

THE THERMAL BEHAVIOR OF MATTER

EXERCISES

Section 17.1 Gases

- 11. INTERPRET** This problem involves the ideal gas law, which we can use to find the volume of 1 mol of Martian atmosphere given its temperature and pressure.

DEVELOP Apply Equation 17.2 $PV = nRT$ with $P = 0.01P_E$, $T = 215\text{ K}$, $n = 1.0\text{ mol}$, and $R = 8.314\text{ J/K}$.

EVALUATE Solving the ideal gas law for the volume and inserting the given quantities gives

$$V = \frac{nRT}{P} = \frac{(1.0\text{ mol})(8.314\text{ J/K})(215\text{ K})}{(0.01)(1.01 \times 10^5\text{ Pa})} = 1.8\text{ m}^3$$

ASSESS The dimensions of this expression are

$$\frac{\text{mol}(\text{J/K})\text{K}}{\text{N/m}^2} = \frac{\text{mol}(\text{N} \cdot \text{m})}{\text{N/m}^2} = \text{mol} \cdot \text{m}^3$$

but a mole is a dimensionless number, so the final dimensions are m^3 , as expected for a volume.

- 12. INTERPRET** We are dealing with an ideal gas. Given the pressure, temperature, and volume, we are to find the number of gas molecules.

DEVELOP We shall use the ideal gas law, $pV = NkT$ (Equation 17.1), to find the number of molecules N .

EVALUATE Inserting the given quantities gives

$$N = \frac{PV}{kT} = \frac{(2.1 \times 10^5\text{ Pa})(9.5 \times 10^{-3}\text{ m}^3)}{(1.38 \times 10^{-23}\text{ J/K})(390\text{ K})} = 3.7 \times 10^{23}$$

ASSESS One mole has $N_A = 6.02 \times 10^{23}$ molecules. Thus, we have about 0.6 mole of molecules in the system.

- 13. INTERPRET** This problem involves an ideal gas, so we can apply the ideal-gas law to find the pressure of the gas at the given temperature and volume.

DEVELOP In terms of moles, the ideal-gas law is given by Equation 17.2, $PV = nRT$. The volume is

$V = 2.2\text{ L} = 2.2 \times 10^{-3}\text{ m}^3$, and $T = -130^\circ\text{C} = 143\text{ K}$.

EVALUATE Solving for the pressure and inserting the given quantities gives

$$P = \frac{nRT}{V} = \frac{(2.9\text{ mol})(8.314\text{ J/K})(143\text{ K})}{2.2 \times 10^{-3}\text{ m}^3} = 1.6 \times 10^6\text{ Pa}$$

ASSESS This is about 15 times the standard atmospheric pressure.

- 14. INTERPRET** You want to verify that the tank you purchased contains the amount of argon that it is supposed to. You can treat it as an ideal gas.

DEVELOP You know the volume, the temperature, and the pressure of the gas. Using the ideal gas law

(Equation 17.2), you can find the number of moles in the tank: $n = pV / RT$, where $R = 8.314\text{ J/K} \cdot \text{mol}$ is the universal gas constant.

EVALUATE Remembering that the temperature needs to be in Kelvin and the volume needs to be in m^3 , the number of moles is:

$$n = \frac{pV}{RT} = \frac{(14 \text{ MPa})(6.88 \text{ L})}{(8.314 \text{ J/K} \cdot \text{mol})(20^\circ\text{C} + 273)} \left(\frac{1 \text{ m}^3}{10^3 \text{ L}} \right) = 40 \text{ mol}$$

This is 5 mol less than the vendor claimed, so you did not get your money's worth.

ASSESS If the right amount of gas were in the tank, then you would have measured a pressure of nearly 16 MPa. Notice that at no point did we have to consider the specific properties of argon. The ideal-gas law applies to any gas. The units work out since $1 \text{ Pa} \cdot \text{m}^3 = 1 \text{ N} \cdot \text{m} = 1 \text{ J}$.

15. **INTERPRET** This problem involves an ideal gas, so we can apply the ideal gas law. We are to find the volume of an ideal gas given its temperature and pressure, then find its new temperature if the pressure is increased and the volume is cut in half.

DEVELOP In terms of moles, the ideal gas law takes the form of Equation 17.2, $pV = nRT$. For part (a), $p = 1.5 \text{ atm} = 1.5 \times 1.01 \times 10^5 \text{ Pa} = 1.515 \times 10^5 \text{ Pa}$, $T = 250 \text{ K}$, and $n = 2.0$. For part (b), we can take the ratio of the ideal gas law applied to part (a) and that applied to part (b) to get

$$\frac{p_a V_a}{p_b V_b} = \frac{nRT_a}{nRT_b} = \frac{T_a}{T_b}$$

which we can solve for T_b given that $V_b = V_a/2$ and $P_b = 4.0 \text{ atm}$. Note that we have used the fact that the number of moles does not change.

EVALUATE (a) The volume V_a of the gas is

$$V = \frac{nRT}{P} = \frac{(2.0 \text{ mol})[8.314 \text{ J/(mol} \cdot \text{K)}](250 \text{ K})}{1.515 \times 10^5 \text{ Pa}} = 2.7 \times 10^{-2} \text{ m}^3 = 27 \text{ L}$$

(b) Upon compressing the gas and increasing its pressure, the new temperature is

$$T_b = \left(\frac{4.0 \text{ atm}}{1.5 \text{ atm}} \right) \left(\frac{0.5V_1}{V_1} \right) (250 \text{ K}) = 330 \text{ K}$$

to two significant figures.

ASSESS As expected, the temperature increases if we compress the gas.

16. **INTERPRET** We treat air molecules as ideal gas. Given the pressure, temperature, and volume, we want to find the number of air molecules.

DEVELOP We shall use the ideal gas law, $pV = NkT$, given in Equation 17.1, to find the number of molecules.

EVALUATE The number of air molecules is

$$N = \frac{PV}{kT} = \frac{(10^{-10} \text{ Pa})(10^{-3} \text{ m}^3)}{(1.38 \times 10^{-23} \text{ J/K})(273 \text{ K})} = 3 \times 10^7$$

ASSESS One mole has $N_A = 6.02 \times 10^{23}$ molecules. Thus, we have about 4.4×10^{-17} mole of molecules in the system.

17. **INTERPRET** In this problem we want to compare the thermal speeds of two different molecules that are at different temperatures.

DEVELOP The thermal speed of a molecule is given by Equation 17.4:

$$v_{\text{th}} = \sqrt{\frac{3kT}{m}}$$

where T is the temperature and m is the mass of the gas molecule. We can use this to compare the thermal speeds of the two molecules in question.

EVALUATE Comparing the thermal speeds for H_2 ($m_{\text{H}_2} = 2u$) and SO_2 ($m_{\text{SO}_2} = 64u$) at the given temperatures, we find

$$\frac{v_{\text{th}}(\text{H}_2)}{v_{\text{th}}(\text{SO}_2)} = \sqrt{\frac{T_{\text{H}_2}}{T_{\text{SO}_2}} \frac{m_{\text{SO}_2}}{m_{\text{H}_2}}} = \sqrt{\frac{75 \text{ K}}{350 \text{ K}} \frac{64u}{2u}} = 2.62$$

So hydrogen is faster.

ASSESS The thermal speed of a gas molecule is proportional to \sqrt{T} and inversely proportional to \sqrt{m} .

Section 17.2 Phase Changes

- 18. INTERPRET** This problem involves the latent heat of fusion, which is the energy it takes to liberate the molecules that compose the ice to form water. We are asked to find the energy required to melt a 63-g ice cube.

DEVELOP The energy required for a solid-liquid phase transition at the normal melting point of water (0°C) is (Equation 17.5) $Q = mL_f$, where $m = 0.063 \text{ kg}$ and $L_f = 334 \text{ kJ/kg}$ (from Table 17.1).

EVALUATE The heat required to melt the ice cube is

$$Q = mL_f = (0.063 \text{ kg})(334 \text{ kJ/kg}) = 21 \text{ kJ}$$

ASSESS This is equivalent to 5.0 kcal.

- 19. INTERPRET** This problem is about melting, and it involves the heat of fusion. We want to identify the substance in question given its mass and the energy required to melt it.

DEVELOP Using Equation 17.5, we find the heat of transformation from solid to liquid to be

$$L_f = \frac{Q}{m} = \frac{200 \text{ J}}{0.008 \text{ kg}} = 25 \text{ kJ/kg}$$

Compare this result with the data given in Table 17.1 to determine the substance.

EVALUATE The closest figure in Table 17.1 is the heat of fusion for lead $L_f = 25 \text{ kJ/kg}$, so the substance is lead.

ASSESS The heat of fusion L_f is a chemical property of a material. Thus, knowing L_f allows us to identify the material.

- 20. INTERPRET** This problem involves the latent heat of vaporization, which is the energy required to pass from the liquid to the gas phase. Given the latent heat of vaporization (Table 17.1) and the energy required to vaporize the substance, we are to calculate the mass of the substance.

DEVELOP Assuming the vaporization takes place at the normal boiling point for oxygen at atmospheric pressure, we may use Equation 17.5, $Q = L_v m$, where $L_v = 213 \text{ kJ/kg}$ (from Table 17.1) and $Q = 920 \text{ kJ}$.

EVALUATE The mass of oxygen in the sample is

$$m = \frac{Q}{L_v} = \frac{920 \text{ kJ}}{213 \text{ kJ/kg}} = 4.3 \text{ kg}$$

ASSESS Given that O_2 is approximately 32 g/mol , this corresponds (at standard temperature and pressure) to a volume of

$$V = \frac{nRT}{p} = \frac{(4300 \text{ g})(8.134 \text{ J/K})(273 \text{ K})}{(32 \text{ g/mol})(10^5 \text{ Pa})} = 3.0 \text{ m}^3$$

- 21. INTERPRET** This problem is about phase change of CO_2 from gas to solid. We are asked to find the heat (i.e., thermal energy) that must be extracted from the given amount of CO_2 to solidify it, which is the same as the heat required to sublime it (turn it from solid into gas).

DEVELOP The relevant latent heat of transformation in this process is the latent heat of sublimation, which is $L_s = 573 \text{ kJ/kg}$. Use this in Equation 17.5 to find the heat needed to solidify CO_2 .

EVALUATE Equation 17.5 gives $Q = mL_s = (0.295 \text{ kg})(573 \text{ kJ/kg}) = 169 \text{ kJ}$.

ASSESS This is the heat that must be extracted to turn CO_2 gas into frozen CO_2 , or dry ice. To revert to the gaseous state, the same amount of heat must be absorbed.

Section 17.3 Thermal Expansion

- 22. INTERPRET** This problem involves the thermal expansion of a one-dimensional object, so we can use the linear expansion formula to find the total distance a 250-m rail would expand for a given temperature change.

DEVELOP Equation 17.7,

$$\alpha = \frac{\Delta L/L}{\Delta T}$$

gives the relationship between temperature change and linear expansion. For aluminum, $\alpha = 24 \times 10^{-6} \text{ K}^{-1}$ and the temperature change is $\Delta T = 29.5^\circ\text{C} - (-12.0^\circ\text{C}) = 41.5^\circ\text{C} = 41.5 \text{ K}$.

