16

TEMPERATURE AND HEAT

EXERCISES

Section 16.1 Heat, Temperature, and Thermodynamic Equilibrium

11. **INTERPRET** This problem involves converting a temperature range from the Celsius scale to the Fahrenheit scale.
DEVELOP We are interested in converting an amount of degrees from one temperature scale to another. Since we are not interested in what theses quantities are relative to a particular temperature (e.g., melting point of water) we only need to multiply by the conversion factor found in Equation 16.2: $T_\mathrm{F} = \frac{9}{5}T_\mathrm{C} + 32$, to find the temperature range ΔT_F in Fahrenheit.
EVALUATE Inserting ΔT_C = 3.0°C to 4.2°C gives

$$\Delta T_\mathrm{F} = \frac{9}{5}(3°\mathrm{C}) = 5.4°\mathrm{F} \text{ to } \Delta T_\mathrm{F} = \frac{9}{5}(4.2°\mathrm{C}) = 7.6°\mathrm{F}$$

ASSESS When converting temperature ranges between scales we omit the addition/subtraction factor since we are interested in definite temperature amounts, not relative ones.

12. **INTERPRET** This problem involves converting temperature from the Fahrenheit scale to the Celsius scale.
DEVELOP We assume that the Mexican meteorologist predicts the same temperature, but expresses it on the Celsius scale. Therefore, apply Equation 16.2:

$$T_\mathrm{F} = \frac{9}{5}T_\mathrm{C} + 32$$

EVALUATE Inserting $T_\mathrm{F} = 14^\circ\mathrm{F}$ and solving the above equation for the Celsius temperature, we obtain

$$T_\mathrm{C} = \frac{5}{9}(T_\mathrm{F} - 32) = \frac{5}{9}(14^\circ\mathrm{F} - 32^\circ\mathrm{F}) = -10^\circ\mathrm{C}$$

ASSESS This is a reasonable temperature for a cold winter night.

13. **INTERPRET** This problem involves converting temperature from the Fahrenheit scale to the Celsius scale.
DEVELOP The two temperature scales are related by Equation 16.2:

$$T_\mathrm{F} = \frac{9}{5}T_\mathrm{C} + 32$$

EVALUATE Inserting T_F = 75°F and solving the above equation for the Celsius temperature, we obtain

$$T_\mathrm{C} = \frac{5}{9}(T_\mathrm{F} - 32) = \frac{5}{9}(75°\mathrm{F} - 32°\mathrm{F}) = 24°\mathrm{C}$$

ASSESS This is a useful result to remember since 24°C or 75°F is a typical room temperature.

14. **INTERPRET** This problem involves converting a temperature difference from the Fahrenheit scale to the Celsius scale.
DEVELOP Apply Equation 16.2 at the two different (and arbitrary) temperatures, then take the difference:

$$T_{F,2} = \frac{9}{5}T_{C,2} + 32$$

$$T_{F,1} = \frac{9}{5}T_{C,1} + 32$$

$$T_{F,2} - T_{F,1} \equiv \Delta T_F = \frac{9}{5}\Delta T_C$$

EVALUATE $\Delta T_F = 27^\circ\text{F}$ gives $\Delta T_C = (5/9)(27^\circ\text{F}) = 15^\circ\text{C}$.

ASSESS Note that a temperature difference and a temperature reading are not the same, even though both are specified in the same units.

15. **INTERPRET** Given both Fahrenheit and Celsius scales, we want to know when T_F and T_C are numerically equivalent.

DEVELOP The two temperature scales are related by Equation 16.2:

$$T_F = \frac{9}{5}T_C + 32$$

The condition that the readings are numerically equivalent is

$$T_F = \frac{9}{5}T_C + 32 = T_C$$

EVALUATE The above equation can be solved to give

$$T_C = -\frac{5}{4}(32) = -40 = T_F$$

ASSESS This is the only temperature in which both scales yield the same reading: $-40°\text{F} = -40°\text{C}$.

16. **INTERPRET** This problem involves converting temperature from Kelvin to Celsius, and then converting from Celsius to Fahrenheit.

DEVELOP To convert from Kelvin to Celsius, apply Equation 16.1 $T_C = T - 273.15$. To convert Celsius to Fahrenheit, use Equation 16.2, $T_F = (9/5)T_C + 32$.

EVALUATE The temperature 77.3 K in degrees Celsius is $T_C = 77.3 - 273.15 = -196°\text{C}$, which in Fahrenheit is $T_F = (9/5)(-196) + 32 = -321°\text{F}$.

ASSESS As a benchmark, it can be useful to know that liquid nitrogen is at approximately −200°C.

17. **INTERPRET** This problem involves converting temperature from the Celsius scale to the Fahrenheit scale.

DEVELOP The two temperature scales are related by Equation 16.2: $T_F = (9/5)T_C + 32$.

EVALUATE Solving the above equation for the Fahrenheit temperature, we obtain

$$T_F = \frac{9}{5}(39.5) + 32 = 103°\text{F}$$

ASSESS The temperature is way above the normal body temperature of 98.6°F (or 37°C). Call the doctor immediately!

Section 16.2 Heat Capacity and Specific Heat

18. **INTERPRET** We find the heat capacity of a large concrete block. We know the mass of the block and its specific heat.

DEVELOP The specific heat of concrete is given in Table 16.1 as $c = 880\ \text{J/kg}\cdot\text{K}$. To find the heat capacity, we multiply this specific heat by the mass (recalling that $1\ \text{ton} = 1000\ \text{kg}$).

EVALUATE The heat capacity of the block is

$$C = mc = (55{,}000\ \text{kg})(880\ \text{J/kg}\cdot\text{K}) = 4.8\times10^7\ \text{J/K}$$

ASSESS This is a large value, but then it takes a large amount of heat to change the temperature of a 55-ton block of concrete.

19. **Interpret** We are to find the energy necessary to change the temperature of an object by a given amount. This involves the heat capacity of the object and the temperature change.

Develop Apply Equation 16.3, $Q = mc\Delta T$. The mass of the aluminum block is $m = 2.2$ kg, the specific heat (from Table 16.1) is $c = 900\ \text{J}/(\text{kg}\cdot\text{K})$, and the temperature change is $\Delta T = 18°\text{C} = 18\ \text{K}$ (see Equation 16.1).

Evaluate Inserting the given quantities gives

$$Q = (2.2\ \text{kg})\left[90\times10^{1}\ \text{J}/(\text{kg}\cdot\text{K})\right](18\ \text{K}) = 36\ \text{kJ}$$

Assess The heat released by the aluminum if it cooled by 18°C would have the same value.

20. **Interpret** Given information about heat, mass, and temperature change of a material, we are asked to find the specific heat of the material.

Develop Apply Equation 16.3 $Q = mc\Delta T$. The mass of the object is m = 1.0 kg, the heat required is Q =7500 J, and the temperature change is ΔT =3.0°C = 3.0 K (see Equation 16.1), so we can solve for the specific heat c.

Evaluate Inserting the given quantities gives

$$Q = mc\Delta T$$

$$c = \frac{Q}{m\Delta T} = \frac{7500\ \text{J}}{(1.0\ \text{kg})(3.0\ \text{K})} = 2500\ \text{J}/(\text{kg}\cdot\text{K})$$

Assess This is a very large value for c; higher than for most solids.

21. **Interpret** The problem involves calculating the average power output of the human body, given the information about the energy acquired in a day from an average diet. Recall that power is energy per unit time.

Develop In a single day, the energy gained from the diet is

$$\Delta E = \left(2\times10^{6}\ \text{cal}\right)(4.184\ \text{J/cal}) = 8.37\times10^{6}\ \text{J}$$

where we have used the conversion factor 1 cal = 4.184 J (see Appendix C). If the body expends all this energy (and does not store any of it), then the energy expended must be this same value (by conservation of energy). Therefore, the average power output of the body is $\bar{P} = \Delta E/\Delta t$, where Δt = (1 day)(86,400 s/day) = 86,400 s.

Evaluate The average power output is

$$\bar{P} = \frac{\Delta E}{\Delta t} = \frac{8.37\times10^{6}\ \text{J}}{86,400\ \text{s}} = 96.9\ \text{W} = 100\ \text{W}$$

to a single significant figure.

Assess The average power output by the human body at rest is about 80 W, the same as a bright light bulb, so this result seems reasonable.

22. **Interpret** We want to know how far one has to walk to expend the energy contained in a slice of chocolate cake.

Develop Energy expended is the power times the time, while the time required is the distance divided by the speed. Therefore, $E = P\cdot d/v$. Recall that $1\ \text{kcal} = 4184\ \text{J}$.

Evaluate The distance required to burn off $390\ \text{kcal}$ is

$$d = \frac{Ev}{P} = \frac{(390\ \text{kcal})(3\ \text{km/h})}{200\ \text{W}}\left[\frac{4184\ \text{J}}{1\ \text{kcal}}\right]\left[\frac{1\ \text{h}}{3600\ \text{s}}\right] = 6.8\ \text{km}$$

Assess This seems like a reasonable amount of exercise for burning off a slice of chocolate cake.

23. **Interpret** We are asked to find the heat (i.e., thermal energy) required to change the temperature of an object, which we can solve using the specific heat and the mass of the object. We are also to find the time taken to heat this object with the given power input.

Develop The heat required to change the temperature of the skillet by the ΔT = 110°C = 110 K is $Q = mc\Delta T$. The mass of the skillet is m = 3.4 kg and the specific heat of iron is given in Table 16.1 as $c = 447\ \text{J}/(\text{kg}\cdot\text{K})$. To find the time to heat the pan for part **(b)**, recall that average power is the energy per unit time, or $\bar{P} = Q/\Delta t$, which we can solve given Q and $\bar{P} = 2.0\ \text{kW}$.

Evaluate (**a**) Inserting the given quantities into Equation 16.3 gives

$$Q = mc\Delta T = (3.4\text{ kg})[447\text{ J}/(\text{kg}\cdot K)](110\text{ K}) = 1.7\times10^5\text{ J}$$

(b) The time interval Δt required to deliver this amount of thermal energy is

$$\Delta t = \frac{Q}{\bar{P}} = \frac{1.67\times10^5\text{ J}}{2.0\times10^3\text{ W}} = 84\text{ s}$$

ASSESS This is a reasonable time to heat a small skillet.

Section 16.3 Heat Transfer

24. **INTERPRET** This problem is about converting heat loss expressed in Btu/h to SI units.

DEVELOP One Btu (British thermal unit) is equal to 1054 J (see Appendix C), which is the amount of heat that is needed to raise the temperature of 1 lb of water from 63°F to 64°F.

EVALUATE The conversion to SI units is

$$1.00\left(\frac{\text{Btu}}{\text{h}}\right) = \left(\frac{1.00\text{ Btu}}{\text{h}}\right)\overbrace{\left(\frac{1054\text{ J}}{\text{Btu}}\right)}^{=1}\overbrace{\left(\frac{1.00\text{ h}}{3.60\times10^3\text{ s}}\right)}^{=1} = 0.293\text{ W}$$

ASSESS Our result shows that 1 W is about 3.4 Btu/h. The power output of air conditioners is commonly given in terms of Btu/h.

25. **INTERPRET** We're asked to compare the heat-loss rate through equal slabs of wood and Styrofoam.

DEVELOP The rate of heat loss from conduction is given by Equation 16.5, $H = -kA\Delta T/\Delta x$. The values of thermal conductivity, k, come from Table 16.2: pine wood $k = 0.11\text{ W/m}\cdot\text{K}$; Styrofoam $k = 0.029\text{ W/m}\cdot\text{K}$. The temperature difference is $\Delta T = 25^\circ\text{C} = 25\text{ K}$. Since we're not given the area of the slabs, we'll write the answers as heat loss per unit area. We are interested in the magnitude of the heat-loss rate, so we will express the absolute value of this quantity.

