

EXERCISES

Sections 19.2 and 19.3 The Second Law of Thermodynamics and Its Applications

- 11. INTERPRET** This problem requires us to calculate the efficiency of reversible heat engines that operate between the given temperatures.
- DEVELOP** The maximum efficiency of a reversible engine, operating between two absolute temperatures, $T_h > T_c$, is given by Equation 19.3, $e_{\text{Carnot}} = 1 - T_c/T_h$. Apply this to each part of this problem to find the corresponding efficiencies.
- EVALUATE** (a) $e = 1 - T_c/T_h = 1 - (273 \text{ K})/(373 \text{ K}) = 26.8\%$.
- (b) $e = 1 - T_c/T_h = \Delta T/T_h = (21 \text{ K})/(298 \text{ K}) = 7.05\%$.
- (c) With room temperature at $e = 1 - T_c/T_h = 1 - (293 \text{ K})/(1273 \text{ K}) = 77.0\%$.
- ASSESS** The engine with the largest difference in reservoir temperature has the largest efficiency.

- 12. INTERPRET** This problem is an exercise in calculating the thermal efficiency of a heat engine, given the temperature of the hot and cold reservoirs.
- DEVELOP** Assuming this cosmic heat engine were a reversible engine, its efficiency would be that of a Carnot engine, given by Equation 19.3:

$$e_{\text{Carnot}} = 1 - \frac{T_c}{T_h}$$

EVALUATE Substituting the values given in the problem, we obtain

$$e_{\text{Carnot}} = 1 - \frac{T_c}{T_h} = 1 - \frac{2.7 \text{ K}}{5800 \text{ K}} = 1 - 4.66 \times 10^{-4} \approx 99.95\%$$

ASSESS The engine efficiency is almost 100%. This is too good to be true.

- 13. INTERPRET** This problem involves a reversible Carnot engine that runs between the boiling and melting point of He. We are given the engine's efficiency and are asked to find the temperature of its cold reservoir (i.e., the melting point of He).
- DEVELOP** Apply Equation 19.3, which gives the efficiency of a Carnot engine. We are given $e_{\text{Carnot}} = 0.777$ and $T_h = 4.25 \text{ K}$, so we can solve for T_c , which will be the melting point of He.
- EVALUATE** The melting point of He is

$$e_{\text{Carnot}} = 1 - \frac{T_c}{T_h}$$

$$T_c = (1 - e_{\text{Carnot}})T_h = (1 - 0.777)(4.25 \text{ K}) = 0.948 \text{ K}$$

ASSESS This agrees with the melting point of He found in the literature, so it seems to be a reasonable result.

- 14. INTERPRET** this problem is about a Carnot engine that operates via the Carnot cycle. We are given the thermal energy absorbed per cycle and the work done per cycle by the engine, and we are asked to find its efficiency.
- DEVELOP** By definition, the efficiency of an engine is

$$e = \frac{W}{Q_h}$$

where W is the work done by the engine and Q_h is the heat absorbed from the hot reservoir per cycle. For part (b), the work W is defined as the work done by the gas, so $W = 350 \text{ J}$ in this case. The heat transferred is $Q = Q_h - Q_c$. Because there is no net change in the internal energy of the engine over a cycle, the first law of thermodynamics gives (note that W in Chapter 19 is defined contrary to W in Chapter 18!)

$$W = Q_h - Q_c$$

For part (c), Equation 19.2 gives

$$\frac{T_h}{T_c} = \frac{Q_h}{Q_c}$$

So, we can solve for T_h , given that $T_c = 10^\circ\text{C} = 283 \text{ K}$.

EVALUATE (a) From the equation above, the efficiency of the engine is

$$e = \frac{W}{Q_h} = \frac{360 \text{ J}}{950 \text{ J}} = 37.9\%$$

(b) The heat rejected each cycle is

$$Q_c = Q_h - W = 950 \text{ J} - 360 \text{ J} = 590 \text{ J}$$

(c) The temperature of the hot reservoir is

$$T_h = T_c \left(\frac{Q_h}{Q_c} \right) = (283 \text{ K}) \left(\frac{950 \text{ J}}{590 \text{ J}} \right) = 456 \text{ K} = 183^\circ\text{C}$$

ASSESS The maximum temperature T_h is greater than T_c , as our calculation confirms. Note that Carnot's theorem applies to the ratio of absolute temperatures.

- 15. INTERPRET** We are to find the coefficient of performance of a reversible refrigerator that operates between 0°C and 35°C .

DEVELOP For a refrigerator, the coefficient of performance is given by Equation 19.4,

$$\text{COP}_{\text{refrigerator}} = \frac{Q_c}{Q_h - Q_c}$$

Use Equation 19.2, $Q_c / Q_h = T_c / T_h$, to convert this to an expression involving temperature:

$$\text{COP}_{\text{refrigerator}} = \frac{Q_c}{Q_h - Q_c} = \frac{Q_c}{Q_h} \left(\frac{1}{1 - Q_c / Q_h} \right) = \frac{T_c}{T_h} \left(\frac{1}{1 - T_c / T_h} \right) = \frac{T_c}{T_h - T_c}$$

which we can solve given that $T_c = 0^\circ\text{C} = 273 \text{ K}$ and $T_h = 35^\circ\text{C} = 308 \text{ K}$.

EVALUATE Inserting the given quantities into the expression for the COP gives

$$\text{COP}_{\text{refrigerator}} = \frac{T_c}{T_h - T_c} = \frac{273 \text{ K}}{308 \text{ K} - 273 \text{ K}} = 7.80$$

ASSESS Notice that the temperatures are absolute temperatures (i.e., Kelvin).

- 16. INTERPRET** This problem requires us to find the work done by a refrigerator to freeze the given quantity of water. The heat of transformation (Chapter 17) is involved in the liquid-to-solid phase change.

DEVELOP The amount of heat that must be extracted in order to freeze the water is (Equation 17.5)

$$Q_c = mL_f = (0.88 \text{ kg})(334 \text{ kJ / kg}) = 294 \text{ kJ}$$

The work consumed by the refrigerator while extracting this heat is given by Equation 19.4, $W = Q_c / \text{COP}$.

EVALUATE Inserting the given values, we obtain

$$W = \frac{Q_c}{\text{COP}} = \frac{294 \text{ kJ}}{4.1} = 72 \text{ kJ}$$

ASSESS A COP of 4.1 means that each unit of work can transfer 4.1 units of heat from inside the refrigerator. A smaller COP would mean that more work is required to freeze the water.

- 17. INTERPRET** We want to know if the human body can be considered as a heat engine, which has stringent limits on its efficiency.

DEVELOP If the human body were a heat engine, the maximum efficiency it could attain would be given by the efficiency for a Carnot engine (Equation 19.3): $e = 1 - T_c / T_h$.

EVALUATE Body temperature is $T_h = 37^\circ\text{C} = 310\text{ K}$. So if we assume the ambient temperature is $T_c = 20^\circ\text{C} = 293\text{ K}$, the maximum efficiency would be

$$e = 1 - \frac{293\text{ K}}{310\text{ K}} = 5\%$$

So under normal circumstances, the human body is far too efficient at 25% to be a heat engine.

ASSESS We often say the body “burns” calories, which sounds like it’s just releasing random heat from the food we eat. But actually the process is more specific. Energy-storing molecules interact with other molecules to cause precise chemical reactions that result in, for example, a muscle contracting or a neuron producing a current. Not all of the stored energy is converted to useful work, however. Some of it ends up as heat.

Section 19.4 Entropy and Energy Quality

- 18. INTERPRET** This problem requires us to calculate the entropy change involved in melting the given quantity of ice at 0°C .

DEVELOP Because the ice is at 0°C , no temperature change is involved in melting it. Therefore, Equation 19.6 takes the form

$$\Delta S = \frac{\Delta Q}{T}$$

The change in heat is just the latent heat of water, which is given by Equation 17.5 ($\Delta Q = mL_f$) and Table 17.1 ($L_f = 334\text{ kJ/kg}$). The temperature is $T = 0^\circ\text{C} = 273\text{ K}$.

EVALUATE Inserting the given quantities into the expression for entropy change gives

$$\Delta S = \frac{\Delta Q}{T} = \frac{mL_f}{T} = \frac{(1.9\text{ kg})(334\text{ kJ/kg})}{273\text{ K}} = 2.3\text{ kJ/K}$$

ASSESS Thus, the water has more entropy than the ice, so it has a greater capacity to do work.

- 19. INTERPRET** The problem concerns the entropy increase associated with metabolizing a hamburger.

DEVELOP We’ll assume the energy in the burger, Q , flows into the body as heat. Therefore, the entropy change from state 1 (burger ingested) to state 2 (burger metabolized) is given by Equation 19.6: $\Delta S_{12} = \int_1^2 dQ / T$.

EVALUATE The body temperature, $T = 37^\circ\text{C} = 310\text{ K}$, remains constant throughout, so

$$\Delta S_{12} = \frac{1}{T} \int_1^2 dQ = \frac{Q}{T} = \frac{450\text{ kcal}}{310\text{ K}} \left(\frac{4.184\text{ kJ}}{1\text{ kcal}} \right) = 6.1\text{ kJ/K}$$

ASSESS Although we assumed the burger’s energy went into heat, the answer would be the same if the body used some of the energy to do work. In either case, the burger’s energy is no longer available to do work once it has been metabolized.

- 20. INTERPRET** This problem asks us to calculate the entropy increase upon heating up a given mass of water from 5°C to 95°C .

DEVELOP The change in heat content of the water is given by Equation 16.3, $Q = mc\Delta T$, where $c = 4.184\text{ kJ/(kg} \cdot \text{K)}$ for water (see Table 16.1). For a substance like water with constant specific heat (in this case, at constant pressure), we can differentiate this expression to find $dQ = mc\,dT$, so (from Equation 19.6) the change in entropy is

$$\Delta S = \int_1^2 \frac{dQ}{T} = mc \int_{T_1}^{T_2} \frac{dT}{T} = mc \ln \left(\frac{T_2}{T_1} \right)$$

The initial and final temperatures are $T_1 = 5^\circ\text{C} = 278\text{ K}$ and $T_2 = 90^\circ\text{C} = 363\text{ K}$, respectively.

EVALUATE Inserting the given values, we have

$$\Delta S = mc \ln \left(\frac{T_2}{T_1} \right) = (0.35 \text{ kg})[4.184 \text{ kJ}/(\text{kg} \cdot \text{K})] \ln \left(\frac{363 \text{ K}}{278 \text{ K}} \right) = 390 \text{ J/K}$$

to two significant figures.

ASSESS The final entropy of the system has increased.

- 21. INTERPRET** This problem requires us to find the mass of a block of lead, given its entropy increase associated with its solid-to-liquid phase transition (i.e., melting it).

DEVELOP The lead starts at its melting-point temperature, so there is no change in temperature associated with the solid-to-liquid phase change. Therefore, Equation 19.6 for entropy change takes the form

$$\Delta S = \frac{\Delta Q}{T}$$

The heat change is given by Equation 17.5, $\Delta Q = mL_f$, where $L_f = 24.7 \text{ kJ/kg}$ (see Table 17.1). Insert this into the expression for entropy change and solve for the mass m .

EVALUATE From the above equation, we find the mass of the lead block to be

$$\Delta S = \frac{mL_f}{T}$$

$$m = \frac{T\Delta S}{L_f} = \frac{(600 \text{ K})(950 \text{ J/K})}{24.7 \text{ kJ/kg}} = 23.1 \text{ kg}$$

ASSESS As expected, the mass of the block is proportional to the change in entropy.

- 22. INTERPRET** We are to find the energy that becomes unavailable during an irreversible isothermal process, given the entropy increase.

DEVELOP The energy that becomes unavailable is given by $E = T_{\min} \Delta S$ (see section Entropy and the Availability of Work), where T_{\min} is the lowest temperature available to the system. The change in entropy ΔS is $\Delta S = 40 \text{ J/K}$, and the temperature is $T = T_{\min} = 425 \text{ K}$.

EVALUATE Inserting the given quantities in the expression for unavailable work gives

$$E = T_{\min} \Delta S = (425 \text{ K})(40 \text{ J/K}) = 17 \text{ kJ}$$

ASSESS Since this is an isothermal process, the temperature is constant, and we use the given minimum temperature throughout.

- 23. INTERPRET** We're asked to find the probability that 6 molecules are distributed in different ways inside a box. This has relevance to the statistical interpretation of entropy.

DEVELOP Considering a single molecule, the probability that it is located on the left side or the right side of the box is $1/2$. Considering 2 molecules, the probability for one particular left-right arrangement (microstate) is $1/4$. Another way to say this is that there are 4 different ways to sort the molecules between the two sides. For 6 molecules, there are $2^6 = 64$ ways to sort, so the probability for one particular arrangement (microstate) is $1/64$. We now have to count how many of these microstates match the following macrostates (see Figs. 19.18 and 19.19).

EVALUATE (a) There's one microstate in which all of the molecules are found on the left side of the box, and another one where all are found on the right side of the box. So the probability of this macrostate is $1/64 + 1/64 = 1/32$.

