WAVE MOTION

14

EXERCISES

Section 14.1 Waves and Their Properties

11. **INTERPRET** This problem asks us to find the time between two successive crests of a wave, given the wavelength and the wave speed.

DEVELOP The distance between wave crests is one wavelength λ. The time it takes a single crest to travel this distance is the wave's period T. The wave speed (or phase velocity) is the distance a crest travels per unit time, so

$$v = \frac{\lambda}{T}$$

which is Equation 14.1.

EVALUATE Since we are given $v = 5.3$ m/s and $\lambda = 18$ m, we can solve for T. The result is

$$T = \frac{\lambda}{v} = \frac{18\text{ m}}{5.3\text{ m/s}} = 3.4\text{ s}$$

ASSESS There is no need to memorize this equation, as it is easily reconstructed by considering the units.

12. **INTERPRET** This problem is about wave propagation. Given the speed and frequency of the ripples, we are asked to compute the period and the wavelength of the wave.

DEVELOP Equation 14.1 relates the speed of the wave to its period, frequency, and wavelength:

$$v = \frac{\lambda}{T} = \lambda f$$

Apply this equation to solve the problem.

EVALUATE Equation 14.1 gives **(a)** $T = 1/f = 1/(5.1\text{ Hz}) = 0.20\text{ s},$ and **(b)**

$$\lambda = v/f = (35\text{ cm/s})/(5.1\text{ Hz}) = 6.9\text{ cm}$$

ASSESS The unit of frequency is Hz, with $1\text{ Hz} \equiv 1\text{ s}^{-1}$. If the frequency is kept fixed, then increasing the wavelength will increase the speed of propagation.

13. **INTERPRET** This problem involves calculating the wavelength of a wave, given its wave speed and its frequency.

DEVELOP Apply Equation 14.1, $v = \lambda f$, to find the wavelength λ.

EVALUATE Solving Equation 14.1 for λ and inserting the given quantities gives a wavelength of

$$\lambda = \frac{v}{f} = \frac{3.00\times10^8\text{ m/s}}{95.8\times10^6\text{ s}^{-1}} = 3.13\text{ m}$$

ASSESS This wavelength seems reasonable, given the size of FM broadcasting towers.

14. **INTERPRET** This problem is an exercise in calculating the wavelength of a wave, given the wave speed and the frequency.

DEVELOP The wave speed can be calculated from the distance and the travel time, which gives $v = d/t$, where $d = 1.350\times10^6$ m and $t = 4.64\text{ min} = 278.4\text{ s}$. Because we are given the frequency f, the wavelength may be calculated by Equation 14.1, $\lambda = v/f$.

EVALUATE Inserting the wave speed derived above into Equation 14.1 gives a wavelength of

$$\lambda = \frac{v}{f} = \frac{d}{tf} = \frac{1.350\times10^6\ \text{m}}{(278.4\ \text{s})(3.17\ \text{Hz})} = 1.53\times10^3\ \text{m} = 1.53\ \text{km}$$

ASSESS Scientific notation was used simply to show the significant figures.

15. **INTERPRET** We are asked to find the wavelengths of ultrasound waves used in medical imaging.

DEVELOP The wavelength is related to the speed and frequency through Equation 14.1: $\lambda = v/f$.

EVALUATE **(a)** For fetal imaging, the ultrasound wavelength is

$$\lambda = \frac{v}{f} = \frac{1500\ \text{m/s}}{8.0\times10^6\,\text{Hz}} = 0.19\ \text{mm}$$

(b) For adult kidney imaging, the ultrasound wavelength is

$$\lambda = \frac{v}{f} = \frac{1500\ \text{m/s}}{3.5\times10^6\,\text{Hz}} = 0.43\ \text{mm}$$

ASSESS The wavelength gives us a sense of the image resolution. Both techniques should be able to discriminate objects that are at least a millimeter-wide.

Section 14.2 Wave Math

16. **INTERPRET** This problem is an exercise in calculating wave properties (i.e., "wave math"). We are asked to find the wave number and angular frequency of a wave, given its period and wavelength.

DEVELOP From the section on wave math, we see that the wave number k is given by $k = 2\pi/\lambda$. From dimensional analysis, we can relate the angular frequency ω, which has units of rad / s, to the frequency f, which has units of rev / s, by using the conversion factor $1 = 2\pi$ rad / rev. Thus, $(\omega\ \text{rad / s}) = (f\ \text{rev / s})(2\pi\ \text{rad / rev})$, or $\omega = 2\pi f$. From Equation 13.1 (or again by dimensional analysis), we have $f = 1/T$, so $\omega = 2\pi/T$.

EVALUATE Inserting the given quantities, we find **(a)** $k = 2\pi/(10.4\ \text{m}) = 0.604\ \text{m}^{-1}$ and **(b)** $\omega = 2\pi/(4.5\ \text{s}) = 1.4\ \text{s}^{-1}$.

ASSESS The wave number is reported to three significant figures because the wavelength is given to three significant figures, whereas the period is given to two significant figures, so the angular frequency is reported to two significant figures.

17. **INTERPRET** We are given a function that describes a traveling sinusoidal wave and asked to compute various physical quantities associated with the wave.

DEVELOP Consider a traveling wave of the form given in Equation 14.3:

$$y(x,t) = A\cos(kx \pm \omega t)$$

The amplitude of the wave is A; its wavelength is given by Equation 14.2, $\lambda = 2\pi/k$; its period is given by Equation 13.5, $T = 2\pi/\omega$. The speed of propagation is $v = \lambda f = \omega/k$, and the direction of propagation is $+x$ if the argument is $kx - \omega t$ and $-x$ if the argument is $kx + \omega t$.

EVALUATE **(a)** Comparing $y = 1.6\cos(0.67x + 30t)$ with Equation 14.3, we find the amplitude to be $A = 1.6$ cm.

(b) The wavelength is

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{0.67\,\text{cm}^{-1}} = 9.4\ \text{cm}$$

(c) Equation 13.5 gives

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{30\ \text{s}^{-1}} = 0.21\ \text{s}$$

(d) The speed of the wave is

$$v = \frac{\omega}{k} = \frac{30\ \text{s}^{-1}}{0.67\ \text{cm}} = 45\ \text{cm / s}$$

(e) A phase of the form $kx + \omega t$ describes a wave propagating in the negative x-direction.

Assess This problem demonstrates that the wave function of the form given in Equation 14.3 contains all the information about the amplitude, the wavelength, the period, the phase, and the speed of propagation of a wave. Thus, once the wave function is given, all these quantities can be calculated.

18. **Interpret** This problem is about the ultrasound wave. Given its frequency, and wavelength, we want to find its angular frequency, wave number, and wave speed.

Develop The relationships between the speed of the wave, its wave number, frequency, and wavelength are given by Equations 13.6, 14.1, and 14.2:

$$f=\frac{\omega}{2\pi},\quad v=\frac{\lambda}{T}=\lambda f,\quad k=\frac{2\pi}{\lambda}$$

Evaluate **(a)** Equation 13.6 gives $\omega=2\pi f=2\pi\left(4.86\times10^{6}\ \text{Hz}\right)=3.05\times10^{7}\ \text{s}^{-1}$.

(b) Equation 14.2 gives $k=2\pi/\lambda=2\pi/\left(0.313\times10^{-3}\ \text{m}\right)=2.01\times10^{4}\ \text{m}^{-1}$.

(c) Using Equation 14.1, the speed of the ultrasound wave is

$$v=f\lambda=\left(4.86\times10^{6}\ \text{Hz}\right)\left(0.313\times10^{-3}\ \text{m}\right)=1.52\times10^{3}\ \text{m/s}.$$

Assess The speed of the wave can also be computed as

$$v=\frac{\omega}{k}=\frac{3.05\times10^{7}\ \text{s}^{-1}}{2.01\times10^{4}\ \text{m}^{-1}}=1.52\times10^{3}\ \text{m/s}$$

Thus, we see that the pairs f, λ and ω, k are equivalent ways to describe the same wave.

19. **Interpret** This problem is an exercise in wave math.

Develop Inspection of Fig. 14.35 reveals that the wavelength is 8 cm, the amplitude is 1.5 cm, and the velocity is $v=\Delta x/\Delta t=(2\ \text{cm})/(2.6\ \text{s})=0.769\ \text{cm/s}$. The phase constant is zero (since $y=A$ at $t=0$ and $x=0$) and the wave is traveling in the positive x-direction. Thus, $k=2\pi/\lambda=0.785\ \text{cm}^{-1}$, and $\omega=kv=0.604\ \text{s}^{-1}$.

Evaluate Inserting these values into Equation 14.3 gives

$$y(x,t)=(1.5\ \text{cm})\cos\left[\left(0.785\ \text{cm}^{-1}\right)x-\left(0.604\ \text{s}^{-1}\right)t\right]$$

Assess The negative sign is used because the wave moves in the positive x-direction.

20. **Interpret** We must find the wave speed of a water wave when the depth is less than the wavelength.

Develop The wave equation relates the second partial derivative of the y with respect to x to the second partial derivative of y with respect to t. Let's assume that the vertical displacement, y, is a function with the form: $y=f(u)$, where $u=x-vt$. This guarantees that we are dealing with a wave traveling in the horizontal x-direction.

Evaluate By the chain rule, the first derivative of y with respect to x is

$$\frac{\partial y}{\partial x}=\frac{\partial f}{\partial u}\frac{\partial u}{\partial x}=\frac{\partial f}{\partial u}$$

since $\partial u/\partial x=1$. The second derivative of y with respect to x is similarly

$$\frac{\partial^2 y}{\partial x^2}=\frac{\partial^2 f}{\partial u^2}\frac{\partial u}{\partial x}=\frac{\partial^2 f}{\partial u^2}$$

Doing the same for the derivatives with respect to time, t, gives

$$\frac{\partial^2 y}{\partial t^2}=\frac{\partial}{\partial t}\left[\frac{\partial f}{\partial u}\frac{\partial u}{\partial t}\right]=-v\frac{\partial^2 f}{\partial u^2}\frac{\partial u}{\partial t}=v^2\frac{\partial^2 f}{\partial u^2}$$

where we have used the fact that $\partial u/\partial t=-v$. Plugging these expressions into the given wave equation:

$$\frac{\partial^2 y}{\partial x^2}=\frac{\partial^2 f}{\partial u^2}=\frac{1}{gh}\frac{\partial^2 y}{\partial t^2}=\frac{v^2}{gh}\frac{\partial^2 f}{\partial u^2}\quad\rightarrow\quad v=\sqrt{gh}$$

Assess The dimensions work out correctly, since $[g]=\text{m/s}$ and $[h]=\text{m}$.

Section 14.3 Waves on a String

21. **Interpret** Given the tension in the cable and the linear mass density (mass per unit length) of the cable, we want to find the speed at which a transverse wave propagates along the crystal.

Develop The relationship between the speed of propagation, the tension, and the mass per unit length of a cable is given by Equation 14.5, $v=\sqrt{F/\mu}$. Given F and μ, we can calculate v.

Evaluate The speed of the transverse wave in the cables is

$$v=\sqrt{\frac{F}{\mu}}=\sqrt{\frac{2.5\times10^8\text{ N}}{4100\text{ kg/m}}}=250\text{ m/s}$$

to two significant figures.

Assess Increasing the tension results in a greater acceleration of the disturbed cables, and hence the wave propagates more rapidly.

22. **Interpret** This problem is an exercise in wave math for a transverse wave traveling on a string. We are given its frequency and amplitude and the tension mass per unit length for the string, and are asked to find wave speed.

Develop Apply Equation 14.6, $v=\sqrt{F/\mu}$ to find the wave speed, where the tension is $F = 21$ N and $m = 15$ g/m = 0.015 kg/m is the mass per unit length of the string.

