

## HEAT, WORK, AND THE FIRST LAW OF THERMODYNAMICS

# 18

### EXERCISES

#### Section 18.1 The First Law of Thermodynamics

- 11. INTERPRET** We identify the system as the water in the insulated container. The problem involves calculating the work done to raise the temperature of a system, so the first law of thermodynamics comes into play.

**DEVELOP** Because the container is a perfect thermal insulator, no heat enters or leaves the water inside of it. Thus,  $Q = 0$ , and the first law of thermodynamics in Equation 18.1 gives  $\Delta E_{\text{int}} = Q + W = W$  where  $W$  is the work done by shaking the container. The change in the internal energy of the water is determined from its temperature rise,  $\Delta E_{\text{int}} = mc\Delta T$  (see comments in Section 16.1 on internal energy). Combining these two expressions for the internal energy change allows us to find the work done.

**EVALUATE** The work done on the water is

$$\begin{aligned} W &= \Delta E_{\text{int}} = mc\Delta T \\ &= (2.5 \text{ kg})[4.184 \text{ kJ}/(\text{kg} \cdot \text{K})](2.0 \text{ K}) = 21 \text{ kJ} \end{aligned}$$

**ASSESS** According to the convention adopted for the first law of thermodynamics, positive work signifies that work is done on the water.

- 12. INTERPRET** This problem involves finding the mechanical work that is required to raise the given mass of water by  $3.0^\circ\text{C}$ , which we can calculate by using the first law of thermodynamics.

**DEVELOP** The change in the internal energy of the water is  $\Delta E_{\text{int}} = mc\Delta T$ , where  $\Delta T = 3.0^\circ\text{C} = 3.0 \text{ K}$  and the work done on it is  $W = 9.0 \text{ kJ}$ . Inserting this into the first law of thermodynamics (Equation 18.1) gives  $Q = \Delta E_{\text{int}} - W = mc\Delta T - W$ . For part (b), note that if the water had been in perfect thermal isolation, no heat would have been transferred, so  $Q' = 0$  and we can solve for the work  $W'$  as for part (a).

**EVALUATE** (a) Inserting the given quantities into the expression for the heat transferred gives

$$Q = mc\Delta T - W = (0.50 \text{ kg})[4.184 \text{ kJ}/(\text{kg} \cdot \text{K})](3.0 \text{ K}) - 9.0 \text{ kJ} = -2.7 \text{ kJ}$$

(b) If  $Q = 0$ , then the work needed is

$$W = mc\Delta T = (0.50 \text{ kg})[4.184 \text{ kJ}/(\text{kg} \cdot \text{K})](3.0 \text{ K}) = 6.3 \text{ kJ}$$

**ASSESS** The fact that  $Q < 0$  for part (a) indicates that heat is transferred from the water to the environment, as expected.

- 13. INTERPRET** The system of interest is the gas that undergoes expansion. The problem involves calculating the change in internal energy of a system, so the first law of thermodynamics comes into play.

**DEVELOP** The heat added to the gas is  $Q = Pt = (40 \text{ W})(40 \text{ s}) = 1600 \text{ J}$ . In addition, the system does  $750 \text{ J}$  of work on its surroundings, so the work done by the surroundings on the system is  $W = -750 \text{ J}$ . The change in internal energy can be found by using the first law of thermodynamics, given in Equation 18.1.

**EVALUATE** Using Equation 18.1, we find

$$\Delta E_{\text{int}} = Q + W = 1600 \text{ J} - 750 \text{ J} = 850 \text{ J}$$

**ASSESS** Since  $\Delta E_{\text{int}} > 0$ , we conclude that the internal energy has increased.

- 14. INTERPRET** This problem involves calculating the rate of heat flow into the system, given the rate at which the internal energy is increasing and the rate at which the system is doing work on its surroundings.

**DEVELOP** The dynamic form of the first law of thermodynamics (Equation 18.2) is

$$\frac{dQ}{dt} = \frac{dE_{\text{int}}}{dt} - \frac{dW}{dt}$$

The problem states that  $dE_{\text{int}}/dt = 65 \text{ W}$  and  $dW/dt = -175 \text{ W}$  (where the negative sign is appropriate because the system is doing work).

**EVALUATE** Inserting the given quantities gives

$$\frac{dQ}{dt} = 65 \text{ W} + 175 \text{ W} = 240 \text{ W}$$

**ASSESS** The rate of heat flow into the system is positive because the system is doing work on its surroundings and increasing its internal energy.

- 15. INTERPRET** This problem is about heat and mechanical energy, which are related by the first law of thermodynamics. The system of interest is the automobile engine.

**DEVELOP** Since we are dealing with rates, we make use of the dynamic form of the first law of thermodynamics, Equation 18.2:

$$\frac{dE_{\text{int}}}{dt} = \frac{dQ}{dt} + \frac{dW}{dt}$$

If we assume that the engine system operates in a cycle, then  $dU/dt = 0$ . The engine's mechanical power output  $dW/dt$  can then be calculated once the heat output is known.

**EVALUATE** The above conditions yield  $(dQ/dt)_{\text{out}} = 68 \text{ kW}$  and  $(dW/dt) = 0.17 (dQ/dt)_{\text{in}}$ . Equation 18.2 then gives

$$\frac{dW}{dt} = -\frac{dQ}{dt} = \left(\frac{dQ}{dt}\right)_{\text{out}} - \left(\frac{dQ}{dt}\right)_{\text{in}} = \left(\frac{dQ}{dt}\right)_{\text{out}} - \frac{1}{0.17} \frac{dW}{dt}$$

or

$$\frac{dW}{dt} = \frac{(dQ/dt)_{\text{out}}}{1 - (0.17)^{-1}} = \frac{68 \text{ kW}}{1 - (0.17)^{-1}} = -14 \text{ kW}$$

**ASSESS** We find the mechanical power output  $dW/dt$  to be proportional to the heat output,  $(dQ/dt)_{\text{out}}$ . In addition,  $dW/dt$  also increases with the percentage of the total energy released in burning gasoline that ends up as mechanical work. The mechanical output is negative because the system is doing work on its surroundings.

## Section 18.2 Thermodynamic Processes

- 16. INTERPRET** This problem involves an ideal gas that changes from an initial pressure-volume state to a final pressure-volume state by traversing the given pressure-volume curve. From this, we are to find the work done by the gas during this process.

**DEVELOP** The work done by the gas is given by the negative of Equation 18.3, which is

$$W_{\text{by gas}} = \int_{V_1}^{V_2} p(V) dV$$

This is the area under the line segment AB in Figure 18.19. We can find this area from simple geometrical arguments, without having to invoke calculus.

**EVALUATE** The work done by the gas is the area of the trapezoid under line AB, which is

$$W_{\text{by gas}} = W = \frac{1}{2}(P_1 + P_2)(V_2 - V_1) = \frac{1}{2}(P_1 + 2P_1)(2V_1 - V_1) = \frac{3}{2}P_1V_1$$

**ASSESS** The work can also be obtained from Equation 18.3. On the path AB,

$$p(V) = P_1 + \frac{(V - V_1)(P_2 - P_1)}{(V_2 - V_1)}$$

so

$$W_{\text{by gas}} = \int_{V_1}^{V_2} p(V) dV = P_1(V_2 - V_1) + \left( \frac{P_2 - P_1}{V_2 - V_1} \right) \frac{1}{2} (V_2 - V_1)^2 = \frac{3}{2} P_1 V_1.$$

- 17. INTERPRET** The expansion of the ideal gas involves two stages: an isochoric (constant-volume) process and an isobaric (constant-pressure) process. We are asked to find the total work done by the gas.

**DEVELOP** For an isochoric process,  $\Delta V = 0$  so  $W_1 = 0$  (see Equation 18.3 and Table 18.1). On the other hand, for an isobaric process, the work done is  $W_2 = p\Delta V$  which is the negative of Equation 18.7 because we are interested in the work done by the gas. (Equation 18.7 gives the work done on the gas.)

**EVALUATE** Summing the two contributions to find the total work gives

$$W_{\text{tot}} = W_1 + W_2 = p_2(V_2 - V_1) = 2p_1(2V_1 - V_1) = 2p_1V_1$$

**ASSESS** In the  $pV$  diagram of Fig. 18.19, the area under  $AC$  is zero, and that under  $CB$ , a rectangle, is  $2p_1V_1$ . The work done by the gas is the area under the  $pV$  curve.

- 18. INTERPRET** This problem involves the isothermal (i.e., the temperature is constant) expansion of a balloon as it rises. We are asked to find the work done by the helium as the balloon rises.

**DEVELOP** During an isothermal expansion, the work done *by* a given amount of ideal gas is

$$W = nRT \ln(V_2 / V_1)$$

which is the negative of Equation 18.3 because that form expresses the work done *on* the gas.

**EVALUATE** Inserting the given quantities yields

$$W = (0.50 \text{ mol})[8.314 \text{ J / (mol} \cdot \text{K)}](295 \text{ K})\ln(4.0) = 1.7 \text{ kJ}$$

**ASSESS** The result is positive because positive work is done *by* the gas to expand its volume.

- 19. INTERPRET** The constant temperature of 300 K indicates that the process is isothermal. We are to find the increase in volume of the balloon and the work done by the gas, given the pressure difference in experiences.

**DEVELOP** We assume the gas to be ideal and apply the ideal gas law given in Equation 17.2:  $pV = nRT$ . For an isothermal process,  $T = \text{constant}$ , so we obtain  $p_1V_1 = p_2V_2$ , from which we can find the fractional volume increase. The total work done by the gas can be calculated using the negative of Equation 18.4:

$$W = nRT \ln\left(\frac{V_2}{V_1}\right)$$

because we are interested in the work done by the gas, not on the gas.

**EVALUATE** (a) For the isothermal expansion process, the volume increases by a factor of

$$\frac{V_2}{V_1} = \frac{p_1}{p_2} = \frac{100 \text{ kPa}}{75 \text{ kPa}} = \frac{4}{3}$$

(b) Using Equation 18.4, the work done by the gas is

$$W = nRT \ln\left(\frac{V_2}{V_1}\right) = (0.30 \text{ mol})[8.314 \text{ J / (mol} \cdot \text{K)}](300 \text{ K})\ln\left(\frac{4}{3}\right) = 220 \text{ J}$$

to two significant figures.

**ASSESS** Because  $V_2 > V_1$ , we find the work to be positive,  $W > 0$ . This makes sense because the gas inside the balloon must do positive work to expand outward.

- 20. INTERPRET** This problem involves an ideal gas that we compress to half its volume in an isothermal process. We are to find the work needed to do this.

**DEVELOP** In an isothermal compression of a fixed quantity of ideal gas, work is done on the gas, so we use Equation 18.4,  $W = -nRT \ln(V_2 / V_1)$ , where  $V_1 = 2V_2$  and  $T = 290 \text{ K}$  for this problem.

**EVALUATE** Inserting the given quantities into Equation 18.4 gives

$$W = -nRT \ln(V_2 / V_1) = -(3.3 \text{ mol})[8.314 \text{ J / (mol} \cdot \text{K)}](290 \text{ K}) \ln(1/2) = +5.5 \text{ kJ}.$$

**ASSESS** The work is positive because positive work must be expended by an external agent other than the gas to compress the gas.

- 21. INTERPRET** The thermodynamic process here is adiabatic, so no heat flows between the system (the gas) and its environment. We are to find the volume change needed to double the temperature.

**DEVELOP** In an adiabatic process,  $Q = 0$ , so the first law of thermodynamics (Equation 18.1) becomes  $\Delta U = W$ . The temperature and volume are related by Equation 18.11b:

$$T_1 V_1^{\gamma-1} = \text{constant} = T_2 V_2^{\gamma-1}$$

The temperature doubles, so  $T_2 = 2T_1$  and  $\gamma = 1.28$ , and so we can solve for the fractional change in volume.

**EVALUATE** From the equation above, we have

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1} \Rightarrow \frac{V_2}{V_1} = \left( \frac{T_1}{T_2} \right)^{1/(\gamma-1)}$$

Thus, for the temperature to double, the volume change must be

$$\frac{V_2}{V_1} = \left( \frac{T_1}{T_2} \right)^{1/(\gamma-1)} = \frac{1}{2}^{1/(1.28-1)} = 0.084$$

**ASSESS** We see that increasing the temperature along the adiabat is accompanied by a volume decrease. In addition, since  $pV^\gamma = \text{constant}$ , the final pressure is also increased:

$$p_2 = p_1 \left( \frac{V_1}{V_2} \right)^\gamma = p_1 \left( \frac{1}{0.084} \right)^{1.28} = 24 p_1$$

- 22. INTERPRET** The thermodynamic process here is adiabatic, so no heat flows between the system (the gas) and its environment. We are to find the temperature increase as the gas volume is reduced.

**DEVELOP** In an adiabatic process,  $Q = 0$ , so the first law of thermodynamics (Equation 18.1) becomes  $\Delta U = W$ . The temperature and volume are related by Equation 18.11b:

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$

The final volume is 1/4 of the initial, so  $V_2 = V_1/4$  and  $\gamma = 1.4$ , so we can solve for the change in temperature.

**EVALUATE** From the equation above, we have

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1} \Rightarrow T_2 = T_1 \left( \frac{V_1}{V_2} \right)^{\gamma-1} = (291 \text{ K})(4)^{1.4-1} = 507 \text{ K}$$

Thus, the temperature change is

$$\Delta T = T_2 - T_1 = 507 \text{ K} - 291 \text{ K} = 216 \text{ K}$$

**ASSESS** We see that decreasing the volume along the adiabat is accompanied by a temperature increase. In addition, since  $pV^\gamma = \text{constant}$ , the final pressure is also increased:

$$p_2 = p_1 \left( \frac{V_1}{V_2} \right)^\gamma = p_1 (4)^{1.4} = 6.96 p_1$$

- 23. INTERPRET** This problem involves an ideal gas that we compress in an isothermal process to 7% of its volume. We are to find the work needed to do this.

**DEVELOP** In an isothermal compression of a fixed quantity of ideal gas, work is done on the gas, so we use Equation 18.4,  $W = -nRT \ln(V_2 / V_1)$ , where  $V_2 / V_1 = 0.07$  for this problem.