EVALUATE **(a)** The heat loss through the wood slab is

$$|H/A| = (0.11\text{ W/m}\cdot\text{K})\frac{25\text{ K}}{0.050\text{ m}} = 55\text{ W/m}^2$$

(b) The heat loss through the Styrofoam slab is

$$|H/A| = (0.029\text{ W/m}\cdot\text{K})\frac{25\text{ K}}{0.050\text{ m}} = 14.5\text{ W/m}^2$$

ASSESS The Styrofoam is a better insulator, since it lets less heat escape.

26. **INTERPRET** You want to convince a client that Styrofoam is a very effective insulator.

DEVELOP You decide to compare the insulation of a 2-inch Styrofoam to that of a concrete wall. Since the area and temperature difference will be the same in both cases, you only need to consider the thermal resistance per unit area, or $\mathcal{R}$-factor of each wall, as defined in Equation 16.8: $\mathcal{R} = \Delta x/k$. From Table 16.2 the thermal conductivity of Styrofoam and concrete are, respectively, $k_\text{S} = 0.029\text{ W/m}\cdot\text{K}$, and $k_\text{c} = 1\text{ W/m}\cdot\text{K}$.

EVALUATE In order for a concrete wall to have the same $\mathcal{R}$-factor as 2-inch Styrofoam, its thickness must be:

$$\Delta x_\text{c} = \frac{k_\text{c}}{k_\text{S}}\Delta x_\text{S} = \frac{1\text{ W/m}\cdot\text{K}}{0.029\text{ W/m}\cdot\text{K}}(2\text{ in}) = 69\text{ in}$$

ASSESS One of the reasons Styrofoam is such a good insulator is that it is full of little air pockets, which have a very low heat conductivity: $k_\text{air} = 0.026\text{ W/m}\cdot\text{K}$.

27. **INTERPRET** This problem involves calculating the rate of heat conduction through the concrete slab, given the temperature difference between the two sides of the slab and the dimensions of the slab.

DEVELOP Take the downward direction as the positive x-direction. We assume a steady flow of heat through the area $A = 4.5\text{ m}\times17\text{ m} = 76.5\text{ m}^2$, with no flow through the edges. The rate of heat flow is given by Equation 16.5:

$$H = -kA\frac{\Delta T}{\Delta x}$$

The temperature difference is $\Delta T = T_{\text{outside}} - T_{\text{inside}} = 20°\text{C} - 30°\text{C} = -10°\text{C} = -10\text{ K}$ (see Equation 16.1) and $\Delta x = x_{\text{outside}} - x_{\text{inside}} = 0.17\text{ m}$.

EVALUATE From Table 16.2, we find the thermal conductivity of concrete to be $k = 1\text{ W}/(\text{m}\cdot\text{K})$. Thus, the rate of heat conduction is

$$H_{\text{floor}} = -kA\frac{\Delta T}{\Delta x} = -\left[1\text{ W}/(\text{m}\cdot\text{K})\right]\left(76.5\text{ m}^2\right)\frac{(-10\text{ K})}{0.17\text{ m}} = 5\text{ kW}$$

which is reported to a single significant figure because the thermal conductivity of concrete is given to one significant figure.

ASSESS The energy loss through the floor by conduction is substantial. That's why carpeting can prevent heat loss and keeps the house warm during the winter season.

28. **INTERPRET** For this problem, we are to find the thermal resistance per unit area (the $\mathcal{R}$ -factor) of a wall given the temperature difference and the rate of heat flow.

DEVELOP Equation 16.5 for the rate of heat flow per square foot through a slab, written in terms of the thermal resistance of the slab (Equation 16.6), is $H = -\Delta T/R$. Dividing each side by the area A and using Equation 16.8 ($\mathcal{R} = RA$) gives

$$\frac{H}{A} = -\frac{\Delta T}{RA} = -\frac{\Delta T}{\mathcal{R}}$$
$$\mathcal{R} = -\frac{\Delta T}{H/A}$$

Given that the heat flow per square meter is

$$\frac{H}{A} = \left(\frac{0.040\text{ Btu}}{\text{h}\cdot\text{ft}^2}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right)\left(\frac{1\text{ ft}}{0.3048\text{ m}}\right)^2\left(\frac{1054\text{ J}}{\text{Btu}}\right) = 0.1261\text{ W/m}^2$$

and

$$\Delta T = -1°\text{F}\left(\frac{5}{9}\right) = -\frac{5}{9}°\text{C} = -\frac{5}{9}\text{ K}$$

where we have used $\Delta T_F = 5\Delta T_C/9$ (see Problem 16.16), we can calculate $\mathcal{R}$.

EVALUATE The $\mathcal{R}$ -factor is

$$\mathcal{R} = -\frac{\Delta T}{H/A} = \frac{5\text{ K}}{9\left(0.1261\text{ W/m}^2\right)} = 4.4\text{ m}^2\cdot\text{K/W}$$

ASSESS There temperature difference is negative because the heat flow is in the direction of decreasing temperature.

29. **INTERPRET** This problem is an exercise in calculating the $\mathcal{R}$-factors for various materials of 1-inch thickness.

DEVELOP The $\mathcal{R}$-factor of a material is given by Equation 16.8:

$$\mathcal{R} = RA = \frac{\Delta x}{k}$$

where R is the thermal resistance and k is the thermal conductivity of a material having a thickness Δx. We will calculate the $\mathcal{R}$-factors in SI units, using Δx = 1 in = 25.4 mm = 0.0254 m.

EVALUATE Using Table 16.2, with $k_{\text{air}} = 0.026\text{ W}/(\text{m}\cdot\text{K})$ for air, we have

$$\mathcal{R}_{\text{air}} = \frac{0.0254\text{ m}}{0.026\text{ W}/(\text{m}\cdot\text{K})} = 0.98\text{ m}^2\cdot\text{K/W}$$

Similarly, with $k_{\text{concrete}} = 1$, $k_{\text{fiberglass}} = 0.042$ $\bar{k}_{\text{glass}} = 0.8$, $k_{\text{Styrofoam}} = 0.029$ and $k_{\text{pine}} = 0.11$ [all in units of W/(m·K)], the $\mathcal{R}$-factors are

$$\mathcal{R}_{\text{concrete}} = \frac{0.0254\text{ m}}{1\text{ W}/(\text{m}\cdot\text{K})} = 0.03\text{ m}^2\cdot\text{K/W}$$

$$\mathcal{R}_{\text{fiberglass}} = \frac{0.0254\ \text{m}}{0.042\ \text{W}/(\text{m}\cdot\text{K})} = 0.60\ \text{m}^2\cdot\text{K/W}$$

$$\mathcal{R}_{\text{glass}} = \frac{0.0254\ \text{m}}{0.8\ \text{W}/(\text{m}\cdot\text{K})} = 0.03\ \text{m}^2\cdot\text{K/W}$$

$$\mathcal{R}_{\text{Styrofoam}} = \frac{0.0254\ \text{m}}{0.029\ \text{W}/(\text{m}\cdot\text{K})} = 0.88\ \text{m}^2\cdot\text{K/W}$$

$$\mathcal{R}_{\text{pine}} = \frac{0.0254\ \text{m}}{0.11\ \text{W}/(\text{m}\cdot\text{K})} = 0.23\ \text{m}^2\cdot\text{K/W}$$

ASSESS The $\mathcal{R}$-factor of a material is inversely proportional to the thermal conductivity. Good thermal insulators such as Styrofoam or wood have large $\mathcal{R}$-factors.

30. **INTERPRET** Given an object's surface area and temperature, we are to find the rate of radiative heat loss. We will use the Stefan-Boltzmann law.

DEVELOP The Stefan-Boltzmann law (Equation 16.9) for radiative power is $P = e\sigma AT^4$, where the Stefan-Boltzmann constant is $\sigma = 5.67\times10^{-8}\ \text{W}/\left(\text{m}^2\cdot\text{K}^4\right)$. The temperature of the horseshoe is $T = 880°\text{C} = 1153\ \text{K}$ (see Equation 16.1), and the area is $A = 50\ \text{cm}^2 = 5.0\times10^{-3}\ \text{m}^2$. We do not know the emissivity e, so we will approximate it by $e = 1.0$.

EVALUATE Inserting the given quantities into the Stefan-Boltzmann law gives

$$P = e\sigma AT^4 = (1.0)\left[5.67\times10^{-8}\ \text{W}/\left(\text{m}^2\cdot\text{K}^4\right)\right]\left(5.0\times10^{-3}\ \text{m}^2\right)(1153\ \text{K})^4 = 501\ \text{W}$$

ASSESS This is equivalent to the wattage of an outdoor floodlight. The radiative heat must be less than this because $e \le 1$, and the heat loss would decrease linearly with e.

Section 16.4

31. **INTERPRET** This is an energy-balance problem involving a stove. We are given the energy loss per unit time per degree temperature difference, and the temperature difference. Note that we are not given the heat-loss mechanism(s), although we can assume it is primarily convection and radiation. We wish to find the rate of energy loss, which, by energy balance, must be the power required to maintain the temperature.

DEVELOP The thermal energy leaving the oven is $H_T\Delta T$, which must be balanced by the power P supplied to the oven in order to maintain thermal-energy balance. We multiply the energy-loss rate per degree by the temperature difference in degrees. We can, therefore, write

$$P = H_T\Delta T$$

EVALUATE Inserting $H_T = 16\ \text{W}/°\text{C}$ and $\Delta T = (240°\text{C} - 18°\text{C}) = 222°\text{C}$ gives

$$P = (16\ \text{W}/°\text{C})(222°\text{C}) = 3.6\ \text{kW}$$

ASSESS 3.6 kW is a reasonable power requirement for an oven.

32. **INTERPRET** This is an energy balance problem. You know the rate of energy loss per degree of temperature difference between the inside and the outside of your house. You just need to calculate what the maximum loss would be for the coldest winter days, and compare that to the power supplied by the heating system.

DEVELOP The coldest temperature difference will be $\Delta T = 20°\text{C} - (-15°\text{C}) = 35°\text{C}$.

EVALUATE The heat-loss rate in your house on the coldest days is

$$H = (1.3\ \text{kW}/°\text{C})(35°\text{C}) = 45.5\ \text{kW}$$

No, you should not buy the 40kW heating system, since you need 5.5 kW more power.

ASSESS Instead of buying a more powerful heating system, it is often cost-effective to add more insulation to reduce the heat-loss rate.

33. **INTERPRET** This problem involves radiative heat loss and the Stefan-Boltzmann law.

DEVELOP Apply the Stefan-Boltzmann law, Equation 16.9, which is $P = e\sigma AT^4$. The power is P = 100 W, the temperature is T = 3000 K, and σ = 5.67 × 10^{-8} W/(m^2·K^4), so we can solve for the area A. We will assume that the emissivity is $e \approx 1$.

EVALUATE Inserting the given quantities into the Stefan-Boltzmann law gives

$$P = e\sigma AT^4$$
$$A = \frac{P}{e\sigma T^4} = \frac{100\ \text{W}}{(1)\left[5.67\times10^{-8}\ \text{W}/\left(\text{m}^2\cdot\text{K}^4\right)\right](3000\ \text{K})^4} = 2\times10^{-5}\ \text{m}^2$$

ASSESS This is about 20 square millimeters, which seems reasonable for the total area of a light bulb filament.

34. **INTERPRET** We want to find the net radiation from a human body.

DEVELOP The Stefan-Boltzmann law tells us the rate of radiation emitted by an object of area A and temperature $T: P = e\sigma AT^4$. But this is also the rate at which radiation is absorbed by this object from its surroundings at ambient temperature, T_{amb}. These temperatures have to be expressed in Kelvin, so

$T = 33^\circ\text{C} + 273 = 306\ \text{K}$, and $T_{\text{amb}} = 20^\circ\text{C} + 273 = 293\ \text{K}$.