Evaluate The wave speed is $v=\sqrt{F/\mu}=\sqrt{(21\text{ N})/(0.015\text{ kg/m})}=37\text{ m/s}$.

Assess The wave speed for a transverse wave on a string does not depend on the wave amplitude or its frequency. The power carried by the wave, however, does depend on the wave amplitude and frequency; see Equation 14.7.

(a) The wave speed is **(b)** From the second equation in Section 14.3, $u_{\text{max}}=\omega A=2\pi(44\text{ Hz})(1.2\text{ cm})=3.32\text{ m/s}$.

23. **Interpret** This problem involves transverse waves on a spring. We are given the initial string tension and wave speed, and are asked to find the wave speed if the string tension is increased to a new value.

Develop The relationship between the wave speed, the tension, and the mass per unit length of a string is given by Equation 14.6, $v=\sqrt{F/\mu}$. Given the initial speed of the transverse wave and the initial tensile force on the string, we can find the mass per unit length μ. Because this quantity remains essentially constant as we increase the tension on the string, we can use this value to find the wave speed that results when the tensile force is increased to a new value. Thus, solving for μ, we find

$$\mu=\frac{F_1}{v_1^2}$$

where $F_1 = 15$ N and $v_1 = 11$ m/s. Insert this mass per unit length back into Equation 14.6 along with the new tensile force $F_2 = 50$ N to find the new wave speed v_2.

Evaluate The new wave speed is

$$v_2=\sqrt{\frac{F_2}{\mu}}=v_1\sqrt{\frac{F_2}{F_1}}=(11\text{ m/s})\sqrt{\frac{50\text{ N}}{15\text{ N}}}=20\text{ m/s}$$

Assess Our result indicates that, keeping μ fixed, increasing the tension will result in a greater propagation speed.

24. **Interpret** This problem requires the use of wave math to find the mass per unit length of the string in order to find the total mass of the string. We can find the mass per unit length, given the propagation speed of a wave on the string and the tension in the string.

Develop The wave speed is $v = L/t$, where $L = 1.75$ m, is the length of the string and $t = 565\text{ ms} = 0.565$ s. The relationship between the speed of propagation, the tension, and the mass per unit length of the medium is given by Equation 14.6, $v=\sqrt{F/\mu}$. Combine these equations and solve for the mass per unit length μ of the string, with which we can find the total mass M of the string using $M=\mu L$.

Evaluate The mass per unit length of the string is

$$\mu = \frac{F}{v^2} = \frac{F}{(L/t)^2} = \frac{Ft^2}{L^2}$$

So, the total mass of the string is

$$M = \mu L = \frac{Ft^2}{L} = \frac{(75.1\text{ N})(0.565\text{ s})^2}{17.5\text{ m}} = 1.37\text{ kg}$$

ASSESS Checking the units of the final expression for the mass, we have

$$\frac{\text{N}\cdot\text{s}^2}{\text{m}} = \frac{(\text{kg}\cdot\text{m/s}^2)\text{s}^2}{\text{m}} = \text{kg}$$

as expected.

25. **INTERPRET** This problem requires us to find the average power carried by a wave propagating along a rope. We are given the mass per unit length of the rope and its tension, and the frequency and amplitude of the wave, from which we can find the average power.

DEVELOP The average power transmitted by transverse traveling waves in a string is given by Equation 14.7, $\bar{P} = \frac{1}{2}\mu\omega^2 A^2 v$. The speed of propagation can be obtained by using Equation 14.6, $v = \sqrt{F/\mu}$.

EVALUATE Using the values given in the problem statement, we find the average power is

$$\begin{aligned}\bar{P} &= \frac{1}{2}\mu\omega^2 A^2 v = \frac{1}{2}\mu\omega^2 A^2\sqrt{\frac{F}{\mu}} = \frac{1}{2}(0.28\text{ kg/m})\left(2\pi\,\text{rad}\times 3.7\text{ s}^{-1}\right)^2(0.06\text{ m})^2\sqrt{\frac{550\text{ N}}{0.28\text{ kg/m}}}\\ &= 12\text{ W}\end{aligned}$$

ASSESS The wave power is proportional to the speed of propagation. It is also proportional to the square of the amplitude and the square of the angular frequency.

Section 14.4 Sound Waves

26. **INTERPRET** This problem involves the wave intensity of spherical waves whose power and range differ in two different areas.

DEVELOP We are told there is a signal power boost which occurs for a given intensity, meaning we can use Equation 14.8, $I = P/4\pi r^2$, to find how the range has increased from one location to the other. Setting these two intensities equal to each other will allow us to find the relationship between the two ranges.

EVALUATE From equating the intensities we find

$$\frac{P_1}{4\pi r_1^2} = \frac{P_2}{4\pi r_2^2}$$

$$\frac{r_2}{r_1} = \sqrt{\frac{P_2}{P_1}} = \sqrt{\frac{3.0\text{ W}}{0.60\text{ W}}} = 2.2$$

Where location 1 is the urban area and location 2 is the rural area. Plugging in the given values for the power we find the range is increased by a factor of 2.

ASSESS The ratio of ranges goes as the square root of the ratio of powers due to the spherical nature of the traveling waves.

27. **INTERPRET** This problem involves finding the wave speed of sound waves traveling through air under standard conditions.

DEVELOP Use Equation 14.9, $v = \sqrt{\gamma P/\rho}$ relates the speed of sound v to pressure P and density ρ. For air, $\gamma = 7/5$.

EVALUATE Inserting the given quantities, we find the speed is

$$v = \sqrt{\gamma P/\rho} = \sqrt{\frac{7\left(101\times 10^3\text{ N/m}^2\right)}{5\left(1.20\text{ kg/m}^3\right)}} = 343\text{ m/s}.$$

ASSESS This is the accepted value for the speed of sound at standard temperature and pressure.

28. **INTERPRET** For this problem, we are to find the error involved in starting the clock for 100-m races upon hearing the starting gun, as opposed to starting the clock when seeing the smoke of the gun. To find the error, we consider the speed of sound in air under standard conditions.

DEVELOP As calculated in the previous problem, the speed of sound under standard conditions is 343 m/s, so we can calculate how long it takes for this sound signal to travel 100 m from the starting gun to the timer's ear.

EVALUATE The sound signal takes a time $t = (100\text{ m})/(343\text{ m/s}) = 0.294$ s. This error is unacceptable in modern sprint races.

ASSESS To cover the same distance, a light signal takes $t_L = (100\text{ m})/(3.00 \times 10^8\text{ m/s}) = 0.33 \times 10^{-6}$ s, which is an insignificant timing error for sprint races.

29. **INTERPRET** This problem involves the sound of speed in a gaseous medium. We are given the specific heat ratio, pressure, and density of the gas.

DEVELOP Equation 14.9, $v = \sqrt{\dfrac{\gamma p}{\rho}}$, gives the speed of sound in gaseous media. From the information provided on nitrous oxide we can determine the speed of sound waves traveling through it.

EVALUATE Evaluating Equation 14.9 for the speed of sound in nitrous oxide we find

$$v = \sqrt{\frac{\gamma p}{\rho}} = \sqrt{\frac{(1.31)\left(1.95\times10^4\text{ N/m}^2\right)}{\left(0.352\text{ kg/m}^3\right)}} = 269\text{ m/s}$$

ASSESS This is approximately 80% of the speed sound travels in air at room temperature and atmospheric pressure.

30. **INTERPRET** This problem requires us to determine whether a gas is monatomic or diatomic based on the speed of sound in the gas.

DEVELOP From the information in the problem statement, we can solve Equation 14.9 for the constant γ. For diatomic gases, $\gamma = 7/5$. For monatomic gases, $\gamma = 5/3$.

EVALUATE Solving for γ in Equation 14.8, we find

$$\gamma = \rho v^2 / P = (1.0\text{ kg/m}^3)(327\text{ m/s})^2 / (7.6\times10^4\text{ N/m}^2) = 1.41$$

very close to the value for an ideal diatomic gas.

ASSESS Actually, $\gamma - 7/5 = 6.96\times10^{-3}$ for this gas, so γ is very close to the value expected (i.e., $7/5$) for an ideal diatomic gas.

31. **INTERPRET** For this problem, we are to find the frequency of a sound wave in a gaseous medium of the underwater habitat and compare this frequency to that of sound waves in air under standard conditions.

DEVELOP To compute the frequency, we first calculate the speed of sound in the underwater habitat using Equation 14.9, $v = \sqrt{\gamma P/\rho}$. Once v is known, we use Equation 14.1, $v = \lambda f$, to find the frequency.

EVALUATE The speed of sound is

$$v = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{1.61\left(6.2\times10^5\text{ N/m}^2\right)}{4.5\text{ kg/m}^3}} = 471\text{ m/s}$$

Therefore, the frequency of 0.37-m wavelength sound waves is

$$f = \frac{v}{\lambda} = \frac{471\text{ m/s}}{0.37\text{ m}} = 1273\text{ Hz}$$

to four significant figures.

ASSESS In "normal air," the frequency would be about

$$f = \frac{v}{\lambda} = \frac{343\text{ m/s}}{0.37\text{ m}} = 927\text{ Hz}$$

Section 14.5 Interference

32. **INTERPRET** This problem concerns the beats that you hear from the airplanes two engines running at slightly different frequencies.

DEVELOP The sound from the two engines will interfere with each other, as shown in Fig. 14.20. As described in the text, the resulting sound wave will be proportional to

$$y(t) \propto \cos\left[\tfrac{1}{2}(\omega_1 - \omega_2)t\right]$$

It's this term that causes the peaks (or beats) in the sound intensity.

EVALUATE A peak occurs whenever the argument of the above cosine function is a multiple of π, so

$$\Delta t = \frac{\pi}{\frac{1}{2}(\omega_1 - \omega_2)} = \frac{2\pi}{(980\text{ rpm} - 975\text{ rpm})}\left[\frac{60\text{ rpm}}{2\pi\text{ rad/s}}\right] = 12\text{ s}$$

ASSESS You might wonder why there's a peak in intensity when the cosine term above is equal to -1. It's because the intensity of a wave (Equation 14.6) is proportional to the square of the displacement:

$$I \propto y^2 \propto \cos^2\left[\tfrac{1}{2}(\omega_1 - \omega_2)t\right]$$

The intensity will, therefore, peak with a frequency of $\omega_1 - \omega_2$.

33. **INTERPRET** This problem is about wave interference. Given the condition for the second calm region where waves interfere destructively, we want to compute the wavelength of the ocean wave.

DEVELOP The condition for destructive interference is a phase difference of $k_2\Delta r = (2\pi/\lambda_2)\Delta r = 3\pi$, or an odd multiple of π= 180°. The second node occurs when the path difference is three half-wavelengths, or $\Delta r \equiv AP - BP = 3\lambda_2/2$.

EVALUATE From Example 14.5, we have $\Delta r = 8.1$ m, so the wavelength is

$$\lambda_2 = \frac{2}{3}\Delta r = \frac{2}{3}(8.1\text{ m}) = 5.4\text{ m}$$

ASSESS Comparing with Example 14.5, we expect the wavelength in this case to be shorter since at the same distance away from the source the calm region encountered here is the second one.

Section 14.7 Standing Waves

34. **INTERPRET** This problem involves finding the longest wavelength of a standing wave on a string and the lowest frequency, given the wave speed.

DEVELOP The fundamental mode represents the longest wavelength (largest λ) and lowest frequency (smallest f). From Equation 14.13, $L = m\lambda/2$, we see that the longest wavelength occurs for the smallest value of m (= 1). Use Equation 14.1 to find the frequency of this wave.