EVALUATE Solving the expression for ΔL and inserting the given quantities gives

$$\Delta L = \alpha L \Delta T = (24 \times 10^{-6} \text{ K}^{-1})(250.42 \text{ m})(41.5 \text{ K}) = 25 \text{ cm}$$

ASSESS The rail expands by approximately 0.1% its original length.

- 23. INTERPRET** This is a problem in thermal expansion. We know the initial volume, the material, and the change in temperature, and we need to find the new volume.

DEVELOP The change in volume is given by Equation 17.6,

$$\beta = \frac{\Delta V}{V \Delta T}$$

The initial volume is $V = 1.00 \text{ L}$. The material is ethyl alcohol, which has a volume coefficient of expansion of $\beta = 75 \times 10^{-5} \text{ K}^{-1}$, and the change in temperature is $\Delta T = -19 \text{ K}$. The new volume V' of the liquid is

$$V' = V + \Delta V = \Delta V (1 + \beta \Delta T)$$

EVALUATE Inserting the given quantities gives

$$V' = V (1 + \beta \Delta T) = (1.00 \text{ L}) \left[1 + (75 \times 10^{-5})(-19 \text{ K}) \right] = 0.986 \text{ L}$$

ASSESS This can be observed in your own refrigerator: Seal a volume of liquid at room temperature, put it in the refrigerator, and observe the deformation of the container when it cools.

- 24. INTERPRET** This problem involves thermal expansion (volume expansion in this case). We are given the diameter of a Pyrex sphere and are to find its new diameter due to a given temperature change.

DEVELOP The volume V of the marble is related to its radius r by $V = 4\pi r^3/3$. The change in volume is given by Equation 17.6,

$$\beta = \frac{\Delta V}{V \Delta T}$$

where $\beta = 3\alpha = 3(3.2 \times 10^{-6} \text{ K}^{-1}) = 9.6 \times 10^{-6} \text{ K}^{-1}$ (see Table 17.2 and Problem 71) and $\Delta T = 65^\circ\text{C} = 65 \text{ K}$. The new volume is

$$V' = V + \Delta V = V (1 + \beta \Delta T)$$

so the new diameter will be

$$V' = \frac{4}{3}\pi r'^3 = \frac{4}{3}\pi r^3 (1 + \beta \Delta T)$$

$$d' = d \sqrt[3]{1 + \beta \Delta T}$$

where we have used $d = 2r$.

EVALUATE Inserting the given quantities into the expression above gives

$$d' = d \sqrt[3]{1 + \beta \Delta T} = (1.00000 \text{ cm}) \sqrt[3]{1 + (9.6 \times 10^{-6} \text{ K}^{-1})(65 \text{ K})} = 1.00021 \text{ cm}$$

ASSESS Because we are dealing with a sphere, we can also consider the linear expansion of the diameter using α .

- 25. INTERPRET** This problem deals with thermal expansion of a steel washer. The quantity of interest is the diameter of the washer, so the relevant quantity is the coefficient of linear expansion, α .

DEVELOP The coefficient of linear expansion is defined as (see Equation 17.7):

$$\alpha = \frac{\Delta L / L}{\Delta T}$$

For steel, its value is (see Table 17.2) $\alpha = 12 \times 10^{-6} \text{ K}^{-1}$. Solve this equation for ΔT .

EVALUATE From the equation above, we get

$$\Delta T = \frac{\Delta L}{\alpha L} = \frac{9.55 \text{ mm} - 9.52 \text{ mm}}{(12 \times 10^{-6} \text{ K}^{-1})(9.52 \text{ mm})} = 263 \text{ K}$$

Since the initial temperature is 0°C we must heat the washer to 263°C .

ASSESS Since α is very small, a large increase in temperature results in a small increase in the washer's diameter.

- 26. INTERPRET** This problem involves the thermal expansion of a one-dimensional object, so we can use the linear expansion formula to find the total distance a 5000-km rail would expand for a 40°C temperature change.

DEVELOP Equation 17.7:

$$\alpha = \frac{\Delta L / L}{\Delta T}$$

gives the relationship between temperature change and linear expansion. For steel, $\alpha = 12 \times 10^{-6} \text{ K}^{-1}$, and the temperature change is $\Delta T = 37^\circ\text{C} - (-3^\circ\text{C}) = 40^\circ\text{C} = 40 \text{ K}$.

EVALUATE Solving the expression for ΔL and inserting the given quantities gives

$$\Delta L = \alpha L \Delta T = (12 \times 10^{-6} \text{ K}^{-1})(5000 \text{ km})(40 \text{ K}) = 2.4 \text{ km}$$

ASSESS This is a significant expansion, but it is spread over the 5000-km distance.

EXAMPLE VARIATIONS

- 27. INTERPRET** We're dealing with an ideal gas, and we're given the amount of gas, the temperature, and the pressure.

DEVELOP Because we're given the number of moles n , we'll use the ideal gas law in the form of Equation 17.2, $pV = nRT$, to find the volume.

EVALUATE Solving for V gives

$$V = \frac{nRT}{p} = \frac{(1.00 \text{ mol})(8.314 \text{ J/K} \cdot \text{mol})(254.15 \text{ K})}{3.37 \times 10^4 \text{ Pa}} = 62.7 \times 10^{-3} \text{ m}^3$$

Also expressed as 62.7 L.

ASSESS Although the reduction in temperature leads to a smaller volume occupied by the air, the larger decrease in pressure at this height leads to the more than double expansion in volume when compared to atmospheric pressure and temperature.

- 28. INTERPRET** We're dealing with an ideal gas, and we're given the volume of gas, the temperature, and the pressure.

DEVELOP Because we're looking for the number of moles n per liter, we'll use the ideal gas law in the form of Equation 17.2, $pV = nRT$, to find the density.

EVALUATE Solving for n/V gives

$$\frac{n}{V} = \frac{p}{RT} = \frac{7.64 \times 10^4 \text{ Pa}}{(8.314 \text{ J/K} \cdot \text{mol})(267.65 \text{ K})} = 34.3 \text{ mol/m}^3$$

Also expressed as 34.3 mmol/L. Performing this calculation for the density at STP as determined in Example 17.1 we obtain 44.6 mmol/L. Thus at this height, the gas density is 77.0% of the STP density.

ASSESS The decrease in pressure at this height leads to a less concentrated gas sample when compared to atmospheric pressure and temperature.

- 29. INTERPRET** We're dealing with an ideal gas, and we're given the amount of gas, the temperature, and the pressure.

DEVELOP Because we're given the number of moles n , we'll use the ideal gas law in the form of Equation 17.2, $pV = nRT$, to find the volume.

EVALUATE Solving for V gives

$$V = \frac{nRT}{p} = \frac{(1.00 \text{ mol})(8.314 \text{ J/K} \cdot \text{mol})(93.15 \text{ K})}{(1.47)(1.013 \times 10^5 \text{ Pa})} = 5.2 \times 10^{-3} \text{ m}^3$$

Also expressed as 5.2 L.

ASSESS Both the increase in pressure and decrease in temperature of this environment work to decrease the volume of the gas by a factor of more than 4 that of STP.

30. **INTERPRET** Treating this medium as an ideal gas, and we're given the density and temperature of the gas. We are to estimate and compare this pressure to that found on Earth's atmosphere.

DEVELOP Because we're given the number of particles per cubic centimeter, we'll use the ideal gas law in the form of Equation 17.1, $pV = NkT$, to find the pressure.

EVALUATE Solving for p gives

$$p = \frac{NkT}{V} = \frac{(10^8)(1.38 \times 10^{-23} \text{ J/K})(10^6 \text{ K})}{(10^{-6} \text{ m}^3)} \approx 10^{-3} \text{ Pa}$$

When compared to the $\sim 10^5 \text{ Pa}$ found on Earth's atmosphere, we find that $p_{\text{corona}} \sim 10^{-8} p_{\text{Earth}}$.

ASSESS Here we have used the given particle density to express the quantity N/V .

31. **INTERPRET** This problem involves both a temperature rise and a phase change. We identify water as the substance involved.

DEVELOP To determine whether the water can all reach 0°C , and if any ice remains in the mixture, we must first calculate the energy required to warm the ice up to that temperature and to melt it all. These are both calculated with the use of Equation 16.3 and Equation 17.5, respectively. If the amount of energy required to cool the water, also calculated using Equation 16.3, is less than either one of these quantities we will know the answer to those questions.

EVALUATE We begin by evaluating the energy Q_i to heat the ice and melt it all

$$Q_i = m_{\text{ice}}c_{\text{ice}}\Delta T_{\text{ice}} + m_{\text{ice}}L_f = (0.066 \text{ kg})(2.05 \text{ kJ/kg} \cdot \text{K})(10 \text{ K}) + (0.066 \text{ kg})(334 \text{ kJ/kg})$$

$$Q_i = 1.35 \text{ kJ} + 22.0 \text{ kJ} = 23.35 \text{ kJ}$$

Cooling the water to 0°C would extract energy Q_w given by

$$Q_w = m_{\text{water}}c_{\text{water}}\Delta T_{\text{water}} = (1.0 \text{ kg})(4.184 \text{ kJ/kg} \cdot \text{K})(15 \text{ K}) = 62.8 \text{ kJ}$$

This is enough energy to both cool the ice and melt it all away, leaving us with 39.4 kJ of energy. We can think of this energy being left over and being used to heat the now 1066 g of water which is at 0°C . We can calculate the final temperature it would reach by expressing the change in temperature using Equation 16.3

$$\Delta T = (Q_w - Q_i) / [c_{\text{water}}(m_{\text{water}} + m_{\text{ice}})] = 8.8 \text{ K}$$

Meaning the final water temperature would be 8.8°C .

ASSESS Although only a portion of the water reaches 0°C as it warms the ice, we can use this approach to describe the energy exchange process accurately.

32. **INTERPRET** This problem involves both a temperature rise and a phase change. We identify water as the substance involved.

DEVELOP To determine what the minimum amount of ice that would leave the final mixture at 0°C , we must calculate amount of ice that would require the same amount of energy to warm-up and melt, that is available from the cooling water. These are both calculated with the use of Equation 16.3 and Equation 17.5, respectively.

EVALUATE We begin by evaluating the energy Q_w to cool the water to 0°C

$$Q_w = m_{\text{water}}c_{\text{water}}\Delta T_{\text{water}} = (1.0 \text{ kg})(4.184 \text{ kJ/kg} \cdot \text{K})(15 \text{ K}) = 62.8 \text{ kJ}$$

We can then equate this to the energy Q_i to heat the ice and melt it all, and solve for m_{ice} .

$$Q_w = Q_i = m_{\text{ice}}(c_{\text{ice}}\Delta T_{\text{ice}} + L_f)$$

$$m_{\text{ice}} = \frac{Q_w}{(c_{\text{ice}}\Delta T_{\text{ice}} + L_f)} = \frac{62.8 \text{ kJ}}{(2.05 \text{ kJ/kg} \cdot \text{K})(10 \text{ K}) + (334 \text{ kJ/kg})} = 0.177 \text{ kg}$$

The minimum mass of ice that could be added to the water in Example 17.4 in order to end up with a final temperature of 0°C is 177 g.

ASSESS This answer makes sense since in Example 17.4 only a small amount of ice remains.

