21

GAUSS'S LAW

EXERCISES

Section 21.1 Electric Field Lines

11. **INTERPRET** This problem involves associating electric field lines to charges. Given the electric field lines and the magnitude of a single charge, we are to find the net charge.

DEVELOP The number of field lines emanating from (or terminating on) the positive (or negative) charges is the same (i.e., 14)—the charges have the same magnitude. The field lines are pointing toward the middle charge, which means that a positive charge placed in this field will experience a force attracting it to the central charge. Thus, the central charge must be negative (i.e., −1 μC). The field lines point outward from the outer charges, so they are positive charges. Because the same number of field lines emanate from the outer charges as from the central charge, they must have the same magnitude, so the outer charges are +1 μC. Sum the charges to find the net charge.

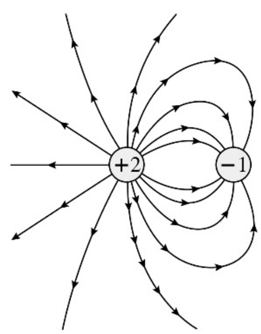
EVALUATE The net charge is thus $+1\ \mu\text{C} + 1\ \mu\text{C} - 1\ \mu\text{C} = +1\ \mu\text{C}$.

ASSESS If the magnitude of the central charge were less (greater) than 1 μC, then fewer (more) lines would terminate on this charge.

12. **INTERPRET** This problem is an exercise in drawing electric field lines to represent the field strength of a charge configuration.

DEVELOP We follow the methodology illustrated in Fig. 21.3. There are 16 lines emanating from charge $+2q$ (eight for each unit of $+q$). Similarly, we have eight lines ending on $-q$.

EVALUATE The field lines of the charge configuration are shown in the figure below.



ASSESS Our sketch is similar to Fig. 21.3 **(f)** with twice the number of lines of force.

13. **INTERPRET** In this problem we are asked to identify the charges based on the pattern of the field lines and the given net charge.

DEVELOP From the direction of the lines of force (away from positive and toward negative charge) one sees that A and C are positive charges and B is a negative charge. Eight lines of force terminate on B, eight originate on C, but only four originate on A, so the magnitudes of B and C are equal, while the magnitude of A is half that value.

EVALUATE Based on the reasoning above, we may write $Q_C = -Q_B = 2Q_A$. The total charge is $Q = Q_A + Q_B + Q_C = Q_A$, so $Q_C = 2Q = -Q_B$.

ASSESS The magnitude of the charge is proportional to the number of field lines emerging from or terminating at the charge.

Section 21.2 Electric Flux and Field

14. **INTERPRET** This problem is an exercise in finding the flux, given the electric field, a surface area, and the relative orientation of the surface with respect to the electric field.

DEVELOP The electric flux is defined by Equation 21.1, $\Phi = \vec{E}\cdot\vec{A} = EA\cos\theta,$ where $\vec{A}$ is a vector whose magnitude is the surface area and whose direction is normal (i.e., perpendicular) to the surface, and θ is the angle between $\vec{A}$ and $\vec{E}.$ Given that the magnitude of $\vec{A}$ is $A = 1.5\ \text{m}^2$ and $E = 1600\ \text{N/C},$ we can find the flux for the given angles.

EVALUATE **(a)** For $\theta = 90^\circ,$ the flux is

$$\Phi = EA\cos\theta = (1600\ \text{N/C})(1.5\ \text{m}^2)\cos(0^\circ) = 2.4\ \text{kN}\cdot\text{m}^2/\text{C}$$

(b) For $\theta = 45^\circ,$ the flux is

$$\Phi = EA\cos\theta = (1600\ \text{N/C})(1.5\ \text{m}^2)\cos(45^\circ) = 1.7\ \text{kN}\cdot\text{m}^2/\text{C}$$

(c) For $\theta = 90^\circ,$ the flux is

$$\Phi = EA\cos\theta = (1600\ \text{N/C})(1.5\ \text{m}^2)\cos(90^\circ) = 0$$

ASSESS Note that the angles given in the problem statement are between the plane of the surface and the electric field, whereas the angle θ is the angle between $\vec{A},$ which is perpendicular to the surface, and the electric field.

15. **INTERPRET** This problem involves calculating the electric flux through the surface of a sphere.

DEVELOP The general expression for the electric flux Φ is given by Equation 21.1: $\Phi = \vec{E}\cdot\vec{A} = EA\cos\theta,$ where θ is the angle between the normal vector $\vec{A}$ and the electric field $\vec{E}.$ The magnitude of the normal vector $\vec{A}$ is the surface area of the sphere: $A = 4\pi r^2.$

EVALUATE For a sphere, with $\vec{E}$ parallel or antiparallel to $\vec{A},$ Equation 21.1 gives

$$\Phi = \pm 4\pi r^2 E = \pm 4\pi\left(\frac{0.11}{2}\text{m}\right)^2(42\ \text{kN/C}) = \pm 1.6\ \text{kN}\cdot\text{m}^2/\text{C}$$

ASSESS The flux Φ is positive if $\vec{A}$ points outward and is negative if $\vec{A}$ points inward.

16. **INTERPRET** This problem involves calculating the electric flux through the surface of a cube.

DEVELOP The general expression for the electric flux Φ is given by Equation 21.1: $\Phi = \vec{E}\cdot\vec{A} = EA\cos\theta,$ where θ is the angle between the normal vector $\vec{A}$ and the electric field $\vec{E}.$ The magnitude of the normal vector $\vec{A}$ is the surface area of the cube: $A = s^2$, where s is the side length of the cube.

EVALUATE **(a)** For the orientation shown in **(a)** of GOT IT? 21.2, with $\vec{E}$ perpendicular to the normal of face B and parallel to the normal of face C, Equation 21.1 gives

$$\Phi_B = EA\cos\theta_B = EA\cos 90^\circ = 0$$
$$\Phi_C = EA\cos\theta_C = EA\cos 0^\circ = Es^2 = (1.75\ \text{kN/C})(1.25\ \text{m})^2 = 2.73\ \text{kN}\cdot\text{m}^2/\text{C}$$

(b) For the orientation shown in **(b)** of GOT IT?, the electric field makes a 45° angle with the normal of faces B and C. Thus,

$$\Phi_B = EA\cos\theta_B = Es^2\cos 45^\circ = (1.75\ \text{kN/C})(1.25\ \text{m})^2\cos 45^\circ = 1.93\ \text{kN}\cdot\text{m}^2/\text{C}$$
$$\Phi_C = EA\cos\theta_C = Es^2\cos 45^\circ = (1.75\ \text{kN/C})(1.25\ \text{m})^2\cos 45^\circ = 1.93\ \text{kN}\cdot\text{m}^2/\text{C}$$

ASSESS The flux Φ is positive if $\vec{A}$ points outward, and is negative if $\vec{A}$ points inward.

17. **INTERPRET** This problem involves calculating the electric flux through the curved surface of the half-cylinder shown in Fig. 21.8.

DEVELOP The general expression for the electric flux Φ is given by Equation 21.1: $\Phi = \vec{E}\cdot\vec{A} = EA\cos\theta,$ where θ is the angle between the normal vector $\vec{A}$ and the electric field $\vec{E}.$ The quantity $A_n = A\cos\theta$ can also be interpreted as the surface area that's normal to $\vec{E}.$ With the latter interpretation, we find the normal surface to be the rectangular $2r\times l$ area with $A_n = 2rl.$

EVALUATE Using Equation 21.1, we find the flux through the half-cylinder to be

$$\Phi = EA\cos\theta = E(2rl) = (6.3\times10^3\ \text{N / C})2(0.036\ \text{m})(0.17\ \text{m}) = 77\ \text{N}\cdot\text{m}^2\ /\ \text{C}$$

ASSESS The problem could indeed be solved without doing an integral.

Section 21.3 Gauss's Law

18. **INTERPRET** We are to find the total electric flux through a surface surrounding a given charge q. This is the simplest application of Gauss's law; integration is not required.

DEVELOP According to Gauss's law (Equation 21.3), the flux through a closed surface is proportional to the charge enclosed by that surface:

$$\Phi = \frac{q_{\text{enclosed}}}{\epsilon_0}$$

The total charge is $q_{\text{enclosed}} = q_e \times 10^{12}$, where $q_e = 1.60\times10^{-19}$ C is the charge of a single electron.

EVALUATE Inserting the given quantities into Gauss's law gives

$$\Phi = \frac{1.60\times10^{-19}\times10^{12}\ \text{C}}{8.85\times10^{-12}\ \text{C}^2/\left(\text{N}\cdot\text{m}^2\right)} = 1.81\times10^4\ \text{N}\cdot\text{m}^2/\text{C}$$

ASSESS We could also use Gauss's law to find the approximate electric field in the area surrounding this sock.

19. **INTERPRET** This problem is about applying Gauss's law to find the electric flux through a closed surface.

DEVELOP Gauss's law is given in Equation 21.3 and states that the flux through any closed surface is proportional to the charge enclosed:

$$\Phi = \oint \vec{E}\cdot d\vec{A} = \frac{q_{\text{enclosed}}}{\epsilon_0}$$

EVALUATE For the surfaces shown, the results are as follows:

(a) $q_{\text{enclosed}} = q + (-2q) = -q \quad \Rightarrow \quad \Phi = -q/\epsilon_0$

(b) $q_{\text{enclosed}} = q + (-2q) + (-q) + 3q + (-3q) = -2q \quad \Rightarrow \quad \Phi = -2q/\epsilon_0$

(c) $q_{\text{enclosed}} = 0 \quad \Rightarrow \quad \Phi = 0$

(d) $q_{\text{enclosed}} = 3q + (-3q) = 0 \quad \Rightarrow \quad \Phi = 0$

ASSESS The flux through the closed surface depends only on the charge enclosed, and is independent of the shape of the surface.

20. **INTERPRET** This problem involves applying Gauss's law to find the electric flux through a sphere that encloses a given charge.

DEVELOP Gauss's law (Equation 21.3) states that the electric flux through a closed surface is proportional to the charge enclosed. Specifically,

$$\Phi = \oint \vec{E}\cdot d\vec{A} = \frac{q_{\text{enclosed}}}{\epsilon_0}$$

So, given q_{enclosed}, we can find Φ.

EVALUATE The electric flux is

$$\Phi = \frac{q_{\text{enclosed}}}{\epsilon_0} = \frac{(7.6-5.3)\ \mu\text{C}}{8.85\times10^{-12}\ \text{C}^2\ /\ (\text{N}\cdot\text{m}^2)} = 260\ \text{kN}\cdot\text{m}^2\ /\ \text{C}$$

ASSESS The electric flux does not depend on the size of the sphere, only on the charge enclosed.