EVALUATE **(a)** For $m = 1$, $\lambda = 2L = 5.0$ m.

(b) Inserting the known quantities into Equation 14.1, we find $f_1 = v/\lambda_1 = (56\text{ m/s})/(5\text{ m}) = 11$ Hz.

ASSESS The wavelength is twice the length of the string because the wave has a node midway between the endpoints of each wavelength (see the top two panels of Fig. 14.27).

35. **INTERPRET** This problem is about standing-wave modes in a string that is either clamped at both ends or clamped at a single end with the opposite end free.

DEVELOP If the string is clamped at both ends, the amplitudes there must be zero. If L is the length of the string, then the standing waves must satisfy the condition given in Equation 14.13:

$$L = \frac{m\lambda}{2} \quad m = 1, 2, 3, \ldots$$

It follows that the frequencies of the standing-wave modes of a string fixed at both ends are all the (positive) integer multiples of the fundamental frequency,

$$f_m^e = \frac{v}{\lambda_m} = m\left(\frac{v}{2L}\right) = mf_1^e \quad m = 1, 2, 3, \ldots$$

where $f_1^e = v/(2L)$ is the fundamental frequency. However, if only one end of the string is fixed, then from Fig. 14.28, the last wavelength will be either a quarter-wavelength or three-quarters of a wavelength. In other words, an odd number of quarter-wavelengths must fit within the total length *L*. Mathematically, this is expressed as

$$L = \frac{(2m+1)\lambda}{4} \quad m = 0,1,2,3,...$$

and the corresponding frequencies are

$$f_m^0 = \frac{v}{\lambda_m} = (2m+1)\left(\frac{v}{4L}\right) = mf_1^0, \quad m = 0,1,2,...$$

where $f_1^0 = v/(4L) = f_1^e/2$ is the fundamental frequency.

EVALUATE **(a)** With both ends fixed, the next higher frequency above the fundamental frequency is

$$f_2^e = 2f_1^e = 2(160\text{ Hz}) = 320\text{ Hz}$$

(b) The fundamental frequency for the string fixed at one end is

$$f_1^0 = (2\cdot 0+1)\frac{v}{4L} = \left(\frac{1}{2}\right)\left(\frac{v}{2L}\right) = \frac{1}{2}f_1^e = \frac{1}{2}(160\text{ Hz}) = 80\text{ Hz}$$

which is one-half the fundamental frequency of the string fixed at both ends.

(c) In this case, the standing-wave frequencies are only the odd multiples of the fundamental frequency; therefore, the second standing-wave mode has frequency

$$f_2^o = (2\cdot 1+1)f_1^o = 3f_1^o = 3(80\text{ Hz}) = 240\text{ Hz}$$

ASSESS When the string is clamped at both ends, it can accommodate an integer number of half-wavelengths. However, if it's clamped only at one end and the other end is free, then the string can accommodate only an odd number of quarter-wavelengths.

36. **INTERPRET** This problem involves standing waves on a string. We are given a string tensioned so that the fundamental frequency is 83 Hz, and we are to find the fundamental frequency of the string if it is clamped at its midpoint.

DEVELOP From Equation 4.1, we know that the wave speed is related to the frequency by $v = \lambda f$. The wave speed depends on the tension of the string, but not on its length. We also know that an integer number of wavelengths must fit into the string length (Equation 14.13). For the total length, the fundamental wavelength is

$$L = \frac{1\cdot\lambda_1}{2} \Rightarrow \lambda_1 = 2L$$

so

$$v = \lambda_1 f_1 \text{ or } f_1 = v/\lambda_1$$

If the string is clamped in the middle, the new string length is $L/2$, so the new fundamental wavelength is $\lambda_1' = 2(L/2) = L$. Insert this wavelength into the relationship between the wave speed and the frequency to find the new fundamental frequency.

EVALUATE **(a)** Inserting the new fundamental wavelength into the expression for the fundamental frequency gives

$$f_1' = \frac{v}{\lambda_1'} = \frac{1}{\lambda_1'}(\lambda_1 f_1) = \frac{1}{L}(2Lf_1) = 2f_1 = 2(83\text{ Hz}) = 166\text{ Hz}$$

ASSESS The key to this problem is that the wave speed remains constant because the tension in the string does not change.

37. **INTERPRET** Using a crude model, we can estimate the length of a person's vocal tract by the lowest pitch they can generate.

DEVELOP Treating the vocal tract as a pipe closed at one end, the air inside the vocal tract will form standing waves that satisfy $|\sin kL| = 1$, or

$$L = \frac{m\lambda}{4}, \quad m = 1,3,5,7....$$

To write this in terms of frequency $(f = v/\lambda)$, we need the speed of sound waves at body temperature. From Equation 14.9, we know that $v = \sqrt{\gamma P/\rho}$, and in Chapter 17, we will learn that for most gases the pressure and temperature are related such that $P/\rho \propto T$, where the temperature is in Kelvin. The speed of sound at standard temperature $(20°\text{C} = 293\text{ K})$ is 343 m/s, so at body temperature $(37°\text{C} = 310\text{ K})$ the speed of sound is

$$v = (343\text{ m/s})\sqrt{\frac{310\text{ K}}{293\text{ K}}} = 353\text{ m/s}$$

EVALUATE Using the given fundamental mode $(m = 1)$, we can solve for the length of the vocal tract:

$$L = \frac{\lambda}{4} = \frac{v}{4f} = \frac{353\text{ m/s}}{4(620\text{ Hz})} = 14\text{ cm}$$

ASSESS This seems like a reasonable length for the vocal tract. And indeed, an outside reference says the average length of the human vocal tract is about 17 cm in males and 14 cm in females.

Section 14.8 The Doppler Effect and Shock Waves

38. **INTERPRET** This problem involves a source that is emitting sound waves and that is moving toward the receiver. Therefore, the wavelength and frequency of the sound wave detected is shifted by the Doppler effect. We can use this to calculate the frequency of the sound waves detected by the receiver.

DEVELOP For a source moving toward the receiver, the shifted wavelength is given by Equation 14.15, with the negative sign used in the denominator. The source is moving at 18 m/s, so $u = 18\text{ m/s}$. From Problem 14.27, we know that the speed of sound in standard conditions in air is $v = 343\text{ m/s}$.

EVALUATE Inserting the given quantities into Equation 14.15, we find the new frequency to be

$$f' = \frac{f}{1 - u/v} = \frac{360\text{ Hz}}{1 - (18\text{ m/s})/(343\text{ m/s})} = 380\text{ Hz}$$

ASSESS The minus sign in the denominator is due to the fact that the car is approaching the observer.

39. **INTERPRET** This problem involves the Doppler effect. We want to find the frequency perceived by an observer that is moving toward the source such that the source is at rest with respect to the medium through which the wave moves.

DEVELOP The Doppler-shifted frequency perceived by the firefighter moving toward the siren is given by Equation 14.16:

$$f' = f(1 + u/v)$$

where we use the positive sign because the observer is approaching the source at a speed $u = 100\text{ km/h} = 27.8\text{ m/s}$. From Problem 14.27, we know that the speed of sound in air under standard conditions is 343 m/s.

EVALUATE Inserting the given quantities into Equation 14.16, the frequency perceived by the firefighter is

$$f' = f\left(1 + \frac{u}{v}\right) = 87\text{ Hz}\left[1 + (27.8\text{ m/s})/(343\text{ m/s})\right] = 94\text{ Hz}$$

ASSESS As expected, because the firefighter is moving toward the sound source, the frequency he perceives is higher than that when he is at rest (i.e., $f' > f$). On the other hand, $f' < f$ if he were to move away from the source.

40. **INTERPRET** This problem involves the Doppler effect for a source that is approaching the observer. We are given the frequency shift and are asked to find the speed at which the source is approaching.

DEVELOP Apply Equation 14.15, with the negative sign in the denominator because the source is approaching the observer. Solving this equation for the speed u at which the source is approaching gives

$$u = v(1 - f/f')$$

where $f = 1400$ Hz, $f' = 1510$ Hz, and (from Problem 14.27) $v = 343$ m/s.

EVALUATE Inserting the given quantities into the expression for u gives

$$u = v(1 - f/f') = (343 \text{ m/s})(1 - (1400 \text{ Hz})/(1510 \text{ Hz})) = 25 \text{ m/s} = 90 \text{ km/h}$$

ASSESS This speed is quite high for a truck.

41. **INTERPRET** This problem is about using the Doppler effect for light to deduce the galaxy's motion relative to Earth.

DEVELOP The formula for the Doppler shift for light is different than for sound, but when the relative velocity u of the source with respect to the observer is very small compared to the wave speed c for light, the result is the same as Equations 14.14a and 14.14b.

EVALUATE For the galaxy described in this problem, the observed wavelength is greater (red-shifted) than the laboratory wavelength, so the galaxy is receding with speed

$$\frac{u}{c} = \frac{\lambda'}{\lambda} - 1 = \frac{708 \text{ nm}}{656 \text{ nm}} - 1 = 7.93\times10^{-2}$$

$$u = 0.0793c \approx 2.38\times10^{7} \text{ m/s}$$

ASSESS The red shift observed in light from distant galaxies is an indication that the universe is expanding, as suggested by the Big Bang theory.

EXAMPLE VARIATIONS

42. **INTERPRET** This is a problem about a simple harmonic wave—that is, a wave with sinusoidal shape.

DEVELOP Choosing the wave crest to be located at $x = 0$ when $t = 0$, and modeling the wave as traveling in the positive *x*-direction, we use the minus sign in Equation 14.3, $y(x,t) = A\cos(kx - \omega t)$, to express the motion of the wave.

EVALUATE The through-to-crest time given is the half the full period T, so $T = 6.18\text{s}$. The distance separating crests is the wavelength λ, so $\lambda = 59.6\text{m}$. The total vertical displacement which the surfer experiences is the twice the amplitude A, so $A = 2.14\text{m}$. Knowing these we can calculate the wave speed v using Equation 14.1, along with the wavevector k and angular frequency ω, using the wavelength and period, respectively.

$$v = \frac{\lambda}{T} = \frac{59.6\text{m}}{6.18\text{s}} = 9.64 \text{ m/s}$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{59.6\text{m}} = 0.105 \text{ m}^{-1}$$

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{6.18\text{s}} = 1.02\text{s}^{-1}$$

The wave can thus be described as

$$y(x,t) = 2.14\cos(0.105x - 1.02t)$$

ASSESS We can use the fact that $\omega = kv = \left(0.105\text{m}^{-1}\right)(9.64\text{m/s}) = 1.02 \text{ s}^{-1}$ to verify our result.

43. **INTERPRET** This is a problem about a simple harmonic wave—that is, a wave with sinusoidal shape.

DEVELOP We want to determine the period of the wave based on the wavelength, since we'd like to determine the time difference between maxima (crests) of a traveling wave. Since this wave is traveling in water, we can determine its wave speed using Equation 14.11: $v = \sqrt{\lambda g / 2\pi}$. Knowing the wavelength and the wave speed, we can use Equation 14.1 to determine the period.

EVALUATE Using Equation 14.11 for the given wavelength to express the wave speed, we find the period is equal to

$$T = \frac{\lambda}{v} = \sqrt{\frac{2\pi\lambda}{g}} = \sqrt{\frac{(2\pi)(78.2)}{(9.8\text{m/s}^2)}} = 7.08\text{s}$$

ASSESS We can see these waves are dispersive since their speed depends on the wavelength.

44. **INTERPRET** This is a problem about a simple harmonic wave—that is, a wave with sinusoidal shape.

DEVELOP We want to determine the speed at which sound waves of known wavelength and frequency travel on the Martian atmosphere. We can thus use Equation 14.1 to find the sound speed.