**EVALUATE** Inserting the given quantities into Equation 18.4 gives

$$W = -nRT \ln(V_2/V_1) = -p_1 V_1 \ln(V_2/V_1) = -(1.013 \times 10^5 \text{ Pa})(6.3 \text{ m}^3) \ln(0.070) = 1.7 \times 10^6 \text{ J.}$$

**ASSESS** The work is positive because positive work must be expended by an external agent other than the gas to compress the gas.

### Section 18.3 Specific Heats of an Ideal Gas

- 24. INTERPRET** This problem involves finding the molar specific heat at constant volume and constant pressure for the given gas mixture, which contains the given quantities of diatomic  $\text{O}_2$  and monatomic Ar.

**DEVELOP** This problem is similar to Example 18.5, so we will use the same approach as outlined there.

**EVALUATE** Because  $\text{O}_2$  has 5 degrees of freedom, so its average energy per molecule is  $5kT/2$ . For Ar, the average energy per molecule is  $3kT/2$  because it is monatomic. The total internal energy is therefore

$$E_{\text{int}} = n_{\text{O}_2} \frac{5}{2} RT + n_{\text{Ar}} \frac{3}{2} RT = \left[ \frac{5}{2}(2.5 \text{ mol}) + \frac{3}{2}(3.0 \text{ mol}) \right] RT = (10.75 \text{ mol}) RT$$

From Equation 18.6, the molar specific heat at constant volume is

$$C_V = \frac{1}{n} \frac{dE_{\text{int}}}{dT} = \frac{(10.75 \text{ mol}) R}{(2.5 + 3.0) \text{ mol}} = 2.0R$$

to two significant figures. From Equation 18.9, the molar specific heat at constant pressure is

$$C_P = C_V + R = 3.0R$$

**ASSESS** We find that the molar specific heat at constant volume is between that of a diatomic molecule ( $2.5R$ ) and a monatomic molecule ( $1.5R$ ), as expected since we have a mixture of these two types of molecules.

- 25. INTERPRET** The problem involves the specific heat of a mixture of gases. We want to know what fraction of the molecules is monatomic, given the specific-heat ratio.

**DEVELOP** The internal energy of a mixture of two ideal gases is

$$E_{\text{int}} = f_1 N \bar{E}_1 + f_2 N \bar{E}_2$$

where  $f_i$  is the fraction of the total number of molecules,  $N$ , of type  $i$ , and  $\bar{E}_i$  is the average energy of a molecule of type  $i$ . Classically,  $\bar{E} = g \left( \frac{1}{2} kT \right)$ , where  $g$  is the number of degrees of freedom (the equipartition theorem). From Equation 18.6, the molar specific heat at constant volume is

$$C_V = \frac{1}{n} \frac{dE_{\text{int}}}{dT} = nR \frac{d}{dT} \left( f_1 g_1 \frac{1}{2} T + f_2 g_2 \frac{1}{2} T \right) = \frac{1}{2} R (f_1 g_1 + f_2 g_2)$$

Suppose that the temperature range is such that  $g_1 = 3$  for the monatomic gas and  $g_2 = 5$  for the diatomic gas, as discussed in Section 18.3. Then

$$C_V = \frac{1}{2} R (3f_1 + 5f_2) = R(2.5 - f_1)$$

where  $f_2 = 1 - f_1$  since the sum of the fractions of the mixture is unity. Now,  $C_V$  can also be specified by the ratio

$$\gamma = \frac{C_P}{C_V} = \frac{C_V + R}{C_V} = 1 + \frac{R}{C_V}$$

$$C_V = \frac{R}{\gamma - 1}$$

Equating the two expressions allows us to solve for  $f_1$ .

**EVALUATE** Solving, we find

$$2.5 - f_1 = \frac{1}{\gamma - 1} = \frac{1}{0.62} \Rightarrow f_1 = 88.7\%$$

**ASSESS** From the equation above, we see that the specific-heat ratio can be written as

$$\gamma = 1 + \frac{1}{2.5 - f_1}$$

In the limit where all the gas molecules are monatomic,  $f_1 = 1$  and  $\gamma = 1.62$ . On the other hand, if all the molecules are diatomic, then  $f_1 = 0$  and the specific-heat ratio is  $\gamma = 1.4$ . The equation yields the expected results in both limits.

- 26. INTERPRET** For this problem, we are to find the specific-heat ratio of a gas consisting of the given ratio of different gas molecules.

**DEVELOP** By generalizing the result of the previous problem, the molar specific heat of a mixture of three gases is

$$C_V = f_1 C_{V1} + f_2 C_{V2} + f_3 C_{V3}.$$

For each gas and the mixture, use  $C_V = R(\gamma - 1)$  to obtain

$$(\gamma - 1)^{-1} = f_1(\gamma_1 - 1)^{-1} + f_2(\gamma_2 - 1)^{-1} + f_3(\gamma_3 - 1)^{-1}$$

which we can evaluate to find  $\gamma$ .

**EVALUATE** When the data specified in the problem is inserted, one gets

$$\frac{1}{\gamma - 1} = \frac{0.55}{0.29} + \frac{0.35}{0.40} + \frac{0.10}{0.67} = 2.92$$

$$\gamma = 1.34$$

**ASSESS** Because we have some triatomic molecules, the specific-heat ratio is less than that for a pure diatomic gas (see previous problem).

- 27. INTERPRET** The thermodynamic process is adiabatic, and we want to know the temperature change when work is done on a monatomic gas and a diatomic gas.

**DEVELOP** In an adiabatic process,  $Q = 0$ ; so from the first law of thermodynamics (Equation 18.1),  $\Delta E_{int} = W$ , where  $W$  is the work done on the gas. From Equation 18.6,  $\Delta E_{int} = nC_V \Delta T$ , the change in temperature is

$$\Delta T = \frac{\Delta E_{int}}{nC_V} = \frac{W}{nC_V}$$

If the work done per mole on the gas is  $W/n = 5.5 \text{ kJ/mol}$ , then  $\Delta T = (5.5 \text{ kJ/mol})/C_V$ .

**EVALUATE (a)** For an ideal monatomic gas,  $C_V = \frac{3}{2}R = \frac{3}{2}[8.314 \text{ J/(mol} \cdot \text{K)}]$ , so  $\Delta T = 441 \text{ K}$ .

**(b)** For an ideal diatomic gas (with five degrees of freedom),  $C_V = \frac{5}{2}R$ , so  $\Delta T = 260 \text{ K}$ .

**ASSESS** Since the diatomic gas has a greater specific heat  $C_V$ , its temperature change is less than that of the monatomic gas.

### EXAMPLE VARIATIONS

- 28. INTERPRET** The constant temperature tells us we're dealing with an isothermal process.

**DEVELOP** Equation 18.4 determines the work:  $-W = nRT \ln(V_2/V_1)$ . Here  $-W$  is just what we're after: the work done by the gas in the bubble. To use this equation, we need the quantity  $nRT$  and the volume ratio  $V_2/V_1$ . We know  $p$  and  $V$  at this depth, so we can use the ideal gas law  $pV = nRT$  to get  $nRT$  and the bubble volume just before it reaches the surface. Then we'll have everything we need to apply Equation 18.4.

**EVALUATE** The ideal gas law gives  $nRT = pV = \frac{4}{3}\pi r^3 p$ . The number of moles  $n$  doesn't change, and  $R$  is a constant, so  $pV$  is itself constant in the isothermal process. That means  $p_1 V_1 = p_2 V_2$ , showing that the volume expands by a factor of 2.85 as the pressure drops from 2.85 atm to 1 atm at the surface—so  $V_2/V_1 = 2.85$ . Then Equation 18.4 gives

$$-W = nRT \ln\left(\frac{V_2}{V_1}\right) = \frac{4}{3}\pi r^3 p \ln 2.85$$

Using the 8.8-mm bubble radius and the 2.85-atm pressure gives 0.863 J for the work.

**ASSESS** The work  $-W$  done by the gas is positive because an expanding bubble pushes water outward and ultimately upward, raising the ocean's gravitational potential energy.

**29. INTERPRET** The constant temperature tells us we're dealing with an isothermal process.

**DEVELOP** Equation 18.4 determines the work:  $-W = nRT \ln(V_2/V_1)$ . Here  $-W$  is just what we're after: the work done by the gas in the bubble. This is because in an isothermal process it will equal the amount of heat absorbed by the bubble,  $Q$ . To use this equation, we need the quantity  $nRT$  and the volume ratio  $V_2/V_1$ . We know  $p$  and  $V$  at this depth, so we can use the ideal gas law  $pV = nRT$  to get  $nRT$  and the bubble volume just before it reaches the surface. Then we'll have everything we need to apply Equation 18.4.

**EVALUATE** The ideal gas law gives  $nRT = pV = \frac{4}{3}\pi r^3 p$ . The number of moles  $n$  doesn't change, and  $R$  is a constant, so  $pV$  is itself constant in the isothermal process. That means  $p_1V_1 = p_2V_2$ , showing that the volume expands by a factor of  $(417\text{ kPa}/101.3\text{ kPa}) = 4.12$  as the pressure drops in traveling to the surface—so  $V_2/V_1 = 4.12$ . Then Equation 18.4 gives

$$Q = -W = nRT \ln\left(\frac{V_2}{V_1}\right) = \frac{4}{3}\pi r^3 p \ln 4.12$$

Using the 0.79-cm bubble radius and the 417 kPa pressure gives 1.22 J for the work and thus the heat absorbed.

**ASSESS** If the work  $-W$  done by the gas is positive, while it maintains a constant temperature, it must absorb an equal amount of heat  $Q$ .

**30. INTERPRET** The constant temperature tells us we're dealing with an isothermal process.

**DEVELOP** Equation 18.4 determines the work:  $W = -nRT \ln(V_2/V_1)$ . Here  $W$  is just what we're after: the work done onto the air as it compresses. To use this equation, we need the quantity  $nRT$  and the volume ratio  $V_2/V_1$ . We know  $p$  and  $V$  at this depth, so we can use the ideal gas law  $pV = nRT$  to get  $nRT$  and the air volume at this depth. Then we'll have everything we need to apply Equation 18.4.

**EVALUATE** The number of moles  $n$  doesn't change, and  $R$  is a constant, so  $pV$  is itself constant in the isothermal process. That means  $p_1V_1 = p_2V_2$ , showing that  $V_1/V_2 = p_2/p_1 = 3.46$ , and the volume compresses by a factor of 3.46—so  $V_2/V_1 = 1/3.46$ . Then Equation 18.4 gives

$$W = -nRT \ln\left(\frac{V_2}{V_1}\right) = -p_1V_1 \ln\left(\frac{1}{3.46}\right)$$

Using the 5.25 L of air at normal atmospheric pressure gives 660 J for the work on the air.

**ASSESS** The work  $W$  done onto the gas is positive because the final volume is less than the initial volume.

**31. INTERPRET** The constant temperature tells us we're dealing with an isothermal process.

**DEVELOP** Equation 18.4 determines the work:  $-W = nRT \ln(V_2/V_1)$ . Here  $-W$  is just what we're after: the work done by the gas in the balloon. This is because in an isothermal process it will equal the amount of heat absorbed by the balloon,  $Q$ . To use this equation, we need the quantity  $nRT$  and the volume ratio  $V_2/V_1$ . We don't know  $p$  and  $V$  after the pump has been turned on, but we do know the pressure before the pump is turned on and we know the factor by which the gas volume expands. We also know the amount of heat absorbed, so we know the amount of work done onto the gas. Thus, we can use the fact that this is an isothermal expansion, as well as Equation 18.4 to solve for the final pressure in the chamber and for the initial volume the air occupies.

**EVALUATE** The ideal gas law gives  $nRT = pV$ . The number of moles  $n$  doesn't change, and  $R$  is a constant, so  $pV$  is itself constant in the isothermal process. That means  $p_1V_1 = p_2V_2$ , showing that if the volume expands by a factor of 3.50, the pressure drops by a factor of 3.50. This means the final pressure in the chamber is equal to  $(101.3\text{ kPa})/3.50 = 28.9\text{ kPa}$ . Then Equation 18.4 gives

$$Q = -W = p_1V_1 \ln\left(\frac{V_2}{V_1}\right) = p_1 \frac{4}{3}\pi r_1^3 \ln 3.50$$

Using the 147 J absorbed by the air as it expands and the normal atmospheric pressure of the chamber before the pump is turned on, gives 6.5 cm for the initial radius  $r_1$  of the balloon. Thus, the balloon's original diameter is 13.0 cm.

**ASSESS** Since gas volume expands by a factor of 3.50, the radius expands by the cubic root of that factor, and the final diameter of the balloon is 19.7 cm.

- 32. INTERPRET** The problem involves a cyclic process with three separate stages of the cycle: adiabatic ( $AB$ ), isochoric ( $BC$ ), and isothermal ( $CA$ ). We are to calculate the net work done on the gas over the entire cycle.

**DEVELOP** The work done *on* the gas in each segment of the cycle is summarized in Table 18.1. For the adiabatic process (path  $AB$ ), the work done on the gas is given by Equation 18.12:

$$W_{AB} = \frac{p_B V_B - p_A V_A}{\gamma - 1}$$

Where we note that  $p_B = p_A (V_A/V_B)^\gamma$ . For the isochoric process (path  $BC$ ),  $\Delta V = 0$  so  $W = 0$ . Finally, for the isothermal process ( $CA$ ), the work done on the gas is given by Equation 18.4:

$$W_{CA} = -nRT_A \ln \left( \frac{V_A}{V_C} \right)$$

**EVALUATE** Using the equations above, we obtain

$$W_{AB} = \frac{p_A (V_A/V_B)^\gamma V_B - p_A V_A}{\gamma - 1} = \frac{p_A V_A (3^{\gamma-1} - 1)}{\gamma - 1} = \frac{(89.2 \text{ kPa})(8.26 \text{ L})(3^{0.40} - 1)}{0.40} = 1017 \text{ J} \quad (\text{adiabatic})$$

$$W_{BC} = 0 \quad (\text{isochoric})$$

$$W_{CA} = -nRT_A \ln \left( \frac{V_A}{V_C} \right) = -p_A V_A \ln \left( \frac{V_A}{V_C} \right) = -(89.2 \text{ kPa})(8.26 \text{ L}) \ln(3) = -810 \text{ J} \quad (\text{isothermal})$$

Adding up all the contributions, the net work done *on* the gas is

$$W_{ABCA} = W_{AB} + W_{BC} + W_{CA} = 1017 \text{ J} + 0 \text{ J} - 810 \text{ J} = 207 \text{ J}$$

**ASSESS** The final answer is positive because we've done net work on the gas; that's always the case in going counterclockwise around a cyclic path in a  $pV$  diagram.

