23

ELECTROSTATIC ENERGY AND CAPACITORS

EXERCISES

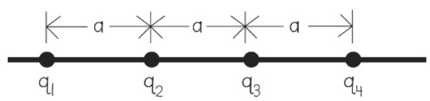
Section 23.1 Electrostatic Energy

11. **INTERPRET** We are to find the work required to assemble a linear sequence of charges.

DEVELOP We use the technique described for assembling the three charges in Fig. 23.1. For this problem, we have four charges, to be arranged as shown in the figure below. Number the charges $q_i = 75\ \mu\text{C},\ i = 1,\ 2,\ 3,\ 4,$ as they are spaced along the line at a = 5.0 cm intervals. There are six pairs, so

$$W = \sum_{\text{pairs}} \frac{kq_i q_j}{r_{ij}}$$

which we can evaluate to find the work W.



EVALUATE Evaluating the expression above gives

$$W = k\left(\frac{q_1q_2}{a} + \frac{q_1q_3}{2a} + \frac{q_1q_4}{3a} + \frac{q_2q_3}{a} + \frac{q_2q_4}{2a} + \frac{q_3q_4}{a}\right)$$

$$= \frac{kq^2}{a}\left(1 + \tfrac{1}{2} + \tfrac{1}{3} + 1 + \tfrac{1}{2} + 1\right) = \frac{13kq^2}{3a} = \frac{13\left(9.0\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(75\ \mu\text{C}\right)^2}{3.0\times 0.050\ \text{m}} = 4.4\ \text{kJ}$$

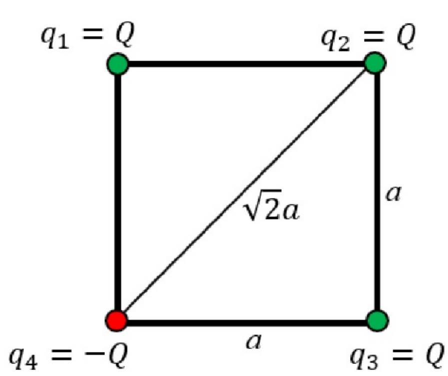
ASSESS The work required does not depend on how the charge configuration is assembled, only on its final state.

12. **INTERPRET** This problem is similar to the preceding problem, except that the final geometry of the assembled point charges is different (and the point charges do not all have the same charge). We can thus apply the same strategy as in Exercise 23.11 to find the electrostatic energy for this collection of point charges.

DEVELOP We again use the strategy outlined in the discussion accompanying Fig. 23.1. The point charges are arranged on the corners of a square. Three of them have charge $q_1 = q_2 = q_3 = +Q$, and the fourth has charge $q_4 = -Q.$ Make a sketch of the final charge distribution and label the charges (see the figure below). Again, there are six pairs of charges, so the work W required to assemble them is

$$W = \sum_{\text{pairs}} \frac{kq_i q_j}{r_{ij}}$$

which we can evaluate.



EVALUATE Evaluating the expression for work gives

$$W = k\left(\frac{q_1q_2}{a} + \frac{q_1q_3}{\sqrt{2}a} + \frac{q_1q_4}{a} + \frac{q_2q_3}{a} + \frac{q_2q_4}{\sqrt{2}a} + \frac{q_3q_4}{a}\right)$$
$$= \frac{kq^2}{a}\left(1 + \frac{1}{\sqrt{2}} - 1 + 1 - \frac{1}{\sqrt{2}} - 1\right) = 0$$

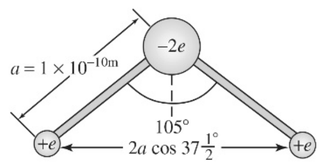
ASSESS This is not a particularly convenient method of calculating energies. Fortunately, the charge on a single electron is small enough that we can usually approximate real charge distributions as continuous and integrate.

13. **INTERPRET** For this problem, we are to find the work required to assemble a crude model of a water molecule. Note that if the work is negative, then energy is released in forming the molecule.

DEVELOP In this approximation, electrostatic potential energy of the water molecule (i.e., the work required to assemble the molecule) is

$$U = W = \sum_{\text{pairs}} \frac{kq_iq_j}{r_{ij}}$$

The two oxygen-hydrogen pairs have separation $a = 10^{-10}$ m, while the hydrogen-hydrogen pair has separation $2a\cos(37.5°) = 1.59a$.



EVALUATE Evaluating this expression gives

$$U = 2ke\left(\frac{-2e}{a}\right) + ke\left(\frac{e}{1.59a}\right) = -\frac{3.37ke^2}{a}$$
$$= -3.37\left(9.0\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19} \text{ C}\right)^2 = -7.76\times10^{-18} \text{ J} = -48.5 \text{ eV}.$$

ASSESS Because the potential energy is negative, assembling this molecules releases energy (or does work). Note that the electrostatic potential energy of the assembled molecule is with respect to the constituents being infinitely far apart, so the work done equates to the change in potential energy caused by bringing the charges together from infinity.

Section 23.2 Capacitors

14. **INTERPRET** This problem is about a parallel-plate capacitor. We are given the plate separation and the charges on the plates and are asked to find the electric field between the plates, the potential difference between the plates, and the energy stored in the capacitor.

DEVELOP The electric field between two closely spaced, oppositely charged, parallel conducting plates is approximately uniform (directed from the positive to the negative plate), with strength (see Equation 21.8)

$$E = \frac{\sigma}{\epsilon_0} = \frac{q}{\epsilon_0 A}$$

Since the electric field is uniform, the potential difference between the plates is given by Equation 22.1b, $V = Ed$, where d is the plate separation. Finally, the energy stored in the capacitor can be calculated using Equation 23.3: $U = CV^2/2 = qV/2$ where $q = CV$.

EVALUATE **(a)** Using the equation above, the electric field is

$$E = \frac{q}{\epsilon_0 A} = \frac{1.1\,\mu\text{C}}{\left[8.85\times10^{-12} \text{ C}^2/\left(\text{N}\cdot\text{m}^2\right)\right](0.25 \text{ m})^2} = 2.0 \text{ MV/m}$$

(b) The potential difference is

$$V = Ed = (1.99 \text{ MV/m})(0.0050 \text{ m}) = 9.9 \text{ kV}$$

(c) The energy stored is

$$U = \frac{1}{2}CV^2 = \frac{1}{2}qV = \frac{1}{2}(1.1\ \mu\text{C})(9.94\ \text{kV}) = 5.5\ \text{mJ}$$

ASSESS Note that the final results are given to two significant figures, as warranted by the data. When used as intermediate results, however, three significant figures are retained. For completeness, the capacitance of the capacitor is

$$C = \frac{\epsilon_0 A}{d} = \frac{\left[8.85\times10^{-12}\ \text{C}^2/\left(\text{N}\cdot\text{m}^2\right)\right](0.25\ \text{m})^2}{5.0\times10^{-3}\ \text{m}} = 1.1\times10^{-10}\,\text{F} = 0.11\,\text{nF}$$

The value is typical of a capacitor.

15. **INTERPRET** We are to find the work required to charge a capacitor with the given charge, then find the additional work required to double the charge.

DEVELOP The separation between capacitor plates is much smaller than the linear dimensions of the plates, so the discussion in Section 23.2 applies. From Equation 23.3, we see that the work is

$$W = \frac{1}{2}CV^2$$

where V is the final voltage and may be expressed using Equation 23.1, $C = Q/V$. This gives

$$W = \frac{1}{2}C\left(\frac{Q}{C}\right)^2 = \frac{1}{2}\frac{Q^2}{C}$$

The capacitance can be expressed in terms of the geometry of the capacity (Equation 23.2), $C = \epsilon_0 A/d$, which leads to

$$W = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\frac{Q^2 d}{\epsilon_0 A}$$

EVALUATE (**a**) The work required to transfer $Q = 7.2\ \mu\text{C}$ is

$$W = \frac{1}{2}\frac{Q^2 d}{\epsilon_0 A} = \frac{1}{2}\frac{(7.2\ \mu\text{C})(0.0012\ \text{m})}{\left[8.85\times10^{-12}\ \text{C}^2/\left(\text{N}\cdot\text{m}^2\right)\right](0.050\ \text{m})^2} = 1.4\ \text{J}$$

(**b**) The additional work required to double the charge on each plate is

$$\Delta W = \frac{(2Q)^2 d}{2\epsilon_0 A} - W = 3W = 4.2\ \text{J}$$

ASSESS This energy is stored in the capacitor and can be released by electrically connecting the two capacitor faces.

16. **INTERPRET** This problem is about the energy stored in a parallel-plate capacitor. We are to find the charge on each plate (oppositely charged, of course) needed to store the given energy, and the resulting electric potential between the plates.

DEVELOP Using the expression from Problem 23.15 for the work (i.e., change in potential energy) required to charge the plates, we can solve for the charge:

$$U = \frac{Q^2 d}{2\epsilon_0 A} \quad \Rightarrow \quad Q = \sqrt{\frac{2\epsilon_0 AU}{d}}$$

Once Q is known, the potential difference V between the plates is

$$U = \frac{1}{2}CV^2 = \frac{1}{2}(CV)V = \frac{1}{2}QV \quad \Rightarrow \quad V = \frac{2U}{Q}$$

EVALUATE **(a)** Using the values given in the problem statement, we find the charge to be

$$Q = \sqrt{\frac{2\epsilon_0 AU}{d}} = \sqrt{\frac{2\left[8.85\times10^{-12}\ \text{C}^2/\left(\text{N}\cdot\text{m}^2\right)\right](0.050\ \text{m})^2\left(15\times10^{-3}\ \text{J}\right)}{1.2\times10^{-3}\ \text{m}}} = 0.74\,\mu\text{C}$$

(b) The potential difference is

$$V=\frac{2U}{Q}=\frac{2\left(15\times10^{-3}\text{ J}\right)}{0.744\times10^{-6}\text{ C}}=40\text{ kV}$$

Assess Since the electric field between the plates is $E=\sigma/\epsilon_0$ the potential can also be found using

$$V=\frac{E}{d}=\frac{\sigma}{\epsilon_0 d}=\frac{Qd}{\epsilon_0 A}=40\text{ kV}$$

17. **Interpret** We are given the charge and voltage of a capacitor and are to find the capacitance.
Develop Apply Equation 2.31, $C=Q/V$.
Evaluate From Equation 23.1, $C=Q/V=(1\ \mu\text{C})/(72\text{ V})=14\text{ nF}$.