EVALUATE Since the power emitted is a loss of heat, we'll treat it as negative, whereas power absorbed is positive. We're told to treat the body as a perfect emitter/absorber $(e = 1)$, so the body's net radiation transfer is

$$\begin{aligned}P_{\text{net}} &= e\sigma A(T_{\text{amb}}^4 - T^4)\\ &= (1)(5.67\times10^{-8}\ \text{W}/\text{m}^2\cdot\text{K}^4)(1.4\ \text{m}^2)\left[(293\ \text{K})^4 - (306\ \text{K})^4\right] = -110\ \text{W}\end{aligned}$$

ASSESS Over the course of a day, this radiation loss corresponds to about 2300 kilocalories. This is an overestimate, since wearing clothes will affect the energy balance by keeping a warm air buffer next to the body.

EXAMPLE VARIATIONS

35. **INTERPRET** This is a problem involving two masses coming together to reach an equilibrium temperature.

DEVELOP Our system consists of two masses: iron and water. These are initially at different temperatures, and when brought together, they reach thermal equilibrium. We want to find the equilibrium temperature achieved for the given initial temperatures of the two. Since all the heat lost by the hot pan is gained by the cold water, with no heat transfer to the container or its surroundings, we know: $-Q_{\text{Ir}} = Q_{\text{Wa}}$. Expressing each side of this equation using Equation 16.3, we find

$$-m_{\text{Ir}}c_{\text{Ir}}(T - T_{\text{Ir}}) = m_{\text{Wa}}c_{\text{Wa}}(T - T_{\text{Wa}})$$

EVALUATE We now solve for the equilibrium temperature T

$$T = \frac{m_{\text{Ir}}c_{\text{Ir}}T_{\text{Ir}} + m_{\text{Wa}}c_{\text{Wa}}T_{\text{Wa}}}{m_{\text{Ir}}c_{\text{Ir}} + m_{\text{Wa}}c_{\text{Wa}}}$$

Using the given values of m_{Ir}, T_{Ir}, m_{Wa}, and T_{Wa}, and taking c_{Ir} and c_{Wa} from Table 16.1, we get T = 24.1° C.

ASSESS The larger mass and higher specific heat of the water makes its temperature change a lot less than that experienced by the pan.

36. **INTERPRET** This is a problem involving two masses coming together to reach an equilibrium temperature.

DEVELOP Our system consists of two masses: aluminum and water. These are initially at different temperatures, and when brought together, they reach thermal equilibrium. We want to find the amount of water necessary to reach the given equilibrium temperature when the two come together. Since all the heat lost by the hot pan is gained by the cold water, with no heat transfer to the container or its surroundings, we know: $-Q_{\text{Al}} = Q_{\text{Wa}}$. Expressing each side of this equation using Equation 16.3, we find

$$-m_{\text{Al}}c_{\text{Al}}(T - T_{\text{Al}}) = m_{\text{Wa}}c_{\text{Wa}}(T - T_{\text{Wa}})$$

EVALUATE We now solve for the mass of water m_{Wa}

$$m_{\text{Wa}} = \frac{m_{\text{Al}}c_{\text{Al}}(T_{\text{Al}} - T)}{c_{\text{Wa}}(T - T_{\text{Wa}})}$$

Using the given values of m_{Al}, T_{Al}, T_{Wa}, and T, and taking c_{Al} and c_{Wa} from Table 16.1, we get a minimum mass m_{Al} = 8.22 kg.

ASSESS The larger mass and higher specific heat of the water makes its temperature change a lot less than that experienced by the pan.

37. **INTERPRET** This is a problem involving two masses coming together to reach an equilibrium temperature.

DEVELOP Our system consists of two objects: fuel assemblies and water. These are initially at different temperatures, and when brought together, they reach thermal equilibrium. We want to find by how much the temperature of the water increases once it reaches the given equilibrium temperature. Since all the heat lost by the fuel rods is gained by the cold water, with no heat transfer to the container or its surroundings, we know: $-Q_{\text{Fr}} = Q_{\text{Wa}}$. Expressing each side of this equation using Equation 16.3, we find

$$-(248)m_{\text{Fr}}c_{\text{Fr}}(T - T_{\text{Fr}}) = m_{\text{Wa}}c_{\text{Wa}}(T - T_{\text{Wa}})$$

EVALUATE We now solve for the equilibrium temperature T

$$T = \frac{(248)m_{\text{Fr}}c_{\text{Fr}}T_{\text{Fr}} + m_{\text{Wa}}c_{\text{Wa}}T_{\text{Wa}}}{(248)m_{\text{Fr}}c_{\text{Fr}} + m_{\text{Wa}}c_{\text{Wa}}}$$

Using the given values of m_{Fr}, T_{Fr}, c_{Fr}, m_{Wa}, and T_{Wa}, and taking c_{Wa} from Table 16.1, we get T = 17.0 ° C. This means the water temperature changed by 2.0 ° C.

ASSESS The larger mass and higher specific heat of the water makes its temperature change a lot less than that experienced by the fuel assemblies.

38. **INTERPRET** This is a problem involving two masses coming together to reach an equilibrium temperature.

DEVELOP Our system consists of two masses: copper and zinc. These are initially at different temperatures, and when brought together, they reach thermal equilibrium. We want to find the amount of zinc present in the alloy that would result in the given equilibrium temperature when the two come together. We assume all heat transfer occur between the two masses, meaning: $-Q_{\text{Cu}} = Q_{\text{Zn}}$. Expressing each side of this equation using Equation 16.3, we find

$$-m_{\text{Cu}}c_{\text{Cu}}(T - T_{\text{Cu}}) = m_{\text{Zn}}c_{\text{Zn}}(T - T_{\text{Zn}})$$

EVALUATE We now solve for the mass of water m_{Zn}

$$m_{\text{Zn}} = \frac{m_{\text{Cu}}c_{\text{Cu}}(T_{\text{Cu}} - T)}{c_{\text{Zn}}(T - T_{\text{Zn}})}$$

Using the given values of m_{Cu}, T_{Cu}, c_{Cu}, T_{Zn}, c_{Zn}, and T, we get a mass m_{Zn} = 223 kg. This means that the percent of the alloy's mass made up of zinc is equal to 22.8%.

ASSESS The large portion of copper makes this alloy more malleable and golden looking.

39. **INTERPRET** The concept here is energy balance, now with the greenhouse as the system of interest. We're given *R*-factors, suggesting that the energy loss is by conduction through walls and glazing. The energy input is sunlight.

DEVELOP As we saw in Example 16.4, the $\mathcal{R}$-factor determines a heat-loss rate that is related directly to area and temperature difference and inversely to the *R*-factor. So we have

$$H_{\text{w}} = \frac{A_{\text{w}}\Delta T}{\mathcal{R}_{\text{w}}} = \left(\frac{435}{45}\right)\Delta T = (9.67\,\text{Btu/h/°F})\Delta T$$

for the heat loss through the walls and

$$H_{\text{g}} = \frac{A_{\text{g}}\Delta T}{\mathcal{R}_{\text{g}}} = \left(\frac{285}{2.1}\right)\Delta T = (136\,\text{Btu/h/°F})\Delta T$$

for the heat loss through the glass, giving a total heat loss H = (145 Btu/h/°F)ΔT. Meanwhile, the energy input through the entire 285 ft^2 of glass is (35.6 Btu/h/ft^2)(285 ft^2) = 10,150 Btu/h. Our plan is to equate energy input and loss and then solve for ΔT.

EVALUATE Equating loss and gain, and solving for ΔT gives

$$\Delta T = \frac{(10{,}150\,\text{Btu/h})}{(145.4\,\text{Btu/h/°F})} = 69.8\,°\text{F}$$

So when it's –10.5 °F outside, the greenhouse is at 59.3 °F.

ASSESS Our calculation assumes that solar input remains constant; in a real greenhouse the temperature would fluctuate as the Sun's angle changes and clouds pass over.

40. **INTERPRET** The concept here is energy balance, now with the greenhouse as the system of interest. We're given *R*-factors, suggesting that the energy loss is by conduction through walls and glazing. The energy input is sunlight.

DEVELOP As we saw in Example 16.4, the $\mathcal{R}$ -factor determines a heat-loss rate that is related directly to area and temperature difference and inversely to the *R*-factor. So we have

$$H_\text{w} = \frac{A_\text{w}\Delta T}{\mathcal{R}_\text{w}} = \left(\frac{51.5}{9.56}\right)\Delta T = (5.39\,\text{W/K})\Delta T$$

for the heat loss through the walls and

$$H_\text{g} = \frac{A_\text{g}\Delta T}{\mathcal{R}_\text{g}} = \left(\frac{32.3}{0.21}\right)\Delta T = (154\,\text{W/K})\Delta T$$

for the heat loss through the glass, giving a total heat loss H = (159 W/K)ΔT. Meanwhile, the energy input through the entire 32.3 ft^2 of glass is (112 W/m^2)(32.3 m^2) = 3618 W. Our plan is to equate energy input and loss and then solve for ΔT.

EVALUATE Equating loss and gain, and solving for ΔT gives

$$\Delta T = \frac{(3618\,\text{W})}{(159.2\,\text{W/K})} = 23\,\text{K} = 23°\ \text{C}$$

So in order for it to be 0° C or above inside the greenhouse, the minimum outdoor temperature is –23° C.

ASSESS Our calculation assumes that solar input remains constant; in a real greenhouse the temperature would fluctuate as the Sun's angle changes and clouds pass over.

41. **INTERPRET** This problem involves the absorption and emission of radiation by an object, which we can consider behaves like a blackbody. We want to determine the object's temperature.

DEVELOP We are told the asteroid behaves like a blackbody (e = 1), meaning it is a perfect emitter and a perfect absorber. Thus, the given solar energy absorption rate is equal to the radiated energy rate. We can then apply Equation 16.9 to calculate the surface temperature of the asteroid.

EVALUATE Solving for T from Equation 16.9 and evaluating gives

$$T = \left(\frac{(P/A)}{(e\sigma)}\right)^{1/4} = \left(\frac{(96.2\,\text{W/m}^2)}{\left[5.67\times10^{-8}\,(\text{W/m}^2\cdot\text{K}^4)\right]}\right)^{1/4} = 203\ \text{K}$$

Where we have expressed the power per unit area radiated from the average rate of solar energy absorbed for every square meter on the asteroid surface given.

ASSESS Due to the vacuum of space, radiation is the dominant energy-loss mechanism for the asteroid.

42. **INTERPRET** This problem involves the absorption and emission of radiation by objects in the path of a red dwarf star's light. We want to determine the distance from the star over which these planets will have a certain temperature range.

DEVELOP We are told the luminosity of the star is a small fraction of our Sun's, so we can determine the intensity of the spherical waves reaching the planets using Equation 14.8: $I = P/4\pi r^2$, and the value for the Sun's power output found on the inside front cover of the text book: $P = 3.85\times10^{26}$ W. This intensity is what reaches planets at different distances r from the star, and gets absorbed by their cross-sectional area. Treating this as an energy balance problem like in the "The Greenhouse Effect and Global Warming Application," we equate this absorbed energy to the emitted radiation.

EVALUATE Expressing this energy balance we find

$$\pi R_\text{P}{}^2 S = \sigma 4\pi R_\text{P}{}^2 T^4$$
$$S = 4\sigma T^4$$

Where S is equal to the intensity which reaches the planets at various distances from the dwarf star. Plugging in the star's luminosity from Equation 14.8 and solving for the distance r we find

$$\frac{P}{4\pi r^2} = 4\sigma T^4$$

$$r = \sqrt{\frac{P}{16\pi\sigma T^4}} = \sqrt{\frac{(0.00522)(3.85\times10^{26}\ \text{W})}{16\pi\left[5.67\times10^{-8}\ (\text{W/m}^2\cdot\text{K}^4)\right]T^4}}$$

Here, we plug in 0° C and 100° C to find the range of distances from TRAPPIST-1 that mark its habitable zone. We find that it is between 1.91×10^9 m and 3.56×10^9 m from the star.

