MAGNETISM: FORCE AND FIELD

26

EXERCISES

Section 26.2 Magnetic Force and Field

11. **INTERPRET** This problem involves the magnetic force exerted on a moving electron.

DEVELOP The magnetic force on a charge q moving with velocity $\vec{v}$ is given by Equation 26.1: $\vec{F}_B = q\vec{v} \times \vec{B}$. The magnitude of $\vec{F}_B$ is

$$F_B = \left|\vec{F}_B\right| = \left|q\vec{v} \times \vec{B}\right| = |q| vB\sin\theta$$

EVALUATE **(a)** The magnetic field is minimum when $\sin\theta = 1$ (the magnetic field perpendicular to the velocity). Thus,

$$B_{\min} = \frac{F_B}{|q| v} = \frac{5.7\times10^{-15}\ \text{N}}{\left(1.6\times10^{-19}\ \text{C}\right)\left(1.8\times10^{7}\ \text{m/s}\right)} = 1.98\times10^{-3}\ \text{T} = 20\ \text{G}$$

(b) For $\theta = 45°$, the magnetic field is

$$B = \frac{F_B}{|q| v\sin\theta} = \frac{5.7\times10^{-15}\ \text{N}}{\left(1.6\times10^{-19}\ \text{C}\right)\left(1.8\times10^{7}\ \text{m/s}\right)\sin 45°} = \sqrt{2}\,B_{\min} = 28\ \text{G}$$

ASSESS The magnetic force on the electron is very tiny. The magnetic field required to produce this force can be compared to Earth's magnetic field, which is about 1 G.

12. **INTERPRET** This problem involves force on an electron that moves through a magnetic field. From Newton's second law, we can relate this force to the acceleration experienced by the electron.

DEVELOP The magnetic force on a moving charge is given by Equation 26.1, which in scalar form is

$$F = qvB\sin\theta$$

Using Newton's second law (for constant mass, Equation 4.3, $F = ma$), we can solve for the speed v. From the vector form of Equation 26.1, we see that the force is perpendicular to the velocity of the particle. Therefore, this force does no work on the particle (see Equation 6.11); so, from the work-energy theorem (Equation 6.14), we know that the particle's kinetic energy does not change.

EVALUATE **(a)** Solving for the velocity gives

$$F = ma = qvB\sin\theta$$

$$v = \frac{ma}{qB\sin\theta} = \frac{\left(9.11\times10^{-31}\ \text{kg}\right)\left(6.5\times10^{15}\ \text{m/s}^2\right)}{\left(1.6\times10^{-19}\ \text{C}\right)\left(0.20\ \text{T}\right)\sin\left(90°\right)} = 1.9\times10^{5}\ \text{m/s}$$

(b) Because the particle's kinetic energy does not change, its speed does not change.

ASSESS The magnetic force does cause the particle's velocity to change, but not its speed. In other words, only the direction of the particle's velocity changes, but the magnitude of the velocity (i.e., its speed) does not change.

13. **INTERPRET** In this problem we are asked to find the magnetic force on a proton moving at various angles with respect to a magnetic field.

DEVELOP The magnetic force on a charge q moving with velocity $\vec{v}$ is given by Equation 26.1: $\vec{F}_B = q\vec{v} \times \vec{B}$.

The magnitude of $\vec{F}_B$ is

$$F_B = |\vec{F}_B| = |q\vec{v}\times\vec{B}| = |q|vB\sin\theta$$

The charge of the proton is $q = 1.6\times10^{-19}$ C.

EVALUATE **(a)** When $\theta = 90°,$ the magnitude of the magnetic force is

$$F_B = qvB\sin(90°) = (1.6\times10^{-19}\text{ C})(2.5\times10^{5}\text{ m/s})(0.50\text{ T}) = 2.0\times10^{-14}\text{ N}$$

(b) When $\theta = 30°,$ the force is

$$F_B = qvB\sin(30°) = (1.6\times10^{-19}\text{ C})(2.5\times10^{5}\text{ m/s})(0.50\text{ T})\sin(30°) = 1.0\times10^{-14}\text{ N}$$

(c) When $\theta = 0°,$ the force is $F_B = qvB\sin(0°) = 0.$

ASSESS The magnetic force is a maximum $F_{B,\text{max}} = |q|vB$ when $\theta = 90°$ and a minimum $F_{B,\text{min}} = 0$ when $\theta = 0°.$

14. **INTERPRET** We're asked to calculate the maximum magnetic force on an electron moving in the Earth's magnetic field at the surface. We then compare that to the gravitational force on the same electron.

DEVELOP We're given the electron's kinetic energy, which we can equate to a velocity by

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(10^3\text{eV})}{(9.11\times10^{-31}\text{kg})}\left[\frac{1.6\times10^{-19}\text{J}}{1\text{ eV}}\right]} = 1.87\times10^{7}\text{m/s}$$

The maximum magnetic force occurs when the velocity is perpendicular to the Earth's magnetic field: $F = evB$ from Equation 26.1 with $\sin\theta = 90°.$

EVALUATE The maximum magnetic force on the electron is then

$$F = evB = (1.6\times10^{-19}\text{C})(1.87\times10^{7}\text{m/s})(0.5\times10^{-4}\text{T}) = 1.5\times10^{-16}\text{N}$$

By comparison, the weight of an electron at the Earth's surface is

$$F_g = mg = (9.11\times10^{-31}\text{kg})(9.8\text{ m/s}^2) = 8.9\times10^{-30}\text{N}$$

ASSESS The gravitational force is 10^{13} times smaller than the magnetic force. This has partly to do with the electron moving near the speed of light, but even so, gravity is very weak in comparison to both electric and magnetic forces.

15. **INTERPRET** This problem involves the speed of a given charge if it is to pass through the velocity selector undeflected. A velocity selector contains an electric and a magnetic field that are perpendicular to each other (see Fig. 26.5).

DEVELOP In the presence of both electric and magnetic fields, the force on a moving charge is (see Equation 26.2):

$$\vec{F} = \vec{F}_E + \vec{F}_B = q(\vec{E} + \vec{v}\times\vec{B})$$

Because $\vec{E}$ is perpendicular to $\vec{B},$ as shown in Fig. 26.5, the forces due to each field on a charged particle are antiparallel. Thus, the condition for a charged particle to pass undeflected through the velocity selector is that the net force on it is zero, or $\vec{F}_E = -\vec{F}_B.$

EVALUATE Substituting the values given in the problem statement, we obtain

$$0 = q(E - vB\sin\theta)$$

$$v = \frac{E}{B\sin\theta} = \frac{20\text{ kN / C}}{(0.058\text{ T})\sin(90°)} = 340\text{ km / s}$$

ASSESS Only particles with this speed would pass undeflected through the mutually perpendicular fields; at any other speed, particles would be deflected. Note also that the particle velocity must be perpendicular to the magnetic field for this result to hold.

Section 26.3 Charged Particles in Magnetic Fields

16. **INTERPRET** This problem involves finding the radius of orbit of a proton moving perpendicular to a magnetic field.

DEVELOP Apply Equation 26.3, $r = vm/(eB)$ to find the radius r.

EVALUATE Inserting the given quantities gives

$$r = \frac{mv}{eB} = \frac{\left(1.67\times10^{-27}\ \text{kg}\right)(175\ \text{km/s})}{\left(1.6\times10^{-19}\ \text{C}\right)\left(6.46\times10^{-2}\ \text{T}\right)} = 2.83\ \text{cm}$$

ASSESS To verify that the units of this expression are correct, note that a tesla can be expressed as

$$\text{T} = \frac{\text{N}}{\text{C}\cdot\text{m/s}} = \frac{\text{kg}\cdot\text{m/s}^2}{\text{C}\cdot\text{m/s}} = \frac{\text{kg}}{\text{C}\cdot\text{s}}$$

Using kg/(C·s) instead of T in Equation 26.3 gives units of distance, as expected.

17. **INTERPRET** This problem is about an electron undergoing circular motion in a uniform magnetic field. We want to know its period, or the time it takes to complete one revolution.

DEVELOP Using Equation 26.3, the radius of the circular motion is $r = mv/\left(|e|B\right)$. Therefore, the period of the motion is

$$T = \frac{2\pi r}{v} = \frac{2\pi}{v}\frac{mv}{|e|B} = \frac{2\pi m}{|e|B}$$

EVALUATE Substituting the values given in the problem statement, we find the period to be

$$T = \frac{2\pi m}{|e|B} = \frac{2\pi\left(9.11\times10^{-31}\ \text{kg}\right)}{\left(1.6\times10^{-19}\ \text{C}\right)\left(1.15\times10^{-4}\ \text{T}\right)} = 311\ \text{ns}$$

to two significant figures.

ASSESS The period is independent of the electron's speed and orbital radius. However, it is inversely proportional to the magnetic field strength

18. **INTERPRET** We are to find the magnetic field strength given the frequency of the radiation emitted by electrons in the field. We can assume that the electrons are moving in a circular path in the field, as for a cyclotron.

DEVELOP Apply Equation 26.4 ($f = qB/2\pi m$) for cyclotron motion, and solve for B.

EVALUATE The magnetic field has a strength of

$$B = \frac{2\pi fm}{e} = \frac{2\pi(42\ \text{MHz})\left(9.11\times10^{-31}\ \text{kg}\right)}{\left(1.6\times10^{-19}\ \text{C}\right)} = 1.5\times10^{-3}\ \text{T} = 15\ \text{G}$$

ASSESS This is not a very strong field. The Earth's magnetic field is 1 G, and a typical refrigerator magnet produces a magnetic field of about 100 G.

19. **INTERPRET** This problem involves finding the magnetic field strength required for the given frequency of electrons moving in a circular path through the field. In addition, given the maximum radius of the electron path, we are to find the maximum electron energy (i.e., kinetic energy).

DEVELOP For part **(a)**, apply Equation 26.4, which gives the frequency of motion as a function of magnetic field. For part **(b)**, we use Equation 26.3, $r = mv/(qB)$ to find the kinetic energy $K = mv^2/2$ that corresponds to radius $r = (2.72\ \text{mm})/2 = 1.36\ \text{mm}$.

EVALUATE **(a)** A cyclotron frequency of 2.45 GHz for electrons implies a magnetic field strength of

$$B = \frac{2\pi fm}{e} = \frac{2\pi(2.45\ \text{GHz})\left(9.11\times10^{-31}\ \text{kg}\right)}{\left(1.6\times10^{-19}\ \text{C}\right)} = 87.6\ \text{mT}$$

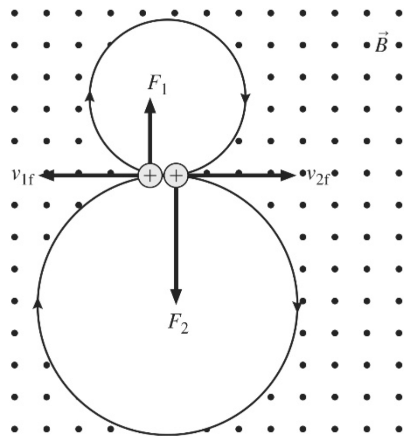
(b) Solving Equation 26.3 for v and inserting this into the expression for kinetic energy gives

$$K = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{rqB}{m}\right)^2 = \frac{r^2q^2B^2}{2m} = \frac{\left(1.36\times10^{-3}\,\text{m}\right)^2\left(1.6\times10^{-19}\,\text{C}\right)^2\left(87.6\times10^{-3}\,\text{T}\right)^2}{2\left(9.11\times10^{-31}\,\text{kg}\right)} = 1.99\times10^{-16}\,\text{J} = 1.25\,\text{keV}$$

ASSESS The electron's kinetic energy could also be expressed in terms of the cyclotron frequency directly, $K = (2\pi frm)^2/(2m) = 2m(\pi rf)^2$, with the same result.

20. **INTERPRET** This problem is about two protons undergoing circular motion and colliding head-on.

DEVELOP In an elastic head-on collision between particles of equal mass, the particles exchange velocities: $v_{1\text{f}} = v_{2\text{i}}$ and $v_{2\text{f}} = v_{1\text{i}}$. The uniform magnetic field is perpendicular to both their velocities, so the magnetic force $(F = evB)$ will induce circular motion with radius given in Equation 26.3: $r = mv/eB$. See the figure below, where the magnetic field points out of the page.



In the figure, we have arbitrarily given the second proton a greater post-collision speed $(v_{2\text{f}} > v_{1\text{f}})$, which means $F_2 > F_1$ and $r_2 > r_1$. However, from Equation 26.4, we know that the time it takes for each proton to complete its circle is independent of velocity. The two protons will return to the collision point at the same time after one period: $T = 2\pi m/eB$.

EVALUATE The protons will collide again after

$$T = \frac{2\pi m}{eB} = \frac{2\pi(1.67\times10^{-27}\,\text{kg})}{(1.60\times10^{-19}\,\text{C})(5\times10^{-2}\,\text{T})} = 1.31\ \mu\text{s}$$

ASSESS In solving the problem, we have ignored Coulomb repulsion between the two protons.

Section 26.4 The Magnetic Force on a Current

21. **INTERPRET** This problem involves finding the force on a wire that is perpendicular to the given magnetic field and that carries the given current.

DEVELOP Apply Equation 26.5

$$\vec{F} = I\vec{l}\times\vec{B}$$

which, in scalar form, is $F = IlB\sin\theta$.

EVALUATE Inserting the given quantities into the expression above gives

$$F = IlB\sin\theta = (12\ \text{A})(0.695\ \text{m})(0.0385\ \text{T})\sin(90^\circ) = 0.321\ \text{N}$$

ASSESS The direction of this force is given by the right-hand rule, crossing the current $\vec{l}$ into the magnetic field $\vec{B}$.

22. **INTERPRET** In this problem we are asked about the magnetic field strength given the force per unit length exerted on a wire in the field. We are also to find the maximum force this wire can experience in this magnetic field if we were to reorient the wire in the field.