- 33. INTERPRET** This problem involves both a temperature rise and a phase change. We identify water as the substance involved.

DEVELOP To determine whether the water can all reach 0°C , and if any ice remains in the mixture, we must first calculate the energy required to warm the ice up to that temperature and to melt it all. These are both calculated with the use of Equation 16.3 and Equation 17.5, respectively. If the amount of energy required to cool the water, also calculated using Equation 16.3, is less than either one of these quantities we will know the answer to those questions.

EVALUATE We begin by evaluating the energy Q_i to heat the ice and melt it all

$$Q_i = m_{\text{ice}}c_{\text{ice}}\Delta T_{\text{ice}} + m_{\text{ice}}L_f = (17.3 \times 10^6 \text{ kg})(2.05 \text{ kJ/kg} \cdot \text{K})(10 \text{ K}) + (17.3 \times 10^6 \text{ kg})(334 \text{ kJ/kg})$$

$$Q_i = 3.55 \times 10^{11} \text{ J} + 5.78 \times 10^{12} \text{ J} = 6.14 \times 10^{12} \text{ J}$$

Cooling the water to 0°C would extract energy Q_w given by

$$Q_w = m_{\text{water}}c_{\text{water}}\Delta T_{\text{water}} = (1.85 \times 10^8 \text{ kg})(4.184 \text{ kJ/kg} \cdot \text{K})(6.40 \text{ K}) = 4.95 \times 10^{12} \text{ J}$$

Where we have used the density of water, 1000 kg/m^3 , to calculate the mass. This is enough energy to cool the ice but not enough to melt it all away. After the mixture reaches 0°C we are left with

$$4.95 \times 10^{12} \text{ J} - 3.55 \times 10^{11} \text{ J} = 4.60 \times 10^{12} \text{ J}$$

From Equation 17.5, the amount of ice melted is then

$$m_{\text{melted}} = \frac{Q}{L_f} = \frac{4.60 \times 10^{12} \text{ J}}{3.34 \times 10^5 \text{ J/kg}} = 13.77 \times 10^6 \text{ kg}$$

Thus, there are $3.53 \times 10^6 \text{ kg}$ of ice left.

ASSESS The remaining energy was enough to melt ~80% of the ice that falls into the lake.

- 34. INTERPRET** This problem involves both a temperature rise and a phase change. We identify water as the substance involved.

DEVELOP To determine what the minimum amount of ice that would leave the final mixture at 0°C , we must calculate amount of ice that would require the same amount of energy to warm-up and melt, that is available from the cooling water. These are both calculated with the use of Equation 16.3 and Equation 17.5, respectively.

EVALUATE We begin by evaluating the energy Q_w to cool the water to 0°C

$$Q_w = m_{\text{water}}c_{\text{water}}\Delta T_{\text{water}} = (1.85 \times 10^8 \text{ kg})(4.184 \text{ kJ/kg} \cdot \text{K})(6.40 \text{ K}) = 4.95 \times 10^{12} \text{ J}$$

We can then equate this to the energy Q_i to heat the ice and melt it all, and solve for m_{ice} .

$$Q_w = Q_i = m_{\text{ice}}(c_{\text{ice}}\Delta T_{\text{ice}} + L_f)$$

$$m_{\text{ice}} = \frac{Q_w}{(c_{\text{ice}}\Delta T_{\text{ice}} + L_f)} = \frac{4.95 \times 10^{12} \text{ J}}{(2.05 \text{ kJ/kg} \cdot \text{K})(10 \text{ K}) + (334 \text{ kJ/kg})} = 14.0 \times 10^6 \text{ kg}$$

The minimum mass for the iceberg in the preceding problem that will ensure an equilibrium temperature of 0°C is $14.0 \times 10^6 \text{ kg}$.

ASSESS This answer makes sense since in the preceding problem only ~20% of the ice remains.

PROBLEMS

- 35. INTERPRET** The system of interest is the solar corona, which we treat as an ideal gas. The quantity of interest is the number density of air molecules.

DEVELOP The number density implied by the ideal gas law (Equation 17.1) is

$$pV = NkT \rightarrow \frac{N}{V} = \frac{p}{kT}$$

EVALUATE Applying the above equation to the solar corona, we obtain

$$\left(\frac{N}{V}\right)_{\text{corona}} = \frac{p}{kT} = \frac{3 \times 10^{-2} \text{ Pa}}{(1.38 \times 10^{-23} \text{ J/K})(2 \times 10^6 \text{ K})} = 1 \times 10^{15} \text{ m}^{-3}$$

If we assume the Earth's atmosphere has standard temperature and pressure, the particle density is

$$\left(\frac{N}{V}\right)_{\text{STP}} = \frac{p}{kT} = \frac{1.013 \times 10^5 \text{ Pa}}{(1.38 \times 10^{-23} \text{ J/K})(273 \text{ K})} = 2.7 \times 10^{25} \text{ m}^{-3}$$

So the corona is over 10 billion times less dense than on Earth.

ASSESS Scientists are still not entirely certain how the corona ends up being so hot.

- 36. INTERPRET** We're asked to figure out how much a balloon expands as it rises to an altitude of lower pressure and temperature.

DEVELOP We'll neglect the tension in the balloon material, so the helium gas will expand until its outward pressure on the balloon matches the inward pressure from the surrounding air. The balloon's temperature will also come into equilibrium with its surroundings. The helium inside the balloon obeys the ideal-gas law at both the initial and final altitudes, and since the number of molecules is constant, we have:

$$\frac{p_i V_i}{T_i} = \frac{p_f V_f}{T_f}$$

EVALUATE Solving for the final volume,

$$V_f = \frac{p_i T_f}{p_f T_i} V_i = \frac{(1.0 \text{ atm}) (263 \text{ K})}{(0.70 \text{ atm}) (293 \text{ K})} (7.4 \text{ L}) = 9.5 \text{ L}$$

ASSESS If the balloon's pressure and temperature are in equilibrium with its surroundings, you might wonder why it rises. By Archimedes' principle from Chapter 15, the balloon will rise until its weight is equal to the weight of the air it displaces. If we neglect the contribution from the balloon material, the final volume will be $V_f = m_{\text{He}} N / \rho_{\text{air}}$, where m_{He} is the mass of a single helium atom, N is the total number of helium atoms, and ρ_{air} is the air density, which depends on altitude.

- 37. INTERPRET** The object of interest is the cylinder compressed with air. We are given the pressure, temperature, and volume, and want to find the number of moles (i.e., the number of air molecules) in the cylinder.

DEVELOP We shall treat the air as an ideal gas (although this is somewhat risky at 180 atm) and use the ideal gas law $PV = nRT$ given in Equation 17.2, to find the number n of moles. The volume of the cylinder is

$$V = \pi \left(\frac{d^2}{4} \right) h = \pi (0.10 \text{ m})^2 (1.0 \text{ m}) = 0.01 \pi \text{ m}^3$$

EVALUATE (a) Applying the ideal gas law gives

$$n = \frac{pV}{RT} = \frac{(180 \text{ atm})(1.013 \times 10^5 \text{ Pa/atm})(0.01 \pi \text{ m}^3)}{(8.314 \text{ J/K} \cdot \text{mol})(293 \text{ K})} = 235 \text{ mol}$$

where we have used $T = 20^\circ\text{C} = 293 \text{ K}$ as the room temperature.

(b) If the pressure is $p' = 1 \text{ atm}$, then the volume would be

$$V' = \frac{nRT}{p'} = \left(\frac{p}{p'}\right)V = \left(\frac{180 \text{ atm}}{1 \text{ atm}}\right)(0.01\pi \text{ m}^3) = 5.65 \text{ m}^3$$

ASSESS When temperature is held constant, $PV = \text{constant}$ for an ideal gas. Therefore, decreasing the pressure increases the volume in a proportional amount.

- 38. INTERPRET** You're arguing that a whipped cream can exploded because of an error on the part of the manufacturer.

DEVELOP It's true that the pressure is the relevant parameter, not the temperature. You can assume that the can is in danger of exploding when the pressure is above the value p_{max} . You can estimate this value from the manufacturer's claim that a full can should not exceed a temperature of $T_{\text{max}} = 55^\circ\text{C}$. By the ideal-gas law, the maximum pressure is related to this temperature as well as to the initial volume and initial number of moles of the propellant:

$$p_{\text{max}} = \frac{n_i RT_{\text{max}}}{V_i}$$

You can't calculate the maximum pressure, but you can compare it to the pressure, p , of the can that exploded, which had $T = 60^\circ\text{C}$, $V > 2V_i$, and $n < n_i$.

EVALUATE The ratio of the pressure to the maximum pressure is

$$\frac{p}{p_{\text{max}}} = \frac{n}{n_i} \frac{V_i}{V} \frac{T}{T_{\text{max}}} < (1) \left(\frac{1}{2}\right) \left(\frac{333 \text{ K}}{328 \text{ K}}\right) = 0.51$$

The can exploded when its pressure was about half the maximum pressure, so the manufacturer appears to be at fault.

ASSESS By the above reasoning, a half-full can could supposedly withstand temperatures as high as about 370°C . However, at such a high temperature, the whipped cream will vaporize, thus adding its pressure to that of the propellant.

- 39. INTERPRET** The object of interest is the flask filled with air, which we treat as an ideal gas. We explore the effect of changing temperature and pressure. The maximum pressure in the flask will occur when the gas inside the flask, which is initially at STP, is heated to the boiling point of water (100°C). To find the number of moles that escape when the flask is opened, we consider that the gas escapes so fast that the temperature of the gas can be considered to be constant on this timescale.

DEVELOP When the flask is immersed in boiling water, its volume remains fixed. Therefore, the ideal-gas law $pV = nRT$ (Equation 17.2) applied at each temperature gives

$$\left. \begin{aligned} p_1 V &= nRT_1 \\ p_2 V &= nRT_2 \end{aligned} \right\} \frac{p_2}{p_1} = \frac{T_2}{T_1}$$

The initial conditions of the gas are $p_1 = 1 \text{ atm}$, $V = 3.80 \text{ L} = 3.80 \times 10^{-3} \text{ m}^3$, and $T_1 = 293 \text{ K}$. The maximum pressure in the flask occurs when $T_2 = 100^\circ\text{C} = 373 \text{ K}$. For part (b), we first calculate the number of moles of gas initially in the flask. Applying the ideal-gas law again, we find the number of molecules to be

$$n_1 = \frac{p_1 V}{RT_1} = \frac{(1 \text{ atm})(1.013 \times 10^5 \text{ Pa / atm})(3.8 \times 10^{-3} \text{ m}^3)}{[8.314 \text{ J / (K} \cdot \text{mol)}](293 \text{ K})} = 0.158 \text{ mol}$$

When the flask is opened at $T_2 = 373 \text{ K}$, the pressure rapidly decreases to $p_2 = 1 \text{ atm}$, so the quantity of gas remaining in the flask is

$$n_2 = \frac{p_1 V_1}{RT_2} = \left(\frac{T_1}{T_2}\right)n_1$$

So the number of moles that escaped from the flask is $\Delta n = n_1 - n_2$. After the flask is closed and cooled back down to $T_3 = 20^\circ\text{C} = 293 \text{ K}$, we apply the ideal-gas law again, using n_2 to find the new pressure. This gives

$$p_3 = \frac{n_2 RT_1}{V} = \left(\frac{n_2}{n_1} \right) p_1$$

EVALUATE (a) From the equation above, we find the maximum pressure reached in the flask to be

$$p_2 = \left(\frac{T_2}{T_1} \right) p_1 = \left(\frac{373 \text{ K}}{293 \text{ K}} \right) (1 \text{ atm}) = 1.27 \text{ atm}$$

(b) After opening the flask, the quantity of gas left in the flask is

$$n_2 = \left(\frac{T_1}{T_2} \right) n_1 = \left(\frac{293 \text{ K}}{373 \text{ K}} \right) (0.158 \text{ mol}) = 0.124 \text{ mol}$$

Therefore, the amount that escaped is $\Delta n = n_1 - n_2 = 0.0334 \text{ mol}$.