Assess The capacitance is the charge per unit voltage.

18. **Interpret** We are to derive an alternate form of the units of ϵ_0, the permittivity constant.
Develop The value of ϵ_0 is $8.85\times10^{-12}\text{ C}^2/\left(\text{N}\cdot\text{m}^2\right)$ with units $\text{C}^2/\left(\text{N}\cdot\text{m}^2\right)$. However, because $C=Q/V$, 1 F (farad) is equivalent to 1 C/V.
Evaluate From the above, we see that the units of ϵ_0 are

$$\text{C}^2/\left(\text{N}\cdot\text{m}^2\right)=\left[\text{C}/(\text{N}\cdot\text{m})\right](\text{C/m})=(\text{C/J})(\text{C/m})=\text{C}/(\text{V}\cdot\text{m})=\text{F/m}$$

Assess To see that the result makes sense, we can use Equation 23.2 and write

$$\epsilon_0=\frac{Cd}{A}$$

The units of C are F, while the units of d/A are m^{-1}.

19. **Interpret** We are given the separation of a parallel-plate capacitor, its charge, and its voltage and are to find its capacitance.
Develop Because d (the plate separation) $<<$ r (the radius of the capacitor), we apply Equation 23.2, $C=\varepsilon_0 A/d$, with $A=\pi r^2$.
Evaluate Inserting the given quantities gives

$$C=\frac{\varepsilon_0\pi r^2}{d}=\frac{\left(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2\right)\pi(0.18\text{ m})^2}{0.0025}=360\text{ pF}$$

to two significant figures.
Assess This is a typical value for a capacitance.

20. **Interpret** For a parallel-plate capacitor, we are given the plate spacing, the charge, and the voltage and are to find the plate area.
Develop Combining Equations 23.1 and 23.2 for capacitance gives

$$C=\frac{Q}{V}=\frac{\epsilon_0 A}{d}\quad\Rightarrow\quad A=\frac{Qd}{\epsilon_0 V}$$

Evaluate Inserting the given quantities gives

$$A=\frac{Qd}{\epsilon_0 V}=\frac{(2.9\times10^{-6}\text{ C})(1.2\times10^{-3}\text{ m})}{\left[8.85\times10^{-12}\text{ C}^2/(\text{N}\cdot\text{m}^2)\right](230\text{ V})}=1.7\text{ m}^2$$

Assess If we take the plate to be square, then each side has a length of 1.3 m, which is indeed much greater than the distance of 1.2 mm between the plates. Note: Expressing voltage in its SI units of $\text{N}\cdot\text{m/C}$ gives the proper units.

21. **Interpret** We are given the capacitance and the voltage of a capacitor and are to find the stored energy.
Develop Apply Equation 23.3, $U=CV^2/2$.
Evaluate From Equation 23.3,

$$U_C = \frac{1}{2}CV^2 = \frac{1}{2}(35\ \text{F})(1.5\ \text{V})^2 = 39\ \text{J}$$

ASSESS This is the energy it would take to lift 1.0 liter of water through a height of 40 cm.

$$U = mgh$$

$$h = \frac{U}{mg} = \frac{39\ \text{J}}{(1.0\ \text{kg})(9.8\ \text{m/s}^2)} = 40\ \text{cm}$$

Section 23.3 Using Capacitors

22. **INTERPRET** This problem involves calculating the equivalent capacitance for the two given capacitors connected in series or in parallel.

DEVELOP Apply Equation 23.5 to find the equivalent series capacitance and Equation 23.6b for the equivalent parallel capacitance.

EVALUATE In parallel, the capacitance is

$$C = C_1 + C_2 = 1.4\ \mu\text{F} + 2.6\ \mu\text{F} = 4.0\ \mu\text{F}$$

In series, the capacitance is

$$C = \frac{C_1C_2}{C_1 + C_2} = 0.91\ \mu\text{F}$$

ASSESS Connecting the capacitors in parallel results in a higher equivalent capacitance than connecting them in series.

23. **INTERPRET** This problem requires us to find the equivalent capacitance of the given arrangement of individual capacitors, which combines series and parallel connections.

DEVELOP For part **(a)**, compute the equivalent capacitance of C_2 and C_3 (which are in parallel), then combine this in series with C_1 to find the overall capacitance. For part **(b)**, note that the charge on C_2 and C_3 must be the same as on C_1, which must be the same as the total charge (see discussion accompanying Fig. 23.8), so

$$Q_\text{T} = Q_1 = Q_2 + Q_3$$

In addition, the total charge and capacitance must satisfy Equation 23.1,

$$C_\text{T} = \frac{Q_T}{V_\text{T}}$$

where V_T = 12.0 V. Finally, the voltage drop across C_2 must be the same as across C_3, so

$$V_2 = Q_2/C_2 = V_3 = Q_3/C_3$$

$$Q_2 = Q_3\frac{C_2}{C_3}$$

EVALUATE (**a**) Capacitors C_2 and C_3 combined in parallel give an equivalent capacitance of

$$C_{2,3} = C_1 + C_2 = 1.0\ \mu\text{F} + 2.0\ \mu\text{F} = 3.0\ \mu\text{F}$$

Combining this in series with C_1 gives

$$C_\text{T} = \frac{C_1C_{2,3}}{C_1 + C_{2,3}} = \frac{(2.0\ \mu\text{F})(3.0\ \mu\text{F})}{2.0\ \mu\text{F} + 3.0\ \mu\text{F}} = 1.2\ \mu\text{F}$$

(**b**) Knowing C_T and V_T, we can find Q_T, which must be the same as Q_1 (see Example 23.3)

$$C_\text{T} = \frac{Q_T}{V_\text{T}} = \frac{Q_1}{V_\text{T}}$$

$$Q_1 = C_TV_T = (1.2\ \mu\text{F})(12.0\ \text{V}) = 14.4\ \mu\text{C}$$

Knowing Q_1, we can solve for Q_2 and Q_3 using the expressions above. The result is

$$Q_1 = Q_2 + Q_3 = Q_3\frac{C_2}{C_3} + Q_3 = Q_3\left(1+\frac{C_2}{C_3}\right)$$

$$Q_3 = \frac{Q_1}{1+C_2/C_3} = \frac{14.4\ \mu\text{C}}{1+1/2} = 9.60\ \mu\text{C}$$

Finally, Q_2 is

$$Q_2 = Q_3\frac{C_2}{C_3} = (9.60\ \mu\text{C})\frac{1}{2} = 4.80\ \mu\text{C}$$

(**c**) The voltage on C_1 can be found using Equation 23.1. The result is

$$V_1 = \frac{Q_1}{C_1} = \frac{14.4\ \mu\text{C}}{2.0\ \mu\text{F}} = 7.2\ \text{V}$$

$$V_2 = V_3 = V_\text{T} - V_1 = 12.0\ \text{V} - 7.2\ \text{V} = 4.8\ \text{V}$$

ASSESS The voltage across the parallel capacitors may also be found using Equation 23.1:

$$V_2 = V_3 = \frac{Q_2}{C_2} = \frac{Q_3}{C_3} = 4.8\ \text{V}$$

24. **INTERPRET** This problem requires us to find all possible equivalent capacitances that can be obtained by connecting three capacitors in different ways (serial, parallel, and combinations thereof).

DEVELOP The equivalent capacitance C is maximized when all capacitors are connected in parallel and is given by (Equation 23.5)

$$C = C_1 + C_2 + C_3$$

On the other hand, C is minimized when the capacitors are connected in series. For three capacitors connected in series, the equivalent capacitance C is given by (Equation 23.6)

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \Rightarrow C = \frac{C_1C_2C_3}{C_1C_2 + C_1C_3 + C_2C_3}$$

Intermediate values are obtained when one capacitor is in parallel with the other two in series, or one in series with the other two in parallel.

EVALUATE **(a)** When all capacitors are in parallel, we have

$$C = C_1 + C_2 + C_3 = 1.3\ \mu\text{F} + 2.3\ \mu\text{F} + 3.3\ \mu\text{F} = 6.9\ \mu\text{F}$$

(b) When all are in series, the equivalent capacitance is

$$C = \frac{C_1C_2C_3}{C_1C_2 + C_1C_3 + C_2C_3} = 0.66\ \mu\text{F}$$

(c) When one is in parallel with the other two in series, the possible values are

$$C = C_1 + \frac{C_2C_3}{C_2 + C_3} = 2.7\ \mu\text{F}$$

$$C = C_2 + \frac{C_1C_3}{C_1 + C_3} = 3.2\ \mu\text{F}$$

$$C = C_3 + \frac{C_1C_2}{C_1 + C_2} = 4.1\ \mu\text{F}$$

Similarly, when one is in series with the other two in parallel, the equivalent capacitance is

$$\frac{1}{C} = \frac{1}{C_i} + \frac{1}{C_j + C_k} \Rightarrow C = \frac{C_i(C_j + C_k)}{C_i + C_j + C_k}$$

Therefore, the possible values are

$$C = \frac{C_1(C_2 + C_3)}{C_1 + C_2 + C_3} = 1.1\ \mu\text{F}$$

$$C = \frac{C_2(C_1 + C_3)}{C_2 + C_1 + C_3} = 1.5\ \mu\text{F}$$

$$C = \frac{C_3(C_1 + C_2)}{C_3 + C_1 + C_2} = 1.7\ \mu\text{F}$$

ASSESS With three capacitors, each having two options (parallel or series), there are eight possible outcomes $(=2^3)$.

Section 23.4 Energy in the Electric Field

25. **INTERPRET** This problem involves finding the uniform electric field that carries the given energy density.

DEVELOP Apply Equation 23.7, $(E = \sqrt{2u/\epsilon_0})$, which relates the field strength and the electric energy density.

EVALUATE Inserting the given energy density into Equation 23.7 gives

$$E = \sqrt{2u/\epsilon_0} = \sqrt{\frac{2(3.0\ \text{J/m}^3)}{8.85\times10^{-12}\ \text{F/m}}} = 8.2\times10^5\ \text{V/m}$$

ASSESS The manipulation of units is facilitated by the relations V = J/C and F = C/V. Thus,

$$(\text{J/m}^3)/(\text{F/m}) = \frac{\text{VC/m}^3}{\text{C}/(\text{V}\cdot\text{m})} = \text{V/m}^2$$

26. **INTERPRET** This problem involves the volume required for storing a given amount of electrostatic energy.