(b) It's a bit harder to find the number of microstates with half the molecules on one side, half on the other. So let's label the molecules A, B, C, D, E, and F, and let's identify a microstate by the 3 molecules on the left-hand side. So for example, (ABC) is the microstate with A, B, C on the left-hand side, and the others on the right. We can switch out C in three different ways: (ABD), (ABE), and (ABF). Similarly we can switch out B in three different ways and switch out A in three different ways. That gives us 10 microstates. We count another 10 microstates if we start with (DEF), and switch out D, then E, then F. The total number is 20 microstates.

So the probability of the macrostate with the molecules split evenly between the sides is $20/64$.

ASSESS It is 20 times more likely that the 6 molecules will be spread out evenly between the two sides of the box vs. all on one side. In general, if there are n molecules, the probability that k of them will be on one side and $(n-k)$ on the other side is given by the coefficients from the binomial theorem:

$$P = \binom{n}{k} \cdot \frac{1}{2^n} = \left[\frac{n!}{k!(n-k)!} \right] \cdot \frac{1}{2^n}$$

In the case above, $n = 6$ and $k = 3$, so the probability is $P = 6! / (3! 3!) 2^6 = 20 / 64$, as we found.

EXAMPLE VARIATIONS

- 24. INTERPRET** This problem is about a Carnot engine, which operates via the Carnot cycle.

DEVELOP Equation 19.3, $e_{\text{Carnot}} = 1 - (T_c/T_h)$, relates the two temperatures and the efficiency. Here $Q_h = 2.84$ kJ, $Q_c = 1.31$ kJ, and $T_c = 22.5^\circ\text{C}$ or 296 K. The first law of thermodynamics relates work and heat flows. So our plan is to use the first law to find the work, then find the efficiency, and then use Equation 19.3 to find T_h .

EVALUATE Since there's no change in internal energy over one cycle, the first law requires that the work W done by the engine be equal to the net heat absorbed—namely, 2.84 kJ $- 1.31$ kJ. So $W = 1.53$ kJ. The efficiency is the ratio of work delivered to heat extracted, so $e = W / Q_h = 1.53$ kJ / 2.84 kJ = 0.539 . Knowing the efficiency, we solve Equation 19.3 for T_h :

$$T_h = \frac{T_c}{1 - e} = \frac{295.7 \text{ K}}{1 - 0.539} = 641 \text{ K} = 368^\circ\text{C}$$

ASSESS The engine rejects somewhat less than half the 2.84 kJ as waste heat, so we should expect efficiency somewhat over 50%. And T_h must be greater than T_c , as our calculation confirms.

- 25. INTERPRET** This problem is about a Carnot engine, which operates via the Carnot cycle.

DEVELOP Equation 19.3, $e_{\text{Carnot}} = 1 - (T_c/T_h)$, relates the two temperatures and the efficiency. Here $P_w = 48.4$ kW, $P_c = 41.7$ kW, and $T_h = 625$ K. The first law of thermodynamics relates work and heat flows. So our plan is to use the first law to find the rate of energy extracted from the reservoir, since the time interval considered is the same for all heat energy exchange processes. Then we can find the efficiency, and then use Equation 19.3 to find T_c .

EVALUATE Since there's no change in internal energy over one cycle, the first law requires that the work W done by the engine be equal to the net heat absorbed. Thus, the difference between the rate of energy extracted from the hot reservoir and the rejected heat is equal to mechanical output power—namely, $P_w = P_h - P_c$.

So: $P_h = 90.1$ kW. The efficiency is the ratio of work delivered to heat extracted, which is also equal to the ratio of the rates at which the work is done and heat is extracted, so $e = P_w / P_h = 0.573$. Knowing the efficiency, we solve Equation 19.3 for T_c :

$$T_c = T_h(1 - e) = 625 \text{ K}(1 - 0.573) = 289 \text{ K} = 16^\circ\text{C}$$

ASSESS Knowing the rates at which the engine process and outputs energy allows us to calculate the efficiency in the same way we would if we had the exact amount of energy converted during the time intervals considered.

- 26. INTERPRET** This problem is about a Carnot engine, which operates via the Carnot cycle.

DEVELOP Equation 19.3, $e_{\text{Carnot}} = 1 - (T_c/T_h)$, relates the two temperatures and the efficiency. Here $P_h = 610$ MW, $P_c = 233$ MW, and $T_c = 320$ K. The first law of thermodynamics relates work and heat flows. So our plan is to use the first law to find the rate of energy generated as electric output, since the time interval considered is the same for all heat energy exchange processes. Then we can find the efficiency, and then use Equation 19.3 to find T_h .

EVALUATE Since there's no change in internal energy over one cycle, the first law requires that the work W done by the engine be equal to the net heat absorbed. Thus, the difference between the rate of energy extracted from the hot reservoir and the rejected heat is equal to electric output power—namely, $P_w = P_h - P_c = 377$ MW. The efficiency is the ratio of work delivered to heat extracted, which is also equal to the ratio of the rates at which the work is done and heat is extracted, so $e = P_w / P_h = 0.62$. Knowing the efficiency, we solve Equation 19.3 for T_h :

$$T_h = \frac{T_c}{1-e} = \frac{320\text{ K}}{1-0.62} = 840\text{ K}$$

ASSESS The fluid in the towers reaches $\sim 570^\circ\text{C}$, which is upward of 1000°F ! The sunlight heats up molten salt which is stored and accessed when needed to heat water and generate steam like other power plants.

27. INTERPRET This problem is about a Carnot engine, which operates via the Carnot cycle.

DEVELOP Equation 19.3, $e_{\text{Carnot}} = 1 - (T_c/T_h)$, relates the two temperatures and the efficiency.

EVALUATE Knowing the water temperature deep down and at the surface level, we calculate an efficiency of

$$e = 1 - \frac{T_c}{T_h} = \frac{278\text{ K}}{298\text{ K}} = 0.067$$

6.7%.

ASSESS Recall this is the theoretical maximum, so the thermal efficiency is bound to be lower during actual operation. Nevertheless, this technology is still developing and could prove very useful since it is a continuously available renewable energy source.

28. INTERPRET This problem asks about the loss of energy quality during an irreversible and therefore entropy-increasing process—namely, an adiabatic free expansion.

DEVELOP During an irreversible process where the entropy of a system increases by ΔS , energy $E = T_{\min}\Delta S$ becomes unavailable to do work, where T_{\min} is the lowest temperature available to the system. During an adiabatic free expansion, with the temperature constant, the entropy change is given by

$$\Delta S = nR \ln\left(\frac{V_2}{V_1}\right)$$

Our plan is to calculate ΔS and multiply by T_{\min} to find that unavailable energy.

EVALUATE Because the temperature doesn't change, T_{\min} is the temperature we're given, and we find the unavailable energy is equal to

$$E = T\Delta S = nRT \ln\left(\frac{V_2}{V_1}\right) = (52.8\text{ mol})(8.314\text{ J/K}\cdot\text{mol})(282\text{ K}) \ln\left(\frac{445,000\text{ L}}{6.88\text{ L}}\right) = 1.37\text{ MJ}$$

Here we set $V_2 = 445,000\text{ L}$ because although the final volume includes both the larger chamber and the original cylinder, the final volume is much larger.

ASSESS This is the work we could have extracted from a reversible isothermal expansion. By letting the gas undergo an irreversible process, we gave up the possibility of extracting this work.

29. INTERPRET This problem asks about the loss of energy quality during an irreversible and therefore entropy-increasing process—namely, an adiabatic free expansion.

DEVELOP During an irreversible process where the entropy of a system increases by ΔS , energy $E = T_{\min}\Delta S$ becomes unavailable to do work, where T_{\min} is the lowest temperature available to the system. During an adiabatic free expansion, with the temperature constant, the entropy change is given by

$$\Delta S = nR \ln\left(\frac{V_2}{V_1}\right)$$

Our plan is to express ΔS in terms of the two vessel volumes, multiply by T_{\min} to obtain that unavailable energy, and solve for the fuel tank's volume V_1 .

EVALUATE Because the temperature doesn't change, T_{\min} is the temperature we're given. Solving for the initial volume from the expression for the unavailable energy we obtain

$$\begin{aligned} E = T\Delta S &= nRT \ln\left(\frac{V_2}{V_1}\right) \\ e^{\frac{E}{nRT}} &= \left(\frac{V_2}{V_1}\right) \\ V_1 &= V_2 e^{-\frac{E}{nRT}} \end{aligned}$$

Which after plugging in the given values, results in 122 L for the fuel tank's volume.

ASSESS We see from this expression that the initial and final volumes are proportional to each other and related via an exponential quantity dependent on the unavailable energy, temperature, and amount of gas.

- 30. INTERPRET** This problem asks about the loss of energy quality during an irreversible and therefore entropy-increasing process—namely, an adiabatic free expansion.

DEVELOP During an irreversible process where the entropy of a system increases by ΔS , energy $E = T_{\min} \Delta S$ becomes unavailable to do work, where T_{\min} is the lowest temperature available to the system. During an adiabatic free expansion, with the temperature constant, the entropy change is given by

$$\Delta S = nR \ln \left(\frac{V_2}{V_1} \right)$$

Our plan is to calculate ΔS and multiply by T_{\min} to find that unavailable energy.

EVALUATE Because the temperature doesn't change, T_{\min} is the temperature we're given, and we find the unavailable energy is equal to

$$E = T \Delta S = nRT \ln \left(\frac{V_2}{V_1} \right)$$

Here we set $V_2 = 2V_1$ because the final volume includes both identical cylinders, and obtain an expression for the energy that becomes unavailable to do work equal to $W_{\text{unavailable}} = nRT \ln 2$.

ASSESS This is the work we could have extracted from a reversible isothermal expansion. By letting the gas undergo an irreversible process, we gave up the possibility of extracting this work.

- 31. INTERPRET** This problem asks about the loss of energy quality during an irreversible and therefore entropy-increasing process—namely, an adiabatic free expansion.

DEVELOP During an irreversible process where the entropy of a system increases by ΔS , energy $E = T_{\min} \Delta S$ becomes unavailable to do work, where T_{\min} is the lowest temperature available to the system. During an adiabatic free expansion, with the temperature constant, the entropy change is given by

$$\Delta S = nR \ln \left(\frac{V_2}{V_1} \right)$$

Our plan is to express ΔS in terms of the two vessel volumes, multiply by T_{\min} to obtain that unavailable energy, and solve for volume of the second cylinder. We will write $V_2 = V_1 + V_t$, where V_t is the second cylinder volume we are after.

EVALUATE Because the temperature doesn't change, T_{\min} is the temperature we're given. Solving for the initial volume from the expression for the unavailable energy we obtain

$$E = T \Delta S = nRT \ln \left(\frac{V_1 + V_t}{V_1} \right)$$

$$e^{\frac{E}{nRT}} = \left(\frac{V_1 + V_t}{V_1} \right)$$

$$V_t = V_1 \left(e^{\frac{E}{nRT}} - 1 \right)$$

Here we recognize that $nRT = P_1 V_1$, and thus our final expression for the second cylinder volume is equal to

$$V_t = V_1 \left(e^{\frac{E}{P_1 V_1}} - 1 \right)$$

Which after plugging in the given values, results in 16.08 L.

ASSESS We see from this expression that the initial and final volumes are proportional to each other and related via an exponential quantity dependent on the unavailable energy, as well as the pressure and volume of the first chamber.

PROBLEMS

- 32. INTERPRET** This problem is about a Carnot engine, its work, efficiency, and power output.

DEVELOP For a cyclic operation, the change in internal energy is zero, $\Delta U = 0$. From the first law of thermodynamics, we have $W = Q_h - Q_c$. Once the work W is known, its efficiency can be obtained as $e = W/Q_h$. For a Carnot engine, $Q_c/Q_h = T_c/T_h$.

EVALUATE (a) The work done by the engine during each cycle is

$$W = Q_h - Q_c = 745 \text{ J} - 458 \text{ J} = 287 \text{ J}$$

(b) The efficiency of the engine is

$$e = \frac{W}{Q_h} = \frac{287 \text{ J}}{745 \text{ J}} = 38.5\%$$

(c) For a Carnot engine,

$$T_c = \left(\frac{Q_c}{Q_h} \right) T_h = \left(\frac{458 \text{ J}}{745 \text{ J}} \right) (592 \text{ K}) = 364 \text{ K}$$

(d) The work from (a) is per cycle, so the mechanical power output of the engine is this divided by the cycle period, which is equivalent to multiplying by the frequency ($f = 1/T$):

$$P_{\text{out}} = \frac{W}{T} = Wf = (287 \text{ J/cycle})(18.6 \text{ cycles/s}) = 5.34 \text{ kW}$$

ASSESS The cool reservoir temperature is cooler than the hot reservoir temperature ($T_c < T_h$), as expected. And the efficiency can be verified by using Equation 19.3: $e = 1 - T_c/T_h = 38.5\%$.

- 33. INTERPRET** This problem requires us to find the thermodynamic efficiency of a nuclear power plant in winter and in summer, when the temperature of its cold reservoir is 0°C and 25°C , respectively.

DEVELOP From Equation 19.3, the thermodynamic efficiency of a Carnot engine is

$$e_{\text{Carnot}} = 1 - \frac{T_c}{T_h}$$

where the temperatures are in Kelvin.