(c) After sealing the flask and cooling it to 293 K, the pressure of the gas in the flask is

$$p_3 = \frac{n_2 RT_1}{V} = \left(\frac{n_2}{n_1} \right) p_1 = \left(\frac{0.124 \text{ mol}}{0.158 \text{ mol}} \right) (1 \text{ atm}) = 0.785 \text{ atm}$$

ASSESS As expected, the pressure in the flask is greatest for part (a), when the temperature is highest and there are the most moles of gas in the flask, and the pressure is the lowest for part (c), when the reverse is true. Pressure is proportional to the number of molecules in the volume; so after some gas molecules escape from the flask, the pressure decreases.

- 40. INTERPRET** We want to know how long it will take to warm up a frostbitten hand. We can calculate the total energy needed and divide by the rate at which energy is absorbed from the water bath.

DEVELOP We model the frostbitten hand as pure ice with mass m . A certain amount of heat will be needed to first melt the ice: $Q_1 = L_f m$, which is Equation 17.5, with $L_f = 334 \text{ kJ/kg}$ for water. Afterward, the water will have to be warmed from the freezing temperature ($T = 0^\circ\text{C}$) to the normal body temperature ($T = 37^\circ\text{C}$). This will require $Q_2 = mc\Delta T$, where $c = 4.184 \text{ kJ/kg}\cdot\text{K}$ from Table 16.1. The time to supply the full heat will be: $t = (Q_1 + Q_2) / P$.

EVALUATE The melting and warming steps will require energies of:

$$Q_1 = L_f m = (334 \text{ kJ/kg})(0.15 \text{ kg}) = 50.1 \text{ kJ}$$

$$Q_2 = mc\Delta T = (0.15 \text{ kg})(4.184 \text{ kJ/kg}\cdot\text{K})(37 \text{ K}) = 23.22 \text{ kJ}$$

Using the rate at which heat is conducted into the hand,

$$t = \frac{Q_1 + Q_2}{P} = \frac{50.1 \text{ kJ} + 23.22 \text{ kJ}}{750 \text{ W}} = 98 \text{ s}$$

ASSESS This seems reasonable, assuming the bath supplies heat to the hand at a constant rate. In fact, the outside of the hand will come into near temperature equilibrium with the surrounding water, thus slowing the rate of heat flow to the inner part of the hand.

- 41. INTERPRET** This problem involves finding the time required to transform the given amount of 100°C water to gas with the given rate of heating.

DEVELOP From Equation 17.5, the energy absorbed during the vaporization of water at its boiling point is $Q = L_v m$. If this energy is supplied in a time t , then the power must be $P = Q/t$, so $t = Q/P$. Given that $P = 1200 \text{ W}$, we can solve for the time t .

EVALUATE The time it takes to boil away the water is

$$t = \frac{Q}{P} = \frac{L_v m}{P} = \frac{(2257 \text{ kJ/kg})(2 \text{ kg})}{1.2 \text{ kW}} = 3.76 \times 10^3 \text{ s} = 63 \text{ min}$$

ASSESS This seems like a reasonable time, and it explains why it is a good idea to occasionally add water to steamers when cooking with steam.

- 42. INTERPRET** This problem is about melting, and it involves heat of fusion. We want to know how much ice would melt by exploding a 1-megaton nuclear bomb in the Greenland ice cap.

DEVELOP A 1-megaton nuclear device releases about $Q = 4.184 \times 10^{15} \text{ J}$ of energy. The heat of fusion for water is $L_f = 334 \text{ kJ/kg}$, so we can use Equation 17.5 $Q = L_f m$ to find the mass m of ice that is melted.

EVALUATE The amount of ice at the normal melting point of 0°C that melts is

$$m = \frac{Q}{L_f} = \frac{4.184 \times 10^{15} \text{ J}}{334 \text{ kJ/kg}} = 1.25 \times 10^{10} \text{ kg}$$

ASSESS The amount of ice melted by this explosion corresponds to a lake approximately 100-m deep, 1-km long, and 100-m wide.

43. **INTERPRET** This problem involves the latent heat of fusion, with which we can calculate the time it takes to melt a hole through a piece of steel.

DEVELOP From Equation 17.5, the torch must provide the energy $Q = L_f m$. The metal's heat of fusion L_f is given, and mass m can be calculated from the given dimensions of the hole and density of steel. At the rate $P = Q/t$, where P is the power applied by the torch, it will take a time $t = Q/P$ to melt the hole.

EVALUATE Inserting the given quantities into the expression above gives

$$t = \frac{Q}{P} = \frac{L_f \rho V}{P} = \frac{(268 \text{ kJ/kg})(7970 \text{ kg/m}^3) [\pi (0.01 \text{ m})^2 (0.09 \text{ m})]}{(0.45)(2.35 \text{ kW})} = 57.1 \text{ s}$$

ASSESS While less than 50% of the energy provided goes into melting the metal, it takes only less than 1 minute to create the hole.

44. **INTERPRET** This problem is about melting, so heat of fusion is involved. We want to know how much energy is needed to melt a given quantity of ice.

DEVELOP We can look up the surface area of Lake Superior: $A = 82,000 \text{ km}^2$. The total mass of ice is then

$$m = \rho V = \rho Ah = (917 \text{ kg/m}^3)(8.2 \times 10^{10} \text{ m}^2)(1.3 \text{ m}) = 9.78 \times 10^{13} \text{ kg}$$

If we can assume the ice is initially at 0°C , the energy needed to melt it will be $Q = L_f m$, where $L_f = 334 \text{ kJ/kg}$ from Table 17.1.

EVALUATE (a) Substituting the values, the heat required to thaw Lake Superior is:

$$Q = L_f m = (334 \text{ kJ/kg})(9.78 \times 10^{13} \text{ kg}) = 3.27 \times 10^{19} \text{ J} \approx 33 \text{ EJ}$$

Where we have used the prefix exa (10^{18}).

(b) If the ice melts in 3 weeks, the average power must be

$$\bar{P} = \frac{Q}{t} = \frac{3.27 \times 10^{19} \text{ J}}{3 \cdot 7 \cdot 24 \cdot 60 \cdot 60 \text{ s}} = 1.8 \times 10^{13} \text{ W} = 18 \text{ TW}$$

ASSESS The power per unit area (or intensity) in this case is 219 W/m^2 . In Chapter 16, we learned that the Earth absorbs energy from the Sun at a rate of 960 W/m^2 averaged over its cross-sectional area. Considering that the Sun is shining on Lake Superior only a fraction of the 3-week period, the calculated power is reasonable.

45. **INTERPRET** This problem involves the latent heat of fusion, with which we can calculate the time it takes to freeze water that is at 0°C if we remove energy at the rate given.

DEVELOP From Equation 17.5, the refrigerator must extract the energy $Q = L_f m$, where $m = 0.75 \text{ kg}$ and $L_f = 334 \text{ kJ/kg}$ (from Table 17.1). At the rate $P = Q/t$, where P is the power applied by the refrigerator, it will take a time $t = Q/P$ to freeze the water.

EVALUATE Inserting the given quantities into the expression above gives

$$t = \frac{Q}{P} = \frac{L_f m}{P} = \frac{(334 \text{ kJ/kg})(0.75 \text{ kg})}{0.097 \text{ kW}} = 2.58 \times 10^3 \text{ s} = 43 \text{ min}$$

ASSESS This is a good figure to keep in mind if you need to make ice cubes for a party.

46. **INTERPRET** You're asked to determine if a highly absorbing black carbon can cause a layer of Arctic ice to completely melt.

DEVELOP The ice has a thickness of $d = 2.5 \text{ m}$. You assume that the only energy input is from absorbed sunlight:

$$E = (1 - a)SA t$$

where a is the fraction of light reflected away, $S = 300 \text{ W/m}^2$ is the solar intensity, A is the exposed surface area of ice, and t is the time, which we will assume is a quarter of a year for the summer season. The ice will melt if $E \geq Q = L_f m$, where $L_f = 334 \text{ kJ/kg}$ and $m = \rho A d$.

EVALUATE Combining the above relations, the criteria for the ice to melt can be written as:

$$a \leq 1 - \frac{L_f \rho d}{S t} = 1 - \frac{(334 \text{ kJ/kg})(917 \text{ kg/m}^3)(2.5 \text{ m})}{(300 \text{ W/m}^2)\left(\frac{1}{4} 365 \times 86400 \text{ s}\right)} = 68\%$$

So the ice shouldn't melt completely under normal conditions when $a = 90\%$, but the presence of black carbon could cause complete melting, assuming $a = 50\%$ in this case.

ASSESS The parameter a is called the albedo. An object with zero albedo would absorb all incoming radiation, like a perfect blackbody.

- 47. INTERPRET** This problem involves raising the temperature of ice to the melting point and then changing the phase, so both specific heat and heat of fusion are involved.

DEVELOP The energy needed to raise the temperature is given by Equation 16.3, $Q_1 = mc_{\text{ice}}\Delta T$, with $\Delta T = T_{\text{final}} - T_{\text{initial}} = 0^\circ\text{C} - (-10^\circ\text{C}) = 10^\circ\text{C} = 10 \text{ K}$. Equation 17.5, $Q_2 = L_f m$, gives the heat of fusion required to change the phase from solid to liquid. Use Table 16.1 to find the specific heat c_{ice} of ice and Table 17.1 to find the latent heat of fusion L_f for water. The total energy required to go from ice at -10°C to water at 0°C is the sum of Q_1 and Q_2 .

EVALUATE Adding up the two energies, we obtain

$$Q_{\text{tot}} = Q_1 + Q_2 = mc_{\text{ice}}\Delta T + mL_f$$

$$= (12.5 \text{ kg}) \left\{ [2.05 \text{ kJ/(kg} \cdot \text{K)}](10 \text{ K}) + 334 \text{ kJ/kg} \right\} = \overbrace{0.256 \text{ MJ}}^{\text{warm water}} + \overbrace{4.18 \text{ MJ}}^{\text{melt ice}} = 4.43 \text{ MJ}$$

ASSESS About 94% of the energy is actually used to melt the ice, only 6% for raising the temperature.

- 48. INTERPRET** This problem involves raising the temperature of water, which involves the specific heat of water, then changing its phase from liquid to gas, which involves the latent heat of vaporization. Using these two concepts, we are to find the initial temperature of the water given that the 90% of the energy needed to boil the water away is used to change the phase, with only 10% being used to raise its temperature.