DEVELOP For a uniform electric field, Equation 23.8 can be written as

$$U = \frac{1}{2}\varepsilon_0 E^2 \times \text{volume}$$

Knowing U and E allows us to find the volume.

EVALUATE Substituting the values given in the problem statement gives the volume as

$$\text{volume} = \frac{2U}{\varepsilon_0 E^2} = \frac{2(3.6\ \text{MJ})}{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(32\ \text{kV/m})^2} = 0.79\ \text{km}^3$$

ASSESS This is a very large volume occupied by a car battery. In reality, not all the stored energy goes into creating the field.

27. **INTERPRET** This problem involves finding the maximum electrical energy density possible in air.

DEVELOP From Appendix C, we find that the energy content of gasoline is 44×10^6 J/kg, and the density of gasoline is $670\ \text{kg/m}^3$, so the equivalent energy density is

$$u_{\text{gas}} = (44\times10^6\ \text{J/kg})(670\ \text{kg/m}^3) = 2.95\times10^{10}\ \text{J/m}^3$$

EVALUATE From Equation 23.7, the field strength giving the same electrostatic energy density is

$$E = \sqrt{2u/\epsilon_0} = \sqrt{\frac{2(2.95\times10^{10}\ \text{J/m}^3)}{8.85\times10^{-12}\ \text{F/m}}} = 8.16\times10^{10}\ \text{V/m}$$

which greatly exceeds the breakdown field in air.

ASSESS Gasoline is actually a very dense form of energy storage, which is one reason it is hard to replace!

28. **INTERPRET** In this problem we are asked to find the electric energy stored in a proton by assuming it to be a uniformly charged sphere.

DEVELOP For this model of the proton, the field strength at the surface is $E = ke/R^2$ (from spherical symmetry and Gauss's law). Thus, the energy density in the surface electric field is

$$u = \frac{1}{2}\epsilon_0 E^2 = \frac{1}{2}\frac{1}{4\pi k}\left(\frac{ke}{R^2}\right)^2 = \frac{ke^2}{8\pi R^4}$$

EVALUATE With $R = 1\text{ fm} = 1\times 10^{-15}\text{ m},$ the energy density is

$$u = \frac{ke^2}{8\pi R^4} = \frac{(9\times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(1.6\times 10^{-19}\text{ C})^2}{8\pi(1\times 10^{-15}\text{m})^4} = 9.17\times 10^{30}\text{ J/m}^3 = 57\text{ keV/fm}^3$$

ASSESS The energy density is enormous, given the small size of the proton.

Example Variations

29. **Interpret** This problem is about an electric circuit—in this case, an assemblage of three capacitors.
Develop To handle such circuit problems, we find combinations of series and parallel components, and then simplify the circuit by treating each combination as a single component.
Evaluate We begin by noting that C_2 and C_3 are in parallel, so the equivalent capacitance is given by Equation 23.5: $C_{23} = C_2 + C_3 = 6.9\,\mu\text{F}$. In Fig. 23.9b the redrawn circuit shows this combination of the two individual capacitors. Next, we see that C_1 is in series with C_{23}, so their equivalent capacitance follows from Equation 23.6b:

$$C_{123} = \frac{C_1 C_{23}}{C_1 + C_{23}} = \frac{(6.8\,\mu\text{F})(6.9\,\mu\text{F})}{6.8\,\mu\text{F} + 6.9\,\mu\text{F}} = 3.4\,\mu\text{F}$$

Assess These are common commercially available capacitances and are often used when building circuits in an electronic course.

30. **Interpret** This problem is about an electric circuit—in this case, an assemblage of three capacitors.
Develop To handle such circuit problems, we find combinations of series and parallel components, and then simplify the circuit by treating each combination as a single component.
Evaluate We begin by noting that C_2 and C_3 are in parallel, so the equivalent capacitance is given by Equation 23.5: $C_{23} = C_2 + C_3 = 4.0\,\mu\text{F}$. In Fig. 23.9b the redrawn circuit shows this combination of the two individual capacitors. Next, we see that C_1 is in series with C_{23}, so their equivalent capacitance follows from Equation 23.6b:

$$C_{123} = \frac{C_1 C_{23}}{C_1 + C_{23}} = \frac{(12\,\mu\text{F})(4.0\,\mu\text{F})}{12\,\mu\text{F} + 4.0\,\mu\text{F}} = 3.0\,\mu\text{F}$$

Since the charge that accumulates on C_{23} will accumulate on C_{123}, then: $Q_{123} = Q_{23} \rightarrow V_{23}C_{23} = V_{123}C_{123}$. Meaning that if a voltage of 48 V is desired across C_2, the same voltage will be across C_3 and thus C_{23}, making the necessary voltage across points *A* and *B* (i.e. the voltage across the equivalent capacitor C_{123}) equal to:

$$V_{123} = V_{23}\frac{C_{23}}{C_{123}} = 48\,\text{V}\left(\frac{4.0\,\mu\text{F}}{3.0\,\mu\text{F}}\right) = 64\text{ V}$$

Assess These are common commercially available capacitances and are often used when building circuits in an electronic course.

31. **Interpret** This problem is about an electric circuit—in this case, an assemblage of four capacitors.
Develop To handle such circuit problems, we find combinations of series and parallel components, and then simplify the circuit by treating each combination as a single component.
Evaluate We will refer to the capacitors in Fig. 23.14 as follows: $C_1 = 3.3\,\mu\text{F}$, $C_2 = 2.7\,\mu\text{F}$, $C_3 = 1.8\,\mu\text{F}$, $C_4 = 5.6\,\mu\text{F}$. We begin by noting that C_2 and C_3 are in parallel, so the equivalent capacitance is given by Equation 23.5: $C_{23} = C_2 + C_3 = 4.5\,\mu\text{F}$. Next we see that C_1 is in series with C_{23} and C_4, so their equivalent capacitance follows from Equation 23.6b:

$$C_{1234} = \left(\frac{1}{C_1} + \frac{1}{C_{23}} + \frac{1}{C_4}\right)^{-1} = \frac{C_1 C_{23} C_4}{(C_1 C_{23}) + (C_{23}C_4) + (C_1 C_4)}$$

Plugging in the given capacitances we obtain $C_{1234} = 1.4\,\mu\text{F}$.
Assess These are common commercially available capacitances and are often used when building circuits in an electronic course.

32. **Interpret** This problem is about an electric circuit—in this case, an assemblage of four capacitors.
Develop To handle such circuit problems, we find combinations of series and parallel components, and then simplify the circuit by treating each combination as a single component.
Evaluate We will refer to the capacitors in Fig. 23.14 as follows: $C_1 = 3.3\,\mu\text{F}$, $C_2 = 2.7\,\mu\text{F}$, $C_3 = 1.8\,\mu\text{F}$, $C_4 = 5.6\,\mu\text{F}$. We begin by noting that C_2 and C_3 are in parallel, so the equivalent capacitance is given by Equation 23.5: $C_{23} = C_2 + C_3 = 4.5\,\mu\text{F}$. Next we see that C_1 is in series with C_{23} and C_4, so their equivalent capacitance follows from Equation 23.6b:

$$C_{1234} = \left(\frac{1}{C_1} + \frac{1}{C_{23}} + \frac{1}{C_4}\right)^{-1} = \frac{C_1 C_{23} C_4}{(C_1 C_{23}) + (C_{23} C_4) + (C_1 C_4)}$$

Plugging in the given capacitances we obtain $C_{1234} = 1.4\ \mu\text{F}$. Since these three capacitors are all in series, they all accumulate the same amount of charge. This means we can find the potential difference across C_2 by noting that: $Q_{1234} = Q_{23} \rightarrow V_{23} C_{23} = V_{1234} C_{1234}$. This is because the voltage across C_2 is the same voltage across C_3 and thus C_{23}. Knowing the total voltage applied between points *A* and *B* (i.e., V_{1234}), we find that the voltage across the 2.7-μF capacitor is equal to: $V_{23} = V_{1234}\dfrac{C_{1234}}{C_{23}} = 75\,\text{V}\left(\frac{1.42\ \mu\text{F}}{4.5\ \mu\text{F}}\right) = 24\,\text{V}$. Using Equation 23.3 we find that the energy stored in this capacitor is equal to: $U_C = \frac{1}{2}CV^2 = \frac{1}{2}(2.7\ \mu\text{F})(23.7\ \text{V})^2 = 0.76\ \text{mJ}$.

ASSESS These are common commercially available capacitances and are often used when building circuits in an electronic course.

33. **INTERPRET** This problem asks for the work done in rearranging a charge distribution, which we know is equal to the change in stored electric energy. Here we start with a charged sphere already assembled and rearrange the charge by shrinking the sphere to a smaller radius.

DEVELOP We have spherical symmetry, so the field and thus the stored energy outside the original radius R_1 don't change. Therefore, we need to find the energy stored in the new field created when the sphere shrinks. Figure 23.10 shows a sketch of the situation before and after the sphere shrinks. Here the field varies with position, so Equation 23.8: $U = \frac{1}{2}\epsilon_0 \int E^2 dV$ gives the stored energy. Our plan is to evaluate the field in the region $R_2 < r < R_1$ and use the result in Equation 23.8. Then we can evaluate the expression for the case where the shell shrinks to half its original radius: $\left(R_2 = \frac{1}{2}R_1\right)$.

EVALUATE Evaluating the energy integral using thin spherical shells for our volume element we arrive at:

$$U = \frac{1}{2}\epsilon_0 \int E^2 dV = \frac{1}{2}\epsilon_0 \int_{R_1}^{R_2} \left(\frac{kQ}{r^2}\right)^2 4\pi r^2 dr = \frac{kQ^2}{2}\int_{R_1}^{R_2} r^{-2} dr$$

$$U = \frac{kQ^2}{2}\left(-\frac{1}{r}\right)\Bigg|_{R_1}^{\frac{1}{2}R_1} = \frac{kQ^2}{2}\left(\frac{2}{R_1} - \frac{1}{R_1}\right) = \frac{kQ^2}{2R_1}$$

ASSESS Here $R_2 < R_1$, so the stored energy is positive and indicates that this much work had to be done to shrink the sphere.

