28

ALTERNATING-CURRENT CIRCUITS

EXERCISES

Section 28.1 Alternating Current

11. **INTERPRET** We're asked to express how the voltage in North America varies with time.

DEVELOP The time-dependence for AC voltage is given by Equation 28.3: $V = V_p \sin(\omega t)$. In North America, the peak voltage and angular frequency are

$$V_p = \sqrt{2}V_{rms} = \sqrt{2}(120\text{ V}) = 170\text{ V}$$
$$\omega = 2\pi f = 2\pi(60\text{ Hz}) = 377\text{ s}^{-1}$$

EVALUATE Plugging the given values into the voltage equation gives

$$V = (170\text{ V})\sin\left[(377\text{ s}^{-1})t\right]$$

ASSESS The peak voltage and angular frequency are both larger than the rms voltage and frequency.

12. **INTERPRET** We're given the AC current in terms of a sinusoidal function and asked to deduce the rms current and the frequency of the current.

DEVELOP As shown in Equation 28.3, the AC current can be written as

$$I = I_p \sin(\omega t + \phi_I)$$

where I_p is the peak current amplitude, ω is the angular frequency, and ϕ_I is the phase constant. Comparison of the current with Equation 28.3 shows that its amplitude and angular frequency are $I_p = 693$ mA and $\omega = 7.54\,(\text{ms})^{-1}$.

EVALUATE **(a)** Applying Equation 28.1 gives $I_{rms} = I_p/\sqrt{2} = 490$ mA.

(b) Similarly, using Equation 28.2, we have $f = \omega/2\pi = 1.20$ kHz.

ASSESS The phase ϕ_I is zero in this problem. Note that since the rms (root-mean-square) current is obtained by squaring the current, taking its time average, and then taking the square root, it is smaller than the peak current by a factor of $\sqrt{2}$.

13. **INTERPRET** We are to find the phase constants for a series of signals plotted as voltage versus dimensionless time.

DEVELOP The phase constant is a solution of Equation 28.3 for $t = 0$; that is, $V(0) = V_p \sin(\phi_V)$. Since $\sin(\phi_V) = \sin(-\phi_V \pm \pi)$, one must also consider the slope of the sinusoidal signal function at $t = 0$. In addition, the conventional range for ϕ_V usually runs from −180° to +180°, or $-\pi \le \phi_V \le \pi$. Thus, $\phi_V = \sin^{-1}\left[V(0)/V_p\right]$ when $(dV/dt)_0 \ge 0$, but $\phi_V = -\sin^{-1}\left[V(0)/V_p \pm \pi\right]$ when $(dV/dt)_0 \le 0$.

EVALUATE For signal **(a)** in Fig. 28.25, we guess that $V(0) \approx V_p/\sqrt{2}$ (since that curve next crosses zero about halfway between $\pi/2$ and π) and the slope at zero is positive, so $\phi_a = \sin^{-1}\left(1/\sqrt{2}\right) = \pi/4$ or 45°. This signal is $V_p \,\text{in}(\omega t + \phi_a) = V_p \sin(\omega t + \pi/4)$, which leads a signal with zero phase constant by 45°. For the other signals,

(b) $V(0) = 0$ and $(dV/dt)_0 > 0$, so $\phi_b = 0$;

(c) $V(0) = V_p$, $(dV/dt)_0 = 0$, so $\phi_c = \sin^{-1}(1) = -\sin^{-1}(1) + \pi = \pi/2$ or 90°

(d) $V(0) = 0$ and $(dV/dt)_0 < 0$, so $\phi_d = \pm\pi$ or ±180°; and

(e) $V(0) = -V_p$ and $(dV/dt)_0 = 0$, so $\phi_e = \sin^{-1}(-1) = -\sin^{-1}(-1) - \pi = -\pi/2$ or −90°.

Assess We used $\left|\sin^{-1}\left[V(0)/V_\mathrm{p}\right]\right| \le \pi/2$ or 90°, as is common on most electronic calculators, since the sine function is one-to-one only in such a restricted range.

Section 28.2 Circuit Elements in AC Circuits

14. **Interpret** In this problem, we want to find the rms current in a capacitor connected to an AC power source.

Develop The amplitude of the current in a capacitor is given by Equation 28.5:

$$I_\mathrm{p} = \frac{V_\mathrm{p}}{X_C} = \frac{V_\mathrm{p}}{1/\omega C} = \omega C V_\mathrm{p}$$

Using Equation 28.1, the corresponding rms current is $I_\mathrm{rms} = \omega C V_\mathrm{rms}$.

Evaluate Substituting the values given in the problem statement, we find the rms current is

$$I_\mathrm{rms} = \omega C V_\mathrm{rms} = (2\pi \times 50\ \mathrm{Hz})(1\ \mu\mathrm{F})(230\ \mathrm{V}) = 72\ \mathrm{mA}$$

Assess The capacitive reactance is $X_C = 1/\omega C = 3.18\times 10^3\ \Omega$. In this circuit, the current in the capacitor leads the voltage across the capacitor by 90°.

15. **Interpret** We are to find the rms current in each element of an *RLC* circuit connected across the given emf source.

Develop Apply the equations in Table 28.1 and convert them to rms values using Equations 28.1 and 28.2.

Evaluate The equations in Table 28.1 (expressed in rms values) give

$$I_{R,\mathrm{rms}} = V_\mathrm{rms}/R = (6.3\ \mathrm{V})/(140\ \Omega) = 45\ \mathrm{mA}$$
$$I_{C,\mathrm{rms}} = V_\mathrm{rms}\omega C = (6.3\ \mathrm{V})2\pi(50\ \mathrm{Hz})(47\ \mu\mathrm{F}) = 93\ \mathrm{mA}$$
$$I_{L,\mathrm{rms}} = V_\mathrm{rms}/(\omega L) = (6.3\ \mathrm{V})/\left[2\pi(50\ \mathrm{Hz})(220\ \mathrm{mH})\right] = 91\ \mathrm{mA}$$

Assess These values are realistic for *RLC* circuits.

16. **Interpret** This problem involves the capacitive reactance of the given capacitor at various angular frequencies.

Develop From Equation 28.5, we see that the capacitive reactance is

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

Evaluate **(a)** For $f = 50$ Hz, the capacitive reactance is

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi(50\ \mathrm{Hz})(6.8\times 10^{-6}\ \mathrm{F})} = 470\ \Omega$$

to two significant figures.

(b) For $f = 1.0$ kHz, the capacitive reactance is

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi(1.0\ \mathrm{kHz})(6.8\times 10^{-6}\ \mathrm{F})} = 23\ \Omega$$

(c) Similarly, for $f = 20$ kHz, the capacitive reactance is

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi(20\ \mathrm{kHz})(6.8\times 10^{-6}\ \mathrm{F})} = 1.2\ \Omega$$

Assess One can see that a capacitor has the greatest effect (largest reactance) at low frequency.

17. **Interpret** This problem deals with the minimum safety voltage of a capacitive circuit.

Develop Take the minimum safe voltage to be equal to the peak voltage, and use Equation 28.5 to find the peak voltage.

Evaluate **(a)** For a frequency $f = 50$ Hz, the minimum safe voltage is

$$V_\mathrm{p} = I_\mathrm{p} X_C = \frac{\sqrt{2}(2.2\ \mathrm{A})}{2\pi(50\ \mathrm{Hz})(33\ \mu\mathrm{F})} = 300\ \mathrm{V}$$

(b) For $f = 1$ kHz, the minimum safe voltage is

$$V_p = I_p X_C = \frac{\sqrt{2}(2.2\text{ A})}{2\pi(1000\text{ Hz})(33\,\mu\text{F})} = 15\text{ V}$$

ASSESS The results are given to two significant figures. The safe voltage is based on the peak voltage, which seems reasonable. Notice that the capacitor has the greatest effect (largest reactance) at low frequency.

18. **INTERPRET** This problem involves the capacitance of a capacitor that's connected across an AC power source.

DEVELOP The fact that the capacitor and the resistor both pass the same current implies that

$$I_p = \frac{V_p}{R} = \frac{V_p}{X_C} \quad \Rightarrow \quad R = X_C = \frac{1}{\omega C}$$

Therefore, the capacitance is $C = 1/\omega R$.

EVALUATE Inserting the values given, we obtain

$$C = \frac{1}{\omega R} = \frac{1}{2\pi fR} = \frac{1}{2\pi(50\text{ Hz})(2.9\text{ k}\Omega)} = 1.1\,\mu\text{F}$$

ASSESS Since $R = X_C$, the greater the value of resistance R, the greater the capacitive reactance, and, thus, the smaller the capacitance.

19. **INTERPRET** We are to find the frequency of an inductive circuit given the rms inductance, emf, and current.

DEVELOP Apply Equation 28.7, $I_p = V_p/(\omega L)$ and Equation 28.2, $\omega = 2\pi f$. Because

$$\frac{V_{rms}}{I_{rms}} = \frac{V_p}{I_p}$$

we can use the rms values instead of the peak values in these expressions.

EVALUATE Combining the expressions above gives

$$f = \frac{\omega}{2\pi} = \frac{V_p}{2\pi I_p L} = \frac{V_{rms}}{2\pi I_{rms} L} = \frac{10\text{ V}}{2\pi(2.0\text{ mA})(50\text{ mH})} = 16\text{ kHz}$$

ASSESS The inductance and frequency are inversely proportional.

Section 28.3 *LC* Circuits

20. **INTERPRET** We are to find the resonant frequency of an *LC* circuit.

DEVELOP Using Equations 28.2 and 28.10, the resonant frequency can be written as

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi\sqrt{LC}}$$

EVALUATE Substituting the values given for capacitance and inductance, the resonant frequency is

$$f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(1.7\text{ mH})(0.22\,\mu\text{F})}} = 8.2\text{ kHz}$$

ASSESS The mechanical analog of the *LC* circuit is the mass-spring system whose angular frequency is $\omega = \sqrt{k/m}$. Thus, the correspondence between the two systems is: $L \leftrightarrow m$ and $C \leftrightarrow 1/k$.

21. **INTERPRET** Given the oscillation frequency of an *LC* circuit and its capacitance, we are to find the inductance.

DEVELOP The inductance and capacitance are related to the frequency of an *LC* circuit by Equation 28.10, $\omega = (LC)^{-1/2}$. The angular frequency is related to the oscillation frequency f as $\omega = 2\pi f$.

EVALUATE Solving the expression above for the inductance and inserting the given quantities gives

$$L = \frac{1}{\omega^2 C} = \frac{(2\pi \cdot 256\text{ Hz})^{-2}}{68.0\,\mu\text{F}} = 5.68\text{ mH}$$

ASSESS The inductance and capacitance are inversely proportional for a given frequency.

22. **INTERPRET** You're helping your sister build her radio and need to determine what variable capacitor to use so as to cover the AM radio band.