EVALUATE Using Equation 14.1 for the given wavelength and frequency, we find the sound speed is equal to

$$v = \lambda f = (0.506\,\mathrm{m})\left(482\,\mathrm{s}^{-1}\right) = 244\,\mathrm{m/s}$$

ASSESS The speed of sounds is lower on Mars than on Earth due to the atmosphere being less dense.

45. **INTERPRET** This is a problem about a simple harmonic wave—that is, a wave with sinusoidal shape.

DEVELOP We want to determine the wavelength of sound waves, knowing their speed and frequency, as they travel in water. Knowing the speed at which they travel in air, we want to compare their wavelengths in these two media. With Equation 14.1 we can find the sound wave's wavelength in water and air.

EVALUATE Using Equation 14.1 for the given speed and frequency, we find the wavelength is water and air are equal to

$$\lambda_{\text{water}} = \frac{v_{\text{water}}}{f} = \frac{(1480\,\mathrm{m/s})}{\left(14.5\,\mathrm{s}^{-1}\right)} = 102\,\mathrm{m/s}$$

$$\lambda_{\text{air}} = \frac{v_{\text{air}}}{f} = \frac{(343\,\mathrm{m/s})}{\left(14.5\,\mathrm{s}^{-1}\right)} = 23.7\,\mathrm{m/s}$$

The wavelength in air of sound with the same frequency is 4.32 times larger than it is in water

$$(\ \lambda_{\text{water}} = 4.32\lambda_{\text{air}}\ )$$

ASSESS This relative proportionality can vary depending on the temperature/density of the two media.

46. **INTERPRET** This is a problem about the Doppler effect in sound from a moving source.

DEVELOP Equation 14.15, $f' = f/(1 \pm u/v)$, relates the original and shifted frequencies to the source speed u, so our plan is to solve this equation for u. We'll use the minus sign because the source is approaching. We'll also need the sound speed v, which the problem statement gave as 343 m/s.

EVALUATE Solving Equation 14.15 for u gives

$$u = v\left(1 - \frac{f}{f'}\right) = (343\ \mathrm{m/s})\left(1 - \frac{494\ \mathrm{Hz}}{523\ \mathrm{Hz}}\right) = 19.0\ \mathrm{m/s}$$

The car was traveling at 68.5 km/h.

ASSESS The vehicle speed is approximately 6% of the sound speed, consistent with the roughly 6% percent change in the sound frequency.

47. **INTERPRET** This is a problem about the Doppler effect in sound from a moving source.

DEVELOP Equation 14.15, $f' = f/(1 \pm u/v)$, relates the original and shifted frequencies to the source speed u, so our plan is to solve for the perceived frequency f'. We'll use the minus sign because the source is approaching. We'll also need the sound speed v, which the problem statement gave as 343 m/s.

EVALUATE Solving Equation 14.15 for f' gives

$$f' = \frac{f}{\left(1 - \dfrac{u}{v}\right)} = \frac{352\,\mathrm{Hz}}{\left(1 - \dfrac{26.4\,\mathrm{m/s}}{343\,\mathrm{m/s}}\right)} = 381\ \mathrm{Hz}$$

Where we have converted the vehicle speed into m/s.

ASSESS The vehicle speed is approximately 8% of the sound speed, consistent with the roughly 8% percent change in the sound frequency.

48. **INTERPRET** This is a problem about the Doppler effect in light from a moving source.

DEVELOP Equation 14.15, $f' = f/(1 \pm u/v)$, relates the original and shifted frequencies to the source speed u, so our plan is to express it in terms of wavelength, and solve for the source speed u. We'll use consider the source moves toward us as it oscillates back and forth, and we'll use the speed of light $c = 3.0 \times 10^8\ \mathrm{m/s}$ for the wave speed v.

EVALUATE Expressing Equation 14.15 in terms of λ, and using the speed of light as the wave speed we find

$$u = c\left(1 - \frac{f}{f'}\right) = c\left(1 - \frac{\lambda'}{\lambda}\right) = c\left(1 - \frac{\lambda - \Delta\lambda}{\lambda}\right)$$

$$u = \frac{c}{\lambda}\left(\lambda - (\lambda - \Delta\lambda)\right) = \frac{c}{\lambda}(\Delta\lambda) = \frac{\left(3.0\times10^{8}\ \text{m/s}\right)}{(676.8\,\text{nm})}\left(3.52\times10^{-7}\ \text{nm}\right) = 15.6\,\text{cm/s}$$

Where we have assumed the shift in the wavelength is negative since the shift in the frequency will be positive for a source moving toward us.

ASSESS We obtain a number on the order of 10 cm/s, as expected.

49. **INTERPRET** This is a problem about the Doppler effect in light from a moving source.
DEVELOP Equation 14.15, $f' = f/(1 \pm u/v)$, relates the original and shifted frequencies to the source speed u, so our plan is to express it in terms of wavelength, and solve for the change in the shifted wavelength λ'. We know the source moves toward us, and the wave speed v is that of light, $c = 3.0\times10^{8}\ \text{m/s}$.
EVALUATE Expressing Equation 14.15 in terms of λ, and using the speed of light as the wave speed we find

$$u = c\left(1 - \frac{f}{f'}\right) = c\left(1 - \frac{\lambda'}{\lambda}\right) = c\left(1 - \frac{\lambda - \Delta\lambda}{\lambda}\right)$$

$$u = \frac{c}{\lambda}\left(\lambda - (\lambda - \Delta\lambda)\right) = \frac{c}{\lambda}(\Delta\lambda)$$

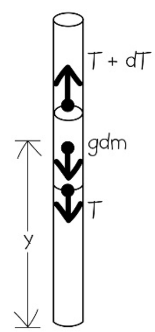
$$\Delta\lambda = \frac{u\lambda}{c} = \frac{\left(64.8\times10^{3}\ \text{m/s}\right)(656.28\,\text{nm})}{\left(3.0\times10^{8}\ \text{m/s}\right)} = 0.142\,\text{nm}$$

Where we have assumed the shift in the wavelength is negative since the shift in the frequency will be positive for a source moving toward us.

ASSESS The wavelength shift is proportional to the ratio between the source speed and the wave speed.

PROBLEMS

50. **INTERPRET** This problem involves Newton's second law and wave motion on a cable that is hanging vertically under the influence of gravity. We are asked to find the wave speed as a function of position on the cable.
DEVELOP The tension in the cable can be found by integrating Newton's second law, applied to a small element of the cable that is at rest. With quantities defined in the sketch below $0 = T + dT - T - g\ dm$, or $dT = g\ dm$. For a uniform cable, $dm = \mu\ dy$ where the linear density μ is a constant, so $T = \mu g y$ (the constant of integration is zero for y measured from the bottom of the cable). Insert this result into Equation 14.6 to find the wave speed.



EVALUATE It follows from Equation 14.6 that $v = \sqrt{T/\mu} = \sqrt{gy}$.

ASSESS The higher you go on the cable, the more cable is supported beneath your position. Thus, the tension in the cable must increase as you go up the cable to counteract gravity acting on an ever-increasing section of the cable. Thus, the tensile force in the cable must also increase as you go up the cable.

51. **INTERPRET** This problem involves the speed of waves traveling on a rope. We are given the rope's breaking tension and the linear mass density.

DEVELOP Equation 14.6, $v = \sqrt{\frac{F}{\mu}},$ gives the speed of waves traveling on a string. We are told the breaking tension of the rope, meaning we know the maximum amount of force the string can support. This, in turn, is the force that will result in the maximum speed of transmission of waves on the rope.

EVALUATE Evaluating Equation 14.6 for the maximum speed of transmission, we find

$$v = \sqrt{\frac{T_{\max}}{\mu}} = \sqrt{\frac{390 \text{ N}}{0.0765 \text{ kg / m}}} = 71.4 \text{ m / s}$$

ASSESS The maximum speed of transmission on this rope is approximately 257 km / h.

52. **INTERPRET** This problem involves finding the intensity of a spherical wave as a function of the distance from the localized wave source.

DEVELOP From Equation 14.8, $I = P/\left(4\pi r^2\right),$ we see that the wave intensity from a localized source falls off as $r^{-2},$ where r is the distance from the localized source.

EVALUATE Inserting the given power $P = 54$ W and $r = 17$ m, we find the wave intensity is

$$I = \frac{54 \text{ W}}{4\pi\left(17 \text{ m}\right)^2} = 15 \text{ mW / m}^2$$

to two significant figures.

ASSESS Although the intensity falls off as r^{-2}, the total power crossing any given spherical surface centered at the wave source is constant.

53. **INTERPRET** This problem is about the power emitted by a localized wave source that emits uniformly in all directions. We are given its intensity at a certain distance and are asked to find the power of the source.

DEVELOP Assuming the light bulb emits uniformly in all directions, the intensity at a distance r from the light bulb that has an average power output P is given by Equation 14.8, $I = P/(4\pi r^2).$ Solve this equation for $P.$

EVALUATE From Equation 14.8, we have

$$P = 4\pi r^2 I = 4\pi(4.2 \text{ m})^2(0.36 \text{ W / m}^2) = 80 \text{ W}$$

ASSESS The light source is an 80-watt light bulb. Note that the intensity decreases with the square of the distance.

54. **INTERPRET** This problem involves the wave intensity of plane waves whose initial power remains constant and radii vary.

DEVELOP The beam emitted from a laser is collimated, meaning we can treat the electromagnetic waves as plane waves, and use Equation 14.8, $I = P/A = P/\pi r^2$, to express their intensity. We are told how the beam radius changes as it is emitted from the laser r_{e} , passes through the lens r_{l} , and once it has arrived at the moon's surface r_{m} .

EVALUATE Evaluating Equation 14.8 for the three given radii we find

$$I_{\text{e}} = \frac{P}{\pi r_{\text{e}}^2} = \frac{\left(120\times10^9 \text{ W}\right)}{\pi\left(3.50\times10^{-3} \text{ m}\right)^2} = 3.1 \text{PW / m}^2$$

$$I_{\text{l}} = \frac{P}{\pi r_{\text{l}}^2} = \frac{\left(120\times10^9 \text{ W}\right)}{\pi\left(1.75 \text{m}\right)^2} = 12 \text{GW / m}^2$$

$$I_{\text{m}} = \frac{P}{\pi r_{\text{m}}^2} = \frac{\left(120\times10^9 \text{ W}\right)}{\pi\left(3.25\times10^3 \text{ m}\right)^2} = 3.6 \text{kW / m}^2$$

These all exceed the brightness of sunlight on Earth (1 kW/m^2).

ASSESS These all consider the power remains constant throughout, when in reality, dispersion in the telescope optics and absorption in the atmosphere will reduce the laser power.

55. **INTERPRET** This problem involves the superposition of waves. We want to show that the superposition of two harmonic waves results in a third harmonic wave.

DEVELOP Using the identity

$$\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha-\beta}{2}\right)\cos\left(\frac{\alpha+\beta}{2}\right)$$

we find that the superposition of the two waves y_1 and y_2 gives

$$y = y_1 + y_2 = A\cos(kx-\omega t) + A\cos(kx-\omega t+\phi) = 2A\cos\left(\frac{\phi}{2}\right)\cos\left(kx-\omega t+\tfrac{1}{2}\phi\right)$$

EVALUATE We see that from the expression above that the superposition of the two harmonic waves results in a third harmonic wave. Writing $y = A_s\cos(kx-\omega t+\phi_s)$ and comparing with the expression above, we find the amplitude and the phase to be

$$A_s = 2A\cos\left(\frac{\phi}{2}\right) \qquad \phi_s = \frac{\phi}{2}$$

ASSESS Let's check our results by considering the following limits: **(i)** $\phi = 0$: In this case, $y_1 = y_2$, and the resultant amplitude is simply $A_s = A + A = 2A$. **(ii)** $\phi = \pi$: In this case, we have [using the identity $\cos(\theta+\pi) = -\cos\theta$]

$$y_2 = A\cos(kx-\omega t+\pi) = -A\cos(kx-\omega t) = -y_1$$

and therefore, $y = y_1 + y_2 = 0$.