34. **INTERPRET** This problem asks for an estimate to a proton's radius. Using the work done in rearranging a charge distribution, which we know is equal to the change in stored electric energy, we can determine the radius. This is done by considering the original radius to be infinite, and the final radius to be the value we are after.

DEVELOP We have spherical symmetry, so the field and thus the stored energy outside the original radius R_1 don't change. Therefore, we need to find the energy stored in the new field created when the sphere shrinks. Here the field varies with position, so Equation 23.8: $U = \frac{1}{2}\epsilon_0 \int E^2 dV$ gives the stored energy. Our plan is to evaluate the field in the region $R_2 < r < R_1$ and use the result in Equation 23.8. Then we can evaluate the expression for the case where the shell shrinks to a radius R_2 from an original radius of $R_1 = \infty$.

EVALUATE Evaluating the energy integral using thin spherical shells for our volume element we arrive at:

$$U = \frac{1}{2}\epsilon_0 \int E^2 dV = \frac{1}{2}\epsilon_0 \int_{R_1}^{R_2} \left(\frac{kQ}{r^2}\right)^2 4\pi r^2 dr = \frac{kQ^2}{2}\int_{R_1}^{R_2} r^{-2} dr$$

$$U = \frac{kQ^2}{2}\left(-\frac{1}{r}\right)\Bigg|_{\infty}^{R_2} = \frac{kQ^2}{2}\left(\frac{1}{R_2} - 0\right) = \frac{kQ^2}{2R_2}$$

Solving for R_2 and using the elementary charge for *Q* and the given energy stored in a proton's electric field for *U* we obtain a radius of

$$R_2 = \frac{kQ^2}{2U} = \frac{\left(9.0\times10^9\,\mathrm{N\cdot m^2/C^2}\right)\left(1.6\times10^{-19}\,\mathrm{C}\right)^2}{2\left(130\times10^{-15}\,\mathrm{J}\right)} = 0.9\ \mathrm{fm}$$

ASSESS Although this is an unrealistic estimate, this value is close to the accepted value of approximately 1 femtometer.

35. **INTERPRET** This problem asks us to calculate the electrostatic energy contained within a sphere containing a charge spread uniformly through its volume.

DEVELOP Example 21.3 gives the electric field inside a uniformly charged sphere as

$$E_{\mathrm{in}} = Qr/4\pi\epsilon_0 R^3$$

We can use Equation 23.8: $U = \frac{1}{2}\epsilon_0\int E^2 dV$, to obtain the stored energy by integrating from 0 to R.

EVALUATE Evaluating the energy integral using thin spherical shells for our volume element we arrive at:

$$U = \frac{1}{2}\epsilon_0\int E^2 dV = \frac{1}{2}\epsilon_0\int_0^R\left(\frac{Qr}{4\pi\epsilon_0 R^3}\right)^2 4\pi r^2 dr = \frac{Q^2}{8\pi\epsilon_0}\frac{1}{R^6}\int_0^R r^4 dr$$

$$U = \frac{Q^2}{8\pi\epsilon_0}\left(\frac{1}{5R^6}r^5\right)\Bigg|_0^R = \frac{Q^2}{8\pi\epsilon_0}\left(\frac{1}{5R}\right) = \frac{Q^2}{40\pi\epsilon_0 R}$$

Or equivalently: $kQ^2/10R$.

ASSESS The energy contained in the space surrounding the sphere can be determined by performing the same calculation using the electric field found outside the uniformly charged sphere: $E_{\mathrm{out}} = Q/4\pi\epsilon_0 r^2$, integrating from R to infinity. This results in $Q^2/8\pi\epsilon_0 R$, meaning the total stored energy due to the uniformly charged sphere is equal to: $3Q^2/20\pi\epsilon_0 R$.

36. **INTERPRET** This problem asks us to estimate to a proton's radius using the preceding problem's results.

DEVELOP Example 21.3 gives the electric field inside and outside a uniformly charged sphere as

$$E_{\mathrm{in}} = Qr/4\pi\epsilon_0 R^3\ ;\ E_{\mathrm{out}} = Q/4\pi\epsilon_0 r^2$$

We can use Equation 23.8: $U = \frac{1}{2}\epsilon_0\int E^2 dV$, to obtain the stored energy by integrating from 0 to infinity.

EVALUATE Evaluating the energy integral using thin spherical shells for our volume element we arrive at:

$$U = \frac{1}{2}\epsilon_0\int E^2 dV = \frac{1}{2}\epsilon_0\left[\int_0^R\left(\frac{Qr}{4\pi\epsilon_0 R^3}\right)^2 4\pi r^2 dr + \int_R^\infty\left(\frac{Q}{4\pi\epsilon_0 r^2}\right)^2 4\pi r^2 dr\right] = \frac{Q}{8\pi\epsilon_0}\left[\frac{1}{R^6}\int_0^R r^4 dr + \int_R^\infty \frac{1}{r^2}dr\right]$$

$$U = \frac{Q^2}{8\pi\epsilon_0}\left[\left(\frac{1}{5R^6}r^5\right)\Bigg|_0^R + \left(-\frac{1}{r}\right)\Bigg|_R^\infty\right] = \frac{Q^2}{8\pi\epsilon_0}\left(\frac{6}{5R}\right) = \frac{3Q^2}{20\pi\epsilon_0 R} = \frac{3}{5}\frac{kQ^2}{R}$$

Using the elementary charge for Q and the given energy stored in a proton's electric field for U we obtain a radius of:

$$R = \frac{3kQ^2}{5U} = \frac{3\left(9.0\times10^9\,\mathrm{N\cdot m^2/C^2}\right)\left(1.6\times10^{-19}\,\mathrm{C}\right)^2}{5\left(130\times10^{-15}\,\mathrm{J}\right)} = 1\,\mathrm{fm}$$

ASSESS While this is still an estimate, this approach considers the total stored energy in the field due to a uniformly charged proton, and results in something closer to the accepted value of approximately 1 femtometer.

PROBLEMS

37. **INTERPRET** This problem involves finding an expression of the work required to assemble the given charge configuration, and using this expression to find the relative charge on one of the charges with respect to the initial charge.

DEVELOP The work necessary to position Q_x is

$$W_x = \frac{kQ_0Q_x}{a} = \frac{2kQ_0^2}{a}$$

while the work necessary to bring up Q_y is

$$W_y = \frac{kQ_0Q_y}{a} + \frac{kQ_xQ_y}{\sqrt{2}a} = \frac{kQ_0Q_y\left(1+\sqrt{2}\right)}{a}$$

Given that $W_y = 2W_x,$ we can solve for Q_y in terms of Q_0.

EVALUATE $W_y = 2W_x$ gives

$$\frac{kQ_0Q_y\left(1+\sqrt{2}\right)}{a} = \frac{4kQ_0^2}{a} \quad\Rightarrow\quad Q_y = \frac{4Q_0}{\sqrt{2}+1}$$

ASSESS More explicitly, we can write $Q_y = 1.66Q_0$, so Q_y is a little less than twice Q_0.

38. **INTERPRET** This problem involves finding the work required to create the given charge distribution. The work is equal to the energy stored in the system.

DEVELOP When a charge q (assumed positive) is on the inner sphere, the potential difference between the spheres is (Equation 22.1a)

$$V = -\int_a^b \vec{E}\cdot d\vec{r} = -\int_a^b \frac{kqdr}{r^2} = kq\left(\frac{1}{a}-\frac{1}{b}\right)$$

where we have used the fact (from Gauss's law; see Example 21.1) that the field outside a spherical charge distribution is the same as a point charge at the center of the sphere. To transfer an additional charge dq from the outer sphere requires work $dW = V\ dq.$

EVALUATE The total work required to transfer charge Q (leaving the spheres oppositely charged) is

$$W = \int_0^Q V\ dq = \int_0^Q kq\left(\frac{1}{a}-\frac{1}{b}\right)dq = \frac{1}{2}kQ^2\left(\frac{1}{a}-\frac{1}{b}\right)$$

ASSESS Since $U = Q^2/2C,$ this shows that the capacitance of this spherical capacitor is

$$C = \frac{1}{k\left(a^{-1}-b^{-1}\right)} = \frac{ab}{k\left(b-a\right)}$$

Note that capacitance depends only on the geometry of the system, and is independent of V and Q.

39. **INTERPRET** This problem requires us to find the charge on a pair of parallel, square conducting plates (i.e., a parallel-plate capacitor), given the energy density in the electric field between the plates.

DEVELOP Combine Equation 23.7, which relates the electric field to the energy density,

$$E = \pm\sqrt{2u/\epsilon_0}$$

with Equation 21.8, which gives the electric field near the surface of a charged conducting plate,

$$E = \sigma/\epsilon_0$$

EVALUATE Eliminating the unknown electric field E and solving for the surface charge density σ gives

$$\sigma = \pm\epsilon_0\sqrt{2u/\epsilon_0} = \pm\sqrt{2u\epsilon_0}$$

Using the given surface area, the total charge on a plate is

$$q = \sigma A = \pm A\sqrt{2u\epsilon_0} = \pm(0.12\text{ m})^2\sqrt{2(2.8\text{ kJ/m}^3)\left[8.85\times10^{-12}\text{ C}^2/(\text{N}\cdot\text{m}^2)\right]} = \pm 3.2\ \mu\text{C}$$

ASSESS Notice that we do not know the sign on the charge because of the charge symmetry involved.

40. **INTERPRET** The problem involves the capacitance of a cell membrane, given the potential difference and charge across the membrane.

DEVELOP In general, the capacitance is $C = Q/V$ (Equation 23.1). The charge in this case is $Q = 1.8\times10^6 e.$

EVALUATE Given the values for the cell membrane, the capacitance is

$$C = \frac{Q}{V} = \frac{\left(1.8\times10^6\right)\left(1.6\times10^{-19}\text{ C}\right)}{65\text{ mV}} = 4.4\text{ pF}$$

ASSESS Does this make sense? What if we assumed the cell membrane was a parallel-plate capacitor with a dielectric? Then Equation 23.4 applies: $C = \kappa\varepsilon_0 A/d$. For the membrane thickness, let $d = 10$ nm. For the cell surface area, let $A = (10\ \mu\text{m})^2$. And for the dielectric constant, let's use the one for water since the cell is largely made up of water: $\kappa = 80$. With these choices, the capacitance comes out to be 7 pF, so our answer seems to make sense.