21. **INTERPRET** This problem involves applying Gauss's law to find the electric flux through the surface of a cube that encloses a given charge.

DEVELOP Gauss's law, given in Equation 21.3, states that the flux through any closed surface is proportional to the charge enclosed:

$$\Phi = \oint \vec{E}\cdot d\vec{A} = \frac{q_{\text{enclosed}}}{\varepsilon_0}$$

The symmetry of the situation guarantees that the flux through one face is $1/6$ the flux through the whole cubical surface.

EVALUATE The flux through one face of a cube is

$$\Phi_{\text{face}} = \frac{1}{6}\oint_{\text{cube}} \vec{E}\cdot d\vec{A} = \frac{1}{6}\frac{q_{\text{enclosed}}}{\varepsilon_0} = \frac{3.6\ \mu\text{C}}{6\left[8.85\times10^{-12}\ \text{C}^2/\left(\text{N}\cdot\text{m}^2\right)\right]} = 68\ \text{kN}\cdot\text{m}^2/\text{C}$$

ASSESS Because the flux through each surface is the same, the total flux through the cube is simply $\Phi = 6\Phi_{\text{face}}$, which is proportional to q_{enclosed}.

Section 21.4 Using Gauss's Law

22. **INTERPRET** This problem involves using Gauss's law to find the strength of the electric field a given distance from the center of a uniformly charged sphere. Note that the symmetry in this problem is spherical symmetry, which we can exploit in solving the problem.

DEVELOP As discussed in Example 21.1 and in the text just following this example, the electric field of a spherically symmetric charge distribution of total charge Q is the same as that of a point charge Q at the center of the sphere. Thus, the field is proportional to $1/r^2$ for $r > R$, where R is the radius of the spherical charge distribution, which means

$$\left.\begin{aligned} E(r_1) &\propto \frac{1}{r_1^2} \\ E(r_2) &\propto \frac{1}{r_2^2} \end{aligned}\right\} \frac{E(r_1)}{E(r_2)} = \frac{r_2^2}{r_1^2}$$

EVALUATE Inserting the given quantities into the expression above and solving for $E(r_2)$ gives

$$E(r_2) = E(r_1)\frac{r_1^2}{r_2^2} = (90\ \text{kN/C})\frac{(5.0\ \text{cm})^2}{(10\ \text{cm}+5.0\ \text{cm})^2} = 10\ \text{kN/C}$$

ASSESS As expected, the magnitude of the electric field has decreased because we are farther from the charge.

23. **INTERPRET** This problem involves applying Gauss's law to calculate the electric field. Our charge distribution has spherical symmetry.

DEVELOP The charge distribution is exactly that considered in Example 21.1. This example derives Equations 21.4 and 21.5, which express the strength of the electric field inside and outside the sphere of radius $R = 20$ cm. The result is

$$E = \begin{cases} \dfrac{1}{4\pi\varepsilon_0}\dfrac{Qr}{R^3}, & r \le R \\[2ex] \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{r^2}, & r \ge R \end{cases}$$

which we can evaluate at the points given.

EVALUATE **(a)** At $r = 10$ cm, (i.e., inside the sphere), we can use the upper formula, which gives

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Qr}{R^3} = \frac{(18\ \mu\text{C})(0.1\ \text{m})}{4\pi\left[8.85\times10^{-12}\ \text{C}^{-2}/\left(\text{N}\cdot\text{m}^2\right)\right](0.2\ \text{m})^3} = 2.0\ \text{MN/C}$$

(b) At $r = R$, we can use either the upper or the lower formula. The result is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2} = \frac{(18\ \mu\text{C})}{4\pi\left[8.85\times10^{-12}\ \text{C}^{-2}/\left(\text{N}\cdot\text{m}^2\right)\right](0.2\ \text{m})^2} = 4.0\ \text{MN/C}$$

(c) At $r = 2R = 40$ cm (i.e., outside the sphere), we use the lower formula, which gives

$$E=\frac{1}{4\pi\varepsilon_0}\frac{Q}{(2R)^2}=\frac{(18\ \mu\text{C})}{4\pi\left[8.85\times10^{-12}\ \text{C}^{-2}/\left(\text{N}\cdot\text{m}^2\right)\right](0.4\ \text{m})^2}=1.0\ \text{MN/C}$$

ASSESS Inside the solid sphere, where $r < R,$ the electric field increases linearly with r. On the other hand, outside the sphere, where $r < R,$ the field strength decreases as $1/r^2$. Gauss's law can be applied in this problem because the charge configuration is spherically symmetric.

24. **INTERPRET** This problem involves a charge distribution that is spherically symmetric, so we can apply Gauss's law to find the electric field at the various points. Also, because the total electric field may be thought of as the electric field due to the central point charge and the electric field due to the charge distribution, we can apply the superposition principle as well.

DEVELOP The total electric field is the superposition of the field due to the point charge and the field due to the charge on the spherical shell. Both charge distributions are spherically symmetric about the same center. We will consider the fields due to each charge distribution individually, then sum them by invoking the superposition principle to find the net field. Consider first the field inside the shell with no point charge ($r < R = 10$ cm). The field E_{sphere} here is zero, as explained in Example 21.2, so we have

$$\vec{E}_{\text{sphere}}(r<10\ \text{cm})=0$$

Consider now the field E_{pc} due to the point charge, which is simply given by Equation 20.3

$$\vec{E}_{\text{pc}}=\frac{kq_{\text{pc}}}{r^2}\hat{r}$$

For this problem, $q_{\text{pc}} = 15$ nC. The total field is the sum of these two

$$\vec{E}=\vec{E}_{\text{sphere}}+\vec{E}_{\text{pc}}$$

For part **(c)**, we are considering a point outside the sphere, so the result of Example 21.1 tells us that the field here due to the charge distribution on the sphere is the same as if the total charge were located at the center. Thus

$$\vec{E}_{\text{sphere}}(r>10\ \text{cm})=\frac{kq_{\text{sphere}}}{r^2}\hat{r}$$

For this problem, $q_{\text{sphere}} = -22$ nC. The expression for the field due to the point charge is the same as given above, so we can find the net field again by summing these two contributions.

EVALUATE **(a)** For $r = 2.2$ cm, the net electric field is

$$\vec{E}=\vec{E}_{\text{sphere}}+\vec{E}_{\text{pc}}=0+\frac{kq_{\text{pc}}}{r^2}\hat{r}=\frac{\left(9.0\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(15\ \text{nC})}{(0.022\ \text{m})^2}\hat{r}=(280\ \text{kN/C})\hat{r}$$

to two significant figures.

(b) For $r = 5.6$ cm, the net electric field is

$$\vec{E}=\vec{E}_{\text{sphere}}+\vec{E}_{\text{pc}}=0+\frac{kq_{\text{pc}}}{r^2}\hat{r}=\frac{\left(9.0\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(15\ \text{nC})}{(0.056\ \text{m})^2}\hat{r}=(43\ \text{kN/C})\hat{r}$$

(c) For $r = 14$ cm, the net electric field is

$$\vec{E}=\vec{E}_{\text{sphere}}+\vec{E}_{\text{pc}}=\frac{kq_{\text{sphere}}}{r^2}\hat{r}+\frac{kq_{\text{pc}}}{r^2}\hat{r}=\frac{\left(9.0\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(-22\ \text{nC}+15\ \text{nC})}{(0.14\ \text{m})^2}\hat{r}=(-3.2\ \text{kN/C})\hat{r}$$

ASSESS For parts **(a)** and **(b)**, the net field is oriented outward. For part **(c)**, the net field is oriented inward because the net charge for $r < 15$ cm is negative.

25. **INTERPRET** In this problem we are given the field strength at two different points outside the charge distribution and asked to determine the symmetry possessed by the configuration.

Develop The symmetry of the charge distribution can be determined by noting that the electric field strength decreases as $1/r^2$ for a spherically symmetric charge distribution, and as $1/r$ for a line charge (see Examples 21.4 and 21.1).

Evaluate We write $E = Cr^{-n}$, for some constant C. This gives

$$\frac{E_2}{E_1} = \left(\frac{r_1}{r_2}\right)^n \quad \Rightarrow \quad \ln\left(\frac{E_2}{E_1}\right) = n \ln\left(\frac{r_1}{r_2}\right)$$

Inserting the values given, we obtain

$$n = \frac{\ln(E_2/E_1)}{\ln(r_2/r_1)} = \frac{\ln(55/43)}{\ln(23/18)} = 1.00$$

Thus, we conclude that the charge distribution possesses line symmetry.

Assess The $1/r$ dependence characteristic of line symmetry can be readily verified by taking the field strength to be of the form $E = 2k\lambda/r$.

26. **Interpret** We are given the force on an electron near a sheet of charge, and are asked to find the surface charge density on the sheet. This problem has planar symmetry, and we will use the result for the electric field near an infinite plane of charge.

Develop The force on a charge is $F = qE$, and $E = \dfrac{\sigma}{2\epsilon_0}$ near a plane of charge. We note that the electron (a negative charge) is repelled, so the charge on the plane must be negatively charged as well. The charge on the electron is $q = 1.6\times10^{-19}$ C, and the magnitude of the force felt by the electron is $F = 2.7\times10^{-12}$ N. We solve for σ.

Evaluate $F = qE = \dfrac{q\sigma}{2\epsilon_0} \rightarrow \sigma = \dfrac{2\epsilon_0 F}{q} = 3.0\times10^{-4}\ \mathrm{C/m^2}$. The charge on the plate must be negative, so $\sigma = -3.0\times10^{-4}\ \mathrm{C/m^2}$.

Assess We could also get the sign of the charge sheet by being more careful about F and E. They are both vectors!

27. **Interpret** We are to find the electric field produced by a uniformly charged sheet. This problem has planar symmetry, so we will use the result for the electric field near an infinite plane of charge (Example 21.6 and Equation 21.7).

Develop Example 21.6 derives the electric field near a uniform sheet of charge with charge density σ. The result is (Equation 21.7)

$$E = \frac{\sigma}{2\varepsilon_0}$$

The charge density given in the problem is $\sigma = 89\ \mathrm{pC/cm^2} = 89\times10^{-12}\ \mathrm{C}/(0.01\ \mathrm{m})^2$.