- 33. INTERPRET** The problem involves a cyclic process with three separate stages of the cycle: adiabatic ( $AB$ ), isochoric ( $BC$ ), and isothermal ( $CA$ ). We are to calculate the gas pressure at the start of the cycle knowing the net work done on the gas over the entire cycle.

**DEVELOP** The work done *on* the gas in each segment of the cycle is summarized in Table 18.1. For the adiabatic process (path  $AB$ ), the work done on the gas is given by Equation 18.12:

$$W_{AB} = \frac{p_B V_B - p_A V_A}{\gamma - 1}$$

Where we note that  $p_B = p_A (V_A/V_B)^\gamma$ . For the isochoric process (path  $BC$ ),  $\Delta V = 0$  so  $W = 0$ . Finally, for the isothermal process ( $CA$ ), the work done on the gas is given by Equation 18.4:

$$W_{CA} = -nRT_A \ln \left( \frac{V_A}{V_C} \right)$$

**EVALUATE** Using the equations above, we obtain

$$W_{AB} = \frac{p_A (V_A/V_B)^\gamma V_B - p_A V_A}{\gamma - 1} = \frac{p_A V_A (2^{\gamma-1} - 1)}{\gamma - 1} \quad (\text{adiabatic})$$

$$W_{BC} = 0 \quad (\text{isochoric})$$

$$W_{CA} = -nRT_A \ln \left( \frac{V_A}{V_C} \right) = -p_A V_A \ln 2 \quad (\text{isothermal})$$

Adding up all the contributions, the net work done *on* the gas is

$$W_{ABCA} = W_{AB} + W_{BC} + W_{CA} = p_A V_A \left( \frac{(2^{\gamma-1} - 1)}{\gamma - 1} - \ln 2 \right)$$

Solving for the initial pressure, and plugging in the given values for the initial volume, work done on the gas during the cycle, and the specific heat ratio, we obtain



$$p_A = \frac{W_{ABCA}}{V_A \left( \frac{(2^{\gamma-1} - 1)}{\gamma - 1} - \ln 2 \right)} = 165 \text{ kPa}$$

**ASSESS** Since the gas returns to its initial temperature and volume at the end of the cycle it will also return to its initial pressure.

- 34. INTERPRET** The problem involves a cyclic process with four separate stages of the cycle: isochoric ( $AB$ ), adiabatic ( $BC$ ), isobaric ( $CD$ ), and isothermal ( $DA$ ). We are to find an exact expression for the net work done on the gas over the entire cycle, and evaluate it to three significant figures.

**DEVELOP** The work done *on* the gas in each segment of the cycle is summarized in Table 18.1. For the isochoric process (path  $AB$ ),  $\Delta V = 0$  so  $W = 0$ . For the adiabatic process (path  $BC$ ) the work done on the gas is given by Equation 18.12:

$$W_{BC} = \frac{p_C V_C - p_B V_B}{\gamma - 1}$$

Where we note that  $p_C = p_B (V_B/V_C)^\gamma$ . For the isobaric process (path  $CD$ ), the work done is

$$W_{CD} = -p_C (V_D - V_C)$$

Finally, for the isothermal process ( $DA$ ), the work done on the gas is given by Equation 18.4:

$$W_{DA} = -nRT_A \ln \left( \frac{V_A}{V_D} \right)$$

**EVALUATE** Using the equations above, we obtain

$$W_{AB} = 0 \quad (\text{isochoric})$$

$$W_{BC} = \frac{p_B (V_B/V_C)^\gamma V_C - p_B V_B}{\gamma - 1} = \frac{p_B V_B (5^{\gamma-1} - 1)}{\gamma - 1} \quad (\text{adiabatic})$$

$$W_{CD} = -p_C (V_D - V_C) \quad (\text{isobaric})$$

$$W_{DA} = -nRT_A \ln \left( \frac{V_A}{V_D} \right) = -p_A V_A \ln \left( \frac{V_A}{V_D} \right) \quad (\text{isothermal})$$

Adding up all the contributions, the net work done *on* the gas is

$$W_{ABCA} = W_{AB} + W_{BC} + W_{CD} + W_{DA} = \frac{p_B V_B (5^{\gamma-1} - 1)}{\gamma - 1} - p_C (V_D - V_C) - p_A V_A \ln(2 \times 5^\gamma)$$

Next, we find the relationships between the various pressures and volumes throughout the cycle and express all quantities in terms of the initial pressure and volume. We first note the volumes  $V_A$  and  $V_B$  are after the isochoric process, and the pressure doubles since the temperature doubles, meaning  $p_B = 2p_A$ . Then we recall that  $p_C = p_B (V_B/V_C)^\gamma = 2p_A 5^\gamma$ . Lastly, we note that the product of pressure and volume before and after the isothermal process are equal, meaning we can write  $p_D V_D = p_A V_A$ . Since the path  $CD$  was isobaric, this means  $p_D = p_C$ , and thus  $V_D = (V_A / (2 \times 5^\gamma))$ . We can now rewrite the work done throughout the entire cycle in terms of the initial pressure and volume. We plug in the given specific heat ratio and simplify to find

$$W_{ABCA} = \frac{2p_A V_A (5^{\gamma-1} - 1)}{\gamma - 1} - 2p_A 5^\gamma \left( \left( \frac{V_A}{2 \times 5^\gamma} \right) - \left( \frac{V_A}{5} \right) \right) - p_A V_A \ln(2 \times 5^\gamma)$$

$$W_{ABCA} = \left[ 7 \times 5^{2/5} - 6 - \ln(2 \times 5^{7/5}) \right] p_A V_A$$

Numerically evaluating this to three significant figures gives  $4.38 p_A V_A$ .

**ASSESS** Since the process is cyclic, its pressure, volume, and temperature will return to the initial values at the end of each cycle. We can thus express all intermediate quantities in terms of these using the equations which govern these energy exchange processes.

- 35. INTERPRET** The problem involves a cyclic process with four separate stages of the cycle: isochoric ( $AB$ ), adiabatic ( $BC$ ), isobaric ( $CD$ ), and isothermal ( $DA$ ). We are to find the initial pressure knowing the work done *on* the gas in one complete cycle.

**DEVELOP** The work done *on* the gas in each segment of the cycle is summarized in Table 18.1. For the isochoric process (path  $AB$ ),  $\Delta V = 0$  so  $W = 0$ . For the adiabatic process (path  $BC$ ) the work done on the gas is given by Equation 18.12:

$$W_{BC} = \frac{p_C V_C - p_B V_B}{\gamma - 1}$$

Where we note that  $p_C = p_B (V_B/V_C)^\gamma$ . For the isobaric process (path  $CD$ ), the work done is

$$W_{CD} = -p_C (V_D - V_C)$$

Finally, for the isothermal process ( $DA$ ), the work done on the gas is given by Equation 18.4:

$$W_{DA} = -nRT_A \ln \left( \frac{V_A}{V_D} \right)$$

**EVALUATE** Using the equations above, we obtain

$$W_{AB} = 0 \quad (\text{isochoric})$$

$$W_{BC} = \frac{p_B (V_B/V_C)^\gamma V_C - p_B V_B}{\gamma - 1} = \frac{p_B V_B (5^{\gamma-1} - 1)}{\gamma - 1} \quad (\text{adiabatic})$$

$$W_{CD} = -p_C (V_D - V_C) \quad (\text{isobaric})$$

$$W_{DA} = -nRT_A \ln \left( \frac{V_A}{V_D} \right) = -p_A V_A \ln \left( \frac{V_A}{V_D} \right) \quad (\text{isothermal})$$

Adding up all the contributions, the net work done *on* the gas is

$$W_{ABCA} = W_{AB} + W_{BC} + W_{CD} + W_{DA} = \frac{p_B V_B (5^{\gamma-1} - 1)}{\gamma - 1} - p_C (V_D - V_C) - p_A V_A \ln(2 \times 5^\gamma)$$

Next, we find the relationships between the various pressures and volumes throughout the cycle and express all quantities in terms of the initial pressure and volume. We first note the volumes  $V_A$  and  $V_B$  are after the isochoric process, and the pressure grows by a factor of  $373/273$  since the temperature increases by this factor, meaning  $p_B = 1.37 p_A$ . Then we recall that  $p_C = p_B (V_B/V_C)^\gamma = 1.37 p_A 8^\gamma$ . Lastly, we note that the product of pressure and volume before and after the isothermal process are equal, meaning we can write  $p_D V_D = p_A V_A$ . Since the path  $CD$  was isobaric, this means  $p_D = p_C$ , and thus  $V_D = (V_A / (1.37 \times 8^\gamma))$ . We can now rewrite the work done throughout the entire cycle in terms of the initial pressure and volume. We plug in the given specific heat ratio and simplify to find

$$W_{ABCA} = \frac{1.37 p_A V_A (8^{\gamma-1} - 1)}{\gamma - 1} - 1.37 p_A 8^\gamma \left( \left( V_A / (1.37 \times 8^\gamma) \right) - (V_A / 8) \right) - p_A V_A \ln(1.37 \times 8^\gamma)$$

$$910 \text{ J} = \left[ 1.37 \frac{1.4 \times 8^{0.4} - 1}{0.4} - 1 - \ln(1.37 \times 8^{1.4}) \right] (2.0 \text{ L}) p_A$$

Numerically evaluating this and solving for the initial pressure we find  $p_A = 136 \text{ kPa}$ .

**ASSESS** Since the process is cyclic, its pressure, volume, and temperature will return to the initial values at the end of each cycle. We can thus express all intermediate quantities in terms of these using the equations which govern these energy exchange processes.

## PROBLEMS

- 36. INTERPRET** The constant temperature of  $440 \text{ K}$  indicates that the process is isothermal. We are given the amount of work done by the gas, and are asked to find the heat it absorbs and how many moles of gas there are.

**DEVELOP** Apply the ideal gas law given in Equation 17.2:  $pV = nRT$ . For an isothermal process,  $T = \text{constant}$ , so we obtain  $p_1 V_1 = p_2 V_2$ . Since  $\Delta E_{\text{int}} = 0$  for an isothermal process, the heat absorbed is the negative of the total work done on the gas (Equation 18.4).

$$Q = -W = nRT \ln \left( \frac{V_2}{V_1} \right)$$

For this problem, the gas does 3.3 kJ of work on its surroundings, so the surroundings do  $-3.3$  kJ of work on the gas, so  $W = -3.3$  kJ.

**EVALUATE** (a) Using the equation above, the heat absorbed is  $Q = -W = 3.3$  kJ.

(b) Solving the expression above for the number  $n$  of moles gives

$$n = \frac{-W}{RT \ln(V_2/V_1)} = \frac{3.3 \text{ kJ}}{[8.314 \text{ J/(mol} \cdot \text{K)}](440 \text{ K}) \ln(10)} = 0.39 \text{ mol}$$

**ASSESS** The heat absorbed by the gas is equal to the work done by the gas on its surrounding as it expands, and there is no change in temperature.

- 37. INTERPRET** We're asked to find the rate that heat is produced in the body when cycling. This involves the first law of thermodynamics.

**DEVELOP** We're dealing with the rates of work and heat production, so we'll use Equation 18.2:

$\frac{dE_{\text{int}}}{dt} = \frac{dQ}{dt} + \frac{dW}{dt}$ . If the body is releasing stored food energy, that corresponds to a *decrease* in the body's internal energy:  $dE_{\text{int}}/dt = -500 \text{ W}$ . Likewise, the mechanical power quoted is for work done *by* the body, so  $dW/dt = -120 \text{ W}$ . We are looking for the rate at which the body produces heat, which is technically heat that it is losing ( $-dQ$ ).

**EVALUATE** The rate of heat production is the negative heat absorbed, so

$$-\frac{dQ}{dt} = -\frac{dE_{\text{int}}}{dt} + \frac{dW}{dt} = -(-500 \text{ W}) + (-120 \text{ W}) = 380 \text{ W}$$

**ASSESS** All the signs can be confusing, but essentially the body burns stored energy and some of it is used to do work and the rest is released as heat.

- 38. INTERPRET** This problem deals with isothermal compression, so the temperature is constant in this process. We are to find the temperature at which the given quantity of ideal gas is compressed from 4.0 L to 3.5 L.

**DEVELOP** Apply Equation 18.4,

$$Q = -W = nRT \ln(V_2/V_1)$$

where  $W$  is the work done by the gas,  $V_1 = 4.0 \text{ L}$ , and  $V_2 = 3.5 \text{ L}$ . Since we do 68 J of work on the gas to compress it,  $W = 68 \text{ J}$ , so we can solve this expression for the temperature  $T$ .

**EVALUATE** The temperature of the ideal gas is

$$T = \frac{-W}{nR \ln(V_2/V_1)} = \frac{-68 \text{ J}}{(0.35 \text{ mol})[8.314 \text{ J/(mol} \cdot \text{K)}] \ln(3.5/4.0)} = 175 \text{ K}$$

**ASSESS** This gas is about 118 K below room temperature.

- 39. INTERPRET** Assume the air inside the spherical bubble behaves like an ideal gas at constant temperature, so the process is isothermal. We need to find the diameter of the bubble at maximum pressure and the work done on the gas in compressing it.