DEVELOP The frequency of your sister's radio receiver will be set by the *LC* circuit. Recall that the inductor that she wound around the cardboard tube has an inductance of 450 μH. The *LC* circuit oscillates at the angular frequency $\omega = 1/\sqrt{LC},$ so you need to find the minimum and maximum capacitance in order to cover the minimum and maximum frequencies $\left(f = \omega/2\pi\right)$ of the AM band.

EVALUATE The minimum frequency is 550 kHz, which corresponds to a capacitance of

$$C_{\text{max}} = \frac{1}{(2\pi f_{\text{min}})^2 L} = \frac{1}{(2\pi \cdot 550\,\text{kHz})^2 (450\ \mu\text{H})} = 190\ \text{pF}$$

The maximum frequency is 1600 kHz, which corresponds to a capacitance of

$$C_{\text{min}} = \frac{1}{(2\pi f_{\text{max}})^2 L} = \frac{1}{(2\pi \cdot 1600\,\text{kHz})^2 (450\ \mu\text{H})} = 22\ \text{pF}$$

So your sister needs a variable capacitor with a range of 22 to 190 pF.

ASSESS The capacitance is small, but it is typical for small electronic applications.

23. **INTERPRET** We are to find the inductance and peak voltage of an *LC* circuit given its oscillation period and peak current.

DEVELOP Using Equation 28.2, the oscillation frequency is

$$\omega = 2\pi f = \frac{2\pi}{T}$$

The inductance can be calculated from Equation 28.10: $L = 1/(\omega^2 C) = T^2/(4\pi^2 C)$

EVALUATE (a) The inductance is

$$L = \frac{T^2}{4\pi^2 C} = \frac{(5.0\ \text{ms})}{4\pi^2 (20\ \mu\text{F})} = 32\ \text{mH}$$

(b) Figure 28.11 and the expressions for the electric and magnetic energies for the *LC* circuit in the text imply that $\frac{1}{2}CV_{\text{p}}^2 = \frac{1}{2}LI_{\text{p}}^2$, so

$$V_{\text{p}} = I_{\text{p}}\sqrt{L/C} = (25\ \text{mA})\sqrt{(31.7\text{mH})\,/(20\ \mu\text{F})} = 1.0\ \text{V}$$

ASSESS The results are given to two significant figures, as warranted by the data.

Section 28.4 Driven *RLC* Circuits and Resonance

24. **INTERPRET** We are to find the capacitance of the given *RLC* circuit and then find its impedance at the two given frequencies.

DEVELOP The capacitance can be found from the relation between resonance frequency and the inductance and capacitance:

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

Knowing the capacitance, use Equation 28.12 to find the impedance, using $X_L = \omega L$ and $X_C = 1/(\omega C)$.

EVALUATE **(a)** From the expression for resonance in an *RLC* circuit,

$$C = 1/(\omega_0^2 L) = (2\pi \times 5.0\ \text{kHz})^{-2}(18\ \text{mH})^{-1} = 56\ \text{nF}$$

(b) At resonance, $X_L - X_C = 0$, so $Z = R = 75\ \Omega$.

(c) At 4 kHz, $X_L - X_C = \omega L - 1/(\omega C) = (2\pi)(4.0\ \text{kHz})(18\ \text{mH}) - [(2\pi)(4.0\ \text{kHz})(56\ \text{nF})]^{-1} = -260\ \Omega$,

so

$$Z = \sqrt{R^2 + (X_L - X_c)^2} = Z = \sqrt{(75\,\Omega)^2 + (-260\,\Omega)^2} = 270\ \Omega$$

ASSESS The impedance is frequency-dependent, so its value is different for different frequencies.

25. **INTERPRET** We are to find the impedance of an *LRC* circuit at a given frequency.

DEVELOP From Equation 28.12, we know that $Z = \sqrt{R^2 + (X_L - X_C)^2}$, where $X_c = 1/(\omega C)I$ and $X_L = \omega L$.

The given frequency is $f = 10 \times 10^3\ \text{Hz} \Rightarrow \omega = 20\pi \times 10^3\ \text{s}^{-1}$. The values of the circuit elements are $R = 1.5\ \text{k}\Omega$, $C = 6.0 \times 10^{-6}\ \text{F}$, and $L = 55 \times 10^{-3}\ \text{H}$.

EVALUATE Inserting the given values into the expression for the impedance gives

$$Z=\sqrt{R^2+\left(X_L-X_C\right)^2}=\sqrt{R^2+\left(\omega L-\frac{1}{\omega C}\right)^2}$$

$$=\sqrt{(1.5\ \text{k}\Omega)^2+\left[\left(2\pi\times 10\ \text{kHz}\right)\left(55\ \text{mH}\right)=\frac{1}{\left(20\pi\times 10\ \text{kHz}\right)\left(6.0\ \mu\text{F}\right)}\right]^2}=3.8\ \text{k}\Omega$$

ASSESS Note that, at this frequency, the capacitor has almost no effect compared to the other two circuit elements.

26. **INTERPRET** For a series *RLC* circuit, we are to find the frequency at which the impedance is a *minimum* and the value of that impedance.

DEVELOP From Equation 28.12, we know that the impedance Z is $Z=\sqrt{R^2+\left(X_L-X_C\right)^2}$, where $X_C=1/\left(\omega C\right)$ and $X_L=\omega L.$ The frequency at which Z is minimum will be when $X_C=X_L.$ At that resonance frequency, the impedance is $Z=R.$ The component values in this circuit are $R=18\ \text{k}\Omega,$ $C=14\ \mu\text{F},$ and $L=0.20\ \text{H}.$

EVALUATE (**a**) The minimum-impedance frequency is

$$X_C=X_L \quad \Rightarrow \frac{1}{\omega C}=\omega L$$

$$\omega=\frac{1}{\sqrt{LC}} \quad \Rightarrow f \quad =\frac{1}{2\pi\sqrt{LC}}=\frac{1}{2\pi\sqrt{\left(0.20\ \text{H}\right)\left(14\ \mu\text{F}\right)}}=95\ \text{Hz}$$

(b) At this frequency, the impedance is $Z=R=18\ \text{k}\Omega.$

ASSESS At resonance, the effects of the inductor and the capacitor cancel out, leaving only resistance.

27. **INTERPRET** We are to find the peak current through an *RLC* circuit at resonance, for three values of R in the circuit. We shall use the fact that at resonance, $Z=R.$

DEVELOP The peak voltage is $V_p=120\ \text{V}.$ The value of R is $R=10\ \text{K}\Omega.$ Since $Z=R$ at resonance, the peak current will be $I_p=V_p/R.$

EVALUATE For resistance $\frac{1}{2}R,$ $I_p=V_p/R=\left(120\ \text{V}\right)/\left(5.0\ \text{k}\Omega\right)=24\ \text{mA}.$ Similarly, for resistance $R,$ $I_p=\left(120\ \text{V}\right)/\left(10\ \text{k}\Omega\right)=12\ \text{mA},$ and for resistance $2R,$ $I_p=\left(120\ \text{V}\right)/\left(20\ \text{k}\Omega\right)=6.0\ \text{mA}.$

ASSESS At frequencies off the resonance peak, these calculations become somewhat more complicated. But at resonance, $Z=R,$ so everything becomes easy.

Sections 28.5 Power in AC Circuits and 28.6 Transformers and Power Supplies

28. **INTERPRET** We are to find the power consumption of a device, given its rms current and the current phase.

DEVELOP The average power consumed by an AC circuit is given by Equation 28.14, $P_{\text{ave}}=V_{\text{rms}}I_{\text{rms}}\cos\phi.$

EVALUATE Inserting the given values into the expression above gives

$$P_{\text{ave}}=V_{\text{rms}}I_{\text{rms}}\cos\phi=(230\ \text{V})(4.1\ \text{A})\cos(32^\circ)=800\ \text{W}$$

ASSESS The maximum power for this would be $(230\ \text{V})(4.1\ \text{A})=943\ \text{W},$ which would require operating at a different frequency. Thus, at 32° phase, the power is about 85% of its maximum value.

29. **INTERPRET** We shall use the average power of a lamp, as well as the rms voltage and the power factor, to calculate the rms current that it draws.

DEVELOP Use Equation 28.14, $P_{\text{ave}}=V_{\text{rms}}I_{\text{rms}}\cos\phi,$ with $\cos\phi=0.79$ being the given power factor. $P_{\text{ave}}=60\ \text{W}$ and $V_{\text{rms}}=230\ \text{V},$ so we simply solve for $I_{\text{rms}}.$

EVALUATE Solving for the rms current gives

$$I_{\text{rms}}=\frac{P_{\text{ave}}}{V_{\text{rms}}\cos\phi}=330\ \text{mA}$$

ASSESS The power factor actually matters with fluorescent lamps. With incandescent lamps, the impedance is almost entirely resistive, so the power factor is almost exactly one.

30. **INTERPRET** We are to compare the power consumption of two circuits that have the same current and voltage; but one that is purely resistive and the other has voltage leading current. The difference in the power usage by these two circuits will be due to the difference in power factors between the two circuits.

DEVELOP The average power consumption of a circuit is (Equation 28.14) $P_{\text{ave}} = I_{\text{rms}}V_{\text{rms}}\cos\phi.$ In the first circuit, the power factor is $\cos\phi = 1$, since the circuit is purely resistive. In the second, $\phi = 20°$. In each case, $I_{\text{rms}} = 20\ \text{A}$ and $V_{\text{rms}} = 240\ \text{V}.$

EVALUATE For the first circuit,

$$P_{\text{ave}} = I_{\text{rms}}V_{\text{rms}}\cos\phi = I_{\text{rms}}V_{\text{rms}} = (20\ \text{A})(240\ \text{V}) = 4.8\ \text{kW}$$

For the second circuit,

$$P_{\text{ave}} = I_{\text{rms}}V_{\text{rms}}\cos\phi = (20\ \text{A})(240\ \text{V})\cos\left(20^\circ\right) = 4.5\ \text{kW}.$$

ASSESS This is a fairly direct application of a power calculation.

31. **INTERPRET** The problem concerns isolation transformers that are used for safety.

DEVELOP The turns ratio is defined in Equation 28.15: $N_1 / N_2 = V_1 / V_2.$

EVALUATE Since the input and output voltages have the same magnitude, the turns ratio is 1.

ASSESS It might seem like a waste to have a transformer that doesn't "transform," but the isolation transformer can provide an ungrounded power supply separate from the main utility line. An ungrounded power supply is said to be "floating," that is, the voltage is alternating, but the zero of the voltage is not specified, as it is for a grounded power supply. If a patient were to touch a live wire in a medical device connected to an isolation transformer, he or she would provide a "ground." However, since this is the only connection to ground for the medical device, there is no circuit, and, therefore, no current flows through the patient.