DEVELOP Equation 26.5 gives the magnetic force on a straight current-carrying wire in a uniform magnetic field,

$\vec{F} = I\vec{l} \times \vec{B}$. The magnitude of the force is $F = IlB\sin\theta$.

EVALUATE **(a)** From the magnitude of the force per unit length, $\vec{F}/l = 0.15$ N/m and the given data, we find the magnetic field strength to be

$$B = \frac{F}{Il\sin\theta} = \frac{0.31 \text{ N/m}}{(15 \text{ A})\sin(25°)} = 49 \text{ mT}$$

(b) By placing the wire perpendicular to the field ($\sin\theta = 1$), a maximum force per unit length of

$$\frac{F}{l} = IB = (15 \text{ A})(48.9 \text{ mT}) = 0.73 \text{ N/m}$$

could be attained.

ASSESS From the definition of cross product between two vectors, we see that the magnetic force $\vec{F}$ is perpendicular to both the current direction $\vec{l}$ and the magnetic field $\vec{B}$, and the magnitude of $\vec{F}$ is a maximum when $\vec{l} \perp \vec{B}$.

23. **INTERPRET** Two forces are involved in this problem: the magnetic force and the gravitational force. We want to find the maximum current the wire can carry in the presence of a magnetic field that will result in the forces being of equal magnitude.

DEVELOP The magnetic force is given by the scalar form of Equation 26.5 with $\theta = 90°$, and the force due to gravity is $F = mg$. A magnetic force equal in magnitude to the weight of the wire requires that

$$F_B = F_g \quad \Rightarrow \quad IlB = mg$$

since the wire is perpendicular to the field.

EVALUATE The equation above implies that the current is

$$I = \frac{mg}{Bl} = \frac{(2.44\,\text{kg})(9.8 \text{ m/s}^2)}{(1.52\,\text{T})(3.15\,\text{m})} = 5.00 \text{ A}$$

ASSESS The shorter and heavier the wire, the more current it can handle before the forces are no longer balanced at the magnetic field strength present

24. **INTERPRET** Two forces are involved in this problem: the magnetic force and the gravitational force. We want to find the magnetic field strength such that the two forces are equal in magnitude.

DEVELOP The magnetic force is given by the scalar form of Equation 26.5 with $\theta = 90°$, and the force due to gravity is $F = mg$. A magnetic force equal in magnitude to the weight of the wire requires that

$$F_B = F_g \quad \Rightarrow \quad IlB = mg$$

since the wire is perpendicular to the field.

EVALUATE The equation above implies that the field strength is

$$B = \frac{mg}{Il} = \frac{(m/l)g}{I} = \frac{(75 \text{ g/m})(9.8 \text{ m/s}^2)}{6.2 \text{ A}} = 0.12 \text{ T}$$

ASSESS This field strength is much greater than the typical value of 0.01 T produced by a bar magnet.

Section 26.5 Origin of the Magnetic Field

25. **INTERPRET** For this problem, we are given the current carried by a wire that forms a loop. Given the magnetic field strength at the loop center, we are to find the radius of the loop.

DEVELOP This problem is dealt with in Example 26.3, so apply that result (Equation 16.9) here with $x = 0$ (since we are in the plane of the loop).

EVALUATE **(a)** Solving Equation 26.9 (with $x = 0$) for the loop radius a gives

$$B = \frac{\mu_0 I}{2a} \quad \Rightarrow \quad a = \frac{\mu_0 I}{2B} = \frac{(4\pi\times10^{-7} \text{ N/A}^2)(6.71 \text{ A})}{2(42.8\,\mu\text{T})} = 9.85\,\text{cm}$$

(b) At $x = 10$ cm, the field strength is

$$B = \frac{\mu_0 I a^2}{2(x^2 + a^2)^{3/2}} = \left(\frac{\mu_0 I}{2a}\right)\frac{1}{(1 + x^2/a^2)^{3/2}} = (42.8\,\mu\text{T})\frac{1}{\left[1 + (1.0 \text{ cm}/9.85 \text{ cm})^2\right]^{3/2}} = 14.8\,\mu\text{T}$$

ASSESS The current in the loop is high (6.71 A!) yet the magnetic field it produces is quite small (~ 0.4 G).

26. INTERPRET This problem involves finding the magnetic field on the axis of a current-carrying loop.

DEVELOP As shown in Example 26.4, the magnetic field at a point *P* on the axis of a circular loop of radius *a* carrying current *I* is (Equation 26.9):

$$B = \frac{\mu_0 I a^2}{2\left(x^2 + a^2\right)^{3/2}}$$

EVALUATE **(a)** At the center, $x = 0$, so the field strength is

$$B = \frac{\mu_0 I}{2a} = \frac{\left(4\pi \times 10^{-7}\ \text{N000 A}^2\right)(670\ \text{mA})}{2(1.5\ \text{cm})} = 28\ \mu\text{T}$$

(b) At $x = 25$ cm on the axis, we have

$$B = \frac{\mu_0 I a^2}{2\left(x^2 + a^2\right)^{3/2}} = \frac{\left(4\pi \times 10^{-7}\ \text{N/A}^2\right)(670\ \text{mA})(1.5\ \text{cm})^2}{2\left[(25\ \text{cm})^2 + (1.5\ \text{cm})^2\right]^{3/2}} = 6.0\ \text{nT}$$

ASSESS The direction of the field is along the axis. The field strength is greatest at the center of the loop since this point is closest to the current.

27. INTERPRET This problem is similar to the preceding one, except that here we consider the effect of not one, but several current-carrying loops that are positioned very close together on the same axis. We are given the current in each loop and the radius and are to find the magnetic field strength at the center of the loops.

DEVELOP Using the principle of superposition, the total magnetic field at the center of the loops will be the sum of the magnetic field from each loop. The number *n* of loops involved is $n = L/(2\pi a)$, where $L = 2.2$ m and $2a = 6.5$ cm. From Problem 29, we see that the magnetic field due to a single loop at the center of the loops is $B = \mu_0 I/(2a)$.

EVALUATE The total magnetic field is

$$B = n\left(\frac{\mu_0 I}{a}\right) = \left(\frac{L}{2\pi a}\right)\left(\frac{\mu_0 I}{a}\right) = \frac{\mu_0 I L}{2\pi a^2} = \frac{\left(4\pi \times 10^{-7}\ \text{N/A}^2\right)(3.7\ \text{A})(2.2\ \text{m})}{2\pi(3.25\ \text{cm})^2\ \text{mT}} = 1.5\ \text{mT}$$

ASSESS For this approximation to be valid, the loop radius must be much, much larger than the separation between the loops.

28. INTERPRET This problem involves finding the magnetic field strength at a given distance from a current-carrying wire.

DEVELOP Equation 26.10 of Example 26.4 gives the magnetic field strength at a distance *r* from an infinitely long straight wire:

$$B = \frac{\mu_0 I}{2\pi r}$$

The expression is applicable if *r* is much smaller than the length of the wire.

EVALUATE Solving the equation above for the current, we find

$$I = \frac{2\pi r B}{\mu_0} = \frac{2\pi(2.2\ \text{cm})(67\ \mu\text{T})}{4\pi \times 10^{-7}\ \text{N/A}^2} = 7.4\ \text{A}$$

ASSESS The current is proportional to the magnetic field strength. Note that the magnetic field lines are concentric circles, as illustrated in Fig. 26.19.

29. INTERPRET This problem involves two long parallel wires separated by 1 cm and carrying the given current (note that the current is in the same direction for both wires). We are to find the force between these wires.

DEVELOP Apply Equation 26.11. To find the force per unit length, simply divide through by the length *l* of the wires:

$$\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi d}$$

EVALUATE Inserting the given quantities gives

$$\frac{F}{l}=\frac{\mu_0 I_1 I_2}{2\pi d}=\frac{\left(4\pi\times10^{-7}\text{ N/A}^2\right)(15\text{ A})^2}{2\pi(0.01\text{ m})}=5\text{ mN/m}$$

to a single significant figure.

ASSESS If the currents were in the opposite directions, the force would be zero.

Section 26.6 Magnetic Dipoles

30. **INTERPRET** We are to find the magnetic field strength produced by the Earth's magnetic dipole.

DEVELOP Apply Equation 26.12, which gives the magnetic field strength B at a distance x along the axis of a magnetic dipole moment μ as

$$B=\frac{\mu_0}{2\pi}\frac{\mu}{x^3}$$

EVALUATE Substituting the values given, we find the field strength to be

$$B=\frac{\mu_0}{2\pi}\frac{\mu}{R_E^3}=\frac{\left(4\pi\times10^{-7}\text{ N/A}^2\right)\left(8\times10^{22}\text{ A}\cdot\text{m}^2\right)}{2\pi\left(6.37\times10^6\text{ m}\right)^3}=6.2\times10^{-5}\text{ T}=0.62\text{ G}$$

ASSESS The main component of the Earth's magnetic field is a dipole. The magnetic field near the surface of the Earth is about 0.5 G.

31. **INTERPRET** We are to find the strength of the magnetic dipole moment of the given current loop, and the magnitude of the torque it would experience when placed at 40° in the given magnetic field.

DEVELOP To find the magnetic dipole moment, apply Equation 26.13, $\vec{\mu}=NI\vec{A}$, which in scalar form is $\mu=NIA$. The torque on this loop in a magnetic field B = 2.12 T is given by Equation 26.15.

EVALUATE **(a)** The strength of the magnetic dipole moment is

$$\mu=NIA=(1)(1.25\text{ A})(0.18\text{ m})^2=4.05\times10^{-2}\text{ A}\cdot\text{m}^2$$

(b) The torque on the current loop is

$$\tau=\left|\vec{\mu}\times\vec{B}\right|=\mu B\sin\theta=\left(4.05\times10^{-2}\text{ A}\cdot\text{m}^2\right)(2.12\text{ T})\sin(65°)=7.78\times10^{-2}\text{ N}\cdot\text{m}.$$

ASSESS The maximum torque occurs at θ = 90°, as expected.

32. **INTERPRET** This problem involves an electric motor. We are asked to find the magnetic field strength, given the torque, the current, and the area of the current loop.

DEVELOP The maximum torque on a plane circular coil follows from Equation 26.15 (i.e., for $\theta=90°$):

$$\tau_{\text{max}}=\mu B=NI\pi R^2 B$$

EVALUATE Solving the expression above for B gives

$$B=\frac{\tau_{\text{max}}}{\mu}=\frac{\tau_{\text{max}}}{NI\pi R^2}=\frac{2.0\text{ N}\cdot\text{m}}{(250)(3.1\text{ A})\pi(2.7\text{ cm})^2}=1.1\text{ T}$$

ASSESS This field strength is rather high, but it is reasonable for producing the torque needed to rotate the motor.

Section 26.8 Ampère's Law

33. **INTERPRET** This problem involves Ampère's law for magnetism, which we will use to find the current in a wire, given the magnitude of the line integral of the magnetic field.

DEVELOP Apply Equation 26.17, where the left-hand side is 9.2 μT·m.

EVALUATE Inserting the given quantity for the integral and solving for the current gives

$$9.2\ \mu\text{T}\cdot\text{m}=\oint\vec{B}\cdot d\vec{r}=\mu_0 I_{\text{wire}}$$

$$I_{\text{wire}}=\frac{9.2\ \mu\text{T}\cdot\text{m}}{4\pi\times10^{-7}\text{ N/A}}=7.3\text{ A}$$

ASSESS The current within the area defined by the line integral is directly proportional to the value of the line integral.

34. **INTERPRET** This problem involves an application of Ampère's law, which we can use to find the current enclosed by the loop. The magnetic field is antisymmetric about the horizontal axis through center of the Ampèrian loop, so the contribution to the integral from the top and bottom part of the loop is the same.

DEVELOP Applying Ampère's law (Equation 26.17) to the loop shown in Fig. 26.39 (going clockwise) gives

$$\oint \vec{B} \cdot d\vec{r} = \mu_0 I_{\text{encircled}}$$
$$2aB = \mu_0 I_{\text{encircled}}$$

where $a = 20$ cm is the length of the top and bottom of the loop. Note that the vertical sides of the loop give no contribution to the line integral because they are perpendicular to the field.

EVALUATE Thus, the encircled current is

$$I_{\text{encircled}} = \frac{2Ba}{\mu_0} = \frac{(75\ \mu\text{T})(2\times 0.20\ \text{m})}{4\pi\times 10^{-7}\ \text{N/A}^2} = 24\ \text{A}$$

ASSESS As explained in the text, the current flows along the boundary surface between the regions of oppositely directed $\vec{B}$, positive into the page in Fig. 26.39, for a clockwise circulation around the loop.

35. **INTERPRET** This problem is similar to Example 26.7. We can apply Ampère's law to find the strength of the magnetic field inside and at the surface of the wire with the given dimensions and carrying the given current.

DEVELOP Because the wire has 2.5-mm^2 cross-sectional area, its radius is $R = \sqrt{2.5\ \text{mm}^2/\pi} = 0.892\ \text{mm}.$ For part (a), since current is uniform within the wire, the fraction of current contained within $r = 0.180\ \text{mm}$ of the wire's axis is

$$I = I_0 \frac{\pi r^2}{\pi R^2} = I_0 \left(\frac{r}{R}\right)^2$$

We can insert this into Ampère's law (Equation 26.17) to find the strength of the magnetic field at $r.$ For part (b), we are to find the magnetic field strength at the surface of the wire $(R),$ so the current enclosed is simply op $I_0 = 20.0$ A. For part (c), we are to find the field strength at $r > R,$ so the current enclosed is again $I_0 = 20.0$ A.