**DEVELOP** Apply the ideal gas law given in Equation 17.2:  $pV = nRT$ . For an isothermal process,  $T = \text{constant}$ , which leads to  $p_1V_1 = p_2V_2$ . Since the volume of a spherical bubble of diameter  $d$  is

$$V = \frac{4\pi}{3} \left( \frac{d}{2} \right)^3 = \frac{\pi d^3}{6}$$

the relationship between the diameter and the pressure is

$$p_1 \left( \frac{\pi d_1^3}{6} \right) = p_2 \left( \frac{\pi d_2^3}{6} \right)$$

$$\frac{d_2}{d_1} = \left( \frac{p_1}{p_2} \right)^{1/3}$$

**EVALUATE** (a) Using the equation above, we find the diameter at the maximum pressure to be

$$d_2 = \left(\frac{p_1}{p_2}\right)^{1/3} d_1 = \left[\frac{(80 + 760) \text{ mm of Hg}}{(125 + 760) \text{ mm of Hg}}\right]^{1/3} (1.52 \text{ mm}) = 1.49 \text{ mm}$$

(b) The work done *on* the air is given by Equation 18.4, or

$$\begin{aligned} W_{\text{on air}} &= -nRT \ln\left(\frac{V_2}{V_1}\right) = -p_1 V_1 \ln\left(\frac{p_1}{p_2}\right) = p_1 V_1 \ln\left(\frac{p_2}{p_1}\right) \\ &= (840 \text{ mm of Hg}) \left(\frac{101.3 \text{ kPa}}{760 \text{ mm of Hg}}\right) \frac{\pi}{6} (1.52 \text{ mm})^3 \ln\left(\frac{885 \text{ mm of Hg}}{840 \text{ mm of Hg}}\right) \\ &= 10.7 \text{ } \mu\text{J} \end{aligned}$$

**ASSESS** Positive work is done by the blood in compressing the air bubble.

- 40. INTERPRET** This problem involves compressing a gas in an isothermal process, so the temperature is constant. We are to find how much work it takes to reduce the volume by a factor of 15, given the work it takes to decrease the volume by a factor of two.

**DEVELOP** Apply Equation 18.4 to both situations. To compress a gas by a factor of two, we have

$$-W_2 = nRT \ln(2)$$

and to compress it by a factor of 15 gives

$$-W_{15} = nRT \ln(15)$$

Given that  $T$  is constant, we can take the ratio of these expressions to find the work  $W_{15}$  needed to compress the gas by a factor of 15.

**EVALUATE** The work needed to compress the gas by a factor of 15 is

$$W_{15} = W_2 \frac{\ln(15)}{\ln(2)} = (0.8 \text{ kJ}) \frac{\ln(15)}{\ln(2)} = 3.1 \text{ kJ}$$

**ASSESS** Note that the work required is logarithmic in volume.

- 41. INTERPRET** The thermodynamic process here is adiabatic, with no heat flowing between the system (the gas) and its environment. We are to find the specific-heat ratio  $\gamma$ , given the fraction increase in the pressure of the gas.

**DEVELOP** In an adiabatic process,  $Q = 0$ , and the first law of thermodynamics becomes  $\Delta E_{\text{int}} = W$ . The pressure and volume are related by Equation 18.11a:  $PV^\gamma = \text{constant}$ . This implies

$$p_1 V_1^\gamma = p_2 V_2^\gamma \Rightarrow \frac{p_2}{p_1} = \left(\frac{V_1}{V_2}\right)^\gamma$$

So, we can solve for  $\gamma$ .

**EVALUATE** Taking the natural logarithm on both sides of the above equation to solve for  $\gamma$ , we obtain

$$\ln\left(\frac{p_2}{p_1}\right) = \gamma \ln\left(\frac{V_1}{V_2}\right) \Rightarrow \gamma = \frac{\ln(p_2/p_1)}{\ln(V_1/V_2)} = \frac{\ln(2.51)}{\ln(2)} = 1.33$$

**ASSESS** The value of  $\gamma$  indicates that the gas consists of polyatomic molecules (see Problems 25 and 26).

- 42. INTERPRET** The problem involves compressing a gas adiabatically.

**DEVELOP** For an adiabatic process, the initial pressure and volume are related to the final pressure and volume through Equation 18.11a:  $p_1 V_1^\gamma = p_2 V_2^\gamma$ . We can use this to rewrite Equation 18.12 for the work done on the gas in going from the initial to the final volume:

$$W = \frac{p_2 V_2 - p_1 V_1}{\gamma - 1} = \frac{p_1 V_1}{\gamma - 1} \left[ \left(\frac{V_1}{V_2}\right)^{\gamma-1} - 1 \right]$$

**EVALUATE** (a) The final pressure in this case is

$$p_2 = p_1 \left( \frac{V_1}{V_2} \right)^\gamma = (98.5 \text{ kPa}) \left( \frac{6.25 \text{ L}}{4.18 \text{ L}} \right)^{1.40} = 173 \text{ kPa}$$

(b) The work done on the gas in compressing it from  $V_1 = 6.25 \text{ L}$  to  $V_2 = 4.18 \text{ L}$  is

$$W = \frac{(98.5 \text{ kPa})(6.25 \text{ L})}{1.40 - 1} \left[ \left( \frac{6.25 \text{ L}}{4.18 \text{ L}} \right)^{1.40 - 1} - 1 \right] = 269 \text{ J}$$

**ASSESS** The volume decreases by a factor of 1.5, but the pressure increases by a factor of 1.76. The pressure increases due to both the increase in the density ( $N/V$ ) and an increase in the internal energy of the gas ( $\Delta E_{\text{int}} = nC_V \Delta T$ ).

- 43. INTERPRET** This problem involves a cyclic process. The three processes that make up the cycle are: isothermal ( $AB$ ), isochoric ( $BC$ ), and isobaric ( $CA$ ). We are given the pressure at point  $A$  and are to find the pressure at point  $B$  and the net work done on the gas.

**DEVELOP** Along the isotherm  $AB$   $T = \text{constant}$ , so the ideal gas law (Equation 17.2,  $pV = nRT$ ) gives  $p_A V_A = p_B V_B$ . For an isothermal process, the work  $W$  done on the gas is (Equation 18.4):

$$W = -nRT \ln \left( \frac{V_2}{V_1} \right)$$

**EVALUATE** (a) If  $AB$  is an isotherm, with  $V_A = 5 \text{ L}$  and  $V_B = 1 \text{ L}$ , then the ideal gas law gives

$$P_B = \left( \frac{V_A}{V_B} \right) P_A = \left( \frac{5}{1} \right) (60 \text{ kPa}) = 300 \text{ kPa}$$

(b) The work  $W$  done on the gas in the isothermal process  $AB$  is

$$W_{AB} = -nRT_A \ln \left( \frac{V_B}{V_A} \right) = -p_A V_A \ln \left( \frac{V_B}{V_A} \right) = -(300 \text{ J}) \ln \left( \frac{1.0 \text{ L}}{5.0 \text{ L}} \right) = 483 \text{ J}$$

The process  $BC$  is isochoric (constant volume) so  $W_{BC} = 0$ . The process  $CA$  is isobaric (constant pressure) so the work done by the gas is (see Table 18.1)

$$W'_{CA} = p_A (V_A - V_C) = (60 \text{ kPa})(5.0 \text{ L} - 1.0 \text{ L}) = 240 \text{ J}$$

or the work done on the gas is  $W_{CA} = -240 \text{ J}$ . The total work done on the gas is the sum of these three contributions, or

$$W_{ABCA} = W_{AB} + W_{BC} + W_{CA} = 483 \text{ J} + 0 - 240 \text{ J} = 243 \text{ J} \approx 240 \text{ J}$$

to two significant figures.

**ASSESS** Since the process is cyclic, the system returns to its original state, there's no net change in internal energy, so  $\Delta E_{\text{int}} = 0$ . This implies that  $Q = -W_{ABCA} = -240 \text{ J}$ . That is, 240 J of heat must come out of the system.

- 44. INTERPRET** This problem is similar to the previous one, except that the isotherm curve  $AB$  is now to be considered as an adiabat. We are given the specific heat ratio and are to find the pressure change in going from point  $A$  to point  $B$ , and the net work done by the cycle.

**DEVELOP** For an adiabat, Equation 18.11a is valid. Applying this to points  $A$  and  $B$ , and taking the ratio, gives

$$\frac{p_A}{p_B} = \left( \frac{V_B}{V_A} \right)^\gamma$$

which allows us to find the pressure at point  $B$ . The calculation for part (b) then proceeds as for Problem 18.43, with the help of Equation 18.12, which gives the work done on the gas in an adiabatic process.

**EVALUATE** (a) The pressure at point  $B$  is

$$p_B = p_A \left( \frac{V_A}{V_B} \right)^\gamma = (60 \text{ kPa}) \left( \frac{5.0 \text{ L}}{1.0 \text{ L}} \right)^{1.4} = 570 \text{ kPa}$$

to two significant figures.

(b) The work done on the gas between points  $A$  and  $B$  is

$$W_{AB} = \frac{P_B V_B - P_A V_A}{\gamma - 1} = \frac{(571 \text{ kPa})(1.0 \text{ L}) - (60 \text{ kPa})(5.0 \text{ L})}{0.4} = 678 \text{ J}$$

The process BC is isochoric (constant volume), so  $W_{BC} = 0$ . The process CA is the same as for Problem 18.41, so  $W_{CA} = -240 \text{ J}$ . Summing all three contributions gives a net work done on the gas of

$$W_{ABCA} = W_{AB} + W_{BC} + W_{CA} = 678 \text{ J} + 0 - 240 \text{ J} = 440 \text{ J}.$$

to two significant figures.

**ASSESS** As in the preceding problem, heat must flow out of the system to keep the internal energy unchanged in the cyclic process; here the quantity of heat is 440 J.

- 45. INTERPRET** We identify the thermodynamic process here as adiabatic compression.

**DEVELOP** In an adiabatic process,  $Q = 0$ , and the first law of thermodynamics becomes  $\Delta E_{\text{int}} = W$ . The temperature and volume are related by Equation 18.11b:

$$TV^{\gamma-1} = \text{constant}$$

From the equation above, we obtain

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1} \Rightarrow T_2 = T_1 \left( \frac{V_1}{V_2} \right)^{\gamma-1}$$

where  $V_1/V_2$  is the compression ratio (for  $T$  and  $V$  at maximum compression).

**EVALUATE** Inserting the values given gives

$$T_2 = T_1 \left( \frac{V_1}{V_2} \right)^{\gamma-1} = (307 \text{ K})(8.5)^{0.4} = 723 \text{ K} = 450^\circ\text{C}$$

**ASSESS** Note that the temperature  $T$  appearing in the gas laws is the absolute temperature. The higher the compression ratio  $V_1/V_2$ , the greater the temperature at the maximum compression, and hence, the higher the thermal efficiency.

- 46. INTERPRET** This problem involves an adiabatic process that results in a change of pressure and volume. We are to find the fractional change in volume given that the pressure increases by a factor of 2.

**DEVELOP** Apply Equation 18.11a,  $pV^\gamma = \text{constant}$ . Using subscripts 1 and 2 to indicate before and after the process, respectively, this gives

$$p_1 V_1^\gamma = \text{constant} = p_2 V_2^\gamma$$

$$\frac{V_2}{V_1} = \left( \frac{p_1}{p_2} \right)^{1/\gamma}$$

which we can solve for the fractional change in volume, using  $\gamma = 1.4$  and  $p_2 = 2p_1$ .

**EVALUATE** Inserting the given quantities into the expression above gives

$$\frac{V_2}{V_1} = \left( \frac{p_1}{p_2} \right)^{1/\gamma} = (0.5)^{1/1.4} = 0.61$$

**ASSESS** Thus, the new volume is almost 50% of the original volume, which seems reasonable given that the pressure has doubled.

- 47. INTERPRET** We identify the thermodynamic process here as adiabatic compression. We are to find the temperature and pressure in the cylinder when it is at maximum compression.

**DEVELOP** In an adiabatic process,  $Q = 0$ , and the first law of thermodynamics becomes  $\Delta E_{\text{int}} = W$ . Because we are dealing with an adiabatic process, the temperature and volume are related by Equation 18.11b:

$$TV^{\gamma-1} = \text{constant}$$

Applying this expression before (subscript 1) and after (subscript 2) compression gives

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1} \Rightarrow T_2 = T_1 \left( \frac{V_1}{V_2} \right)^{\gamma-1}$$

where  $V_1/V_2 = 10.8$  is the compression ratio (for  $T$  and  $V$  at maximum compression) and  $\gamma = 1.40$ . In addition, since  $pV^\gamma = \text{constant}$  (Equation 18.11a), the final pressure is  $p_2 = p_1 (V_1/V_2)^\gamma$ .

**EVALUATE** (a) Substituting the values given in the problem statement, we find the air temperature at the maximum compression to be

$$T_2 = T_1 \left( \frac{V_1}{V_2} \right)^{\gamma-1} = (342 \text{ K})(10.8)^{0.40} = 866 \text{ K}$$

(b) The corresponding pressure is

$$p_2 = p_1 (V_1/V_2)^\gamma = (1.50)(101.3 \text{ kPa})(10.8)^{1.4} = 4.25 \text{ MPa} = 42.0 \text{ atm}$$

**ASSESS** The higher the compression ratio  $V_1/V_2$ , the greater the temperature and pressure at the maximum compression, and hence, a higher thermal (fuel) efficiency.

- 48. INTERPRET** The thermodynamic process here is adiabatic, so no heat flows between the system (the gas) and its environment. We are to find the change in temperature and volume as the pressure is reduced.

**DEVELOP** In an adiabatic process,  $Q = 0$ , so the first law of thermodynamics (Equation 18.1) becomes  $\Delta E_{\text{int}} = W$ . The temperature, pressure, and volume are related by Equation 18.11:

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}, \quad p_1 V_1^\gamma = p_2 V_2^\gamma$$

The initial pressure is  $p_1 = 1.0 \text{ atm}$ , and the final pressure is  $p_2 = 0.340 \text{ atm}$ . With  $\gamma = 5/3$ , we can solve for the new volume and temperature.

**EVALUATE** (a) From the equation above, we have

$$V_2 = V_1 (p_1/p_2)^{1/\gamma} = (1.75 \times 10^3 \text{ m}^3) \left( \frac{1.0 \text{ atm}}{0.34 \text{ atm}} \right)^{3/5} = 3.43 \times 10^3 \text{ m}^3$$

(b) The final temperature is

$$T_2 = T_1 \left( \frac{V_1}{V_2} \right)^{\gamma-1} = (285 \text{ K}) \left( \frac{3.43 \times 10^3 \text{ m}^3}{1.75 \times 10^3 \text{ m}^3} \right)^{(5/3)-1} = 185 \text{ K} = -88^\circ \text{C}$$

**ASSESS** We see that decreasing the pressure along the adiabat is accompanied by a temperature decrease. In addition, since  $pV^\gamma = \text{constant}$ , the final volume increases.