32. **INTERPRET** We're trying to determine what transformer the exchange student needs to run her hairdryer in Germany.

DEVELOP The exchange student needs a step-down transformer that goes from Germany's 230 V to the 120 V used by her hairdryer. The number of turns in the primary and secondary coils are related by Equation 28.15: $N_2 / N_1 = V_2 / V_1.$ Power, $P = IV,$ is ideally conserved in the transformer, so the currents in the primary and secondary coils should be related by: $I_2 / I_1 = V_1 / V_2.$

EVALUATE **(a)** Given the number of turns in the primary, the number of turns in the secondary is

$$N_2 = N_1 \frac{V_2}{V_1} = (460)\left(\frac{120\ \text{V}}{230\ \text{V}}\right) = 240$$

(b) Given the maximum primary current, the maximum secondary current will be

$$I_2 = I_1 \frac{V_1}{V_2} = (6.0\ \text{A})\left(\frac{230\ \text{V}}{120\ \text{V}}\right) = 11.5\ \text{A}$$

This is below the threshold 12.5 A of her hairdryer, so the transformer will not work.

ASSESS The emf per turn in the secondary is set by the number of turns and the current in the primary. Therefore, to lower the voltage, the secondary should have fewer turns than the primary, as we have found. By contrast, the reduced voltage of the secondary requires more current in order to conserve power. (Of course, some power will be lost in the transformer to resistive heating in the coils.)

EXAMPLE VARIATIONS

33. **INTERPRET** We're being asked about the relation between AC voltage and current in capacitors and inductors. We want to determine the capacitance in the circuit, given the rms current and voltage, as well as the necessary inductance to obtain the same current.

DEVELOP The angular frequency is related to the oscillation frequency f as $\omega = 2\pi f.$ Equations 28.5, $I_{C_\text{p}} = V_{C_\text{p}}\omega C,$ and 28.7, $I_{L_\text{p}} = V_{L_\text{p}} / (\omega L),$ relate the peak current and peak voltage in the two devices. We can apply these to the rms current and voltage as well, since they are each equally proportional to their respective peak quantities.

EVALUATE **(a)** For the capacitor, we know the voltage and current. Equation 28.5 then gives

$$C = \frac{I_{C_{\text{rms}}}}{\omega V_{C_{\text{rms}}}} = \frac{295\ \text{mA}}{(2\pi \times 60\ \text{Hz})(120\ \text{V})} = 6.52\ \mu\text{F}$$

(b) For an inductor to pass the same current, it must have the same reactance; comparing Equations 28.5 and 28.7 shows that

$$L = \frac{1}{\omega^2 C} = \frac{1}{(2\pi \times 60\ \text{Hz})^2 (6.52\ \mu\text{F})} = 1.08\ \text{H}$$

ASSESS Our expression for L shows that a larger capacitor would require a smaller inductor for the same current.

EVALUATE **(a)** For the capacitor, we know the voltage and current. Equation 28.5 then gives

$$C = I_{\text{Crms}}/\omega V_{\text{Crms}} = (485\,\text{mA})\big/\big((2\pi\times 50\,\text{Hz})(230\,\text{V})\big) = 6.71\,\mu\text{F}$$

(b) For an inductor to pass the same current, it must have the same reactance; comparing Equations 28.5 and 28.7 shows that

$$L = \frac{1}{\omega^2 C} = \frac{(2\pi\times 50\,\text{Hz})^{-2}}{6.71\ \mu\text{F}} = 1.51\ \text{H}$$

ASSESS Our expression for L shows that a larger capacitor would require a smaller inductor for the same current.

34. **INTERPRET** We're being asked about the relation between AC voltage and current in capacitors and inductors. We want to determine the frequency of the sources, knowing that the rms current through the capacitor and inductor are equivalent.

DEVELOP The inductance and capacitance are related to the frequency of an LC circuit by Equation 28.10, $\omega = (LC)^{-1/2}$. The angular frequency is related to the oscillation frequency f as $\omega = 2\pi f$. Equations 28.5, $I_{Cp} = V_{Cp}\omega C$, and 28.7, $I_{Lp} = V_{Lp}/\omega L$, relate the peak current and peak voltage in the two devices. We can apply these to the rms current and voltage as well since they are each equally proportional to their respective peak quantities.

EVALUATE Since the current is the same across both inductor and capacitor, their respective inductances are also equivalent, meaning we can determine the source frequency with the use of Equation 28.10

$$f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(0.470\,\mu\text{F})(144\,\mu\text{H})}} = 19.3\,\text{kHz}$$

ASSESS The frequency at which the reactance of both inductor and capacitor is the same is the resonant frequency of the LC circuit.

35. **INTERPRET** We're being asked about the relation between AC voltage and current in capacitors and inductors. We want to determine the transmitter frequency and inductance, knowing the rms voltage and current, along with the capacitance.

DEVELOP The inductance and capacitance are related to the frequency of an LC circuit by Equation 28.10, $\omega = (LC)^{-1/2}$. The angular frequency is related to the oscillation frequency f as $\omega = 2\pi f$. Equations 28.5, $I_{Cp} = V_{Cp}\omega C$, and 28.7, $I_{Lp} = V_{Lp}/\omega L$, relate the peak current and peak voltage in the two devices. We can apply these to the rms current and voltage as well since they are each equally proportional to their respective peak quantities. We are told the circuit is at resonance, so the reactance of both capacitor and inductor will be equivalent.

EVALUATE **(a)** For the capacitor, we know the voltage, current, and capacitance. Equation 28.5 then gives

$$f = I_{\text{Crms}}/2\pi C V_{\text{Crms}} = (22.0\,\text{A})\big/\big((2\pi)(6800\,\text{pF})(480\,\text{V})\big) = 1.07\,\text{MHz}$$

(b) Using the transmitter frequency, we evaluate the inductance

$$L = \frac{1}{\omega^2 C} = \frac{(2\pi\times 1.07\,\text{MHz})^{-2}}{6800\ \text{pF}} = 3.24\ \mu\text{H}$$

ASSESS A transmitter frequency of approximately 1 MHz places it in the AM radio band.

36. **INTERPRET** We're being asked about the relation between AC voltage and current in capacitors and inductors. We want to find a pair of values for C and L that will meet the specifications necessary for the radio transmitter frequency and current.

DEVELOP The inductance and capacitance are related to the frequency of an LC circuit by Equation 28.10, $\omega = (LC)^{-1/2}$. The angular frequency is related to the oscillation frequency f as $\omega = 2\pi f$. Equations 28.5, $I_{Cp} = V_{Cp}\omega C$, and 28.7, $I_{Lp} = V_{Lp}/\omega L$, relate the peak current and peak voltage in the two devices. We can apply these to the rms current and voltage as well since they are each equally proportional to their respective peak quantities. We are told the values from which we can choose from, so we can check all solutions and ensure that the corresponding frequency falls within the FM band and the current is below 500 mA rms.

EVALUATE **(a)** Using Equations 28.5 and 28.10, we can express the current as

$$I_{\text{rms}} = \sqrt{\frac{C}{L}} V_{\text{rms}}$$

Meaning we can check which pairs will not exceed 500 mA rms. From the listed values, only one pair exceeds this current ($C = 2.2$ pF and $L = 1.3$ μH). However, out of the remaining three pairs, only one will result in a frequency within the FM broadcast band: $C = 1.3$ pF and $L = 2.4$ μH.

(b) These values result in a frequency and an rms current of

$$f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(1.3\ \text{pF})(2.4\ \mu\text{H})}} = 90.1\ \text{MHz}$$

$$I_{\text{rms}} = \sqrt{\frac{C}{L}} V_{\text{rms}} = \sqrt{\frac{(1.3\,\text{pF})}{(2.4\,\mu\text{H})}}(480\,\text{V}) = 353\,\text{mA}$$

ASSESS We had to check the pair met both requirements, but if we didn't have a set number of inductors and capacitors to choose from, we could keep in mind that at a fixed frequency, the inductance required for a given current scales inversely with the capacitance required for the same current.

37. **INTERPRET** This is a problem about the current and voltage in a series *RLC* circuit, where we treat the speaker as *R*. We want to determine the capacitance that will maximize the current at a particular frequency, as well as the value of *R* knowing something about the circuit parameters at a given frequency.

DEVELOP The peak current is at the resonant frequency of Equation 28.10, $\omega = (LC)^{-1/2}$. Equation 28.12 relates peak voltage and current to the component values and the frequency, so in (b) we can solve for *R*.

EVALUATE **(a)** Using Equations 28.10, we obtain a capacitance of

$$C = \frac{1}{\omega^2 L} = \frac{(2\pi \cdot 1.25\,\text{kHz})^{-2}}{1.80\ \text{mH}} = 9.01\ \mu\text{F}$$

(b) Equation 28.12 shows that we'll have half the peak current when *Z* is twice the value $Z = R$ that it has at resonance. So, we want $Z = \sqrt{R^2 + (X_L - X_C)^2} = 2R$ at the frequency $\omega_2 = 2\pi(525\,\text{Hz})$.

$$R = \frac{1}{\sqrt{3}}\left|\omega_2 L - \frac{1}{\omega_2 C}\right| = 16.0\,\Omega$$

ASSESS If we knew the speaker's resistance from the start, we could have used it to determine the capacitance since the same voltage resulted in half the resonance current.

38. **INTERPRET** This is a problem about the current and voltage in a series *RLC* circuit, where we treat the speaker as *R*. We want to determine the resonance frequency and the maximum rms voltage at this frequency knowing the maximum power dissipated.

DEVELOP The peak current is at the resonant frequency of Equation 28.10, $\omega = (LC)^{-1/2}$. Equation 28.14 relates rms voltage and current to the time averaged power, so in (b) we can solve for the rms maximum voltage at this resonant frequency.

EVALUATE **(a)** The maximum frequency will be found at the resonant frequency, which is equal to

$$f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(6.80\ \mu\text{F})(2.70\ \text{mH})}} = 1.17\,\text{kHz}$$

(b) Equation 28.14 shows that the maximum power dissipation occurs when $\langle P \rangle = I_{\text{rms}} V_{\text{rms}} = V_{\text{rms}}{}^2 / R$, so the maximum rms voltage that can be applied to the system at this frequency is equal to

$$V_{\text{rms}} = \sqrt{R\langle P \rangle} = \sqrt{(8.00\,\Omega)(45.0\,\text{W})} = 19.0\,\text{V}$$

ASSESS We would have to use *Z* in our calculation for (b) if we were not operating at the resonance frequency.

39. **INTERPRET** This is a problem about the current and voltage in a parallel *RLC* circuit, where we are given the resistance *R* and capacitance *C*. We are told an ammeter reads a minimum current when connected in parallel with *R* at a particular frequency, and we are interested in the value of *L* and the minimum value of the ammeter reading.