ASSESS These planets orbit 100 times closer to their star than the Earth does to our Sun, which makes sense due its much lesser luminosity.

PROBLEMS

43. **INTERPRET** This problem is about finding the pressure at different temperatures, given its pressure at a reference temperature and that the volume is held constant.

DEVELOP For a constant-volume system, the pressure will be linear in temperature (see Fig. 16.3). Therefore, we can write

$$\frac{p}{T} = \frac{p_{\text{ref}}}{T_{\text{ref}}} \quad \Rightarrow \quad p = \left(\frac{T}{T_{\text{ref}}}\right)p_{\text{ref}}$$

If we use the given values at the normal melting point of ice, then the pressure-temperature relationship is

$$p = \left(\frac{T}{T_{\text{ref}}}\right)p_{\text{ref}} = T\left(\frac{101\ \text{kPa}}{273.15\ \text{K}}\right)$$

EVALUATE **(a)** When the temperature is the normal boiling point of water $T = 100°\text{C} = 373.15$ K, the pressure is

$$p = (373.15\ \text{K})\left(\frac{101\ \text{kPa}}{273.15\ \text{K}}\right) = 138\ \text{kPa}$$

(b) If the temperature is the normal boiling point of oxygen (90.2 K), then

$$p = (90.2\ \text{K})\left(\frac{101\ \text{kPa}}{273.15\ \text{K}}\right) = 33.4\ \text{kPa}$$

(c) If the temperature is the normal boiling point of mercury (630 K), then

$$p = (630\ \text{K})\left(\frac{101\ \text{kPa}}{273.15\ \text{K}}\right) = 233\ \text{kPa}$$

ASSESS These results hold only if the volume is held constant while the temperature varies.

44. **INTERPRET** This problem involves the change in pressure for a constant-volume system that changes in temperature, given the initial pressure and temperature and the temperature change.

DEVELOP Because this is a constant-volume process, we known from Fig. 16.3 that pressure and temperature are linearly related, so we can write

$$p = p_0 + T\frac{dp}{dT}$$

where dp/dT is the slope of the curve. Because the pressure is zero when the temperature is zero, $p_0 = 0$. The slope of the curve is then just the ratio of pressure to temperature at any point, which is constant, which we can obtain from the given pressure and temperature of the triple point of water:

$$\frac{p_{\text{triple}}}{T_{\text{triple}}} = \text{constant}$$

where $p_{\text{triple}} = 55$ kPa and $T_{\text{triple}} = 273.16$ K. Thus, the change in pressure is

$$p = T\frac{p_{\text{triple}}}{T_{\text{triple}}}$$

EVALUATE For a one-Kelvin change in temperature ($T = 1$ K), the pressure will change by

$$p = (1\text{ K})\frac{55\text{ kPa}}{273.16\text{ K}} = 2.0\times10^2\text{ Pa/K}$$

ASSESS The result is reported to two significant figures to reflect the precision of the data.

45. **INTERPRET** In this problem, we are asked to calculate the boiling point of SO_2, given the height difference between the liquid levels in a constant-volume gas thermometer.

DEVELOP The thermometric equation for an ideal constant-volume gas thermometer is (see Problem 16.39)

$$p = \left(\frac{T}{T_{\text{ref}}}\right)p_{\text{ref}}$$

where T is measured on the Kelvin scale. Since the pressure in the constant-volume gas thermometer shown is proportional to h, the temperature at the boiling point of SO_2 is

$$T = T_3\frac{p}{p_3} = T_3\frac{h}{h_3}$$

EVALUATE From the equation above, we find the boiling point of SO_2 to be

$$T = (273.16\text{ K})\left(\frac{57.7\text{ mm}}{60.4\text{ mm}}\right) = 261\text{ K} = -12.2°\text{C}$$

ASSESS For a constant-volume gas thermometer, p/T is constant. Since pressure can be measured in mm of mercury $(p = \rho gh)$, it is also true that h/T is constant.

46. **INTERPRET** This problem involves calculating the minimum work done climbing a mountain, which is work done against gravity. We are then to convert this energy to kcal.

DEVELOP The work done against gravity is $W = mgh$ (Equation 7.3), which gives the result in joules. To convert this to kcal, use the conversion factor $1\text{ kcal} = 4.184\text{ kJ}$ (Appendix C).

EVALUATE The minimum number of calories, Q, burned off climbing the mountain is

$$Q = (67\text{ kg})(9.8\text{ m/s}^2)(1600\text{ m})\left(\frac{1\text{ kcal}}{4.184\text{ kJ}}\right) = 250\text{ kcal}$$

ASSESS Much more energy than this is required in reality due to the many loss mechanisms, such as friction, slippage, etc.

47. **INTERPRET** This problem involves calculating the amount of energy a body uses to run a marathon and, assuming that fat is converted to energy with 100% efficiency, converting this energy to an equivalent mass of fat.

DEVELOP The energy expended in running a marathon for a person with the given mass is

$$\Delta Q = (125\text{ kcal/mi})(26.2\text{ mi}) = 3.28\times10^3\text{ kcal}$$

Knowing the amount of energy per gram of fat allows us to answer the question.

EVALUATE Since typical fats contain about 9 kcal per gram, ΔQ is equivalent to the energy content of

$$\frac{3.28\times10^3\text{ kcal}}{9\text{ kcal/g}} = 364\text{ g}$$

or about 13 oz of fat.

ASSESS Running a marathon is a good way to burn the fat stored in the body.

48. **INTERPRET** This problem involves calculating the temperature rise in the lake due to the given power input from the Sun. We are to assume that all the Sun's power is absorbed by the lake water, so the energy absorbed will go to raising the temperature via Equation 16.3.

DEVELOP The energy absorbed by the lake water is $Q = P_T\Delta t$, so $\Delta t = Q/P_T$ (where the total power $P_T = PA$, with $P = 200\text{ W/m}^2$ and $A = \pi r^2$). From Equation 16.3, a rise in the temperature of the lake water from 10°C to 20°C requires an energy $Q = mc\Delta T$, where $\Delta T = 10°\text{C}$, $c = 4184$ J/kg, and $m = \pi A d$ with $\rho = 1.0\times10^3\text{ kg/m}^3$ and $d = 10$ m.

EVALUATE Inserting the expression for Q and P_T into the expression for Δt gives

$$\Delta t = \frac{Q}{P_T} = \frac{mc\Delta T}{PA} = \frac{\rho dAc\Delta T}{PA} = \frac{\rho dc\Delta T}{P}$$

$$= \frac{(1.0\times10^3\ \text{kg/m}^3)(10\ \text{m})(4184\ \text{J/kg}\cdot\text{K})(10\ \text{C}°)}{200\ \text{W/m}^2} = 2.09\times10^6\ \text{s} = 24.2\ \text{d}$$

ASSESS Checking the units of this expression, we find that they work out to be units of time (i.e., s), as expected.

49. **INTERPRET** We are interested in the energy needed to raise the temperature of a system. We can solve this problem using the specific heat of the given substances.

DEVELOP The energy Q required to increase the temperature by ΔT is given by Equation 16.3: $Q = mc\Delta T$, where c is the specific heat and m is the mass of the material. The specific heats of some common materials can be found in Table 16.1.

EVALUATE **(a)** When just the pan is heated, with $c_{\text{Cu}} = 386\ \text{J}/(\text{kg}\cdot\text{K})$, the energy required is

$$\Delta Q = m_{\text{Cu}}c_{\text{Cu}}\Delta T = (0.77\ \text{kg})[386\ \text{J}/(\text{kg}\cdot\text{K})](90\ \text{K} - 14\ \text{K}) = 22.6\ \text{kJ}$$

(b) If the pan contains water and both are heated to the same temperature as in part (a), we then have

$$\Delta Q = (m_{\text{Cu}}c_{\text{Cu}} + m_{\text{H}_2\text{O}}c_{\text{H}_2\text{O}})\Delta T = 22.6\ \text{kJ} + (1.1\ \text{kg})[4184\ \text{J}/(\text{kg}\cdot\text{K})](76\ \text{K}) = 372\ \text{kJ}$$

(c) With $m_{\text{Hg}} = 4\ \text{kg}$ of mercury replacing the water,

$$\Delta Q = (m_{\text{Cu}}c_{\text{Cu}} + m_{\text{Hg}}c_{\text{Hg}})\Delta T = 22.6\ \text{kJ} + (4.2\ \text{kg})[140\ \text{J}/(\text{kg}\cdot\text{K})](76\ \text{K}) = 67.3\ \text{kJ}$$

ASSESS The energy required is proportional to the specific heat c. In this problem,

$$c_{\text{Hg}}[140\ \text{J}/(\text{kg}\cdot\text{K})] < c_{\text{Cu}}[386\ \text{J}/(\text{kg}\cdot\text{K})] < c_{\text{H}_2\text{O}}[4184\ \text{J}/(\text{kg}\cdot\text{K})]$$

50. **INTERPRET** This problem involves finding the specific heat of an unknown substance.

DEVELOP Because the heat energy transferred to both substances is the same, $\Delta Q = m_{\text{w}}c_{\text{w}}\Delta T_{\text{w}} = m_{\text{x}}c_{\text{x}}\Delta T_{\text{x}}$, or

$$c_{\text{x}} = c_{\text{w}}\left(\frac{m_{\text{w}}}{m_{\text{x}}}\right)\left(\frac{\Delta T_{\text{w}}}{\Delta T_{\text{x}}}\right) = c_{\text{w}}\left(\frac{\Delta T_{\text{w}}}{\Delta T_{\text{x}}}\right)$$

where the final equality comes from the fact that we are considering equal masses of water and the unknown substance.

EVALUATE **(a)** Inserting the given quantities in the expression above gives

$$c_{\text{x}} = [4184\ \text{J}/(\text{kg}\cdot\text{K})]\left(\frac{29^\circ\text{C} - 20^\circ\text{C}}{95^\circ\text{C} - 20^\circ\text{C}}\right) = 502\ \text{J}/(\text{kg}\cdot\text{K})$$

which is the value listed in Table 16.1 for steel.

(b) The rate of heating is

$$\Delta Q/\Delta t = m_{\text{w}}c_{\text{w}}\Delta T_{\text{w}}/\Delta t = (0.10\ \text{kg})[4184\ \text{J}/(\text{kg}\cdot\text{K})](9\ \text{K})/(60\ \text{s}) = 63\ \text{W}$$

ASSESS Notice that, for part (a), the difference in degrees Celsius is the same as the difference in Kelvin (see Equation 16.1).

51. **INTERPRET** You wish to know how long it will take a microwave to heat a cup of water to the boiling temperature.

DEVELOP The heat needed to bring the water to the point of boiling can be found with Equation 16.3: $Q = mc\Delta T$, where $c = 4184\ \text{J/kg}\cdot\text{K}$ from Table 16.1. The mass of 300 mL of water can be found from the density: $\rho = 1\ \text{g/cm}^3 = 1\ \text{g/mL}$. The temperature change is $\Delta T = 100°\text{C} - 15°\text{C} = 85°\text{C}$. Note: We don't have to convert to Kelvin, since the change in degrees Celsius is the same as the change in Kelvin. The time it takes the water to absorb this much heat comes from the energy divided by the power.

EVALUATE The time to heat the water to the boiling temperature is

$$t=\frac{Q}{P}=\frac{\rho Vc\Delta T}{P}=\frac{(1\text{ g / mL})(300\text{ mL})(4184\text{ J / kg}\cdot\text{K})(85\text{ K})}{(900\text{ W})}=119\text{ s}$$

ASSESS Approximately two minutes to bring the water to boil sounds about right.

52. **INTERPRET** This problem involves calculating the time it takes to heat an object, given its specific heat, its mass, and the power supplied.