41. **INTERPRET** In this problem, we are asked to compare the amount of energy stored in two different capacitors.

DEVELOP The energy stored in a capacitor can be calculated using Equation 23.3: $U = CV^2/2$.

EVALUATE The energy stored in each capacitor is

$$U_1 = \frac{1}{2}C_1V_1^2 = \frac{1}{2}(1.0\ \mu\text{F})(250\ \text{V})^2 = 31\ \text{mJ}$$

$$U_2 = \frac{1}{2}C_2V_2^2 = \frac{1}{2}(470\ \text{pF})(3.0\ \text{kV})^2 = 2.1\ \text{mJ}$$

Thus, the energy stored in capacitor 2 is about 15 times less than that stored in capacitor 1.

ASSESS The general expression for the ratio of the energies stored in two capacitors is

$$\frac{U_2}{U_1} = \frac{C_2V_2^2/2}{C_1V_1^2/2} = \left(\frac{C_2}{C_1}\right)\left(\frac{V_2}{V_1}\right)^2$$

so the energy stored is linear in capacitance but quadratic in voltage.

42. **INTERPRET** In this problem we are asked to compare the amount of charge and energy stored in three different capacitors.

DEVELOP The charge stored in a capacitor is $Q = CV$ (Equation 23.1), and the energy stored is $U = CV^2/2$ (Equation 23.3).

EVALUATE **(a)** The charge stored is

$Q_1 = C_1V_1 = (0.01\ \mu\text{F})(300\ \text{V}) = 3\ \mu\text{C}$ for the first capacitor

$Q_2 = C_2V_2 = (0.1\ \mu\text{F})(100\ \text{V}) = 10\ \mu\text{C}$ for the second

$Q_3 = C_3V_3 = (30\ \mu\text{F})(5\ \text{V}) = 150\ \mu\text{C} = 200\ \mu\text{C}$ to one significant figure for the third

(b) To one significant figure, the energy stored in each capacitor is

$$U_1 = \frac{1}{2}C_1V_1^2 = \frac{1}{2}(3\ \mu\text{C})(300\ \text{V}) = 450\ \mu\text{J} = 500\ \mu\text{J}$$

$$U_2 = \frac{1}{2}C_2V_2^2 = \frac{1}{2}(0.1\ \mu\text{F})(100\ \text{V})^2 = 500\ \mu\text{J}$$

$$U_3 = \frac{1}{2}C_3V_3^2 = \frac{1}{2}(30\ \mu\text{F})(5\ \text{V})^2 = 375\ \mu\text{J} = 400\ \mu\text{J}$$

(c) The cost effectiveness *eff*, measured in J/¢ and to a single significant figure, is

$$eff_1 = \frac{450\ \mu\text{J}}{25\ ¢} = 18\times10^{-6}\ \text{J/¢} = 2\times10^{-5}\ \text{J/¢}$$

$$eff_2 = \frac{500\ \mu\text{J}}{35\ ¢} = 14\times10^{-6}\ \text{J/¢} = 1\times10^{-5}\ \text{J/¢}$$

$$eff_3 = \frac{375\ \mu\text{J}}{88\ ¢} = 4.3\times10^{-6}\ \text{J/¢} = 0.4\times10^{-5}\ \text{J/¢}$$

ASSESS Notice that, for part **(c)**, more significant figures were retained for the results of part **(b)** because these are intermediary results for part **(c)**. The first capacitor is the most cost effective of the three.

43. **INTERPRET** We are to find the voltage across the given capacitor and the power it discharges if all its energy is discharged in the given time.

DEVELOP Solve Equation 23.3 for voltage to find the voltage across the capacitor. Use the definition of average power, $\overline{P} = E/\Delta t$, to find the average power.

EVALUATE **(a)** From Equation 23.3, $V = \sqrt{2U/C} = \sqrt{2(950\ \text{J})/(100\ \mu\text{F})} = 4.4\ \text{kV}$.

(b) $\overline{P} = \Delta U/\Delta t = (280\ \text{J})/(3.1\ \text{ms}) = 90\ \text{kW}$.

ASSESS This is like connecting yourself to 1500 60-W light bulbs for 3.1 ms. Care to try?

44. **INTERPRET** This problem involves finding the power consumption of a camera flashtube per flash and the average power consumed if the flash is used every 10 s. From this power, we are to find the capacitor required to power the flash.

DEVELOP Equation 6.15 gives the average power, $\overline{P} = \Delta W / \Delta t$, where ΔW is the work done (or the energy used). Apply this to find the power consumed by the flash for the different time intervals Δt. Once we find the average power required, we apply Equation 23.3, $U = CV^2/2$, to find the capacitance required.

EVALUATE **(a)** The power delivered to the tube during the flash is

$$\overline{P}_{\text{flash}} = \frac{\Delta W}{\Delta t} = \frac{5.5\ \text{J}}{1.0\ \text{ms}} = 5.5\ \text{kW}$$

(b) The energy stored in a capacitor is $U = \frac{1}{2}CV^2$. Thus, the capacitance needed to supply the flash energy is

$$C = \frac{2U}{V^2} = \frac{2(5.5\ \text{J})}{(210\ \text{V})^2} = 249\ \mu\text{F}$$

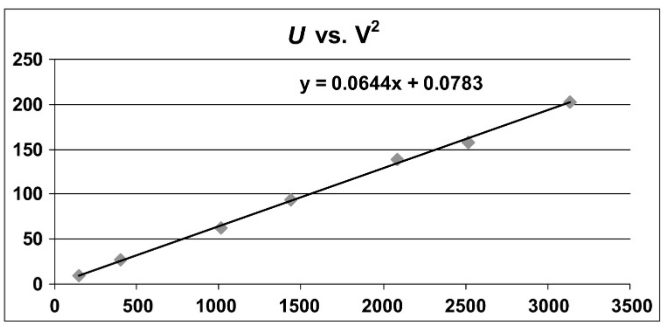
(c) The average power consumption during the 10-s interval is $\overline{P} = (5.5\ \text{J})/(10\ \text{s}) = 0.55\ \text{W}$.

ASSESS The average power $\overline{P}$ is 10^{-4} times P_{flash}, which is the same ratio as that for the time intervals (1 ms to 10 s).

45. **INTERPRET** In this problem, we are given the data of the energy stored in a capacitor at different voltages. From the data we are asked to deduce the capacitance.

DEVELOP The energy stored in a capacitor can be calculated using Equation 23.3: $U = CV^2/2$. Thus, plotting U versus V^2 should give a straight line with slope $C/2$.

EVALUATE The plot is shown below.



The slop is $0.0644\ \text{kJ/V}^2 = 64.4\ \text{C/V} = 64.4\ \text{F}$, from which we find the capacitance to be 2(64.4 F) ≈ 129 F.

ASSESS The energy stored increases quadratically with voltage. The general expression for the ratio of the energies stored in two different voltages is

$$\frac{U_2}{U_1} = \frac{CV_2^2/2}{CV_1^2/2} = \left(\frac{V_2}{V_1}\right)^2$$

46. **INTERPRET** In this problem we want to connect capacitors of known capacitance and voltage rating to obtain the desired equivalent capacitance and voltage rating.

DEVELOP In parallel, the voltage across each element is the same, so to increase the voltage rating of a combination of equal capacitors, series connections must be considered. The general result of Problem 22 shows that the voltage across n capacitors in series, each of which is rated at voltage V, is simply nV. The voltage across capacitors in parallel does not change, so we can combine capacitors in series to adjust the voltage and capacitors in parallel to adjust the overall capacitance (without changing the voltage rating).

EVALUATE **(a)** To obtain the desired voltage rating, we must use two capacitors in series so that the voltage becomes

$$V_{\text{series pair}} = V_1 + V_2 + 50\ \text{V} + 50\ \text{V} = 100\ \text{V}\ .$$

However, the overall capacitance is now

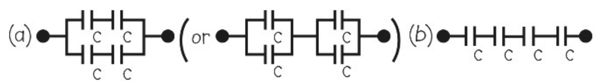
$$C_{\text{series pair}} = \frac{C_1C_2}{C_1+C_2} = \frac{(2.0\ \mu\text{F})(2.0\ \mu\text{F})}{2.0\ \mu\text{F}+2.0\ \mu\text{F}} = 1.0\ \mu\text{F}$$

and we need to increase the total capacitance to twice that of just two in series, without altering the voltage rating. This can be accomplished with a parallel combination of two $C_{\text{series pair}}$. This gives a voltage rating of $V_{\text{series pair}}$ and a capacitance of $2C_{\text{series pair}}$ = 2.0 μF. Thus, the equivalent capacitance would be as shown in part (a) of the figure below.

(b) For an equivalent capacitance of 0.5 μF and a voltage of 50 V, four capacitors in series are required, which gives a rating $V_{\text{tot}} = 4V = 4(50\ \text{V}) = 200$ V, and capacitance

$$C^{-1} = 4(2.0\ \mu\text{F})^{-1}$$
$$C = 0.50\ \mu\text{F}$$

ASSESS Schematically the connections described look like the following:



One may use Equations 23.5 and 23.6 to verify that the configurations indeed have the desired capacitance and voltage rating as specified in the problem statement.

47. **INTERPRET** This problem involves two capacitors in series. We are to find an expression for the voltage across each capacitor.

DEVELOP From Equation 23.5, we have

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

Furthermore, the charge on each capacitor must be the same, as argued in the text accompanying Fig. 23.8, so, from Equation 23.1, $Q = CV_1 = CV_2 = CV$. In addition, the voltages across each capacitor must sum to the voltage across the equivalent capacitor, so $V = V_1 + V_2$.

EVALUATE Combining the expressions above to solve for V_1 in terms of V, C_1, and C_2 gives

$$V_1 = V - V_2 = V - Q/C_2 = V - \frac{CV}{C_2} = V - \frac{V}{\left(C_1^{-1}+C_2^{-1}\right)C_2} = V\left(1 - \frac{C_1}{C_2+C_1}\right) = \frac{VC_2}{C_2+C_1}$$

Solving for V_2 likewise gives

$$V_2 = V - V_1 = V - Q/C_1 = V - \frac{CV}{C_1} = V - \frac{V}{\left(C_1^{-1}+C_2^{-1}\right)C_1} = V\left(1 - \frac{C_2}{C_2+C_1}\right) = \frac{VC_1}{C_2+C_1}$$

ASSESS These expressions agree with that given in the problem statement.