EVALUATE Inserting the given temperatures for summer and winter gives

$$e_{\text{summer}} = 1 - \frac{298 \text{ K}}{570 \text{ K}} = 47.7\%$$

$$e_{\text{winter}} = 1 - \frac{273 \text{ K}}{570 \text{ K}} = 52.1\%$$

ASSESS The plant is more efficient in winter than in summer because there is a greater heat difference. However, as explained in Section 19.3, irreversible processes, transmission losses, and so on make actual efficiencies less than the theoretical maxima.

- 34. INTERPRET** You want to calculate the minimum coefficient of performance for a heat pump that's to be installed in a new house.

DEVELOP For a heat pump, what you want is heat, and what you put in is work in the form of electricity, so $\text{COP} = Q_h / W$. In this case, you are dealing with rates, that is, the rate that at which house needs to be heated, H , and the electric power, P , that the solar voltaic system supplies to the heat pump. The coefficient of performance can, therefore, be written as $\text{COP} = H/P$.

EVALUATE The minimum COP needed to keep the house warm on a cold day is

$$\text{COP} = \frac{H}{P} = \frac{6.45 \text{ kW}}{2.24 \text{ kW}} = 2.88$$

ASSESS This is entirely reasonable for a heat pump. It might seem impossible to obtain more heat energy than you put in, but the heat pump is not generating heat but only "moving" it from outside to inside (see Fig. 19.12).

- 35. INTERPRET** This problem involves a nuclear power plant and asks us to calculate the rate of energy extraction, the efficiency, and the highest temperature the plant attains.

DEVELOP From Equation 16.3, $Q = mc\Delta T$, we see that to raise the temperature of the cooling water by 8.5 K, heat must be exhausted to it at a rate of

$$\frac{dQ_c}{dt} = c \left(\frac{dm}{dt} \right) \Delta T = [4.184 \text{ kJ/(kg} \cdot \text{K)}] (2.8 \times 10^4 \text{ kg/s}) (8.5 \text{ K}) = 996 \text{ MW}$$

We take this to be the rate of all the heat rejected by the power plant. Since the rate of work output dW/dt is also given, the heat input to the plant (extracted from its fuel) is

$$\frac{dQ_h}{dt} = \frac{dQ_c}{dt} + \frac{dW}{dt}$$

where we have used the first law of thermodynamics (see Problem 19.14). In terms of the rates, the efficiency of the plant is

$$e = \frac{dW/dt}{dQ_h/dt}$$

If we consider the plant to operate like a Carnot engine, then its highest temperature can be calculated using $Q_c/Q_h = T_c/T_h$ (from Equation 19.2).

EVALUATE (a) Substituting the values given, we obtain

$$\frac{dQ_h}{dt} = \frac{dQ_c}{dt} + \frac{dW}{dt} = 996 \text{ MW} + 750 \text{ MW} = 1.75 \text{ GW}$$

(b) The plant's efficiency (from the definition of efficiency in terms of rates) is

$$e = \frac{dW/dt}{dQ_h/dt} = \frac{750 \text{ MW}}{1.75 \text{ GW}} = 43.0\%$$

(c) With the assumption that the plant operates like an ideal Carnot engine, then

$$\frac{T_h}{T_c} = \frac{Q_h}{Q_c} = \frac{dQ_h/dt}{dQ_c/dt} = \frac{1.75 \text{ GW}}{996 \text{ MW}} = 1.75$$

(Note that the energy rate per cycle and the energy rate per second are proportional.) If $T_c = 15^\circ\text{C} = 288 \text{ K}$, then

$$T_h = 1.75T_c = 1.75(288 \text{ K}) = 505 \text{ K} = 232^\circ\text{C}$$

ASSESS The actual highest temperature would be somewhat greater than this, because the actual efficiency is always less than the Carnot efficiency.

- 36. INTERPRET** This problem requires us to calculate the efficiency of an electrical power plant, given the temperature of its hot and cold reservoirs. From this, we are to find the power discharged as waste heat and the number of houses we could heat with this waste heat.

DEVELOP The maximum efficiency of the plant as a function of the temperature of its hot and cold reservoirs is given by Equation 19.3

$$e_{\text{Carnot}} = 1 - \frac{T_c}{T_h}$$

which we are to compare with the actual efficiency which is

$$e = \frac{W}{Q_h} = 0.29$$

The actual efficiency may also be expressed as $e = 1 - Q_c/Q_h$ (Equation 19.1). In this chapter, W is defined as the work done *by* the system (as compared to W in the first law of thermodynamics, Equation 17.1, which is the work done *on* the system). Thus, W is equal to the net heat,

$$W = Q_h - Q_c \Rightarrow Q_h = Q_c + W$$

where Q_c is the heat rejected by the system to the cold reservoir and Q_h is the heat absorbed by the system from the hot reservoir. Using this result to eliminate Q_h in the expression for the plant's actual efficiency gives

$$Q_c = W \frac{(1-e)}{e}$$

Differentiating this with respect to time gives the waste power dQ_c/dt in terms of the power output $dW/dt = 800 \text{ MW}$ and the actual efficiency of the plant e :

$$\frac{dQ_c}{dt} = \frac{dW}{dt} \left(\frac{1-e}{e} \right)$$

Finally, the number of houses that can be warmed with this waste power is simply the waste power divided by the power requirement of a single house, or

$$N_{\text{houses}} = \frac{dQ_c/dt}{P_{\text{house}}}$$

where $P_{\text{house}} = 23 \text{ kW}$.

EVALUATE (a) The maximum possible efficiency for the power plant is

$$e_{\text{Carnot}} = 1 - \frac{T_c}{T_h} = 1 - \frac{273 \text{ K} + 30^\circ\text{C}}{273 \text{ K} + 280^\circ\text{C}} = 1 - \frac{303 \text{ K}}{553 \text{ K}} = 45.2\% > e$$

(b) The waste power is

$$\frac{dQ_c}{dt} = \frac{dW}{dt} \left(\frac{1-e}{e} \right) = (800 \text{ MW}) \left(\frac{1-0.29}{0.29} \right) = 2.2 \text{ GW}$$

(c) The number houses that could be heated by the waste power is

$$N_{\text{houses}} = \frac{dQ_c/dt}{P_{\text{house}}} = \frac{2.2 \times 10^9 \text{ W}}{23 \times 10^3 \text{ W}} = 9.4 \times 10^4$$

ASSESS As expected, the maximum efficiency e_{Carnot} is greater than the actual efficiency e .

37. INTERPRET This problem asks us to find the rate (i.e., kg/s) at which all the power plants in the United States use cooling water. We are given the actual efficiency of the power plants and the temperature rise in the cooling water.

DEVELOP For a cyclic operation, the change in internal energy is zero, $\Delta E_{\text{int}} = 0$. From the first law of thermodynamics, we have $W = Q_h - Q_c$, where W is the work done by the system (contrary to the definition of W in Chapter 18), Q_h is the heat absorbed by the system, and Q_c is the heat rejected by the system. Therefore, the total rate at which heat is exhausted by all the power plants is

$$\frac{dQ_c}{dt} = \frac{d}{dt}(Q_h - W) = \frac{dW}{dt} \left(\frac{1}{e} - 1 \right) = (2 \times 10^{11} \text{ W}) \left(\frac{1}{33\%} - 1 \right) = 4.06 \times 10^{11} \text{ W}$$

The mass rate of flow at which water could absorb this amount of energy, with only a 5°C temperature rise, is

$$\frac{dQ_c}{dt} = \frac{dm}{dt} c_{\text{water}} \Delta T$$

where $c_{\text{water}} = 4184 \text{ J/(kg}\cdot\text{K)}$ (see Table 16.1). This equation can be solved to give the mass rate of cooling water used.

EVALUATE Solving the equation above for dm/dt , we obtain

$$\frac{dm}{dt} = \frac{dQ_c}{dt} c_{\text{water}} \Delta T = \frac{4.06 \times 10^{11} \text{ W}}{[4184 \text{ J/(kg}\cdot\text{K)}](5 \text{ K})} = 2 \times 10^7 \text{ kg/s}$$

or about 1 Mississippi (a self-explanatory unit of river flow).

ASSESS To absorb the power output of $2 \times 10^{11} \text{ W}$ with only an increase of temperature of 5°C , we expect the mass flow rate to be large.

38. INTERPRET For this problem, we are to show that the overall efficiency of a two-stage heat engine is the same as a single-stage engine operating between T_h and T_c of the two-stage engine.

DEVELOP Let the heat exhausted Q_i by the first engine equal the heat input to the second. Then,

$Q_h = W_1 + Q_i$ and $Q_i = W_2 + Q_c$, where Q_c is the heat exhausted to the environment. For a single-stage engine operating between T_h and T_c , the actual efficiency is the total work $W_1 + W_2$ divided by Q_h (Equation 19.1), which we can evaluate to compare with the Carnot efficiency of the two-stage engine, which is $e = 1 - T_c/T_h$ (Equation 19.3).

EVALUATE The actual efficiency of the single-stage engine is

$$e = \frac{W_1 + W_2}{Q_h} = \frac{Q_h - Q_i + Q_i - Q_c}{Q_h} = 1 - \frac{Q_c}{Q_h} = 1 - \frac{T_c}{T_h}$$

which is the Carnot efficiency of the two-stage engine.

ASSESS In the last equality, we used Equation 19.2, $T_c/T_h = Q_c/Q_h$.

- 39. INTERPRET** This problem involves finding the COP of a freezer for which the highest and the lowest temperatures are $T_h = 35^\circ\text{C} = 308\text{ K}$ and $T_c = -18^\circ\text{C} = 255\text{ K}$, respectively. In addition, we are to find how much water at 0°C the freezer can freeze in 1 hour.

DEVELOP The coefficient of performance (COP) of a reversible freezer is given by Equation 19.4:

$$\text{COP} = \frac{Q_c}{W} = \frac{Q_c}{Q_h - Q_c} = \frac{T_c}{T_h - T_c}$$

where we have used Equation 19.2, $T_c/T_h = Q_c/Q_h$ for the last equality. Once the COP is known, we can solve for Q_c and the amount of water the freezer can freeze in 1 hour, which is $m = Q_c/L_f$ with $L_f = 334\text{ kJ/kg}$ (see Equation 17.5 and Table 17.1).

EVALUATE (a) The COP of the freezer is

$$\text{COP} = \frac{T_c}{T_h - T_c} = \frac{255\text{ K}}{308\text{ K} - 255\text{ K}} = 4.81$$

(b) The heat rejected in 1 hour is $Q_c = \text{COP} \times W = 4.81 \times (26.4\text{ kWh}) = 457\text{ MJ}$, so the water we can freeze is

$$m = \frac{Q_c}{L_f} = \frac{457\text{ MJ}}{334\text{ kJ/kg}} = 1.37 \times 10^3\text{ kg}$$

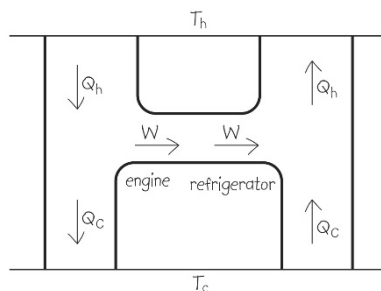
also written as 1.37 tonnes.

ASSESS The higher the freezer's COP, the more electrical energy is needed to freeze the same amount of water.

- 40. INTERPRET** We are to use an energy-flow diagram to analyze whether using a refrigerator to cool the low-temperature reservoir of a Carnot can increase its overall efficiency.

DEVELOP In order to lower the temperature of the cold reservoir, the combination must remove more heat from the cold reservoir than it puts into the reservoir. Since both the engine and the refrigerator are at the Carnot efficiency, they are reversible.

EVALUATE As shown in the figure below, the *best* that this can do is zero work output while the temperatures stay the same. If the temperature of the cold reservoir is to become lower, then the heat extracted from the cold reservoir must be larger, which will require more work than the engine produces.



ASSESS Might as well do nothing at all!

- 41. INTERPRET** This problem requires us to find the monthly cost of using all the incoming electrical energy to power a heat pump with $\text{COP} = 3.4$ to heat a house. We are given that the electrical energy costs \$230 per month in the winter.

DEVELOP The same electrical energy W used for direct conversion in electric heating would produce heat $Q_h = W + Q_c$. Using Equation 19.4 allows us to express this as

$$Q_h = W + Q_c = W + Q_h (1 - 1/\text{COP})$$

$$Q_h = W (\text{COP})$$

Thus, the heat pump can produce a factor COP more heat than if the electrical energy is converted directly to heat.

EVALUATE Because the heat pump is a factor COP more efficient, the cost will be reduced by this same factor, so the monthly heating bill would be

$$\frac{\$230}{\text{COP}} = \frac{\$230}{3.4} = \$68$$

ASSESS The savings are significant, which is why electrical heating is not recommended.

- 42. INTERPRET** This problem requires us to find the power needed to run a refrigerator that leaks heat at the given rate to its cold reservoir (i.e., the environment). The refrigerator operates as a reversible heat engine, so we know its COP.