DEVELOP The minimum current running through *R* will be a result of the circuit operating at resonance, such that the *LC* loop is constantly storing energy back and forth between the magnetic field of the inductor and the electric field of the capacitor. This in turn results in little current drawn from the supply and thus minimum current crossing the resistor *R*. We can use this frequency to determine the value of *L* and use Equation 28.12 to determine the value of the current at resonance.

EVALUATE **(a)** Using the resonance frequency given, we determine the value of *L* to be

$$L = \frac{1}{\omega^2 C} = \frac{(2\pi \times 855\,\text{Hz})^{-2}}{1.50\ \mu\text{F}} = 23.1\ \text{mH}$$

(b) At this frequency, Equation 28.12 gives a minimum ammeter reading of

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{R} = \frac{(18\,\text{V})}{(1.50\,\text{k}\Omega)} = 12.0\,\text{mA}$$

ASSESS When a voltage supply operating at the resonance frequency is connected across a parallel *RLC* circuit, we can think of the parallel *LC* components as an open circuit, with a minimum current flowing through *R*.

40. **INTERPRET** This is a problem about the current and voltage in a parallel *RLC* circuit, for which we want to express the impedance *Z* with the use of a phasor diagram, as was done for a series *RLC* circuit in Section 28.4.

DEVELOP In the case of a parallel *RLC* circuit, the voltage across each element is the same, meaning a single phasor of length V_p represents the voltage. Just like in the case of a series *RLC* circuit, the resistor voltage and current are in phase, but the capacitor current leads the voltage and the inductor current lags, each by 90°. We can look at Fig. 28.16 to develop an analogous phasor diagram for the case of a parallel *RLC* circuit.

EVALUATE **(a)** From the phasor diagram we find we can express the peak current as: $I_\text{p} = \sqrt{I_{R\text{p}}^2 + \left(I_{L\text{p}} - I_{C\text{p}}\right)^2}$. Expressing this in terms of the common voltage V_p and the resistance and reactances gives

$$I_\text{p} = \sqrt{\frac{V_\text{p}^2}{R^2} + \left(\frac{V_\text{p}}{X_L} - \frac{V_\text{p}}{X_C}\right)^2}$$

Solving for V_p gives

$$V_\text{p} = \frac{I_\text{p}}{\sqrt{\frac{1}{R^2} + \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2}} = \frac{V_\text{p}}{Z}$$

Where we have obtained an impedance of

$$Z = \sqrt{\frac{1}{R^2} + \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2}$$

(b) Looking at our expression for the impedance, we see *Z* has its maximum value, namely *R*, at the resonant frequency, where $X_L = X_C$ and the term in parentheses is zero. At other frequencies *Z* is lower, so there's more current flowing through *R*, resulting in a higher reading at the ammeter.

ASSESS If we had been given a different frequency in the previous problem, we could have obtained the inductance and current from this circuit analysis.

PROBLEMS

41. **INTERPRET** We are to find the rms inductor current for two different reactive circuits.

DEVELOP Apply Equation 28.7, $I_{L,\text{rms}} = V_\text{rms} / (\omega L)$

EVALUATE **(a)** Inserting the given values into the expression for rms current gives

$$I_{L,\text{rms}} = \frac{V_\text{rms}}{\omega L} = \frac{230\ \text{V}}{2\pi(50\ \text{Hz})(1.7\ \text{H})} = 430\ \text{mA}$$

(b) A similar calculation with North American values gives 190 mA.

ASSESS The result is reported to two significant figures.

42. **INTERPRET** This problem is about capacitive and inductive reactances and how they depend on the frequency.

DEVELOP From Equations 28.5 and 28.7, the capacitive and inductive reactances are

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi fC}, \qquad X_L = \omega L = 2\pi fL$$

EVALUATE **(a)** From the above equation, the frequency of the applied voltage is

$$f = \frac{1}{2\pi X_C C} = \frac{1}{2\pi(1.0\ \text{k}\Omega)(3.0\ \mu\text{F})} = 53\ \text{Hz} \approx 50\ \text{Hz}$$

(b) Equating $X_L = X_C$ implies

$$L = \frac{X_C}{\omega} = \frac{X_C}{2\pi f} = \frac{1.0\ \text{k}\Omega}{2\pi(53\ \text{Hz})} = 3.0\ \text{H}$$

(c) Doubling ω doubles X_L and halves X_C, so X_L would be four times X_C at $f = 106$ Hz.

ASSESS The capacitive reactance X_C is inversely proportional to ω, whereas the inductive reactance X_L is proportional to ω. A larger capacitor has lower reactance and a larger inductor has higher reactance.

43. **INTERPRET** This is an exercise in dimensional analysis. We are to show that the units of capacitive and inductive reactance are ohms.

DEVELOP Inductance is defined as the ratio of flux to current (Equations 27.3), and capacitance as that of charge to potential difference (Equation 23.1).

EVALUATE Thus, the units of inductive reactance $(X_L = \omega L)$ are $\text{s}^{-1}(\text{T}\cdot\text{m}^2)/\text{A} = \text{V}/\text{A} = \Omega$ (the middle step follows from Faraday's law), and for capacitive reactance $(X_C = 1/\omega C)$ the units are $\text{V}/\text{C s}^{-1} = \text{V}/\text{A} = \Omega$ (the middle step following from the definition of current).

ASSESS The units work out as expected.

44. **INTERPRET** We're asked to express the time-varying potential of an alpha wave in the human brain.

DEVELOP To use Equation 28.3, $V = V_\text{p}\sin(\omega t)$, we need to convert the rms voltage to the voltage amplitude, as well as the frequency to the angular frequency:

$$V_\text{p} = \sqrt{2}V_\text{rms} = \sqrt{2}(31.8\ \mu\text{V}) = 45.0\ \mu\text{V}$$
$$\omega = 2\pi f = 2\pi(9.84\ \text{Hz}) = 61.8\ \text{s}^{-1}$$

EVALUATE Plugging the given values into the voltage equation gives

$$V = (45.0\ \mu\text{V})\sin\left[(61.8\ \text{s}^{-1})t\right]$$

ASSESS We may also write the angular frequency as rad/s, but radians are dimensionless, so it's not obligatory.

45. **INTERPRET** This problem involves a capacitive circuit consisting of two capacitors connected in parallel across a emf source. We are given one capacitance and are asked to find the other, and we are also asked to find frequency at which the rms current decreases to the given value.

DEVELOP Capacitors in parallel add, so the reactance of the combination is

$$X_C = \left[\omega(C_1 + C_2)\right]^{-1}$$

and, from the generalized version of Ohm's law (Equation 28.12 with $Z = X_C$) the rms current is $I_{C,\text{rms}} = \omega(C_1 + C_2)V_\text{rms}$, which allows us to find C_2 ($C_1 = 2.2$ nF).

EVALUATE **(a)** At a frequency of 1.0 kHz,

$$C_1 + C_2 = \frac{(3.4\ \text{mA})}{2\pi(1.0\ \text{kHz})(10\ \text{V})} = 54.1\ \text{nF}$$

Thus,

$$C_2 = (54.1 - 2.2)\ \text{nF} = 52\ \text{nF}$$

(b) Dividing the rms currents at the two frequencies, we get $f_2/f_1 = I_{\text{rms},2}/I_{\text{rms},1}$, or

$$f_2 = \frac{1.2\text{ mA}}{3.4\text{ mA}}(1.0\text{ kHz}) = 350\text{ Hz}$$

ASSESS The results are reported to two significant figures, as warranted by the data.

46. **INTERPRET** The problem concerns a device that uses capacitors to measure electric signals in the body. We want to know what's the minimum capacitance needed to measure beta waves in the brain.

DEVELOP The reactance is inversely proportional to the capacitance: $X_C = 1/\omega C$ (Equation 28.5). We have to remember to convert the frequency to angular frequency.

EVALUATE Given the maximum reactance for a certain frequency, the minimum capacitance for the electrode is

$$C \geq \frac{1}{\omega X_C} = \frac{1}{2\pi f X_C} = \frac{1}{2\pi(25\text{ Hz})(10\text{ M}\Omega)} = 640\text{ pF}$$

ASSESS We can check if this makes sense. Imagine the electrodes are placed over 1-mm-thick cotton fabric with a dielectric constant of near unity $(\kappa \approx 1)$. Then by Equation 23.4, the area of the electrodes would roughly be

$$A = \frac{Cd}{\kappa\epsilon_0} \approx \frac{(640\text{ pF})(1\text{ mm})}{\left(8.85\times10^{-12}\frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)} \approx 7\text{ cm}^2$$

This seems reasonable for the size of an electrode, so the capacitance we found is plausible.

47. **INTERPRET** This problem asks for the inductance that satisfies the resonance condition for a given range of capacitances and frequencies.

DEVELOP Using Equations 28.2 and 28.10, the resonant frequency can be written as

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi\sqrt{LC}}$$

which can be solved to give

$$L = \frac{1}{\omega^2 C} = \frac{1}{4\pi^2 f^2 C}$$

EVALUATE Using either condition, $f_1 = 88\text{ MHz}$ with $C_1 = 16.4\text{ pF}$, or $f_2 = 108\text{ MHz}$ with $C_2 = 10.9\text{ pF}$, we find the inductance to be

$$L = \frac{1}{4\pi^2 f_1^2 C_1} = \frac{1}{4\pi^2(88.0\text{ MHz})^2(16.4\text{ pF})} = 0.199\ \mu\text{H}$$

$$L = \frac{1}{4\pi^2 f_2^2 C_2} = \frac{1}{4\pi^2(108\text{ MHz})^2(10.9\text{ pF})} = 0.199\ \mu\text{H}$$

ASSESS For a given inductance *L*, the capacitance is inversely proportional to f^2. Thus, lower capacitance covers the higher end of the frequency band.

48. **INTERPRET** This problem involves an *LC* circuit for which we are to find the peak inductor current given the peak capacitor voltage. We are also to find the delay between the peak voltage and peak current.

DEVELOP From Example 28.3, we find that the peak current and voltage in an *LC* circuit are related by

$$I_\text{p} = \omega q_\text{p} = \omega C V_\text{p} = CV_\text{p}\big/\sqrt{LC} = V_\text{p}\big/\sqrt{L/C}$$

From the discussion accompanying Equation 28.6, we find that the voltage peak across an inductor precedes the current peak by $\omega\Delta t = 90° = \pi/2$.

EVALUATE (**a**) Inserting the given values into the expression for peak current gives

$$I_\text{p} = (190\text{ V})\sqrt{(0.025\ \mu\text{F})/(340\ \mu\text{H})} = 1.6\text{ A}$$

(**b**) Solving for Δt, we find that the voltage peaks

$$\Delta t = \frac{\pi}{2\omega} = \frac{\pi\sqrt{LC}}{2} = \frac{\pi}{2}\sqrt{(0.025\ \mu\text{F})(340\ \mu\text{H})} = 4.6\ \mu\text{s}$$

before the current peaks, where we have used Equation 28.10, $\omega = (LC)^{-1/2}$.