DEVELOP Equation 16.3 $Q_1 = mc\Delta T$ gives the energy needed to raise the water's temperature, where $\Delta T = 100^\circ\text{C} - T$ where T is the initial temperature of the water. The energy needed to boil the water (i.e., change it from the liquid phase to the gas phase) is $Q_2 = mL_v$. The quantities L_v and c can be found in Tables 17.1 and 16.1, respectively. The problem statement says that $Q_1 = Q_2/10$, so

$$mc\Delta T = mc(100^\circ\text{C} - T) = \frac{mL_v}{10}$$

$$T = (100^\circ\text{C}) - \frac{L_v}{10c}$$

we can solve for the initial water temperature T .

EVALUATE Inserting the specific heat and latent heat of vaporization into the expression above gives

$$T = (100^\circ\text{C}) - \frac{2257 \text{ kJ/kg}}{10.0[4.184 \text{ kJ/(kg} \cdot \text{K)}]} = 46.1^\circ\text{C}$$

ASSESS Much more heat is required to boil the water away (i.e., change its phase from liquid to gas) than to raise its temperature from 46°C to 100°C .

- 49. INTERPRET** We are asked to find the time it takes a certain amount of water to boil away after being heated.
- DEVELOP** In Problem 54 of Chapter 16 we used the volume V , the density ρ , and the specific heat of seawater to find the time to heat the water to its boiling point. Now we want to find the time it would take that amount of water to boil away entirely if the heat continues to be delivered at the same rate. Using Equation 17.5, we note that at the rate $P = Q/t$, where P is the power applied, it will take a time $t = Q/P$ to boil the water.
- EVALUATE** Using the latent heat of vaporization of water found in Table 17.1 and the given quantities from Problem 54 of Chapter 16 we calculate the heat and divide by the power to obtain a duration of

$$t = \frac{Q}{P} = \frac{L_v \rho V}{P} = \frac{(2257 \text{ kJ/kg})(1030 \text{ kg/m}^3)(650 \text{ m}^3)}{(33 \times 10^6 \text{ W})} = 12.7 \text{ h}$$

Adding to this the 2.1 h necessary to heat the water to its boiling point we find it would take 14.8 h from the injection of the 10°C water to boil entirely away.

ASSESS With no replenishment of water it would take less than a day for it to all evaporate.

- 50. INTERPRET** Our system consists of both ice and warm water. We want to know how much ice at 0°C is needed to bring the water to 0°C . This process involves cooling the water (i.e., using the specific heat of water) and melting the ice (i.e., using the latent heat of fusion of ice).

DEVELOP Assume that the only heat transfer is between the water and the ice. To cool the water to 0°C , the amount of heat that must be extracted is (using Equation 16.3)

$$Q_1 = m_w c_w \Delta T = (14 \text{ kg})(4.184 \text{ kJ/kg} \cdot \text{K})(31^\circ\text{C} - 0^\circ\text{C}) = 1.82 \text{ MJ}$$

The amount of heat used to melt ice at 0°C is, from Equation 17.3, $Q_2 = L_f m_{\text{ice}}$. For the ice to just melt and not increase in temperature, we must have $Q_1 = Q_2$.

EVALUATE To bring the water temperature down to 0°C , the minimum mass of ice needed is thus

$$m = \frac{\Delta Q}{L_f} = \frac{1.82 \text{ MJ}}{334 \text{ kJ/kg}} = 5.4 \text{ kg}$$

ASSESS Note that the punch would be diluted with 5.4 kg of meltwater. To avoid this dilution, you could use less ice at a temperature below 0°C .

- 51. INTERPRET** This problem involves mixing an equal mass of ice and water and letting the mixture reach equilibrium. We are to calculate at what temperature the water must be if the final mixture is to contain equal amounts of ice and water.

DEVELOP Assume that all the heat gained by the ice was lost by the water, with no heat transfer to the container or the surroundings. An equilibrium mixture of ice and water (at atmospheric pressure) must be at 0°C , and if the masses of ice and water start out and remain equal, there is no net melting or freezing. Thus any energy spent raising the temperature of the ice and melting it must be balanced by energy lost by the water as its temperature lowers and it freezes. These energies are given by Equations 16.3 (for the temperature change), $Q_1 = mc$ and (for the melting or freezing) $Q_2 = L_f m$. We can therefore sum these energies for both ice and water and equate the result, which gives

$$Q_{\text{ice}} = \overbrace{m_{\text{ice}} c_{\text{ice}} (0^\circ\text{C} - T_{\text{ice}})}^{Q_1^{\text{ice}}} + \overbrace{L_f m_{\text{ice}}}^{Q_2^{\text{ice}}} = -Q_w = \overbrace{m_w c_w (T_w - 0^\circ\text{C})}^{-Q_1^w} + \overbrace{L_f m_w}^{-Q_2^w}$$

where $T_{\text{ice}} = -10^\circ\text{C}$. Given that $m_{\text{ice}} = m_w$, we can solve for the water temperature T_w .

EVALUATE Solving for T_w and inserting the given quantities gives

$$T_w = \frac{c_{\text{ice}} (-T_{\text{ice}})}{c_w} = \frac{2.050 \text{ kJ/(kg} \cdot \text{K)}}{4.184 \text{ kJ/(kg} \cdot \text{K)}} (10^\circ\text{C}) = 4.9^\circ\text{C}$$

ASSESS Because any thermal energy spent melting the ice is balanced by freezing the water, only the specific heats of water and ice come into play.

- 52. INTERPRET** The evaporation of sweat removes heat from the body. We're asked to find how much heat is removed from a marathon runner during a race.

DEVELOP It takes 2.4 MJ to evaporate a kg of water, and we're told that a marathoner typically sweats 3 L per hour. We assume that all of this sweat is evaporated and that all of the needed heat comes from the marathoner's body.

EVALUATE During a 3-hour marathon, the amount of heat expelled due to sweating is:

$$Q = (2.4 \text{ MJ/kg}) \left(\frac{1 \text{ kg}}{1 \text{ L}} \right) (3 \text{ L/h})(2.9 \text{ h}) = 21 \text{ MJ}$$

ASSESS This is equivalent to 5000 kilocalories. However, running for 2.9 hours should only burn about 2500 kilocalories. That would mean the marathoner is losing more than twice the amount of energy he/she is consuming. That wouldn't make sense, so we might assume that some of the sweat falls off before evaporating.

- 53. INTERPRET** This problem involves the latent heat of fusion of water, which is the energy required per unit mass to change ice to water (or vice versa, but with the opposite sign). We can use this concept to find the energy required to melt 25 kg of ice in 10 min, and from there find the power.

DEVELOP If the melting occurs at atmospheric pressure and if the ice is at 0°C, the energy required to melt the ice is given by Equation 17.3, $Q = L_f m$, where L_f is the latent heat of fusion, which is given in Table 17.1. To melt the ice in a time $t = 10 \text{ min}$ would require a power $P = Q/t$.

EVALUATE The power required is

$$P = \frac{Q}{t} = \frac{L_f m}{t} = \frac{(334 \text{ kJ/kg})(25 \text{ kg})}{10 \times 60 \text{ s}} = 14 \text{ kW}$$

ASSESS This is equivalent to the power needed to light 140 100-W bulbs.

- 54. INTERPRET** This problem involves both a temperature rise and a phase change, so both the specific heat and the latent heat (of vaporization of water) will enter into the calculation. The object of interest is the water in the microwave oven.

DEVELOP The energy needed to raise the temperature of the water is given by Equation 16.3, $Q = mc\Delta T$. Equation 17.5, $Q = L_v m$, gives the heat needed to vaporize water. In 20 minutes, the total heat energy transferred to the water (if we ignore energy absorbed by a container or lost to the surroundings) is

$$Q_{\text{tot}} = Pt = (0.6 \text{ kW})(20 \times 60 \text{ s}) = 720 \text{ kJ}$$

Out of this total energy, the energy consumed in raising the water's temperature to the normal boiling point is

$$Q_1 = mc\Delta T = (0.4 \text{ kg})[4.184 \text{ kJ/(kg} \cdot \text{K)}](100^\circ\text{C} - 20^\circ\text{C}) = 134 \text{ kJ}$$

and the difference $Q_2 = Q_{\text{tot}} - Q_1 = 586 \text{ kJ}$ is left to vaporize some of the water. The amount of water vaporized can be found by using Equation 17.5, $Q_2 = L_v m$, where L_v is the latent heat of vaporization from Table 17.1.

EVALUATE Using Equation 17.5, the mass m_v of water vaporized is

$$m_v = \frac{Q_2}{L_v} = \frac{586 \text{ kJ}}{2257 \text{ kJ/kg}} = 0.26 \text{ kg}$$

Therefore, only $\Delta m = m - m_v = 0.40 \text{ kg} - 0.26 \text{ kg} = 0.14 \text{ kg}$ of boiling water is all that remains (to two significant figures).

ASSESS The excess heat from the microwave oven vaporizes the water. This is precisely what causes your food to dry out when you heat it in the microwave oven for too long.

- 55. INTERPRET** This problem involves a change in temperature and a phase change (liquid to gas) of water. We will apply the concepts of specific heat and heat of vaporization to find temperature of the system in equilibrium, and whether all the water boils away.

DEVELOP To find the equilibrium temperature, consider that the lava must first warm the water from 24.0°C to 100°C, which will require an energy $Q_1 = m_w c_w \Delta T_w$, with $\Delta T_w = 76 \text{ K}$, $m_w = 20.4 \text{ Mg}$, and $c_w = 4.184 \text{ kJ/(kg} \cdot \text{K)}$ (see Table 16.1). Next, for the lava to boil the water, it would require an energy $Q_2 = L_{vw} m_w$, with $L_v = 2257 \text{ kJ/kg}$ (see Table 17.1.) The sum of these two energies must equate to the energy *lost* by the lava $Q_3 = m_l c_l \Delta T_l$, with $\Delta T_l = 884 \text{ K}$, $m_l = 15.4 \text{ Mg}$, and $c_l = 1.04 \text{ kJ/(kg} \cdot \text{K)}$.

EVALUATE Calculating each amount of energy we find

$$Q_1 = m_w c_w \Delta T_w = 6.49 \times 10^6 \text{ kJ}$$

$$Q_2 = L_{vw} m_w = 4.60 \times 10^7 \text{ kJ}$$

$$Q_3 = m_l c_l \Delta T_l = 1.42 \times 10^7 \text{ kJ}$$

Since the energy provided by the cooling lava Q_3 is larger than the energy required to bring the water to a boiling point Q_1 , it will all reach 100°C , but since $Q_3 < Q_1 + Q_2$, it will not all boil away.

ASSESS The energy remaining from the cooling lava after the water has reached its boiling point will only evaporate $\sim 17\%$ of the pool's water.

- 56. INTERPRET** This problem involves heat transfer between ice and water. Some of the ice may melt and some of the water may freeze, which will involve the latent heat of fusion of water. In addition, the ice and the water will change temperature, so the specific heats of ice and water are involved.