48. **INTERPRET** You want to know the working voltage for two capacitors connected in series.

DEVELOP The voltage across two capacitors in series is just the sum of the voltage across each capacitor. Your company's new hire applied this same logic to the working voltage, which is the maximum that the capacitors can handle. However, the working voltage basically tells you the maximum charge that a capacitor can handle: $Q_{\max} = CV_{\text{work}}$. Since the charge on each capacitor in the series is the same (see Fig. 23.8), the working voltage will be set by the smallest maximum charge of the capacitors in the series.

EVALUATE The maximum charges on the two capacitors are

$$Q_1 = C_1V_1 = (0.1\ \mu\text{F})(50\ \text{V}) = 5.0\ \mu\text{C}$$
$$Q_2 = C_2V_2 = (0.2\ \mu\text{F})(200\ \text{V}) = 40\ \mu\text{C}$$

Thus, the most charge that the series can handle is $5.0\ \mu\text{C}$, which means the working voltage is

$$V_{\text{work}} = \frac{Q_2}{C_1} + \frac{Q_2}{C_2} = (5.0\ \mu\text{C})\left(\frac{1}{0.1\ \mu\text{F}} + \frac{1}{0.2\ \mu\text{F}}\right) = 75\ \text{V}$$

The new hire was wrong: 75 V is the safe limit.

ASSESS What this tells you is that you should try to match the maximum allowable charge of capacitors that you put in series.

49. **INTERPRET** For this problem, we are to find the capacitance and working voltage of a capacitor, given its geometry and dielectric material.

DEVELOP Apply Equation 22.4 and use Table 23.1 for the dielectric constant of polyethylene.

EVALUATE **(a)** From Equation 23.4 and Table 23.1, one obtains

$$C = \kappa\frac{\varepsilon_0 A}{d} = (2.3)\frac{\left(8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2}\right)\left(53\times10^{-4}\ \mathrm{m^2}\right)}{30\times10^{-6}\ \mathrm{m}} = 3.6\ \mathrm{nF}$$

(b) Dielectric breakdown in polyethylene occurs at a field strength of 50 kV/mm, corresponding to a maximum voltage for this capacitor of

$$V = Ed = \left(50\times10^{6}\ \mathrm{V/m}\right)\left(30\times10^{-6}\ \mathrm{m}\right) = 1.5\ \mathrm{kV}$$

ASSESS The results are given to two significant figures, as warranted by the data.

50. **INTERPRET** This problem involves a capacitor using polystyrene as the dielectric between the conductor plates.

DEVELOP From Equation 23.4, the capacitance of two parallel plates with a dielectric in between is $C = \kappa C_0 = \kappa\epsilon_0 A/d$. As for part (b), the maximum voltage for a parallel-plate capacitor is the breakdown field times the distance between the plates: $V_{\max} = E_{\mathrm{break}}d$.

EVALUATE (a) The dielectric constant of polystyrene is given in Table 23.1, so its thickness is:

$$d = \frac{\kappa\epsilon_0 A}{C} = \frac{(2.6)\left[8.85\times10^{-12}\ \mathrm{C^2/(N\cdot m^2)}\right]\left(\pi(15\ \mathrm{cm})^2\right)}{560\ \mathrm{pF}} = 2.9\ \mathrm{mm}$$

(b) The breakdown field of polystyrene is given in Table 23.1, so the maximum voltage is:

$$V_{\max} = (25\ \mathrm{MV/m})(2.9\ \mathrm{mm}) = 73\ \mathrm{kV}$$

ASSESS These values seem reasonable for the thickness and working voltage of a capacitor. Notice that, to increase the working voltage for a given geometry, one would like a material with both a large dielectric constant and a large breakdown field.

51. **INTERPRET** In this problem, we are given the capacitance per unit area and the dielectric strength for a capacitor and are asked to find the plate separation.

DEVELOP If we assume that the inner and outer surfaces of the membrane act like a parallel-plate capacitor, with the space between the plates filled with material of dielectric constant $\kappa = 3$, then we can use Equation 23.4 to find the separation *d*.

EVALUATE The capacitance per unit area is $C/A = \kappa\epsilon_0/d$. Thus, $d = 3\left(8.85\ \mathrm{pF/m}\right)\big/\left(1\ \mu\mathrm{F/cm^2}\right) = 2.7\ \mathrm{nm}$.

ASSESS This result is about an order of magnitude larger than the Bohr radius, which gives an idea of the thickness of the membrane in terms of atoms (biological molecules being largely hydrogen and carbon).

52. **INTERPRET** You want to see if "trimmer" capacitors can provide the extra capacitance you are looking for.

DEVELOP Placing the trimmer capacitors in parallel with the cheap 2-μF capacitors will result in a combined capacitance of $C = C_{\mathrm{trim}} + C_{2\mu\mathrm{F}}$.

EVALUATE Since you want 2.0 μF of capacitance and the cheap capacitors only provide between 1.7 and 1.9 μF, the trimmer capacitors have to supply between

$$C_{\mathrm{trim}} = C - C_{2\mu\mathrm{F}} = 2.0\ \mu\mathrm{F} - \begin{cases}1.7\ \mu\mathrm{F} = 300\ \mathrm{nF}\\ 1.9\ \mu\mathrm{F} = 100\ \mathrm{nF}\end{cases}$$

Since these values are both within the variable limits of the trimmer capacitors (25 and 350 nF), they will work for this application.

ASSESS One way to make a variable capacitor is to allow the plate separation, *d*, to be changed, say, with a small screw. Another way is to vary the area, *A*, by shifting the two plates so that there is greater or less overlap.

53. **INTERPRET** We're asked to find the total electric-field energy in a cubical region with a variable field.

DEVELOP The electric-field energy for a variable field is given by Equation 23.8: $U = \frac{1}{2}\varepsilon_0 \int E^2 dV$. For the cubical region in this case, $dV = dxdydz$, and the integration for each variable is from 0 to 1 m.

EVALUATE Performing the integration, the energy stored in the field is

$$U = \frac{1}{2}\varepsilon_0 \int_0^{1\text{m}} dz \int_0^{1\text{m}} dy \int_0^{1\text{m}} \left[E_0\left(\frac{x}{x_0}\right)\right]^2 dx$$

$$= \frac{1}{2}\left(8.85\times10^{-12}\,\frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)(1\text{ m})^2(16\text{ kV/m})^2\left[\frac{\frac{1}{3}(1\text{ m})^3}{(7\text{ m})^2}\right] = 7.7\ \mu\text{J}$$

ASSESS The value seems reasonable. One can check that the units work out by using the fact that $1\text{ V}\cdot\text{C} = 1\text{ J} = 1\text{ N}\cdot\text{m}$.

54. **INTERPRET** This problem involves electrostatic energy contained within a spherical region. Our system has spherical symmetry.

DEVELOP We are told the electric field in this region has the form $E = E_0R/r$ so the energy density is

$$u(r) = \frac{1}{2}\epsilon_0E^2 = \frac{\epsilon_0}{2}\left(\frac{E_0R}{r}\right)^2 = \frac{\epsilon_0E_0{}^2R^2}{2r^2}$$

The energy within the spherical region of radius R can be found by integrating this expression over the volume.

EVALUATE With thin spherical shells of radius r for volume elements, $dV = 4\pi r^2dr$, the integral for the energy is

$$U = \int_{\text{sphere}} udV = \int_0^R\left(\frac{\epsilon_0E_0{}^2R^2}{2r^2}\right)4\pi r^2dr = 2\pi\epsilon_0E_0{}^2R^2\int_0^R dr = 2\pi\epsilon_0E_0{}^2R^3$$

ASSESS The result shows that U is directly proportional to R. This means that the stored energy increases if the same amount of charge Q is distributed over a greater volume.

55. **INTERPRET** This problem involves a spherical charge distribution outside of which we are to find the total energy contained in the electric field.

DEVELOP This problem is the same Example 23.5 if we let $R_2 = R$ and $R_1 \to \infty$ (i.e., we can consider that we are compressing an infinitely separated charge distribution to one that is on the surface of the sphere of radius R_2).

EVALUATE The energy is thus

$$U = \frac{kQ^2}{2R}$$

ASSESS The energy is quadratic in charge so, for example, it would take four times the energy to assemble twice the charge.

56. **INTERPRET** We want to model the Earth's surface and the ionosphere as a parallel-plate capacitor. Knowing the separation distance and potential difference we want to find its capacitance and the total energy stored.

DEVELOP Since the capacitor is made up of two spherical surfaces, we apply Equation 23.2, $C = \epsilon_0A/d$, with $A = 4\pi R_E{}^2$, where we use the radius of Earth found in Appendix E.

EVALUATE Inserting the given quantities gives

$$C = \frac{\epsilon_0 4\pi R_E{}^2}{d} = \frac{(8.85\text{ pF/m})4\pi\left(6.37\times10^6\text{ m}\right)^2}{60\text{ km}} = 75\text{ mF}$$

To find the energy stored, we evaluate Equation 23.3 using the given potential difference to find

$$U_C = \frac{1}{2}CV^2 = \frac{1}{2}(75\text{ mF})(400\text{ kV})^2 = 6\text{ GJ}$$

ASSESS These are estimates since the field is constantly varying due to fluctuations in the Sun's activity and illumination as Earth rotates.

57. **INTERPRET** This problem involves finding the change in electrostatic potential energy between two separated, charged water drops and a single drop with the same charge.

DEVELOP The initial electrostatic energy of two isolated spherical drops, with charge Q on their surfaces and radii R, is given by $U_\mathrm{i} = 2\left(\frac{1}{2}kQ^2/R\right)$ (see Problem 55 and Example 23.5). Together, a drop of charge 2Q, radius $2^{1/3}R$, and energy $U_\mathrm{f} = \frac{1}{2}k(2Q)^2/\left(2^{1/3}R\right) = 2^{2/3}kQ^2/R$ is created. The difference in potential energy is $W = U_\mathrm{f} - U_\mathrm{i}$.