Evaluate Inserting the given quantities into the expression above gives

$$E = \frac{\sigma}{2\varepsilon_0} = \frac{(89\times10^{-12}\ \mathrm{C})/(0.01\ \mathrm{m})^2}{2\left[8.85\times10^{-12}\ \mathrm{C^2}/(\mathrm{N\cdot m^2})\right]} = 50\ \mathrm{kN/C}$$

Assess Although this seems to be a small charge per area at first, the resulting field is quite large. Remember that a Coulomb is a very large charge.

28. **Interpret** We are to find the uniform surface charge density that will produce the given electric field at points nearby. This problem has planar symmetry, so we will use the result for the electric field near an infinite plane of charge (Example 21.6).

Develop Example 21.6 derives the electric field near a uniform sheet of charge with charge density σ. The result is

$$E = \frac{\sigma}{2\epsilon_0}$$

The electric field given in the problem is $E = 1.7\times10^3$ N/C, so we can solve for σ.

EVALUATE Inserting the given quantities into the expression above gives

$$E = \frac{\sigma}{2\epsilon_0}$$

$$\sigma = 2E\epsilon_0 = 2(1.7\times10^3 \text{ N/C})\left[8.85\times10^{-12} \text{ C}^2/(\text{N}\cdot\text{m}^2)\right] = 3.0\times10^{-8} \text{ C/m}^2$$

ASSESS This is a small charge density in terms of the Coulomb, but the Coulomb is a relatively large charge.

Section 21.5 Field of Arbitrary Charge Distribution

29. **INTERPRET** We are given a charge distribution with approximate line symmetry, so we can apply Gauss's law to compute the electric field. We are to consider two cases: the first at a distance that is small compared to the length of the rod and the second at a distance much, much greater than the length of the rod.

DEVELOP Close to the rod, but far from either end, the rod appears infinite, so the electric field strength is (see Example 21.4) $E = 2k\lambda/r$. For part (b), we are considering a position $r = 32$ m that is over an order of magnitude larger than the size of the rod (0.49 m). Therefore, the rod may be treated as a point charge, so the electric field will be given by Equation 20.3.

EVALUATE **(a)** Substituting the values given in the problem statement, we obtain

$$E = \frac{2k\lambda}{r} = \frac{2kQ/l}{r} = \frac{2\left(9.0\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2\right)(2.0\ \mu\text{C})/(0.49\text{ m})}{0.014\text{ m}} = 5.2\times10^6 \text{ N/C}$$

(b) Far away $(r >> L)$, the rod appears like a point charge, so

$$E \approx \frac{kq}{r^2} = \frac{\left(9.0\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2\right)(2.0\ \mu\text{C})}{(32\text{ m})^2} = 17.6 \text{ N/C}$$

ASSESS At a distance much, much greater than the characteristic size of any charge distribution (line charge, in this case), the field always resembles that of a point charge.

30. **INTERPRET** We are to determine the approximate field strength near a charged piece of paper with a given surface charge density. We will assume that the paper is flat and is large compared to the distance involved.

DEVELOP As derived in Example 21.6, the electric field due to an infinite sheet of charge is

$$E = \frac{\sigma}{2\epsilon_0}$$

A piece of paper is not infinite, but the problem states that it is an ordinary piece of paper, so we'll assume that the paper is large compared to the 1-cm distance from the paper. This approximation allows us to use the equation above for the infinite case. The surface charge density on the paper is $\sigma = 48 \text{ nC/m}^2$.

EVALUATE Inserting the given quantities gives

$$E = \frac{\sigma}{2\epsilon_0} = \frac{48 \text{ nC/m}^2}{2\left[8.85\times10^{-12} \text{ C}^2/(\text{N}\cdot\text{m}^2)\right]} = 2.7 \text{ kN/C}$$

ASSESS The infinite sheet of charge is not a useless construct. If you are close enough to any flat surface, it *looks* infinite, and you can use the infinite case as a very good approximation.

31. **INTERPRET** We are to approximate the electric field strength near the center of a flat, charged disk and far from the charged disk. In both cases, we will choose appropriate approximations.

DEVELOP The area of the disk is given as $A = 0.14 \text{ m}^2$, so the radius of the disk must be

$$\pi r^2 = A \quad \Rightarrow \quad r = \sqrt{\frac{A}{\pi}} = 21 \text{ cm}.$$

The point $r = 0.1$ cm from the center of the disk is 2 orders of magnitude smaller than the disk radius, so the disk will appear to be an infinite plane. Thus, we can use the result of Example 21.6,

$$E = \frac{\sigma}{2\epsilon_0} = \frac{Q}{2\epsilon_0 A}$$

The point $r = 250$ cm from the disk is one order of magnitude larger than the disk radius, so the disk will look more like a point charge and we can use Equation 20.3

$$E = \frac{kQ}{r^2}$$

The charge on the disk is $Q = 5.0\,\mu\text{C}$.

EVALUATE **(a)** Inserting the given quantities into the expression for a flat plane gives

$$E = \frac{Q}{2\epsilon_0 A} = \frac{5.0\,\mu\text{C}}{2\left[8.85\times10^{-12}\ \text{C}^2/\left(\text{N}\cdot\text{m}^2\right)\right](0.14\ \text{m})^2} = 2.0\times10^{6}\,\text{N/C}$$

(b) Inserting the given quantities into the expression for the point charge gives

$$E = \frac{kQ}{r^2} = \frac{\left(9.0\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(5.0\,\mu C)}{(2.5\ \text{m})^2} = 7.2\times10^{3}\,\text{N/C}$$

ASSESS If you are far enough from anything, it looks like a point charge; and if you are close enough, it looks like an infinite plane.

Section 21.6 Gauss's Law and Conductors

32. **INTERPRET** This problem requires us to find the electric field strength "just" outside a conducting sphere with a given charge density. Although the problem contains spherical symmetry, we can also consider it to contain planar symmetry, since the surface of the sphere will appear to be an infinite plane for distances from the surface that are much, much less than the sphere's radius (consider the surface of Earth as an example of this effect).

DEVELOP Because we are not given the sphere's radius, we cannot calculate the total charge on the sphere, so we are obliged to approximate the electric field by considering the surface of the sphere as an infinite plane. Under these conditions, we can use Equation 21.7 to find the electric field. However, there is one caveat that we must take into consideration that differentiates this problem from that of Example 21.7, namely, that we are considering a conducting sphere here, so the field inside it is zero! This means that the electric field that emanates from both sides of our conducting sheet in Example 21.7 must now emanate only in a single direction away from the surface of the sphere (i.e., away from the center). Thus, we expect the electric field to have twice the strength of that given in Example 21.7, or

$$E = \frac{\sigma}{\epsilon_0}$$

To see that this is so, consider a conducting sphere of radius r with surface charge σ. The total charge on the sphere is

$$Q = 4\pi r^2 \sigma$$

From Example 21.1, we know that the electric field outside a spherical charge distribution is

$$E = \frac{Q}{4\pi\epsilon_0 r^2} = \frac{\sigma}{\epsilon_0}$$

which is what we derived above by considering the physics involved.

EVALUATE Inserting the surface charge density into the expression above for electric field, we find

$$E = \frac{2.3\,\mu\text{C}/\text{m}^2}{8.85\times10^{-12}\ \text{C}^2/(\text{N}\cdot\text{m}^2)} = 260\ \text{kN}/\text{C}$$

ASSESS Because the charge density is positive, we can also say that the electric field lines point radially away from the center of the sphere.

33. **INTERPRET** This problem involves finding the charge distribution of a conductor using Gauss's law. The charge distribution is spherically symmetric.

DEVELOP As explained in Section 21.6, the electric field inside a conductor of arbitrary geometry is zero. In addition, the net charge must reside on the conductor surface.

EVALUATE **(a)** Because the net charge inside the conductor is zero, the volume charge density inside the conductor is also zero.

(b) If the sphere is electrically isolated, the charge will be uniformly distributed (i.e., spherically symmetric), so the surface charge density is just the total charge divided by the surface area of the sphere. This gives

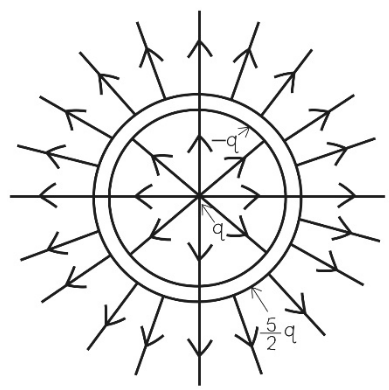
$$\sigma = \frac{Q}{4\pi R^2} = \frac{4.0\ \mu\text{C}}{4\pi(0.019\ \text{m})^2} = 8.8\times10^{-4}\ \text{C/m}^2$$

ASSESS Since charges are mobile, the presence of other charges near the conductor will cause the charges on the surface to move, so the equilibrium charge distribution will not be spherically symmetric.

34. **INTERPRET** This problem requires us to sketch the electric field lines for a spherically symmetric charge distribution.

DEVELOP The field inside the shell is just due to the point charge (8 electric field lines radiating outward). The field outside the sphere is just that of a point charge $q + \frac{3}{2}q = \frac{5}{2}q$, so there are (5/2) × 8 = 20 lines of force radiating outward.

EVALUATE See figure below.



ASSESS There is a charge $-q$ spread uniformly over the inner surface of the shell, so the field inside the conducting material that makes up the spherical shell is zero.

35. **INTERPRET** This problem is about finding the electric field near the surface of a conducting plate. The approximate plane symmetry of the system allows us to make use of Gauss's law.

DEVELOP The net charge of $Q = 20\ \mu\text{C}$ must distribute itself over the outer surface of the plate, in accordance with Gauss's law for conductors (see Section 21.6). The outer surface of the plate consists of two plane square surfaces on each face, plus the edges and corners. Symmetry arguments imply that for an isolated plate, the charge density on each face is the same, but not necessarily uniform because the edges and corners also have charge. If the thickness of the plate is much, much less than its length and width, we can assume that the edges and corners have negligible charge and that the density on the faces is approximately uniform. With this assumption, the surface charge density is the total charge divided by the area of both faces,

$$\sigma = \frac{Q}{2A} = \frac{20\ \mu\text{C}}{2(0.74\ \text{m})^2} = 18.3\ \mu\text{C/m}^2$$

Given this surface charge density, we can apply Equation 21.8 to find the electric field near the surface (but not near an edge) of the conducting plate.

EVALUATE Inserting the surface charge density into Equation 21.8, we find the field strength near the plate to be

$$E = \frac{\sigma}{\varepsilon_0} = \frac{18.3\ \mu\text{C}/\text{m}^2}{\left[8.85\times10^{-12}\ \text{C}^2/\left(\text{N}\cdot\text{m}^2\right)\right]} = 2.1\ \text{MN}/\text{C}$$

ASSESS Note the distinction between a charged conducting plate and a uniformly charged plate. In the latter, charges are not free to move and the electric field is (see Example 21.6) $E = \sigma/(\varepsilon_0)$.