DEVELOP Assume that all the heat lost by the water is gained by the ice. The temperature of the water drops and that of the ice rises. If either reaches 0°C , a change of phase occurs, freezing or melting, depending on which reaches 0°C first. To cool to 0°C , the amount of heat the water would lose is

$$Q_{w,1} = m_w c_w \Delta T_w = (1.0 \text{ kg})[4.184 \text{ kJ} / (\text{kg} \cdot \text{K})](7.0 \text{ K}) = 29.3 \text{ kJ}$$

On the other hand, to warm the ice to 0°C , it would gain an energy

$$Q_i = m_i c_i \Delta T_i = (1.0 \text{ kg})[2.050 \text{ kJ} / (\text{kg} \cdot \text{K})](34 \text{ K}) = 69.7 \text{ kJ}$$

Evidently, the water reaches 0°C first, so we will assume it freezes and the ice simply warms up. Because all the energy lost by the water in cooling and then changing phase to ice is assumed to be gained by the ice, we must have $Q_i = Q_{w,1} + Q_{w,2}$, where $Q_{w,2} = L_f m$ (Equation 17.3) is the energy it takes to freeze the water. We can, therefore, solve for the mass m of water that is changed to ice.

EVALUATE Therefore, the mass of water m that freezes is

$$m = \frac{Q_i - Q_{w,1}}{L_f} = \frac{40.4 \text{ kJ}}{334 \text{ kJ} / \text{kg}} = 0.12 \text{ kg}$$

The final mixture is thus at 0°C and contains 1.12 kg of ice and $1.0 \text{ kg} - 0.12 \text{ kg} = 0.88 \text{ kg}$ of water.

ASSESS The equilibrium temperature in this case is 0°C , at which where water and ice coexist.

- 57. INTERPRET** This problem involves the linear thermal expansion of steel. We are to find the temperature at which the hole in the steel plate will become large enough to allow the marble to pass through it.

DEVELOP The diameter of the hole expands with the coefficient of linear expansion of steel. Using Equation 17.7

$$\alpha \Delta T = \frac{\Delta L}{L}$$

with $L = 1.000$ and $L_0 = 0.998$, we can solve for ΔT .

EVALUATE Solving for T and inserting the given quantities gives

$$\Delta T = \frac{\Delta L}{L\alpha} = \frac{0.002 \text{ cm}}{(0.998 \text{ cm})(12 \times 10^{-6} \text{ K}^{-1})} = 167 \text{ K}$$

So, the plate must be heated to 167 K above room temperature.

ASSESS Note that we have treated the problem as if we were considering a steel disk of initial diameter 0.998 cm instead of a hole in a steel plate. The treatment is valid because such a hole must expand at the same rate as the disk because if we put the disk in the hole, it must fit at all temperatures.

- 58. INTERPRET** This problem is about thermal expansion. Since it involves volume, the relevant quantity is the coefficient of volume expansion β whose value can be used to identify the necessary amount of ethyl alcohol.

DEVELOP Neglecting any expansion in the glass itself, we note only the vertical dimension (height) of ethyl alcohol volume in the thermometer is subject to change. We can thus express the liquid's change in volume as $\Delta V = \pi(d/2)^2(1 \text{ mm})$, where d is the constant diameter of the thermometer. Along with the volume expansion coefficient of ethyl alcohol found in Table 17.2, the desired change in volume can be used to find the necessary amount liquid in the thermometer by applying Equation 17.6,

$$\beta = \frac{\Delta V/V}{\Delta T}$$

EVALUATE Substituting the values given in the problem statement, we have

$$V = \frac{\Delta V}{\beta \Delta T} = \frac{\pi(0.167/2 \text{ mm})^2(1 \text{ mm})}{(75 \times 10^{-5} \text{ K}^{-1})(1 \text{ K})} = 29.2 \text{ mm}^3$$

ASSESS Since the expansion occurs in one dimension, we might be tempted to express this change in terms of the linear expansion coefficient alpha. However, liquids don't expand proportionately in all directions. The increase in volume is directly proportional to the length in this case, and beta is the appropriate expansion coefficient for the liquid.

- 59. INTERPRET** This problem involves the linear expansion of Pyrex and steel. We are to find the temperature at which the diameter of the Pyrex tube is $2 \mu\text{m}$ greater than the diameter of the steel ball.

DEVELOP Since the coefficient of linear expansion of steel is greater than that of Pyrex glass, the unit must be cooled to provide clearance. The difference in the contraction of steel and Pyrex must be twice the given clearance on one side, so

$$|\Delta L_{\text{steel}}| - |\Delta L_{\text{pyrex}}| = 2 \mu\text{m} = (\alpha_{\text{steel}} - \alpha_{\text{pyrex}})L|\Delta T|$$

which we can solve for ΔT (find the values for α_{steel} and α_{pyrex} in Table 17.2).

EVALUATE Solving for ΔT and inserting the known quantities gives

$$|\Delta T| = \frac{2.0 \times 10^{-4} \text{ cm}}{(\alpha_{\text{steel}} - \alpha_{\text{pyrex}})L} = \frac{2.0 \times 10^{-4} \text{ cm}}{[(12 - 3.2) \times 10^{-6} \text{ K}^{-1}](1.0 \text{ cm})} = 22.7 \text{ K}$$

Since we must cool the system by this amount, the final temperature of the system will be

$$T = 380 \text{ K} - 23 \text{ K} = 357 \text{ K}.$$

ASSESS Reversing this process is a good technique to create tightly fitted parts.

- 60. INTERPRET** You're in charge of specifying the size of an expansion tank for a car's fuel system. This will involve calculating the thermal expansion of gasoline.

DEVELOP The amount that a material expands due to an increase in temperature is given in Equation 17. 6: $\Delta V / V = \beta \Delta T$. For gasoline, the coefficient of volume expansion is $\beta = 95 \times 10^{-5} \text{ K}^{-1}$ from Table 17.2.

EVALUATE The car you are working on has a gas tank with a volume of $V = 50 \text{ L}$. You assume the tank is initially filled with gasoline at 10°C . Once the gasoline comes into equilibrium with the outside temperature of 35°C , the volume will have expanded by

$$\Delta V = V\beta\Delta T = (50 \text{ L})(95 \times 10^{-5} \text{ K}^{-1})(25 \text{ K}) = 1.2 \text{ L}$$

Your expansion tank needs to be at least this big.

ASSESS The expansion tank size is a little over 2% of the main tank's size, which seems reasonable. Notice that gasoline has the highest coefficient of volume expansion of all the liquids in Table 17.2.

- 61. INTERPRET** This problem involves the linear expansion of a rod as it is heated, so the coefficient of linear expansion will come into play. We are asked to calculate the height d of the apex of the triangle formed by the rod that cracks upon expanding because it is fixed between two immovable walls.

DEVELOP If the two straight pieces in Fig. 17.11 are of equal length, the Pythagorean theorem gives

$$d = \sqrt{(L/2)^2 - (L_0/2)^2}$$

where $L = L_0(1 + \alpha\Delta T)$ is the total expanded length of the rod.

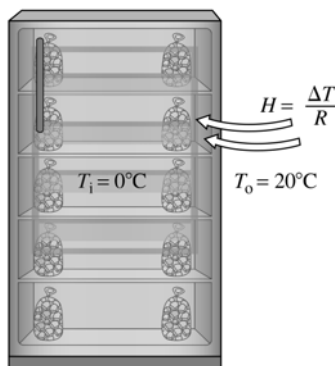
EVALUATE Inserting the expression for L gives

$$d = \frac{L_0}{2} \sqrt{2\alpha\Delta T + \alpha^2\Delta T^2}$$

ASSESS Checking the limits, we see that for $\Delta T \rightarrow 0$ K, $d \rightarrow 0$, as expected.

- 62. INTERPRET** You have to quickly calculate how much ice to buy to keep your parent's refrigerator from warming up during a temporary blackout.

DEVELOP For simplicity, let's imagine that you will be placing bags of ice along the interior walls of the refrigerator, as shown in the figure below. In this way, the heat from the outside room will flow directly into the ice at a rate of $H = \Delta T / R$, where $R = 0.12$ K / W is the thermal resistance of the fridge walls, and $\Delta T = 20^\circ\text{C} - 0^\circ\text{C}$ is the temperature between the room and the ice.



The heat will melt the ice in $t = L_f m / H$, so you want to buy enough ice such that this time equals the 15 hours it will supposedly take for the electric power to resume at your home.

EVALUATE Solving for the required mass of ice gives

$$m = \frac{tH}{L_f} = \frac{t\Delta T}{L_f R} = \frac{(10 \cdot 3600 \text{ s})(20 \text{ K})}{(334 \text{ kJ / kg})(0.12 \text{ K / W})} = 18 \text{ kg}$$

ASSESS You might have some difficulty stuffing 18 1-kg bags of ice in the fridge. Note that we haven't taken into account the fact that the fridge interior will not start out at 0°C , so some extra heat will flow between your food and the ice.

- 63. INTERPRET** This problem involves the temperature change and phase change (solid-liquid) of Glauber salt. Given its thermal parameters, we are to calculate how long the salt takes to cool to 60°F , and how long the salt takes to solidify at 90°F .

DEVELOP In cooling from 95°F to 90°F , the liquid expels a heat $Q_1 = mc_{\text{liq}}\Delta T_{\text{liq}}$ (Equation 16.3). Changing phase at 90°F from liquid to solid expels a further amount $Q_2 = L_f m$ (Equation 17.5). Finally, cooling the solid salt from 90°F to 60°F expels the heat $Q_3 = mc_{\text{sol}}\Delta T_{\text{sol}}$. Thus, the total heat expelled is

$$Q = Q_1 + Q_2 + Q_3 = mc_{\text{liq}}\Delta T_{\text{liq}} + L_f m + mc_{\text{sol}}\Delta T_{\text{sol}}$$

Given that the house loses heat at a rate of $P = 20,000$ Btu/h, the time t it takes to cool to 60°F is $t = Q/P$.

EVALUATE (a) Inserting the given quantities gives

$$\begin{aligned} t &= \frac{mc_{\text{liq}}\Delta T_{\text{liq}} + L_f m + mc_{\text{sol}}\Delta T_{\text{sol}}}{P} \\ &= \left(\frac{5.0 \times 2000 \text{ lbs}}{2.0 \times 10^4 \text{ Btu/h}} \right) \left\{ (5^\circ\text{F}) [0.68 \text{ Btu / (lb} \cdot ^\circ\text{F)}] + 104 \text{ Btu / lb} + [0.46 \text{ Btu / (lb} \cdot ^\circ\text{F)}] (30^\circ\text{F}) \right\} \\ &= 61 \text{ h} \end{aligned}$$

(b) The time spent during just the solidification at 90°F is

$$\frac{mL_f}{P} = \frac{(5.0 \times 2000 \text{ lb})(104 \text{ Btu/lb})}{2.0 \times 10^4 \text{ Btu/h}} = 52 \text{ h}$$

ASSESS Most of the time the salt is at 90° because the latent heat of fusion is much greater than the heat liberated by temperature change.

- 64. INTERPRET** For this problem, we are to prove that the coefficient of volume expansion of an ideal gas at constant pressure is the inverse temperature in the Kelvin scale.

DEVELOP As mentioned in the text following Equation 17.6, the coefficient of volume expansion β is defined in general as

$$\beta = \frac{dV/V}{dT} = \frac{1}{V} \frac{dV}{dT}$$

The ideal gas law $pV = NkT$ is given by Equation 17.1. The two equations can be combined to give the proof.