- 49. INTERPRET** This problem involves an adiabatic compression of a gas, for which we know the initial and final temperatures. We are to find the ratio of the final pressure to the initial pressure of the gas.

**DEVELOP** For an adiabatic process, the pressure and volume are related by Equation 18.11a:

$$pV^\gamma = \text{constant}$$

Rearranging this equation and using the ideal gas law (Equation 17.2,  $pV = nRT$ ) gives

$$pV^\gamma = p^{1-\gamma} (pV)^\gamma = p^{1-\gamma} (nRT)^\gamma = \text{constant} \Rightarrow p_1 T_1^{\gamma/(1-\gamma)} = p_2 T_2^{\gamma/(1-\gamma)}$$

**EVALUATE** Inserting the given quantities into the expression above for temperature gives

$$\frac{p_2}{p_1} = \left( \frac{T_2}{T_1} \right)^{\frac{\gamma}{\gamma-1}} = \left( \frac{293 \text{ K}}{33 \text{ K}} \right)^{\frac{5/3}{(5/3)-1}} = 235$$

**ASSESS** The pressure increase is enormous in order to accompany the increase in temperature along the adiabat. The corresponding change in volume is

$$\frac{V_2}{V_1} = \left( \frac{p_1}{p_2} \right)^{\frac{1}{\gamma}} = \left( \frac{1}{235} \right)^{3/5} = 3.78 \times 10^{-2}$$

The final volume is only about 4% of the original.

- 50. INTERPRET** This problem explores how different ways of compressing volume (isothermal, adiabatic, or isobaric) affects the internal energy of the system. The internal energy of  $n$  moles of diatomic gas is  $E_{\text{int}} = \frac{5}{2}nRT$ , by equipartition theorem.

**DEVELOP** In an isothermal process, the temperature  $T$  is kept constant, so  $\Delta E_{\text{int}} = 0$ . In an isobaric process,  $p = \text{constant}$ , and  $T_2/T_1 = V_2/V_1$ . Finally, in an adiabatic process,  $Q = 0$  and

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

**EVALUATE** (a) For isothermal compression,  $\Delta T = 0$ , so  $\Delta E_{\text{int}} = 0$ . There is no change in internal energy.

(b) For isobaric compression,

$$\frac{E_{\text{int}2}}{E_{\text{int}1}} = \frac{5nRT_2/2}{5nRT_1/2} = \frac{T_2}{T_1} = \frac{V_2}{V_1} = \frac{1}{2}.$$

(c) For adiabatic compression,  $\gamma = 7/5$  for diatomic gas,

$$\frac{E_{\text{int}2}}{E_{\text{int}1}} = \frac{5nRT_2/2}{5nRT_1/2} = \frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = (2)^{1.4-1} = 1.32$$

**ASSESS** As the gas volume is halved, the internal energy of the gas decreases if compression is isobaric, but increases if the compression is adiabatic. There is no change if the compression is isothermal.

- 51. INTERPRET** This problem explores how different ways of adding heat (isothermal, isochoric, or isobaric) affects the final temperature of the system. Starting with the given quantity of gas at the given temperature, we are to find the work done by the gas upon adding heat to the gas via these different processes.

**DEVELOP** In an isothermal process, the temperature  $T$  is kept constant, so  $\Delta E_{\text{int}} = 0$ . The first law of thermodynamics (Equation 18.1) therefore gives  $Q = -W$ . In an isochoric process,  $\Delta V = 0$  and  $W = 0$  (see Equation 18.7), so the first law of thermodynamics gives  $Q = \Delta E_{\text{int}} = nC_V\Delta T$ . Finally, in an isobaric process,  $\Delta p = 0$  and

$$Q = nC_p\Delta T = n(C_V + R)\Delta T$$

Use these expressions to solve for  $\Delta T$  and  $W$  for each case.

**EVALUATE** (a) With  $\Delta E_{\text{int}} = 0$ ,  $W = -Q = -1.75 \text{ kJ}$ . For an isothermal process, the temperature is constant so  $T_2 = 255 \text{ K}$ .

(b) Solving the expression above for  $\Delta T$ , we find for an isochoric process

$$\Delta T = \frac{Q}{nC_V} = \frac{1.75 \text{ kJ}}{(3.5 \text{ mol})(5R/2)} = 24 \text{ K}$$

Therefore,  $T_2 = 255 \text{ K} + \Delta T = 279 \text{ K}$ . Because the volume does not change,  $W = 0$ .

(c) In an isobaric process,

$$\Delta T = \frac{Q}{nC_p} = \frac{Q}{n(C_V + R)} = \frac{1.75 \text{ kJ}}{(3.5 \text{ mol})(5R/2 + R)} = 17 \text{ K}$$

and  $T_2 = 255 \text{ K} + 17 \text{ K} = 272 \text{ K}$ . The work done by the gas is

$$W = p\Delta V = nR\Delta T = \frac{R}{C_p}Q = \frac{R}{(7R/2)}Q = \frac{2(1.75 \text{ kJ})}{7} = 500 \text{ J}$$

**ASSESS** Comparing all three cases, we find

$\Delta T$ : isothermal < isobaric < isochoric

$W$ : isochoric < isobaric < isothermal

The results agree with that illustrated in Table 18.1.

- 52. INTERPRET** This problem is an exercise to show the functional relationship in terms of pressure and volume between an adiabatic process and an isothermal process.

**DEVELOP** The equations of an adiabat and an isotherm, passing through the point  $(V_0, p_0)$  in the  $pV$  diagram are

$$p(V) = p_0V_0^\gamma/V^\gamma \text{ and } p(V) = p_0V_0/V$$

respectively. Differentiate these expressions and evaluate the results at  $(V_0, p_0)$  to find the desired relationship.



**EVALUATE** Differentiating the above expression for an adiabat and evaluating the result at  $V = V_0$  and  $p = p_0$  gives

$$\left(\frac{dp}{dV}\right)_{\text{adiabat}} = p_0 V_0^\gamma \left(-\gamma V^{-\gamma-1}\right) \Big|_{V=V_0}^{p=p_0} = \frac{-\gamma p_0}{V_0}$$

Differentiating the expression above for an isotherm and evaluating the result at  $V = V_0$  gives

$$\left(\frac{dp}{dV}\right)_{\text{isotherm}} = -\frac{p_0 V_0}{V^2} \Big|_{V=V_0} = -\frac{p_0}{V_0}$$

Comparing the two results gives

$$\left(\frac{dp}{dV}\right)_{\text{adiabat}} = \gamma \left(\frac{dp}{dV}\right)_{\text{isotherm}}$$

**ASSESS** We have found the expression given in the problem statement. Because  $\gamma > 0$ , we see that the slope of the  $pV$  curve for an adiabat is greater than that for an isotherm, so the pressure of an adiabatic process changes more rapidly with volume compared with an isothermal process, as shown in Fig. 18.11.

- 53. INTERPRET** The problem involves a cyclic process with three separate stages: adiabatic, isochoric, and isothermal. We are given the initial volume and pressure of the gas, and its adiabatic exponent  $\gamma$ , and are to find the pressure at points B and C and the net work done on the gas.

**DEVELOP** For the adiabatic process AB,  $Q = 0$ , and the first law of thermodynamics becomes  $\Delta E_{\text{int}} = -W$ . The pressure and volume are related by Equation 18.11a:  $pV^\gamma = \text{constant}$ , which gives

$$p_A V_A^\gamma = p_B V_B^\gamma \Rightarrow p_B = \left(\frac{V_A}{V_B}\right)^\gamma p_A$$

Point C lies on an isotherm (constant temperature) with A, so the ideal-gas law (Equation 17.2) yields

$$p_C = \frac{p_A V_A}{V_C}$$

To find the net work done on the gas, sum the contributions from each stage of the cycle. The contribution  $W_{AB}$  may be found from Equation 18.12 for an adiabatic process:

$$W_{AB} = \frac{p_B V_B - p_A V_A}{\gamma - 1}$$

For an isochoric (constant-volume) process,  $W_{BC} = 0$  (see Equation 18.7), and for the isothermal process

$$W_{CA} = -nRT_A \ln\left(\frac{V_A}{V_C}\right)$$

**EVALUATE (a)** From the equation above, the pressure at point B is

$$p_B = \left(\frac{V_A}{V_B}\right)^\gamma p_A = (250 \text{ kPa}) \left(\frac{1}{3}\right)^{1.28} = 61.3 \text{ kPa}$$

**(b)** The pressure at point C is

$$p_C = p_A \left(\frac{V_A}{V_C}\right) = \frac{250 \text{ kPa}}{3} = 83 \text{ kPa}$$

**(c)** The net work done by the gas is

$$W_{ABCA} = W_{AB} + W_{BC} + W_{CA}$$

The work  $W_{AB}$  for the adiabatic segment is

$$W_{AB} = \frac{p_B V_B - p_A V_A}{\gamma - 1} = \frac{(61.3 \text{ kPa})(3.0 \text{ m}^3) - (250 \text{ kPa})(1.00 \text{ m}^3)}{0.28} = -236 \text{ kJ}$$

$W_{BC}$  is for an isochoric process and equals zero. Finally,  $W_{CA}$  is for an isothermal process and is

$$W_{CA} = -nRT_A \ln\left(\frac{V_A}{V_C}\right) = (250 \text{ kJ}) \ln\left(\frac{1}{3}\right) = 275 \text{ kJ}$$

Summing these contributions gives

$$W_{ABCA} = W_{AB} + W_{BC} + W_{CA} = -236 \text{ kJ} + 0 + 275 \text{ kJ} = 39 \text{ kJ}$$

So, the work done *on* the gas is 39 kJ.

**ASSESS** Since the process is cyclic, the system returns to its original state; there's no net change in internal energy, so  $\Delta E_{int} = 0$ . This implies that  $Q = -W_{ABCA} = -39 \text{ kJ}$ . That is, 39 kJ of heat must come *out* of the system.

- 54. INTERPRET** This problem involves a cyclic process with an adiabatic compression, an isobaric compression, and an isothermal expansion. We are to find the net work done on the gas and its minimum volume.

**DEVELOP** The  $pV$  diagram for this problem is different than that of Fig. 18.14; only point A is the same (see figure below). Note that  $p_A V_A = 400 \text{ J}$  and  $V_A = 2V_B$ . To find the net work done on the gas, sum up the contributions from each stage. For the adiabatic stage AB, the work done on the gas is given by Equation 18.12:

$$W_{AB} = \frac{p_B V_B - p_A V_A}{\gamma - 1}$$

For the isobaric process BC, the work done on the gas is given by Equation 18.7:

$$W_{BC} = -p_B \Delta V = -p_B (V_C - V_B)$$

Via the adiabatic process, we can find the product  $p_B V_B$  (using Equation 18.11), which gives

$$p_B = p_A \left( \frac{V_A}{V_B} \right)^\gamma$$

$$p_B V_B = p_A V_A \left( \frac{V_A}{V_B} \right)^{\gamma-1}$$

Via the isothermal process CA, we know that C is at the same temperature as A. We also know that  $p_C = p_B$  because BC is isobaric. Thus, from the ideal gas law (Equation 17.2), we have

$$p_C V_C = p_B V_C = p_A V_A$$

Therefore, the work  $W_{BC}$  is

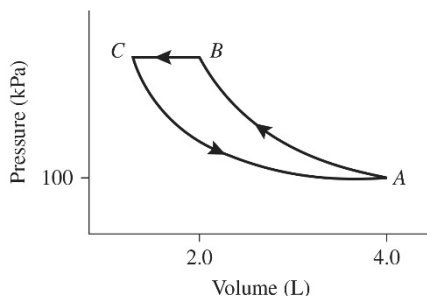
$$W_{BC} = p_A V_A \left[ \left( \frac{V_A}{V_B} \right)^{\gamma-1} - 1 \right]$$

The work done on the gas for the isothermal process CA is given by Equation 18.4, which gives (using the ideal gas law  $pV = nRT$ )

$$W_{CA} = -p_A V_A \ln \left( \frac{V_A}{V_C} \right)$$

The minimum volume of the gas occurs at point C, and is given by

$$V_C = \frac{p_A V_A}{p_B} = V_B \left( \frac{V_B}{V_A} \right)^{\gamma-1}$$



**EVALUATE** (a) Summing the work done on the gas for each process gives

$$W_{AB} = \frac{p_B V_B - p_A V_A}{\gamma - 1} = p_A V_A \frac{(V_A/V_B)^{\gamma-1} - 1}{\gamma - 1} = \frac{(400 \text{ J})(2^{0.4} - 1)}{0.4} = 319.5 \text{ J}$$

where we have used the expression above relating  $p_B V_B$  to  $p_A V_A$  from the adiabatic process. Continuing, we have for process BC

$$W_{BC} = p_A V_A \left[ \left( \frac{V_A}{V_B} \right)^{\gamma-1} - 1 \right] = (400 \text{ J})(2^{0.4} - 1) = 127.8 \text{ J}$$

and for process CA we have

$$W_{CA} = -p_A V_A \ln \left( \frac{V_A}{V_C} \right) = -p_A V_A \ln \left[ \left( \frac{V_A}{V_B} \right)^{\gamma} \right] = -(400 \text{ J})(1.4) \ln(2) = -388.2 \text{ J}$$

Summing these, the total work done on the gas is  $W_{\text{tot}} = 319.5 \text{ J} + 127.8 \text{ J} - 388.2 \text{ J} = 59.1 \text{ J}$ , or about 59 J.

(b) The minimum volume is

$$V_C = V_B \left( \frac{V_B}{V_A} \right)^{\gamma-1} = (2.0 \text{ L}) \left( \frac{1}{2} \right)^{0.4} = 1.5 \text{ L}$$

**ASSESS** The total work is positive, so work is done on the gas in this cycle.

- 55. INTERPRET** We are to find the work done in the given heat cycle, which consists of an isochoric doubling in pressure, and adiabatic compression, an isochoric cooling to 300 K, and an isothermal expansion.

**DEVELOP** The net (or total) work done on the gas is the sum of the work done for each process in the cycle (see figure below), which we give here:

(AB) It is heated at constant volume until the pressure is doubled. Because  $\Delta V = 0$ ,  $W_{AB} = 0$  (see Equation 18.7).