EXAMPLE VARIATIONS

36. **INTERPRET** This problem is about a charge distribution with spherical symmetry.

DEVELOP We want to find the fields both inside and outside the distribution, so we will use two spherical Gaussian surfaces to enclose the charge within those regions.

EVALUATE We already know that the flux through a spherical Gaussian surface is $\Phi = 4\pi r^2 E$ when we have spherical symmetry, so that's the flux for both surfaces. The outer surface encloses the charge $+q$ at the center and $+2q$ on the shell, so the net charge enclosed is $q_{\text{enc}} = 3q$ for $r > R$. The inner surface encloses only the point charge $+q$, so $q_{\text{enc}} = q$ for $r < R$. Now we find the field by equating the flux $\Phi = 4\pi r^2 E$ to $q_{\text{enc}}/\epsilon_0$.

The result for $r > R$ is $E = 3q/4\pi\epsilon_0 r^2$. For $r < R$ with enclosed charge $+q$, the result is $E = q/4\pi\epsilon_0 r^2$.

ASSESS We've seen that the field outside a spherically symmetric charge distribution is that of an equivalent point charge at the center, which contains the total enclosed charge.

37. **INTERPRET** This problem is about a charge distribution with spherical symmetry.

DEVELOP We want to find the magnitude and sign of the point charge q, surface charge Q, and the magnitude and direction of the electric field at the outer surface of the shell. We are given the magnitude and direction of the field at two regions, so we will use two spherical Gaussian surfaces to enclose the charge within those regions.

EVALUATE We already know that the flux through a spherical Gaussian surface is $\Phi = 4\pi r^2 E$ when we have spherical symmetry, so that's the flux for both surfaces. The outer surface encloses the charge q at the center and Q on the shell, so the net charge enclosed is $q_{\text{enc}} = q + Q$ for $r > R$. The inner surface encloses only the point charge q, so $q_{\text{enc}} = q$ for $r < R$. Now we find the field by equating the flux $\Phi = 4\pi r^2 E$ to $q_{\text{enc}}/\epsilon_0$.

The result for $r > R$, where $r = 23.8$ cm, gives

$$E(r = 23.8\text{cm}) = \frac{q+Q}{4\pi\epsilon_0 r^2} \rightarrow q + Q = 4\pi\left(8.85\times10^{-12}\text{C}^2/\text{N}\cdot\text{m}^2\right)(23.8\text{cm})^2\left(2.68\ \text{MN/C}\right).$$

$$q + Q = 16.9\ \mu\text{C}$$

For $r < R$, where $r = 7.50$ cm, we get

$$E(r = 7.50\ \text{cm}) = \frac{q}{4\pi\epsilon_0 r^2} \rightarrow q = 4\pi\left(8.85\times10^{-12}\text{C}^2/\text{N}\cdot\text{m}^2\right)(7.50\ \text{cm})^2\left(-1.17\ \text{MN/C}\right).$$

$$q = -732\,\text{nC}$$

Where we have used a negative value for the electric field since we are told it points inward in this region. From these we find that the surface charge is equal to $Q = 17.6\ \mu\text{C}$.

Finally, to evaluate the electric field at the surface we calculate the value at $r = R$ using the total charge enclosed

$$E(r = 15.0\ \text{cm}) = \frac{q+Q}{4\pi\epsilon_0 r^2} = \frac{16.9\ \mu\text{C}}{4\pi\left(8.85\times10^{-12}\text{C}^2/\text{N}\cdot\text{m}^2\right)(15.0\ \text{cm})^2} = 6.75\ \text{MN/C}$$

Which due to its positive sign, points radially outward.

ASSESS We've seen that the field outside a spherically symmetric charge distribution is that of an equivalent point charge at the center, which contains the total enclosed charge.

38. **INTERPRET** This problem is about a charge distribution with linear symmetry.

DEVELOP We want to find the fields both inside and outside the distribution, so we will use two cylindrical Gaussian surfaces to enclose the charge within those regions.

EVALUATE We already know that the flux through a cylindrical Gaussian surface is $\Phi = 2\pi r l E$ when we have linear symmetry, so that's the flux for both surfaces. The outer surface encloses the charge density $+\lambda$ at the center and -2λ on the surface, so the net charge enclosed is $q_{\text{enc}} = -\lambda l$ for $r > R$. The inner surface encloses only the charge density $+\lambda$ at the center, so $q_{\text{enc}} = \lambda l$ for $r < R$. Now we find the field by equating the flux $\Phi = 2\pi r l E$ to $q_{\text{enc}}/\epsilon_0$.

The result for $r > R$ is $E = -\lambda l/2\pi\epsilon_0 l = -\lambda/2\pi\epsilon_0$. For $r < R$ with enclosed density is $+\lambda$, the result is $E = \lambda/2\pi\epsilon_0$.

ASSESS We've seen that the field outside a linearly symmetric charge distribution is that of an equivalent point charge at the center, which contains the total enclosed charge.

39. **INTERPRET** This problem is about a charge distribution with linear symmetry.

DEVELOP We want to find the magnitude and sign of the line charge densities λ_1, λ_2, and the magnitude and direction of the electric field at the outer surface of the tube. We are given the magnitude and direction of the field at two regions, so we will use two cylindrical Gaussian surfaces to enclose the charge within those regions.

EVALUATE We already know that the flux through a cylindrical Gaussian surface is $\Phi = 2\pi r l E$ when we have linear symmetry, so that's the flux for both surfaces. The outer surface encloses the charge density λ_1 from the tube at the center and λ_2 on the surface, so the net charge enclosed is $q_{\text{enc}} = (\lambda_1 + \lambda_2)l$ for $r > R$. The inner surface encloses only the charge density λ_1 at the center, so $q_{\text{enc}} = \lambda_1 l$ for $r < R$. Now we find the field by equating the flux $\Phi = 2\pi r l E$ to $q_{\text{enc}}/\epsilon_0$.

The result for $r > R$, where $r = 23.8$ cm, gives

$$E(r = 23.8\,\text{cm}) = \frac{\lambda_1 + \lambda_2}{2\pi\epsilon_0 r} \rightarrow \lambda_1 + \lambda_2 = 2\pi\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)(23.8\,\text{cm})\left(2.68\text{ MN/C}\right).$$

$$\lambda_1 + \lambda_2 = 35.4\,\mu\text{C/m}$$

For $r < R$, where $r = 7.50$ cm, we get

$$E(r = 7.50\,\text{cm}) = \frac{\lambda_1}{2\pi\epsilon_0 r} \rightarrow \lambda_1 = 2\pi\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)(7.50\,\text{cm})\left(-1.17\text{ MN/C}\right).$$

$$\lambda_1 = -4.88\,\mu\text{C/m}$$

Where we have used a negative value for the electric field since we are told it points inward in this region. From these we find that the surface charge density is equal to $\lambda_2 = 40.3\,\mu\text{C/m}$.

Finally, to evaluate the electric field at the surface we calculate the value at $r = R$ using the total charge enclosed

$$E(r = 15.0\text{ cm}) = \frac{\lambda_1 + \lambda_2}{2\pi\epsilon_0 r} = \frac{35.4\,\mu\text{C/m}}{2\pi\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)(15.0\text{ cm})} = 4.25\,\text{MN/C}$$

Which due to its positive sign, points radially outward.

ASSESS We've seen that the field outside a linearly symmetric charge distribution is that of an equivalent point charge at the center, which contains the total enclosed charge.

40. **INTERPRET** An infinite line has line symmetry, so we can apply Gauss's law to find the electric field.

DEVELOP We want to find the fields a certain distance from the distribution, so we will use a cylindrical Gaussian surfaces to enclose the charge.

EVALUATE We already know that the flux through a cylindrical Gaussian surface is $\Phi = 2\pi r l E$ when we have linear symmetry. This surface encloses the charge density λ given, so the net charge enclosed is $q_{\text{enc}} = \lambda l$ for $r = 85.0$ cm. We find the field by equating the flux $\Phi = 2\pi r l E$ to $q_{\text{enc}}/\epsilon_0$.

$$E = \frac{\lambda}{2\pi\epsilon_0 r} = \frac{292\,\mu\text{C/m}}{2\pi\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)(85.0\text{ cm})} = 6.18\text{ MN/C}$$

ASSESS We've seen that the field outside a linearly symmetric charge distribution is that of an equivalent point charge at the center, which contains the total enclosed charge.

41. **INTERPRET** A finite line has line symmetry, so we can apply Gauss's law to express the electric field, and find the total charge on the rod.

DEVELOP We want to find the total charge on the rod knowing the field strength a certain distance from the distribution, so we will use a cylindrical Gaussian surfaces to enclose the charge.

EVALUATE We already know that the flux through a cylindrical Gaussian surface is $\Phi = 2\pi rlE$ when we have linear symmetry. This surface encloses the charge density λ given, so the net charge enclosed is $q_{\text{enc}} = \lambda l$ for l = 3.26 m and r = 5.12 cm. We find the field by equating the flux $\Phi = 2\pi rlE$ to $q_{\text{enc}}/\epsilon_0$.

$$E = \frac{q_{\text{enc}}}{2\pi\epsilon_0 rl} \to q_{\text{enc}} = 2\pi\epsilon_0 rlE = 2\pi\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)(5.12\text{ cm})(3.26\text{ m})\left(7.88\text{ kN/C}\right) = 7.31\,\mu\text{C}$$

ASSESS We've seen that the field outside a linearly symmetric charge distribution is that of an equivalent point charge at the center, which contains the total enclosed charge.

42. **INTERPRET** An infinitely long rod has line symmetry and a uniform volume charge density, so we can apply Gauss's law to express the electric field inside and outside the rod.

DEVELOP We want to find the field strength at certain distances from the distribution knowing the volume charge density, so we will use two cylindrical Gaussian surfaces to enclose the charge within those regions.