EVALUATE Inserting the given quantities gives

$$\Delta U = \frac{\left(2^{2/3}-1\right)kQ^2}{R} = \frac{(0.587)\left(9.0\times10^9\ \mathrm{N\cdot m^2/C^2}\right)\left(2.2\times10^{-8}\ \mathrm{C}\right)^2}{2.35\times10^{-3}\ \mathrm{m}} = 1.1\times10^{-3}\ \mathrm{J}$$

ASSESS In eV, this is

$$\Delta U = \frac{5.4\times10^{-4}\ \mathrm{J}}{1.6\times10^{-19}\ \mathrm{J/eV}} = 3.4\times10^{15}\ \mathrm{eV}$$

58. **INTERPRET** This problem involves finding the electrostatic energy contained within a volume around a charged wire. This system has line symmetry.

DEVELOP If the wire length is much, much greater than its radius, and if the volume we are considering is many radii away from either end, then the system has line symmetry (i.e., we can consider it to be an infinite wire). In this case, the electric field outside the wire is radially away from the axis (for $\lambda > 0$) with magnitude $E_r = 2k\lambda/r$ (see Equation 21.6). The energy density in a cylindrical shell of radius r, length L, and volume $dV = 2\pi rLdr$, is

$$u(r) = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2(4\pi k)}\left(\frac{2k\lambda}{r}\right)^2 = \frac{k\lambda^2}{2\pi r^2}$$

Integrate this expression over the volume of the cylinder to obtain the energy.

EVALUATE Thus, the energy in the space mentioned in this problem is

$$U = \int_{\text{cylinder}} u(r)\,dV = \int_{\mathrm{R}}^{3\mathrm{R}} \left(\frac{k\lambda^2}{2\pi r^2}\right) 2\pi rLdr = k\lambda^2 L\ln(3)$$

$$= \left(9.0\times10^9\ \mathrm{N\cdot m^2/C^2}\right)(32\ \mu\mathrm{C/m})^2(1.0\ \mathrm{m})\ln(3) = 10.1\ \mathrm{J}$$

ASSESS The energy density (energy/volume) decreases as $1/r^2$. This means that more energy is concentrated in the volume closer to the wire.

59. **INTERPRET** For this problem, we are asked to find the time required for lightning strikes to deplete a reservoir of energy, given the charge transferred, the electric potential energy difference (per charge), and the frequency of the lightning strikes.

DEVELOP The energy in the thunderstorm of Example 23.4 is about 1.4×10^{11} J, while the energy in a lightning flash is $qV = (30\ \mathrm{C})(30\ \mathrm{MV}) = 9.0\times10^8\ \mathrm{J}$. Thus, there is energy for about $\left(1.4\times10^{11}\right)/\left(9.0\times10^8\right) = 156$ flashes.

EVALUATE At a rate of one flash every 5 s, there is enough energy to last $156\times(5\ \mathrm{s}) = 13\ \mathrm{min}$.

ASSESS This seems like a reasonable time frame for a summer thunderstorm.

60. **INTERPRET** We are to find the capacitance of the given pair of coaxial cylinders.

DEVELOP From Example 22.4, we can see that the magnitude of the potential difference between two points distances a and b from a cylinder with charge per unit length λ is

$$\Delta V = \left|\frac{\lambda}{2\pi\epsilon_0}\ln\left(\frac{a}{b}\right)\right| = \frac{\lambda}{2\pi\epsilon_0}\ln\left(\frac{b}{a}\right)$$

for $b > a$. The total charge on the inner cylinder is $Q = \lambda L,$ and the definition of capacitance is $C = Q/V$.

EVALUATE Combining these expressions to find the capacitance gives

$$C = \frac{Q}{V} = \frac{\lambda L}{\frac{\lambda}{2\pi\epsilon_0}\ln\left(\frac{b}{a}\right)} = \frac{2\pi\epsilon_0 L}{\ln\left(\frac{b}{a}\right)}$$

ASSESS The capacitance depends only on the geometry, as expected. Note that this result assumes that $a, b \ll L$, which is used in the derivation of the expression for the voltage difference.

61. **INTERPRET** We are to find the capacitance of a sphere surrounded by a concentric shell.

DEVELOP This geometry is the same as for Problem 23.36, for which we found the potential between the spheres to be

$$V = kQ\left(a^{-1} - b^{-1}\right)$$

from which we can find the capacitance using Equation 23.1, $C = Q/V$.

EVALUATE The capacitance is

$$V = kQ\left(a^{-1} - b^{-1}\right) = \frac{Q(b-a)}{4\pi\epsilon_0 ab} = \frac{Q}{C}$$

$$C = \frac{4\pi\epsilon_0 ab}{(b-a)}$$

ASSESS The capacitance depends only on the geometry of the capacitor, as expected.

62. **INTERPRET** The object of interest is a spherical capacitor, and we are to explore the limit when the separation between the concentric spheres is much, much smaller than the radii of the spheres (i.e., $b - a \ll a$).

DEVELOP The capacitance for a spherical capacitor can be derived by noting that the potential difference between two concentric conducting spheres is

$$V = kQ\left(\frac{1}{a} - \frac{1}{b}\right) = \frac{kQ(b-a)}{ab} \quad \Rightarrow \quad C = \frac{Q}{V} = \frac{ab}{k(b-a)}$$

Take the limit $d \equiv b - a \ll a$ to show that C reduces to that of a parallel-plate capacitor.

EVALUATE With $d \equiv b - a \ll a,$ we find

$$C = \frac{ab}{k(b-a)} = \frac{4\pi\epsilon_0 ab}{d} = \frac{4\pi\epsilon_0 a(a+d)}{d} \approx \frac{4\pi\epsilon_0 a^2}{d} = \frac{\epsilon_0 A}{d}$$

which is the result of Equation 23.2, with $A = 4\pi a^2$ being the area of the spherical plates.

ASSESS The limit $d \equiv b - a \ll a$ means that the radius of the sphere is much greater than the separation between the two spheres. Thus, the interface between the spheres is very similar to two parallel plates.

63. **INTERPRET** This problem involves a spherical, uniform charge density. We are to find the fraction of the total energy contained within the sphere.

DEVELOP In Problem 23.60, we found that the energy within just such a charged sphere is

$$U_{\text{inside}} = \frac{kQ^2}{10R}$$

In Problem 23.61, we found that the energy outside such a sphere is

$$U_{\text{outside}} = \frac{kQ^2}{2R}$$

Divide U_{inside} by the sum to find the fraction of energy inside the sphere.

EVALUATE The fraction of energy inside the sphere is

$$f_{\text{inside}} = \frac{U_{\text{inside}}}{U_{\text{inside}} + U_{\text{outside}}} = \frac{kQ^2/(10R)}{kQ^2/(10R) + kQ^2/(2R)} = \frac{1}{1+5} = \frac{1}{6}$$

ASSESS Ignoring gravity, this result is independent of the size of the sphere.

64. **INTERPRET** This problem involves a charged capacitor into which we insert a dielectric material so that it occupies half the volume of the capacitor. We are to find the new capacitance, the stored energy, and the force on the dielectric in this configuration.

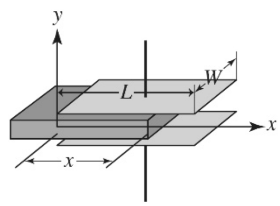
DEVELOP Use the coordinate system defined in the figure below. In so far as fringing fields can be neglected, the electric field between the plates is a uniform $E = V/d$ (but when the dielectric is inserted, $V \neq V_0$ and E depends on x). In fact, on the left side, where the slab has penetrated, $E = (1/\kappa)(\sigma_L/\epsilon_0)$ and on the right, $E = \sigma_R/\epsilon_0$, where σ_L and σ_R are the charge densities on the left and right sides, respectively. Thus, $\sigma_L = \kappa\epsilon_0 E$ and $\sigma_R = \epsilon_0 E$, and the charge can be written (in terms of geometrical values taken from Fig. 23.19) as

$$Q = \sigma_L wx + \sigma_R w(L-x) = \epsilon_0 Ew(\kappa x + L - x) = \epsilon_0 (V/d) w(\kappa x + L - x).$$

When the battery is disconnected, the capacitor is isolated and the charge on it is a constant, $Q = Q_0$, and we can use Equation 23.3 $U = CV^2/2$ to find the energy stored in the capacitor. The force on a part of an isolated system is related to the potential energy of the system by Equation 8.9. The force on the slab is therefore

$$F_x = -\frac{dU}{dx} = -\frac{d}{dx}\left(\frac{U_0 L}{\kappa x + L - x}\right) = \frac{U_0 L(\kappa - 1)}{(\kappa x + L - x)^2}$$

in the direction of increasing x (so as to pull the slab into the capacitor).



EVALUATE **(a)** From Equation 23.1,

$$C = \frac{Q}{V} = C_0 \frac{(\kappa x + L - x)}{L}$$

where $C_0 = \epsilon_0 A/d$ and $A = Lw$. Inserting $x = L/2$, we find

$$C = C_0 \frac{\kappa + 1}{2}$$

(b) The stored energy is

$$U = \frac{Q^2}{2C} = \frac{q_0^2 L}{2C_0(\kappa x + L - x)} = \frac{U_0 L}{\kappa x + L - x}$$

where $U_0 = Q_0^2/(2C_0) = C_0 V_0^2/2$. For $x = L/2$, the energy is $C_0 V_0^2/(\kappa + 1)$.

(c) For $x = L/2$, the magnitude of the force is

$$F_x = \frac{2C_0 V_0^2 (\kappa - 1)}{L(\kappa + 1)^2}$$

ASSESS Notice that the results depend only on geometrical factors and the dielectric strength of the material. It turns out that if we rewrite the force, for any value of x, in terms of the voltage for that x, using $q_0 = C_0 V_0 = CV = C_0 V(\kappa x + L - x)/L$, the expression can be used in the succeeding problem. Thus,

$$F_x = \frac{C_0 V_0^2 L(\kappa - 1)}{2(\kappa x + L - x)^2} = \frac{C_0}{2}\left(\frac{V}{L}\right)^2 L(\kappa - 1) = \frac{C_0 V^2 (\kappa - 1)}{2L}$$

65. **INTERPRET** This problem is about the effect of inserting a dielectric material into a parallel-plate capacitor, which is connected to a battery and so remains at a constant voltage.

DEVELOP We first note that the capacitance depends on the configuration and electrical properties of the plates and insulating materials, not on the external connections. Thus, we can use the result from the previous problem:

$$C = C_0 \frac{\kappa x + L - x}{L}$$

If the capacitor remains connected to a battery, the voltage is constant, $V = V_0$.