56. **INTERPRET** This is an exercise in wave math. Given the mathematical expression for the wave, we are asked to find the amplitude, wavelength, period, wave speed, and power for the wave.

DEVELOP Comparing the given expression for the wave with Equation 14.3, we identify A = 1.75 cm, k = 0.211 cm^{-1}, and ω = 466 rad/s. Equation 14.2 $k = 2\pi/\lambda$ allows us to find the wavelength, and Equation 14.4, $v = \omega/k$ allows us to find the wave speed. To calculate the power in the wave, we apply Equation 14.7 $\bar{P} = \mu\omega^2 A^2 v/2$ and use Equation 14.6 $v = \sqrt{F/\mu}$ to eliminate μ. The result is

$$\bar{P} = \frac{1}{2}\left(\frac{F}{v^2}\right)\omega^2 A^2 v = \frac{F\omega^2 A^2}{2v}$$

EVALUATE **(a)** The amplitude of the wave is A = 1.75 cm.

(b) The wavelength is

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{0.211\ \text{cm}^{-1}} = 29.8\ \text{cm}$$

(c) The period is related to the frequency by

$$T = 1/f = 2\pi/\omega = 2\pi/(466\ \text{s}^{-1}) = 13.5\ \text{ms}$$

(d) The wave speed is

$$v = \frac{\omega}{k} = \frac{466\ \text{rad/s}}{0.211\ \text{cm}^{-1}} = 22.1\ \text{m/s}$$

(e) The power is

$$\bar{P} = \frac{F\omega^2 A^2}{2v} = \frac{(32.8\ \text{N})(466\ \text{rad/s})^2(0.0175\ \text{m})^2}{2(22.1\ \text{m/s})} = 49.4\ \text{W}$$

ASSESS The mathematical expression for a wave carries a significant amount of information.

57. **INTERPRET** For this problem, we are to calculate how the propagation speed of a transverse wave on a spring is affected as the spring is stretched, increasing the tension force in the spring.

DEVELOP The speed of propagation can be obtained by using Equation 14.6: $v = \sqrt{F/\mu}$. We regard the spring as a stretched string with tension, $F = k\Delta x = k(L - L_0)$. In addition, its linear mass density is $\mu = m/L$.

EVALUATE Equation 14.5 gives the speed of transverse waves as

$$v=\sqrt{\frac{F}{\mu}}=\sqrt{\frac{k\Delta x}{m/L}}=\sqrt{\frac{k\left(L-L_0\right)}{m/L}}=\sqrt{\frac{kL\left(L-L_0\right)}{m}}$$

ASSESS There are two effects here that affect the speed of propagation. The first is the amount of stretching, Δx. This makes the speed of propagation go up as $\sqrt{\Delta x}$. The second effect is the change in mass per unit length, characteristic of the inertia of the spring. The mass density decreases as the spring is stretched. This makes it easier for the wave to propagate on the spring.

58. **INTERPRET** This problem involves the propagation of a transverse wave on a stretched spring.

DEVELOP From the solution to Problem 57,

$$mv^2=kL\left(L-L_0\right)$$

$$k=\frac{mv^2}{L\left(L-L_0\right)}$$

With v_1 and v_2 given for L_1 and L_2, k may be eliminated by division, before solving for L_0:

$$\left(\frac{v_2}{v_1}\right)^2=\frac{L_2\left(L_2-L_0\right)}{L_1\left(L_1-L_0\right)}$$

$$L_0=\frac{L_1^2\left(v_2/v_1\right)^2-L_2^2}{L_1\left(v_2/v_1\right)^2-L_2}$$

EVALUATE **(a)** The unstretched length of the spring is

$$L_0=\frac{(35\text{ cm})^2\left((12\text{ m/s})/(4.2\text{ m/s})\right)^2-(63\text{ cm})^2}{(35\text{ cm})\left((12\text{ m/s})/(4.2\text{ m/s})\right)^2-(63\text{ cm})}=27\text{ cm}$$

(b) From either pair of values of wave speed and length,

$$k=\frac{mv^2}{L\left(L-L_0\right)}=\frac{(0.40\text{ kg})(4.2\text{ m/s})^2}{(0.35\text{ m})(0.35\text{ m}-0.271\text{ m})}=\frac{(0.40\text{ kg})(12\text{ m/s})^2}{(0.63\text{ m})(0.63\text{ m}-0.271\text{ m})}=255\text{ N/m}$$

ASSESS Notice that the units in the expression for the spring constant come out to $\text{kg}\cdot\text{m/s}^2$, which is a newton.

59. **INTERPRET** This problem involves spherical sound waves emitted from a localized source. Therefore, the sound waves propagate in all directions (4p steradians) from the source. The total power carried by the sound waves is constant (ignoring loss mechanisms), but the intensity (i.e., power per unit area) decreases as the waves get farther from the source, much as an image on a balloon becomes larger as you blow up the balloon.

DEVELOP As a function of distance, the intensity of spherical waves emitted from a point source is given by Equation 14.8:

$$I=\frac{P}{A}=\frac{P}{4\pi r^2}$$

Ignoring loss mechanisms, the power P carried by the waves remains constant (ignoring loss mechanisms), so we can relate the intensity measured at the two distances as follows:

$$P=I_1\left(4\pi r_1^2\right)=I_2\left(4\pi r_2^2\right)$$

Solving this for r_2 gives

$$r_2=\pm r_1\sqrt{\frac{I_1}{I_2}}$$

We will use the positive square root since we are interested in the distance the person needs to walk directly away from the source, which is $d = r_2 - r_1$.

EVALUATE Inserting the given values for the intensities and distance, we find

$$r_2 = (12\text{ m})\sqrt{\frac{690\text{ mW/m}^2}{260\text{ mW/m}^2}} = 20\text{ m}$$

Thus, the person needs to walk a distance $d = r_2 - r_1 = 20\text{ m} - 12\text{ m} = 8\text{ m}$ away from the source.

ASSESS The intensity falls off as the inverse square of the distance. The further you walk away from the source, the weaker is the intensity. Note that if we use the negative square root for r_2, we find the distance the person needs to walk directly toward the source is $d = 20\text{ m} - 8\text{ m} = -28\text{ m}$.

60. **INTERPRET** This problem involves a localized source that emits waves (light waves, in this case) uniformly in all directions. We are given the relationship between the intensities of the waves as a function of distance from the source, from which we are to calculate the distance at which a particular intensity is measured.

DEVELOP Ignoring loss mechanisms, the total power carried by the waves is constant, irrespective of the distance from the source. Therefore, Equation 14.8 allows us to relate the intensities measured at the two points as follows:

$$P = I_1\left(4\pi r_1^2\right) = I_2\left(4\pi r_2^2\right)$$

$$\frac{I_1}{I_2} = \frac{r_2^2}{r_1^2}$$

From the problem statement, we know that $I_1 = 1.52\, I_2$ and $r_2 = r_1 + \Delta r\left(\text{with}\, \Delta\text{r} = 20\text{ m}\right)$, so we can solve the expression above for r_1.

EVALUATE Inserting the relationships above into the expression relating the intensities and distances, we find

$$\frac{I_1}{I_2} r_1^2 = r_2^2 = \left(r_1 + \Delta r\right)^2$$

$$1.52 r_1^2 = r_1^2 + \Delta r^2 + 2 r_1 \Delta r$$

$$r_1 = \Delta r\left(\frac{5}{13}\right)\left(5 \pm \sqrt{38}\right) = (20\text{ m})\left(\frac{5}{13}\right)\left(5 + \sqrt{38}\right) = 86\text{ m}$$

where we have taken the positive solution of the quadratic because both observers are on the same side of the source.

ASSESS The negative root, $r_1 = (20\text{ m})\left(\frac{5}{13}\right)\left(5 - \sqrt{38}\right) = -9.0\text{ m}$, corresponds to observers on opposite sides of the source (i.e., with the lamp in Fig. 14.37 between the two observers).

61. **INTERPRET** This problem is about the relationship between the wave speed of a transverse wave on a spring and the amount the spring is stretched. As the spring is stretched, the tensile force increases, which causes the wave speed to increase. Given the relationship between the wave speed and the spring distortion (i.e., the amount by which it is stretched), we are to calculate the spring's equilibrium length.

DEVELOP The wave speed can be obtained from Equation 14.6: $v = \sqrt{F/\mu}$. We regard the spring as a stretched string with tension $F_1 = k\Delta x = k\left(L_1 - L_0\right)$, where L_0 is its equilibrium length. In addition, its linear mass density is $\mu_1 = m/L_1$. When the spring is stretched to $L_2 = 2L_1$, the tension becomes $F_2 = k\left(L_2 - L_0\right) = k\left(2L_1 - L_0\right)$ and the linear mass density becomes $\mu_2 = m/L_2 = m/\left(2L_1\right)$. Therefore, since the speed of the transverse wave on the spring stretched to a total length L_2 is triple that of the spring stretched to a total length L_1, we can write

$$3v_1 = v_2$$
$$3\sqrt{\frac{F_1}{\mu_1}} = \sqrt{\frac{F_2}{\mu_2}}$$
$$3\sqrt{\frac{k(L_1 - L_0)}{m/L_1}} = \sqrt{\frac{k(2L_1 - L_0)}{m/(2L_1)}}$$

which we can solve for the unstretched length L_0.

EVALUATE Solving the above expression gives

$$3\sqrt{L_1(L_1 - L_0)} = \sqrt{2L_1(2L_1 - L_0)}$$
$$9(L_1 - L_0) = 2(2L_1 - L_0)$$
$$L_0 = L_1\left(\frac{5}{7}\right)$$

ASSESS There are two effects here that affect the speed of propagation. The first one is the amount of stretching, Δx. This makes the speed of propagation go up as $\sqrt{\Delta x}$. The second effect is the change in mass per unit length, which is characteristic of the inertia of the spring. The linear mass density decreases as the spring is stretched, which makes it easier for the wave to propagate along the spring.

62. **INTERPRET** This problem requires us to calculate the time it takes for a wave to travel a given distance along a cable whose tension varies as a function of distance as per Problem 14.50. Because the tension varies with position, so does the wave velocity, so we will need to integrate an expression for time to find the total time it takes for the wave to travel the given distance.

DEVELOP The wave speed for the cable of Problem 50 is $v = dy/dt = \sqrt{gy}$, where y is the distance from the bottom of the cable. The time for a transverse wave to propagate from the bottom to the top of the cable ($y = 0$ to $y = L$) is

$$t = \int dt = \int_0^L \frac{dy}{v} = \int_0^L \frac{dy}{\sqrt{gy}}$$

EVALUATE Performing the integration, we find

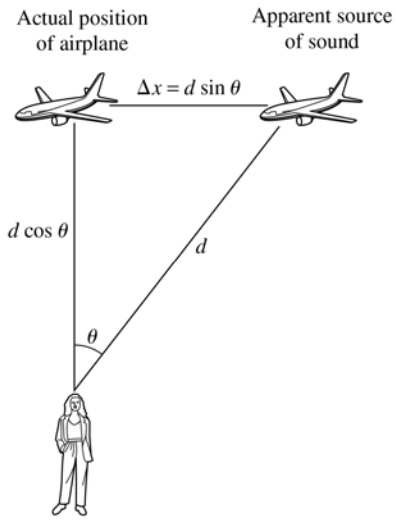
$$t = \int_0^L \frac{dy}{\sqrt{gy}} = 2\sqrt{\frac{y}{g}}\Bigg|_0^L = 2\sqrt{\frac{L}{g}}$$

as given in the problem statement.