EVALUATE We already know that the flux through a cylindrical Gaussian surface is $\Phi = 2\pi rlE$ when we have linear symmetry. This surface encloses the charge density ρ given, so the net charge enclosed is $q_{\text{enc}} = \rho\pi r^2 l$, where r is the distance from the center of the rod. We find the fields by equating the flux $\Phi = 2\pi rlE$ to $q_{\text{enc}}/\epsilon_0$. Inside and outside the rod we find the fields are given by, respectively:

$$E_{\text{in}} = \frac{\rho\pi r^2 l}{2\pi\epsilon_0 rl} = \frac{\rho r}{2\epsilon_0}\,;\; E_{\text{out}} = \frac{\rho\pi R^2 l}{2\pi\epsilon_0 rl} = \frac{\rho R^2}{2\epsilon_0 r}$$

ASSESS The amount of charge enclosed by the inner surface is a fraction of the entire charge and depends on the distance r, while the amount of charge enclosed by the outer surface is the entire charge enclosed up to $r = R$.

43. **INTERPRET** A finite long rod has line symmetry and a uniform volume charge density, so we can apply Gauss's law to express the electric field at a certain distance from the rod and obtain the total charge on the rod.

DEVELOP We want to find the field strength at a given distance outside the distribution knowing the magnitude of the field at a distance inside the distribution. We first find the total charge from the given value for the field inside the rod, and use that to obtain the value for the field outside, using the results obtained in the preceding problem.

EVALUATE Using the field inside the rod found in the preceding problem, we find the total charge is equal to:

$$E_{\text{in}} = \frac{\rho r}{2\epsilon_0} = \frac{q_{\text{enc}}}{\pi R^2 l}\frac{r}{2\epsilon_0} \to q_{\text{enc}} = \frac{2\pi R^2\epsilon_0 lE}{r}$$

$$q_{\text{enc}} = \frac{2\pi(1.27\,\text{cm})^2\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)(75.0\,\text{cm})(286\,\text{kN/C})}{(6.84\,\text{mm})} = 281\,\text{nC}$$

Using the field outside the rod found in the preceding problem, we find the field 3.60 cm from the rod's axis is equal to:

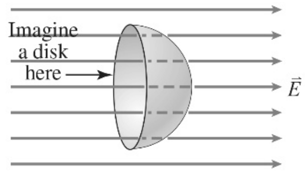
$$E_{\text{out}} = \frac{\rho R^2}{2\epsilon_0 r} = \frac{q_{\text{enc}}}{2\pi\epsilon_0 rl} = \frac{281\,\text{nC}}{2\pi\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)(3.60\,\text{cm})(75.0\,\text{cm})} = 187\,\text{kN/C}$$

ASSESS The amount of charge enclosed by the inner surface is a fraction of the entire charge and depends on the distance r, while the amount of charge enclosed by the outer surface is the entire charge enclosed up to $r = R$.

PROBLEMS

44. **INTERPRET** This problem requires us to consider the geometry of the situation to find the electric flux through the hemispherical surface depicted in Fig. 21.33.

DEVELOP All of the electric field lines going through the hemisphere also go through an equatorial disk covering its edge (see figure below). Therefore, the flux through the disk (normal in the direction of $\vec{E}$) equals the flux through the hemisphere.



EVALUATE Since $\vec{E}$ is uniform, the flux through the disk is just $\pi R^2 E.$

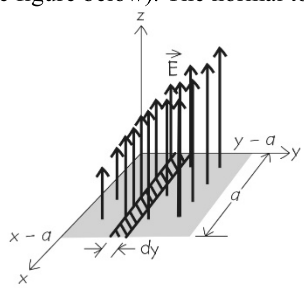
ASSESS Gauss's law gives the same result, since the flux through the closed surface, consisting of the hemisphere plus the disk, is zero. See Section 21.3.

45. **INTERPRET** This problem is about finding the electric flux through a given surface.

DEVELOP The electric flux through a surface is given by Equation 21.2:

$$\Phi = \int_{\text{surface}} \vec{E} \cdot d\vec{A}$$

Since the electric field depends only on *y*, we break up the square into strips of area $d\vec{A} = \pm a\, dy\, \hat{k}$ of length a parallel to the *x*-axis and width *dy* (see figure below). The normal to the surface is $\pm\hat{k}.$



EVALUATE The integral of Equation 21.2 gives

$$\Phi = \int_{\text{surface}} \vec{E} \cdot d\vec{A} = \int_0^a (E_0 y/a)\hat{k} \cdot (\pm a dy\, \hat{k}) = \pm E_0 \int_0^a y dy = \pm\frac{1}{2} E_0 a^2$$

ASSESS Our result can be compared to the case where the field strength is constant. In that case, the flux through the surface would be $\Phi = \pm E_0 a^2.$

46. **INTERPRET** From the given electric field, we are to determine the volume charge density in a region, which we can do by applying Gauss's law.

DEVELOP The electric field in the region is given as $\vec{E} = ax\hat{i},$ where $a = 32\ \text{N}/(\text{C}\cdot\text{m}).$ Since the field does not depend on *y* or *z*, we must conclude that there is no variation in the *y*- or *z*-directions. Use a Gaussian surface that is a right prism with a 1-m square base, with the base *A* of the prism at $x = 0.$ Gauss's law states that

$$\int \vec{E} \cdot d\vec{A} = \frac{q_{\text{enclosed}}}{\varepsilon_0}$$

EVALUATE For this particular electric field, the field is only in the $\hat{i}$ direction, so the only surfaces on the prism that have a flux are the surfaces at $x' = 0$ and $x' = x.$ The flux through the surface at $x' = 0$ is zero, since the field there is zero. The flux through the surface at $x' = x$ is

$$\Phi = \int \vec{E}(x) \cdot d\vec{A} = E(x)A = axA$$

The flux through this surface is also the flux through the entire Gaussian surface, since the rest of the surface has zero flux, so

$$\int \vec{E} \cdot d\vec{A} = axA = \frac{q_{\text{enclosed}}}{\varepsilon_0} \;\Rightarrow\; q_{\text{enclosed}} = a\varepsilon_0 Ax$$

The volume of the prism is $V = Ax,$ so the charge density is

$$\rho = \frac{q_{\text{enclosed}}}{V} = \frac{a\varepsilon_0 Ax}{Ax} = a\varepsilon_0 = \left[32\ \text{N}/(\text{C}\cdot\text{m})\right]\left[8.85\times10^{-12}\ \text{C}^2/\left(\text{N}\cdot\text{m}^2\right)\right] = 2.83\times10^{-10}\ \text{C}/\text{m}^3$$

ASSESS Notice that the units come out to be coulombs per unit volume, as expected. This is the electric field and charge density we would have inside a thick slab of charged material, such as in Fig. 21.34.

47. **INTERPRET** We're asked to estimate the electric field outside red blood cells of two different species, given the enclosed charges and the cell radii.

DEVELOP We are told to approximate the red blood cells as perfect spheres. The charge from an excess of electrons $(Q = Ne)$ is spread uniformly around the surface. The symmetry implies Gauss's law can be used to find the electric field. Outside a charged sphere, the electric field is $E = kQ/r^2$, which is the same as for a point charge at the origin.

EVALUATE For rabbit RBCs:

$$E = \frac{kQ}{r^2} = \frac{\left(9.0\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left(4.4\times10^6\right)\left(1.6\times10^{-19}\,\text{C}\right)}{\left(30\times10^{-6}\ \text{m}\right)^2} = 7.0\ \text{MN/C}$$

For human RBCs:

$$E = \frac{kQ}{r^2} = \frac{\left(9.0\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left(15\times10^6\right)\left(1.6\times10^{-19}\,\text{C}\right)}{\left(36\times10^{-6}\ \text{m}\right)^2} = 17\ \text{MN/C}$$

ASSESS Because the charge is from electrons, the electric fields point in toward the center of the cells.

48. **INTERPRET** We are given a charge distribution with spherical symmetry, so we can apply Gauss's law to find the electric field.

DEVELOP The balloon can be regarded as a spherical shell with charge residing on the outer surface. Gauss's law (Equation 21.3) is

$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{\text{enclosed}}}{\epsilon_0}$$

For part **(a)**, the surface over which we perform the integration is inside the balloon (r = 50 cm), so $q_{\text{enclosed}} = 0$ and Gauss's law gives

$$\oint \vec{E}\cdot d\vec{A} = EA = \frac{q_{\text{enclosed}}}{\epsilon_0} = 0$$
$$E = 0$$

For part **(b)**, we note that outside the balloon, the field is like that of a point charge at the center that carries the total charge. Therefore, the electric field is given by Equation 20.3, which gives

$$\left.\begin{aligned} E_1 &= \frac{kQ}{R^2} \\ E_2 &= \frac{kQ}{r_2^2} \end{aligned}\right\} E_2 = E_1\frac{R^2}{r_2^2}$$

where R = 70 cm, r_2 = 190 cm, and E_1 = 26 kN/C. Notice that the expression for E_1 also allows us to find the total charge Q on the balloon for part **(c)**.

EVALUATE **(a)** Inside the balloon, E = 0 as discussed above.

(b) Inserting the given quantities into the expression for E_2 gives

$$E_2 = E_1\left(\frac{R}{r}\right)^2 = \left(26\ \text{kN/C}\right)\left(\frac{70\ \text{cm}}{190\ \text{cm}}\right)^2 = 3.5\ \text{kN/C}$$

(c) Using the given field strength at the surface, we find a net charge

$$Q = \frac{E_0R^2}{k} = \frac{\left(26\ \text{kN/C}\right)\left(0.70\ \text{m}\right)^2}{\left(9.0\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)} = 1.4\ \mu\text{C}$$

ASSESS The electric field inside a spherical shell is identically zero. Outside the shell, the field decreases as $1/r^2$.

49. **INTERPRET** This problem involves a uniform, spherically symmetric charge distribution. We are to find the second location where the electric field has the given value and find the net charge on the sphere. Doing so will involve Gauss's law.

DEVELOP Example 21.1 uses Gauss's law to derive expressions for the electric fields inside and outside a spherical charge distribution. We are given the field strength inside the sphere (at $r_1 = R/3$), so we can equate it to the field strength outside the sphere at r_2 and solve for the position. Explicitly, we have

$$E_{\text{inside}} = \frac{Qr_1}{4\pi\epsilon_0 R^3} = E_{\text{outside}} = \frac{Q}{4\pi\epsilon_0 r_2^2}$$

$$\frac{1}{3R^2} = \frac{1}{r_2^2}$$

To find the net charge on the sphere, we can use the expression for E_{inside} and solve for Q.

EVALUATE **(a)** Solving the expression above for r_2 gives $r_2 = R\sqrt{3} = (3.0\text{ cm})\sqrt{3} = 5.2\text{ cm}$.