(BC) It is compressed adiabatically until its volume is  $\frac{1}{4}$  the initial value. This is analogous to the process AB of Problem 18.50, so the result is

$$W_{BC} = p_B V_B \frac{(V_B/V_C)^{\gamma-1} - 1}{\gamma - 1}$$

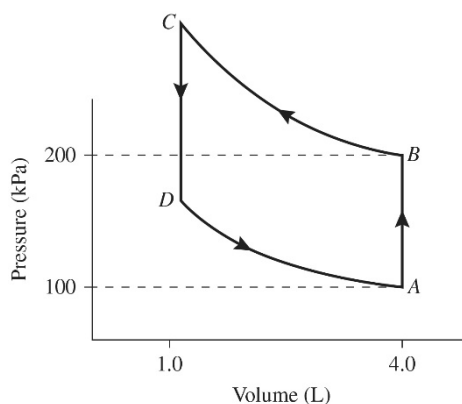
where  $V_B/V_C = 1/4$  and  $p_B V_B = 800 \text{ J}$  (see figure below).

(CD) It is cooled at constant volume to a temperature of 300 K. No work is done because  $\Delta V = 0$ , so  $W_{CD} = 0$ .

(DA) It is expanded isothermally until it returns to the original state. This is analogous to the process CA of Problem 18.46, so we use that result:

$$W_{DA} = -p_A V_A \ln \left( \frac{V_A}{V_D} \right)$$

where  $V_A/V_D = 4$  and  $p_A V_A = 400 \text{ J}$  (see figure).



**EVALUATE** Summing the nonzero contributions to the work gives

$$\begin{aligned} W_{\text{tot}} &= W_{BC} + W_{DA} = p_B V_B \frac{(V_B/V_C)^{\gamma-1} - 1}{\gamma-1} - p_A V_A \ln\left(\frac{V_A}{V_D}\right) \\ &= (800 \text{ J}) \frac{4^{0.4} - 1}{0.4} - (400 \text{ J}) \ln(4) = 930 \text{ J} \end{aligned}$$

**ASSESS** Work is done on the gas. From the figure above, we see that the area under the entire curve is negative, because the gas goes around the cycle counterclockwise. As per Fig. 18.6, the work done by the gas is the negative of the area under the  $pV$  curve.

**56. INTERPRET** We're asked to derive the relation between pressure and temperature in an adiabatic process.

**DEVELOP** We just need to combine Equations 18.11a and 18.11b:  $pV^\gamma = \text{constant}$ , and  $TV^{\gamma-1} = \text{constant}$ .

**EVALUATE** Isolating the volume in both equations and matching the exponents gives

$$\left. \begin{aligned} V^{\gamma(\gamma-1)} &= \frac{\text{constant}}{p^{(\gamma-1)}} \\ V^{\gamma(\gamma-1)} &= \frac{\text{constant}}{T^\gamma} \end{aligned} \right\} \quad p^{-(\gamma-1)} T^\gamma = p^{1-\gamma} T^\gamma = \text{constant}$$

The “constant” term is just a placeholder. You can raise the constant to a power and it's still just a constant. Or you can multiply the constant by another constant, and the result is still just a constant.

**ASSESS** For  $\gamma > 1$ , the result says the pressure and temperature will increase or decrease together during an adiabatic process.

**57. INTERPRET** The pump handle is pressed rapidly, so you can assume that there's no time for heat to flow into or out of the gas in the pump. This means that the process is adiabatic.

**DEVELOP** You want to check that the temperature rise in the pump is less than  $75^\circ\text{C}$ . Equation 18.11b tells you how to relate the final temperature and volume to the initial temperature and volume:  $TV^{\gamma-1} = T_0V_0^{\gamma-1}$ . Therefore, the temperature rise is

$$\Delta T = T - T_0 = T_0 \left[ \left( \frac{V_0}{V} \right)^{\gamma-1} - 1 \right]$$

The gas in the cylinder is air ( $\gamma = 1.4$ ), initially at  $T_0 = 18^\circ\text{C} = 291 \text{ K}$ .

**EVALUATE** Since the cross-sectional area of the cylinder remains unchanged during the compression, the volume ratio is just equal to the ratio in the cylinder's height:

$$\Delta T = T_0 \left[ \left( \frac{V_0}{V} \right)^{\gamma-1} - 1 \right] = (291 \text{ K}) \left[ \left( \frac{35 \text{ cm}}{20 \text{ cm}} \right)^{1.4-1} - 1 \right] = 73 \text{ K}$$

This does just meet your  $75^\circ\text{C}$  criteria.

**ASSESS** There's no obvious way to avoid this temperature rise. The cylinder can be made of a material with a high thermal conductivity, such as aluminum, so that heat can flow out as fast as possible.

**58. INTERPRET** We're given experimental data and are asked to estimate the work involved in inflating a human lung.

**DEVELOP** We want calculate the work done *by* the gas in expanding the lungs:  $\Delta W = p\Delta V$ . In the given graph, we can take the volume change between successive points as  $\Delta V$ , and the pressure at the midpoint between successive points as the average pressure  $\bar{p}$ . To find the total work done, we sum over these small individual expansions.

**EVALUATE** There are seven main volume expansions that correspond to pressure increases of 5 kPa. We add the work involved in each expansion to get the total work. (Note: to save space, we leave off the units until the end of the sum):

$$\begin{aligned}
 W &= \sum \bar{p} \Delta V = \sum \frac{1}{2} (p_i + p_{i-1}) (V_i - V_{i-1}) \\
 &= \left[ \frac{1}{2} (0.5 + 5) (125 - 0) + \frac{1}{2} (5 + 10) (260 - 125) + \frac{1}{2} (10 + 15) (520 - 260) \right. \\
 &\quad + \frac{1}{2} (15 + 20) (800 - 520) + \frac{1}{2} (20 + 25) (1100 - 800) + \frac{1}{2} (25 + 30) (1350 - 1100) \\
 &\quad \left. + \frac{1}{2} (30 + 35) (1550 - 1350) \right] \text{ kPa} \cdot \text{mL} \\
 &= 30 \text{ J}
 \end{aligned}$$

**ASSESS** It may seem counterintuitive that the pressure increases as the volume increases, but in this case air is rushing into the lungs from the outside. When a person breathes, his/her diaphragm contracts to open up the lung cavity. This lowers the pressure on the lungs, so he/she inflates like two balloons. The air rushing into the lungs does the work to expand the lung volume; the diaphragm is only indirectly responsible. Afterward, the diaphragm expands to squeeze on the lungs, which increase the pressure and thus force air out.

- 59. INTERPRET** For this problem, we are given the heat transferred from a compressed gas to the surroundings and the change in internal energy of the gas, and we are to find the work done on this air in the process.

**DEVELOP** Apply the first law of thermodynamics, Equation 18.1,  $\Delta E_{\text{int}} = Q + W$ , where  $Q = -158 \text{ J}$ . The internal energy change can be calculated using the constant-volume specific heat for a diatomic gas, which is  $C_V = 5R/2$  (see Equation 18.13). Thus,

$$\Delta E_{\text{int}} = nC_V \Delta T$$

where  $n = 1.00 \text{ mol}$  and  $\Delta T = 48.6 \text{ K}$ .

**EVALUATE** Combining the above expressions and inserting the given quantities gives

$$W = \Delta E_{\text{int}} - Q = nC_V \Delta T - Q = (1.00 \text{ mol}) \left( \frac{5}{2} \right) [8.314 \text{ J/(mol} \cdot \text{K)}] (48.6 \text{ K}) + 158 \text{ J} = 1.17 \text{ kJ}$$

**ASSESS** Since the work is done on the gas, we obtain a positive quantity when solving for it using Equation 18.1.

- 60. INTERPRET** The thermodynamic process here involves two stages: isothermal compression followed by an adiabatic compression. We are to find the final temperature of the gas.

**DEVELOP** During the first stage, the gas is compressed isothermally, so  $\Delta T = 0$  and there is no change in the temperature of the system (i.e.,  $T_0 = T_1$ ), but the volume is reduced from  $V_0$  to  $V_1 = V_0 / 2$ . During the next stage of adiabatic compression, the temperature and volume are related by Equation 18.11b:

$$TV^{\gamma-1} = \text{constant}$$

which we apply to points 1 and 2 (i.e., before and after the adiabatic compression, respectively) to find

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1} \Rightarrow T_2 = T_1 \left( \frac{V_1}{V_2} \right)^{\gamma-1}$$

where the final temperature is  $T_2$

**EVALUATE** Substituting the values given, we find  $T_2$  to be

$$T_2 = T_1 \left( \frac{V_1}{V_2} \right)^{\gamma-1} = T_0 \left( \frac{V_0/2}{V_0/3} \right)^{7/5-1} = (273 \text{ K}) \left( \frac{3}{2} \right)^{0.4} = 321 \text{ K}$$

**ASSESS** Since  $TV^{\gamma-1} = \text{constant}$  for an adiabatic process, the compression results in an increase in the final temperature.

- 61. INTERPRET** This problem involves an adiabatic compression of a gas, for which we know the initial temperature and pressure and the final pressure. We are to find the final temperature of the gas.

**DEVELOP** For an adiabatic process, the temperature and pressure are related by Equation 18.11a:

$$pV^{\gamma} = \text{constant}$$

Rearranging this equation and using the ideal-gas law (Equation 17.2,  $pV = nRT$ ) gives

$$pV^{\gamma} = p^{1-\gamma} (pV)^{\gamma} = p^{1-\gamma} (nRT)^{\gamma} = \text{constant}$$

$$pT^{\gamma/(1-\gamma)} = \text{another constant}$$

Applying this to the gas both before and after the compression gives  $T = T_0 (P_0 / P)^{(1/\gamma)-1}$ .

**EVALUATE** Inserting the given quantities into the expression above for temperature gives

$$T = T_0 (P_0 / P)^{(1/\gamma)-1} = (275 \text{ K}) \left( \frac{100 \text{ kPa}}{240 \text{ kPa}} \right)^{(1/1.3)-1} = 337 \text{ K}$$

or  $3.4 \times 10^2 \text{ K}$  to two significant figures.

**ASSESS** Because the gas is compressed adiabatically, its temperature rises.

- 62. INTERPRET** The problem involves a cyclic process with three separate stages of the cycle: isochoric ( $AB$ ), isobaric ( $BC$ ), and isothermal ( $CA$ ). We are to calculate the net work done on the gas over the entire cycle, and the heat transfer over segment  $AB$ .

**DEVELOP** The work done *on* the gas in each segment of the cycle is summarized in Table 18.1. For the isochoric process (path  $AB$ ),  $\Delta V = 0$  so  $W = 0$ . For the isobaric process (path  $BC$ ), the work done is

$$W_{BC} = -p_B(V_C - V_B)$$

Finally, for the isothermal process ( $CA$ ), the work done on the gas is given by Equation 18.4:

$$W_{CA} = -nRT_A \ln \left( \frac{V_A}{V_C} \right)$$

Segment  $AB$  is isochoric, so  $Q_{AB} = nC_V\Delta T$  (see Table 18.1). To eliminate  $C_V$ , note that

$$\gamma = \frac{C_P}{C_V} = \frac{C_V + R}{C_V} = 1 + \frac{R}{C_V} \Rightarrow C_V = \frac{R}{\gamma - 1}$$

which follows from the definition of the adiabatic ratio  $\gamma$  (see discussion for Equation 18.11a). Inserting this into the expression for the heat transfer gives

$$Q_{AB} = \frac{nR(T_B - T_A)}{\gamma - 1} = \frac{p_B V_B - p_A V_A}{\gamma - 1}$$

where the second equality follows from the ideal gas law (Equation 17.2),  $pV = nRT$ .

**EVALUATE (a)** Using the equations above, we obtain

$$W_{AB} = 0 \quad (\text{isochoric})$$

$$W_{BC} = -p_B(V_C - V_B) = -(250 \text{ kPa})(1.00 \text{ L} - 5.00 \text{ L}) = 1.00 \text{ kJ} \quad (\text{isobaric})$$

$$W_{CA} = -nRT_A \ln \left( \frac{V_A}{V_C} \right) = -p_A V_A \ln \left( \frac{V_A}{V_C} \right) = -(50.0 \text{ kPa})(5.00 \text{ L}) \ln(5.00) = -402 \text{ J} \quad (\text{isothermal})$$

Adding up all the contributions, the net work done *on* the gas is

$$W_{ABCA} = W_{AB} + W_{BC} + W_{CA} = 0 + 1000 \text{ J} - 402 \text{ J} = 598 \text{ J}$$

**(b)** The heat transferred is

$$Q_{AB} = \frac{p_B V_B - p_A V_A}{\gamma - 1} = \frac{(250 \text{ kPa} - 50 \text{ kPa})(5.0 \text{ L})}{1.4 - 1.0} = 2.5 \text{ kJ}$$

Since  $Q_{AB} > 0$ , heat is transferred into the gas.

**ASSESS** At constant volume, the gas must be heated in order to raise its pressure.

$$\gamma = \frac{C_P}{C_V} = \frac{C_V + R}{C_V} = 1 + \frac{R}{C_V} \rightarrow C_V = \frac{R}{\gamma - 1}$$

- 63. INTERPRET** This problem is similar to the preceding one, except that the order of segments in the cycle are changed. The cycle proceeds along the 350-K isotherm from 50 kPa to 250 kPa ( $AC$ ), then decreases its pressure to 50 kPa via an isochoric process ( $CD$ ), then returns to the starting point via a 50-kPa isobar ( $DA$ ). We are to find the total work done on the gas for the complete cycle and the heat transferred in segment  $CD$ .

**DEVELOP** The formulae are the same as for the previous problem, but the initial and final point for each segment are different. The work done on the gas over segment AC will be the opposite of the result we found for the segment CA in Problem 18.62, so  $W_{AC} = 402 \text{ J}$ . The work done over segment CD is zero because the volume is constant (see Equation 18.7). The work done over segment DA is

$$W_{BC} = -p_D(V_A - V_D)$$

The heat transfer can be calculated as per Problem 18.56, which gives

$$Q_{CD} = \frac{p_D V_D - p_C V_C}{\gamma - 1}$$

**EVALUATE** (a) The total work is

$$W_{ACD} = W_{AC} + \overbrace{W_{CD}}^{=0} + W_{DA} = W_{AC} - p_D(V_A - V_D) = 402 \text{ J} - (50.0 \text{ kPa})(5.00 \text{ L} - 1.00 \text{ L}) = 202 \text{ J}$$

(b) The heat transfer over segment CD is

$$Q_{CD} = \frac{p_D V_D - p_C V_C}{\gamma - 1} = \frac{(50 \text{ kPa} - 250 \text{ kPa})(1 \text{ L})}{0.4} = -500 \text{ J}$$

Because  $Q_{CD} < 0$ , we interpret the result as 500 J of heat transferred out of the gas.