DEVELOP The rate at which heat leaks from the refrigerator is $dQ_c/dt = 320 \text{ W}$. For a reversible refrigerator, the COP is given by Equation 19.4:

$$\text{COP}_{\text{refrigerator}} = \frac{Q_c}{W} = \frac{T_c}{T_h - T_c} = \frac{276 \text{ K}}{27 \text{ K}} = 10.22$$

$$W = \frac{Q_c}{10.22}$$

Differentiating this expression with respect to time gives the power needed to run the refrigerator.

EVALUATE The power needed to run the refrigerator is

$$\frac{dW}{dt} = \frac{dQ_c}{dt} \left(\frac{1}{10.22} \right) = \frac{320 \text{ W}}{10.22} = 31 \text{ W}$$

ASSESS The cooling capacity of the refrigerator is not given, so 320 W is the minimum heat leakage, which means that this power is the minimum power required to run the refrigerator.

- 43. INTERPRET** We are to find the minimum COP required to save money if we switch from an gas furnace to an electrically powered heat pump, considering the cost of gas and of electricity. We will do this by calculating the cost of the heat delivered by both the oil burning heater and the electric heat pump.

DEVELOP The coefficient of performance (COP) is the relationship between the heat sent to the cold reservoir and the work done. Set the heat Q_c to be the same for both heating mechanisms, and solve for COP. The cost of gas is $\$_{\text{gas}} = \frac{\$1.28}{24.4 \text{ kWh}} = \0.0525 kWh^{-1} , and the cost of electricity is $\$_{\text{electric}} = \0.146 kWh^{-1} . The heat delivered is

$$Q_c = W \times \text{COP}$$

(see derivation of Equation 19.4), and we are paying for the work done, so the COP must exceed the ratio of the costs ($\text{COP} > \$_{\text{electric}}/\$_{\text{gas}}$).

EVALUATE The COP must satisfy

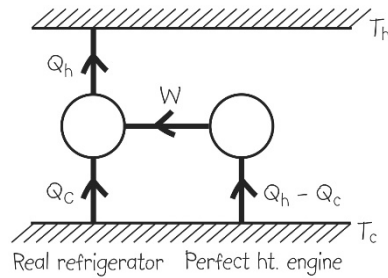
$$\text{COP} > \$_{\text{electric}}/\$_{\text{gas}} = 2.78$$

So in order to be cost-effective, the heat pump must have a COP of greater than 2.78.

ASSESS Most heat pumps have a COP much higher than this value, so it's probably a good idea to switch.

- 44. INTERPRET** For this problem, we are to show that the Clausius statement (i.e., the second law of thermodynamics) would be violated by the existence of a perfect heat engine, which would allow the construction of a perfect refrigerator.

DEVELOP If it were possible to construct a perfect heat engine (one which would extract heat and perform an equivalent amount of work), then it could be coupled to a real refrigerator in such a way that the work output of the engine equals the work input to the refrigerator, as shown in the figure below.



EVALUATE The net effect of this arrangement is to produce a perfect refrigerator (a cyclic device whose sole effect is the transfer of heat, $Q_c + Q_h - Q_c = Q_h$, from a cold reservoir to a hot one), in violation of the Clausius statement of the second law.

ASSESS This completes the proof of the equivalence of the Kelvin-Planck and Clausius statements in Section 19.2.

- 45. INTERPRET** We are asked to find the power usage, and operating cost compared to that of a gas burning heater of a heat pump. We will assume that the heat pump is a Carnot heat pump.

DEVELOP The maximum COP of a heat pump (when its heating, not cooling) is given in Equation 19.4b:

$\text{COP} = T_h / (T_h - T_c)$. In this case, $T_c = -7^\circ\text{C} = 266\text{ K}$ and $T_h = 48^\circ\text{C} = 321\text{ K}$. In theory, the COP for a heat pump is the heat supplied divided by the work input. In terms of rates, that can be written as $\text{COP} = H / P$, where H is the supplied heat rate and P is the electric power consumption.

EVALUATE (a) The power consumption needed to supply heat at 18.8 kW is

$$P = \frac{H}{\text{COP}} = \frac{18.8\text{ kW}}{2.72} = 6.91\text{ kW}$$

(b) Given the utility rate for electric power, the heat pump's hourly operating cost is

$$C_{\text{pump}} = (Pt)R_{\text{utility}} = (6.91\text{ kW})(1\text{h})(11.4\text{¢ / kWh}) = 78.8\text{¢}$$

In comparison, an oil furnace, supplying the same heat, would have an hourly operating cost of

$$C_{\text{gas}} = \frac{18.8\text{ kW}(1\text{h})}{25.3\text{ kWh/CCF}}(\$1.28\text{ / CCF}) = \$0.95$$

Multiplying these hourly costs by 24 to determine the cost per day we obtain \$22.83/day for gas, and \$18.91/day for electricity.

(c) To compare the actual COP to the theoretical maximum COP we calculate the maximal efficiency using Equation 19.4b

$$\text{COP} = \frac{T_h}{T_h - T_c} = \frac{321\text{ K}}{321\text{ K} - 266\text{ K}} = 5.83$$

Making the actual COP 46.6% of the theoretical maximum.

ASSESS The large difference between the actual and theoretical COP will change with the seasons, since it is dependent on the indoor and outdoor temperatures.

- 46. INTERPRET** This problem involves a reversible engine that contains a given volume of monatomic gas. The system goes through the four thermodynamics processes indicated in the problem statement, and we are to find net work done by the system and the net heat added to the system over a complete cycle, and the engine's efficiency as defined by the ratio of the work done by the engine to the heat absorbed over a complete cycle.

DEVELOP The pV diagram for the cycle is as shown below. Let us calculate the work and heat absorbed for each stage of the cycle. For the isothermal expansion AB the change in internal energy of the system is zero ($\Delta U = 0$), so the first law of thermodynamics gives

$$Q_{AB} = W_{AB} = nRT_A \ln(V_B / V_A) = (0.35\text{ mol})[8.314\text{ J / (mol} \cdot \text{K)}](586\text{ K}) \ln\left(\frac{4.84\text{ L}}{2.42\text{ L}}\right) = 1182\text{ J}$$

where Q_{AB} is the heat absorbed and W_{AB} is the work done by the system (note that this definition of W is contrary to the definition used in Chapter 18). The second equality above is from Equation 18.4. For the isovolumic cooling BC , the volume does not change so no work is done. The heat absorbed is given by Equations 18.5 and 18.13, which give

$$W_{BC} = 0 \text{ and}$$

$$Q_{BC} = nC_V \Delta T_{BC} = \frac{3}{2}nR(T_C - T_B) = \frac{3}{2}(0.35 \text{ mol})[8.314 \text{ J/(mol} \cdot \text{K)}](292 \text{ K} - 586 \text{ K}) = -1283.3 \text{ J}$$

For the isothermal compression CD ,

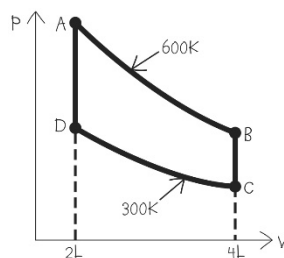
$$Q_{CD} = W_{CD} = nRT_C \ln(V_D/V_C) = (0.35 \text{ mol})[8.314 \text{ J/(mol} \cdot \text{K)}](292 \text{ K}) \ln\left(\frac{2.42 \text{ L}}{4.84 \text{ L}}\right) = -589 \text{ J}$$

and for the isovolumic heating DA ,

$$W_{DA} = 0 \text{ and}$$

$$Q_{DA} = \frac{3}{2}nR(T_A - T_D) = -Q_{BC}$$

For these processes, we are given that $V_B = 2V_A = V_C = 2V_D = 4.84 \text{ L}$, $T_A = T_B = 586 \text{ K}$, and $T_C = T_D = 292 \text{ K}$, so we can sum up the contributions to work and heat to find the total for each for a complete cycle.



EVALUATE (a) The net heat added to the system is

$$Q_{ABCD} = Q_{AB} + \overbrace{Q_{BC} + Q_{DA}}^{=0} + Q_{CD} = 1182 \text{ J} + (-589 \text{ J}) = 593 \text{ J}$$

The net work done by the system is

$$W_{ABCD} = W_{AB} + \overbrace{W_{BC} + W_{DA}}^{=0} + W_{CD} = 1182 \text{ J} + (-589 \text{ J}) = 593 \text{ J}$$

(b) The ratio of the work done to the heat absorbed is

$$\frac{W_{ABCD}}{Q_{AB} + Q_{DA}} = \frac{593 \text{ J}}{1182 \text{ J} + 1283 \text{ J}} = 24.0\%$$

ASSESS A Carnot engine operating between 586 K and 292 K has efficiency $1 - 292/586 = 50.2\%$. This is not a contradiction of Carnot's theorem, because the engine in this problem does *not* absorb and exhaust heat at constant temperatures.

- 47. INTERPRET** Our engine cycle consists of four paths, two of which are isochoric and two of which are isobaric. We are to determine the efficiency, defined as the work done per unit heat absorbed, and compare the result with the efficiency of a Carnot engine operating between the same temperatures. Finally, we need to explain any difference between the two efficiencies.

DEVELOP Label the states in Fig. 19.22 A, B, C, and D going clockwise from the upper left corner. The work done and the heat absorbed during the isobaric segments AB and CD are

$$W_{AB} = p_A(V_B - V_A) = (6 \text{ atm})(6 \text{ L} - 2 \text{ L}) = 24 \text{ L} \cdot \text{atm}$$

$$W_{CD} = p_C(V_D - V_C) = (3 \text{ atm})(2 \text{ L} - 6 \text{ L}) = -12 \text{ L} \cdot \text{atm}$$

and

$$Q_{AB} = nC_P(T_B - T_A) = n \left(\frac{5}{2} R \right) \left(\frac{p_B V_B}{nR} - \frac{p_A V_A}{nR} \right) = \frac{5}{2} (36 - 12) \text{ L} \cdot \text{atm} = 60 \text{ L} \cdot \text{atm}$$

$$Q_{CD} = nC_P(T_D - T_C) = \frac{5}{2} (p_D V_D - p_C V_C) = \frac{5}{2} (6 - 18) \text{ L} \cdot \text{atm} = -30 \text{ L} \cdot \text{atm}$$

where we have assumed an ideal monatomic gas (see Equation 18.13).

For the isochoric segments, we have

$$Q_{BC} = nC_V(T_C - T_B) = \frac{3}{2} (18 - 36) \text{ L} \cdot \text{atm} = -27 \text{ L} \cdot \text{atm}$$

$$Q_{DA} = nC_V(T_A - T_D) = \frac{3}{2} (12 - 6) \text{ L} \cdot \text{atm} = 9 \text{ L} \cdot \text{atm}$$

and $W_{BC} = W_{DA} = 0$. The net heat added for one cycle is therefore

$$\begin{aligned} Q_{ABCD} &= Q_{AB} + Q_{BC} + Q_{CD} + Q_{DA} = (60 - 27 - 30 + 9) \text{ L} \cdot \text{atm} = (12 \text{ L} \cdot \text{atm})(101.3 \text{ J/L} \cdot \text{atm}) \\ &= 1.22 \text{ kJ} \end{aligned}$$

and the net work done is $W = (24 + 0 - 12 + 0) \text{ L} \cdot \text{atm} = 12 \text{ L} \cdot \text{atm} = 1.22 \text{ kJ}$. Note that the first law of thermodynamics, applied to a cyclic process, requires that $W = Q$ when using the definition that W is the work done by the system (which is opposite to the definition used in Chapter 18).

EVALUATE (a) Since the heat absorbed is $Q_+ = (60 + 9) \text{ L} \cdot \text{atm} = 69 \text{ L} \cdot \text{atm}$, the efficiency is

$$e = \frac{W}{Q_+} = \frac{12 \text{ L} \cdot \text{atm}}{69 \text{ L} \cdot \text{atm}} = 17.4\%$$

(b) The maximum and minimum temperatures are $T_B = p_B V_B / nR$ and $T_D = p_D V_D / nR$ so the efficiency of a Carnot engine operating between these temperatures is

$$e_{\text{Carnot}} = 1 - \frac{T_D}{T_B} = 1 - \frac{p_D V_D}{p_B V_B} = 1 - \frac{6 \text{ L} \cdot \text{atm}}{36 \text{ L} \cdot \text{atm}} = 83.3\%$$

This is not a contradiction of Carnot's theorem, because the given engine does not operate between two heat reservoirs at fixed temperatures.

ASSESS The efficiency of a real engine is always less or equal to that of a Carnot engine.

- 48. INTERPRET** For the given Carnot cycle, we are to find the heat absorbed, the heat rejected, and the work done per cycle. We are then to find the efficiency of the engine and the maximum and minimum temperatures, and show that the efficiency as given in Equation 19.1 equates to the Carnot efficiency of Equation 19.3.