(b) Solving for the total charge Q gives

$$Q = \frac{4\pi\epsilon_0 R^3 E_{\text{inside}}}{r_1} = 12\pi\epsilon_0 R^2 E_{\text{inside}} = 12\pi\left[8.85\times10^{-12}\text{ C}^2/(\text{N}\cdot\text{m}^2)\right](0.030\text{ m})^2(16\text{ kN/C}) = 4.8\text{ nC}$$

ASSESS For part (a), we assumed that the second location at which the field had the given value was outside the sphere. Were this not true, we would have found an unphysical answer (i.e., an imaginary field).

50. **INTERPRET** The given charge distribution has spherical symmetry, so we can apply Gauss's law to find the electric field. In addition, because the charge distribution consists of two distinct charge distributions, we can apply the principle of superposition to find the net electric field.

DEVELOP The total electric field is the superposition of the fields due to the point charge and the spherical shell.

$$E = E_{\text{pt}} + E_{\text{shell}}$$

The field is spherically symmetric about the center. Inside the shell, the contribution E_{shell} to the electric field is zero because there is no charge enclosed (remember, this is just for the shell with no point charge inside). Outside the shell, the contribution E_{shell} is like that of a point with charge Q located at the center of the sphere of the shell. Both inside and outside the shell, the contribution E_{pt} is that of a point charge with charge $-2Q$.

EVALUATE **(a)** At $r = R/2 < R$ (inside the shell), the electric field is

$$E = E_{\text{pt}} + E_{\text{shell}} = \frac{k(-2Q)}{(R/2)^2} + 0 = -\frac{8\,kQ}{R^2}$$

Note that the minus sign means the direction is radially inward.

(b) At $r = 2R > R$ (outside shell), the field strength is

$$E = E_{\text{pt}} + E_{\text{shell}} = \frac{k(-2Q+Q)}{(2R)^2} = -\frac{kQ}{4R^2}$$

Again the direction of the field is radially inward.

(c) If $Q_{\text{shell}} = 2Q$, the field inside would be unchanged, but the field outside would be zero by Gauss's law (Equation 21.3) since $q_{\text{enclosed}} = q_{\text{shell}} + q_{\text{pt}} = 2Q - 2Q = 0$.

ASSESS By Gauss's law, the shell produces no electric field in its interior. The field outside a spherically symmetric distribution is the same as if all the charges were concentrated at the center of the sphere.

51. **INTERPRET** We are interested in finding the total charge and exact dimensions of a nonconducting square for which we know the electric field at two different locations. The situation has planar symmetry.

DEVELOP The field strength is specified both at a distance far from the plate, $r_f = 71.0\text{ m}$, and at a distance right above the plate, $r_n = 0$ m. In the case where the field is measured right above the plate, we can assume that the field lines are essentially perpendicular to the plate, which means the electric field strength is given by Gauss's law: $E = \sigma/2\epsilon_0$. The total charge on the plate will be related to the exact size of the plate by: $|Q| = \sigma L^2$. We can find the value for the total charge by applying Gauss's law at distance y_f to express the value of the field produced here, and use that to determine the precise dimensions of the square.

EVALUATE **(a)** Applying Gauss's law, at a distance y_f gives

$$E_\mathrm{f} = \frac{kQ}{r_\mathrm{f}^2} \rightarrow Q = \frac{E_\mathrm{f} r_\mathrm{f}^2}{k} = \frac{(314\ \mathrm{N/C})(71.0\ \mathrm{m})^2}{(9.0\times10^9\ \mathrm{N\cdot m^2/C^2})} = 176\ \mu\mathrm{C}$$

Where we have used spherical symmetry to enclose the charge distribution since the distance from the plate is large in comparison to its approximate size.

(b) From the given field generated right above the charged sheet, we can express the charged density and use it along with the total charge to find the precise dimensions of the square

$$L = \sqrt{\frac{|Q|}{\sigma}} = \sqrt{\frac{|Q|}{2E_\mathrm{n}\epsilon_0}} = \sqrt{\frac{176\ \mu\mathrm{C}}{2(6.80\ \mathrm{MN/C})(8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2})}} = 1.21\ \mathrm{m}$$

ASSESS The value of the field near the charged sheet has no dependence on the distance, but as one moves farther from it, the assumption that the field lines are all pointing in the direction perpendicular to the sheet will no longer be true.

52. **INTERPRET** In this problem we wish to find relationships between certain quantities of a charge distribution, composed of a point charge inside a charged spherical shell.

DEVELOP The field strength is specified at the outer surface of the charged shell, which we know must be due to the total charge enclosed: $Q_\mathrm{T} = Q + q$, where Q is the surface charge of the shell, and q is the point charge inside the shell. Using Gauss's law, we can find an expression for this given field, E_surf, in terms of the total charge and the radius of the sphere. From this we can obtain an expression for the point charge q, and use it to find the value of the field at the inner surface of the sphere.

EVALUATE **(a)** Applying Gauss's law, at a distance R gives

$$E_\mathrm{surf} = \frac{Q_\mathrm{T}}{4\pi\epsilon_0 R^2} \rightarrow Q_\mathrm{T} = 4\pi\epsilon_0 R^2 E_\mathrm{surf} = Q + q$$

$$q = 4\pi\epsilon_0 R^2 E_\mathrm{surf} - Q$$

Where we have used the radius of the sphere since the field magnitude given is at the outer surface of the shell.

(b) To find the field at the inner surface of the sphere, we apply Gauss's law to enclose the point charge q

$$E_\mathrm{inner} = \frac{q}{4\pi\epsilon_0 R^2} = E_\mathrm{surf} - \frac{Q}{4\pi\epsilon_0 R^2}$$

Where we have again used the radius of the sphere since the shell has negligible thickness.

ASSESS Our result makes sense since the field at the inner surface of the sphere is only due to the enclosed charge q, and thus it is equal to the field on the outer surface (due to the total charge) minus the contribution from the charge located over the shell's surface.

53. **INTERPRET** The charge distribution has spherical symmetry, so we can apply Gauss's law to find the electric field. In addition, it is composed of two distinct charge distributions, so we can consider each one separately and use the superposition principle to construct the net electric field. Note that this problem is exactly the same as Problem 21.52, so we will refer to that solution here.

DEVELOP See the solution to Problem 21.52 for the derivation. The total electric field is the superposition of the fields due to the point charge and the spherical shell. The field is spherically symmetric about the center.

EVALUATE **(a)** The field due to the shell is zero inside; so, at $r = 6$ cm, the field is due to the point charge only. Thus,

$$E = \frac{kq}{r^2} = \frac{(9.0\times10^9\ \mathrm{N\cdot m^2/C^2})(4.0\ \mu\mathrm{C})}{(0.06\ \mathrm{m})^2} = 7.5\times10^6\ \mathrm{N/C}$$

The field points radially outward.

(b) Outside the shell, its field is like that of a point charge with a total charge equal to the sum of the charge on the sphere and the point charge; so, at $r = 65$ cm, the field strength is

$$E = \frac{k(q+Q)}{r^2} = \frac{(9.0\times10^9\ \mathrm{N\cdot m^2/C^2})(86\ \mu\mathrm{C})}{(0.65\ \mathrm{m})^2} = 1.8\times10^6\ \mathrm{N/C}$$

The direction of the field is radially outward.

(c) If the charge on the shell were doubled, the charge enclosed inside the sphere would not change, so Gauss's law (Equation 21.3) tells us that the electric field inside the sphere would not change. However, the field outside would be

$$E = \frac{k(3.0\ \mu\mathrm{C} + 2\times83\ \mu\mathrm{C})}{(0.65\ \mathrm{m})^2} = 3.6\ \mathrm{MN/C}$$

which is essentially doubled (to two significant figures).

ASSESS By Gauss's law, the shell produces no electric field in its interior. The field outside a spherically symmetric distribution is the same as that if all the charges were concentrated at the center of the sphere.

54. **INTERPRET** This problem involves a spherically symmetric, uniform charge distribution, so we can apply Gauss's law to find an expression for the electric field.

DEVELOP Make a sketch of the situation (see figure below). Consider a sphere with radius *r*, where $a < r < b$. Gauss's law tells us that the electric field at this surface is

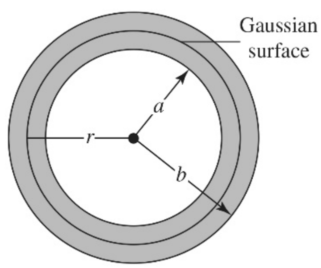
$$\oint \vec{E}\cdot d\vec{A} = EA = \frac{q_{\text{enclosed}}}{\epsilon_0}$$

$$E = \frac{q_{\text{enclosed}}}{A\epsilon_0} = \frac{q_{\text{enclosed}}}{4\pi\epsilon_0 r^2}$$

which is Equation 20.3 for the field of a point charge with charge q_{enclosed}. The charge q_{enclosed} is given by the charge density ρ times the charged volume enclosed by the sphere or radius *r*, or

$$q_{\text{enclosed}} = \frac{4}{3}\pi\left(r^3 - a^3\right)\rho,$$

We can combine these two expressions to express the electric field inside the charged volume.



EVALUATE Inserting q_{enclosed} into the expression for the electric field gives

$$E = \frac{\rho\left(r^3 - a^3\right)}{3\epsilon_0 r^2} = \frac{\rho}{3\epsilon_0}\left(r - a^3/r^2\right)$$

If we let $a \to 0$, and recall that $Q = \rho V = \rho(4\pi R^3/3)$, we get

$$E = \frac{\rho r}{3\epsilon_0} = \frac{Qr}{4\pi\epsilon_0 R^3}$$

which is Equation 21.5 for the electric field inside a uniformly charged sphere of radius *R*.

ASSESS Equation 21.5 for a uniformly charged spherical volume is recovered.

55. **INTERPRET** We are given a charge distribution with approximate line symmetry (provided we consider distances from the wire that are much, much smaller than the length of the wire, and provided we are not near the end of the wire). We can use this symmetry and Gauss's law to compute the electric field.

DEVELOP With the assumption that the electric field is approximately that from an infinitely long, line symmetric charge distribution, we can use the result of Example 21.4 (i.e., Equation 21.6) to express the electric field near the wire:

$$E = \frac{\lambda_{\text{enclosed}}}{2\pi\varepsilon_0 r}$$

where $\lambda_{\text{enclosed}}$ is the charge per unit length enclosed by a cylinder of radius r. We can evaluate this expression to find the electric field at different distances r from the wire.