EVALUATE **(a)** The magnetic field strength at 0.200 mm from the wire's axis is

$$\oint \vec{B} \cdot d\vec{r} = \mu_0 I \quad \Rightarrow \quad B(2\pi r) = \mu_0 I_0 \left(\frac{r}{R}\right)^2$$
$$B = \frac{\mu_0 I_0 r}{2\pi R^2} = \frac{(4\pi\times 10^{-7}\ \text{N/A}^2)(20.0\ \text{A})(0.180\times 10^{-3}\ \text{m})}{2\pi(0.892\times 10^{-3}\ \text{m})^2} = 9.05\times 10^{-4}\ \text{T}$$

or 0.905 mT.

(b) At the surface of the wire, Ampère's law gives

$$\oint \vec{B} \cdot d\vec{r} = \mu_0 I \quad \Rightarrow \quad B(2\pi r) = \mu_0 I_0$$
$$B = \frac{\mu_0 I_0}{2\pi R} = \frac{(4\pi\times 10^{-7}\ \text{N/A}^2)(20.0\ \text{A})}{2\pi(0.892\times 10^{-3}\ \text{m})} = 4.48\times 10^{-3}\ \text{T}$$

or 4.48 mT.

(c) At $r = 0.892\ \text{mm} + 0.250\ \text{mm} = 1.142\ \text{mm},$ Ampère's law gives

$$\oint \vec{B} \cdot d\vec{r} = \mu_0 I \quad \Rightarrow \quad B(2\pi r) = \mu_0 I_0$$
$$B = \frac{\mu_0 I_0}{2\pi r} = \frac{(4\pi\times 10^{-7}\ \text{N/A}^2)(20.0\ \text{A})}{2\pi(1.142\times 10^{-3}\ \text{m})} = 3.50\times 10^{-3}\ \text{T}$$

or 3.50 mT.

ASSESS The field increases linearly with r inside the wire, but falls off as $1/r$ outside the wire.

36. **INTERPRET** This problem is about the magnetic field of a long current-carrying wire of radius R. We want to show that the expression for the current inside the wire (Equation 26.19) reduces to the expression for the current outside the wire (Equation 26.18) at the wire's surface.

DEVELOP Equation 26.18, $B=\mu_0 I/(2\pi r)$ holds for $r \geq R$ while Equation 26.19, $B=\mu_0 Ir/\left(2\pi R^2\right)$ holds for $r \leq R$. Evaluating both at $r = R$ will determine if they give the same result at the surface of the wire.

EVALUATE Inserting $r = R$ into Equation 26.18 gives $B=\mu_0 I/(2\pi R)$. Inserting $r = R$ into Equation 26.19 gives $B=\mu_0 IR/\left(2\pi R^2\right)=\mu_0 I/(2\pi R)$, which is the same result as for Equation 26.18.

ASSESS We expect both equations to give the same result for the magnetic field since the encircled current at $r = R$ is $I_{\text{encircled}} = I$ in both cases.

37. **INTERPRET** We are asked to find the magnetic field strength inside a solenoid given the current-loop density and the current.

DEVELOP Apply Equation 26.21, which gives the field inside the solenoid (i.e., many radii away from the end of the solenoid).

EVALUATE Inserting the given quantities gives

$$B=\mu_0 nI=\left(4\pi\times10^{-7}\ \text{N/A}^2\right)\left(3300\ \text{m}^{-1}\right)(4.1\ \text{kA})=17\ \text{T}$$

ASSESS This is a very strong magnetic field.

EXAMPLE VARIATIONS

38. **INTERPRET** This problem is about charged particles undergoing circular motion in a uniform magnetic field. We want to find the distance separating different isotopes of Cl in a mass spectrometer upon reaching the detector.

DEVELOP Equation 26.3, $r = mv/qB$, shows that the path radius depends on the field and on the particle's mass, charge, and speed. From the applied potential each particle gains kinetic energy $\frac{1}{2}mv^2 = qV$ resulting in a velocity of $v=\sqrt{2qV/m}$, which we can use to the separation distance Δx, where x is the particle's path diameter.

EVALUATE We find that each particle will have a path diameter given by

$$x = 2r = \frac{2}{B}\sqrt{\left(\frac{2V}{q}\right)m}$$

Meaning the path difference between particles is given by their mass difference, as

$$\Delta x = \frac{2}{B}\sqrt{\left(\frac{2V}{q}\right)}\left(\Delta\sqrt{m}\right)$$

Plugging in the given values for the magnetic field and applied potential, as well as the atomic masses for the two Cl isotopes we find

$$\Delta x = \frac{2}{(163\,\text{mT})}\sqrt{\left(\frac{2(3.50\,\text{kV})\left(1.66\times10^{-27}\ \text{kg}\right)}{\left(1.60\times10^{-19}\text{C}\right)}\right)}\left(\sqrt{37}-\sqrt{35}\right)=1.74\,\text{cm}$$

ASSESS The spectrometer's detector array should have enough resolution to distinguish between the distribution of these two species.

39. **INTERPRET** This problem is about charged particles undergoing circular motion in a uniform magnetic field. We want to determine if arsenic is present in the collected mass spectrum for which three ion impacts are detected at given distances.

DEVELOP Equation 26.3, $r = mv/qB$, shows that the path radius depends on the field and on the particle's mass, charge, and speed. From the applied potential each particle gains kinetic energy $\frac{1}{2}mv^2 = qV$ resulting in a velocity of $v=\sqrt{2qV/m}$, which we can use to the separation distance express the particle's path diameter x. Plugging in the mass of arsenic's only stable isotope from Appendix D we can determine if one of the three peaks detected corresponds to As.

EVALUATE We find that an arsenic isotope would have a path diameter given by

$$x_{\mathrm{As}} = 2r = \frac{2}{B}\sqrt{\left(\frac{2V}{q}\right)m_{\mathrm{As}}}$$

From Appendix D we find that the mass of arsenic's only stable isotope is equal to $m_{\mathrm{As}} = 74.92\,\mathrm{au}$. Plugging in the given values for the magnetic field and applied potential, as well as the atomic masses for this isotope we find

$$x_{\mathrm{As}} = \frac{2}{(0.460\,\mathrm{T})}\sqrt{\left(\frac{2(5.75\,\mathrm{kV})(1.24\times10^{-25}\,\mathrm{kg})}{(1.60\times10^{-19}\,\mathrm{C})}\right)} = 41.1\,\mathrm{cm}$$

Which corresponds to one of the peaks found in the mass spectrum, signifying the presence of As in the water.

ASSESS What else was found in this sample of water? You can work backward and solved for the mass of the particles at the other two path diameters and check which isotopes from Appendix D most closely match these values.

40. **INTERPRET** This problem is about a beam of charged particles entering a region with a uniform magnetic field and undergoing circular motion. We want to determine how far into the region the particles make it, and the coordinates of the point where the beam exits the region.

DEVELOP Equation 26.3, $r = mv/qB$, shows that the path radius depends on the field and on the particle's mass, charge, and speed. We are given the electron's initial speed as they travel in the positive *x*-direction and are told they encounter a magnetic field upon entering the region at $x = 0$ which points in the positive *y*-direction. The path radius r and diameter d ($d = 2r$), along with the right-hand rule, will help us determine the maximum distance traveled by the beam along the *x*-direction and the location where it exits the region.

EVALUATE Since the electrons are negatively charged, they will feel a force that points in the direction

$$-(\hat{x}\times\hat{y}) = -\hat{z}$$

meaning the electrons will travel a maximum distance r in the positive *x*-direction as they curve toward the negative *z*-direction, where they will have traveled a distance d before exiting the region and heading in the opposite direction from which they came. So we find that the maximum distance traveled is given by

$$r = \frac{mv}{qB} = \frac{(9.11\times10^{-31}\,\mathrm{kg})(7.18\,\mathrm{Mm/s})}{(1.60\times10^{-19}\,\mathrm{C})(2.86\,\mathrm{mT})} = 1.43\,\mathrm{cm}$$

meaning the exit coordinates of the beam are: ($x = 0$, $y = 0$, $z = -2.86$ cm).

ASSESS A larger initial velocity would result in a greater distance traveled and a lower exit coordinate, while a stronger magnetic field would result in the opposite.

41. **INTERPRET** This problem is about modern mass spectrometer geometries, where a beam of charged particles entering a region with a uniform magnetic field, whose magnitude is varied, travel in an arc to the location of a single detector. For a particular design, we are to find the range of magnetic field strengths that would allow detection of singly ionized atoms in a particular mass range.

DEVELOP Equation 26.3, $r = mv/qB$, shows that the path radius depends on the field and on the particle's mass, charge, and speed. From the applied potential each particle gains kinetic energy $\frac{1}{2}mv^2 = qV$ resulting in a velocity of $v = \sqrt{2qV/m}$. We will solve for the magnetic field that would induce the path radius of this spectrometer for a given mass, in this case ranging from carbon-12 to iron-56.

EVALUATE Solving for the magnetic field strengths needed to for this spectrometer to detect isotopes with masses ranging from 12 to 56 atomic units we find

$$B = \frac{mv}{qr} = \frac{1}{r}\sqrt{\frac{2mV}{q}}$$

$$B_{12} = \frac{1}{(22\,\mathrm{cm})}\sqrt{\frac{2(2.75\,\mathrm{kV})(1.66\times10^{-27}\,\mathrm{kg})}{(1.60\times10^{-19}\,\mathrm{C})}}\sqrt{12} = 0.119\,\mathrm{T}$$

$$B_{56} = \frac{1}{(22\,\text{cm})}\sqrt{\frac{2(2.75\,\text{kV})(1.66\times10^{-27}\,\text{kg})}{(1.60\times10^{-19}\,\text{C})}}\sqrt{56} = 0.257\,\text{T}$$

meaning the magnetic field range needed is $0.119\,\text{T} \le B \le 0.257\,\text{T}$.

ASSESS If there were limitations to the design that would prevent the application of magnetic fields in this range, the distance from apex to detector, as well as the applied voltage, could be modified to meet those requirements.

42. **INTERPRET** We are to find the field inside and outside a current-carrying wire, so we can apply Ampère's Law.

DEVELOP To find the field at the given distances away from the center we will consider the enclosed current by an Ampèrian loop of radius equal to each distance. The magnetic field will be tangent to the current and will match the direction of the enclosing loop, so we can write

$$\oint \vec{B}\cdot d\vec{l} = 2\pi r B = \mu_0 I_{\text{enclosed}}$$

$$B = \frac{\mu_0 I_{\text{enclosed}}}{2\pi r}$$

We must then find the amount of current enclosed at each distance, recalling that the total current is related to the current density J by $J = I/A$, and the enclosed current is equal to the current density times the area enclosed.

EVALUATE **(a)** For a distance smaller than the wire radius, we find that the enclosed current is given by

$$I_{\text{enclosed}} = \frac{I}{\pi R^2}\pi r^2 = I\left(\frac{r}{R}\right)^2$$

and thus the magnetic field at the given distance ($r < R$) is equal to

$$B = \frac{\mu_0 I r}{2\pi R^2} = \frac{(1.26\times10^{-6}\,\text{N/A}^2)(147\,\text{A})(2.50\,\text{mm})}{2\pi(9.27\,\text{mm})^2} = 3.43\,\text{mT}$$

(b) For a distance greater than the wire radius, the enclosed current is equal to the total current, and the magnetic field at the given distance ($r > R$) is equal to

$$B = \frac{\mu_0 I}{2\pi r} = \frac{(1.26\times10^{-6}\,\text{N/A}^2)(147\,\text{A})}{2\pi(7.50\,\text{mm})} = 3.93\,\text{mT}$$

ASSESS Although the magnetic field at the position inside the wire is closer to the current, the source of magnetic field, the Ampèrian loop does not enclose the entire current. At the given a position outside the wire, although the field begins to decrease inversely with the distance, all of the current is enclosed, and we find a greater magnetic field at this location.

43. **INTERPRET** We want to find the maximum current which can flow through a wire before it reaches a certain magnetic field everywhere inside the wire. We can apply Ampère's Law to describe the field inside the wire.

DEVELOP To find the field at a given distance away from the center we will consider the enclosed current by an Ampèrian loop. The magnetic field will be tangent to the current and will match the direction of the enclosing loop, so we can write

$$\oint \vec{B}\cdot d\vec{l} = 2\pi r B = \mu_0 I_{\text{enclosed}}$$

$$B = \frac{\mu_0 I_{\text{enclosed}}}{2\pi r}$$

We are interested in ensuring the field everywhere inside the wire is below the given amount, and since the maximum value for the field inside will be found when $r = R$, the allowed current is given by

$$I \le \frac{2\pi R B_{\text{max}}}{\mu_0}$$

EVALUATE Plugging in the given values for the field and wire radius we find a maximum allowed current of

$$I_{\text{max}} = \frac{2\pi(1.5\,\text{mm})(0.19\,\text{T})}{(1.26\times10^{-6}\,\text{N/A}^2)} = 1.4\,\text{kA}$$

ASSESS Using a wire with a larger cross-sectional area could allow a higher current to pass through the superconductor before increasing the field beyond the critical amount.

44. **INTERPRET** We want to describe the field at various distances from the center of a coaxial cable carrying currents through an inner and outer conductor. We can apply Ampère's Law to describe the field in each region.

DEVELOP To find the field at the given distances away from the center we will consider the enclosed current by an Ampèrian loop of radius equal to each distance. The magnetic field will be tangent to the current and will match the direction of the enclosing loop, so we can write

$$\oint \vec{B} \cdot d\vec{l} = 2\pi r B = \mu_0 I_{\text{enclosed}}$$
$$B = \frac{\mu_0 I_{\text{enclosed}}}{2\pi r}$$

We must then find the amount of current enclosed at each distance, recalling that the total current is related to the current density J by $J = I/A$, and the enclosed current is equal to the current density times the area enclosed. We will choose to define the current flowing through the inner conductor as positive, and through the outer conductor as negative.