**ASSESS** At constant volume, the gas must be cooled to lower its pressure.

- 64. INTERPRET** This problem deals with an adiabatic expansion of a gas mixture. The gas contains monatomic and diatomic molecules; we are given that the volume triples and the pressure decreases by one-fifth because of the adiabatic expansion, and we are to find the fraction of each molecule type.

**DEVELOP** In an adiabatic process (AB),  $Q = 0$ , and the pressure and volume are related by Equation 18.11a:  $pV^\gamma = \text{constant}$ . Applying this to the gas before and after the expansion gives

$$p_1 V_1^\gamma = p_2 V_2^\gamma \Rightarrow \frac{p_2}{p_1} = \left( \frac{V_1}{V_2} \right)^\gamma$$

Taking the natural logarithm of both sides of the expression above and solving for  $\gamma$ , we obtain

$$\ln\left(\frac{p_2}{p_1}\right) = \gamma \ln\left(\frac{V_1}{V_2}\right) \Rightarrow \gamma = \frac{\ln(p_2/p_1)}{\ln(V_1/V_2)} = \frac{\ln(1/5)}{\ln(1/3)} = \frac{-\ln(5)}{-\ln(3)} = 1.46$$

To find the fraction of the molecules that are argon, we use the result from Exercise 25:

$$f_1 = 2.5 - \frac{1}{\gamma - 1}$$

**EVALUATE** Substituting  $\gamma = 1.46$  into the equation above gives

$$f_{\text{Ar}} = 2.5 - \frac{1}{\gamma - 1} = 2.5 - \frac{1}{1.46 - 1} = 0.349 = 34.9\%$$

**ASSESS** In the limit where all the gas molecules are monatomic,  $f_1 = 1$ , and  $\gamma_{\text{monatomic}} = 1.67$ . On the other hand, if all the molecules are diatomic, then  $f_1 = 0$ , and the specific heat ratio is  $\gamma_{\text{diatomic}} = 1.4$ . Our ratio of  $\gamma = 1.46$  is closer to 1.4. This implies that the gas mixture is predominantly diatomic.

- 65. INTERPRET** For this problem, we are to find the ratio of triatomic molecules to monatomic molecules that will result in a gas that has the specific heat at constant volume of a diatomic gas.

**DEVELOP** The specific heat of a mixture of two gases is  $C_V = f_1 C_{V1} + f_2 C_{V2}$ , where the  $f_i$  are the number

fractions of the gases. Gas 1 is monatomic ( $C_V = \frac{3}{2}R$ ), gas 2 is triatomic (with  $C_V = 3R$ , as given in the problem statement), and we wish to have a mixture with  $C_V = \frac{5}{2}R$ , as for a diatomic gas. In this case,

$$\frac{5}{2}R = \frac{3}{2}R f_1 + 3R f_2, \text{ or } 5 = 3f_1 + 6f_2.$$

Given that  $f_1 + f_2 = 1$  and  $f_1 = 6$  mol, we can solve for  $f_2$ .

**EVALUATE** From the equations above, we find  $f_1 = 1/3$  and  $f_2 = 2/3$ . With 6 mol of monatomic gas, one needs 12 mol of triatomic gas.

**ASSESS** There are twice as many triatomic molecules as there are monatomic molecules.

- 66. INTERPRET** This problem is about melting, and it involves heat of fusion. The source of energy is the gravitational potential energy of the rock that is dropped into the water.

**DEVELOP** The gravitational potential energy of the rock melts the ice, and no heat energy is transferred ( $Q = 0$ ) to the environment. The first law of thermodynamics (Equation 18.1) then takes the form  $\Delta E_{int} = W$ . The work is  $W = m_{\text{rock}}gh$ , and the internal energy of the ice is raised  $m_{\text{ice}}L_f$  (see Equation 17.5), where  $L_f = 334 \text{ kJ/kg}$  (from Table 17.1). Therefore, we have

$$m_{\text{rock}}gh = m_{\text{ice}}L_f$$

**EVALUATE** Thus, we find the height from which the rock must be dropped is

$$h = \frac{m_{\text{ice}}L_f}{m_{\text{rock}}g} = \frac{(0.0013 \text{ kg})(334 \text{ kJ/kg})}{(2.5 \text{ kg})(9.8 \text{ m/s}^2)} = 18 \text{ m}$$

**ASSESS** In this problem, the rock did positive work on the ice-water system. Therefore, the work done *on* the system is positive, so  $W > 0$ .

- 67. INTERPRET** This problem deals with an isothermal expansion of a gas from an unknown pressure to the ambient pressure of 1 atm. The expansion extracts heat from the surrounding ice-water bath so that 20 g of ice is created in the process. We are to find the original pressure of the gas.

**DEVELOP** For an isothermal expansion of an ideal gas, Equation 18.4 and the ideal gas law ( $PV = nRT = \text{constant}$  for an isothermal expansion) give

$$Q = nRT \ln\left(\frac{V_2}{V_1}\right) = nRT \ln\left(\frac{p_1}{p_2}\right)$$

From this,

$$p_1 = p_2 e^{Q/nRT}$$

where  $p_1$  is the original pressure of the gas,  $p_2 = 1 \text{ atm}$ , and  $n = 0.70 \text{ mol}$ . The heat  $Q$  is provided by the freezing of 20 g of ice that is already at  $0^\circ\text{C}$ , which is (see Equation 17.5)

$$Q = m_{\text{ice}}L_f = (0.02 \text{ kg})(334 \text{ kJ/kg}) = 6.68 \text{ kJ}$$

**EVALUATE** Inserting the known quantities into the expression for  $p_1$  gives

$$p_1 = (1.0 \text{ atm}) \exp\left\{\frac{6.68 \text{ kJ}}{(0.70 \text{ mol})[8.314 \text{ J/(mol} \cdot \text{K)}](273 \text{ K})}\right\} = (1.0 \text{ atm})e^{4.20} = 67 \text{ atm}$$

to two significant figures.

**ASSESS** Notice that the original quantity of ice is not involved, provided we know the ice-water bath is at  $0^\circ\text{C}$  and we know the quantity of ice that is produced by the energy requirements of the isothermal expansion.

- 68. INTERPRET** The problem concerns the work done in expanding a frog's lung.

**DEVELOP** Equation 18.3,  $W = \int_{V_1}^{V_2} p dV$ , gives the work done by a gas as its volume goes from  $V_1$  to  $V_2$ . We'll plug

the formula for the pressure in the frog's lung,  $p = 10v^3 - 67v^2 + 220v$ , where  $p$  is in Pa and  $v = V/\text{mL}$ .

**EVALUATE** If the frog's lung inflates from 0 to 3.9 mL, the work done by the gas is

$$\begin{aligned} W &= \left[ \int_0^{4.5} (10v^3 - 67v^2 + 220v) dv \right] \text{Pa} \cdot \text{mL} \\ &= \left[ \frac{1}{4} 10(3.9)^4 - \frac{1}{3} 67(3.9)^3 + \frac{1}{2} 220(3.9)^2 \right] \times 10^{-6} \text{ J} = 0.93 \text{ mJ} \end{aligned}$$



**ASSESS** This is much smaller than our result in Problem 18.52, but there we were considering a human lung inflating to over 1500 mL.

- 69. INTERPRET** For this problem, we are to derive the work done by an adiabatic process by applying Equation 18.3, which describes the work done by changes in a volume of gas.

**DEVELOP** For an adiabatic process, pressure and volume are related by Equation 18.11a,  $pV^\gamma = \text{constant}$ .

Applying this to a gas before and after an arbitrary adiabatic process

$$p_1 V_1^\gamma = p V^\gamma$$

$$p = \frac{p_1 V_1^\gamma}{V^\gamma}$$

Insert this expression into Equation 18.3 to find the work done by an adiabatic process.

**EVALUATE** The work in going from a pressure  $p_1$  to a pressure  $p_2$  via an adiabatic process is

$$W_{12} = - \int_{V_1}^{V_2} p dV = - \int_{V_1}^{V_2} \left( p_1 V_1^\gamma \right) \frac{dV}{V^\gamma} = - p_1 V_1^\gamma \left( \frac{V_2^{-\gamma+1} - V_1^{-\gamma+1}}{-\gamma+1} \right) = \frac{p_2 V_2^\gamma V_2^{-\gamma+1} - p_1 V_1^\gamma V_1^{-\gamma+1}}{\gamma-1}$$

$$= \frac{p_2 V_2 - p_1 V_1}{\gamma-1}$$

which is Equation 18.12 (note that we have used  $p_1 V_1^\gamma = p_2 V_2^\gamma$  to obtain the last equality of the first line).

**ASSESS** From the general expression for work done by a change in volume of a gas, we have derived the expression for the work done by an adiabatic change in volume of a gas.

- 70. INTERPRET** For this problem, we want to find expressions for the isothermal and adiabatic compression of two identical gas samples. We then want to find the ratio of the work done through these two processes and to evaluate it for a diatomic gas.

**DEVELOP** The work done to an ideal gas sample when isothermally and adiabatically compressed are given by Equation 18.4 and Equation 18.12, respectively. We can develop these expressions for two identical samples, determine the ratio of these works, and evaluate for the given specific heat ratio.

**EVALUATE** We begin by defining the two works:  $W_{\text{is}}$  and  $W_{\text{ad}}$

$$W_{\text{is}} = -nRT \ln \left( \frac{V_2}{V_1} \right) \quad W_{\text{ad}} = \frac{p_2 V_2 - p_1 V_1}{\gamma-1}$$

Where the quantities are labeled for before (subscript 1) and after (subscript 2) compression. In addition, since  $pV^\gamma = \text{constant}$  (Equation 18.11a) in the adiabatic case, the final pressure can be written as  $p_2 = p_1 (V_1/V_2)^\gamma$ . Applying this substitution and taking the ratio of the work done during the adiabatic compression to the work done during the isothermal compression we find

$$\frac{W_{\text{ad}}}{W_{\text{is}}} = \frac{p_1 (V_1/V_2)^\gamma V_2 - p_1 V_1}{(\gamma-1)nRT \ln(V_1/V_2)} = \frac{p_1 V_1 (2^{\gamma-1} - 1)}{(\gamma-1)nRT \ln 2}$$

Where we have used the fact that  $V_1$  is twice as large as  $V_2$ . Next, we recognize that the product of the initial pressure and volume  $p_1$  and  $V_1$  are equal to  $nRT$ , so our final expression becomes

$$\frac{W_{\text{ad}}}{W_{\text{is}}} = \frac{(2^{\gamma-1} - 1)}{(\gamma-1) \ln 2}$$

Evaluating this for a diatomic gas with  $\gamma = 1.40$  we obtain a ratio of 1.15.

**ASSESS** Therefore, it requires more work done onto the gas to compress it to half its volume adiabatically than isothermally.

- 71. INTERPRET** We are to derive the relationship between temperature and volume for adiabatic processes.

**DEVELOP** For an adiabatic, the relationship between pressure and volume is  $pV^\gamma = \text{constant}$ . Use this and the ideal gas law (Equation 17.2)  $pV = nRT$ , to derive equation 18.11b,  $TV^{\gamma-1} = \text{constant}$ .

**EVALUATE** From the ideal gas law,

$$pV = nRT \Rightarrow p = \frac{nRT}{V}$$

Substitute this into  $pV^\gamma = \text{constant}$  to obtain

$$\frac{nRT}{V} V^\gamma = (nR)TV^{\gamma-1} = \text{constant}$$

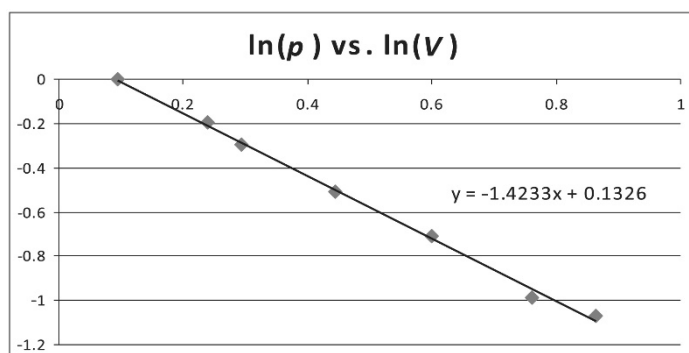
$$TV^{\gamma-1} = \frac{\text{constant}}{nR} = \text{a different constant}$$

**ASSESS** We have shown what was required. Note that the constants involved in Equations 18.11a and 18.11b are different constants.

- 72. INTERPRET** In this problem we are asked to analyze the data of the pressures of an ideal gas as a function of volumes in a thermodynamic process, and deduce whether the process is isothermal or adiabatic.

**DEVELOP** For an adiabatic process, the relationship between pressure and volume is  $pV^\gamma = \text{constant}$ . Taking the logarithm on both sides leads to  $\ln p + \gamma \ln V = \ln c$ , where  $c$  is a constant. Therefore, if the process is adiabatic, plotting  $\ln p$  versus  $\ln V$  gives a straight line with slope equal to  $-\gamma$ . On the other hand, if the process is isothermal, then  $pV = \text{constant}$ . Taking the logarithm on both sides leads to  $\ln p + \ln V = \ln c$ , where  $c$  is a constant. Plotting  $\ln p$  versus  $\ln V$  would give a straight line with slope equal to  $-1$  for the isothermal process.

**EVALUATE** The log-log plot is shown below.



The slope is  $-1.4$ . This implies that the process is adiabatic.

**ASSESS** The value  $\gamma = 1.4$  implies that the gas is diatomic.

- 73. INTERPRET** For this problem, we want to find the amount of work done onto a gas when it has reached a particular pressure throughout a reversible process.