DEVELOP See Fig. 19.5 and the accompanying discussion of the Carnot engine. The heat absorbed is in the isothermal expansion is

$$Q_h = nRT_A \ln \left(\frac{V_B}{V_A} \right) = P_A V_A \ln \left(\frac{V_B}{V_A} \right)$$

where we have used the ideal gas law (Equation 17.2) $pV = nRT$ for the last equality, state $A = (8.000 \text{ atm}, 1.000 \text{ L})$, and state $B = (4.000 \text{ atm}, 2.000 \text{ L})$. The heat rejected during isothermal compression is

$$Q_c = P_C V_C \ln \left(\frac{V_C}{V_D} \right)$$

where state $C = (2.050 \text{ atm}, 3.224 \text{ L})$ and state $D = (4.100 \text{ atm}, 1.612 \text{ L})$. Because the internal energy of the engine does not change, $\Delta U = 0$, so the first law of thermodynamics states that

$$W = Q_h - Q_c$$

where W is the work done by the system. From these results, we can calculate the efficiency using Equation 19.1, $e = W/Q_h$. The maximum and minimum temperatures may be found from the ideal gas law (Equation 17.2),

$$T_A = P_A V_A / nR \text{ and } T_C = P_C V_C / nR$$

EVALUATE (a) The heat absorbed is

$$Q_h = p_A V_A \ln \left(\frac{V_B}{V_A} \right) = (8.000 \text{ atm})(1.000 \text{ L}) [8.314 \text{ J/(mol} \cdot \text{K)}] \ln(2) \left(\frac{101.3 \text{ J}}{\text{atm} \cdot \text{L}} \right) = 561.7 \text{ J}$$

(b) The heat rejected is

$$Q_c = p_C V_C \ln \left(\frac{V_C}{V_D} \right) = (2.050 \text{ atm})(3.224 \text{ L}) [8.314 \text{ J/(mol} \cdot \text{K)}] \ln \left(\frac{3.224 \text{ L}}{1.612 \text{ L}} \right) \left(\frac{101.3 \text{ J}}{\text{atm} \cdot \text{L}} \right) = 464.1 \text{ J}$$

(c) The work done by the engine is

$$W = Q_h - Q_c = 561.7 \text{ J} - 464.1 \text{ J} = 97.66 \text{ J}$$

(d) The efficiency is

$$e = \frac{W}{Q_h} = \frac{97.66 \text{ J}}{561.7 \text{ J}} = 17.39\%$$

(e) The maximum and minimum temperatures are

$$T_h = T_A = \frac{p_A V_A}{nR} = \frac{(8.000 \text{ atm})(1.000 \text{ L})}{(0.20 \text{ mol}) [8.314 \text{ J/(mol} \cdot \text{K)}]} = 487.4 \text{ K} = 490 \text{ K}$$

$$T_c = T_C = \frac{p_C V_C}{nR} = \frac{(2.050 \text{ atm})(3.224 \text{ L})}{(0.20 \text{ mol}) [8.314 \text{ J/(mol} \cdot \text{K)}]} = 402.6 \text{ K} = 400 \text{ K}$$

to two significant figures. The Carnot efficiency is thus

$$e_{\text{Carnot}} = 1 - \frac{T_c}{T_h} = 1 - \frac{402.6 \text{ K}}{487.4 \text{ K}} = 17.39\%$$

which is the same result as for part (d).

ASSESS For a Carnot engine, the actual efficiency is the Carnot efficiency.

These imply a Carnot efficiency of $e = 1 - T_c/T_h = 0.1739$, exactly as before. Equations 19.1 and 19.3 are identical because $Q_c/Q_h = T_c/T_h = 0.8261$, explicitly.

49. INTERPRET This problem involves the increase in entropy as the ice is melted and heated up.

DEVELOP The entropy increase is given by Equation 19.6: $\Delta S_{12} = \int_1^2 dQ/T$. We consider the entropy increase in two steps. First, the heat needed to melt the pond ice is $Q = mL_f$, where $L_f = 334 \text{ kJ/kg}$ from Table 17.1. The temperature is constant during the melting, $T_0 = 273 \text{ K}$. In the second step, the heat input raises the water temperature according to $dQ = mc dT$, where $c = 4.184 \text{ kJ/(kg} \cdot \text{K)}$ from Table 16.1. Here, the temperature is not constant, so we will have to integrate.

EVALUATE The entropy increase during melting is

$$\Delta S_{\text{melt}} = \int_1^2 dQ/T = \frac{mL_f}{T_0} = \frac{(85 \text{ Mg})(334 \text{ kJ/kg})}{273 \text{ K}} = 104 \text{ MJ/K}$$

The entropy increase during warming is

$$\Delta S_{\text{warm}} = \int_{T_0}^{T_1} \frac{mc dT}{T} = mc \ln \left(\frac{T_1}{T_0} \right) = (85 \text{ Mg}) [4.184 \text{ kJ/(kg} \cdot \text{K)}] \ln \left(\frac{293 \text{ K}}{273 \text{ K}} \right) = 25.1 \text{ MJ/K}$$

So the total entropy increase is

$$\Delta S_{\text{tot}} = \Delta S_{\text{melt}} + \Delta S_{\text{warm}} = 104 \text{ MJ/K} + 25.1 \text{ MJ/K} = 129 \text{ MJ/K} \approx 1.3 \times 10^2 \text{ MJ/K}$$

ASSESS As expected, the entropy change is positive in both melting and warming processes.

50. INTERPRET You want to know the rate at which your body's entropy increases during normal metabolism.

DEVELOP The normal calorie intake is about 2000 kcal/day for women and about 2500 kcal/day for men. We'll split the difference and assume a metabolic rate of 2250 kcal/day. Temperature of metabolism rate (or basal

metabolic rate) for males is about 1300 kcal/day and for females is about 1200 kcal/day. Q flows into the body as heat. Therefore, the entropy change from state 1 (burger ingested) and state 2 (burger metabolized) is given by Equation 19.6: $\Delta S_{12} = \int_1^2 dQ/T$.

EVALUATE The body consumes food energy and converts it to work and heat at a constant body temperature of $T = 37^\circ\text{C} = 310\text{ K}$, so the rate of entropy increase is

$$\frac{\Delta S}{\Delta t} = \frac{1}{\Delta t} \int_1^2 \frac{dQ}{T} = \frac{\Delta Q / \Delta t}{T} = \frac{2250 \text{ kcal/day} \left(\frac{4.184 \text{ kJ}}{1 \text{ kcal}} \right)}{310 \text{ K}} \approx 0.4 \text{ mW/K}$$

ASSESS One could say this is the rate at which we create disorder.

- 51. INTERPRET** We are to derive the formula given in the problem statement that describes the entropy change for n moles of an ideal gas that undergoes an isovolumic temperature change from T_1 to T_2 .

DEVELOP From the first law of thermodynamics ($dQ = dU + dW$) and the properties of an ideal gas ($dU = nC_V dT$ and $pV = nRT$), an infinitesimal entropy change is

$$dS = \frac{dQ}{T} = nC_V \frac{dT}{T} + \frac{p}{T} dV = nC_V \frac{dT}{T} + nR \frac{dV}{V}$$

Integrate from state 1 (T_1, V_1) to state 2 (T_2, V_2), and apply the isovolumic constraint to obtain the given formula.

EVALUATE Integrating the expression above gives

$$\Delta S = nC_V \ln\left(\frac{T_2}{T_1}\right) + nR \ln\left(\frac{V_2}{V_1}\right)$$

For an isovolumic process $V_1 = V_2$ so $\Delta S = nC_V \ln(T_2/T_1)$.

ASSESS Of course, we could have started with $dQ = nC_V dT$ at constant volume, but we wanted to display ΔS for a general ideal gas process, for use in other problems.

- 52. INTERPRET** We are to derive the formula given in the problem statement that describes the entropy change for n moles of an ideal gas that undergoes an isobaric temperature change from T_1 to T_2 .

DEVELOP From the first law of thermodynamics ($dQ = dE_{\text{int}} + dW$) and the properties of an ideal gas ($dE_{\text{int}} = nC_V dT$ and $PV = nRT$), an infinitesimal entropy change is

$$dS = \frac{dQ}{T} = nC_V \frac{dT}{T} + \frac{p}{T} dV = nC_V \frac{dT}{T} + nR \frac{dV}{V}$$

Integrate from state 1 (T_1, V_1) to state 2 (T_2, V_2), and use Equation 18.9, $C_p = C_V + R$, to obtain the given formula.

Note that when the pressure is constant, the ideal gas law gives $V_2/V_1 = T_2/T_1$.

EVALUATE Substituting the second equation into the first one yields

$$\Delta S = nC_V \ln\left(\frac{T_2}{T_1}\right) + nR \ln\left(\frac{V_2}{V_1}\right) = n(C_V + R) \ln\left(\frac{T_2}{T_1}\right) = nC_p \ln\left(\frac{T_2}{T_1}\right)$$

ASSESS The same expression can also be obtained by using $dQ = nC_p dT$ at constant pressure. Note that $\Delta S > 0$ if $T_2 > T_1$, as expected.

- 53. INTERPRET** This problem involves the entropy change in an ideal diatomic gas heated under three different conditions: constant volume, constant pressure, and adiabatically.

DEVELOP From Problem 19.51, the entropy change at constant volume is $\Delta S_V = nC_V \ln(T_2/T_1)$, where $C_V = 5R/2$ for a diatomic gas (see discussion after Equation 18.13). From Problem 19.52 the entropy change at constant pressure is

$$\Delta S_P = nC_P \ln(T_2/T_1) = n(C_V + R) \ln(T_2/T_1)$$

For an adiabatic process, consider the discussion accompanying Fig. 19.16. The entropy change is

$$\Delta S = nR \ln \left(\frac{V_2}{V_1} \right) = nR \ln \left[\left(\frac{T_1}{T_2} \right)^{\frac{1}{\gamma-1}} \right] = \frac{nR}{\gamma-1} \ln \left(\frac{T_1}{T_2} \right)$$

where we have used the relation for an adiabatic process $T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$ (Equation 18.11b). For a diatomic gas,

$$\gamma = \frac{C_P}{C_V} = \frac{C_V + R}{C_V} = \frac{5R/2 + R}{5R/2} = \frac{7}{5}$$

EVALUATE (a) At constant volume, the entropy change is

$$\Delta S_V = nC_V \ln(T_2 / T_1) = (6 \text{ mol}) \left(\frac{5}{2} \right) [8.314 \text{ J / (mol} \cdot \text{K)}] \ln \left(\frac{510 \text{ K}}{350 \text{ K}} \right) = 47 \text{ J/K}$$

(b) At constant pressure, the entropy change is

$$\Delta S_P = nC_P \ln(T_2 / T_1) = n(C_V + R) \ln(T_2 / T_1) = (6.00 \text{ mol}) \left(\frac{7}{2} \right) [8.314 \text{ J / (mol} \cdot \text{K)}] \ln \left(\frac{510 \text{ K}}{350 \text{ K}} \right) = 66 \text{ J/K}$$

(c) For an adiabatic process, $Q = 0$ (no heat flow between system and environment) and the entropy change is $\Delta S = 0$.

ASSESS The entropy change is greater in the isobaric (constant-pressure) process than in the isochoric (constant-volume) process since $C_P = C_V$.

54. INTERPRET This problem involves the entropy change that results from mixing the given amount of hot and cold water. We are to find the entropy change for the hot water, the cold water, and the entire system.

DEVELOP When mixing two liquids that are initially at different temperatures, the thermal energy change in both liquids must be the same, assuming all the thermal energy lost by the hot liquid is gained by the cold liquid. Thus,

$$\begin{aligned} \Delta Q_{\text{hot}} + \Delta Q_{\text{cold}} &= 0 \\ m_{\text{hot}} c_{\text{hot}} \Delta T_{\text{hot}} + m_{\text{cold}} c_{\text{cold}} \Delta T_{\text{cold}} &= 0 \end{aligned}$$

For this problem, $m_{\text{hot}} = m_{\text{cold}}$ and $c_{\text{hot}} = c_{\text{cold}}$, because the hot and cold liquids are both water. Thus,

$$\Delta T_{\text{hot}} = -\Delta T_{\text{cold}}$$

Because the equilibrium temperature of the mixed liquid is the same for the (previously) hot and cold portions, we also have

$$\begin{aligned} T_{\text{hot}} + \Delta T_{\text{hot}} &= T_{\text{cold}} + \Delta T_{\text{cold}} \\ T_{\text{hot}} - T_{\text{cold}} &= \Delta T_{\text{cold}} - \Delta T_{\text{hot}} \\ \Delta T_{\text{cold}} - \Delta T_{\text{hot}} &= T_{\text{hot}} - T_{\text{cold}} = 70 \text{ K} \end{aligned}$$

Combined with the previous expression gives $\Delta T_{\text{cold}} = -\Delta T_{\text{hot}} = 35 \text{ K}$ and the equilibrium temperature is $T_{\text{cold}} + \Delta T_{\text{cold}} = (10^\circ\text{C} + 273 \text{ K}) + 35 \text{ K} = 318 \text{ K}$. For this problem, the entropy change given by Equation 19.6 takes the form

$$\Delta S = \int_{T_1}^{T_2} \frac{dQ}{T}$$

From the argument above, we have $dQ = mcdT$, so Equation 19.6 gives

$$\Delta S = mc \int_{T_1}^{T_2} \frac{dT}{T} = mc \ln \left(\frac{T_2}{T_1} \right)$$

where $m = 0.250 \text{ kg}$ and $c = 4184 \text{ J/kg}$, so we can solve for the entropy change for each case.