EVALUATE **(a)** For a distance smaller than the inner conductor radius, we find that the enclosed current is given by

$$I_{\text{enclosed}} = \frac{I}{\pi a^2}\pi r^2 = I\left(\frac{r}{a}\right)^2$$

and thus the magnetic field at the given distance ($r < a$) is equal to

$$B = \frac{\mu_0 I r}{2\pi a^2}$$

(b) For a distance greater than the inner conductor radius, the enclosed current is equal to the total current inside the inner conductor, and the magnetic field at the given distance ($r > a$) is equal to

$$B = \frac{\mu_0 I}{2\pi r}$$

(c) For a distance greater than the inner conductor radius ($r > a$), but in between the inner and outer radii of the outer conductor ($b < r < b + c$), the total enclosed current will depend on the current density of the outer conductor. The cross-sectional area of the outer conductor is equal to $A_{\text{bc}} = \pi(b+c)^2 - \pi b^2 = \pi c(2b+c)$, meaning the enclosed current within the outer conductor is equal to

$$I_{\text{enc. bc}} = \frac{-I\pi\left(r^2 - b^2\right)}{A_{\text{bc}}} = \frac{-I\left(r^2 - b^2\right)}{(b+c)^2 - b^2},$$

making the total enclosed current equal to $I_{\text{enc.}} = I\left(1 - \frac{\left(r^2 - b^2\right)}{(b+c)^2 - b^2}\right) = I\left(\frac{(b+c)^2 - r^2}{c(2b+c)}\right)$, and the total magnetic field in this region equal to

$$B = \frac{\mu_0 I}{2\pi r}\left(\frac{(b+c)^2 - r^2}{c(2b+c)}\right)$$

(d) For a distance beyond the outer conductor ($r > b+c$), the total current enclosed sums to zero, and thus the field is also equal to zero.

ASSESS The field inside the wire will increase as the distance from the center increases, until it crosses the edge of the inner conductor. From there it will begin to decrease until it reaches the edge of the outer conductor and becomes zero beyond this point.

45. **INTERPRET** We want to find the current, the magnetic fields found inside the inner conductor, and the current density inside the outer conductor for the coaxial cable described in the preceding problem.

DEVELOP In the previous problem we found the magnetic fields inside various regions of the coaxial cable by applying Ampère's Law at various distances from the center. We are given the field at a distance r greater than the inner conductor radius ($r > a$), where we found the field is given by $B = \mu_0 I/2\pi r$. Thus, we can determine the current which flows through each conductor and use it to find the fields and current densities in the wire.

EVALUATE **(a)** Using the given dimensions, along with the value of the field found at the given distance r, we find that the magnitude of the current which flows through each conductor is equal to

$$I = \frac{2\pi rB}{\mu_0} = \frac{2\pi(0.125\,\text{cm})(384\,\mu\text{T})}{\left(1.26\times10^{-6}\,\text{N/A}^2\right)} = 2.39\,\text{A}$$

(b) To find the field at a distance r which is less than the inner conductor radius ($r < a$) we use the expression found in the previous problem for this region, and evaluate the field using the current found in part (a), resulting in

$$B = \frac{\mu_0 Ir}{2\pi a^2} = \frac{\left(1.26\times10^{-6}\,\text{N/A}^2\right)(2.39)(0.300\,\text{mm})}{2\pi(0.525\,\text{mm})^2} = 522\,\mu\text{T}$$

(c) To find the current density in the outer conductor, we will evaluate the quotient of the total current and the cross-sectional area of the outer conductor, giving

$$J_{\text{bc}} = \frac{I}{\pi c(2b+c)} = \frac{(2.39\,\text{A})}{\pi(0.210\,\text{mm})\big(2(0.400\,\text{cm})+0.210\,\text{mm}\big)} = 442\,\text{kA/m}^2$$

ASSESS We see that the given magnetic field at a distance greater than then inner conductor radius is lower than the field we calculate for the region inside the inner conductor, as we determined would be the case in the previous problem.

PROBLEMS

46. **INTERPRET** This problem asks us to find the magnetic force exerted on a moving charged particle. We are given the velocity and magnetic field.

DEVELOP The magnetic force on a moving charge can be calculated using Equation 26.1: $\vec{F} = q\vec{v}\times\vec{B}$, which gives

$$\vec{F} = q\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5.0\text{ m/s} & 0 & 3.2\text{ m/s} \\ 9.4\text{ T} & 6.7\text{ T} & 0 \end{vmatrix} = -\hat{i}q(3.2\text{ m/s})(6.7\text{ T}) + \hat{j}q(3.2\text{ m/s})(9.4\text{ T}) + \hat{k}q(5.0\text{ m/s})(6.7\text{ T})$$

EVALUATE **(a)** The force is

$$\vec{F} = \left(-1.1\hat{i} + 1.5\hat{j} + 1.7\hat{k}\right)\times10^{-3}\text{ N}$$

The magnitude and direction can be found from the components, if desired.

(b) The dot product between force and velocity is

$$\begin{aligned}\vec{F}\cdot\vec{v} &\propto \left(-1.072\hat{i} + 1.504\hat{j} + 1.675\hat{k}\right)\left(5.0\hat{i} + 3.2\hat{k}\right)\\ &= (-1.072)(5.0) + (1.675)(3.2) = 0\end{aligned}$$

The dot product between force and the magnetic field is

$$\begin{aligned}\vec{F}\cdot\vec{B} &\propto \left(-1.072\hat{i} + 1.504\hat{j} + 1.675\hat{k}\right)\left(9.4\hat{i} + 6.7\hat{j}\right)\\ &= (-1.072)(9.4) + (1.504)(6.7) = 0\end{aligned}$$

ASSESS The fact that the product $\vec{F}\cdot\vec{v}$ vanishes can also be shown in a general fashion as follows:

$$\vec{F}\cdot\vec{v} = \left(q\vec{v}\times\vec{B}\right)\cdot\vec{v} = q\left(\vec{v}\times\vec{v}\right)\cdot\vec{B} = 0$$

where we have used the vector identity $\left(\vec{A}\times\vec{B}\right)\cdot\vec{C} = \left(\vec{C}\times\vec{A}\right)\cdot\vec{B}$.

47. **INTERPRET** The problem asks us to estimate Jupiter's magnetic dipole moment given the magnetic field strength at its poles.

DEVELOP The magnetic field on the axis of a magnetic dipole far from its center is given by $B = \mu_0 I a^2 / 2x^3$ (see Equation 26.9). If we substitute $\mu = \pi I a^2$ for the magnetic dipole moment, $B = \mu_0 \mu / 2\pi x^3$.

EVALUATE We can assume that Jupiter's poles are both one radii away from the planet's magnetic dipole: $x = 6.91\times10^7\,\text{m}$ (from Appendix E). Given the field at the poles, the magnetic dipole moment is

$$\mu = \frac{2\pi x^3 B}{\mu_0} = \frac{2\pi\left(6.91\times10^7\,\text{m}\right)^3\left(14\times10^{-4}\,\text{T}\right)}{4\pi\times10^{-7}\,\text{T}\cdot\text{m/A}} = 2.3\times10^{27}\,\text{A}\cdot\text{m}^2$$

ASSESS This is a huge dipole moment, but we can imagine it would have to be to produce a planet-wide magnetic field. The units are correct, since the magnetic dipole moment is current multiplied by area. Jupiter's magnetic field is believed to arise from currents in metallic hydrogen found deep beneath the planet's surface. If we assume that Jupiter's dipole moment were due to a single giant current loop with a radius half that of the planet, the loop would have to carry a current of $I = \dfrac{\mu}{A} = 6\times10^{11}\,\text{A}$.

48. **INTERPRET** We are given the force exerted on a first proton of known velocity and the force exerted on a second proton whose velocity direction is given, but not its speed. We are to find the magnetic field vector and the speed of the second proton.

DEVELOP Apply Equation 26.1 to generate two scalar equations relating the components of the force to those of the velocity and magnetic field. For the first proton, this gives

$$\vec{F}_1 = F_1\hat{i} = q\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 3.6\times10^4\ \text{m/s} & 0 \\ B_x & B_y & B_z \end{vmatrix} = \begin{cases} q\left[\left(3.6\times10^4\ \text{m/s}\right)(B_z)-(0)\left(B_y\right)\right]\hat{i} \\ -q\left[(0)(B_z)-(0)(B_x)\right]\hat{j} \\ +q\left[(0)B_y-\left(3.6\times10^4\ \text{m/s}\right)(B_x)\right]\hat{k} \end{cases}$$

which tells us that $B_x = 0$ because F_1 has no z-component. For the second proton, Equation 26.1 gives

$$\vec{F}_2 = F_2\hat{j} = q\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ v_x & 0 & 0 \\ 0 & B_y & B_z \end{vmatrix} = \begin{cases} q\left[(0)(B_z)-(0)\left(B_y\right)\right]\hat{i} \\ -q\left[(v_x)(B_z)\right]\hat{j} \\ +q\left[(v_x)\left(B_y\right)\right]\hat{k} \end{cases}$$

which tells us that $B_y = 0$ because the F_2 has no z-component. We can equate the vector components to find B_z and v_x.

EVALUATE From the equation for F_1, the z-component of the magnetic field is

$$F_1 = q\left(3.6\times10^4\ \text{m/s}\right)(B_z)$$

$$B_z = \frac{F_1}{q\left(3.6\times10^4\ \text{m/s}\right)} = \frac{7.4\times10^{-16}\ \text{N}}{\left(1.6\times10^{-19}\ \text{C}\right)\left(3.6\times10^4\ \text{m/s}\right)} = 0.13\ \text{T}$$

From the equation for F_2, the speed of the second proton is

$$F_2 = q(v_x)(B_z)$$

$$v_x = \frac{F_2}{qB_z} = \frac{2.8\times10^{-16}\ \text{N}}{\left(1.6\times10^{-19}\ \text{C}\right)(0.128\ \text{T})} = 1.4\times10^4\ \text{m/s}$$

Thus, the magnetic field is $\vec{B} = (0.13\ \text{T})\hat{k}$ and the velocity of the second proton is $\vec{v} = \left(1.4\times10^4\ \text{m/s}\right)\hat{i}$.

ASSESS As required, the force is perpendicular to the magnetic field and to the velocity in each case.

49. **INTERPRET** We are asked to approximate the amount of current flowing in the Earth's liquid core that would produce the measured magnetic field at the poles. We assume the current is confined to a single loop whose axis passes through the poles.

DEVELOP The magnetic field from a single current loop was calculated in Example 26.3 for a point on the loop's axis: $B = \mu_0 I a^2 / 2\left(x^2 + a^2\right)^{3/2}$. For the given model of the Earth's field, the radius of the loop, $a = 3000$ km, is not much smaller than the distance to the north pole: $x = 6{,}370$ km.

EVALUATE Solving for the current, we arrive at

$$I = \frac{2B\left(x^2 + a^2\right)^{3/2}}{\mu_0 a^2} = \frac{2(62\ \mu\text{T})\left[(6370\text{ km})^2 + (3000\text{ km})^2\right]^{3/2}}{\left(4\pi \times 10^{-7}\,\text{T}\cdot\text{m/A}\right)(3000\text{ km})^2} = 3.8\text{ GA}$$

ASSESS A single current loop would generate a dipole magnetic field, but the Earth's field is more complicated than that. It is believed that convection of molten iron in the liquid core creates a dynamo that sustains the planet's magnetic field.

50. **INTERPRET** A beam of accelerated electrons is bent by a magnetic field. We are asked to find the magnetic field strength necessary to generate a curved path of a given radius.

DEVELOP The accelerated electrons will obtain a kinetic energy equal to the potential energy gained by traveling through the applied potential: $\frac{1}{2}mv^2 = eV$. Since the beam is perpendicular to the field, it will follow a circular path with radius $r = mv/eB$ (Equation 26.3). We can relate their resultant velocity to the desired radius of curvature and solve for the necessary magnetic field strength. .

EVALUATE Plugging in the velocity in terms of the applied potential and solving for the magnetic field strength

$$v = \sqrt{\frac{2eV}{m}}$$

$$B = \frac{1}{r}\sqrt{\frac{2mV}{e}} = \frac{1}{(7.12\text{cm})}\sqrt{\frac{2\left(9.11\times10^{-31}\text{kg}\right)(25.0\text{kV})}{\left(1.6\times10^{-19}\text{C}\right)}} = 7.47\text{ mT}$$

ASSESS Since the electron speed is proportional to the applied potential difference, so will the strength of the field necessary to curve the path.

51. **INTERPRET** This problem is about a charged particle undergoing circular motion in a magnetic field, and we want to express the radius of the orbit in terms of its charge, mass, kinetic energy, and the magnetic field strength.

DEVELOP From Equation 26.3, the radius of the circular motion is $r = mv/(qB)$. For a nonrelativistic particle, $K = \frac{1}{2}mv^2$, or $v = \sqrt{2K/m}$.

EVALUATE Therefore, the radius of the orbit is

$$r = \frac{mv}{qB} = \frac{m}{qB}\sqrt{\frac{2K}{m}} = \frac{\sqrt{2Km}}{qB}$$

ASSESS Our result indicates that the radius is proportional to $\sqrt{K}$, or v. Thus, the greater the kinetic energy of the particle, the larger its radius.

52. **INTERPRET** The problem asks us to consider the acceleration of deuterium nuclei in a cyclotron.

DEVELOP Particles in a cyclotron get a boost in velocity each time they pass from one dee to the other. The magnetic field holds them in a circular orbit, so they make multiple passes. In order to always be accelerating the particles, the voltage has to be alternated every time they make a half circle of their orbit. In other words, the voltage needs to cycle at the same rate as the particles revolve in the magnetic field, which is just the cyclotron frequency: $f = qB/2\pi m$ (Equation 26.4). In this case, the particles are deuterium nuclei, which have atomic mass 2 and charge $+e$:

$$m = 2(1.66\times10^{-27}\text{kg}) = 3.32\times10^{-27}\text{kg}$$

$$q = +1.60\times10^{-19}\text{C}$$

The frequency does not depend on the speed (energy) of the nuclei, but the radius of their orbit does: $r = mv/eB$ (Equation 26.3). The maximum energy is achieved when the nuclei reach the outer rim of the cyclotron. We can figure out how many orbits it takes to reach this maximum by dividing by the kinetic energy gain of each orbit. We'll assume the nuclei have negligible kinetic energy to begin with.