DEVELOP Apply Equation 16.3, $Q=mc\Delta T$, to find the energy required to heat each house. The time it will take for the furnace to supply this energy is $\Delta t = Q / P = mc\Delta T / P$.

EVALUATE The time required to heat the stone house is

$$\Delta t=\frac{mc\Delta T}{P}=\frac{(70\text{ tons}\times 2000\text{ lb / ton})(0.20\text{ Btu / lb}\cdot^\circ\text{F})(36^\circ\text{F})}{1.0\times 10^5\text{ Btu / h}}=10.0\text{ h}$$

The time required to heat the wood house is

$$\Delta t=\frac{mc\Delta T}{P}=\frac{(17\text{ tons}\times 2000\text{ lb / ton})(0.33\text{ Btu / lb}\cdot^\circ\text{F})(36^\circ\text{F})}{1.0\times 10^5\text{ Btu / h}}=4.0\text{ h}$$

ASSESS Although the English units involve weight instead of mass, the units cancel to give units of time, as expected.

53. **INTERPRET** You want to compare the rate at which water is heated by a microwave in a paper cup to on a stovetop in a pan. The hitch is that the stovetop has to heat the pan too.

DEVELOP The temperature rise per second is equal to the heat absorbed per second divided by the heat capacity:

$$\frac{\Delta T}{\Delta t}=\frac{Q/C_{\text{tot}}}{\Delta t}=\frac{\overline{P}}{C_{\text{tot}}}$$

where $\overline{P}$ is the average power supplied, and $C_{\text{tot}}=C_{\text{H}_2\text{O}}+C_{\text{cnt}}$ is the total heat capacity from both the water and the container. This assumes that the water and container both have the same instantaneous temperature. The water's heat capacity is $C_{\text{H}_2\text{O}}=mc$, where $c=4184$ J/kg$\cdot$K from Table 16.1. For the paper cup used in the microwave oven, $C_{\text{cnt}}\approx 0$, whereas for the pan used on the stove burner, $C_{\text{cnt}}=1.4$ kJ/K.

EVALUATE If you equate the rates at which the temperatures rise,

$$\frac{\overline{P}_{\text{micro}}}{mc}=\frac{\overline{P}_{\text{stove}}}{mc+C_{\text{cnt}}}$$

You can then solve for the mass:

$$m=\frac{C_{\text{cnt}}/c}{\overline{P}_{\text{stove}}/\overline{P}_{\text{micro}}-1}=\frac{(1.4\text{ kJ/K})/(4.184\text{ kJ/kg}\cdot\text{K})}{(1000\text{W})/(625\text{W})-1}=0.56\text{ kg}$$

ASSESS This is a little over half a liter. Your own experience may confirm this. For heating a cup of tea, the microwave oven seems to work faster. But for heating a big bowl of soup, the stove will take less time.

54. **INTERPRET** We are asked to find the time required to change the temperature of an object, which we can solve using the specific heat and the mass of the object, as well as the given power input.

DEVELOP In order for the water to reach its boiling point, the tempertature needs to increase to 100° C. The heat required to change the temperature of the water by the ΔT = 90° C = 90 K is $Q=mc\Delta T$. The mass m of the water is calculated by multiplying the volume V by the density ρ, which for seawater is equal to ρ = 1030 kg/m^3, and the specific heat of water is given in Table 16.1 as $c=4184\text{ J/}(\text{kg}\cdot\text{K})$. To find the time to heat the water recall that average power is the energy per unit time, or $\overline{P}=Q/\Delta t$, which we can solve using the given power.

EVALUATE We can insert the given quantities into Equation 16.3 to obtain the heat and divide by the power to obtain a duration of

$$\Delta t=\frac{Q}{\overline{P}}=\frac{V\rho c\Delta T}{P}=\frac{(650\text{m}^3)(1030\text{kg / m}^3)[4184\text{ J/}(\text{kg}\cdot\text{K})](90\text{ K})}{(33\times 10^6\text{W})}=2.1\text{ h}$$

ASSESS Due to its larger density, seawater takes longer than pure water to reach its boiling point.

55. **INTERPRET** Given the power output of the stove and the amount of time it takes to heat up the water, we want to know how much water is in the kettle. This problem involves specific heat.

DEVELOP The energy supplied by the stove burner heats the kettle and the water in it from 21°C to 100°C, so $\Delta T = 79$ K. If we neglect any heat losses and the heat capacity of the burner, this energy is just the burner's power output times the time:

$$\Delta Q = \bar{P}\Delta t = \left(m_{\rm w}c_{\rm w} + m_{\rm K}c_{\rm K}\right)\Delta T$$

This equation can be used to solve for $m_{\rm w}$.

EVALUATE Since all of these quantities are given except for the mass of the water, we can solve for $m_{\rm w}$:

$$m_{\rm w} = \frac{1}{c_{\rm w}}\left(\frac{\bar{P}\Delta t}{\Delta T} - m_{\rm K}c_{\rm K}\right) = \frac{1}{4184\ \text{J}/(\text{kg}\cdot\text{K})}\left(\frac{(2.2\ \text{kW})(5.9\times 60\ \text{s})}{79\ \text{K}} - (1.0\ \text{kg})\left[447\ \text{J} + (\text{kg}\cdot\text{K})\right]\right)$$
$$= 2.2\ \text{kg}$$

ASSESS We find that $m_{\rm w}$ is proportional to Δt. This makes sense because the more water in the kettle, the more time we would expect it to take to heat up the water.

56. **INTERPRET** We're asked to calculate the time it takes for an ear thermometer to collect enough energy from the radiative heat coming from a small area on the eardrum.

DEVELOP The eardrum will radiate heat according to the Stefan-Boltzmann law from Equation 16.9: $P = e\sigma AT^4$, where it's important to remember that the temperature must be in Kelvin: $T = 37°\text{C} + 273 = 310$ K. The time it takes the thermometer to collect enough energy for a reading will be $t = E/P$.

EVALUATE We'll assume the eardrum is a perfect emitter with $e = 1$.

$$t = \frac{100\ \mu\text{J}}{(1)\left(5.67\times 10^{-8}\,\text{W/m}^2\cdot\text{K}^4\right)\left(1\text{mm}^2\right)(310\text{K})^4} = 0.19\ \text{s}$$

ASSESS How much faster would the reading be for someone with a fever of $40°\text{C}$? In fact, only one hundredth of a second faster.

57. **INTERPRET** The objects of interest are the steel disks of the disk brakes. The problem deals with transformation of energy from the kinetic energy of the car to the thermal energy of the brake disks, which we can calculate knowing the specific heat of the disk-brake material.

DEVELOP By energy conservation, the loss of kinetic energy of the car is equal to the thermal energy gained by the four brakes:

$$Q = \Delta K \quad\Rightarrow\quad 4m_{\text{brake}}c\Delta T = \frac{1}{2}m_{\text{car}}v^2$$

EVALUATE From the equation above, with $v = 85\ \text{km/h} = 23.6\ \text{m/s}$, the change of temperature is

$$\Delta T = \frac{m_{\text{car}}\ v^2/2}{4m_{\text{brake}}c} = \frac{(2100\ \text{kg})(23.6\ \text{m/s})^2}{8(5.8\ \text{kg})\left[502\ \text{J}/(\text{kg}\cdot\text{K})\right]} = 50\ \text{K}$$

ASSESS This is a big increase in temperature. The brakes can get very hot depending on how fast the car was moving initially.

58. **INTERPRET** This is a problem involving two masses coming together to reach an equilibrium temperature.

DEVELOP We can think of our system consisting of two masses of water which are initially at different temperatures. They are brought together and reach a thermal equilibrium. We want to find the ratio of hot water to cold water that would result in the given equilibrium temperature for the given initial temperatures of the hot and cold water supplies. Let us assume that all the heat lost by the hot water is gained by the cold water, with no heat transfer to the container or its surroundings, meaning: $-Q_{\text{hw}} = Q_{\text{cw}}$. Expressing each side of this equation using Equation 16.3, we find

$$-m_{\text{hw}}c_{\text{w}}(T-T_{\text{hw}})=m_{\text{cw}}c_{\text{w}}(T-T_{\text{cw}})$$

EVALUATE Expressing all the temperatures in the Kelvin scale and solving for the ratio of hot to cold water mass needed, one finds

$$\frac{m_{\text{hw}}}{m_{\text{cw}}}=\frac{(T-T_{\text{cw}})}{(T_{\text{hw}}-T)}=\frac{(307.15\text{ K}-285.55\text{ K})}{(324.85\text{ K}-307.15\text{ K})}=1.22$$

ASSESS Since both masses coming to an equilibrium temperature are of liquid water, the result is not dependent on the value of the specific heat.

59. **INTERPRET** Our system consists of two materials, water and copper, which are initially at different temperatures. They are brought together and reach a thermal equilibrium. We want to find the mass of the copper, for which we can use the specific heat of copper.

DEVELOP Let us assume that all the heat lost by the copper is gained by the water, with no heat transfer to the container or its surroundings. Then, $-Q_{\text{Cu}}=Q_{\text{w}}$ (as in Example 16.2). Expressing each side of this equation using Equation 16.3, we find

$$-m_{\text{Cu}}c_{\text{Cu}}\left(T-T_{\text{Cu}}\right)=m_{\text{w}}c_{\text{w}}\left(T-T_{\text{w}}\right)$$

The specific heats of copper and water can be found in Table 16.1.

EVALUATE Expressing all the temperatures in the Kelvin scale and solving for m_{Cu}, one finds

$$m_{\text{Cu}}=\frac{m_{\text{w}}c_{\text{w}}\left(T-T_{\text{w}}\right)}{c_{\text{Cu}}\left(T_{\text{Cu}}-T\right)}=\frac{(1.4\text{ kg})\left[4184\text{ J}/(\text{kg}\cdot\text{K})\right](302\text{ K}-297\text{ K})}{\left[386\text{ J}/(\text{kg}\cdot\text{K})\right](533\text{ K}-302\text{ K})}=0.33\text{ kg}$$

ASSESS Since the water has much greater mass and higher specific heat, its temperature change is less compared to copper.

60. **INTERPRET** You want to know how long it will take your camping stove to bring water to a boil, given the formula for the heat flowing into the water as a function of time.

DEVELOP You can integrate the given power, P, to find the total heat that the water has absorbed. You can then equate that to the amount of energy needed to bring the water up to 100°C using Equation 16.3: $Q = mc\Delta T$, where $c = 4184\text{ J}/\text{kg}\times\text{K}$ for water. From this, you can solve for the boiling time.

EVALUATE The heat absorbed by the water over a given time is:

$$Q=\int_0^t P(t')dt'=at+\frac{1}{2}bt^2=(1.1\text{ kW})t+(1.25\text{ W}/\text{s})t^2$$

You want to know how long it would take until this absorbed heat changes the temperature of the water by $\Delta T=100°\text{C}-10°\text{C}=90°\text{C}$.

$$Q=mc\Delta T=(2.1\text{ kg})(4184\text{ J}/\text{kg}\cdot\text{K})(90\text{ K})=791\text{ kJ}$$

This requires solving a quadratic equation with the quadratic formula from Appendix A:

$$t=\frac{-(1100\text{ W})+\sqrt{(1100\text{ W})^2+4(1.25\text{ W}/\text{s})(791\text{ kJ})}}{2(1.25\text{ W}/\text{s})}=469\text{ s}\approx 8\text{ min}$$

ASSESS There’s 2.1 L of water to boil, so 8 minutes sounds about right.

61. **INTERPRET** This problem involves the thermal resistance of a material, which we can use to calculate the rate of heat lost through the material given the temperature difference between the different sides of the material.