ASSESS The period of the voltage waveform is $4\Delta t = 18\ \mu$s.

49. **INTERPRET** This problem involves an *LC* circuit, whose capacitor is initially fully charged. We are to find the times at which the voltage, energy stored in the capacitor, and the current have reached half their initial/maximum values.

DEVELOP The capacitor voltage is given by $V(t) = q(t)/C$, the electric energy stored in the capacitor is given by $U(t) = q(t)^2/(2C)$, where $q(t) = q_0 \cos\omega t$ (see Equation 28.9), and the current in the circuit is given by $I(t) = dq/dt = -q_0\omega \sin\omega t$. We will solve for the time when each of these quantities reach half their initial/maximum values.

EVALUATE **(a)** The time at which the voltage reaches half its initial value is

$$V(t) = V_0/2$$
$$(q_0/C)\cos\omega t = (q_0/2C) \rightarrow \cos\omega t = 1/2$$
$$\omega t = \pi/3 \rightarrow t = \frac{\pi}{3}\sqrt{LC}$$

(b) The time at which the energy stored in the capacitor reaches half its initial value is

$$U(t) = U_0/2$$
$$\left(q_0^2/2C\right)\cos^2\omega t = \left(q_0^2/4C\right) \rightarrow \cos\omega t = \sqrt{2}/2$$
$$\omega t = \pi/4 \rightarrow t = \frac{\pi}{4}\sqrt{LC}$$

(c) The time at which the current reaches its maximum value is

$$I(t) = I_0$$
$$|-q_0\omega|\sin\omega t = q_0\omega \rightarrow \sin\omega t = 1$$
$$\omega t = \pi/2 \rightarrow t = \frac{\pi}{2}\sqrt{LC}$$

ASSESS From these values we see the energy stored in the capacitor goes below half its initial value before the voltage in the capacitor is halved, and before the current reaches its maximum value.

50. **INTERPRET** This problem involves an *LC* circuit with one capacitor charged to 250 V and which is initially in an open-circuit state. We are to manipulate the two switches shown in Fig. 28.25 to transfer all the energy to the other capacitor. To do this, we will need to store the energy temporarily in the intermediate inductor.

DEVELOP The energy in the inductor is proportional to the current squared, whereas that in the capacitor is proportional to the voltage squared (see Table 28.2). The energy initially stored in the first capacitor is

$$U_E = \frac{1}{2}CV^2 = \frac{1}{2}\left(2420\times10^{-6}\ \text{F}\right)\left(250\ \text{V}\right)^2 = 75.625\ \text{J}.$$

In an *LC* circuit, the current peaks 1/4 cycle after the voltage peaks, so we need to close switch *A* for 1/4 cycle to transfer all the energy from the 2420-μF capacitor to the inductor. Next, we can open switch *A* and close switch *B* for another 1/4 cycle to transfer the energy from the inductor to the 605-μF capacitor.

EVALUATE **(a)** As explained above, we first close switch *A* for one quarter of a period of the *LC* circuit containing the 2420-μF capacitor, or

$$t_A = \tfrac{1}{4}T_A = \tfrac{1}{4}(2\pi/\omega_A) = \tfrac{1}{2}\pi\sqrt{LC_A} = \tfrac{1}{2}\pi\sqrt{(112\ \text{H})\left(2420\times10^{-6}\ \text{F}\right)} = 818\ \text{ms}$$

This transfers 75.625 J to the inductor. Then open switch *A* and close switch *B* for one quarter of a period of the *LC* circuit containing the 605-μF capacitor, or

$$t_B = \tfrac{1}{2}\pi\sqrt{(112\ \text{H})\left(605\times10^{-6}\ \text{F}\right)} = 409\ \text{ms}$$

This transfers 75.625 J to the second capacitor from the inductor. Finally, open switch *B* to maintain the charge on the 605-μF capacitor.

(b) When the second capacitor has 75.625 J of stored energy, its voltage is

$$V = \sqrt{2(75.625\ \mathrm{J})\Big/\left(605\times10^{-6}\,\mathrm{F}\right)} = 500\ \mathrm{V}$$

ASSESS The time to transfer the energy from the inductor to the second (smaller) capacitor is ½ that it takes to transfer the energy from the large capacitor to the inductor. This illustrates the more rapid response of smaller capacitors.

51. **INTERPRET** This problem involves an *LC* circuit with damping due to the resistance. We want to find the number of oscillations the circuit completes before the peak voltage is reduced by half.

DEVELOP For a damped *LC* circuit, Equation 28.11 gives the charge as a function of time. Because $V(t) = q(t)/C$, the voltage as a function of time can be written as

$$V(t) = V_\mathrm{p} e^{-Rt/2L} \cos \omega t$$

The peak voltage decays with time constant $2L/R$. Half the initial peak value is reached after a time $t = (2L/R)\ln 2$ (when $e^{-Rt/2L} = \frac{1}{2}$).

EVALUATE Since the period of oscillation is $T = 2\pi/\omega = 2\pi\sqrt{LC}$, the number of cycles that occur within time *t* is

$$\frac{t}{T} = \frac{(2L/R)\ln 2}{2\pi\sqrt{LC}} = \frac{\ln 2}{\pi R}\sqrt{\frac{L}{C}} = \frac{\ln 2}{\pi(1.7\ \Omega)}\sqrt{\frac{20\ \mathrm{mH}}{0.14\ \mu\mathrm{F}}} = 49$$

ASSESS This oscillation is underdamped. The larger the resistance, the more rapidly the oscillation decays.

52. **INTERPRET** This problem involves a damped *LC* circuit. Given the resistance, the inductance, and the time it takes for the circuit to dissipate half its energy, we are to find the capacitance.

DEVELOP If only half the energy is lost after 15 cycles, the damping is small and the energy varies like the square of Equation 28.11, namely

$$U_\mathrm{tot} = U_\mathrm{p} e^{-Rt/L} \cos^2 \omega t$$

The energy time constant is L/R, which is one-half the charging time constant. After 15 cycles, $t = 15\ T = 15(2\pi/\omega)$ and the fraction of energy remaining is

$$\frac{U_\mathrm{tot}}{U_\mathrm{p}} = \frac{1}{2} = e^{-15RT/L}\cos^2(30\pi) = e^{-15RT/L}$$

Solve this for the capacitance *C* using Equation 28.2 $\omega = (LC)^{-1/2}$.

EVALUATE Take logarithms to get

$$L \ln 2 = 15RT = 30\pi R\sqrt{LC},$$

from which we find

$$C = \left(\frac{\ln 2}{30\pi R}\right)^2 L = \left(\frac{\ln 2}{30\pi\times5.0\ \Omega}\right)^2 (100\ \mathrm{mH}) = 0.22\ \mu\mathrm{F}$$

ASSESS This is a typical capacitance for a damped *LC* circuit.

53. **INTERPRET** This problem involves a series *RLC* circuit at resonance. We want to find the smallest resistance that still keeps the capacitor voltage under its rated value when the circuit is at resonance.

DEVELOP In a series *RLC* circuit at resonance, the peak capacitor voltage is

$$V_{C,\mathrm{p}} = I_\mathrm{p} X_C = \frac{V_\mathrm{p}/R}{\omega_0 C} = \frac{V_\mathrm{p}}{R}\sqrt{\frac{L}{C}}$$

where $\omega_0 = (LC)^{-1/2}$ is the resonant angular frequency.

EVALUATE The condition that $V_{C,\mathrm{p}} \le 350\ \mathrm{V}$ implies

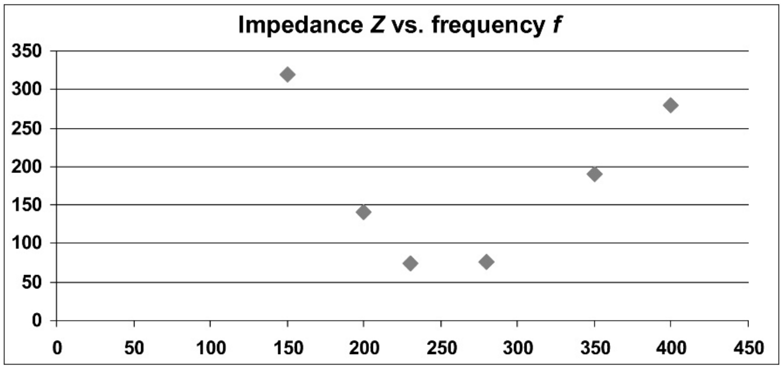
$$R \ge \frac{V_\mathrm{p}}{V_{C,\mathrm{p}}}\sqrt{\frac{L}{C}} = \left(\frac{29\ \mathrm{V}}{350\ \mathrm{V}}\right)\sqrt{\frac{1.7\ \mathrm{H}}{270\ \mu\mathrm{F}}} = 6.6\ \Omega$$

ASSESS Our results shows that $V_{C,\text{p}}$ is inversely proportional to R. This means that a larger resistor would be required if the capacitor has a lower voltage rating.

54. **INTERPRET** We are given the data of the impedance as a function of frequency for a series *RLC* circuit. By analyzing the data, we are asked to estimate the resonant frequency and the resistance.

DEVELOP The impedance is given by $Z = \sqrt{R^2 + (X_L - X_C)^2}$, where $X_C = 1/(\omega C)$ and $X_L = \omega L$. At the resonant frequency ω_0, $X_C = X_L$, the circuit exhibits minimum impedance: $Z = R$. Thus, we plot Z versus f, and located the minimum.

EVALUATE **(a)** The plot of Z versus f is shown below.



The minimum occurs at $f_0 = 250$ Hz, or $\omega_0 = 2\pi f_0 = 2\pi(250) = 500\pi$ rad/s.

(b) From the plot, we estimate the resistance R, or the minimum value of Z, to be about 50 Ω,

ASSESS At resonance, Z is at a minimum, and the current is at a maximum.

55. **INTERPRET** This problem involves analyzing a phasor diagram for a driven *RLC* circuit to find if the driving frequency is above or below resonance. We are also to complete the diagram and use it to find the phase difference between the applied voltage and current.

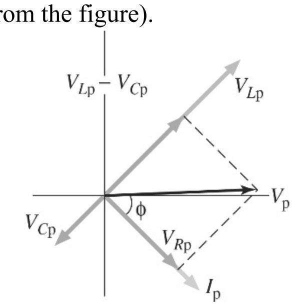
DEVELOP Our diagram has three phasors, $V_{R\text{p}}, V_{L\text{p}},$ and $V_{C\text{p}},$ representing the voltages across the resistor, the inductor, and the capacitor, respectively. Because the resistor voltage is in phase with the current, I_p is in the same direction as $V_{R\text{p}}$. The resonant frequency is $\omega_0 = (LC)^{-1/2}$.