ASSESS The units of this expression are $\sqrt{\dfrac{\text{m}}{\text{m/s}^2}} = \text{s}$, as expected. Notice that the time it takes the wave to travel a given distance increases only as the square root of the distance.

63. **INTERPRET** This problem invlolves finding the speed of the airplane, given its apparent location as the source of sound.

DEVELOP Make a diagram of the situation (see figure below). The travel time for the sound from the airplane, reaching you along a line making an angle of $\theta = 38^\circ$ with the vertical from the airplane's actual location, is $\Delta t = d / v$ where v is the speed of sound in air under standard conditions $(\sim 343\ \text{m/s})$. During this time, the airplane moved a horizontal distance $\Delta x = d \sin\theta$. Once Δt and Δx are known, we can calculate the speed of the airplane using $u = \Delta x / \Delta t$.



EVALUATE Using the values given in the problem statement, the speed of the airplane is

$$u = \frac{\Delta x}{\Delta t} = \frac{d\sin\theta}{d/v} = v\sin\theta = (343\text{ m/s})\sin(38^\circ) = 211\text{ m/s} = 760\text{ km/h}$$

ASSESS The value is reasonable for an airplane's speed. Note that the airplane's altitude, $5.4\text{ km} = d\cos\theta$, is not needed in this calculation.

64. **INTERPRET** This problem is an exercise in calculating sound intensity and pressure amplitude given its intensity level in decibels.

DEVELOP Apply Equation 14.10, which may be written in the form

$$I = I_0 10^{\beta/10}$$

where β is the intensity level in decibels and $I_0 = 10^{-12}\text{ W/m}^2$ is the standard threshold of hearing a 1 kHz. The pressure amplitude may be found from

$$I = \frac{\Delta P_0^2}{2\rho v}$$

$$\Delta P_0 = \sqrt{2I\rho v}$$

where $r = 1.2\text{ kg/m}^3$ is the standard air pressure and $v = 343$ m/s is the standard speed of sound in air.

EVALUATE **(a)** For $\beta = 65$ dB, we find

$$I = \left(10^{-12}\text{ W/m}^2\right)10^{65/10} = 3.2\times10^{-6}\text{ W/m}^2$$

and

$$P = \sqrt{2\left(3.16\times10^{-6}\text{ W/m}^2\right)\left(1.2\text{ kg/m}^3\right)\left(343\text{ m/s}\right)} = 5.1\times10^{-2}\text{ N/m}^2$$

(b) For $\beta = -5$ dB, we find

$$I = \left(10^{-12}\text{ W/m}^2\right)10^{-5/10} = 3.2\times10^{-13}\text{ W/m}^2$$

and

$$P = \sqrt{2\left(3.16\times10^{-13}\text{ W/m}^2\right)\left(1.2\text{ kg/m}^3\right)\left(343\text{ m/s}\right)} = 1.6\times10^{-5}\text{ N/m}^2$$

ASSESS Decibels are used because sound intensity can range over many orders of magnitude, so it is more convenient to express this using a logarithmic scale such as decibels.

65. **INTERPRET** This problem involves converting sound intensity from the decibel scale to the linear scale.

DEVELOP The sound intensity level in decibels is given by Equation 14.10:

$$\beta = 10\log\left(\frac{I}{I_0}\right)$$

where I is the intensity (measured in W/m^2), and $I_0 = 10^{-12}\ W/m^2$ is the standard threshold of hearing a 1 kHz.

EVALUATE If the sound intensity is doubled, then $I' = 2I$. Equation 14.10 shows that

$$\begin{aligned}\beta' &= 10\log\left(\frac{I'}{I_0}\right) = 10\log\left(\frac{2I}{I_0}\right) = 10\log\left(\frac{I}{I_0}\right) + 10\log 2 \\ &= \beta + 3.01\end{aligned}$$

Thus, the decibel level increases by about 3 dB.

ASSESS The problem demonstrates that doubling the intensity corresponds to a 3-dB increase. Human ears, however, do not respond linearly to the intensity change. For each 10-dB increase, you perceive an increase in loudness by roughly a factor of 2.

66. **INTERPRET** This problem involves sound waves emitted uniformly from a localized source. We are to calculate by how much the intensity and decibel level change if an observer doubles her distance from the source.

DEVELOP Apply Equations 14.8 and 14.10 to calculate the change in intensity and decibel level, respectively. Equation 14.8 shows that the intensity decreases by a factor of 4 if the distance from the source is doubled.

EVALUATE **(a)** $I' = I/4$ where I' is the intensity at $r' = 2r$.

(b) From Equation 14.10, we find that

$$\beta' = 10\log\left(\frac{I'}{I_0}\right) = 10\log\left(\frac{I}{4I_0}\right) = \overbrace{10\log\left(\frac{I}{I_0}\right)}^{\beta} + 10\log\left(\frac{1}{4}\right) = \beta - 6.02\ \text{dB}$$

ASSESS Comparing this result with that of Problem 14.65, we see that the results are consistent. In the preceding problem, we found that a two-fold increase in sound intensity equates to a increase of approximately 3 dB, and here we find that a four-fold decrease in sound intensity equates to a decrease by approximately 6 dB.

67. **INTERPRET** This problem deals with the variation with distance of sound intensity measured in decibels. We want to find the distance one must move away from the sound source for the loudness to drop by a factor of 2.

DEVELOP The sound intensity level in decibels is given by Equation 14.10:

$$\beta = 10\log\left(\frac{I}{I_0}\right)$$

where I is the intensity (measured in W/m^2), and $I_0 = 10^{-12}\ W/m^2$ is the standard threshold of hearing a 1 kHz. For the perceived loudness to decrease by a factor of 2, the decibels must decrease by 5. Therefore,

$$\begin{aligned}\beta' - \beta &= -10 \\ 10\log\left(\frac{I'}{I_0}\right) - 10\log\left(\frac{I}{I_0}\right) &= -10 \\ \log\left(\frac{I'}{I}\right) &= -1 \\ \frac{I'}{I} &= 10^{-1}\end{aligned}$$

From Equation 14.8, we see that the sound intensity I drops as the inverse square of the distance from the source, so

$$\frac{I'}{I} = \frac{r^2}{r'^2}$$

where r = 2.0 m. Combining these two expressions allows us to solve for r', which is the distance at which the loudness will decrease by a factor of 2.

EVALUATE Equating the two expressions for the ratio I'/I, we find

$$\frac{r^2}{r'^2} = \frac{1}{10}$$
$$r' = \sqrt{10}r = \sqrt{10}(2.0\text{ m}) = 6.3\text{ m}$$

ASSESS We find that for the loudness perceived to go down by half, the intensity I must decrease by 10 dB, or a factor of 10.

68. **INTERPRET** This problem involves a string whose fundamental vibration frequency is 440 Hz. Given its tension and its length, we are asked to find its total mass. This is an exercise in wave math.

DEVELOP From Equation 14.13, we find that the fundamental wavelength is $\lambda = 2L$ (i.e., $m = 1$). Combining Equations 14.1 and 14.5 and using $\mu = m/L$, we find

$$\lambda f = v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{FL}{m}}$$

which we can solve for the string mass m.

EVALUATE Solving the expression above for m and inserting the given values, we find

$$m = \frac{FL}{\lambda^2 f^2} = \frac{F}{4Lf^2} = \frac{667\text{ N}}{4(0.389\text{ m})(440\text{ Hz})^2} = 2.21\text{ g}$$

ASSESS Let's check the units of this expression for mass:

$$\frac{\text{N}}{\text{m}\cdot\text{s}^{-2}} = \frac{\text{kg}\cdot\text{m}\cdot\text{s}^{-2}}{\text{m}\cdot\text{s}^{-2}} = \text{kg}$$

which is what we expect.

69. **INTERPRET** This problem is about the standing-wave condition and the requirement it imposes on the time it takes for the wave to complete one round trip in the medium. We are to show that this time is an integer number of wave periods.

DEVELOP Because the string is clamped at both ends, the amplitudes there must be zero. If L is the length of the string, then the standing waves must satisfy the condition given in Equation 14.13:

$$L = \frac{m\lambda}{2} \quad m = 1, 2, 3, \ldots$$

On the other hand, the round-trip time for waves on a string of length L and clamped at both ends is

$$t = \frac{2L}{v} = \frac{2L}{\lambda f} = \frac{2LT}{\lambda}$$

using Equation 14.1, $v = \lambda f = \lambda/T$.

EVALUATE Substituting the first equation into the second gives

$$t = \frac{2LT}{\lambda} = \frac{m\lambda T}{\lambda} = mT \quad m = 1, 2, 3, \ldots$$

Therefore, we see that t is an integer multiple of the wave period.

ASSESS The conclusion can also be drawn by examining Fig. 14.28. For example, the wavelength of the fundamental harmonic is $\lambda_1 = 2L$. This gives $t_1 = \lambda_1/v = T$.

70. **INTERPRET** This problem involves standing waves in a pipe that is either closed at one end and open at the opposite end or open at both ends. We are to find the minimum length pipe that will give a fundamental frequency of 22 Hz.

DEVELOP Under standard conditions the speed of sound in air is $v = 343$ m/s. Thus, from Equation 14.1, we find that the wavelength is $\lambda = v/f$, where $f = 22$ Hz is the frequency. From Fig. 14.30, we see that the fundamental wavelength in a half-open pipe is $L = \lambda/4$, so we can solve these two expressions for L. If the pipe is closed, the fundamental wavelength is $L = \lambda/2$, which gives the pipe length for a closed pipe.

EVALUATE **(a)** In the half-closed pipe, the minimum length is

$$L = \frac{\lambda}{4} = \frac{v}{4f} = \frac{343 \text{ m/s}}{4(22 \text{ Hz})} = 3.9 \text{ m}$$

(b) In an open pipe, the minimum length is

$$L = \frac{\lambda}{2} = \frac{v}{2f} = \frac{343 \text{ m/s}}{2(22 \text{ Hz})} = 7.8 \text{ m}$$

ASSESS Checking the units of the expression for pipe length, we find

$$\frac{\text{m/s}}{\text{Hz}} = \frac{\text{m} \cdot \text{s}^{-1}}{\text{s}^{-1}} = \text{m}$$

as expected.

71. **INTERPRET** We are asked to show that the simple harmonic wave with a sinusoidal shape is a solution to the general wave equation.

DEVELOP Equation 14.3 describes a simple harmonic wave:

$$y(x,t) = A\cos(kx - \omega t)$$

where we choose the negative value for a wave traveling in the positive x-direction. We will take the partial derivatives of y with respect to x and t, in order to see if it satisfies Equation 14.5:

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

EVALUATE By the chain rule, the second derivative of y with respect to x is

$$\frac{\partial^2 y}{\partial x^2} = \frac{\partial}{\partial x}\left[-kA\sin(kx - \omega t)\right] = -k^2 A\cos(kx - \omega t) = -k^2 y$$

Similarly, the second derivative of y with respect to time t is

$$\frac{\partial^2 y}{\partial t^2} = \frac{\partial}{\partial t}\left[-(-\omega)A\sin(kx - \omega t)\right] = -\omega^2 A\cos(kx - \omega t) = -\omega^2 y$$

Since both second derivatives are proportional to y, we can equate them to arrive at:

$$\frac{-1}{k^2}\frac{\partial^2 y}{\partial x^2} = \frac{-1}{\omega^2}\frac{\partial^2 y}{\partial t^2} \quad \rightarrow \quad \frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

Where we have used Equation 14.4: $v = \omega / k$. This proves that Equation 14.3 satisfies the wave equation.