EVALUATE For an ideal gas at constant pressure, $V = NkT/p$. This gives $dV/dT = Nk/p$. Substituting the equation into the above expression for β gives

$$\beta = \frac{1}{V} \frac{dV}{dT} = \frac{1}{V} \frac{Nk}{p} = \frac{Nk}{NkT} = \frac{1}{T}$$

ASSESS The unit of β is K^{-1} , which is the inverse temperature in the Kelvin scale.

- 65. INTERPRET** For this problem, we are to show at what temperature between 0°C and 20°C water has its greatest density. We are given the expression for the coefficient of volume expansion as a function of temperature.

DEVELOP We do not actually need to differentiate the density or the volume [$\rho(t) = \text{constant mass}/V(T)$] because Equation 17.6 shows that $dV/dT = \beta V = 0$ when $\beta(T) = 0$. Thus, the maximum density (or minimum volume) occurs for a temperature satisfying $a + bT + cT^2 = 0$, which allows us to solve for T .

EVALUATE The quadratic formula gives

$$T = \frac{-b \pm \sqrt{b^2 - 4ac}}{2c}$$

or, since both a and c are negative,

$$T = \frac{b \mp \sqrt{b^2 - 4|a||c|}}{2c}$$

Canceling a factor of 10^{-5} from the given coefficients, we find

$$T = \frac{1.70 \mp \sqrt{(1.70)^2 - 4(6.43)(0.0202)}^\circ\text{C}}{0.0404} = 3.97^\circ\text{C}$$

ASSESS The second root, 80.2°C , can be discarded because it is outside the range of validity, $0 \leq T \leq 20^\circ\text{C}$, of the original function $\beta(T)$. Thus, the maximum density of water occurs at a temperature close to 4°C . That this represents a minimum volume can be verified by plotting $V(T)$, or from the second derivative,

$$\begin{aligned} \frac{d^2V}{dT^2} &= V \frac{d\beta}{dT} + \beta \frac{dV}{dT} = V \left(\beta^2 + \frac{d\beta}{dT} \right) \\ &= V(\beta^2 + b + 2cT) > 0 \end{aligned}$$

for $T = 3.97^\circ\text{C}$.

- 66. INTERPRET** The problem involves finding the volume of water at a given temperature, given its coefficient of volume expansion as a function of temperature.

DEVELOP In Problem 65, we learned that water's coefficient of volume expansion in the temperature range from 0°C to about 20°C is approximately given by $\beta = a + bT + cT^2$, where T is in Celsius and $a = -6.43 \times 10^{-5}^\circ\text{C}^{-1}$, $b = 1.70 \times 10^{-5}^\circ\text{C}^{-2}$, and $c = -2.02 \times 10^{-7}^\circ\text{C}^{-3}$. Thus, the volume as a function of temperature is

$$\frac{dV}{V} = \beta dT = (a + bT + cT^2) dT$$

Integrate this expression and impose the boundary condition that $V(0^\circ\text{C}) = 1.00000\text{ L}$ to find the volume of water at 12°C .

EVALUATE Integrating the expression above gives

$$\int_{V_1}^{V_2} \frac{dV}{V} = \ln\left(\frac{V_2}{V_1}\right) = \int_{T_1}^{T_2} \beta dT = \int_{T_1}^{T_2} (a + bT + cT^2) dT = a(T_2 - T_1) + \frac{1}{2}b(T_2^2 - T_1^2) + \frac{1}{3}c(T_2^3 - T_1^3)$$

For $T_1 = 0^\circ\text{C}$, $T_2 = 20^\circ\text{C}$, and coefficients a , b , and c given above, the right-hand side is 3.36×10^{-4} . With $V_1 = 1.60000\text{ L}$, exponentiation gives

$$V_2 = V_1 e^{0.000336} = 1.60054\text{ L}$$

ASSESS The fraction of volume change is $(V_2 - V_1)/V_1 = 0.00035$.

- 67. INTERPRET** This problem is about thermal expansion of a solid and a liquid object. We will need to consider how the dimensions of each one expand when heated to determine their respective changes in volume.

DEVELOP When heated, both the glass cylinder and the water inside it will experience an expansion of volume. The solid glass will expand in such a way that its linear dimensions expand equally (See GOT IT? 17.3), so we can calculate its coefficient of volume expansion from its coefficient of linear expansion ($\beta = 3\alpha$). Using Equation 17.6 and the coefficients for Pyrex glass and water found in Table 17.2, we can determine the change in volume experienced by both objects.

EVALUATE Substituting the values given in the problem statement, we have

$$\Delta V_g = 3\alpha_g V_g \Delta T = (3)(3.2 \times 10^{-6}\text{ K}^{-1})(50.000\text{ mL})(25\text{ K}) = 0.012\text{ mL}$$

$$\Delta V_w = \beta_w V_w \Delta T = (5.0 \times 10^{-4}\text{ K}^{-1})(50.000\text{ mL})(25\text{ K}) = 0.625\text{ mL}$$

If the cylinder had a high enough resolution, this would put the water at the mark for $(50.625 - 0.012)\text{ mL} = 50.613\text{ mL}$.

ASSESS Since both experience an expansion from an increase in temperature, but both the water level and the measuring line shift by different amounts, the reflected change will depend on the difference in this shift.

- 68. INTERPRET** This problem deals with simple harmonic motion and the thermal expansion of a brass pendulum.

The quantity of interest is the pendulum's length, so the relevant quantity is the coefficient of linear expansion, α .

DEVELOP N swings of a pendulum clock produce a time reading of $t = N\tau$ where $\tau = 2\pi\sqrt{L/g}$ (Equation 13.15) is the period. If the clock is accurate at $T_1 = 20^\circ\text{C}$, at some other temperature T_2 , the length of the pendulum becomes $L_2 = L_1(1 + \alpha \Delta T)$, where $\Delta T = 17^\circ\text{C} - 20^\circ\text{C} = -3\text{ K}$. Therefore, the ratio of the periods is

$$\frac{\tau_2}{\tau_1} = \sqrt{\frac{L_2}{L_1}} = \sqrt{1 + \alpha \Delta T}$$

The error in the clock is the difference in its time-readings for N swings at T_2 , relative to T_1 :

$$\Delta t = t_2 - t_1 = N\tau_2 - N\tau_1 = N\tau_1 \left(\frac{\tau_2}{\tau_1} - 1 \right)$$

Solve this equation to find the time it takes for the clock to err by 1 minute.

EVALUATE Substituting the ratio of the periods into the expression for Δt and solving for t_1 gives

$$\Delta t = t_1 (\sqrt{1 + \alpha \Delta T} - 1)$$

$$t_1 = \frac{\Delta t}{(\sqrt{1 + \alpha \Delta T} - 1)} = \frac{1.0\text{ min}}{\sqrt{1 + (19 \times 10^{-6}\text{ K}^{-1})(-3\text{ K})} - 1} = 24\text{ d}$$

where we used $\alpha = 19 \times 10^{-6}\text{ K}^{-1}$ for brass from Table 17.2. Because $L_2 < L_1$, $\tau_2 < \tau_1$, $\Delta t = t_2 - t_1 < 0$, and the clock at 17°C is fast.

ASSESS The period of a pendulum, $\tau = 2\pi\sqrt{L/g}$, increases with its length L . Due to thermal expansion, the pendulum length at 20°C is greater than that at 17°C , and consequently, its period is longer.

69. INTERPRET We are asked to derive the given equation for the volume expansion coefficient β .

DEVELOP Equation 17.6 gives the volume expansion coefficient as

$$\beta = \frac{1}{V} \frac{\Delta V}{\Delta T}$$

For infinitesimally small changes, this becomes

$$\beta = \frac{1}{V} \frac{dV}{dT}$$

Using the chain rule in the expression for β gives

$$\beta = \frac{1}{V} \frac{dV}{dT} = \frac{1}{L^3} \frac{dV}{dL} \frac{dL}{dT}$$

Given that

$$\frac{dV}{dL} = \frac{d}{dL}(L^3) = 3L^2$$

and, from Equation 17.7,

$$\frac{dL}{dT} = \alpha L$$

we can evaluate the chain rule expression for β in terms of α .

EVALUATE Inserting the expressions above for the derivatives gives volume expansion coefficient as

$$\beta = \frac{1}{L^3} \frac{dV}{dL} \frac{dL}{dT} = \frac{1}{L^3} (3L^2) (\alpha L) = 3\alpha$$

ASSESS Alternatively, use the binomial approximation for $\Delta V = (L + \Delta L)^3 - L^3 = 3L^2 \Delta L$, keeping only the lowest-order term in ΔL . Since $\Delta V = \beta V \Delta T$ and $\Delta L = \alpha L \Delta T$, one finds $3L^2 (\alpha L \Delta T) = \beta L^3 \Delta T$, or $\beta = 3\alpha$.

70. INTERPRET This problem involves pressure, temperature, and volume. We will assume that the ideal gas law applies and use it to find the volume of 1 mole under the given conditions.

DEVELOP In terms of moles, the ideal gas law (Equation 17.2) is $pV = nRT$ where $R = 8.314 \text{ J/(K} \cdot \text{mol)}$. The pressure is $p = 90 \text{ atm}$, the temperature is $T = 730 \text{ K}$, and we are interested in the volume of 1 mole, so $n = 1.0$. We want to find the volume of 1 mole of gas, and see if this volume is less than 1 L.

EVALUATE Inserting the given quantities into the ideal gas law gives

$$pV = nRT$$

$$V = \frac{nRT}{p} = \frac{(1.0 \text{ mol})[8.314 \text{ J/(K} \cdot \text{mol)}](730 \text{ K})}{(90 \text{ atm})(1.01 \times 10^5 \text{ Pa/atm})} = 6.7 \times 10^{-4} \text{ m}^3 = 0.67 \text{ L}$$

ASSESS This design will work, as 1 liter contains more than a mole at this pressure and temperature.

71. INTERPRET This problem involves the determination of the linear expansion coefficient of a metal wire as it is heated. We analyze the data to see how the vertical distance between the hanging mass and the top varies with temperature.

DEVELOP Pythagorean theorem gives

$$(L/2)^2 - (d/2)^2 + y^2 \Rightarrow y^2 = \frac{1}{4}(L^2 - d^2)$$

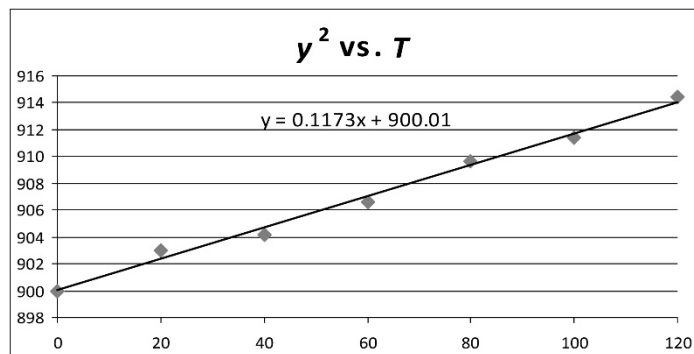
where $L = L_0(1 + \alpha \Delta T)$ is the total expanded length of the wire.