EVALUATE **(a)** From the observation that $V_{L\text{p}} = I_\text{p}\omega L > V_{C\text{p}} = I_\text{p}/\omega C$, we conclude that $\omega^2 > 1/(LC) = \omega_0^2$, which means the frequency is above resonance.

(b) The applied voltage phasor is the vector sum of the resistor, capacitor, and inductor voltage phasors, as shown below. The current is in phase with the voltage across the resistor, which in this case is lagging the applied voltage because

$$\phi = \tan^{-1}\left(\frac{V_{L\text{p}} - V_{C\text{p}}}{V_{R\text{p}}}\right) > 0$$

by approximately 50° (as estimated from the figure).



ASSESS Our circuit is inductive since $V_{L\text{p}} > V_{C\text{p}}$. Note that a positive ϕ means that voltage leads current, and a negative ϕ means voltage lags current. At resonance, $X_L = X_C$ and $\phi = 0$.

56. **INTERPRET** We are to show that the current in an *RLC* circuit at twice the resonance frequency is the same as the current at half the resonance frequency, provided these are half the current at resonance.

DEVELOP At resonance, $X_L = X_C$, and Equation 28.12 takes the form

$$I_{\text{res}} = \frac{V_{\text{p}}}{R}$$

Taking the ratio of the peak current to the current at resonance gives

$$\frac{I_{\text{p}}}{I_{\text{res}}} = \frac{1}{\sqrt{1+\left(X_L - X_C\right)^2 / R^2}} = \left[1 + \frac{L}{R^2 C}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)^2\right]^{-1/2}$$

where $\omega_0 = (LC)^{-1/2}$ is the resonance frequency.

EVALUATE Since this expression does not change when ω/ω_0 is replaced by its reciprocal, the assertion in this problem is true. (That is, $\omega = \frac{1}{2}\omega_0$ and $\omega = 2\omega_0$ give the same I_{p}.)

ASSESS Thus, we have found that if

$$I\left(\omega_0/n\right) = \frac{1}{n} I\left(\omega_0\right)$$

then

$$I\left(\omega_0/n\right) = I\left(n\omega_0\right)$$

57. **INTERPRET** We are to find the power factor and the power dissipation in a series *RLC* circuit.

DEVELOP From the geometry of Fig. 28.16, we find that the power factor of the circuit is

$$\cos\phi = \frac{V_{R\text{p}}}{V_{\text{p}}} = \frac{I_{\text{p}} R}{I_{\text{p}} Z} = \frac{R}{Z}$$

The average power in the circuit is given by Equation 28.14:

$$\langle P \rangle = I_{\text{rms}} V_{\text{rms}} \cos\phi = I_{\text{rms}}\left(I_{\text{rms}} Z\right)\left(R / Z\right) = I_{\text{rms}}^2 R$$

EVALUATE **(a)** Substituting the values given, we find the power factor is

$$\cos\phi = \frac{R}{Z} = \frac{150\ \Omega}{250\ \Omega} = 0.6$$

(b) The above equation gives $\langle P \rangle = I_{\text{rms}}^2 R = (250\ \text{mA})^2 (150\ \Omega) = 9.4\ \text{W}.$

ASSESS Note that the average AC power is given by the same expression as the DC power if the rms current is used. The power factor must be between zero and 1. A purely resistive circuit has a power factor of 1, while a circuit with only capacitance or inductance has a power factor of zero.

58. **INTERPRET** We are to find the resistance and the resonant frequency of a series *RLC* circuit given the power factor and the impedance at 442 Hz.

DEVELOP From the geometry of Fig. 28.16, we find that the power factor of the circuit is

$$\cos\phi = \frac{V_{R\text{p}}}{V_{\text{p}}} = \frac{I_{\text{p}} R}{I_{\text{p}} Z} = \frac{R}{Z}$$

from which we can find the resistance *R*. The reactance of the circuit can be expressed in terms of the inductance, the resonant frequency, and the given values by using Equation 28.12:

$$X_L - X_C = \omega L - \frac{1}{\omega C} = \omega L\left(1 - \frac{1}{\omega^2 LC}\right) = \omega L\left(1 - \frac{\omega_0^2}{\omega^2}\right) = \pm\sqrt{Z^2 - R^2}$$

EVALUATE (**a**) Solving for *R* gives

$$R = Z\cos\phi = (182\ \Omega)(0.764) = 139\ \Omega$$

(**b**) Using the result obtained in **(a)** for *R*, we have

$$\omega L\left(1-\frac{\omega_0^2}{\omega^2}\right)=\pm\sqrt{Z^2-R^2}=\pm\sqrt{(182\ \Omega)^2-(139\ \Omega)^2}=\pm 117.5\ \Omega \quad\Rightarrow\quad \frac{\omega_0^2}{\omega^2}=1\pm\frac{117.5\ \Omega}{\omega L}$$

With $\omega L=(2\pi\times 442\text{ Hz})\left(25.0\times 10^{-3}\text{ H}\right)=69.43\ \Omega,$ we can discard the unphysical solution (with $\omega_0^2/\omega^2<0$) to find

$$\omega_0=\omega\sqrt{1+\frac{117.5}{69.43}}=1.64\omega,$$

or

$$f_0=1.64f=1.64(442\text{ Hz})=725\text{ Hz}$$

Assess An alternative way to show the relationship between the power factor and the impedance and resistance is to use Equations 28.12 and 28.13 and the trigonometric identity $\sec^2\phi=1+\tan^2\phi.$

59. **Interpret** You want to know the percentage of power your company loses during transmission over its electric lines.

Develop For AC circuits, the average power produced is given in Equation 28.14: $\langle P\rangle=I_{\text{rms}}V_{\text{rms}}\cos\phi,$ where $\cos\phi$ is the power factor. The power lost in the transmission lines is I^2R at any given time, but the average power lost will be

$$\langle P_{\text{lost}}\rangle=\langle I^2R\rangle=\langle I^2\rangle R=I_{\text{rms}}^2R$$

where we have used the fact that the resistance is constant over time as well as the definition of root-mean-squared: $I_{\text{rms}}=\sqrt{\langle I^2\rangle}.$

Evaluate **(a)** For a power factor of 1.0, the percentage of power lost in the transmission lines is

$$\frac{\langle P_{\text{lost}}\rangle}{\langle P\rangle}=\frac{I_{\text{rms}}R}{V_{\text{rms}}\cos\phi}=\frac{(200\text{ A})(95\ \Omega)}{(365\text{ kV})(1.0)}=5.2\%$$

(b) For a power factor of 0.8, the same percentage is

$$\frac{\langle P_{\text{lost}}\rangle}{\langle P\rangle}=\frac{I_{\text{rms}}R}{V_{\text{rms}}\cos\phi}=\frac{(200\text{ A})(95\ \Omega)}{(365\text{ kV})(0.8)}=6.5\%$$

Assess In this problem, the current is constant, so the power lost will be the same in both cases. What does change is the amount of AC power produced at the plant. For a power factor of 1.0, the current and voltage are in phase, and the power in the circuit is maximized. But for a lower power factor, the current and voltage are out of phase, so the plant is producing less power for its end-users while still losing the same amount in the transmission lines.

60. **Interpret** You want to know the rms current supplying the motor before and after the power factor increase.

Develop For AC circuits, the average power produced is given in Equation 28.14: $\langle P\rangle=I_{\text{rms}}V_{\text{rms}}\cos\phi,$ where $\cos\phi$ is the power factor. We are given the power supplied (10 hp = 7.46 kW), as well as the rms voltage, so we can determine the initial supply current, and the current supplied after the growth in power factor.

Evaluate **(a)** For a power factor of 0.82, the rms current is

$$I_{\text{rms}}=\frac{\langle P\rangle}{V_{\text{rms}}\cos\phi}=\frac{(7.46\,\text{kW})}{(480\,\text{V})(0.82)}=19\,\text{A}$$

(b) For a power factor of 0.95, the current decreases to

$$I_{\text{rms}}=\frac{\langle P\rangle}{V_{\text{rms}}\cos\phi}=\frac{(7.46\,\text{kW})}{(480\,\text{V})(0.95)}=16\,\text{A}$$

Assess For a power factor of 1.0, the current and voltage are in phase, and the power in the circuit is maximized. But for a lower power factor, the current and voltage are out of phase, so the plant is producing less power for its end-users, while still losing the same amount in the transmission lines.

61. **Interpret** This problem deals with DC power supplies. If the time constant *RC* is long enough, the capacitor voltage will only decrease slightly before the AC voltage from the transformer rises again to fully charge the capacitor.

DEVELOP The scenario is depicted in Fig. 28.23. From the given DC output, we find the load resistance to be $R = (22\text{ V})/(180\text{ mA}) = 122\ \Omega.$ In one period of the input AC $(T = 1/f),$ the capacitor voltage must decay by less than 4%, or $e^{-T/RC} \geq 0.96.$

EVALUATE The above condition implies that

$$C \geq -\frac{T}{R\ln(0.96)} = -\frac{1}{Rf\ln(0.96)} = -\frac{1}{(122\ \Omega)(50\text{ Hz})\ln(0.96)} = 4.0\text{ mF}$$

ASSESS If the capacitance is large enough, the load current and voltage can be made arbitrarily smooth with negligible decay.

62. **INTERPRET** This problem involves a damped *RLC* circuit whose peak voltage across the capacitor decays as given. We are to find the resistance of this circuit.

DEVELOP For the damped oscillations of an *RLC* circuit, the voltage decays according to Equation 28.11,

$$V(t) = q(t)/C = V_\text{p}e^{-Rt/2L}\cos\omega t$$

with frequency given by Equation 28.10. If in 10 cycles $\left[t = 10\ T = 10(2\pi/\omega) = 20\pi\sqrt{LC}\right]$ the peak voltage has decayed from 35 V to 28 V, which gives

$$V(0) = 35\text{ V} = V_\text{p}$$

$$V\left(20\pi\sqrt{LC}\right) = 28\text{ V} = V_\text{p}\exp\left(\frac{-R\left(20\pi\sqrt{LC}\right)}{2L}\right)\cos\left(20\pi\sqrt{LC}\omega\right)$$

Take the ratio and solve for *R*.

EVALUATE The resistance is

$$\ln\left(\frac{35}{28}\right) = \frac{Rt}{2L} = 10\pi R\sqrt{C/L}$$

$$R = (10\pi)^{-1}\sqrt{27\text{ mH}/3.3\ \mu\text{F}} \times \ln(35/28) = 640\text{ m}\Omega$$

ASSESS The result is given to two significant figures, as warranted by the data.

63. **INTERPRET** We have an AC generator connected to a series *RLC* circuit, and we want to know its maximum peak voltage when the circuit is at resonance.