EVALUATE **(a)** The frequency at which the voltage should be alternated is:

$$f = \frac{qB}{2\pi m} = \frac{(1.60\times 10^{-19}\,\text{C})(2.0\,\text{T})}{2\pi(3.32\times 10^{-27}\,\text{kg})} = 15\ \text{MHz}$$

(b) The maximum kinetic energy can be derived from the speed at the cyclotron's radius:

$$K = \tfrac{1}{2}m\left(\frac{rqB}{m}\right)^2 = \frac{(rqB)^2}{2m}$$

$$= \frac{\left[\left(\frac{1}{2}\cdot 90\ \text{cm}\right)\left(1.60\times 10^{-19}\,\text{C}\right)\left(2.0\ \text{T}\right)\right]^2}{2(3.32\times 10^{-27}\,\text{kg})} = 3.1\times 10^{-12}\,\text{J} = 20\ \text{MeV}$$

We've written the answer in eV, as this unit is easier to work with for particles.

(c) Each orbit in the cyclotron accounts for two passes across the potential difference between the dees. Therefore, the kinetic energy gain in each orbit is $\Delta K = 2q\Delta V$, and the number of orbits needed to reach the maximum energy is

$$\frac{K}{\Delta K} = \frac{20\ \text{MeV}}{2(e)(1500\ \text{V})} = \frac{20\ \text{MeV}}{(3000\ \text{eV})} = 6700$$

ASSESS Notice how much easier the final calculation was when we were using eV rather than J. At 15 MHz, the deuterium nuclei will reach the maximum energy in less than half a millisecond.

53. **INTERPRET** In this problem, an electron is moving in a magnetic field with a velocity that has both parallel and perpendicular components to the magnetic field. The path is a spiral.

DEVELOP The radius depends only on the perpendicular velocity component, $r = \frac{mv_\perp}{eB}$. On the other hand, the distance moved parallel to the field is $d = v_\parallel T$, where T is the cyclotron period.

EVALUATE **(a)** The radius of the spiral path is

$$r = \frac{mv_\perp}{eB} = \frac{\left(9.11\times 10^{-31}\ \text{kg}\right)\left(3.1\times 10^{6}\ \text{m/s}\right)}{\left(1.6\times 10^{-19}\ \text{C}\right)\left(0.20\ \text{T}\right)} = 88\ \mu\text{m}$$

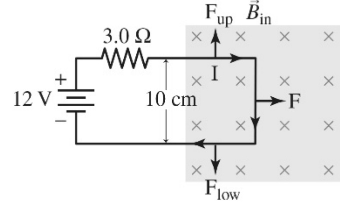
(b) Since $v_\parallel = v_\perp$, the distance moved parallel to the field is

$$d = v_\parallel T = v_\perp\left(\frac{2\pi m}{eB}\right) = 2\pi\left(\frac{mv_\perp}{eB}\right) = 2\pi r = 2\pi\left(88\ \mu\text{m}\right) = 550\ \mu\text{m}$$

ASSESS Since motion parallel to the field is not affected by the magnetic force, with $v_\parallel = v_\perp$, the distance traveled in $t = T$ along the direction of the field is simply $d = 2\pi r$.

54. **INTERPRET** This problem requires us to find the force exerted on a wire that results from the interaction between the current in the wire and the magnetic field.

DEVELOP The current in the wire is given by Ohm's law (macroscopic version, see Table 24.2) $V = IR$. Applying Equation 26.5, $\vec{F} = I\vec{l}\times\vec{B}$, we see that the force on the upper and lower section of the circuit cancel (see sketch below), leaving only the force on the vertical section of the circuit.



EVALUATE Inserting the given quantities into Equation 26.5 we find

$$\vec{F} = I\vec{l}\times\vec{B} = \frac{V}{R}\vec{l}\times\vec{B} = \frac{12\ \text{V}}{3.0\ \Omega}\left[\left(0.10\ \text{m}\right)\left(-\hat{j}\right)\times\left(0.038\ \text{mT}\right)\left(-\hat{k}\right)\right] = \left(15\ \text{mN}\right)\hat{\imath}$$

ASSESS This problem ignores field-fringing effects that would occur at the edge of the field. However, by symmetry these effects cancel because these effects have the opposite influence on the upper and lower sections of the circuit.

55. **INTERPRET** You're designing a prosthetic ankle that uses an electric motor. You need to find the current necessary to achieve the desired torque.

DEVELOP As described in Example 26.5, an electric motor consists of a current loop in a magnetic field. The torque is given by Equation 26.4: $\tau = \left|\vec{\mu}\times\vec{B}\right| = \mu B\sin\theta$. The magnetic dipole moment is equal to $\mu = NIA$ (Equation 26.12).

EVALUATE The torque is maximum when the magnetic dipole moment is perpendicular to the field $(\sin\theta = 1)$. Solving for the current gives

$$I = \frac{\tau_{\max}}{NAB} = \frac{(2.9\ \text{mN}\cdot\text{m})}{(150)\pi(15/2\ \text{mm})^2(200\ \text{mT})} = 0.55\ \text{A}$$

ASSESS Note that the units work out, since $1\ \text{T} = 1\ \text{N}/\text{A}\cdot\text{m}$. The result seems like a reasonable current for this application. But care will be needed to ensure no current leaks out into the surrounding tissue.

56. **INTERPRET** We are to find the current that's needed to produce a magnetic force to raise the current-carrying rod against the force of gravity. In addition, we need to specify the direction required for the current.

DEVELOP An upward magnetic force on the rod equal (in magnitude) to its weight $(= mg)$ is the minimum force necessary to maintain the bar in equilibrium with gravity. The magnetic force is given by Equation 26.5, $\vec{F} = I\vec{l}\times\vec{B}$, which reduces to $F = IlB\sin\theta = IlB$ because the rod is perpendicular to the magnetic field (so $\theta = 90°$).

EVALUATE **(a)** The minimum current is obtained by setting $IlB = mg$, or

$$I = \frac{mg}{lB} = \frac{(0.015\ \text{kg})(9.8\ \text{m}/\text{s}^2)}{(0.20\ \text{m})(0.19\ \text{T})} = 3.9\ \text{A}$$

(b) Using the right-hand rule, we see that, for the force to be upward, the current must flow from A to B.

ASSESS A current of 3.9 A sounds reasonable. The weight of the rod is about $F_g = mg = 0.147\ \text{N}$. To support the weight with an upward magnetic force, we need a strong enough magnetic field and big enough current such that $IlB \ge mg$.

57. **INTERPRET** This problem deals with the Hall effect, which we can use to find the number density of free electrons (i.e., mobile electrons) in copper.

DEVELOP The geometry in this problem is the same as that in the discussion leading to Equation 26.6, which shows that the number density n of charge carriers is

$$n = IB/qV_{\text{H}}t$$

EVALUATE Inserting the given quantities into this expression gives

$$n = IB/qV_{\text{H}}t = \frac{(7.1\ \text{A})(2.9\ \text{T})}{(1.6\times10^{-19}\ \text{C})(1.2\ \mu\text{V})(1.0\ \text{mm})} = 1.1\times10^{23}\ \text{cm}^{-3}$$

ASSESS This is a typical number density for free electrons in a metal.

58. **INTERPRET** This problem is about the change in potential energy of a magnetic dipole moment.

DEVELOP From Equation 26.15, the potential energy of a magnetic dipole in a magnetic field is $U = -\vec{\mu}\cdot\vec{B} = \mu B\cos\theta$. Therefore, the energy required to reverse the orientation of a proton's magnetic moment from parallel $(\theta = 0)$ to antiparallel $(\theta = 180°)$ with respect to the applied magnetic field is

$$\Delta U = U_{\uparrow\downarrow} - U_{\uparrow\uparrow} = -\mu B\cos 180° - (-\mu B\cos 0) = 2\mu B$$

where $\mu = 1.41\times10^{-26}\,\text{A}\cdot\text{m}^2$ is the magnetic dipole moment of the proton.

EVALUATE Substituting the values given, we find the energy to be

$$\Delta U = 2\mu B = 2(1.41\times10^{-26}\,\text{A}\cdot\text{m}^2)(9.4\ \text{T}) = 2.7\times10^{-25}\ \text{J} = 1.7\ \mu\text{eV}$$

ASSESS The potential energy of a dipole moment is a minimum ($U = -\mu B$) when it is parallel to the magnetic field, but a maximum ($U = +\mu B$) when it is antiparallel to the field. Positive work must be done to flip the dipole. In an NMR device, the alignment by the magnetic field of protons and other particles with magnetic dipole moments can be studied by passing radio waves through the sample.

59. **INTERPRET** We are to find the force on a quarter-circle of current-carrying wire in a magnetic field. We will use the equation for magnetic force on a wire, which we will express in differential form and then integrate to determine the net force.

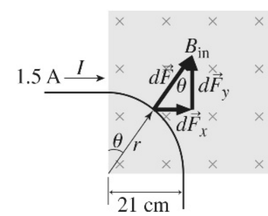
DEVELOP The force on a section of wire *dl* carrying a current *I* in a magnetic field *B* is

$$d\vec{F} = Id\vec{l} \times \vec{B} \quad \Rightarrow \quad dF = IBdl$$

where we have used $\sin\theta = 1$, because $\theta = 90°$ for this problem. Using the right-hand rule, we can see from Fig. 26.42 that the force on the wire will be everywhere radially away from the loop center (see the figure below). The horizontal component of force on each segment *dl* of the wire is given by $dF_x = IBdl\sin\theta$, where θ is the angle between the vertical and a line between the center of curvature and the wire segment *dl*, as shown in the figure below. The vertical component of the force is $dF_y = IBdl\cos\theta$. The total horizontal force on the wire is then

$$\vec{F} = \hat{i}\int_{\theta=0}^{\theta=\pi/2} IB\sin\theta dl + \hat{j}\int_{\theta=0}^{\theta=\pi/2} IB\cos\theta dl$$

In terms of θ, $dl = rd\theta$. The current and field are constant: $I = 1.5$ A and $B = 48\times10^{-3}$ T. The radius of curvature is $r = 0.21$ m.



EVALUATE Evaluating the integral gives

$$\vec{F} = \hat{i}\int_0^{\pi/2} IB\sin\theta rd\theta + \hat{j}\int_0^{\pi/2} IB\cos\theta rd\theta$$

$$= IBr\left[\hat{i}\int_0^{\pi/2}\sin\theta rd\theta + \hat{j}\int_0^{\pi/2}\cos\theta rd\theta\right] = IBr\left(\hat{i} + \hat{j}\right)$$

Inserting the values gives

$$\vec{F} = (1.5\text{ A})\left(48\times10^{-3}\text{ T}\right)(0.21\text{ m})\left(\hat{i} + \hat{j}\right) = (0.015\text{ N})\left(\hat{i} + \hat{j}\right)$$

The magnitude is thus $F = \sqrt{2}(0.015\text{ N}) = 0.021\text{ N}$ and the direction is 45° above horizontal.

ASSESS The symmetry of the problem makes evaluation of the integral straightforward.

60. **INTERPRET** This problem deals with the Hall effect, which we can use to find the maximum value for the Hall potential a phone sensor develops for a given strength of Earth's magnetic field.

DEVELOP The geometry in this problem is the same as that in the discussion leading to Equation 26.6, which shows that the Hall voltage that develops across this sensor is

$$V_H = IB/nqt$$

EVALUATE Inserting the given quantities into this expression gives

$$V_H = IB/nqt = \frac{(625\ \mu\text{A})(27.5\ \mu\text{T})}{\left(2.86\times10^{15}\text{ cm}^{-3}\right)\left(1.6\times10^{-19}\text{ C}\right)(50.0\ \mu\text{m})} = 0.751\ \mu\text{V}$$

ASSESS The silicon used for the sensor is a semiconductor, meaning the electron density and current found across it are smaller than that of conducting material.

61. **INTERPRET** This problem involves finding the magnetic field of a wire that is bent into the given geometrical shape and through which flows the given current.

DEVELOP The wire may be divided into a straight section and the loop, and the current at the loop center will be the superposition of the magnetic fields from these two components. The magnetic field due to the straight section is

$$B_{\text{straight}} = \frac{\mu_0 I}{2\pi a}$$

where a is the loop radius (see Example 26.4 and Equation 26.10). From Example 26.3, the loop contribution to the magnetic field is

$$B_{\text{loop}} = \frac{\mu_0 I}{2a}$$

where we have used $x = 0$ in Equation 26.9. Using the right-hand rule, we see that both contributions are out of the page, which we define as the $\hat{k}$ direction.

EVALUATE Inserting the given quantities and summing the two contributions gives

$$\vec{B} = \vec{B}_{\text{straight}} + \vec{B}_{\text{loop}} = \frac{\mu_0 I}{2\pi a}\hat{k} + \frac{\mu_0 I}{2a}\hat{k} = (1+\pi)\frac{\mu_0 I}{2\pi a}\hat{k}$$

ASSESS The superposition principle greatly simplifies this problem, both analytically and conceptually.

62. **INTERPRET** You want to know what effect a power line overhead will have on your compass reading.

DEVELOP The magnetic field induced by the power line has magnitude given by Equation 26.10: $B = \mu_0 I / 2\pi d$, where d is the distance from the wire to the ground. If the current is moving northward, then by the right-hand rule the induced magnetic field will point westward at ground level. You can add this magnetic field to the Earth's field to see what effect it will have on your orientation.