DEVELOP The total surface area (sides, top, and bottom) of the cooler is $A=2(3.0\times2.0+3.0\times2.3+2.0\times2.3)\text{ m}^2=35\text{ m}^2$. A thickness of 8 cm of Styrofoam of this area has a thermal resistance of $R = Dx/(kA)$ (Equation 16.6), and the heat flow Equation 16.7 gives

$$P=H=\frac{|\Delta T|}{R}=kA\frac{|\Delta T|}{\Delta x}$$

EVALUATE Using $k = 0.029$ W/(m·K) from Table 16.2 gives

$$P = kA\frac{|\Delta T|}{\Delta x} = \frac{(0.029)(35\ \text{m}^2)(20°\text{C} - 4.0°\text{C})}{0.080\ \text{m}} = 2.0\times10^2\ \text{W}$$

to two significant figures.

ASSESS The power requires is equivalent to about three 60-W light bulbs.

62. **INTERPRET** This problem is about conductive heat flow. We want to find the heat-flow rate along an iron rod. Note that the rod is insulated so no heat is lost out through the sides of the rod, only through the ends of the rod.

DEVELOP We assume a uniform variation of temperature along the length of the rod and no heat flow through its sides. The heat-flow rate is given by Equation 16.5:

$$H = -kA\frac{\Delta T}{\Delta x}$$

If we let the origin of our coordinates system be at the hot-water end of the rod, we have $\Delta x = x_{\text{cold}} - x_{\text{hot}} = 0.36\ \text{m}$ and $\Delta T = T_{\text{cold}} - T_{\text{hot}} = 0°\text{C} - 100°\text{C} = -100\ \text{K}$

EVALUATE Entering the numerical values, we get

$$H = -kA\frac{\Delta T}{\Delta x} = -\left[80.4\ \text{W}/(\text{m}\cdot\text{K})\right]\pi(0.014\ \text{m})^2\left(\frac{-100\ \text{K}}{0.36\ \text{m}}\right) = 13.75\ \text{W} = 14\ \text{W}$$

to two significant figures. Here, the minus sign signifies a heat flow from the hot to the cold.

ASSESS The flow rate H increases with the temperature gradient, $\Delta T / \Delta x$. With our choice of coordinate system, the fact that $H > 0$ signifies that the heat flows from the hot water to the cold water, as expected.

63. **INTERPRET** You want to see if the power output from the party guests can compensate for the heat loss from the house.

DEVELOP Combined, the 36 people will generate 3600 W of heat. The house will be in energy balance when the inside temperature results in a heat loss that matches what the people produce:

$$P_{\text{loss}} = (320\ \text{W/°C})(T_{\text{inside}} - 8°\text{C}) = P_{\text{people}} = 3600\ \text{W}$$

EVALUATE Solving for the inside temperature

$$T_{\text{inside}} = 8°\text{C} + \frac{3600\ \text{W}}{320\ \text{W/°C}} = 19°\text{C}$$

This is equal to about 66°F, which means the house will remain at a comfortable temperature.

ASSESS If you wanted the house even a little warmer, you could ask some of the people to do a little light exercise to generate more than a 100 W of heat.

64. **INTERPRET** This problem involves thermal energy balance. The heat source is the electric stove and the main heat loss mechanism is radiation (we ignore convection).

DEVELOP From the Stefan-Boltzmann law (Equation 16.9), the net power radiated (emitted at T_1, absorbed at T_2) is

$$P = e\sigma A\left(T_1^4 - T_2^4\right)$$

EVALUATE Inserting the given quantities in the expression above, we find

$$\begin{aligned}P &= e\sigma A\left(T_1^4 - T_2^4\right)\\ &= (1.0)\left[5.67\times10^{-8}\ \text{W}/\left(\text{m}^2\cdot\text{K}^4\right)\right]\left(3.25\times10^{-2}\ \text{m}^2\right)(300\ \text{K})^4\left(3^4 - 1\right) = 1200\ \text{W}\end{aligned}$$

which is 80% of the input power of 1500 W.

ASSESS The actual power loss will be greater than this because of heat loss due to convection.

65. **INTERPRET** This is a problem involving heat transfer to a metal object through radiation and conduction, for which we know both processes result in an equal amount of energy loss.

DEVELOP Since both the radiative energy and the conductive heat loss of the metal are equivalent, we can equate the rate of heat flow given by Equation 16.5 and the radiated power given by Equation 16.9. These two quantities represent the energy flow at the surface boundary between the blackbody-like surface of the metal and the slab. We can then solve for the temperature of the metal block that satisfies these conditions.

EVALUATE Equating Equation 16.5 and Equation 16.9 we find

$$H = P \rightarrow kA\frac{\Delta T}{\Delta x} = e\sigma AT^4$$

Where Δx is equal to the thickness d, e is the emissivity of the metal block (in this case 1), k is the thermal conductivity of the slab, and σ is the Stefan-Boltzmann constant. Here we can note that the temperature difference ΔT between the two faces of the slab is actually equal to T, since one side is in contact with liquid helium at neatly 0 K, and the other is in contact with the metal block at T. Solving for this temperature T we obtain

$$T = \sqrt[3]{k / \sigma \, \Delta x}$$

Evaluating this for a slab 2.85 cm thick made of insulating foam with $k = 0.0166$, we obtain a temperature of

$$T = \sqrt[3]{k / \sigma d} = \sqrt[3]{\frac{(0.0166)}{[5.67\times 10^{-8}\ \text{W/(m}^2\cdot\text{K}^4)](0.0285\,\text{m})}} = 217\,\text{K}$$

ASSESS We've omitted the negative sign in Equation 16.5 since the magnitude of energy lost through both conductive and radiative rates is equivalent.

66. **INTERPRET** A sleeping bag is like the insulation in the walls of a house. It doesn't generate heat, but it slows the rate at which heat leaves your body. You want to check if a certain sleeping bag really can keep you warm at the minimum temperature that its manufacture specifies.

DEVELOP If the outside temperature is –10°F (or –23°C) , there will be a temperature difference of 60°C between the inside and outside of the sleeping bag. We can find the conductive heat flow using Equation 16.5: $H = -kA\Delta T / \Delta x$, where the thermal conductivity of goose down is given in Table 16.2: $k = 0.043\ \text{W/m}^2\cdot\text{K}$. If this heat loss is greater than the 100 W that your body produces, then you will feel cold.

EVALUATE Assuming the sleeping bag conforms to your body, it will have essentially the same surface area as you. The heat flow through the bag is then

$$H = -kA\frac{\Delta T}{\Delta x} = -\left(0.043\ \text{W/m}^2\cdot\text{K}\right)\left(1.5\ \text{m}^2\right)\frac{60°\text{C}}{0.040\ \text{m}} = 97\ \text{W}$$

The heat loss is less than what your body produces, so you will be able to maintain normal body temperature when the outside temperature drops to –10°F.

ASSESS The heat loss is actually 3W less than what your body produces, so you may start to feel a little too warm in the bag. In actuality, though, your body can regulate how much heat it makes.

67. **INTERPRET** Our system consists of two materials, water and an iron horseshoe, which are initially at different temperatures. They are brought together and reach a thermal equilibrium. We want to find the equilibrium temperature.

DEVELOP Let us assume that all the heat lost by the horseshoe is gained by the water, with no heat transfer to the container or its surroundings. In this case, $-Q_{\text{Fe}} = Q_{\text{w}}$ (as in Example 16.2). Using Equation 16.4 gives

$$-m_{\text{Fe}}c_{\text{Fe}}(T - T_{\text{Fe}}) = m_{\text{w}}c_{\text{w}}(T - T_{\text{w}})$$

The specific heats of copper and water can be found in Table 16.1.

EVALUATE Solving for T, one finds

$$\begin{aligned} T &= \frac{m_{\text{Fe}}c_{\text{Fe}}T_{\text{Fe}} + m_{\text{w}}c_{\text{w}}T_{\text{w}}}{m_{\text{Fe}}c_{\text{Fe}} + m_{\text{w}}c_{\text{w}}} \\ &= \frac{(1.1\ \text{kg})\left[0.107\ \text{kcal/}(\text{kg}\cdot°\text{C})\right](550°\text{C}) + (15\ \text{kg})\left[1.0\ \text{kcal/}(\text{kg}\cdot°\text{C})\right](20°\text{C})}{(1.1\ \text{kg})\left[0.107\ \text{kcal/}(\text{kg}\cdot°\text{C})\right] + (15\ \text{kg})\left[(1.0\ \text{kcal/kg}\cdot°\text{C})\right]} = 24°\text{C} \end{aligned}$$

ASSESS The change of water temperature is $\Delta T_{\text{w}} = T - T_{\text{w}} = 24.1°\text{C} - 20°\text{C} = 4.1°\text{C}$, while the change of temperature of the iron horseshoe is $|\Delta T_{\text{Fe}}| = 525.9°\text{C}$. Because there is more water (by mass) and it has a much higher specific heat, its temperature changes less compared to the horseshoe.

68. **INTERPRET** The problem asks for the power output of a microwave given the time it takes to boil a certain quantity of water.

DEVELOP If we assume microwave is 100% efficient, then all the energy it produced in the given time, $P\Delta t$, will be used heat the water: $Q = mc\Delta T$.

EVALUATE Solving for the power gives

$$P = \frac{mc\Delta T}{\Delta t} = \frac{(0.43\text{ kg})(4184\text{ J/kg}\cdot\text{K})(100°\text{C} - 20°\text{C})}{2.5\cdot 60\text{ s}} = 960\text{ W}$$

ASSESS This is about the typical power for a microwave oven. But in reality some of the power is lost, heating the container or the oven walls.

69. **INTERPRET** This problem is about the radiation emitted by a burning log. Given its emissivity and its radiating power, we are to calculate its temperature.

DEVELOP If we neglect the radiation absorbed by the log from its environment (which should be negligible because the temperature of the log is much, much greater than room temperature), then the net power radiated by the log is just that given by the Stefan-Boltzmann law (Equation 16.9): $P = e\sigma AT^4$. Knowing the surface area of the log allows us to determine T.

EVALUATE The surface area of the log is

$$A = \pi dL + \pi d^2/2 = \pi d\,(L + d/2) = \pi(0.15\text{ m})(0.65\text{ m} + 0.075\text{ m}) = 0.342\text{ m}^2$$

Solving for T, we find

$$T = \left(\frac{P}{e\sigma A}\right)^{1/4} = \left(\frac{34\times10^3\text{ W}}{\left[5.67\times10^{-8}\text{ W/}(\text{m}^2\cdot\text{K}^4)\right](0.342\text{ m}^2)}\right)^{1/4} = 1.2\times10^3\text{ K}$$

ASSESS When a burning log is glowing red hot, its temperature is above 1000°C. If the temperature continues to rise, its color will turn orange, then yellow, then white when it emits over a broad range of visible wavelengths.

70. **INTERPRET** This problem involves blackbody radiation. Given a star's surface temperature and its radiating power, we are asked to calculate the radius of the star.

DEVELOP Apply the Stefan-Boltzmann law, Equation 16.9, $P = e\sigma AT^4$. The surface area of the star is $A = 4\pi r^2$, so we can solve for its radius r. If the star behaves as a blackbody, its emissivity is $e = 1.0$.

EVALUATE Solving the Stefan-Boltzmann law for the radius gives

$$r = \pm\frac{1}{T^2}\sqrt{\frac{P}{4\pi e\sigma}} = \frac{1}{(2.3\times10^4\text{ K})^2}\sqrt{\frac{3.4\times10^{30}\text{ W}}{4\pi(1.0)\left[5.67\times10^{-8}\text{ W/}(\text{m}^2\cdot\text{K}^4)\right]}} = 4.1\times10^9\text{m.}$$

ASSESS The two signs for the radius indicate that the radius may be measured in either the positive or the negative direction.

71. **INTERPRET** This problem is about the heat loss through various structural parts of the house via conduction.