EVALUATE **(a)** For $r = 0.50\text{ cm} = 0.0050\text{ m}$, which is between the wire and the pipe, the enclosed charge per unit length is $\lambda_{\text{enclosed}} = \lambda_{\text{wire}}$, and

$$E = \frac{\lambda_{\text{enclosed}}}{2\pi\varepsilon_0 r} = \frac{6.5\text{ nC/m}}{2\pi\left[8.85\times10^{-12}\text{ C}^2/\left(\text{N.m}^2\right)\right](0.0050\text{ m})} = 23\text{ kN/C}$$

The field is (positive) radially away from the axis of symmetry (i.e., from the wire).

(b) For $r = 4.5\text{ cm} = 0.045\text{ m}$, the enclosed charge per unit length is $\lambda'_{\text{enclosed}} = \lambda_{\text{wire}} + \lambda_{\text{pipe}}$, and

$$E' = \frac{\lambda'_{\text{enclosed}}}{2\pi\varepsilon_0 r} = \frac{6.5\text{ nC/m} - 4.2\text{ nC/m}}{2\pi\left[8.85\times10^{-12}\text{C}^2/\left(\text{N.m}^2\right)\right](0.045\text{ m})} = 0.9\text{ kN/C}$$

The field is in the same direction as the field in part **(a)**.

ASSESS Between the wire and the pipe, the enclosed charge per unit length is $\lambda_{\text{enclosed}} = \lambda_{\text{wire}}$, whereas outside the pipe, the enclosed charge is $\lambda'_{\text{enclosed}} = \lambda_{\text{wire}} + \lambda_{\text{pipe}}$. Since the pipe and the wire carry opposite charges, $\lambda'_{\text{enclosed}} < \lambda_{\text{enclosed}}$; so $E' < E$, as we have shown.

56. **INTERPRET** A finite long rod has line symmetry and a uniform volume charge density, so we can apply Gauss's law to express the electric field at certain distances from the rod.

DEVELOP We want to find the radial coordinate for which the strength of the electric field outside the rod is equal to the field found halfway between the rod's axis and its surface. We will use the fields for the inside and outside of the rod found in Problem 21.42.

EVALUATE Evaluating the field found inside, halfway to the surface ($r = R/2$), and equating it to the field found outside the rod at a distance r from the center we find:

$$E_{\text{in}}(r = R/2) = E_{\text{out}}(r)$$
$$\frac{\rho(R/2)}{2\epsilon_0} = \frac{\rho R^2}{2\epsilon_0 r} \to r = 2R$$

ASSESS From these expressions for the field inside and outside the rod we see that the field found inside at a distance $r = R/c$, where $c < 1$, will be equal to the field found outside at a distance $r = cR$.

57. **INTERPRET** A finite long rod has line symmetry and a uniform volume charge density, so we can apply Gauss's law to express the electric field at a certain distance from the rod and obtain the total charge on the rod.

DEVELOP We want to find the field strength at the surface of the rod, knowing the magnitude of the field at a distance inside the distribution. We first find the total charge from the given value for the field inside the rod, and use that to obtain the value for the field at the surface. We will use the fields for the inside and outside of the rod found in Problem 21.42.

EVALUATE **(a)** Evaluating the field found inside, halfway to the surface $(r = R/2)$, we find the total charge is equal to:

$$E_{\text{in}} = \frac{\rho(R/2)}{2\epsilon_0} = \frac{q_{\text{enc}}}{\pi R^2 l}\frac{(R/2)}{2\epsilon_0} \quad \to \quad q_{\text{enc}} = 4\pi R\epsilon_0 l E_{\text{in}}$$

$$q_{\text{enc}} = 4\pi(1.91\ \text{cm})\left[8.85\times10^{-12}\ \text{C}^2/(\text{N}\cdot\text{m}^2)\right](2.10\ \text{m})(585\ \text{kN/C}) = 2.61\ \mu\text{C}$$

(b) Evaluating the field outside the rod at the radius $(r = R)$ using the total enclosed charge, we obtain:

$$E_{\text{out}} = \frac{\rho R}{2\epsilon_0 r} = \frac{q_{\text{enc}}}{2\pi\epsilon_0 Rl} = \frac{4\pi R\epsilon_0 l E_{\text{in}}}{2\pi\epsilon_0 Rl} = 2E_{\text{in}} = 1.17\ \text{MN/C}$$

ASSESS We could have used the expression for the field found inside to obtain the magnitude of the electric field at the surface, since the expressions for both the inside and outside field are equal when $r = R$.

58. **INTERPRET** To suspend a particle above the "charged" floor, the electric force has to counter the gravitational force.

DEVELOP If the floor is painted with charge, then it will have the same field as the sheet of charge in Example 21.6: $E = \sigma/2\epsilon_0$. This is constant, so theoretically, the height at which the particle is placed shouldn't matter.

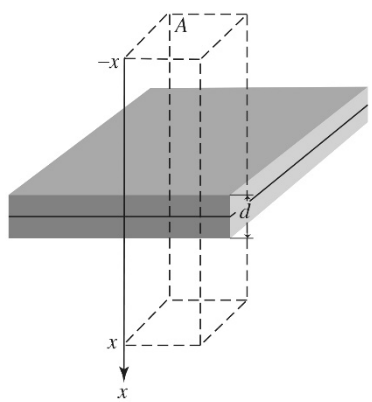
EVALUATE For the electric force, $F = qE$, to counter the gravitational force, $F = mg$, the surface charge density on the painted floor must be

$$\sigma = 2\epsilon_0\frac{mg}{q} = 2\left[8.85\times10^{-12}\ \text{C}^2/(\text{N}\cdot\text{m}^2)\right]\frac{(10\ \text{g})(9.8\ \text{m/s}^2)}{28\ \mu\text{C}} = 62\ \text{nC/m}^2$$

ASSESS Since the particle's charge is positive, the charge on the floor should be positive as well so that the electric force is repulsive. This will be in the direction opposite to that of the gravitational attraction.

59. **INTERPRET** The infinitely large slab has plane symmetry, and we can apply Gauss's law to compute the electric field.

DEVELOP When we take the slab to be infinitely large, the electric field is everywhere normal to the slab's surface and symmetrical about the center plane. We follow the approach outlined in Example 21.6 to compute the electric field. As the Gaussian surface, we choose a box that has area A on its top and bottom and that extends a distance x both up and down from the center of the slab. See figure below.



EVALUATE **(a)** For points inside the slab $|x| \le d/2$, the charge enclosed by our Gaussian box is

$$q_{\text{enclosed}} = \rho V_{\text{enclosed}} = \rho A(2x)$$

Thus, Gauss's law gives

$$\Phi = \int \vec{E}\cdot d\vec{A} = E(2A) = \frac{q_{\text{enclosed}}}{\epsilon_0} \quad \to \quad E = \frac{\rho x}{\epsilon_0}$$

The direction of $\vec{E}$ is away from (toward) the central plane for positive (negative) charge density.

(b) For points outside the slab $|x| > d/2$, the enclosed charge is

$$q_{\text{enclosed}} = \rho V_{\text{enclosed}} = \rho A d$$

Applying Gauss's law again gives

$$E = \frac{\rho d}{2\epsilon_0}$$

ASSESS Inside the slab, the charge distribution is equivalent to a sheet with $\sigma = 2\rho x$. On the other hand, outside the slab, it is equivalent to a sheet with $\sigma = \rho d$.

60. **INTERPRET** We are given some charged concentric spheres, and are asked to find the electric field at various distances from the center. This charge distribution is spherically symmetric and is made of two distinct charge distributions (the inner and outer spheres). We will use Gauss's law and the principle of superposition to find the electric field at the various points.

DEVELOP The central solid sphere has radius R = 10 cm and has a uniformly distributed charge Q = 40 mC. The outer shell has radius R_2 = 20 cm, and holds the same charge, again uniformly distributed. Gauss's law tells us that the field outside a spherical charge distribution is

$$E = \frac{Q}{4\pi\epsilon_0 r^2}$$

inside a spherical shell is E = 0, and inside a uniformly charged sphere the field is

$$E = \frac{Qr}{4\pi\epsilon_0 R^3}$$

EVALUATE **(a)** At r = 5.0 cm, we are inside of the inner sphere, so

$$E = \frac{Qr}{4\pi\epsilon_0 R^3} = \frac{(40\ \mu\text{C})(0.050\ \text{m})}{4\pi\left[8.85\times10^{-12}\ \text{C}^2/\left(\text{N}\cdot\text{m}^2\right)\right](0.10\ \text{m})^3} = 18\times10^6\ \text{N/C}.$$

(b) At r = 15 cm, we are outside the inner sphere but inside the outer sphere, so the outer sphere does not contribute to the field.

$$E = \frac{Q}{4\pi\epsilon_0 r^2} = \frac{40\ \mu\text{C}}{4\pi\left[8.85\times10^{-12}\ \text{C}^2/\left(\text{N}\cdot\text{m}^2\right)\right](0.15\ \text{m})^2} = 1.6\times10^7\ \text{N/C}.$$

(c) At r = 30 cm, we are outside both spheres. The total charge within our spherical Gaussian surface is now 2Q, so and the electric field is

$$E = \frac{2Q}{4\pi\epsilon_0 r^2} = \frac{2(40\ \mu\text{C})}{4\pi\left[8.85\times10^{-12}\ \text{C}^2/\left(\text{N}\cdot\text{m}^2\right)\right](0.30\ \text{m})^2} = 8.0\times10^6\ \text{N/C}.$$

ASSESS The answer in **(c)** is half that in **(b)**! The charge in **(c)** is twice as much, but we are also twice as far away.

61. **INTERPRET** At distances much, much less than 68 cm from the plate, and not near the edges of the plate, it can be approximated by an infinite plane, so we can apply Gauss's law for plane symmetry. For distances much, much greater than 68 cm, the plate looks like a point charge. We can use these limiting cases to find the electric field strength 19 m from the plate.

DEVELOP Close to the nonconducting plate $(x = 1\ \text{cm} << 68\ \text{cm} = a)$, the charge distribution has approximate plane symmetry, so the electric field is given by (see Equation 21.7)

$$E = \frac{\sigma}{2\varepsilon_0}$$

Therefore, the charge on the plate is

$$q = \sigma A = 2\varepsilon_0 EA = 2\varepsilon_0 Ea^2$$

where a is the length of a side of the square plate. At a point sufficiently far from the plate $(r >> a)$, the field strength will resemble that from a point charge, $E = kq/r^2$.

EVALUATE Inserting the values given, we find the charge on the plate is

$$q = 2\varepsilon_0 Ea^2 = 2\left[8.85\times10^{-12}\ \text{C}^2/\left(\text{N.m}^2\right)\right](47\ \text{kN/C})(0.68)^2 = 832\ \text{nC}$$

The field strength at $r = 19\ \text{m} >> 0.68\ \text{cm}$ is like that from a point charge:

$$E=\frac{kq}{r^2}=\frac{\left(9.0\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(385\ \text{nC})}{(19\ \text{m})^2}=9.6\ \text{N/C}$$

ASSESS Far from the finite charge distribution (plane charge, in this case), the field always resembles that of a point charge.