EVALUATE (a) Inserting the expression for L gives

$$y^2 = \frac{1}{4}L_0^2(1 + \alpha \Delta T)^2 - \frac{d^2}{4} \approx \frac{1}{4}L_0^2(1 + 2\alpha \Delta T) - \frac{d^2}{4} = \frac{1}{4}(L_0^2 - d^2) + \frac{1}{2}L_0^2\alpha \Delta T$$

where we drop terms involving α^2 . The expression is of the form $y^2 = a + b(\Delta T)$, so plotting y^2 as a function of ΔT will allow us to determine the expansion coefficient α .

(b) The plot of y^2 versus T is shown below.



The slope is $b = \frac{1}{2}L_0^2\alpha = 0.1173$, from which we find α to be

$$\alpha = \frac{2(0.1173 \text{ cm}^2/\text{C}^\circ)}{L_0^2} = \frac{2(0.1173 \text{ cm}^2/\text{C}^\circ)}{(100 \text{ cm})^2} = 2.35 \times 10^{-5} \text{ C}^{-1}$$

Similarly, the condition $a = \frac{1}{4}(L_0^2 - d^2) = 900.01$ leads to $d = 80.0 \text{ cm}$.

(c) Comparing with Table 17.2, we identify the material to be aluminum.

ASSESS The simple apparatus is very useful for determining the linear expansion coefficient of a metal wire. However, we ignore any stretching due to its “springiness.”

- 72. INTERPRET** This problem involves exploring the impact of global warming on Greenland, and its energy imbalance.

DEVELOP The energy required for a solid-liquid phase transition at the normal melting point of water (0°C) is (Equation 17.5) $Q = mL_f$, where m is the mass of ice and $L_f = 334 \text{ kJ/kg}$ (from Table 17.1).

EVALUATE (a) The heat required to melt 250 Pg ($1 \text{ Pg} = 10^{15} \text{ g} = 10^{12} \text{ kg}$) of ice is

$$Q = mL_f = (250 \times 10^{12} \text{ kg})(334 \text{ kJ/kg}) = 8.35 \times 10^{19} \text{ J} \approx 8.4 \times 10^{19} \text{ J}$$

(b) The rate of energy required to melt the ice is

$$P = \frac{dQ}{dt} = \frac{8.35 \times 10^{19} \text{ J}}{(365.25 \text{ d/year})(86400 \text{ s/d})} = 2.65 \times 10^{12} \text{ W}$$

The area of Greenland covered with ice is approximately $A_{\text{ice}} = 1.71 \times 10^{12} \text{ m}^2$. Therefore, the rate of absorption per unit area covered with ice is

$$S = \frac{P}{A_{\text{ice}}} = \frac{2.65 \times 10^{12} \text{ W}}{1.71 \times 10^{12} \text{ m}^2} = 1.5 \text{ W/m}^2$$

ASSESS The result is larger than the global imbalance (somewhat less than 1 W/m^2) and shows that the impact of global warming is greater in the Arctic.

- 73. INTERPRET** In this problem we derive an expression that relates the speed of sound to temperature.

DEVELOP The speed of sound in a medium is given by Equation 14.9:

$$v_{\text{sound}} = \sqrt{\frac{\gamma P}{\rho}}$$

where γ is a constant characteristic of the gas, P is the pressure, and ρ is the density of the gas. From the ideal gas law, $pV = NkT$ (Equation 17.1), we can write

$$\frac{P}{\rho} = \frac{P}{M/V} = \frac{PV}{M} = \frac{NkT}{M} = \frac{kT}{M/N} = \frac{kT}{m}$$

where M is the mass of the total gas sample, and m is the mass of one molecule.

EVALUATE (a) Combining the two expressions, we obtain the desired result:

$$v_{\text{sound}} = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma kT}{m}}$$

(b) The thermal speed of a molecule is given by Equation 17.4:

$$v_{\text{th}} = \sqrt{\frac{3kT}{m}}$$

The ratio of the speed of sound to the thermal speed is

$$\frac{v_{\text{sound}}}{v_{\text{th}}} = \frac{\sqrt{\gamma kT / m}}{\sqrt{3kT / m}} = \sqrt{\frac{\gamma}{3}}$$

For diatomic molecules where $\gamma = 7/5$, the ratio is

$$\frac{v_{\text{sound}}}{v_{\text{th}}} = \sqrt{\frac{7/5}{3}} = \sqrt{\frac{7}{15}} = 0.683$$

or about 68%.

ASSESS For monatomic molecules where $\gamma = 5/3$, the ratio is

$$\frac{v_{\text{sound}}}{v_{\text{th}}} = \sqrt{\frac{5/3}{3}} = \sqrt{\frac{5}{9}} = 0.745$$

or about 75%.

- 74. INTERPRET** In this problem we are given the Maxwell-Boltzmann distribution function, and asked to derive the most probable speed, which corresponds to the peak of the curve.

DEVELOP The Maxwell-Boltzmann distribution is

$$N(v)\Delta v = 4\pi N \left(\frac{m}{2\pi kT} \right)^{3/2} v^2 e^{-mv^2/2kT} \Delta v$$

The most probable speed v_p is the speed that occurs most often, and this corresponds to the maximum of $N(v)$. The condition $dN(v)/dv = 0$ allows us determine v_p .

EVALUATE Differentiating $N(v)$ with respect to v gives

$$\frac{dN(v)}{dv} = 4\pi N \left(\frac{m}{2\pi kT} \right)^{3/2} e^{-mv^2/2kT} v \left(2 - \frac{mv^2}{kT} \right) = 0$$

Thus, the most probable speed is

$$2 - \frac{mv_p^2}{kT} = 0 \Rightarrow v_p = \sqrt{\frac{2kT}{m}}$$

ASSESS The ratio of the most probable speed to the thermal speed is

$$\frac{v_p}{v_{\text{th}}} = \frac{\sqrt{2kT / m}}{\sqrt{3kT / m}} = \sqrt{\frac{2}{3}} = 0.816$$

The thermal speed (Equation 17.4), which is the average molecular speed, is a factor of $\sqrt{3/2}$, or about 20% greater than the most probable speed—a fact that reflects the long, high-energy “tail” of the Maxwell-Boltzmann distribution.

- 75. INTERPRET** In this problem we compare the temperature predicted by the ideal gas law, $pV = nRT$, to that predicted by van der Waals equation.

DEVELOP At high gas densities, the van der Waals equation modifies the ideal gas law to account for nonzero molecular volume. The equation reads

$$\left(p + \frac{n^2 a}{V^2} \right) (V - nb) = nRT$$

where $a = 0.14 \text{ Pa} \cdot \text{m}^6/\text{mol}^2 = 1.382 \text{ atm} \cdot \text{L}^2/\text{mol}^2$, and $b = 3.91 \times 10^{-5} \text{ m}^3/\text{mol} = 0.0391 \text{ L/mol}$. The conversion facilitates the calculation of the temperature for $n = 1 \text{ mol}$, $p = 10 \text{ atm}$, and $V = 2.0 \text{ L}$.

EVALUATE (a) Using ideal gas law, we find the temperature to be

$$T = \frac{pV}{nR} = \frac{(1.0 \text{ atm})(2.0 \text{ L})}{(1.0 \text{ mol})(0.0821 \text{ L} \cdot \text{atm/mol})} = 244 \text{ K}$$

(b) From van der Waals equation, we have

$$T = \frac{1}{nR} \left(p + \frac{n^2 a}{V^2} \right) (V - nb) = \frac{1}{(1.0)(0.0821)} \left(1.0 + \frac{(1.0)^2 (1.382)}{(2)^2} \right) (2 - 0.0391) = 247 \text{ K}$$

ASSESS The van der Waals equation predicts a slightly higher gas temperature. Note that at low densities, $a/(V/n)^2 \ll p$, and $b \ll V/n$, van der Waals equation reduces to the ideal gas equation.

76. INTERPRET We consider what goes on inside a pressure cooker.

DEVELOP The line connecting the triple point to the critical point in Fig. 17.9 (the liquid-gas boundary) designates all of the situations where the combination of pressure and temperature is right for water to boil.

EVALUATE If the pressure is higher than normal in a pressure cooker, then the temperature at which water boils will be higher as well. By definition, this combination of elevated pressure and temperature must lie on the line connecting the triple point to the critical point.

The answer is (b).

ASSESS If we started with boiling water at normal atmospheric pressure, and suddenly increased the pressure, the water would stop boiling. This corresponds to moving vertically upward from the liquid-gas boundary in the phase diagram of Fig. 17.9. For the water to start boiling again, the temperature will have to increase until the (p, T) combination is again located on the liquid-gas boundary.

77. INTERPRET We consider what goes on inside a pressure cooker.

DEVELOP We can use the ideal gas law to compare the density of steam under atmospheric pressure to the density of steam in a pressure cooker.

EVALUATE The density of steam, ρ , is equal to the mass of an individual water molecule, m , multiplied by the number density of water molecules in the vapor state: $\rho = mN/V$. The number density is related to the temperature and pressure by the ideal gas law: $N/V = p/kT$. Therefore, the ratio of the steam density in a pressure cooker to normal pressure is

$$\frac{\rho_{\text{pc}}}{\rho_{\text{norm}}} = \frac{p_{\text{pc}}}{p_{\text{norm}}} \frac{T_{\text{norm}}}{T_{\text{pc}}} = \frac{(2 \text{ atm})(373 \text{ K})}{(1 \text{ atm})(393 \text{ K})} = 1.9$$

The answer is (c).

ASSESS The increased gas density is due to the fact that more of the water molecules have left the liquid to join the vapor.

78. INTERPRET We consider what goes on inside a pressure cooker.

DEVELOP From the information given we can estimate the slope of the line marking the liquid-gas boundary.

EVALUATE Between 1 and 2 atm, the average slope of the line is $\Delta p / \Delta T = 1/20 \text{ atm/K}$, whereas between 2 and 3 atm, the average slope is $\Delta p / \Delta T = 1/14 \text{ atm/K}$. Since the slope is getting larger as we move to the right on the graph, the line should be concave upward.

The answer is (a).

ASSESS If the boundary line were a straight line, that is, $p = \alpha T$ for some constant α , this would imply that the gas density remains constant as the temperature and pressure increase (see the previous problem). In fact, the gas density increases, reflecting a higher number of water molecules in the gas phase at higher pressures.

79. INTERPRET We consider what goes on inside a pressure cooker.

DEVELOP As the water boils, more and more water molecules go from the liquid to the gaseous state. If the release mechanism is clogged, those additional water molecules will have nowhere to go, so the gas density will rise and pressure in the cooker will rise. If more and more steam is created, then the density inside the cooker will rise. As we showed in Problem 17.72, the density is proportional to the pressure, so the pressure will rise. If the pressure goes up, then the boiling temperature will also go up.

EVALUATE As we showed in Problem 17.72, the gas density is proportional to the pressure, so a clogged mechanism will lead to an increase in pressure, as we'd expect. Now, if the temperature somehow remained constant, the system would no longer be at a boiling point in the phase diagram. However, since heat is constantly being supplied through the burner, the water temperature would eventually have to rise until it reached a new boiling point specified by the elevated pressure. Therefore, both the pressure and temperature would continue to rise.

The answer is (c).

ASSESS This increase in pressure can't last forever. Eventually something will blow. This is why pressure cookers have a reputation for being dangerous. Modern designs incorporate multiple safety valves to deal with the risk of clogging.