ASSESS With a little more effort, one can show that a sum of harmonic waves with different wavelengths will also satisfy the wave equation:

$$y(x,t) = \sum_{i=1}^{N} A_i \cos(k_i x - \omega_i t)$$

The only requirement is that all the individual waves travel at the same speed: $v = \omega_i / k_i$.

72. **INTERPRET** We are asked to show that any function of the form $y = f(x \pm vt)$ is a solution to the wave equation.

DEVELOP Our goal is to show that the partial derivatives of y with respect to x and t satisfy Equation 14.5:

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

EVALUATE It will help to define a new variable $u = x \pm vt$. Then by the chain rule, the first derivative of y with respect to x is

$$\frac{\partial y}{\partial x} = \frac{\partial f}{\partial u}\frac{\partial u}{\partial x} = \frac{\partial f}{\partial u}$$

since $\partial u / \partial x = 1$. Doing the same thing with the second derivative gives

$$\frac{\partial^2 y}{\partial x^2} = \frac{\partial}{\partial u}\left[\frac{\partial f}{\partial u}\right]\frac{\partial u}{\partial x} = \frac{\partial^2 f}{\partial u^2}$$

For the derivatives with respect to time, we use $\partial u / \partial t = \pm v$ to arrive at

$$\frac{\partial^2 y}{\partial t^2} = \frac{\partial}{\partial t}\left[\frac{\partial f}{\partial u}\frac{\partial u}{\partial t}\right] = \pm v\frac{\partial}{\partial u}\left[\frac{\partial f}{\partial u}\right]\frac{\partial u}{\partial t} = v^2\frac{\partial^2 f}{\partial u^2}$$

Combining these two equations gives

$$\frac{\partial^2 y}{\partial x^2} = \frac{\partial^2 f}{\partial u^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

which shows that $y = f(x \pm vt)$ satisfies the wave equation.

ASSESS This basically says that any shape (given by the function *f*) can be turned into a wave if all the points move in unison at the same speed.

73. **INTERPRET** In this problem, we want to find the altitude of a supersonic plane, given its speed and the time at which you hear the sonic boom.

DEVELOP Since the plane is moving at a supersonic speed, shock waves are formed, and the Mach angle is given by $\sin\theta = u/v$. The altitude and distance traveled in level flight are related by $h = (u\Delta t)\tan\theta$ (the shock front moves with the same speed as the aircraft).

EVALUATE The Mach angle is $\theta = \sin^{-1}(1/2) = 30.0^\circ$ for the plane. Its altitude is

$$h = (u\Delta t)\tan\theta = (2\times 343\text{ m/s})(24\text{ s})\tan(30.0^\circ) = 9.5\text{ km}$$

ASSESS The altitude is greater than the typical flying altitude of commercial aircraft (about 3000 m). However, a high-altitude surveillance plane (such as Lockheed U-2) can fly at an altitude greater than 21 km!

74. **INTERPRET** This problem involves standing sound waves in a semi-open pipe (i.e., a pipe with one end closed and the other end open). Given two neighboring standing wave frequencies, we are to find the fundamental frequency and the speed of sound in the pipe.

DEVELOP For a semi-open pipe, an odd number of quarter wavelengths must fit within the pipe length (see Fig. 14.30). Thus,

$$L = (2m-1)\frac{\lambda}{4} \quad m = 1,2,3,\ldots$$

Because the wave velocity is the same for all harmonic frequencies, we can write $v = \lambda_m f_m = \lambda_{m+1} f_{m+1}$ (Equation 4.1). Expressing the pipe length in terms of these frequencies gives

$$L = (2m-1)\frac{\lambda_m}{4} = \left[2(m+1)-1\right]\frac{\lambda_{m+1}}{4}$$

$$(2m-1)\frac{v}{4f_m} = (2m+1)\frac{v}{4f_{m+1}}$$

$$\frac{f_m}{f_{m+1}} = \frac{2m-1}{2m+1}$$

$$m = \frac{f_{m+1}+f_m}{2(f_{m+1}-f_m)}$$

Knowing *m*, we can find the fundamental frequency using

$$f_1 = \frac{f_m}{2m-1}$$

and the wave speed *v* using Equation 4.1.

EVALUATE **(a)** Given that $f_m = 345$ Hz and $f_{m+1} = 483$ Hz, the index *m* is

$$m = \frac{483\text{ Hz} + 345\text{ Hz}}{2(483\text{ Hz} - 345\text{ Hz})} = 3$$

Therefore, the fundamental frequency is

$$f_1 = \frac{f_3}{2\cdot 3-1} = \frac{345\text{ Hz}}{5} = 69.0\text{ Hz}$$

(b) The fundamental wavelength is

$$L = \frac{\lambda_1}{4}$$
$$\lambda_1 = 4L = 4(2.25\ \text{m}) = 9.0\ \text{m}$$

and the wave speed is

$$v = \lambda_1 f_1 = (9.0\ \text{m})(69.0\ \text{Hz}) = 621\ \text{m/s}$$

ASSESS To find the fundamental frequency, we could have used f_4, which gives

$$f_1 = \frac{f_4}{2\cdot 4 - 1} = \frac{483\ \text{Hz}}{7} = 69\ \text{Hz}$$

This is consistent with the result above.

75. **INTERPRET** This problem involves traveling gravitational waves. We want to determine the direction from which they traveled to the detection sites.

DEVELOP The waves are detected at two different locations in the United. States: Livingston, LA and Hanford, WA, which are located in northern and southern U.S. states, respectively. We can estimate their "straight" line separation distance using a great-circle distance L of approximately 3.0×10^6 m, which is oriented diagonally, going from NW to SE. Based on the delay between detection times, we can estimate the vertical (N to S) distance d separating the two sites and use that to determine the relative angle between the direction of the waves' propagation and the Livingston–Hanford line.

EVALUATE Since we are told the southern location detected the signal first, we can determine that the waves traveled from the southern hemisphere of the sky. Based on a delay of $\Delta t = 0.007\,\text{s}$, and the fact that the waves travel at the speed of light, we can estimate that the vertical distance d separating the two detection sites is approximately equal to $d = c\Delta t = \left(3.0\times10^8\ \text{m/s}\right)(0.007\,\text{s}) = 2.1\times10^6\ \text{m}$. Knowing both the vertical and diagonal distances separating the detection sites we find that angle between the direction of the waves' propagation and the Livingston–Hanford line is approximately equal to $\theta = \text{asin}(d/L) = 45°$.

ASSESS Here we have calculated the diagonal distance separating the two detection sites by using the orthodromic distance, the shortest distance between two points on the surface of a sphere. The waves' straight-line path through Earth would be slightly shorter.

76. **INTERPRET** This problem involves the Doppler effect applied twice. First, the heart is moving with respect to the source and second, the heart, which is again moving with respect to the observer, reemits an acoustic wave. Thus, by applying the Doppler effect twice, we can calculate the speed at which the heart wall is moving in order to generate a 100-Hz frequency shift.

DEVELOP The initial wave leaves the stationary ultrasound source with a frequency f, which is then observed by the heart moving at speed u as the frequency $f' = f(1+u/v)$. The heart then acts as a moving source that sends the wave back with frequency f' back to the stationary ultrasound device, where the observed frequency is

$$f'' = \frac{f'}{(1-u/v)} = \frac{f(1+u/v)}{1-u/v}$$

The frequency shift is

$$\Delta f = f'' - f = f\left[\frac{v+u}{v-u} - 1\right] = \frac{2uf}{v-u}$$

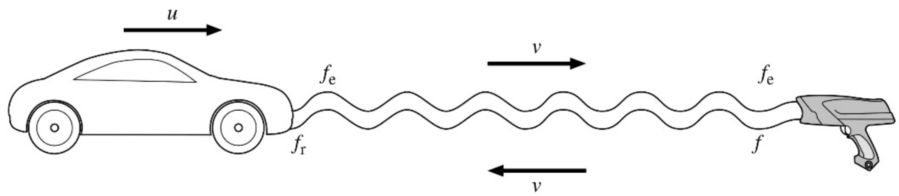
EVALUATE Taking $u \ll v = 1497$ m/s (where we approximate the speed of sound in tissue with the speed of sound in water), we can use the above expression for the frequency shift to write

$$u \approx \frac{v\Delta f}{2f} = \frac{(1497\ \text{m/s})(100\ \text{Hz})}{(2\times5.0\times10^6\ \text{Hz})} = 1.5\ \text{cm/s}$$

ASSESS The result of $u = 1.5$ cm/s $= 1.5\times10^{-2}$ m/s justifies the assumption of $u \ll v$.

77. **INTERPRET** You are trying to get out of a speeding ticket by arguing that the police radar was defective.

DEVELOP The radar works by recording the Doppler shift in high-frequency radio waves that bounce off your car. Let's assume the radar was up the road from your car when it measured your speed, as shown in the figure below.



The reflection of the radio waves should be thought of as two separate interactions. First, the radio waves are "received" on the surface of your car, and second, they are "emitted" by your car's surface back to the radar gun. In this way your car is both a moving observer and a moving source.

The Doppler shift equations (14.14 and 14.15) are approximately valid in the case of radio waves, as long as the velocity of the source/observer is much less than the speed of light $\left(v=3.00\times10^{8}\,\text{m/s}\right)$. This is certainly the case for your car.

EVALUATE As explained above, the car first acts as a moving observer, so it "receives" radio waves with frequency

$$f_r = f\left(1+\frac{u}{v}\right)$$

where we have taken the positive sign because the car is approaching the source in our assumed scenario. This received frequency is immediately "emitted" (i.e., reflected) by the moving car with a further shifted frequency:

$$f_e = \frac{f_r}{1-u/v}$$

where we have taken the negative sign because the car is an approaching source. The police radar picks up this reflected signal and compares it to the original frequency:

$$\Delta f = f_e - f = f\left[\frac{1+u/v}{1-u/v}-1\right] \approx 2f\frac{u}{v}$$

Here we have used the fact that $u \ll v.$ Solving for the car's velocity gives

$$u = \frac{\Delta f}{2f}v = \frac{15.6\text{ kHz}}{2(70\text{ GHz})}\left(3.00\times10^{8}\,\text{m/s}\right) = 120\text{ km/h}$$

The judge should rule that the radar was working properly.

ASSESS You wrongly assumed that there is only one Doppler shift, either from the car as a moving observer or a moving source. This caused your calculation of your car's speed to be twice what it actually was.

78. **INTERPRET** This problem involves the Doppler effect. We are to show that if we are moving toward a wave source that is stationary with respect to the wave medium, then time between wave crests is as given in the problem statement. Furthermore, we are to use this to show that the Doppler-shifted frequency we perceive is given by Equation 14.16 with the + sign.

DEVELOP Period T is the time between wave crests. If we are moving toward the wave source at speed u, it is as if the wave is moving toward us at the speed $v + u$. The time between wave crests is distance divided by speed, so

$$T' = \frac{\lambda}{v+u}$$

We will use this, in conjunction with $T = 1/f$, to verify the positive version of Equation 14.16, $f' = f\left(1+u/v\right)$.

EVALUATE We've already shown the first part just from considering the physics of the problem. For the second part, we take the ratio of the periods for $u = 0$ and $u \neq 0$, which gives

$$\frac{T}{T'} = \frac{\lambda/v}{\lambda/(v+u)} = \frac{f'}{f}$$
$$1+\frac{u}{v} = \frac{f'}{f}$$
$$f' = f(1+u/v)$$

ASSESS We could use the same process to show that if we are moving away from the source, then $f' = f(1-u/v)$.

79. **INTERPRET** You want to verify that a Doppler radar system can measure the velocity of rain drops to the required accuracy.