DEVELOP The peak capacitor voltage $V_{C\text{p}} = I_\text{p}X_C.$ At resonance, the impedance is $Z = R,$ and $I_\text{p} = V_\text{p}/R.$ The capacitive reactance is $X_C = 1/\omega_0 C = \sqrt{L/C}.$

EVALUATE The condition that $V_{C\text{p}} = (V_\text{p}/R)\sqrt{L/C} \leq 600\text{ V}$ implies

$$V_\text{p} \leq (600\text{ V})(1.1\ \Omega)\sqrt{\frac{0.43\ \mu\text{F}}{29\text{ mH}}} = 2.5\text{ V}$$

ASSESS The inductor voltage at resonance is

$$V_{L\text{p}} = I_\text{p}X_L = \frac{V_\text{p}}{R}\omega_0 L = \frac{V_\text{p}}{R}\frac{L}{\sqrt{LC}} = \frac{V_\text{p}}{R}\sqrt{\frac{L}{C}}$$

which is the same as $V_{C\text{p}}$. The two voltages cancel exactly at resonance. Note that $V_{C\text{p}}$ and $V_{L\text{p}}$ are both higher than V_p.

64. **INTERPRET** Starting with the charge on the capacitor in an *LC* circuit, we are to find the current and the voltage, then the energy stored in the capacitor and in the inductor. We are to sum the two to find the total energy and show that it's constant.

DEVELOP We start with $q = q_\text{p}\cos\omega t,$ and differentiate with respect to time to find the current. We also use $q = CV$ to find the voltage. The energy stored in the electric field of the capacitor is $U_C = \tfrac{1}{2}CV^2,$ and the energy stored in the magnetic field of the inductor is $U_L = \tfrac{1}{2}LI^2.$

EVALUATE The current in the *LC* circuit is

$$I = \frac{dq}{dt} = -\omega q_{\mathrm{p}} \sin \omega t$$

The voltage across the capacitance is

$$q = CV$$

$$V = \frac{q}{C} = \frac{q_{\mathrm{p}}}{C} \cos \omega t$$

The electric energy in the capacitor is

$$U_C = \frac{q_{\mathrm{p}}^2}{2C} \cos^2 \omega t$$

The magnetic energy in the inductor is

$$U_L = \tfrac{1}{2} L\omega^2 q_{\mathrm{p}}^2 \sin^2 \omega t.$$

The total energy is

$$U = U_C + U_L = \frac{q_{\mathrm{p}}^2}{2C}\left(\sin^2 \omega t + \cos^2 \omega t\right) = \frac{q_{\mathrm{p}}^2}{2C}$$

ASSESS We have shown that the energy in this circuit is a constant.

65. **INTERPRET** In Example 28.4, we found a frequency at which the current in an *RLC* circuit is half its maximum value. Here, we are to find a second frequency at which the current will be half the maximum. We shall use Equation 28.12 for *Z*.

DEVELOP From Example 28.4, we have $C = 11.5\ \mu\mathrm{F}$, $R = 8.0\ \Omega$, and $L = 22$ mH. We also know that $Z = \sqrt{R^2 + (X_L - X_C)^2}$, where $X_C = 1/(\omega C)$ and $X_L = \omega L$. The current is given by Ohm's law (Equation 28.12), $I = V/Z$, and we are looking for a value of ω such that $Z = 2R$.

EVALUATE Solving for ω gives

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = 2R$$

$$4R^2 = R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2 \quad \Rightarrow \quad 3R^2 = \omega^2 L^2 - 2\frac{L}{C} + \frac{1}{\omega^2 C^2}$$

$$R^2\omega^2 = \omega^4 L^2 - 2\frac{L\omega^2}{C} + \frac{1}{C^2}$$

$$0 = \omega^4 L^2 + \left(-2\frac{L}{C} - 3R^2\right)\omega^2 + \frac{1}{C^2}$$

$$\omega \in \{\pm 3882, \pm 10181\}\ \ \mathrm{rad/s}$$

The sign of ω is irrelevant. We need to convert to frequency using $f = \omega/2\pi$, so $f \in \{618\ \mathrm{Hz}, 1620\ \mathrm{Hz}\}$.

ASSESS The 618-Hz answer was given in the example, so the solution we need is $f = 1620$ Hz.

66. **INTERPRET** We have two capacitors connected first in series and then in parallel with an AC generator, and we want to know their capacitances given that the current drops from 56 mA to 2.8 mA upon going from parallel to series connections.

DEVELOP Equation 28.5 gives the rms current when capacitors are connected to an AC generator,

$$I_{\mathrm{rms}} = V_{\mathrm{rms}}/X_C = \omega C V_{\mathrm{rms}} = 2\pi f C V_{\mathrm{rms}}$$

For the parallel connection, $C_{\mathrm{p}} = C_1 + C_2$, (see Chapter 23), so

$$56\times 10^{-3}\ \mathrm{A} = (2\pi)\left(7.5\times 10^3\ \mathrm{Hz}\right)\left(C_1 + C_2\right)(24\ \mathrm{V})$$

For the series connection, $C_{\mathrm{s}} = C_1 C_2/(C_1 + C_2)$, so Equation 28.5 gives

$$2.8\times10^{-3}\text{ A} = (2\pi)\left(7.5\times10^{3}\text{Hz}\right)\left(\frac{C_1C_2}{C_1+C_2}\right)(24\text{ V})$$

The two equations can be used to solve for C_1 and C_2.

EVALUATE Simplifying the above two equations leads to $C_p = 49.51$ nF and $C_s = 2.476$ nF. The solutions for C_1 and C_2 are

$$C_1 = 2.6\text{ nF},\ C_2 = 46.9\text{ nF}$$

ASSESS The parallel connection yields a greater capacitance, and hence a larger current compared to the series combination.

67. **INTERPRET** This problem involves a series *RLC* circuit for which we are given the current at resonance and at another frequency. We are asked to find the resistance, the inductance, and the capacitance.

DEVELOP At resonance, the impedance is $Z = R$ and the current is $I_p = V_p/Z = V_p/R$ and $X_L - X_C = 0.$ Away from resonance, $Z = V_p/I_p$ and $|X_L - X_C| = \sqrt{Z^2 - R^2}.$

EVALUATE The resonance condition gives

$$R = \frac{V_p}{I_p} = \frac{15.0\text{ V}}{112\text{ mA}} = 134\ \Omega$$

On the other hand, at the other given frequency $\omega_\beta = 2\pi(1.22\,\text{kHz})$, the impedance is

$$Z = \frac{V_p}{I_p} = \frac{15.0\text{V}}{44.8\text{ mA}} = 335\ \Omega = \frac{10R}{4}$$

which gives

$$|X_L - X_C| = \sqrt{Z^2 - R^2} = R\sqrt{(10/4)^2 - 1} = \frac{\sqrt{21}}{2}R$$

With $X_L = \omega L$ and $X_C = 1/\omega C$, we obtain the following conditions:

$$\frac{1}{\omega_0 C} - \omega_0 L = 0, \qquad \left|\frac{1}{\omega_\beta C} - \omega_\beta L\right| = \frac{\sqrt{21}}{2}R$$

These equations can be solved for C and L, with the following result:

$$L = \frac{\sqrt{21}\omega_\beta R}{2\left|\omega_0{}^2 - \omega_\beta{}^2\right|} = \frac{\sqrt{21}\left(2\pi\times1.22\text{ kHz}\right)(134\ \Omega)}{2(2\pi)^2\left|(775\text{ Hz})^2 - (1.22\text{ kHz})^2\right|} = 67.1\text{ mH}$$

$$C = \frac{1}{\omega_0^2 L} = \frac{1}{(2\pi\times775\,\text{Hz})^2(67.1\text{ mH})} = 0.628\ \mu\text{F}$$

ASSESS Above resonance, inductive reactance dominates, with $X_L > X_C$.

68. **INTERPRET** In this problem we are asked to derive the Q factor of an *RLC* circuit that satisfies the criteria given in the problem statement.

DEVELOP To derive the expression for Q, we first need to know the power in the circuit. From Equations 28.12 and 28.14 (with rms values), and

$$\cos\phi = \frac{V_{Rp}}{V_p} = \frac{I_p R}{I_p Z} = \frac{R}{Z}$$

from Fig. 28.16, the average power in a series *RLC* circuit can be written as

$$\langle P\rangle = I_{\text{rms}}V_{\text{rms}}\cos\phi = (V_{\text{rms}}/Z)V_{\text{rms}}(R/Z) = \frac{V_{\text{rms}}^2 R}{Z^2}$$

The above expression shows the power falls to half its resonance value (V_{rms}^2/R) when $Z = \sqrt{2}R,$ or when $|X_L - X_C| = R.$ In terms of the resonant frequency $\omega_0 = (LC)^{-1/2},$ this condition becomes

$$\left|\omega L - \frac{1}{\omega C}\right| = L\left|\omega - \frac{\omega_0^2}{\omega}\right| = R$$

$$\omega^2 - \omega_0^2 = \pm\frac{R}{L}\omega$$

The solutions of these quadratics, with $\omega > 0$, are

$$\omega_\pm = \frac{1}{2}\left[\pm\frac{R}{L} + \sqrt{\frac{R^2}{L^2} + 4\omega_0^2}\right]$$

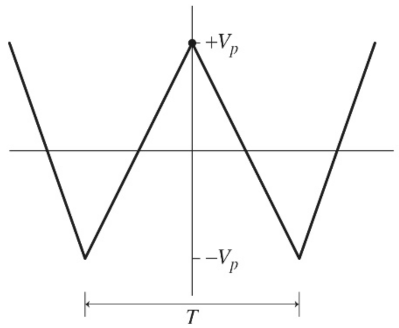
The Q factor is then equal to $\omega_0/\Delta\omega$, where $\Delta\omega = \omega_+ - \omega_-$.

Evaluate If $R/L \ll \omega_0$ (or $R \ll \sqrt{L/C}$), we can neglect the first term under the square root sign compared to the second, which gives $\omega \approx \omega_0 \pm R/2L$. The difference between these two values of ω is $\Delta\omega = R/L$ from which we obtain $Q = \omega_0/\Delta\omega = \omega_0 L/R$.

Assess The Q factor measures the "quality" of oscillation. The smaller the resistance, the higher the Q-factor. In the absence of resistance $(R \to 0)$, the LC circuit can oscillate indefinitely.

69. **Interpret** We want the rms voltage for a triangle wave.

Develop We draw the corresponding triangle wave in the figure below. For simplicity, we've chosen to make the graph symmetric around the $t = 0$ axis. Each cycle takes the time of one period, T.