EVALUATE **(b)** The energy is

$$U = \frac{1}{2}CV_0^2 = \frac{C_0 V_0^2}{2}\left(\frac{\kappa x + L - x}{L}\right)$$

For $x = L/2$, we get $U = C_0 V_0^2(\kappa + 1)/4$.

(c) When the capacitor is connected to a battery, Equation 6.8 $(F_x = -dU/dx)$ for the force does not apply because the battery does work, which changes the energy of the system. However, for particular values of charge and voltage on the capacitor, the force on the slab considered here is the same, regardless of the external connections. In the preceding problem we found that $F_x = \frac{1}{2}C_0 V^2(\kappa - 1)/L$ where V is the particular voltage (and, because of the special form $C(x)$ of the capacitance, the particular charge q does not appear). Since $V = V_0$ in this problem,

$$F_x = \frac{1}{2}\frac{C_0 V_0^2(\kappa - 1)}{L}$$

ASSESS Note that the results are different from the preceding problem, because the battery does work. The force on the slab is to the right, drawing the dielectric between the plates.

66. **INTERPRET** We are to find the capacitance per unit length of two parallel wires whose separation is much, much greater than their radii, so we can assume line symmetry (i.e., infinite wires). The wires carry opposite charge density.

DEVELOP Use the coordinate system shown in the figure below. Apply Equation 22.1a:

$$\Delta V_{AB} = -\int_A^B \vec{E} \cdot d\vec{r}$$

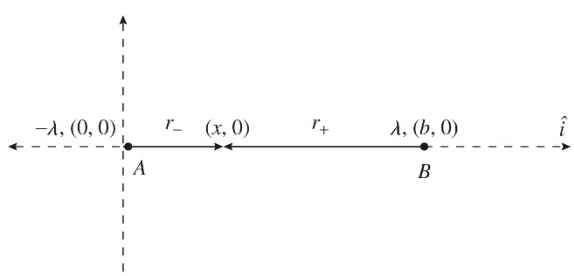
The total electric field is the superposition of the electric fields due to the two wires, each of which is given by Equation 21.6. Summing the two contributions gives

$$\vec{E} = \frac{\lambda}{2\pi\epsilon_0 r_-}(-\hat{i}) + \frac{\lambda}{2\pi\epsilon_0 r_+}(-\hat{i}) = -\frac{\lambda}{2\pi\epsilon_0}\left(\frac{1}{x} + \frac{1}{b-x}\right)(\hat{i})$$

Insert this into the integral and perform the integration to find the voltage difference between the wires. The wire capacitance per unit length can then be found using Equation 23.1, $Q = CV$, which we can transform into per unit length by dividing each side by an arbitrary length L:

$$\frac{Q}{L} = \frac{C}{L}V \quad \Rightarrow \quad \lambda = C_{\mathrm{L}} V$$

where C_{L} is the capacitance per unit length.



EVALUATE Evaluating the integral gives

$$\Delta V_{AB} = -\int_A^B \vec{E} \cdot d\vec{r} = \frac{\lambda}{2\pi\epsilon_0}\int_a^{b-a}\left(\frac{1}{x} + \frac{1}{b-x}\right)dx = \frac{\lambda}{2\pi\epsilon_0}\left[\ln(x) - \ln(b-x)\right]_a^{b-a} = \frac{\lambda}{\pi\epsilon_0}\ln\left(\frac{b-a}{a}\right)$$

Inserting this into the expression for capacitance gives

$$C_L = \pi\epsilon_0 / \ln\left(\frac{b-a}{a}\right)$$

ASSESS The capacitance per unit length depends only on geometrical parameters, and is positive.

67. **INTERPRET** We are to find the energy per unit length stored in the electric field within a uniformly charged rod. We will use Gauss's law to determine the electric field within the rod, and then integrate the energy density to find the total energy.

DEVELOP We use Gauss's law (Equation 21.3)

$$\int \vec{E} \cdot d\vec{a} = \frac{q_{\text{enclosed}}}{\epsilon_0}$$

exploiting the cylindrical symmetry, to find the electric field. Once we have this field, we integrate the energy density $u = \frac{1}{2}\epsilon_0 E^2$ over the cylinder to find the total energy $U = \int u dV$.

EVALUATE

$$\int \vec{E} \cdot d\vec{a} = \frac{q_{\text{enclosed}}}{\epsilon_0} \quad \Rightarrow \quad E(2\pi r L) = \frac{\pi r^2 L \rho}{\epsilon_0} \quad \Rightarrow \quad E = \frac{r\rho}{2\epsilon_0}$$

$$U = \frac{1}{2}\epsilon_0 \int_0^L \int_0^{2\pi} \int_0^R \left(\frac{r\rho}{2\epsilon_0}\right)^2 r dr d\phi dz = \frac{\rho^2}{4\epsilon_0} 2\pi L \int_0^R r^3 dr$$

$$\frac{U}{L} = \frac{\pi\rho^2}{2\epsilon_0}\frac{1}{4}R^4 = \frac{\pi\rho^2 R^4}{8\epsilon_0}$$

ASSESS Increasing the radius increases the energy per length dramatically.

68. **INTERPRET** We are to find the electrostatic energy stored between two parallel plates of a parallel-plate capacitor, and then differentiate to find the force between the plates.

DEVELOP Using Equation 21.8, find the electric field between the plates:

$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{A\epsilon_0}$$

This is constant, so total energy stored in this field is then $U = uV$, where u is the energy density (energy per unit volume). We can find the force by using $F_x = -dU/dx$.

EVALUATE **(a)** The electrostatic potential energy is

$$U = uV = \left(\frac{1}{2}\epsilon_0 E^2\right)(Ax) = \frac{Q^2}{2A\epsilon_0}x$$

(b) The force between the plates is

$$F_x = -\frac{dU}{dx} = -\frac{Q^2}{2A\epsilon_0}$$

This is half the value you would obtain by multiplying the charge on one plate by the field between the plates.

ASSESS The answer we get for **(b)** is half the field times the charge on one plate: but we must remember that the field between the plates is created by *both* charged plates. A charge is not affected by the field it creates. Only the field created by the *other* plate causes a force on each plate, and the other plate creates half the field.

69. **INTERPRET** We are to use the energy stored in a capacitor network at a given voltage to find the capacitance of an unknown capacitor in the network. We shall use the equation for the energy stored in a capacitor, and the rules for adding capacitors in series and in parallel.

DEVELOP We first find the capacitance C_{network} of the entire network in terms of the given values and the unknown capacitance C. Next, we use Equation 23.3, $U = \frac{1}{2}C_{\text{network}}V^2$ with U = 5.8 mJ and V = 100 V and work backward through the network to solve for C.

EVALUATE

(a) See the circuit diagram in the circuit below.

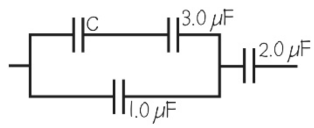
(b)

$$U = \frac{1}{2}C_{\text{network}}V^2 \quad \Rightarrow \quad C_{\text{network}} = \frac{2U}{V^2} = 1.16\ \mu\text{F}$$

$$\frac{1}{C_{\text{network}}} = \frac{1}{C_{\text{left}}} + \frac{1}{2.0\ \mu\text{F}} \quad \Rightarrow \quad C_{\text{left}} = 2.76\ \mu\text{F}$$

$$C_{\text{left}} = C_{\text{top}} + 1.0\ \mu\text{F} \quad \Rightarrow \quad C_{\text{top}} = 1.76\ \mu\text{F}$$

$$\frac{1}{C_{\text{top}}} = \frac{1}{C} + \frac{1}{3.0\ \mu\text{F}} \quad \Rightarrow \quad C = 4.3\ \mu\text{F}$$



ASSESS This trick of "unraveling" the network is often useful. We will use the same trick in dealing with resistor networks later. Note also that three significant figures were retained for the intermediate results, whereas only two significant figures were retained for the final result, as warranted by the data.

70. **INTERPRET** We're considering the energy used by the National Ignition Facility.

DEVELOP The energy stored in a capacitor is given by $U = \frac{1}{2}CV^2$ (Equation 23.3).

EVALUATE We're told that the NIF capacitor system stores 400 MJ at 20 kV, so the capacitance is

$$C = \frac{2U}{V^2} = \frac{2(400\ \text{MJ})}{(20\ \text{kV})^2} = 2\ \text{F}$$

The answer is **(d)**.

ASSESS This is an impressively large capacitance. The advantage of using capacitors in this application is that they can discharge rapidly and thus supply a large amount of power over a short time.

71. **INTERPRET** We're considering the energy used by the National Ignition Facility.

DEVELOP If the voltage is changed, the capacitor system still needs to store the same energy, so the capacitance will change by a factor of $C / C_0 = (V_0 / V)^2$.

EVALUATE Doubling the voltage means the capacitance could drop by ¼ and still store the same 400 MJ. The answer is **(a)**.

ASSESS There's a trade-off between capacitance and voltage. One can think of the capacitor system as the equivalent of a spring storing mechanical energy $\left(U = \frac{1}{2}kx^2\right)$. The capacitance is like the spring constant, *k*, and the voltage is like the displacement, *x*. The same energy can be stored with a weaker spring constant if the spring is compressed or stretched further.

72. **INTERPRET** We're considering the energy used by the National Ignition Facility.

DEVELOP The power is energy divided by time.

EVALUATE We're told that the lasers deliver 2 MJ of energy in 1 ns. So the power is

$$P = \frac{\Delta E}{\Delta t} = \frac{2\times10^6\ \text{J}}{1\times10^{-9}\ \text{s}} = 2\times10^{15}\ \text{W} = 2\ \text{PW}$$

The answer is **(d)**.

ASSESS This is over 100 times the world's average power consumption, but it only lasts for a fraction of a second.

73. **INTERPRET** We're considering the energy used by the National Ignition Facility.

DEVELOP The energy stored in a capacitor is given by $U = \frac{1}{2}CV^2$ (Equation 23.3).

EVALUATE One of the capacitors at NIF stores

$$U = \frac{1}{2}CV^2 = \frac{1}{2}(300\ \mu\text{F})(20\ \text{kV})^2 = 60\ \text{kJ}$$

The answer is **(c)**.

ASSESS If there are 1200 of these capacitors in parallel, they can store 72 MJ. To reach 400 MJ with these type of capacitors would require more than five times this number.