DEVELOP In Problem 14.77, we explained how police radar works and this Doppler radar system is the same. The radar waves reflect off the rain drops, which act both as moving observers and moving sources.
Since the velocity of the rain drops is much less than the speed of light, the total frequency shift measured by the system will be:

$$\Delta f \approx 2f\frac{u}{v}$$

EVALUATE The vendor's 5.0-GHz radar can detect velocities down to

$$u = \frac{\Delta f}{2f}v = \frac{50\text{ Hz}}{2(5.0\text{ GHz})}\left(3.00\times10^{8}\text{m/s}\right) = 1.5\text{ m/s} = 5.4\text{ km/h}$$

No, apparently the vendor's radar is not sufficient.

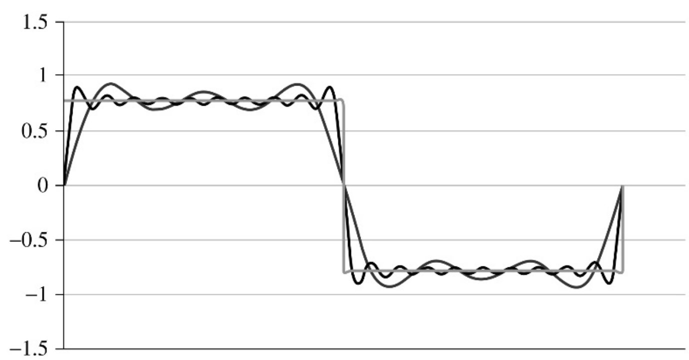
ASSESS The vendor could try to reduce the minimum frequency shift, Δf, that its radar can measure. Equally, it could increase the frequency that its device emits at, since $u \propto 1/f$.

80. **INTERPRET** We're asked to plot the sum of simple harmonic waves, which is meant to be a Fourier representation of a square wave.

DEVELOP As described in Fig. 14.17, a square wave can be built up by summing a series of sine waves with the following form:

$$y(t) = A\sin(\omega t) + \frac{1}{3}A\sin(3\omega t) + \frac{1}{5}A\sin(5\omega t) + \frac{1}{7}A\sin(7\omega t)...$$

EVALUATE We set $A=1$ and $\omega=1$, and let t vary from 0 to 2π. In the figure below, we plot the sum of the first 3 terms and the sum the first 10 terms.



For comparison, we also draw the corresponding square wave:

$$f(t) = \begin{cases} \frac{1}{4}\pi & 0 \le t < \pi \\ -\frac{1}{4}\pi & \pi \le t < 2\pi \end{cases}$$

It's clear that the 10-term sum (with 10 wiggles) comes closer to the square wave than the 3-term sum (with 3 wiggles). To quantify how representative each sum is, we calculate the standard deviation, which is a measure of the "error" between the sum and the square wave:

$$\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left[y(t_i) - f(t_i)\right]^2}$$

Here, the index i designates the individual points in our plot. The standard deviation for the sum of the first 3 terms is 0.23, whereas for the sum of the first 10 terms it is 0.16. This indicates how adding more sine waves will improve the accuracy of the Fourier representation.

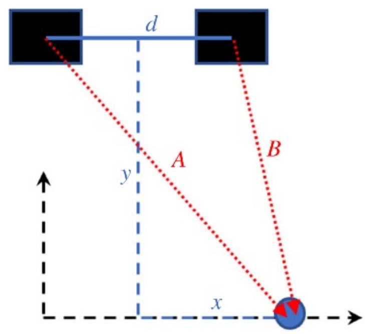
ASSESS Fourier analysis is a common practice in science and engineering. For any given function, $f(t)$, one can find a sum of sine and cosine waves that satisfies:

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nt + \sum_{n=1}^{\infty} b_n \sin nt$$

In the case of the square wave function explored above, the cosine coefficients (the a_n) are all zero. The sine coefficients are zero for n even, but for n odd: $b_n = 1/n$.

81. **INTERPRET** This problem involves the interference of waves. We want to use the properties of destructive interference to determine the frequency of the emitted waves.

DEVELOP Destructive interference of waves occurs when the path lengths from two sources differ by half a wavelength. Looking at the figure below, we see that the location where the sound diminishes is located a distance x away from the perpendicular bisector of the two speakers, which are a vertical distance y away from this spot, and separated by a distance d .



From the given values for these distances we can calculate the path lengths of waves traveling from each source, and find the wavelength that would result in destructive interference at this location. Using the speed of the waves we can then determine their frequency using Equation 14.1, $v = \lambda f$.

EVALUATE The lengths of paths A and B are, respectively, given by

$$A = \sqrt{(x + d/2)^2 + y^2} = 10.72\,\text{m} \qquad B = \sqrt{(x - d/2)^2 + y^2} = 10.05\,\text{m}$$

Meaning the wavelength is then equal to

$$\lambda = 2(A - B) = 1.34\,\text{m}$$

From the given speed of sound in air of 343 m/s we find the frequency of the sound waves is equal to

$$f = \frac{v}{\lambda} = \frac{(343\,\text{m/s})}{(1.34\,\text{m})} = 256\,\text{Hz}$$

ASSESS This frequency is referred to as the philosophical pitch, and is often used for tuning of instruments.

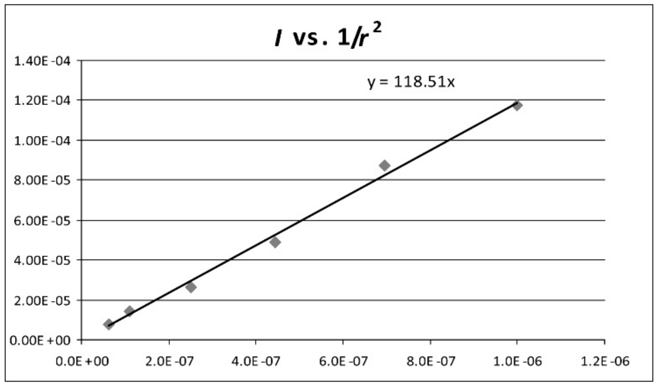
82. **INTERPRET** In this problem you are asked to analyze the data of sound intensity level as function of distance and determine the total sound power of the source.

DEVELOP We first convert the sound intensity levels to actual intensity levels using Equation 14.10, which may be written in the form

$$I = I_0 10^{\beta/10}$$

where β is the intensity level in decibels and $I_0 = 10^{-12}$ W/m^2 is the standard threshold of hearing a 1 kHz. The total power carried by the waves is constant, irrespective of the distance from the source. Therefore, from Equation 14.8, $I = (P/4\pi)(1/r^2)$, plotting I as a function of $1/r^2$ gives a straight line with a slope of $P/4\pi$.

EVALUATE The plot of I vs. $1/r^2$ is shown below.



The slope is 118.51, implying a total sound power of $P = 4\pi(118.51\text{ W}) = 1.5\text{ kW}$.

ASSESS The intensities measured at the two points are related by

$$P = I_1\left(4\pi r_1^2\right) = I_2\left(4\pi r_2^2\right) \quad \Rightarrow \quad \frac{I_1}{I_2} = \frac{r_2^2}{r_1^2}$$

Thus, the change in sound intensity level can be written as

$$\beta_1 - \beta_2 = 10\log\left(\frac{r_2^2}{r_1^2}\right)$$

83. **INTERPRET** We explore the physics of tsunami waves.

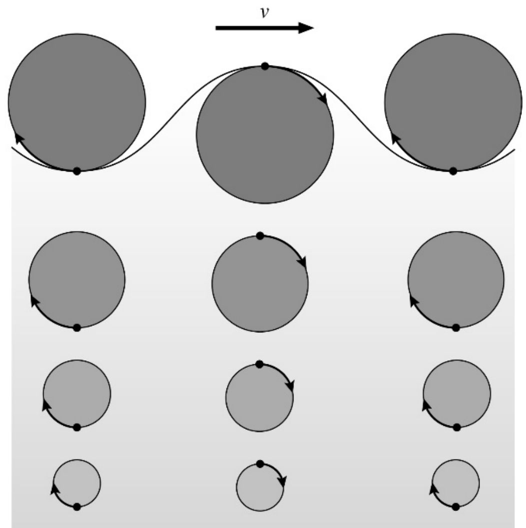
DEVELOP As described in the text, the speed of a tsunami wave is proportional to the square root of the depth: $v = \sqrt{gd}$.

EVALUATE As a tsunami wave approaches the shore, the depth will decrease, as will the speed of the wave. The answer is **(b)**.

ASSESS The slowing down of the wave causes the water to pile up. As we will see in Problem 14.86, the wave amplitude increases as the tsunami approaches the shore.

84. **INTERPRET** We explore the physics of tsunami waves.

DEVELOP If you were to follow the motion of a single water molecule at the surface of the ocean, you would see that it loops around in a nearly circular pattern as each wave goes by. Water molecules beneath the surface also loop around in generally smaller patterns, as shown in the figure below.



This subsurface motion is driven by pressure differences in the water, which are the result of the uneven ocean surface above. As you go deeper beneath the surface, the wave-driven pressure differences become less important. At a certain depth, water molecules no longer move in response to waves at the surface. This depth depends on the wavelength of the waves.

EVALUATE Shallow water waves are those in which subsurface motion continues all the way to the bottom. That is to say, the whole water column moves. For a tsunami to move the whole water column in deep water, its wavelength must be sufficiently long.

The answer is **(a)**.

ASSESS One can find references that say the depth at which waves begin to behave like shallow water waves is at 1/20th of their wavelength. A typical tsunami might have a wavelength of 200 km, which means it would be a shallow water wave in the deepest parts of the ocean (around 10 km).

85. **INTERPRET** We explore the physics of tsunami waves.

DEVELOP The speed of a tsunami wave is proportional to the square root of the depth, so $v = v_0\sqrt{d/d_0}$.

EVALUATE If the depth doubles, the speed will increase to

$$v = v_0\sqrt{\frac{d}{d_0}} = (430\text{ km/h})\sqrt{2} = 610\text{ km/h}$$

The answer is **(b)**.

ASSESS The high speed at which tsunamis travel over deep water is why they often catch people off guard.

86. **INTERPRET** We explore the physics of tsunami waves.

DEVELOP We already know that the speed of the tsunami decreases as it approaches the shore. If the wavelength is also decreasing, then by Equation 14.1, we'd assume that the wave frequency remains roughly constant: $f = v/\lambda$.

EVALUATE It's hard to imagine how the total energy of the wave, E_{tot}, could increase without some external force acting on it. But it's possible that the *rate* at which the wave carries energy to the shore increases. This power will be the total energy in the wave divided by the time it takes for all the wave to reach the shore, which is just the period:

$$P = \frac{E_{\text{tot}}}{T} = E_{\text{tot}} \cdot f$$

We've already argued that the total energy and the frequency are constant, so the power must be constant as well. If we were wrong and one of these quantities is increasing, then this equation says that one of the other quantities would have to change as well. So the only answer that makes sense is that none of these quantities changes.

The answer is **(d)**.

ASSESS Ocean wave theory says that the energy per unit horizontal surface depends only on the density, ρ, and the wave amplitude squared:

$$\sigma_{\text{E}} = \frac{1}{2}\rho g A^2$$

The horizontal area of a wave is the wavelength multiplied by its width, which we will call w. Therefore, the total energy of a tsunami wave would be $E_{\text{tot}} = \sigma_{\text{E}}\lambda w.$ If indeed the total energy is constant, the wave amplitude will be proportional to

$$A \propto \frac{1}{\sqrt{\lambda}} \propto \frac{1}{\sqrt{v}} \propto \frac{1}{\sqrt[4]{d}}$$

If the depth decreases by a factor of 100, the amplitude will increase by a factor of 3.