EVALUATE The induced field will have a magnitude of

$$B = \frac{\mu_0 I}{2\pi d} = \frac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(1.5\text{ kA})}{2\pi(10\text{ m})} = 0.30\text{ G}$$

Let's assume that north is in the positive y-direction, so that this induced field points in the negative x-direction (west). The total field at your location will therefore be

$$\vec{B}_{\text{tot}} = \vec{B}_{\text{E}} + \vec{B} = 0.24\hat{j} - 0.30\hat{i}\text{ G}$$

Your compass needle will point at an angle of $\theta = \tan^{-1}\left(\frac{0.30}{0.24}\right) = 51°$ to the west of true north.

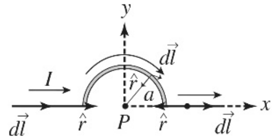
ASSESS The lesson is that you shouldn't use a compass to orient yourself when standing near a power line.

63. **INTERPRET** We are to find the magnetic field at the center of a semicircular current-carrying wire by using the Biot–Savart law.

DEVELOP Use the coordinate system shown in the figure below. The Biot–Savart law (Equation 26.7) written in a coordinate system with origin at P, gives

$$\vec{B}(P) = \frac{\mu_0 I}{4\pi}\int_{wire}\frac{d\vec{l}\times\hat{r}}{r^2}$$

where $\hat{r}$ is a unit vector from an element $d\vec{l}$ on the wire to the field point P. On the straight segments to the left and right of the semicircle, $d\vec{l}$ is parallel to $\hat{r}$ and $-\hat{r}$, respectively, so $d\vec{l}\times\hat{r} = 0.$ On the semicircle, $d\vec{l}$ is perpendicular to $\hat{r}$ and the radius is constant at $r = a$.



EVALUATE Evaluating the integral gives

$$B(P) = \frac{\mu_0 I}{4\pi} \int_{\text{semicircle}} \frac{d\vec{l} \times \hat{r}}{a^2} = \int_0^{\pi} \frac{r d\theta}{a^2} \hat{k} = \frac{\mu_0 I}{4\pi} \left(\frac{\pi a}{a^2} \right) \hat{k} = \frac{\mu_0 I}{4a} \hat{k}$$

where $\hat{k}$ is into the page.

ASSESS Notice that $\hat{r}$ is dimensionless, so the units work out to be $(\text{N/A}^2)(\text{A})/\text{m} = \text{N}/(\text{A}\cdot\text{m}) = \text{T}$.

64. **INTERPRET** This problem deals with the current enclosed by a page, so we can use Ampère's Law.
DEVELOP The geometry in this problem has a magnetic field pointing upward on the left hand side of the page, and pointing downward on the right-hand side of the page. To determine the direction of the current enclosed by the page we can apply the right-hand rule and determine which orientation will result in the observed field directions. To calculate the enclosed current, we must enclose the page with a rectangular loop, and apply Ampère's Law: $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{\text{enclosed}}$. For the segments across the top and bottom of the page, $\vec{B} \perp d\vec{l}$, so they don't contribute to the integral. For the segments left and right of the page, which are parallel to the magnetic field direction, we can measure the length of the page and use it to calculate the enclosed current magnitude.
EVALUATE **(a)** We measure the length of the page to be approximately l = 10.875 in, which results in an enclosed current of

$$\oint \vec{B}\cdot d\vec{l} = 2Bl = \mu_0 I_{\text{enclosed}}$$

$$I_{\text{enclosed}} = \frac{2Bl}{\mu_0} = \frac{2(34\,\mu\text{T})(0.276\,\text{m})}{(1.26\times10^{-6}\,\text{N/A}^2)} = 15\,\text{A}$$

(b) To determine the direction of this current, we point our thumb in the direction of the magnetic field at both edges of the page, and find that our fingers curl toward the page. Therefore the current must flow into the page to generate the observed magnetic field directions.
(c) Lastly, we determine that in order for the magnetic field direction to change as you go across the page left-to-right, the current must be localized near the center, and thus it must cross the page on a thin sheet in the gap between columns.
ASSESS The dimensions you obtain for the measurement of the page length will determine the final answer, but you should find agreement with this result up to two significant figures.

65. **INTERPRET** This problem involves finding the force on a wire loop carrying a current due to the magnetic field of a nearby straight wire also carrying current.
DEVELOP At any given distance from the long, straight wire, the force on a current element in one of the 10-cm segments cancels that on a corresponding element in the other. The force on the near side (parallel currents) is attractive, and that on the far side (antiparallel currents) is repulsive. The force is given by Equation 26.5, $\vec{F} = I\vec{l}\times\vec{B}$, where the magnetic field may be found using Equation 26.10, $B = \mu_0 I/(2\pi y)$, where y is the distance from the straight wire.
EVALUATE Performing the sum and inserting the given quantities gives

$$F = \frac{\mu_0 I_1 I_2 l}{2\pi}\left(\frac{1}{y_{\text{near}}} - \frac{1}{y_{\text{far}}}\right) = \frac{(4\pi\times10^{-7}\ \text{N/A}^2)(25\ \text{A})(0.85\ \text{A})(0.15\ \text{m})}{2\pi}\left(\frac{1}{0.030\ \text{m}} - \frac{1}{0.13\ \text{m}}\right)$$
$$= 1.6\times10^{-5}\ \text{N}$$

or 16 µN toward the wire.
ASSESS Notice that this expression reduces to Equation 26.10 for $y_{\text{far}} \to \infty$, as expected.

66. **INTERPRET** The system is a long conducting rod having a nonuniform current density. We are interested in the magnetic field strength both inside and outside the rod. The problem involves Ampère's law.
DEVELOP The magnetic field of a long conducting rod is approximately cylindrically symmetric, as discussed in Section 26.8. The magnetic field can be found by using Ampère's law (Equation 26.17)

$$\oint \vec{B}\cdot d\vec{r} = \mu_0 I_{\text{encircled}}$$

EVALUATE **(a)** Inside the rod, Ampère's law can be used to find the field, as in Example 26.8, by integrating the current density over a smaller cross-sectional area that corresponds to $I_{\text{encircled}}$ for an Ampèrian loop with $r \le R$. Then,

$$I_{\text{encircled}} = \int_0^r J_0 \frac{r}{R}(2\pi r dr) = \frac{2\pi J_0 r^3}{3R}$$

Here, area elements were chosen to be circular rings of radius r, thickness dr, and area $dA = 2\pi r dr$. Ampère's law thus gives $2\pi r B = \mu_0 I_{\text{encircled}}$, or

$$B = \frac{\mu_0 J_0 r^2}{3R}$$

for $r \le R$.

(b) The field outside $(r \ge R)$ is given by Equation 26.17, and has direction circling the rod according to the right-hand rule. The total current can be related to the current density by integrating over the cross-sectional area of the rod:

$$I = \int \vec{J} \cdot d\vec{A} = \int_0^R J_0 \frac{r}{R}(2\pi dr) = \frac{2\pi J_0 R^2}{3}$$

The magnetic field outside the wire is then

$$B = \frac{\mu_0 I}{2\pi r} = \frac{\mu_0 J_0 R^2}{3r}$$

for $r \ge R$.

ASSESS The magnetic field increases as r^2 inside the rod, but decreases as $1/r$ outside the rod. At $r = R$, both expressions give the same result: $B(r = R) = \mu_0 J_0 R/3$.

67. **INTERPRET** This problem involves Ampère's law, which we can use to find the magnetic field inside and outside a conducting pipe that carries current.

DEVELOP Apply Ampère's law, Equation 26.17 to both situations. Inside the pipe, there is no current enclosed by the Ampèrian loop. Outside the pipe, the current enclosed is I.

EVALUATE **(a)** Because there is no current enclosed inside the pipe, the magnetic field is zero (B = 0) inside the pipe.

(b) Outside the pipe, the field is cylindrically symmetric (provided we are far, compared to the pipe radius, from the pipe ends), so Ampère's law gives

$$\oint \vec{B} \cdot d\vec{r} = B 2\pi r = \mu_0 I_{\text{enclosed}}$$

$$B = \frac{\mu_0 I}{2\pi r}$$

ASSESS The field outside a hollow pipe is just like that outside a wire (see Example 26.7).

68. **INTERPRET** This problem involves finding the magnetic field for two different wire configurations, given the current and the geometrical parameters.

DEVELOP Apply Equation 26.21, $B = \mu_0 n I$, to find the magnetic field inside the solenoid. The maximum loop density is one loop per wire diameter, or $n = 1/d = 2174\ \text{m}^{-1}$. The magnetic field at the center of a flat, circular current loop can be found from Equation 26.9, $B = \mu_0 I/(2a)$, where $2\pi a = 10$ m, or $a = (10\ \text{m})/(2\pi)$.

EVALUATE **(a)** Inserting the given quantities gives the magnetic field B at the solenoid center as

$$B = \mu_0 n I = \left(4\pi \times 10^{-7}\ \text{N/A}^2\right)\left(2174\ \text{m}^{-1}\right)(13\ \text{A}) = 3.6 \times 10^{-2}\ \text{T}$$

(b) The magnetic field at the center of the loop is

$$B = \frac{\mu_0 I}{2a} = \frac{2\pi\left(4\pi \times 10^{-7}\ \text{N/A}^2\right)(13\ \text{A})}{2(10\ \text{m})} = 5.1\ \mu\text{T}$$

ASSESS To verify that the use of Equation 26.21 is justified, we should verify that the solenoid length is much, much greater than its radius. The length of the solenoid is

$$L = d\left(\frac{10\ \text{m}}{2\pi a}\right) = (46\ \text{mm})\frac{10\ \text{m}}{2\pi(0.015\ \text{m})} \approx 5\ \text{cm}$$

which is essentially an order of magnitude larger than the solenoid diameter (anything over a factor of three is about an order of magnitude; see the scale of a $\log_{10}$ plot). Thus, the use of Equation 26.21 is justified. Notice also that the magnetic field in the solenoid is four orders of magnitude larger than that at the center of a single loop.

69. **INTERPRET** In this problem we are asked to derive to expression for the magnetic field of a solenoid by treating it as being made up of a large number of current loops.

DEVELOP Consider a small length of solenoid, dx, to be like a coil of radius R and current $nI\ dx$. Using Equation 26.9, the axial field is

$$dB = \frac{\mu_0 (nIdx)R^2}{2(x^2+R^2)^{3/2}}$$

with direction along the axis according to the right-hand rule. For a very long solenoid, we can integrate this from $x=-\infty$ to $x=+\infty$ to find the total field.

EVALUATE Integrating over dx from $x=-\infty$ to $x=+\infty$, we find the magnetic field to be

$$B_{\text{sol}} = \frac{\mu_0 nIR^2}{2}\int_{-\infty}^{\infty}\frac{dx}{(x^2+R^2)^{3/2}} = \frac{\mu_0 nIR^2}{2}\frac{x}{R^2\sqrt{x^2+R^2}}\Bigg|_{-\infty}^{\infty} = \mu_0 nI$$

This is the expression given in Equation 26.20.

ASSESS For a finite solenoid, a similar integral gives the field at any point on the axis only, for example, at the center of a solenoid of length L,

$$B(0) = \frac{\mu_0 nIL}{\sqrt{L^2+4R^2}}$$

70. **INTERPRET** This problem involves finding the distance from a lightning strike, which we will model as a long, straight wire, at which the magnetic field strength is the same magnitude as the magnetic field of the Earth.

DEVELOP Solve Equation 26.10 of Example 26.4 for the distance y.

EVALUATE Inserting the given quantities, we find

$$y = \frac{\mu_0 I}{2\pi B} = \frac{\left(4\pi\times10^{-7}\ \text{N/A}^2\right)(250\ \text{kA})}{2\pi(50\ \mu\text{T})} = 1.0\ \text{km}$$

ASSESS This seems like a reasonable distance.

71. **INTERPRET** In this problem the magnetic field exerts a torque on protons found in the hydrogen of water and fat molecules. Given the magnetic field strength and proton magnetic dipole we are to find the maximum exerted torque.

DEVELOP The magnetic torque exerted on a dipole moment $\vec{\mu}$ by the magnetic field $\vec{B}$ is given by Equation 26.15 $\vec{\tau} = \vec{\mu}\times\vec{B}.$ The magnitude of $\vec{\tau}$ is

$$\tau = \mu B\sin\theta$$

where θ is the angle between the orientation of the proton's magnetic dipole and the magnetic field direction. The maximum torque exerted on the protons occurs when they lie perpendicular to the field direction, $\theta = 90$.

EVALUATE Substituting the values given in the problem, we find the maximum torque to be

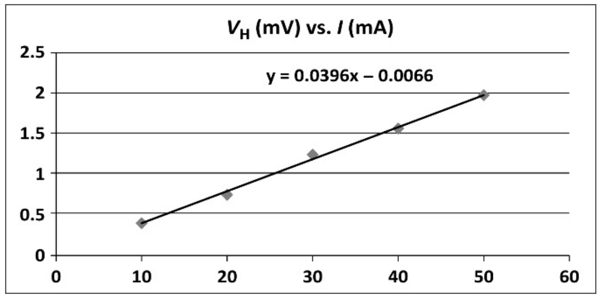
$$\tau = \mu B\sin(90) = \left(1.41\times10^{-26}\,\text{A}\cdot\text{m}^2\right)(2.6\text{T}) = 3.67\times10^{-26}\,\text{N}\cdot\text{m}$$

ASSESS This applied torque will work to align the magnetic dipole moment with the magnetic field.

72. **INTERPRET** This problem deals with the Hall effect. By analyzing the data of the Hall potential as a function of current, we want to find the magnetic field strength.

DEVELOP From Equation 26.6, the Hall potential is $V_{\text{H}} = I(B/nqt)$, where t is the thickness of the conducting material. By plotting V_{H} versus I, we get a straight line with slope B/nqt, which allows us to deduce the field strength B.

EVALUATE The plot is shown below.