**DEVELOP** We are told the pressure and volume of the compressed gas can be related by the equation  $pV^2 = p_0V_0^2$  throughout the reversible process. This behavior tells us the gas is undergoing adiabatic compression, as is seen from Equation 18.11a:  $pV^\gamma = \text{constant}$ . Therefore, we can determine the work done onto the gas using Equation 18.12

**EVALUATE** The work done onto a gas during an adiabatic process is given by

$$W_{\text{ad}} = \frac{p_2V_2 - p_1V_1}{\gamma - 1}$$

Where the quantities are labeled for before (subscript 1) and after (subscript 2) compression. In addition, the final volume can be written as  $V_2 = \sqrt[p_1V_1^2/p_2]{p_1V_1^2/p_2}$ , where in this case  $\gamma = 2$ . Applying this substitution and simplifying the expression for the work done we find

$$W_{\text{ad}} = \frac{p\sqrt{p_0V_0^2/p} - p_0V_0}{2 - 1} = V_0(\sqrt{pp_0} - p_0)$$

Next, we recognize that initial pressure  $p_0$  is half of the final pressure  $p$ , so our final expression becomes

$$W_{\text{ad}} = V_0 p_0 (\sqrt{2} - 1)$$

Evaluating this for the given initial pressure and volume we obtain  $W_{\text{ad}} = 189 \text{ J}$ .

**ASSESS** In the case of this gas, for the given ratio  $\gamma$ , we note that at the moment the gas pressure has increased by a factor of 2 the volume has decreased by a factor of 2.

- 74. INTERPRET** We are to derive an expression for the work done by a non-ideal gas in an isothermal process that follows the van der Waals model. We are given the equation relating pressure and volume for a van der Waals gas.

**DEVELOP** Solving the van der Waals expression for pressure gives

$$p = \frac{nRT}{V - nb} - a\left(\frac{n}{V}\right)^2$$

Insert this expression for pressure into Equation 18.3 and integrate from  $V_1$  to  $V_2$  to find the work done by the gas.

**EVALUATE** Performing the integration gives

$$\begin{aligned} W &= -\int_{V_1}^{V_2} p dV = -\int_{V_1}^{V_2} \left[ \frac{nRT}{V - nb} - a\left(\frac{n}{V}\right)^2 \right] dV = -\left[ nRT \ln(V - nb) + \frac{an^2}{V} \right]_{V_1}^{V_2} \\ &= nRT \ln\left(\frac{V_1 - nb}{V_2 - nb}\right) + an^2 \left( \frac{1}{V_1} - \frac{1}{V_2} \right) \end{aligned}$$

**ASSESS** For an ideal gas,  $a = b = 0$ , and this simplifies to  $W = -nRT \ln(V_2/V_1)$  (Equation 18.4), as expected.

- 75. INTERPRET** An ideal gas expands along a given path, and we are to find the work done by the gas. We can do this by using the general expression for work done by a gas that changes pressure and volume, Equation 18.3.

**DEVELOP** The gas goes from  $(p_1, V_1)$  to  $(p_2, V_2)$  where  $p_2 = 2p_1$  and  $V_2 = 2V_1$ . The path it takes is along

$$p = p_1 \left[ 1 + \left( \frac{V - V_1}{V_1^2} \right)^2 \right]$$

Insert this expression into the integrand of Equation 18.3 and integrate from  $V_1$  to  $V_2$  to find the work done on the gas. The work done by the gas will be the negative of this result.

**EVALUATE** The work done on the gas is

$$\begin{aligned} W &= -\int_{V_1}^{V_2} p dV = -\int_{V_1}^{2V_1} p_1 \left[ 1 + \frac{(V - V_1)^2}{V_1^2} \right] dV = -\frac{p_1}{V_1^2} \int_{V_1}^{2V_1} [V^2 - 2V_1V + 2V_1^2] dV \\ &= -\frac{p_1}{V_1^2} \left[ \frac{1}{3}V^3 - V_1V^2 + 2V_1^2V \right]_{V_1}^{2V_1} = -\frac{p_1}{V_1^2} \left[ \frac{1}{3}(8V_1^3 - V_1^3) - V_1(4V_1^2 - V_1^2) + 2V_1^2(2V_1 - V_1) \right] \\ &= -p_1V_1 \left( \frac{7}{3} - 3 + 2 \right) = -\frac{4}{3}p_1V_1 \end{aligned}$$

so the work done by the gas is  $4p_1V_1/3$ .

**ASSESS** The units are pressure times volume, which is joules, as expected.

- 76. INTERPRET** We calculate the rate  $dT/dy$  at which air cools as it rises, approximating the process as adiabatic.

**DEVELOP** To express  $dT$  in terms of  $dp$ , let's first differentiate the ideal gas law:  $Vdp + pdV = nRdT$ . For adiabatic processes,  $\Delta E_{\text{int}} = -W$ , which we can write in differential form as  $nC_VdT = -pdV$ , and insert into the differentiated ideal gas law to obtain

$$dT = \frac{V}{n(C_V + R)} dp = \frac{V}{nR} \left( \frac{\gamma - 1}{\gamma} \right) dp$$

where we have used  $C_V = R/(\gamma - 1)$ .

The hydrostatic equation shows how the pressure increases with depth:  $dp = \rho g dh$ , but since we are considering altitude (the opposite of depth), we include a negative sign:  $dp = -\rho g dy$ , so that the pressure decreases with increasing altitude.

**EVALUATE** Combining the temperature/pressure relation to hydrostatic equation gives:

$$\frac{dT}{dy} = \frac{-\rho V g}{nR} \left( \frac{\gamma-1}{\gamma} \right) = -\frac{Mg}{R} \left( \frac{\gamma-1}{\gamma} \right)$$

where  $M$  is the molar mass (the mass of 1 mole of the gas). We're told to assume that  $\gamma = 1.4$ , and the average molecular weight is 29u, which is equivalent to saying that the molar mass is  $M = 29 \text{ g/mol}$ . So the adiabatic lapse rate is:

$$\frac{dT}{dy} = -\frac{(0.029 \text{ kg/mol})(9.8 \text{ m/s}^2)}{8.314 \text{ J/K} \cdot \text{mol}} \left( \frac{1.4-1}{1.4} \right) = -9.8 \text{ K/km}$$

**ASSESS** The result seems reasonable: the temperature of an air mass drops by  $10^\circ\text{C}$  for every kilometer it rises. Note that the static temperature gradient of the atmosphere is actually smaller than this (about a  $6^\circ\text{C}$  drop for every kilometer increase in altitude). This is because other effects, like condensation of water molecules, contribute.

- 77. INTERPRET** For this problem, we want to determine the amount of energy wasted as heat during a process, and determine how many homes this amount of energy could power during a given amount of time.

**DEVELOP** Differentiating the first law of thermodynamics results in Equation 18.2, where the rate of change of a system's internal energy can be expressed in terms of the rate of heat entering/leaving the system plus the rate of work done on/by the system.

$$\frac{dE_{\text{int}}}{dt} = \frac{dQ}{dt} + \frac{dW}{dt}$$

We are told the rate of energy extracted from fuel, and the amount of work done by the power plant, so we can determine the amount of energy lost as heat. Once we have this rate, we can multiply it by the time (1 month) to find the number of homes which could be powered during this period.

**EVALUATE** From the above expressions and the given quantities we find a rate of heat loss equal to

$$\frac{dQ}{dt} = \frac{dE_{\text{int}}}{dt} - \frac{dW}{dt} = -3810 \text{ MW} - (-1250 \text{ MW}) = -2560 \text{ MW}$$

We are interested in the magnitude of this quantity. Multiplying by the duration of a month in seconds,

$\Delta t = 2.6 \times 10^6 \text{ s}$ , we obtain a total energy of  $6.66 \times 10^{15} \text{ J}$ . Dividing this available energy by the amount necessary to power a single home during this time period (43.2 GJ), we find there is enough energy lost to heat to power 154,000 homes.

**ASSESS** Since the plant extracts energy from its fuel, and the work is done by the machines in the plant, both the rates used in our expression are negative. This results in a negative rate for heat flow, meaning its energy exits the system, as expected.

- 78. INTERPRET** We are to find the pressure and volume of air within a diving bell, given that the temperature varies in such a way that the pressure and volume are related by the given equation.

**DEVELOP** The equation relating pressure and temperature is

$$p = p_0 \sqrt{\frac{V_0}{V}}$$

where  $V_0 = 16 \text{ m}^3$  and  $p_0 = 1 \text{ atm}$ . The maximum allowable pressure is  $p_{\text{max}} = 1.4 \text{ atm}$ . We can plug these values into the equation, and see what the resulting volume  $V$  is. If it's greater than  $V_{\text{min}} = 8.5 \text{ m}^3$ , the design is ok.

**EVALUATE** Solving the given equation for  $V$  gives

$$p = p_0 \sqrt{\frac{V_0}{V}} \Rightarrow V = \frac{p_0^2 V_0}{p^2} = \frac{(1.0 \text{ atm})^2 (16 \text{ m}^3)}{(1.4 \text{ atm})^2} = 8.2 \text{ m}^3 < 8.5 \text{ m}^3$$

**ASSESS** The design needs work. Note also that since the pressure appears only in a ratio, we do not need to convert to SI units.

- 79. INTERPRET** The problem involves the power needed to extract at 320 K and 1 atm and then compress it adiabatically to 350 atm.

**DEVELOP** For an adiabatic process, the initial pressure and volume are related to the final pressure and volume through Equation 18.11a:  $p_1 V_1^\gamma = p_2 V_2^\gamma$ . We can use this to rewrite Equation 18.12 for the work done on the gas in going from the initial to the final volume:

$$W = \frac{p_2 V_2 - p_1 V_1}{\gamma - 1} = \frac{p_1 V_1}{\gamma - 1} \left[ \left( \frac{V_1}{V_2} \right)^{\gamma-1} - 1 \right]$$

Using  $\gamma = 1.3$ , we obtain

$$\frac{V_1}{V_2} = \left( \frac{p_2}{p_1} \right)^{1/\gamma} = (350)^{1/1.3} = 90.6$$

which then leads to

$$W = \frac{p_1 V_1}{1.3 - 1} \left[ (90.6)^{1.3-1} - 1 \right] = 9.55 p_1 V_1 = 9.55 (n R T_1)$$

On the other hand, the rate of  $\text{CO}_2$  emission is

$$R = 1100 \text{ tons/h} = 1.1 \times 10^6 \text{ kg/h} = 305.6 \text{ kg/s}$$

which is equivalent to  $dn/dt = 6.944 \times 10^3 \text{ mol/s}$  noting that the molar mass of  $\text{CO}_2$  is 44 g. The power required for this process is

$$\frac{dW}{dt} = 9.55 R T_1 \left( \frac{dn}{dt} \right)$$

**EVALUATE** Substituting the values given, we obtain

$$\frac{dW}{dt} = 9.55 R T_1 \left( \frac{dn}{dt} \right) = 9.55 (8.314 \text{ J/mol} \cdot \text{K}) (320 \text{ K}) (6.944 \times 10^3 \text{ mol/s}) = 1.76 \times 10^8 \text{ W} = 0.176 \text{ GW}$$

Since the plant produces electrical energy at a rate of 1.0 GW, the power required is about 18% of the power output.

**ASSESS** This is a substantial fraction of the energy the power plant produces. In addition, it doesn't include the energy cost of separating the  $\text{CO}_2$  from other stack gases or of transporting it to the compression site.

**80. INTERPRET** We consider the physics behind warm winds called Chinooks.

**DEVELOP** We're told that the wind sweeping down from the mountain has no time to exchange heat with the surroundings.

**EVALUATE** No heat exchange means the process is adiabatic.

The answer is **(d)**.

**ASSESS** Recall that air has one of the lowest thermal conductivities in Table 16.2. So it's perhaps not surprising that a large mass of air might need a lot of time to exchange an appreciable amount of heat with its surroundings.

**81. INTERPRET** We consider the physics behind warm winds called Chinooks.

**DEVELOP** For an adiabatic process,  $\Delta E_{\text{int}} = W$  from Equation 18.10. Here,  $W$  is the work done on the air mass.

**EVALUATE** We're told that the pressure increases by 50% as the air descends from the mountain. If the pressure increases, there is positive work being done on the air (the opposite of Figure 8.11), so the change in the internal energy,  $\Delta E_{\text{int}}$ , is positive.

The answer is **(a)**.

**ASSESS** One can verify this result by combining Equations 18.11a and 18.12 to obtain:  $W \propto p_2^{1-1/\gamma} - p_1^{1-1/\gamma}$ . Since  $p_2 > p_1$  and  $\gamma$  is always greater than one, the work done on the air mass is positive.

**82. INTERPRET** We consider the physics behind warm winds called Chinooks.

**DEVELOP** From 18.11a, the pressure and volume in the mountains are related to that in the plains by

$$p_1 V_1^\gamma = p_2 V_2^\gamma. \text{ We'll assume } \gamma = 1.4.$$

**EVALUATE** Given the initial and final pressure, we can solve for the relative change in the volume:

$$\frac{\Delta V}{V_1} = \frac{V_2}{V_1} - 1 = \left( \frac{p_1}{p_2} \right)^{1/\gamma} - 1 = \left( \frac{60 \text{ kPa}}{90 \text{ kPa}} \right)^{1/1.4} - 1 = -25\%$$

This implies the volume decreases by less than 50%.

The answer is **(d)**.

**ASSESS** It makes sense that the volume of air decreases when exposed to the higher pressures at the base of the mountain.

**83. INTERPRET** We consider the physics behind warm winds called Chinooks.

**DEVELOP** One could use the result from the previous problem and apply it to Equation 18.11b:

$TV^{\gamma-1} = \text{constant}$ . But we will use the expression derived in Problem 18.51, relating the pressure and temperature for an adiabatic process,  $p^{1-\gamma}T^\gamma = \text{constant}$ .

**EVALUATE** We know the initial temperature up in the mountains is 260 K, so the temperature down in the plains must be:

$$T_2 = T_1 \left( \frac{p_2}{p_1} \right)^{1-1/\gamma} = (260 \text{ K}) \left( \frac{90 \text{ kPa}}{60 \text{ kPa}} \right)^{1-1/1.4} = 290 \text{ K}$$

The answer is **(c)**.

**ASSESS** As the name implies, the Chinook is a warm wind. In this case, there is a 30°C increase in going from the mountains (−13°C) down to the plains (17°C).