EVALUATE (a) For the hot water, $T_1 = 353 \text{ K}$ and $T_2 = 318 \text{ K}$, so the entropy change is

$$\Delta S_{\text{hot}} = (0.250 \text{ kg}) [4184 \text{ J / (kg} \cdot \text{K)}] \ln \left(\frac{318 \text{ K}}{353 \text{ K}} \right) = -109 \text{ J/K}$$

(b) For the cold water, $T_1 = 283 \text{ K}$ and $T_2 = 318 \text{ K}$, so the entropy change is

$$\Delta S_{\text{cold}} = (0.250 \text{ kg})[4.184 \text{ kJ}/(\text{kg} \cdot \text{K})] \ln\left(\frac{318 \text{ K}}{283 \text{ K}}\right) = 122 \text{ J/K}$$

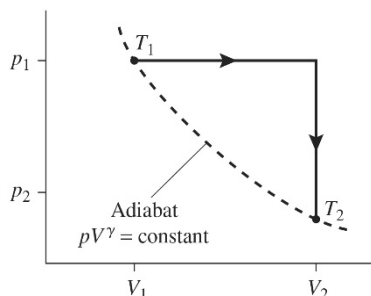
(c) The total entropy change is the sum of these two, so

$$\Delta S_{\text{tot}} = 121.97 \text{ J/K} - 109.22 \text{ J/K} = 12.7 \text{ J/K}$$

ASSESS Note that the results are given to three significant figures because we assumed the data were valid to three significant figures. The total entropy of the system has increased, which conforms with the second law of thermodynamics.

- 55. INTERPRET** You want to find the entropy change in going from one state to another state that lie on the same adiabat.

DEVELOP We're told that the system goes from state p_1, V_1 to state p_2, V_2 , where the two states are related by the adiabatic equation: $p_1 V_1^\gamma = p_2 V_2^\gamma$. Our first inclination would be that the entropy change would be zero, since there is no heat exchange in the adiabatic process that connects these two states. However, we're told that the system goes between the two states in two segments: one a constant-pressure process (going from p_1, V_1 to p_1, V_2) and the other a constant-volume process (going from p_1, V_2 to p_2, V_2). See the figure below.



This is an ideal gas, so the temperatures of the three endpoints are: $T_1 = p_1 V_1 / nR$, $T_{12} = p_1 V_2 / nR$, and $T_2 = p_2 V_2 / nR$. From Table 18.1, the differential heat flows for these two processes are:

$$\begin{aligned} dQ &= nC_p dT && \text{for constant pressure} \\ dQ &= nC_V dT && \text{for constant volume} \end{aligned}$$

Recall that $\gamma = C_p / C_V$.

EVALUATE For the constant pressure process, the entropy change is

$$\Delta S_p = \int_{T_1}^{T_{12}} \frac{nC_p dT}{T} = nC_p \ln\left(\frac{T_{12}}{T_1}\right) = nC_p \ln\left(\frac{V_2}{V_1}\right) = n\gamma C_V \ln\left(\frac{V_2}{V_1}\right)$$

For the constant-volume process, the entropy change is

$$\Delta S_V = \int_{T_{12}}^{T_2} \frac{nC_V dT}{T} = nC_V \ln\left(\frac{T_2}{T_{12}}\right) = nC_V \ln\left(\frac{p_2}{p_1}\right) = nC_V \ln\left(\frac{V_1^\gamma}{V_2^\gamma}\right)$$

The total entropy is the sum:

$$\Delta S = \Delta S_p + \Delta S_V = nC_V \left[\gamma \ln\left(\frac{V_2}{V_1}\right) - \gamma \ln\left(\frac{V_2}{V_1}\right) \right] = 0$$

where we have used the mathematical identity: $\ln(x^a) = a \ln x$. The total entropy change is zero as we expected, since the system could have gone from state 1 to state 2 by an adiabatic process for which $Q = 0$.

ASSESS As explained in the text, the entropy is a state variable, which doesn't depend on how a system arrived at a particular state. Note that this separates entropy from the heat, Q , absorbed or expelled by a system. You can't say that a system contains a particular amount of heat, but you can say that it contains a particular amount of entropy.

- 56. INTERPRET** This problem asks about the energy quality resulting from a thermodynamic process (an adiabatic free expansion in this case) during which entropy has increased. The system consists of a given quantity of ideal gas, and we are given the initial and final volumes occupied by the gas.

DEVELOP From the discussion accompanying Fig. 19.16, we know that the change in entropy during the adiabatic free expansion is

$$\Delta S = nR \ln \left(\frac{V_2}{V_1} \right)$$

where $V_2 = 15V_1$. The energy made unavailable is $E = T\Delta S$.

EVALUATE Substituting the values given in the problem statement, the energy that becomes unavailable to do work in the free expansion of an ideal gas (T remains constant), is

$$E = T\Delta S = nRT \ln \left(\frac{V_2}{V_1} \right) = (6.36 \text{ mol}) [8.314 \text{ J/(mol} \cdot \text{K)}] (305 \text{ K}) \ln (15) = 43.7 \text{ kJ}$$

ASSESS This is the work that could have been recovered from a reversible isothermal expansion. However, due to the irreversible nature of the process, we give up the possibility of extracting this work.

- 57. INTERPRET** This problem asks for the entropy change of the pan-water system, when thermal equilibrium has been reached after a hot pan has been plunged into the given amount of cold water.

DEVELOP Assume all the heat lost by the pan is gained by the water. The equilibrium temperature is given by Equation 16.4, or

$$T_{\text{eq}} = \frac{(2.4 \text{ kg}) [900 \text{ J/(kg} \cdot \text{K)}] (428 \text{ K}) + (3.5 \text{ kg}) [4184 \text{ J/(kg} \cdot \text{K)}] (288 \text{ K})}{(2.4 \text{ kg}) [900 \text{ J/(kg} \cdot \text{K)}] + (3.5 \text{ kg}) [4184 \text{ J/(kg} \cdot \text{K)}]} = 306 \text{ K}$$

Using the result of Problem 54, the change in entropy for the pan is

$$\Delta S_{\text{pan}} = m_{\text{pan}} c_{\text{pan}} \ln \left(\frac{T_{\text{eq}}}{T_{\text{pan}}} \right)$$

Similarly, the change in entropy for the water is

$$\Delta S_{\text{water}} = m_{\text{water}} c_{\text{water}} \ln \left(\frac{T_{\text{eq}}}{T_{\text{water}}} \right)$$

The sum of these two terms is the change of entropy of the pan-water system.

EVALUATE The entropy change of the pan and water together is

$$\begin{aligned} \Delta S &= \Delta S_{\text{pan}} + \Delta S_{\text{water}} = m_{\text{pan}} c_{\text{pan}} \ln \left(\frac{T_{\text{eq}}}{T_{\text{pan}}} \right) + m_{\text{water}} c_{\text{water}} \ln \left(\frac{T_{\text{eq}}}{T_{\text{water}}} \right) \\ &= (2.4 \text{ kg}) [900 \text{ J/(kg} \cdot \text{K)}] \ln \left(\frac{306 \text{ K}}{428 \text{ K}} \right) + (3.5 \text{ kg}) [4184 \text{ J/(kg} \cdot \text{K)}] \ln \left(\frac{306 \text{ K}}{288 \text{ K}} \right) = 160 \text{ J/K} \end{aligned}$$

to two significant figures.

ASSESS The entropy change for the pan is negative, while that of the water is positive. The total entropy change is positive, in accordance with the second law of thermodynamics.

- 58. INTERPRET** This problem involves the efficiency of an engine operating between two temperatures.

DEVELOP We assume the engine is reversible and operates between the two given temperatures ($T_h = 480 \text{ K}$ and $T_c = 273 \text{ K}$). The efficiency can then be computed using Equation 19.3, $e_{\text{Carnot}} = 1 - T_c / T_h$.

EVALUATE (a) Substituting the values given in the problem statement, we find the efficiency to be

$$e_{\text{Carnot}} = 1 - \frac{T_c}{T_h} = 1 - \frac{273 \text{ K}}{480 \text{ K}} = 43.0\%$$

(b) The total heat that the block of ice can absorb as it melts at 273 K is

$$Q = mL_f = (1.0 \times 10^3 \text{ kg}) (334 \text{ kJ/kg}) = 334 \text{ MJ}$$

Then the meltwater temperature will rise and the engine's efficiency will drop. While running at the original efficiency, the engine exhausts heat at the rate

$$\frac{dQ_c}{dt} = \frac{dQ_h}{dt} - \frac{dW}{dt} = \left(\frac{1}{e_{\text{Carnot}}} - 1 \right) \frac{dW}{dt} = \left(\frac{1}{0.43} - 1 \right) (8 \text{ kW}) = 10.6 \text{ kW}$$

(combine the first law with the definition of efficiency). Thus, it can operate between the original temperatures for a time

$$t = \frac{Q}{dQ_c / dt} = \frac{334 \text{ MJ}}{10.6 \text{ kW}} = 3.15 \times 10^4 \text{ s} = 8.75 \text{ h}$$

ASSESS For real engines in which $e < e_{\text{Carnot}}$, heat is exhausted at a greater rate. This shortens the duration for which the engine can maintain its efficiency.

59. INTERPRET This problem is about the increase in entropy as water is vaporized at 100°C .

DEVELOP The entropy increase is given by Equation 19.6: $\Delta S_{12} = \int_1^2 dQ/T$. The heat needed to vaporize the water is $Q = mL_v$, where $L_v = 2257 \text{ kJ/kg}$ from Table 17.1. The temperature is constant during the vaporization process, $T = 373 \text{ K}$.

EVALUATE The entropy increase during the vaporization process is

$$\Delta S = \frac{\Delta Q}{T} = \frac{mL_v}{T} = \frac{(1.7 \text{ kg})(2257 \text{ kJ/kg})}{373 \text{ K}} = 10.3 \text{ kJ/K}$$

ASSESS As expected, the entropy change is positive in both melting and warming processes.

60. INTERPRET We will calculate the efficiency of the Otto cycle, on which gasoline engines are modeled.

DEVELOP The engine absorbs heat (Q_h) during combustion, and expels heat to the environment (Q_c) during the exhaust segment. Both these processes are at constant volume, so $Q = nC_V\Delta T$, and the efficiency is:

$$e_{\text{Otto}} = \frac{W}{Q_h} = 1 - \frac{Q_c}{Q_h} = 1 - \frac{\Delta T_c}{\Delta T_h}$$

We can find the respective temperature changes assuming the gas mixture in the engine is ideal: $T = pV/nR$.

EVALUATE (a) The hot temperature change is between point 2 and point 3 in Fig. 19.24:

$$\Delta T_h = T_3 - T_2 = \frac{1}{nR} \left[3p_2 \left(\frac{1}{5} V_1 \right) - p_2 \left(\frac{1}{5} V_1 \right) \right] = \frac{2p_2 V_1}{5nR}$$

where we use the values for the pressure and volume given in the figure. The cold temperature change is between point 1 and point 4 in the figure, but the pressures aren't given in this case. However, point 1 and point 2 are on the same adiabat ($pV^\gamma = \text{constant}$), so: $p_1 = p_2 \left(\frac{1}{5} \right)^\gamma$, and similarly for point 4 and point 3: $p_4 = 3p_2 \left(\frac{1}{5} \right)^\gamma$. Therefore, the cold temperature change can be written:

$$\Delta T_c = T_4 - T_1 = \frac{1}{nR} [p_4 V_1 - p_1 V_1] = \frac{2p_2 V_1}{5^\gamma nR}$$

From these temperature changes, the efficiency of the Otto cycle is:

$$e_{\text{Otto}} = 1 - \frac{\Delta T_c}{\Delta T_h} = 1 - 5^{1-\gamma}$$

(b) The minimum temperature occurs at point 1 at the end of the exhaust segment:

$$T_{\min} = T_1 = \frac{p_2 V_1}{5^\gamma nR}$$

The maximum temperature occurs at point 3 at the end of combustion:

$$T_{\max} = T_3 = \frac{3p_2 V_1}{5nR} = 3 \cdot 5^{\gamma-1} T_{\min}$$

(c) A Carnot cycle working between minimum and maximum temperatures would have an efficiency of (Equation 19.3):

$$e_{\text{Carnot}} = 1 - \frac{T_{\min}}{T_{\max}} = 1 - \frac{1}{3} 5^{1-\gamma}$$

So, the Carnot cycle efficiency is greater than that of the Otto cycle: $e_{\text{Carnot}} > e_{\text{Otto}}$, as we'd expect since the Carnot cycle has the maximum efficiency for an engine.

ASSESS If we assume $\gamma=1.4$, just for argument sake, then $e_{\text{Otto}} = 47\%$, while $e_{\text{Carnot}} = 82\%$. In this light, gasoline engines are woefully inefficient. Much of the combustion energy is lost as exhaust heat.

61. INTERPRET This problem is about the efficiency of the Otto cycle as a function of the compression ratio.

DEVELOP As argued in the previous problem, the efficiency of the Otto cycle is: $e_{\text{Otto}} = 1 - \Delta T_c / \Delta T_h$. We will write this in terms of the compression ratio, $r = V_1 / V_2$, where V_1 and V_2 are the volumes before and after the adiabatic compression in Fig. 19.24. In this case $V_2 = \frac{1}{3}V_1$, but we'll derive the expression for the general case first.