DEVELOP Follow the approach outlined in Example 16.4. By Equations 16.8, 16.6, and 16.5, the heat-flow rate is related to the $\mathcal{R}$-factor as

$$H = -kA\frac{\Delta T}{\Delta x} = -A\frac{\Delta T}{\Delta x/k} = -A\frac{\Delta T}{\mathcal{R}}$$

The window area here is $A_{\text{window}} = 10(2.5\text{ ft}\times5.0\text{ ft}) = 125\text{ ft}^2$, and the wall area is 125 ft^2 less than in Example 16.4, or $A_{\text{walls}} = 1506\text{ ft}^2 - 125\text{ ft}^2 = 1381\text{ ft}^2$. Thus, the heat lost through these structural parts are:

$$H_{\text{walls}} = \left(\frac{1}{12.37}\frac{\text{Btu}}{\text{h}\cdot\text{ft}^2\cdot{}^\circ\text{F}}\right)\left(1381\ \text{ft}^2\right)\left(50^\circ\text{F}\right) = 5583\ \text{Btu/h}$$

$$H_{\text{roof}} = \left(\frac{1}{31.37}\frac{\text{Btu}}{\text{h}\cdot\text{ft}^2\cdot{}^\circ\text{F}}\right)\left(1164\ \text{ft}^2\right)\left(50^\circ\text{F}\right) = 1855\ \text{Btu/h}$$

$$H_{\text{windows}} = \left(\frac{1}{0.90}\frac{\text{Btu}}{\text{h}\cdot\text{ft}^2\cdot{}^\circ\text{F}}\right)\left(125\ \text{ft}^3\right)\left(50^\circ\text{F}\right) - 4\left(30\ \frac{\text{Btu}}{\text{h}\cdot\text{ft}^2}\right)\left(12.5\ \text{ft}^2\right) = 5444\ \text{Btu/h}$$

where we have included the heat gain by solar energy (1500 Btu/h) in H_{windows}. Thus, the rate of thermal energy loss from the entire house is

$$H_{\text{total}} = \left(5583 + 1855 + 5444\right)\ \text{Btu/h} = 12.88\times10^3\ \text{Btu/h}$$

EVALUATE **(a)** The monthly fuel bill is

$$\left(12.88\times10^3\ \text{Btu/h}\right)\left(24\times30\ \text{h/mo}\right)\left(1\ \text{gal}/10^5\ \text{Btu}\right)\left(\$2.87\ /\text{gal}\right) = \$266/\text{mo}$$

to three significant figures.

(b) The solar gain from the south windows is worth

$$\left(1500\ \text{Btu/h}\right)\left(24\times30\ \text{h/mo}\right)\left(1\ \text{gal}/10^5\ \text{Btu}\right)\left(\$2.87/\text{gal}\right) = \$31/\ \text{mo}$$

ASSESS This is an expensive fuel bill. You probably would want to improve the insulation.

72. **INTERPRET** This is a problem involving the radiative heat loss of a comet.

DEVELOP We are given the emissivity (in this case 1) and measured flux of radiation emitted from the comet. Since we are told the power per square meter, we can simply express the quantity *P/A* and thus, we can apply Equation 16.9 to obtain the temperature of the comet.

EVALUATE Solving for the temperature from Equation 16.9 and evaluating we find

$$T = \sqrt[4]{\frac{(P/A)}{e\sigma}} = \sqrt[4]{\frac{(96.3\ \text{W}/\text{m}^2)}{[5.67\times10^{-8}\ \text{W}/(\text{m}^2\cdot\text{K}^4)]}} = 203\,\text{K}$$

ASSESS Even though this comet has a temperature of –70° C, it is still emitting a measureable amount of energy in the form of blackbody radiation.

73. **INTERPRET** This problem is about radiation received by Pluto from the Sun. Treating Pluto as a blackbody, we want to find its average surface temperature.

DEVELOP Pluto receives energy from the Sun at a rate of $I_\text{S} = 0.876\,\text{W/m}^2$. If we assume that Pluto absorbs the fraction of radiation falling on its cross-sectional area $\left(A_\text{cs} = \pi R_\text{P}^2\right)$, then Pluto's heat input from the Sun is $P_\text{in} = I_\text{S}(\pi R_\text{P}^2)$. It will be radiating away this heat, according to Stefan Boltzmann's law: $P_\text{out} = e\sigma A T^4$, where the area in this case is the total surface area, $A = 4\pi R_\text{P}^2$. The surface temperature, *T*, will settle to a value where the outgoing radiation matches the incoming radiation.

EVALUATE Equating the two powers gives the following for the surface temperature:

$$T = \left(\frac{I_\text{S}}{4\sigma}\right)^{1/4} = \left(\frac{0.876\,\text{W/m}^2}{4\left(5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\right)}\right)^{1/4} = 44\ \text{K}$$

ASSESS Astronomers have recently estimated the temperature on Pluto to be about 43 K, so this answer is in agreement with that. One effect that we didn't account for is Pluto's albedo, that is, how much of the incoming sunlight gets reflected away instead of absorbed.

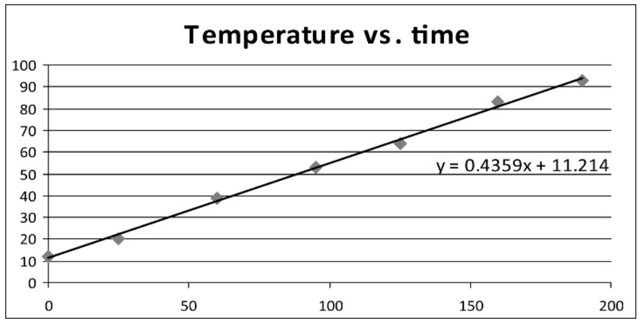
74. **INTERPRET** In this problem you are asked to analyze the data of the temperature of a sample of water as a function of time heated in a microwave oven, and deduce the power of the oven.

DEVELOP Let the energy output of the microwave oven be $U = P\Delta t$. The heat absorbed by water is given by Equation 16.3 $Q = mc\Delta T$, where m = 0.500 kg is the mass of the water sample, c = 4184 J/(kg·K) is the specific heat of water (from Table 16.1), and ΔT is the temperature change. Assuming that all the microwave energy goes into heating the water, we have $Q = U$, or $mc\Delta T = P\Delta t$. This implies

$$\frac{\Delta T}{\Delta t} = \frac{P}{mc}$$

The slope of ΔT versus time is P/mc, which can then be used to determine P.

EVALUATE The plot is shown below.



The slope is P/mc = 0.436, so the power of the microwave oven is

$$P = (0.436\ \text{K/s})(0.500\ \text{kg})(4184\ \text{J/kg}\cdot\text{K}) = 912\ \text{J/s} = 912\ \text{W}$$

ASSESS This is about the typical power for a microwave oven. But in reality some of the power is lost, heating the container or the oven walls.

75. **INTERPRET** This is a heat capacity problem, but with a heat capacity that changes with temperature. We can solve for the heat Q by integrating over T.

DEVELOP The mass is given as m = 1.00 kg and, from Equation 16.3, we have $dQ = mcdT$. The specific heat is $c = c_0 + aT + bT^2$. We can thus integrate from T_1 = 0 °C to T_2 = 100°C to find Q.

EVALUATE **(a)** Performing the integration gives

$$Q = \int_{T_1}^{T_2} mc(T)dT = m\int_T^T \left(c_0 + aT + bT^2\right)dT = m\left[c_0T + \frac{1}{2}aT^2 + \frac{1}{3}bT^3\right]_0^{100}$$

$$= 100c_0 + 5000a + \frac{b}{3}10^6 = 418.76\ \text{kJ}$$

(b) With constant specific heat $c = 4184\ \text{J/kg}\cdot\text{K}$, the heat required would be

$$Q_0 = mc\Delta T = (1.00\ \text{kg})(4184\ \text{J/kg}\cdot\text{K})(100\ \text{K}) = 418.4\ \text{kJ}$$

The difference is $\Delta Q = Q - Q_0 = 0.36$ kJ, or about $(0.36\ \text{kJ})/(418.4\ \text{kJ}) \approx 0.09\%$ higher than that using a constant specific heat.

ASSESS The fact that specific heat tends to increase with temperature is due to the increasing number of excited degrees of freedom (see Chapter 18) that require more energy to cause the same temperature increase.

76. **INTERPRET** This is a heat-capacity problem, but with a heat capacity that changes with temperature. We can solve for the heat Q by integrating over T.

DEVELOP The mass is given as $m = 33$ g, and from Equation 16.3, we have $dQ = mcdT$. The specific heat is $c = 31(T/343\ \text{K})^3\ \text{J}/(\text{g}\cdot\text{K})$. We can thus integrate from $T_1 = 13$ K to $T_2 = 29$ K to find Q.

EVALUATE Performing the integration gives

$$Q = \int_{T_1}^{T_2} m\left(\frac{T}{343\ \text{K}}\right)^3\left(\frac{31\ \text{J}}{\text{g K}}\right)dT = m\left(\frac{1}{343\ \text{K}}\right)^3\left(\frac{31\ \text{J}}{\text{g K}}\right)\int_T^T T^3 dT = m\left(\frac{1}{343\ \text{K}}\right)^3\left(\frac{31\ \text{J}}{\text{g K}}\right)\left[\frac{1}{4}T^4\right]_{T_1}^{T_2}$$

$$= \frac{m}{4}\left(\frac{1}{343\ \text{K}}\right)^3\left(\frac{31\ \text{J}}{\text{g K}}\right)\left(T_2^4 - T_1^4\right) = \frac{33\ \text{g}}{4}\left(\frac{1}{343\ \text{K}}\right)^3\left(\frac{31\ \text{J}}{\text{g K}}\right)\left[(29\ \text{K})^4 - (13\ \text{K})^4\right] = 4.3\ \text{J}$$

ASSESS At more normal temperatures, the specific heat of copper is $c = 386\ \text{J/kg}\cdot\text{K}$; so the heat required to change the temperature of 33 grams of copper by 16°C would be $Q = 204\ \text{J}$. This is substantially greater than that at extremely low temperatures.

77. **INTERPRET** In this problem we explore the greenhouse effect on Mars and Venus. We treat the planets as blackbody to find their average surface temperatures.

DEVELOP The average rate at which the solar energy reaches Earth is $S_0 = 960\ \text{W/m}^2$. Since the rate varies as $1/r^2$, the rates of the solar energy reaching Mars and Venus are given by

$$S_{\text{Venus}} = S_0\left(\frac{r_{SE}}{r_{SV}}\right)^2 = \left(960\ \text{W/m}^2\right)\left(\frac{150\times10^9\,\text{m}}{108\times10^9\,\text{m}}\right)^2 = 1850\ \text{W/m}^2$$

$$S_{\text{Mars}} = S_0\left(\frac{r_{SE}}{r_{SM}}\right)^2 = \left(960\ \text{W/m}^2\right)\left(\frac{150\times10^9\,\text{m}}{228\times10^9\,\text{m}}\right)^2 = 416\ \text{W/m}^2$$

where $r_{SV} = 108\times10^9\,\text{m}$ is the mean distance between Venus and the Sun, and $r_{SM} = 228\times10^9\,\text{m}$ the mean distance between Mars and the Sun (from Appendix E). If we assume that the planets absorb the fraction of radiation falling on its cross-sectional area $\left(A_{\text{cs}} = \pi R^2\right)$, then the heat input from the Sun is $P_{\text{in}} = S(\pi R^2)$. The planets will be radiating away this heat, according to Stefan-Boltzmann's law: $P_{\text{out}} = e\sigma A T^4$, where the area in this case is the total surface area, $A = 4\pi R^2$. The surface temperature, T, will settle to a value where the outgoing radiation matches the incoming radiation.

EVALUATE Equating the two powers gives the following surface temperatures:

$$T_{\text{Venus}} = \left(\frac{S_{\text{Venus}}}{4\sigma}\right)^{1/4} = \left(\frac{1850\ \text{W/m}^2}{4\left(5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\right)}\right)^{1/4} = 301\ \text{K}$$

$$T_{\text{Mars}} = \left(\frac{S_{\text{Mars}}}{4\sigma}\right)^{1/4} = \left(\frac{416\ \text{W/m}^2}{4\left(5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\right)}\right)^{1/4} = 207\ \text{K}$$

ASSESS The measured values are 740 K for Venus and 210 K for Mars. Our results suggest that Mars has very little greenhouse effect, while Venus exhibits a "runaway" greenhouse effect resulting in a very high surface temperature.