The voltage change over half a period is $\Delta V = \pm 2V_\text{p}$, so the slope of the line alternates between $+4V_\text{p}/T$ and $-4V_\text{p}/T$. We can characterize the cycle centered at the origin by

$$V(t) = \begin{cases} V_\text{p} + (4V_\text{p}/T)t & \text{for } -T/2 \le t < 0 \\ V_\text{p} - (4V_\text{p}/T)t & \text{for } 0 \le t < +T/2 \end{cases}$$

Evaluate To find the rms value, we square the voltage and take the average over one period:

$$\begin{aligned}
\langle V^2 \rangle &= \frac{1}{T}\left[\int_{-T/2}^{0}\left(V_\text{p} + \tfrac{4V_\text{p}}{T}t\right)^2 dt + \int_0^{T/2}\left(V_\text{p} - \tfrac{4V_\text{p}}{T}t\right)^2 dt\right] \\
&= \frac{V_\text{p}^2}{T}\left[-\left(-\tfrac{T}{2}\right) - \tfrac{1}{2}\tfrac{8}{T}\left(-\tfrac{T}{2}\right)^2 - \tfrac{1}{3}\left(\tfrac{4}{T}\right)^2\left(-\tfrac{T}{2}\right)^3 + \left(\tfrac{T}{2}\right) - \tfrac{1}{2}\tfrac{8}{T}\left(\tfrac{T}{2}\right)^2 + \tfrac{1}{3}\left(\tfrac{4}{T}\right)^2\left(\tfrac{T}{2}\right)^3\right] \\
&= \tfrac{1}{3}V_\text{p}^2
\end{aligned}$$

Taking the square root gives $V_\text{rms} = V_\text{p}/\sqrt{3}$, as was expected.

Assess One can certainly simplify the calculation by just doing half a cycle, since the triangle wave is symmetric around its midpoint.

70. **Interpret** We are to use the equation for charge on a capacitor in an RLC circuit and the differential equation for an RLC circuit from Kirchhoff's laws to find an expression for ω.

Develop The given equations are $q(t) = q_\text{p}e^{-Rt/2L}\cos\omega t$, and $L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = 0$. We take the derivatives of q, substitute it into the differential equation, and solve the resulting equation for ω.

Evaluate We first calculate the derivatives of $q(t)$:

$$q(t) = q_p e^{-Rt/2L} \cos \omega t$$

$$q'(t) = -\frac{q_p}{2L} e^{-\frac{Rt}{2L}} (R \cos \omega t + 2\omega L \sin \omega t)$$

$$q''(t) = \frac{q_p}{4L^2} e^{-\frac{Rt}{2L}} \left[\left(R^2 - 4L^2\omega^2\right) \cos \omega t + 4\omega RL \sin \omega t \right]$$

Substituting these into the differential equation gives us, after some algebraic steps,

$$\frac{q_\text{p}}{CL} e^{-\frac{Rt}{2L}} \left(-4L + CR^2 + 4CL^2\omega^2\right) \cos \omega t = 0$$

For this equation to be true for all values of *t*, the term in parentheses must be zero.

$$-4L + CR^2 + 4CL^2\omega^2 = 0$$

$$\omega = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$$

ASSESS This reduces to $\omega = (LC)^{-1/2}$ if $R = 0$.

71. **INTERPRET** We are to find the frequency at which the voltage across a capacitor is maximized, and also the value of that maximum voltage. We shall use both the impedance of the *RLC* circuit and the impedance of the capacitor alone.

DEVELOP We are given, in Example 28.4, the component values $L = 2.2$ mH, $C = 11.5$ μF, and $R = 8.0$ Ω. The peak voltage is $V_\text{p} = 20$ V. The peak voltage across the capacitor will be $V_\text{Cp} = I_\text{p} X_C$, where $I_\text{p} = V_\text{p}/Z$ and $X_C = 1/(\omega C)$. We want to find the maximum value of V_Cp and the frequency at which it occurs.

EVALUATE

$$V_\text{Cp} = \left(\frac{V_\text{p}}{\sqrt{R^2 + \left[\omega L - 1/(\omega C)\right]^2}} \right) \frac{1}{\omega C}$$

so we set the derivative equal to zero and solve for ω:

$$\frac{dV_\text{Cp}}{d\omega} = 0 = -\frac{V_\text{p}\left[CR^2 + 2L\left(CL\omega^2 - 1\right)\right]}{\left\{1 + C\omega^2\left[CR^2 + L\left(CL\omega^2 - 2\right)\right]\right\}\sqrt{R^2 + \left[L\omega - 1/(\omega C)\right]^2}}$$

$$= CR^2 + 2L\left(CL\omega^2 - 1\right)$$

$$\omega = \sqrt{\frac{1}{LC} - \frac{R^2}{2L^2}} = \sqrt{\frac{1}{(2.2\text{ mH})(11.5\ \mu\text{F})} - \frac{(8.0\ \Omega)^2}{2(2.2\text{ mH})}} = 5737\ \text{rad/s}$$

$$f = \frac{\omega}{2\pi} = 913\text{ Hz} \approx 910\text{ Hz}$$

to two significant figures. We substitute this value of ω into the equation for V_Cp to find the peak voltage across the capacitor, $V_\text{Cp} = 36$ V.

ASSESS Although the peak voltage on the capacitor is higher than the peak supply voltage, that's ok: the voltage across the inductor will be negative when the capacitor hits this voltage so Kirchhoff's loop law is not violated.

72. **INTERPRET** We are analyzing a filter consisting of an *RC* circuit.

DEVELOP To determine which frequencies can pass through the filter, we consider the voltage across the capacitor, which will be equal to the output voltage, V_out. For a given frequency, the peak current through the *RC* circuit is given by Equation 28.12: $I_\text{p} = V_\text{in,p}/Z$, where in this case $Z = \sqrt{R^2 + X_C^2}$. The peak voltage across the capacitor will be $V_\text{Cp} = I_\text{p} X_C$. Using $X_C = 1/\omega C$, and defining $\omega_{RC} = 1/RC$, we can write the capacitor voltage as

$$V_{Cp} = \frac{V_\text{in,p}}{\sqrt{1 + (\omega/\omega_{RC})^2}}$$

EVALUATE If $\omega << \omega_{RC}$, then $V_{\mathrm{Cp}} = V_{\mathrm{in,p}}$, which implies that $V_{\mathrm{out}} = V_{\mathrm{in}}$. Therefore, low frequencies are passed from the input to the output. By contrast, for $\omega \gg \omega_{RC}$, we have $V_{\mathrm{Cp}} = 0$. This means there is no output at high frequencies. This, then, is a low-pass filter.

The answer is **(a)**.

ASSESS Another way to arrive at this is to recall the short-term and long-term behavior of *RC* circuits from Chapter 25. Over short times ($t << RC$ or equivalently $\omega >> \omega_{RC}$), the capacitor acts like a short-circuit, so current will flow through the capacitor, and there will be no voltage at the output. Over long times ($t >> RC$ or equivalently $\omega << \omega_{RC}$), the capacitor acts like an open circuit, so no current flows in the capacitor, which means the input and output have the same voltage. One might have wrongly guessed that the presence of the capacitor implies a high-pass filter, judging from the Application Loudspeaker Systems in the text. But in that case the capacitor is in series with the output, whereas in this case it is in parallel.

73. **INTERPRET** We are analyzing a filter consisting of an *RC* circuit.

DEVELOP In the previous problem, we argued that the output voltage is the same as the voltage across the capacitor, which has a peak value of

$$V_{Cp} = \frac{V_{\text{in},p} X_C}{\sqrt{R^2 + X_C^2}}$$

EVALUATE If the capacitor's reactance is equal to the resistance $(X_C = R)$, then $V_{\text{Cp}} = V_{\text{in},\,p} / \sqrt{2}$, and the output voltage will be $V_{\text{in}} / \sqrt{2}$.

The answer is **(c)**.

ASSESS Notice that setting the reactance equal to resistance is the same as setting $\omega = 1/RC$, or $\omega = \omega_{RC}$ as defined in the previous problem. This frequency corresponds to a time in between the short- and long-term behavior of the capacitor, so the input is only partially passed to the output.

74. **INTERPRET** We are analyzing a filter consisting of an *RC* circuit.

DEVELOP Since there is no inductance, there is technically no resonance in this circuit. The maximum output voltage is the input voltage $(V_{\text{out}} = V_{\text{in}})$, which occurs when the reactance goes to infinity. This corresponds to zero frequency, or essentially a DC signal.

EVALUATE The output voltage gradually decreases from its maximum at $\omega = 0$ to $V_{\text{out}} = 0$ at very high frequencies. Thus, there is no resonant peak at $\omega = 1/RC$, nor at $\omega = 1/\sqrt{RC}$, but the latter actually has the wrong dimensions for frequency. Since the output voltage is the same as the voltage across the capacitor, it should have the same frequency as the input, but not necessarily the same phase. Indeed, the capacitor voltage lags behind the current by 90° (see Table 28.1), and the current and input voltage have a phase difference given by Equation 28.13, which in this case is $\tan\phi = -1/\omega RC$. So the input and output voltages will differ in phase by $\phi = 90^{\circ} - \tan^{-1}(\omega_{RC}/\omega)$.

The answer is **(d)**.

ASSESS Specifically, the phase difference between the input and the output is $\phi = 90^{\circ} - \tan^{-1}(\omega_{RC}/\omega)$, where $\omega_{RC} = 1/RC$. As $\omega \to \infty$, the two voltages approach 90° out of phase. Conversely, as $\omega \to 0$, the two voltages become more and more in phase.

75. **INTERPRET** We are analyzing a filter consisting of an *RC* circuit.

DEVELOP If the capacitor is replaced by an inductor, the output voltage will now be equal to the voltage across the inductor, which peaks according to

$$V_{L\text{p}} = I_{\text{p}} X_L = \frac{V_{\text{in,p}} X_L}{\sqrt{R^2 + {X_L}^2}} = \frac{V_{\text{in,p}}}{\sqrt{1 + (\omega_{RL}/\omega)^2}}$$

where we have introduced the term $\omega_{RL} = R/L$.

EVALUATE If $\omega << \omega_{RL}$, then $V_{L\text{p}} = 0$, and there's no output voltage. At the other end of the spectrum, $\omega >> \omega_{LC}$, we have $V_{L\text{p}} = V_{\text{in,p}}$, which means $V_{\text{out}} = V_{\text{in}}$. So as opposed to the *RC* circuit, the *LC* circuit is a high-pass filter.

The answer is **(b)**.

ASSESS Recall the short-term and long-term behavior of *RL* circuits from Chapter 27. At short times ($t << L/R$ or equivalently $\omega >> \omega_{RL}$), the inductor acts like an open circuit, so the input and output terminals are at equal voltage. But over long times ($t >> L/R$ or equivalently $\omega << \omega_{RL}$), the inductor begins to behave like a short-circuit, so current will flow through the inductor, and there will be no voltage at the output.