EVALUATE The hot temperature change is between point 2 and point 3 in the figure:

$$\Delta T_h = T_3 - T_2 = \frac{1}{nR}(p_3 V_3 - p_2 V_2) = \frac{V_2}{nR}(p_3 - p_2)$$

The cold temperature change is between point 1 and point 4 in the figure:

$$\Delta T_c = T_4 - T_1 = \frac{1}{nR}(p_4 V_4 - p_1 V_1) = \frac{V_1}{nR}(p_4 - p_1)$$

Since $pV^\gamma = \text{constant}$ for points 1 and 2 and for points 3 and 4, we have $p_1 = p_2 r^{-\gamma}$, and $p_4 = p_3 r^{-\gamma}$. Therefore, the efficiency reduces to

$$e_{\text{Otto}} = 1 - \frac{\Delta T_c}{\Delta T_h} = 1 - \frac{V_1(p_3 r^{-\gamma} - p_2 r^{-\gamma})}{V_2(p_3 - p_2)} = 1 - r^{1-\gamma}$$

ASSESS Since the efficiency increases for larger r , one might assume engineers would try to maximize the compression ratio. In practice, however, the compression ratio cannot be too large, otherwise the fuel pre-ignites, which results in "knocking" that reduces engine performance.

62. INTERPRET This problem is about the efficiency of the Otto cycle when the compression ratio of the gas present has been changed.

DEVELOP As determined in the previous problem, the efficiency of the Otto cycle can be expressed as

$$e_{\text{Otto}} = 1 - \frac{\Delta T_c}{\Delta T_h} = 1 - \frac{V_1(p_3 r^{-\gamma} - p_2 r^{-\gamma})}{V_2(p_3 - p_2)} = 1 - r^{1-\gamma}$$

Where r is the compression ratio and γ is the specific heat ratio of the gas present. We want to find the change in the engine's efficiency when the compression ratio has changed after replacing the gasoline with natural gas.

EVALUATE The power efficiency when using gasoline and natural gas are equal to, respectively

$$e_{\text{gasoline}} = 1 - (8.80)^{1-1.4} = 0.581$$

$$e_{\text{nat. gas}} = 1 - (12.7)^{1-1.4} = 0.638$$

Thus, the efficiency increases from 58.1% to 63.8%.

ASSESS Here we have assumed the change is due entirely to the increased compression ratio, and not reliant on the modifications that took place to allow the engine to run on natural gas.

63. INTERPRET Find the maximum efficiency of a power plant, given the temperature range of its cycle. We will calculate the Carnot efficiency, and compare this with the actual efficiency.

DEVELOP The high temperature $T_h = 950^\circ\text{F} = 783\text{ K}$. The low temperature is $T_c = 90^\circ\text{F} = 305\text{ K}$. The Carnot efficiency is given by $e = 1 - \frac{T_c}{T_h}$.

EVALUATE The maximum efficiency is $e = 1 - \frac{T_c}{T_h} = 61\%$.

ASSESS The actual efficiency of this plant is given as 25%, which is considerably lower due (at least in part) to having to evaporate moisture out of the wood-chip fuel.

- 64. INTERPRET** We are to find the final temperature and entropy change for a system in which two objects at different temperatures are brought into thermal contact and allowed to come to thermal equilibrium.

DEVELOP To find the final temperature, use Equation 16.4, which may be expressed as

$$T_f = \frac{m_W c_W T_{0,W} + m_{Cu} c_{Cu} T_{0,Cu}}{m_W c_W + m_{Cu} c_{Cu}}$$

The entropy change is $dS = dQ/T$ (Equation 19.6), so we can integrate to find the change in entropy for the water and for the copper. The initial temperature and the mass of the copper are $T_{0,Cu} = 80^\circ\text{C} = 353\text{ K}$ and $m_{Cu} = 0.5\text{ kg}$. The initial temperature and the mass of the water are $T_{0,W} = 10^\circ\text{C} = 283\text{ K}$ and $m_W = 1.0\text{ kg}$. The specific heats c_W and c_{Cu} can be found in Table 16.1.

EVALUATE (a) Inserting the given quantities gives

$$\begin{aligned} T_f &= \frac{m_W c_W T_{0,W} + m_{Cu} c_{Cu} T_{0,Cu}}{m_W c_W + m_{Cu} c_{Cu}} \\ &= \frac{(1.0\text{ kg})[4184\text{ J/(kg} \cdot \text{K)}](283\text{ K}) + (0.50\text{ kg})[386\text{ J/(kg} \cdot \text{K)}](353\text{ K})}{(1.0\text{ kg})[4184\text{ J/(kg} \cdot \text{K)}] + (0.50\text{ kg})[386\text{ J/(kg} \cdot \text{K)}]} = 286\text{ K} \end{aligned}$$

(b) The change in entropy for the water and copper are, respectively:

$$\begin{aligned} dS_W &= \frac{dQ_W}{T_W} = \frac{m_W c_W dT_W}{T_W} \\ \Delta S_W &= m_W c_W \int_{T_{0,W}}^{T_f} \frac{dT}{T} = m_W c_W \ln\left(\frac{T_f}{T_{0,W}}\right) = (1.0\text{ kg})[4184\text{ J/(kg} \cdot \text{K)}] \ln\left(\frac{286\text{ K}}{283\text{ K}}\right) = 44.1\text{ J/K} \\ dS_{Cu} &= \frac{dQ_{Cu}}{T_{Cu}} = \frac{m_{Cu} c_{Cu} dT_W}{T_{Cu}} \\ \Delta S_{Cu} &= m_{Cu} c_{Cu} \int_{T_{0,Cu}}^{T_f} \frac{dT}{T} = m_{Cu} c_{Cu} \ln\left(\frac{T_f}{T_{0,Cu}}\right) = (0.50\text{ kg})[386\text{ J/(kg} \cdot \text{K)}] \ln\left(\frac{286\text{ K}}{353\text{ K}}\right) = -40.6\text{ J/K} \end{aligned}$$

The total change in entropy is $\Delta S = \Delta S_W + \Delta S_{Cu} = 3.1\text{ J/K}$.

ASSESS The entropy of the copper actually decreases, but this decrease is more than offset by the increase in entropy of the water. This is an irreversible process, and entropy always increases in irreversible processes.

- 65. INTERPRET** We are asked to calculate the entropy change in an object whose heat capacity is inversely proportional to its temperature.

DEVELOP By definition, the heat capacity relates the heat flowing into an object to the change in its temperature:

$dQ = C dT$. In this case, $C = C_0(T_0/T)$. We can plug this into Equation 19.6 for the entropy change:

$$\Delta S = \int C_0 T_0 dT / T^2.$$

EVALUATE Performing the integration from T_0 to T_1 :

$$\Delta S_{01} = \int_{T_0}^{T_1} \frac{C_0 T_0 dT}{T^2} = C_0 T_0 \left(\frac{-1}{T} \right) \bigg|_{T_0}^{T_1} = C_0 \left(1 - \frac{T_0}{T_1} \right)$$

ASSESS For $T_1 > T_0$, the entropy change is positive. For $T_1 \approx T_0$, the entropy change becomes constant:

$$\Delta S_{01} \approx C_0.$$

- 66. INTERPRET** This problem deals with a Carnot engine for which the temperature of the heat reservoir varies with time. We are to express instantaneous temperature of the hot reservoir as a function of time and find the time it takes for the engine's power to reach zero.

DEVELOP In time dt , the engine extracts heat $dQ_h = -mcdT_h$ from the block, and does work $dW = Pdt$. Equation 19.1 gives the definition of the actual efficiency e :

$$dW = edQ_h = \left(\frac{T_h - T_c}{T_h} \right) (-mcdT_h) = Pdt$$

where for the second equality we have used $e = e_{\text{Carnot}} = 1 - T_c/T_h$. The power is also assumed to be proportional to $T_h - T_c$, so this equation becomes

$$-mc \frac{dT_h}{T_h} = \frac{P_0 dt}{T_{h0} - T_c}$$

Integrating this expression yields T_h as a function of time t . For (b), note that the power output becomes zero when $T_h = T_c$.

EVALUATE (a) Integrating from $t = 0$ and T_{h0} to t and T_h gives

$$\int_{T_{h0}}^{T_h} \frac{dT_h'}{T_h'} = \ln \left(\frac{T_h}{T_{h0}} \right) = - \int_0^t \frac{P_0 dt'}{mc(T_{h0} - T_c)} = \frac{-P_0 t}{mc(T_{h0} - T_c)}$$

or

$$T_h(t) = T_{h0} \exp \left(\frac{-P_0 t}{mc(T_{h0} - T_c)} \right)$$

(b) The power output is zero for $T_h = T_c$. This occurs at time

$$t_0 = \frac{mc(T_{h0} - T_c)}{P_0} \ln \left(\frac{T_{h0}}{T_c} \right)$$

ASSESS We find the instantaneous temperature of the hot block to decrease exponentially with time. At $t = 0$, $T_h(0) = T_{h0}$. However, for $P_0 t \gg mc(T_{h0} - T_c)$ T_h becomes very small. Note that the expression for P was originally assumed to be valid for $T_h \geq T_c$, or for times $t \leq t_0$. If we allow $T_h < T_c$, or $t > t_0$, then $dW = Pdt < 0$ becomes work input to an “engine” which acts like a refrigerator that cools the block.

- 67. INTERPRET** You have an infinite heat reservoir, but a finite cool reservoir. The question is how much work can you obtain with an engine placed between the reservoirs before the cool reservoir is “exhausted.”

DEVELOP The infinite heat reservoir will maintain its temperature, T_h , no matter how much heat, Q_h , you extract from it. The cool reservoir, on the other hand, will not maintain its initial temperature, T_{c0} , as heat from the engine is expelled into it. The temperature will rise in the cool reservoir according to $dQ_c = CdT_c$. But once the cool reservoir temperature is equal to T_h , no more work can be extracted.

EVALUATE You can assume that the engine cycles fast enough that during a single cycle the cool reservoir temperature is approximately constant. To maximize the amount of work that you extract, place a Carnot engine between the reservoirs so that the work extracted during one cycle is:

$$dW = d(Q_h - Q_c) = dQ_c \left[\frac{T_h}{T_c} - 1 \right] = CdT_c \left[\frac{T_h}{T_c} - 1 \right]$$

where $Q_h/Q_c = T_h/T_c$ for a Carnot engine. To find the total work, integrate this expression from T_{c0} to T_h ,

$$W = \int_{T_{c0}}^{T_h} CdT_c \left[\frac{T_h}{T_c} - 1 \right] = CT_h \ln(T_c) - CT_c \Big|_{T_{c0}}^{T_h} = CT_h \ln \left(\frac{T_h}{T_{c0}} \right) - C(T_h - T_{c0})$$

If we let $x = T_h/T_{c0}$, then $W = CT_h(\ln x - 1 + 1/x)$.

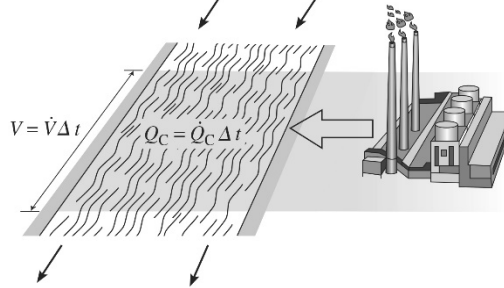
ASSESS The work is proportional to the heat capacity, as you might expect. The heat capacity is a measure of how much heat the cool reservoir can accept, so the larger the heat capacity, the more work that can be extracted. You might worry that the work could be negative for some value of $x = T_h/T_{c0}$. For $x \gg 1$, the work is approximately $W \approx CT_h \ln x$, which is positive. For $x \approx 1$, $\ln x \approx x - 1$, and the work is approximately $W \approx CT_h(x - 2 + 1/x)$, which is positive as well. Therefore, the work extracted is positive for all possible temperature differences.

- 68. INTERPRET** You want to know how much the temperature of a river will increase when heat exhaust from a power plant is absorbed by the water.

DEVELOP Since the plant has an efficiency of $e = W / Q_h = 35\%$, the rate at which it is generating waste heat must be:

$$\dot{Q}_c = \dot{Q}_h - \dot{W} = \dot{W} \left(\frac{1}{e} - 1 \right) = 750 \text{ MW} \left(\frac{1}{0.35} - 1 \right) = 1390 \text{ MW}$$

where the dots here signify that these are rates, for example, $\dot{Q}_c = dQ_c / dt$. The rate at which water is flowing past the plant is given by the volume rate: $\dot{V} = 110 \text{ m}^3/\text{s}$. To better understand how the heat flows into the flowing water, it might help to “freeze” both flows and just imagine what is happening over a short period of time, Δt . In this case, the plant expels a finite amount of heat, $Q_c = \dot{Q}_c \Delta t$, into a volume of water $V = \dot{V} \Delta t$. See the figure below.



This volume of water will rise in temperature according to: $Q_c = mc\Delta T = \rho V c \Delta T$, where ρ is the density of water and c is the specific heat. You can use the equations formulated here to check that the temperature rise is below the environmental regulation.

