant.

EVALUATE From Equation 15.5: $v' = \left(A / A'\right) v$, so if the artery walls thicken and the area decreases: $A' < A$, then the flow speed must increase: $v' > v$.

The answer is **(c)**.

ASSESS If the speed didn't increase, the blood would begin piling up in front of the stenosis.

82. **INTERPRET** We're asked to consider some of the physics of arterial stenosis.

DEVELOP We can determine what will happen to the pressure in the artery by using Bernoulli's equation (Equation 15.6). Assuming negligible change in height, the pressure change at the stenosis will be

$$p' - p = \tfrac{1}{2}\rho\left(v^2 - v'^2\right)$$

EVALUATE Since we have already shown in the previous problem that the flow speed increases $(v' > v)$, the pressure must correspondingly drop.

The answer is **(a)**.

ASSESS This might seem counterintuitive that a constriction in flow might cause the artery to collapse on itself. But when the blood is flowing quickly, the blood molecules take a straighter path through the artery, bumping into the walls less often. This results in less outward pressure on the walls.

83. **INTERPRET** We're asked to consider some of the physics of arterial stenosis.

DEVELOP As pointed out in Problem 15.81, the flow speed in the stenosis is $v' = (A/A')v.$ Since the area is related to the diameter by $A = \frac{\pi}{4}D^2$, the flow speed goes as $v' = (D/D')^2 v.$

EVALUATE If the diameter decreases by half, the flow speed increases by a factor of 4.

The answer is **(e)**.

ASSESS If we use the values from Problem 15.61: $\rho = 1.06\ \text{g/cm}^3$, $p = 10$ kPa, and $v = 30$ cm/s, then the pressure in the stenosis will drop by 7%.