The slope of the best-fit line is 0.0396 V/A, which allows us to solve for B as:

$$B = (0.0396\text{ V/A})nqt = \frac{(0.0396\text{ V/A})(5.0\times10^{-5}\text{ m})}{228\times10^{-6}\text{ m}^3/\text{C}} = 8.69\text{ mT} = 86.9\text{ G}$$

ASSESS The value of the hall potential increases linearly with the magnetic field strength. The higher the B, the greater the potential V_{H}.

73. **INTERPRET** We are to follow Example 26.9 and find the magnetic field inside a slab of thickness d, as a function of the perpendicular distance from the center plane of the slab.

DEVELOP The magnetic field has plane symmetry about the center plane of the slab as in Example 26.9. Ampère's law for a rectangular loop in the x-z plane (perpendicular to $\vec{J} = J_0\hat{j}$) of length ℓ and width $2x$ (extending between $-x$ and $+x$) gives

$$\oint \vec{B}\cdot d\vec{r} = 2\ell B = \mu_0 I_{\text{encircled}} = \mu_0(J_0 2\ell x)$$

EVALUATE Solving for B, we obtain $B = \mu_0 J_0 x$. In the limit where the thickness approaches zero, $d \to 0$, we have surface current $J_{\text{S}} = J_0 d$, and the field strength is, setting $x = d/2$,

$$B = \mu_0(J_S / d)(d / 2) = \frac{1}{2}\mu_0 J_S$$

in agreement which that found in Example 26.9.

ASSESS Outside the slab, we have

$$\oint \vec{B}\cdot d\vec{r} = 2\ell B = \mu_0 I_{\text{encircled}} = \mu_0(J_0 \ell d)$$

and the field strength is $B = \mu_0 J_0 d / 2$.

74. **INTERPRET** We are to find the magnetic field at various points around a loop that carries current. By making the appropriate assumptions, we can directly apply the different formulas from Chapter 26, alleviating us from the need to derive new expressions.

DEVELOP For part **(a)**, we are to find the magnetic field 1.0 mm from a 15-cm-radius wire. At this distance, the wire may be considered to be straight (because $1.0\text{ mm} \ll 150\text{ mm}$), so we can apply Equation 26.10. For part **(b)**, we can apply Equation 26.9 with $x \gg a$, which gives

$$B = \frac{\mu_0 I a^2}{2x^3}$$

EVALUATE **(a)** Inserting the given quantities, we find a magnetic field to be

$$B = \frac{\mu_0 I}{2\pi y} = \frac{\left(4\pi\times10^{-7}\text{ N/A}^2\right)(2.0\text{ A})}{2\pi\left(1.0\times10^{-3}\text{ m}\right)} = 4.0\text{ G}$$

(b) The magnetic field 3 m from the loop, on its axis, is

$$B = \frac{\mu_0 I a^2}{2x^3} = \frac{\left(4\pi\times10^{-7}\text{ N/A}^2\right)(2.0\text{ A})}{2(3.0\text{ m})^3} = 1.1\times10^{-5}\text{ G}$$

ASSESS The field is much weaker at 3 m from the loop than at 1 mm, as expected.

75. **INTERPRET** This problem is about the magnetic field of a current-carrying conducting bar. Symmetry holds approximately in certain limits.

DEVELOP Very near the conductor, but far from any edge, the field is like that due to a large current sheet. On the other hand, very far from the conductor, the field is like that due to a long, straight wire.

EVALUATE **(a)** Approximating the bar by a large current sheet with $J_s = I/w$, Equation 26.19 gives

$$B \approx \frac{\mu_0 I}{2w}$$

(b) Approximating the bar by a long, straight wire. Equation 26.17 gives

$$B \approx \frac{\mu_0 I}{2\pi r}$$

ASSESS The conductor exhibits different approximate symmetries, depending on where the field point is chosen.

76. **INTERPRET** For this problem, we are to find the magnetic field inside and outside a hollow pipe around which current circulates (see sketch below).

DEVELOP The current distribution is similar to a solenoid, where the number of turns per unit length and the current in each turn are related to the total current in the pipe by $nLI_t = I.$ Therefore (see Section 26.8), the field is approximately that of an infinite solenoid.

EVALUATE **(a)** Inside the pipe, the magnetic field is

$$B = \mu_0 n I_t = \frac{\mu_0 I}{L}$$

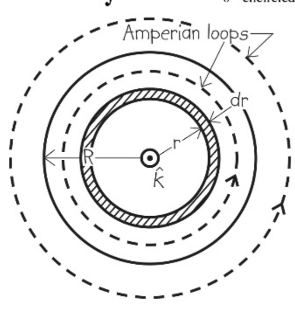
(b) Outside the pipe, the current is zero because $I_{\text{enclosed}} = 0$ for an Ampèrian loop whose plane is perpendicular to the cylinder axis.

ASSESS If we let $L \to \infty$, this is a perfect solenoid (note that I has to tend to infinity also!).

77. **INTERPRET** The system is a solid conducting wire having a nonuniform current density. We are interested in the magnetic field strength both inside and outside the wire. This problem involves Ampère's law.

DEVELOP The total current in the wire can be obtained by integrating the current density over the cross-sectional area (see sketch below). The magnetic field of a long conducting wire is approximately cylindrically symmetric, as discussed in Section 26.8. The magnetic field can be found by using Ampère's law:

$$\oint \vec{B} \cdot d\vec{r} = \mu_0 I_{\text{encircled}}$$



EVALUATE **(a)** Using thin rings as the area elements with $dA = 2\pi r dr,$ the total current in the wire (z-axis out of the page) is

$$I = \int_0^R J\, dA = \int_0^R J_0\left(1 - \frac{r}{R}\right)(2\pi r dr) = 2\pi J_0\left(\frac{r^2}{2} - \frac{r^3}{3R}\right)\Bigg|_0^R = \frac{1}{3}\pi R^2 J_0$$

(b) A concentric Ampèrian loop outside the wire encircles the total current, so Ampère's law gives

$$2\pi r B = \mu_0 I = \mu_0\left(\frac{1}{3}\pi R^2 J_0\right)$$

$$B = \frac{\mu_0 J_0 R^2}{6r}$$

(c) Inside the wire, Ampère's law gives $2\pi rB = \mu_0 I_{\text{encircled}}$. The calculation in part **(a)** shows that within a loop of radius $r < R$,

$$I_{\text{encircled}} = \int_0^r JdA = 2\pi J_0 \left(\frac{r^2}{2} - \frac{r^3}{3R}\right)\Bigg|_0^r = \pi J_0 r^2 \left(1 - \frac{2r}{3R}\right)$$

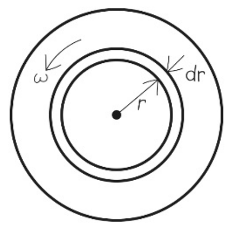
Therefore,

$$B = \frac{\mu_0 J_0 r}{2}\left(1 - \frac{2r}{3R}\right)$$

ASSESS At $r = R$ both equations give $B = \mu_0 J_0 R/6$. The form is the same as that shown in Equation 26.17.

78. **INTERPRET** Given a rotating disk of uniform charge density, we are to find the magnetic field at the disk's center.

DEVELOP The disk may be considered to be composed of rings of radius r, thickness dr, and charge $dq = 2\pi r\sigma dr$. Each ring represents a circular current loop, $dI = dq/T = dq/(2\pi/\omega) = \omega\sigma r dr$, which produces a magnetic field strength $dB = \mu_0 dI/(2r) = \frac{1}{2}\mu_0 \omega\sigma dr$ at the center of the disk, directed out of the page, as sketched below for positive charge density.



EVALUATE Performing the integration gives

$$B = \int_0^a dB = \frac{\mu_0 \omega\sigma}{2}\int_0^a dr = \frac{\mu_0 \omega\sigma a}{2}$$

ASSESS This is the same as for a loop with radius a and with current $I = \omega\sigma a^2$.

79. **INTERPRET** You're designing a system to orient a satellite using the torque that the Earth's magnetic field will induce on a current loop. You want the maximum torque possible, but you are limited to a fixed length of wire.

DEVELOP Let's assume that the loops are circular. The torque on such a loop is given by Equations 26.14 and 26.12:

$$\tau = \left|\vec{\mu} \times \vec{B}\right| = NIAB\sin\theta = \pi r^2 NIB\sin\theta$$

The current will be specified by the satellite's power supply. The magnetic field is that of the Earth's and the angle θ is dependent on the satellite's orientation. What you need to determine is whether one turn $(N = 1)$ or many turns will give more torque, given that the total length of wire is set.

EVALUATE The wire length is related to the size and number of loops by: $l = N(2\pi r)$. Using this to eliminate r from the torque equation gives:

$$\tau = \pi\left(\frac{l}{2\pi N}\right)^2 NIB\sin\theta = \frac{1}{N}\left(\frac{l^2 IB\sin\theta}{4\pi}\right)$$

Since $\tau \propto 1/N$, you'd get more torque from a single turn loop.

ASSESS Although you gain by having more turns, you're losing more from reducing the area of the loop.

80. **INTERPRET** This problem involves a current-carrying bar that is maintained in equilibrium by the force of gravity and the magnetic force. At equilibrium, the two forces cancel exactly.

DEVELOP If the height h is small compared to the length of the rods, we can use Equation 26.11 for the repulsive magnetic force between the horizontal rods (upward on the top rod):

$$F_B = \frac{\mu_0 I^2 L}{2\pi h}$$

The rod is in equilibrium when this equals its weight, $F_g = mg$.

EVALUATE The equilibrium condition $F_B = F_g$ gives

$$h = \frac{\mu_0 I^2 L}{2\pi mg} = \frac{\left(4\pi\times10^{-7}\text{ N/A}^2\right)(67\text{ A})^2(0.95\text{ m})}{2\pi(0.027\text{ kg})\left(9.8\text{ m/s}^2\right)} = 3.2\text{ mm}$$

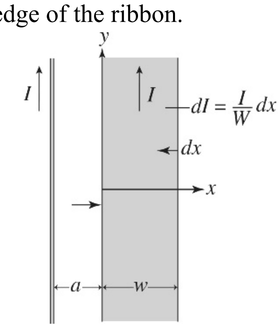
ASSESS The height, *h,* is indeed small compared to 95 cm, so our assumption is justified.

81. **INTERPRET** We are to find the force per unit length between a thin wire and a parallel ribbon, each of which carry a current *I*.

DEVELOP Use the coordinate system shown in the figure below. The magnitude of the force per unit length on a thin strip of ribbon, of width *dx*, carrying current $I\,dx/w$, is given by Equation 26.11:

$$dF = \frac{\mu_0 I(I\,dx/w)L}{2\pi(a+x)} \quad \Rightarrow \quad \frac{dF}{L} = \frac{\mu_0 I(I\,dx/w)}{2\pi(a+x)}$$

where *x* is the distance from the near edge of the ribbon.



EVALUATE Integrating the expression above from $x = 0$ to $x = w$ gives a total force of

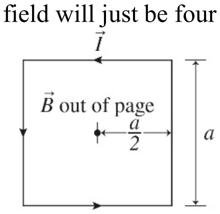
$$\frac{F}{L} = \frac{\mu_0 I^2}{2\pi w}\int_0^w \frac{dx}{a+x} = \frac{\mu_0 I^2}{2\pi w}\ln\left(\frac{a+w}{a}\right)$$

The force is attractive since the currents are parallel.

ASSESS Note that this expression reduces to Equation 26.11 for $w = 0$ [using $\ln(1+\varepsilon) = \varepsilon + \ldots$].

82. **INTERPRET** We're looking for an expression for the magnetic field at the center of a square loop.

DEVELOP In Example 26.4, the Biot-Savart law was used to find the field from an infinite wire. In this case, we have four wires of finite length, but much of the math will be the same. Let's assume the current moves around the loop in a counterclockwise direction, as shown in the figure below. By the right-hand rule, the magnetic field contributions from each of the four wires will all point out of the page. By symmetry, all the contributions are also equal in magnitude, so the total magnetic field will just be four times the field from one of the wire segments.



EVALUATE The only difference in the magnetic field of a finite wire from that of an infinite wire is the limits of the integration. So borrowing from Example 26.4, the magnetic field at a distance of $a/2$ from a wire that extends from $-a/2$ to $a/2$ is

$$B = \frac{\mu_0 I(a/2)}{4\pi}\int_{-a/2}^{a/2}\frac{dx}{\left(x^2+(a/2)^2\right)^{3/2}}$$

$$= \frac{\mu_0 I(a/2)}{4\pi}\left[\frac{x}{(a/2)^2\sqrt{x^2+(a/2)^2}}\right]_{-a/2}^{a/2} = \frac{\mu_0 I}{\sqrt{2}\pi a}$$

where we have used the integral table in Appendix A. The total field is four times this:

$$B_{\text{tot}} = 4B = \frac{2\sqrt{2}\mu_0 I}{\pi a}$$

ASSESS Our result for a single wire segment is smaller by a factor of $1/\sqrt{2}$ than the magnetic field from an infinite wire at the same given distance. But the sum of the contributions from the 4 wires is in fact larger than that from a single infinite wire. It might also be interesting to compare the magnetic field of a square loop to that of a circular loop with the same area, a^2. From Equation 26.9, the field at the center of a circular loop with radius $\frac{a}{\sqrt{\pi}}$ is: $B = \sqrt{\pi}\mu_0 I/2a.$ This means the magnetic field at the center of a square loop is roughly 1% bigger than the field at the center of a circular loop.

83. **INTERPRET** We find the magnetic field at the center of a "real" solenoid of finite length, treating the solenoid as a stack of individual coils. We use the formula for the magnetic field due to a single loop, and integrate.

DEVELOP As before in Problem 26.69, we can divide up the solenoid into infinitesimal loops with current of $nIdx$, where n is the number of turns of wire per length. Using Equation 26.9, the axial field from this infinitesimal loop is

$$dB = \frac{\mu_0 (nIdx)a^2}{2(x^2 + a^2)^{3/2}}$$

Now instead of integrating x from $-\infty$ to ∞, we integrate from $-l/2$ to $l/2$ to obtain the field at the center of the finite length solenoid.