EVALUATE Solving for the temperature change of the water:

$$\Delta T = \frac{Q_c}{\rho V c} = \frac{\dot{Q}_c}{\rho \dot{V} c} = \frac{(1390 \text{ MW})}{(1000 \text{ kg/m}^3)(110 \text{ m}^3/\text{s})(4184 \text{ J/kg} \cdot \text{K})} = 3.0^\circ \text{C}$$

The temperature rise is below the regulated limit.

ASSESS The river makes for a good reservoir, since the flow will constantly bring cool water that has yet to be heated by the plant's exhaust.

- 69. INTERPRET** In this problem we want to continue the derivation for the entropy change when equal masses of hot and cold water are mixed which is discussed in the subsection “Irreversible Heat Transfer”.

DEVELOP We begin with the equation for the entropy change for each mass of water

$$\Delta S_{h,c} = mc \int_{T_{h,c}}^{T_f} \frac{dT}{T}$$

We will perform the integration for the hot and cold water masses and simplify the expressions to interpret the result

EVALUATE (a) The change in entropy for the hot and cold water masses are equal to

$$\Delta S_h = mc \int_{T_h}^{T_f} \frac{dT}{T} = mc (\ln T_f - \ln T_h)$$

$$\Delta S_c = mc \int_{T_c}^{T_f} \frac{dT}{T} = mc (\ln T_f - \ln T_c)$$

The overall entropy change for this system is the sum of these two quantities

$$\Delta S = \Delta S_c + \Delta S_h = mc (2 \ln T_f - \ln T_c - \ln T_h)$$

Here we have equal masses of hot and cold water, so the final temperature is $T_f = (T_c + T_h) / 2$. Substituting this into our expression for the change in entropy we obtain

$$\Delta S = mc (2 \ln [(T_c + T_h) / 2] - \ln T_c - \ln T_h) = mc (\ln [(T_c + T_h) / 2]^2 - \ln T_c T_h) = mc \ln [(T_c + T_h)^2 / 4 T_c T_h]$$

(b) In order for the argument of this logarithm to be greater than 1 the numerator should be greater than the denominator. Expanding this expression shows

$$(T_c + T_h)^2 > 4 T_c T_h$$

$$T_c^2 + 2T_c T_h + T_h^2 > 4T_c T_h$$

$$T_c^2 - 2T_c T_h + T_h^2 > 0$$

$$(T_h - T_c)^2 > 0$$

$$T_h > T_c$$

Meaning $T_h \neq T_c$ and the logarithm argument is greater than 1, signifying ΔS is positive.

ASSESS We choose to write $(T_h - T_c)^2 > 0$, and not the other way around, since we define $T_h > T_c$ on our temperature scale.

- 70. INTERPRET** We are to find the change in entropy for a sample of copper at low temperatures, where the specific heat changes with temperature.

DEVELOP The specific heat of copper at low temperatures is given as $c = 31\left(\frac{T}{343 \text{ K}}\right)^3 \text{ J/(g} \cdot \text{K)}$. From Equation 19.6, we have $dS = \frac{dQ}{T}$, and from Equation 16.3 we have $dQ = mc dT$. We find the change in entropy by integrating from $T_1 = 25 \text{ K}$ to $T_2 = 10 \text{ K}$. The mass of the copper is $m = 40 \text{ g}$.

EVALUATE

$$\begin{aligned} dS &= \frac{dQ}{T} \\ \Delta S &= \int_{T_1}^{T_2} \frac{mc}{T} dT = \int_{T_1}^{T_2} \frac{m \left(31 \left(\frac{T}{343} \right)^3 \right)}{T} dT = \frac{31m}{343^3} \int_{T_1}^{T_2} T^3 dT \\ &= \frac{31m}{343^3} \left[\frac{1}{4} (T_2^4 - T_1^4) \right] = \frac{31(0.040 \text{ kg})}{343^3} \left[\frac{1}{4} (10^4 - 25^4) \text{ K}^3 \right] = -2.9 \times 10^{-3} \text{ J/K} \end{aligned}$$

ASSESS This change is negative because the temperature goes down. To satisfy the second law of thermodynamics, somewhere in this cooling process, the entropy of something else must have gone up by more than 2.9 mJ/K .

- 71. INTERPRET** We are to find the entropy change for a sample of gas with the given temperature change, where the specific heat of the gas changes with temperature.

DEVELOP We are given an equation for the molar specific heat: $c_p = a + bT + cT^2$, where $a = 33.6 \text{ J/(mol} \cdot \text{K)}$, $b = 2.93 \times 10^{-3} \text{ J/(mol} \cdot \text{K}^2)$, and $c = 2.13 \times 10^{-5} \text{ J/(mol} \cdot \text{K}^3)$. The amount of gas is 2 moles, and the temperature changes from $T_1 = 293 \text{ K}$ to $T_2 = 473 \text{ K}$. From the definition of the molar specific heat (see discussion preceding Equation 18.3), $Q = nC_p \Delta T$, which we use to express the heat change in terms of temperature and specific heat. Insert this into Equation 19.6 to find the entropy change.

EVALUATE

$$\begin{aligned} dS &= \frac{dQ}{T} \\ \Delta S &= \int_{T_1}^{T_2} \frac{nC_p}{T} dT = n \int_{T_1}^{T_2} \frac{(a + bT + cT^2)}{T} dT = n \int_{T_1}^{T_2} \left(\frac{a}{T} + b + cT \right) dT \\ &= n \left[a \ln \left(\frac{T_2}{T_1} \right) + b(T_2 - T_1) + \frac{c}{2} (T_2^2 - T_1^2) \right] = 36.2 \text{ J/K} \end{aligned}$$

ASSESS The entropy increases as the temperature increases, as we would expect.

- 72. INTERPRET** We're asked to consider the statistics of N molecules distributed among two halves of a closed box. This is the general case of Figs. 19.18 and 19.19, as well as Exercise 19.23.

DEVELOP The molecules have equal probability of being in the left side or right side of the box. If each molecule were different from the rest, then we could distinguish each arrangement (or microstate) of the molecules in the

box. But the molecules are identical, so they can be switched (or permuted), and the overall macrostate will be the same.

EVALUATE (a) Let's assume that X_N is the number of microstates for N molecules. If we now add one more molecule to the box, then for each of the X_N microstates, the new molecule can either be put in the left-hand or right-hand side of the box. That means the new number of microstates is $X_{N+1} = 2X_N$. Since a single molecule in the box has $X_1 = 2$ microstates, the number of microstates for any N must be $X_N = 2^N$.

(b) Of the 2^N microstates, we'll call $Y_{N/2}$ the number that have half of the molecules on one side of the box. We showed in Exercise 19.23 for $N = 6$ how one can do permutations of the molecules to find all the combinations where the molecules are split equally between the two sides. For any even N , the number of combinations that will have $N/2$ on one side of the box is given by the coefficients from the binomial theorem:

$$Y_{N/2} = \binom{N}{N/2} = \frac{N!}{(N/2)!(N/2)!}$$

(c) The probability of a given macrostate is the number microstates that match the macrostate divided by the total number of microstates. So, the probability of having an equal number of molecules on each side of the box is:

$$P_N^{N/2} = \frac{Y_{N/2}}{X_N} = \frac{N!}{2^N [(N/2)!]^2}$$

There are two microstates with all of the particles on one side of the box, all on the left, or all on the right, each with $P_N^N = 1/2^N$. So the ratio of these probabilities is:

$$\frac{2P_N^N}{P_N^{N/2}} = \frac{2[(N/2)!]^2}{N!}$$

(d) For $N=4$, this probability ratio is

$$\frac{2P_4^4}{P_4^2} = \frac{2[2!]^2}{4!} = \frac{2 \cdot 2^2}{24} = \frac{1}{3}$$

For $N=100$, the factorials become unwieldy. We can use Stirling's approximation for large n : $\ln n! \approx n \ln n - n$.

$$\ln \left[\frac{50!^2}{100!} \right] \approx 2(50 \ln 50 - 50) - (100 \ln 100 - 100) = -100 \ln 2$$

So the probability ratio is approximately

$$\frac{2P_{100}^{100}}{P_{100}^{50}} = \frac{2[50!]^2}{100!} \approx 2e^{-100 \ln 2} = 2^{-99} \approx 10^{-30}$$

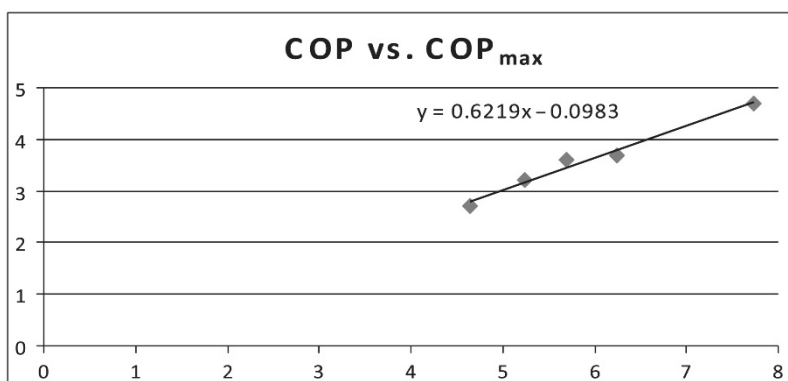
ASSESS This says that for 100 molecules, it's 10^{30} times more likely that the molecules will be evenly distributed in the box versus all of them on one side. One can imagine how impossible it would be for a room full of 10^{23} molecules to suddenly evacuate one side for the other.

73. INTERPRET We are given the data of the COP of a heat pump at different outdoor temperatures T_c . We would like to compare the pump's COP with its theoretical maximum.

DEVELOP The maximum COP of a heat pump (when its heating, not cooling) is given in Equation 19.4b:

$$\text{COP}_{\max} = T_h / (T_h - T_c). \text{ In our case, } T_h = 52^\circ\text{C} = 325 \text{ K.}$$

EVALUATE (a) A plot of COP versus $T_h / (T_h - T_c)$ gives a straight line. (b) The plot is shown below. The best-fit line has a slope of 0.62, showing that the heat pump achieves 62% of the theoretical maximum COP.



ASSESS The COP of the pump goes up as the difference between T_h and T_c gets smaller.

74. INTERPRET We will consider the energy consumption of a typical refrigerator.

DEVELOP To get a sense of how the refrigerator works, we can look at Fig. 19.6. In the course of a day, an amount of heat, $Q_c = 30$ MJ, is drawn from the fridge's cold interior. But this requires work to be done, $W = 10$ MJ. Specifically, electricity is needed to pump refrigerant through the system.

EVALUATE Both the heat drawn from the fridge interior and the work done by the electrical energy are expelled as heat: $Q_h = Q_c + W$. So, the work effectively ends up as waste heat rejected to the kitchen environment. The answer is (d).

ASSESS This might sound wasteful: turning high-quality electrical energy into heat that gets dumped out of the backside of your fridge. But according to Clausius statement of the second law of thermodynamics, it's impossible to construct a perfect refrigerator whose sole effect is to transfer heat from a cooler object to a hotter one. An external energy source is needed. However, it's not necessary to use electrical energy. For example, solar refrigerators use sunlight to evaporate water and thus draw heat from the fridge interior.

75. INTERPRET We will consider the energy consumption of a typical refrigerator.

DEVELOP The COP for a refrigerator is defined as what we want (heat drawn from fridge contents) divided by what we put in (work from electricity), that is, $COP = Q_c / W$.

EVALUATE We're told that it takes 10 MJ of electrical energy to draw 30 MJ of heat from the fridge contents, so $COP = 30 / 10 = 3$.

The answer is (c).

ASSESS This is a typical COP for a refrigerator. However, refrigerators are often rated not by COP, but by the average amount of electrical energy, W , they use in a year. Guidelines are adjusted for the volume of the fridge interior, which is easier to measure than, say, the amount of heat drawn from the fridge contents, which probably varies widely.

76. INTERPRET We will consider the energy consumption of a typical refrigerator.

DEVELOP We're told the coal-fired power plant has an efficiency of $e = 40\%$. In contrast to the COP, the efficiency is defined as what we want (electrical energy to do work) divided by what we put in (heat from the burning of coal), that is, $e = W / Q_h$.

EVALUATE To make 10 MJ of electrical energy, the power plant has to burn enough fuel to generate $Q_h = W / e = 25$ MJ.

The answer is (b).

ASSESS Notice what this says: it takes 25 MJ of heat from coal burning to extract 30 MJ of heat from the fridge contents.

77. INTERPRET We will consider the energy consumption of a typical refrigerator.

DEVELOP As we argued in Problem 19.74, the heat drawn from the fridge interior and the work from the electricity are both expelled as heat into the relatively warmer environment: $Q_h = Q_c + W$.

EVALUATE Over the course of a day, the refrigerator expels $Q_h = 40$ MJ of heat into the surrounding kitchen. The answer is (c).

ASSESS It might be interesting to compare this to the waste heat expelled by the power plant in order to supply the 10 MJ of electricity the refrigerator needs. In the plant's case, the exhaust is $Q_c = Q_h - W = 15$ MJ.