78. **INTERPRET** We are to show that the equation for conductive heat loss through a cylindrical surface is as given. To do this, we consider the differential form of Equation 16.5 and integrate the heat loss through thin cylindrical shells.

DEVELOP The rate of heat flow is given by $H = -kA\left(dT/dr\right)$. Consider the heat loss through thin cylindrical shell of thickness dr and length L, which is

$$H = -kA\frac{dT}{dr} = -k(2\pi rL)\frac{dT}{dr}$$

$$\frac{H}{r}dr = -2\pi kL\,dT$$

We can integrate this expression to find an expression for the heat loss through the macroscopic cylinder.

EVALUATE Performing the integration gives

$$H\int_{R_1}^{R_2}\frac{1}{r}dr = -2\pi kL\int_{T_1}^{T_2}dT$$

$$H\ln\left(\frac{R_2}{R_1}\right) = -2\pi kL(T_2 - T_1)$$

$$H = \frac{2\pi kL(T_1 - T_2)}{\ln(R_2/R_1)}$$

ASSESS We have shown what was required.

79. **INTERPRET** You want to check whether the Sun's recent increase in power output can explain the rise in the global average temperature. This is your friend's argument against human-induced global warming.

DEVELOP From the Application "The Greenhouse Effect and Global Warming," you were told that the Earth currently absorbs energy from the Sun at a rate of $S = 960\ \text{W/m}^2$, averaged over the cross-sectional area of the planet, πR_E^2. Using energy balance arguments and a assuming the Earth's emissivity is 1, a formula was derived for the Earth's average temperature:

$$T = \left(\frac{S}{4e\sigma}\right)^{1/4} = \left(\frac{960\ \text{W/m}^2}{4(1)\left(5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\right)}\right)^{1/4} = 255\ \text{K} = -18°\text{C}$$

This is too cold. The global average temperature is actually about 15°C, due to the greenhouse effect. Effectively, the greenhouse effect reduces the Earth's emissivity to about 0.61. Let's assume that the emissivity has been constant since the start of the industrial era. Then $T \propto S^{1/4}$, and we can verify if the change in the solar flux can account for the measured temperature change since the start of the industrial era.

EVALUATE The solar flux has increased by 0.05% since pre-industrial time, which can be expressed as $S = (1.0005)S_{\text{pre}}$. The temperature should correspondingly be higher due to this change:

$$T = T_{\text{pre}}\left(\frac{S}{S_{\text{pre}}}\right)^{1/4} \quad\rightarrow\quad \Delta T = T - T_{\text{pre}} = \left(1-(1.0005)^{-1/4}\right)T = 1.2496\times10^{-4}\,T$$

In Kelvin, the current global average temperature is $T = 288$ K, so the temperature change from the solar flux increase is $\Delta T = 0.036^{\circ}\text{C}$. This only accounts for about 4% of the measured temperature increase (0.85°C), so your friend is wrong.

ASSESS The argument for human-induced global warming is that the temperature increase is due to a decrease in the effective emissivity. Rising levels of greenhouse gases since the beginning of the industrial era allow less of the infrared radiation from the Earth's surface to be emitted into space.

80. **INTERPRET** This problem involves converting units from English to SI. Specifically, we are to convert from $\mathcal{R}$-factor to $\text{m}^2\cdot\text{K/W}$.

DEVELOP The units of $\mathcal{R}$ are $\text{ft}^2\cdot{}^{\circ}\text{F}\cdot\text{h/Btu}$. Use the conversion factors $1 = 0.3048\ \text{m/ft}$, $1 = 1.8^{\circ}\text{F/K}$, and $1 = 1054\ \text{J/Btu}$ from Appendix C.

EVALUATE Inserting these conversion factors gives

$$19\frac{\text{ft}^2\cdot{}^{\circ}\text{F}\cdot\text{h}}{\text{Btu}} = \left(19\frac{\text{ft}^2\cdot{}^{\circ}\text{F}\cdot\text{h}}{\text{Btu}}\right)\overbrace{\left(\frac{0.3048\ \text{m}}{1\ \text{ft}}\right)^2}^{=1}\overbrace{\left(\frac{1\ \text{K}}{1.8^{\circ}\text{F}}\right)}^{=1}\overbrace{\left(\frac{3600\ \text{s}}{1\ \text{h}}\right)}^{=1}\overbrace{\left(\frac{1\ \text{Btu}}{1054\ \text{J}}\right)}^{=1} = 3.35\ \text{m}^2\cdot\text{K/W}$$

So, the insulation will not be sufficient.

ASSESS Notice that the units in the above expression cancel to give the correct result.

81. **INTERPRET** This is a problem involving the thermal energy balance of a home gaining and losing energy through a window. We are interested in the lowest outdoor temperature that would maintain a particular temperature inside.

DEVELOP The home gains energy at a given rate and loses energy at a rate dependent on the temperature difference between the inside and outside. We can equate the heat-loss rate to the energy-input rate and solve for the temperature difference. Knowing the desired indoor temperature, we can then determine the minimum outdoor temperature the house can handle.

EVALUATE The heat-loss rate can be expressed as $60(W/{}^{\circ}\text{C})(\Delta T)$, which when equated to the energy-input rate of 2.1 kW results in a temperature difference equal to 35°C. This means that if the temperature is to be 20°C indoors, the minimum outdoor temperature is -15°C.

ASSESS We are told this is a well-insulated home, so it is not surprising that the heating system can work with such a large temperature difference.

82. **INTERPRET** We're asked to compute the temperature inside a greenhouse given a time-varying solar input.

DEVELOP We'll assume the Sun's energy only enters through the windows $\left(A_{\mathrm{w}}=250\,\mathrm{ft}^2\right)$, in which case the rate of heat gain from the Sun is

$$P_{\text{in}}=SA_{\mathrm{w}}=\left(40\ \text{Btu/h/ft}^2\cdot\sin^2\left(\tfrac{\pi}{24}t\right)\right)\left(250\ \text{ft}^2\right)=1.0\times10^4\,\text{Btu/h}\cdot\sin^2\left(\tfrac{\pi}{24}t\right)$$

The rate of heat loss was computed in Example 16.7:

$$P_{\text{out}}=H_{\text{tot}}=(149\ \text{Btu/h/°F})(T-T_{\text{out}})$$

where T is the indoor temperature, and we assume that the outdoor temperature remains constant throughout the day: $T_{\text{out}}=15°\text{F}$. The net heat exchange will cause the indoor temperature to change according to

$$P_{\text{in}}-P_{\text{out}}=\frac{dQ}{dt}=C\frac{dT}{dt}=(1500\ \text{Btu/°F})\frac{dT}{dt}$$

This is a linear first-order differential equation. We set $y=T-T_{\text{out}}$, such that:

$$\frac{dy}{dt}+Ay=B\sin^2(\omega t)$$

where $A=0.0993\ \text{h}^{-1}$, $B=6.67°\text{F/h}$, and $\omega=\pi/24\text{h}$.

EVALUATE One can solve the differential equation with a computer program or a calculator. We will solve it analytically. If we multiply both sides of the equation by e^{At}, then the solution for $y(t)$ has the form

$$y(t)=e^{-At}\left[\int e^{At}\cdot B\sin^2(\omega t)\,dt+D\right]$$

where D is an integration constant. One can find the integral in a table:

$$y(t)=\frac{BA}{A^2+4\omega^2}\left[\sin^2(\omega t)-\frac{2\omega}{A}\sin(\omega t)\cos(\omega t)+\frac{2\omega^2}{A^2}\right]+De^{-At}$$

We will neglect the exponential term because it will decay away, so we are left with

$$y(t)=\frac{B}{2A}\left[1-\frac{A^2\cos\omega t+2A\omega\sin\omega t}{A^2+4\omega^2}\right]$$

To find the maximum and minimum of $y(t)$, we take the derivative and set it to zero. The extrema occur when $\tan\omega t=2\omega/A$, which corresponds to $\omega t=1.21$ and $\omega t=4.35$. Substituting these values back into the original equation, we find the minimum and maximum values of $y(t)$ are $22°\text{F}$ and $45°\text{F}$, respectively. Adding these values to the outdoor temperature, the minimum and maximum indoor temperatures are $37°\text{F}$ and $60°\text{F}$.

ASSESS The average temperature in the greenhouse is $48.5°\text{F}$, which is $33.5°\text{F}$ above the outdoor temperature. Notice that this is exactly half the temperature difference found in Example 16.7 $(\Delta T=67°\text{F})$. This makes sense, since the average solar input in this problem is half of what it was in Example 16.7:

$$\langle S\rangle=\left\langle 40\ \text{Btu/h/ft}^2\cdot\sin^2\left(\tfrac{\pi}{24}t\right)\right\rangle=20\ \text{Btu/h/ft}^2$$

Here, we've used the fact that the average of $\sin^2$ is ½.

83. **INTERPRET** We consider the physical properties of fiberglass insulation.

DEVELOP The thermal resistance, which measures the level of insulation, is proportional to the inverse of the thermal conductivity. So a low thermal conductivity implies a high level of insulation.

EVALUATE From Table 16.1, glass has a thermal conductivity of around $k=0.8\ \text{W/m}\cdot\text{K}$, whereas air trapped between the fibers has $k=0.026\ \text{W/m}\cdot\text{K}$. So the air seems to be the more important element as far as the insulating quality is concerned.

The answer is (c).

ASSESS The logic here also applies to double pane (and even triple pane) windows. Having a thin layer of air between thin sheets of glass provides much better insulation than having a thick solid sheet of glass.

84. **INTERPRET** We consider the physical properties of fiberglass insulation.

DEVELOP Aluminum foil has a very high thermal conductivity, $k = 237$ W/m·K, so it's definitely not being used to reduce heat loss by conduction. It will help prevent air from flowing through the fiberglass, but that's usually not a problem in an attic or a wall, where the air is pretty still.

EVALUATE Aluminum is a good reflector of radiation, so it will reflect back radiation emitted from the fiberglass. This will help to reduce heat loss from radiation.

The answer is (c).

ASSESS The reflectivity is a measure of how good a material is at reflecting radiation. It is equal to $1-e$, where e is the emissivity. Since e is a measure of absorption as well as emission, we can understand that a good reflector is a bad absorber. Aluminum foil has an emissivity of 0.03, which is why it is a good reflector.

85. **INTERPRET** We consider the physical properties of fiberglass insulation.

DEVELOP We're told that a 6-inch fiberglass has an $\mathcal{R}$-factor of 19.

EVALUATE As defined in Equation 16.8: $\mathcal{R} = \Delta x / k$. So doubling the thickness to 12 inches should double the $\mathcal{R}$-factor to 38.

The answer is (a).

ASSESS For the most part, two sheets of a 6-inch fiberglass should provide the same insulation (i.e., equal $\mathcal{R}$-factor) as one sheet of a 12-inch fiberglass.

86. **INTERPRET** We consider the physical properties of fiberglass insulation.

DEVELOP Squeezing a fiberglass sheet will reduce the amount of air trapped between the glass fibers. By cramming two sheets into the space of one, we would essentially be replacing trapped air with glass fibers.

EVALUATE As we argued in Problem 16.78, the trapped air is providing a large part of the insulation thanks to its low thermal conductivity. Therefore, squeezing the air out will reduce the overall $\mathcal{R}$-factor.

The answer is (c).

ASSESS One might imagine that the best insulation would be a layer of air, with only a thin shell to keep it in place. In fact, that's the logic behind double-pane windows. However, if the air layer is too thick, you start to have convection, which vastly reduces the insulation quality.