EVALUATE Performing the integration with help from the tables in Appendix A, we get

$$B = \frac{\mu_0 nIa^2}{2}\int_{-l/2}^{l/2} \frac{dx}{(x^2 + a^2)^{3/2}} = \frac{\mu_0 nIa^2}{2} \frac{x}{a^2\sqrt{x^2 + a^2}}\Bigg|_{-l/2}^{l/2} = \frac{\mu_0 nIl}{\sqrt{l^2 + 4a^2}}$$

ASSESS We can check this formula by letting $l \to \infty,$ in which case the magnetic field becomes $B = \mu_0 nI,$ as was already given for an infinite solenoid in Equation 26.21.

84. **INTERPRET** In this problem we want to express the magnetic field generated by two Helmholtz coils. We are to then show the first and second derivatives of the magnetic field are equal to zero at the origin. Lastly, we will plot the geometric description of the vector field to show the uniformity in the region near the origin.

DEVELOP From Example 26.3 we know the magnetic field generated by a current-carrying coil, and can modify the result for the given dimensions and locations of the two coils in this problem. We are told the current in both coils is the same and flows in the same direction, as well as their radius and position along the x-axis. We will take the sum of the two fields generated along the x-axis in the region between the two coils.

EVALUATE **(a)** Substituting the given dimensions and positions into the expression obtain in Example 26.3 we find

$$B(x) = \frac{\mu_0 IR^2}{2}\left(\frac{1}{\left((R/2 - x)^2 + R^2\right)^{3/2}} + \frac{1}{\left((R/2 + x)^2 + R^2\right)^{3/2}}\right)$$

where the first and second terms correspond to the coil above and below the origin, respectively.

(b) To calculate the derivatives, we first rewrite the magnetic field as

$$B(x) = \alpha\left(\left((R/2 - x)^2 + R^2\right)^{-3/2} + \left((R/2 + x)^2 + R^2\right)^{-3/2}\right)$$

where we have grouped constants together as α since they are independent of the variable x. For the first derivative we find:

$$\frac{dB}{dx} = \frac{-3\alpha}{2}\left(-2(R/2 - x)\left((R/2 - x)^2 + R^2\right)^{-5/2} + 2(R/2 + x)\left((R/2 + x)^2 + R^2\right)^{-5/2}\right)$$

$$\frac{dB(x=0)}{dx} = \frac{-3\alpha}{2}\left(-2(R/2)\left((R/2)^2 + R^2\right)^{-5/2} + 2(R/2)\left((R/2)^2 + R^2\right)^{-5/2}\right) = 0$$

For the second derivative we find:

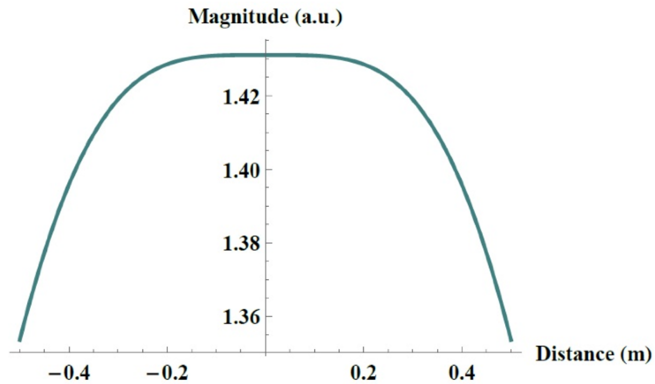
$$\frac{d^2B}{dx^2}=\alpha\left[\begin{array}{l}\left(\frac{15}{4}\left((-2(R/2-x))^2\left((R/2-x)^2+R^2\right)^{-7/2}\right)-\frac{3}{2}\left(2\left((R/2-x)^2+R^2\right)^{-5/2}\right)\right)+\\ \left(\frac{15}{4}\left((2(R/2+x))^2\left((R/2+x)^2+R^2\right)^{-7/2}\right)-\frac{3}{2}\left(2\left((R/2+x)^2+R^2\right)^{-5/2}\right)\right)\end{array}\right]$$

$$\frac{d^2B(x=0)}{dx^2}=\alpha\left[\begin{array}{l}\left(\frac{15}{4}\left((-2(R/2))^2\left((R/2)^2+R^2\right)^{-7/2}\right)-\frac{3}{2}\left(2\left((R/2)^2+R^2\right)^{-5/2}\right)\right)+\\ \left(\frac{15}{4}\left((2(R/2))^2\left((R/2)^2+R^2\right)^{-7/2}\right)-\frac{3}{2}\left(2\left((R/2)^2+R^2\right)^{-5/2}\right)\right)\end{array}\right]$$

$$=\alpha\left[\frac{15}{4}\left(2R^2\left(\frac{5R^2}{4}\right)^{-7/2}\right)-\frac{3}{2}\left(4\left(\frac{5R^2}{4}\right)^{-5/2}\right)\right]=\alpha R^{-5}\left[\frac{15}{2}\left(\frac{4}{5}\right)^{7/2}-6\left(\frac{4}{5}\right)^{5/2}\right]=0$$

(c) To see the region of uniform field, we set $R = 1$ and plot the quantity $2B/\mu_0 I$ in the region: $-½ < x < ½$ in the figure below

$$\frac{2B}{\mu_0 I}=\left(\frac{1}{\left(1+(1/2-x)^2\right)^{3/2}}+\frac{1}{\left(1+(1/2+x)^2\right)^{3/2}}\right)$$



Showing a continuous and uniform field in the region above and below the origin

ASSESS The region of uniformity extends almost half-way to the location of each coil and then quickly decays as one moves further from the origin.

85. INTERPRET We're asked to determine whether the clamps holding the conductors in place are adequate to the task. This means finding whether or not the 100 N they can withstand is more or less than the magnetic force between the two conductors.

DEVELOP Equation 26.11 gives the magnetic force between parallel conducting wires:

$$F=\frac{\mu_0 I_1 I_2 l}{2\pi d}$$

In our case the two currents have the same magnitude, $I = 15$ kA, the distance d is 0.30 m, and the length of wire held by each clamp is $l = 1$ m. So the force on each meter of the conductors is

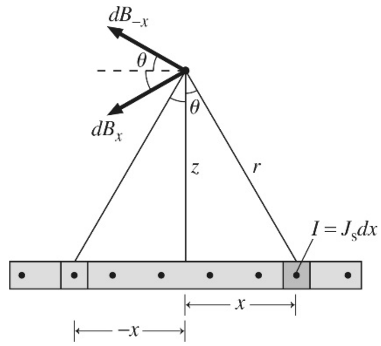
$$F=\frac{\mu_0 I^2 l}{2\pi d}=\frac{(4\pi\times10^{-7}\ \text{N/A}^2)(15\times10^3\ \text{A})^2(1\ \text{m})}{2\pi(0.30\ \text{m})}=150\ \text{N}$$

Since this is more than the clamps can withstand, the design is inadequate.

ASSESS We calculated the force on a meter of the conductors because the clamps are spaced 1 meter apart. Although the conductors aren't strictly wires, cylindrical rods would behave the same way.

86. INTERPRET We're asked to derive the formula for the magnetic field above a current sheet. We won't use Ampère's law, as in Example 26.8, but instead we will integrate over an infinite number of wires.

DEVELOP Let's assume the sheet is in the *x*-*y* plane, and the current per unit width, J_s, is in the *y*-direction. We consider this sheet current to be made up of an infinite number of line currents with $I = J_s dx$. We'll calculate the magnetic field that these line currents produce at a point at $x = 0$ and at a height *z* above the sheet. See the figure below.



From Equation 26.10, each line current will generate a magnetic field at the chosen point that is inversely proportional to the distance:

$$dB_x = \frac{\mu_0 J dx}{2\pi r} = \frac{\mu_0 J_s dx}{2\pi\sqrt{x^2+z^2}}$$

where we use the subscript *x* to denote the location of the line current. It's clear that the field contribution, dB_{+x}, from the line current at +*x* will have the same magnitude as the field contribution, dB_{-x}, from the line current at –*x*. By the symmetry of the situation, the *z*-component of these two fields will cancel out, and we're left with only their *x*-component: $dB_{\pm x}\cos\theta$. From the figure, we can see that $\cos\theta = z/r$. We combine the contribution from the two line currents at +*x* and –*x* into one term:

$$dB = dB_{+x}\cos\theta + dB_{-x}\cos\theta = \frac{\mu_0 J_s dx}{2\pi r} = \frac{\mu_0 J_s z dx}{\pi\left(x^2+z^2\right)}$$

EVALUATE To find the total field, we integrate *dB* from 0 to ∞:

$$B = \int dB = \frac{\mu_0 J_s z}{\pi}\int_0^\infty \frac{dx}{\left(x^2+z^2\right)} = \frac{\mu_0 J_s z}{\pi}\left[\frac{1}{z}\tan^{-1}\left(\frac{x}{z}\right)\right]_0^\infty = \frac{\mu_0 J_s}{\pi}\left[\frac{\pi}{2}-0\right] = \tfrac{1}{2}\mu_0 J_s$$

where we have used the integral table in Appendix A, as well as the table of selected values of tan θ.

ASSESS Assuming the current in the sheet points out of the paper as in the figure above, the magnetic field will point to the left above the sheet and to the right below the sheet. This all agrees with the derivation in Example 26.8, but notice how much easier this problem is when you use Ampère's law.

87. **INTERPRET** You want to consider the possible effect that magnets used in magnet therapy might have on blood flow.

DEVELOP You first have to estimate the typical current in a blood vessel. Each blood vessel carries a small charge, $q = 2$ pC, and is moving at a speed of $v = 12$ cm/s. There are roughly 5 billion blood cells per mL moving through a vessel of diameter 3 mm. Plugging these values into Equation 24.2, the current flowing in the vessel is

$$I = nAqv = \left(\frac{5\times10^9}{\text{mL}}\right)\left[\pi\left(\tfrac{1}{2}\cdot 3\text{ mm}\right)^2\right](2\text{ pC})(12\text{ cm/s}) = 8.5\text{ mA}$$

You can compute the Hall effect that a bar magnet would cause inside a current-carrying blood vessel.

EVALUATE From Equation 26.6, the Hall potential is $V_H = IB/nqt$, where *t* is the thickness of the conducting material. In the case of blood, we can assume *t* is just the diameter of the blood vessel, in which case the Hall potential is

$$V_H = \frac{IB}{nqd} = \frac{(8.5\text{ mA})(10\text{ mT})}{\left(5\times10^9/\text{mL}\right)(2\text{ pC})(3\text{ mm})} = 3\ \mu\text{V}$$

This is roughly 10,000 times smaller than the electric potentials of bioelectric activity.

ASSESS A more straightforward way to calculate the Hall effect would be with $V_H \approx vBd$, which gives roughly the same answer.

88. **INTERPRET** We consider the magnetic field generated by a toroid.

DEVELOP The magnetic field is symmetric around the axis of the toroid. We can therefore imagine an Ampèrian loop that is a circle with radius *r* extending from the toroid's axis (as drawn in Fig. 26.52b). The magnetic field should be parallel to the tangent of the circle, so by Ampère's law,

$$\oint \vec{B} \cdot d\vec{r} = 2\pi rB = \mu_0 I_{\text{encircled}}$$

EVALUATE For an Ampèrian loop inside the donut "hole," there is no encircled current, so $B = 0$. Within the region bounded by the coils, an Ampèrian loop will encircle the wires on the inner half of the toroid. In this case, the encircled current is nonzero, and therefore so is the magnetic field. Lastly, at radii greater than the outer radius of the toroid, the Ampèrian loop will encircle the inner part of the toroid, where the currents flow in one direction, but it will also encircle the outer part of the toroid, where the same currents flow in the opposite direction. Therefore the total current will be zero, and the magnetic field outside the coils will be zero.

The answer is **(b)**.

ASSESS The toroid field is confined to inside the coils just like the infinite solenoid field in Fig. 26.34.

89. **INTERPRET** We consider the magnetic field generated by a toroid.

DEVELOP As we explained in the previous problem, the magnetic field is symmetric around the axis of the toroid.

EVALUATE By the right-hand rule, it's clear that the magnetic field lines have to be in the plane of the page. That rules out choices **(a)** and **(b)**. If the field lines were straight and pointing radially, that would seem to contradict Gauss's law for magnetism, Equation 26.14. One could imagine a sphere centered around where the field was radiating outward. The magnetic flux through this sphere would presumably be non-zero, as if there were a magnetic monopole at the center of the toroid. Ruling out that possibility, we're left with circular field lines, which agrees with the arguments made in the previous problem.

The answer is **(d)**.

ASSESS As described in Fig. 26.8, charged particles will spiral around magnetic field lines. Therefore, inside a toroid, charged particles will orbit essentially in a circle as they spiral around the field lines. This is how the million degree fuel in a future fusion reactor will presumably be confined.

90. **INTERPRET** We consider the magnetic field generated by a toroid.

DEVELOP From Ampère's law, we know that the magnetic field is proportional to the enclosed current, which in the case of the toroid is proportional to the number of coils: $B \propto I_{\text{enclosed}} \propto N$.

EVALUATE If the number of turns is doubled, the magnetic field should double as well.

The answer is **(a)**.

ASSESS To increase the magnetic field in a solenoid or toroid, it is often easier to wind more turns in the wire than to increase the current.

91. **INTERPRET** We consider the magnetic field generated by a toroid.

DEVELOP To find the field magnitude, we can use the Ampèrian circles that were introduced in Problem 26.88.

EVALUATE At a given radius, the magnetic field inside the coils will be

$$B = \frac{\mu_0 NI}{2\pi r}$$

The answer is **(d)**.

ASSESS We see here that the difference between the magnetic field in a solenoid and in a toroid is that in the former the field is uniform (Equation 26.21) but in the latter it is not $(B \propto 1/r)$.