62. **INTERPRET** This problem involves a uniform charge distribution with spherical symmetry. Close to the surface of the sphere (a distance much, much less than the radius of the sphere), the electric field will resemble that from a uniformly charged conducting plate.

DEVELOP There is a nonzero field outside the shell, because the net charge within is not zero. Therefore, from Equation 21.8, there is a surface charge density $\sigma=\varepsilon_0 E$ on the outer surface of the shell, which is uniform if we ignore the possible presence of other charges and conducting surfaces outside the shell. Gauss's law (with reasoning similar to Example 21.7) requires that the charge on the shell's outer surface equal the point charge within, so

$$\sigma A=\sigma\left(4\pi R^2\right)=q$$

EVALUATE **(a)** The surface charge density is

$$\sigma=\frac{q}{4\pi R^2}=\frac{250\ \text{nC}}{4\pi(0.20\ \text{m})^2}=500\ \text{nC/m}^2$$

(b) The field strength at the outer surface of the sphere is

$$E=\frac{\sigma}{\epsilon_0}=\frac{497\ \text{nC/m}^2}{8.85\times10^{-12}\ \text{C}^2/\left(\text{N}\cdot\text{m}^2\right)}=56\ \text{kN/C}$$

ASSESS Notice that the units for part **(b)** cancel correctly to give N/C.

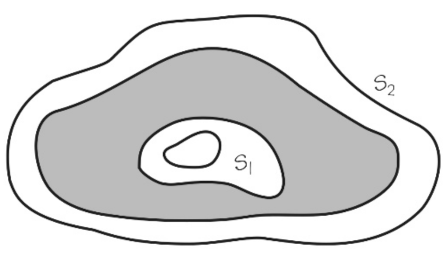
63. **INTERPRET** This problem involves an irregular-shaped conductor in electrostatic equilibrium. The conductor is hollow and we are to show there is zero field in the cavity. This geometry is nonsymmetric, so we cannot apply the formulas developed for circular, plane, or line charge distributions.

DEVELOP Review Section 21.6 to see why the electric field within a conducting medium, in electrostatic equilibrium, is always zero. In addition, the net charge must reside on the conductor surface.

EVALUATE **(a)** When there is no charge inside the cavity, the flux through any closed surface within the cavity (see S_1 in figure below) is zero, so the electric field is also zero. If it were not, there would be a nonzero electric field acting on the surface charges, which would therefore move and the conductor would not be in equilibrium.

(b) If the surface charge density on the outer surface (and also the electric field there) is to vanish, then the net charge inside a Gaussian surface S_2 containing the conductor must be zero. Thus, the point charge in the cavity must be $-Q$, so that

$$q_{\text{enclosed}}=Q+(-Q)=0.$$



ASSESS An alternative approach is to say that the total charge on the conductor is $q_c=q_{\text{inner}}+q_{\text{outer}}=Q.$ Requiring $q_{\text{outer}}=0$ means that $q_{\text{inner}}=Q.$ Since electric field inside the cavity vanishes, the point charge placed inside must be $q=-q_{\text{inner}}=-Q.$

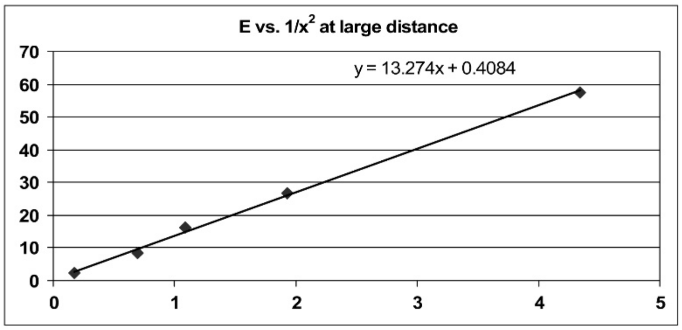
64. **INTERPRET** In this problem we are given the electric field strength as a function of distance from a square plate with uniform charge density. We are asked to use the data provided to determine the plate dimension and the total charge on the plate.

DEVELOP Close to the nonconducting plate, the charge distribution has approximate plane symmetry, so the electric field is given by (see Equation 21.7):

$$E = \frac{\sigma}{2\epsilon_0}$$

Therefore, the charge on the plate is $q = \sigma A = 2\epsilon_0 EA = 2\epsilon_0 Ea^2$, where a is the length of a side of the square plate. At a point sufficiently far from the plate $(r \gg a)$ the field strength will resemble that from a point charge, $E = kq/r^2$.

EVALUATE **(a)** Using the five data points for large values of x, we plot below the electric field strength as a function of $1/x^2$.



From $E = kq(1/r^2)$, we see that the value of the slope of the straight line, $13.274\ \text{N}\cdot\text{m}^2/\text{C}$, corresponds to kq. Thus, the amount of charge carried by the plate is

$$q = \frac{13.274}{k} = \frac{13.274\ \text{N}\cdot\text{m}^2/\text{C}}{9\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2} = 1.47\ \text{nC}$$

(b) At very small value of x, the field strength approaches $E = 5870\ \text{N/C}$. With

$$E = \frac{\sigma}{2\epsilon_0} = \frac{q}{2\epsilon_0 A} = \frac{q}{2\epsilon_0 s^2}$$

we find the side length of the plate to be

$$s = \sqrt{\frac{q}{2\epsilon_0 E}} = \sqrt{\frac{1.47\times10^{-9}\ \text{C}}{2(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(5870\ \text{N/C})}} = 0.119\ \text{m} = 11.9\ \text{cm}$$

ASSESS The charge density of the plate is

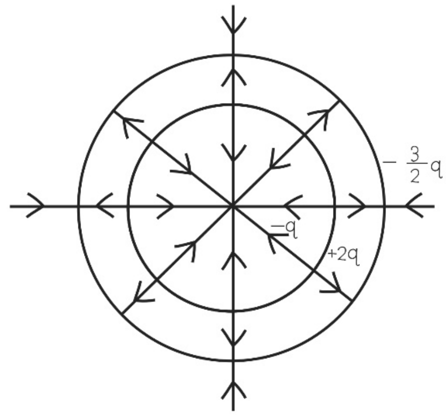
$$\sigma = \frac{q}{A} = \frac{q}{s^2} = \frac{1.47\times10^{-9}\ \text{C}}{(0.119\ \text{m})^2} = 1.04\times10^{-7}\ \text{C/m}^2$$

which can also be calculated using $\sigma = 2\epsilon_0 E$.

65. **INTERPRET** We are given a spherically symmetric charge distribution, so Gauss's law can be applied in this problem.

DEVELOP The field from the given charges is spherically symmetric, so (from Gauss's law) is like that of a point charge located at the center with magnitude equal to the net charge enclosed by a sphere of radius equal to the distance to the field point.

EVALUATE Thus, the electric field is $E = -kq/r^2$ inside the first shell (8 lines radially inward, see figure below), $E = +kq/r^2$ between the first and second shells (8 lines radially outward), and $E = -kq/(2r^2)$ outside the second shell (4 lines radially inward).



ASSESS The direction of the electric field depends on the net charge enclosed. When $q_{\text{enclosed}} > 0$, $\vec{E}$ is radially outward. On the other hand, $\vec{E}$ is radially inward when $q_{\text{enclosed}} < 0$. The discontinuity in electric field across the shell is due to a net surface charge density on the shell.

66. **INTERPRET** The charge distribution is spherically symmetric and may be treated as two distinct charge distributions, so we can apply Gauss's law and the principle of superposition to find the electric field at the various points.

DEVELOP Separate the charge distribution into the point charge at the center of the sphere with $q_{\text{pc}} = q$ and the charge distributed over the surface of the spherical shell with charge $q_{\text{shell}} = 2q$. Using the superposition principle, the total electric field E_{tot} is the sum of the electric field from each charge distribution:

$$E_{\text{tot}} = E_{\text{pc}} + E_{\text{shell}}$$

Inside the sphere, the charge enclosed by a concentric Gaussian surface is just the point charge, so the electric field will simply be that due to a point charge (see Equation 20.3), $E_{\text{pc}} = kq_{\text{pc}}/r^2$. Outside the sphere, a concentric Gaussian surface will enclose the point charge and the spherical charge distribution. As shown in Example 21.1, the field outside any spherically symmetric charge distribution is that due to a point charge at the center that carries the entire charge (see Equation 21.4): $E_{\text{shell}} = kq_{\text{shell}}/r^2$.

EVALUATE **(a)** For $r = R/2$, we are inside the sphere, so the electric field is given by

$$E_{\text{tot}} = E_{\text{pc}} + \overbrace{E_{\text{shell}}}^{=0} = E_{\text{pc}} = \frac{kq_{\text{pc}}}{\left(\frac{1}{2}R\right)^2} = \frac{4kq}{R^2}$$

(b) For $r = 2R$, we are outside the sphere, so the electric field is given by

$$E_{\text{tot}} = E_{\text{pc}} + E_{\text{shell}} = \frac{kq_{\text{pc}}}{(2R)^2} + \frac{kq_{\text{shell}}}{(2R)^2} = \frac{k(q+2q)}{4R^2} = \frac{3kq}{4R^2}$$

ASSESS Instead of using the superposition principle in this way, you can consider the entire charge distribution. For part **(a)**, a concentric Gaussian surface encloses only the point charge, so

$$E_{\text{tot}} = \frac{kq_{\text{enclosed}}}{\left(\frac{1}{2}R\right)^2} = \frac{kq_{\text{pc}}}{\left(\frac{1}{2}R\right)^2} = \frac{4kq}{R^2}$$

For part **(b)**, a concentric Gaussian surface encloses the central point charge and the spherical charge (which may be considered as a point charge at the center), so the electric field is

$$E_{\text{tot}} = \frac{kq_{\text{enclosed}}}{(2R)^2} = \frac{k\left(q_{\text{pc}} + q_{\text{shell}}\right)}{(2R)^2} = \frac{3kq}{4R^2}$$

which agrees with the results above.

67. **INTERPRET** The charge distribution has spherical symmetry, so we can apply Gauss's law. Also, since the charge distribution is nonuniform, integration is needed to find the electric field.

DEVELOP The charge inside a sphere of radius $r \le a$ is $q(r) = \int_0^r \rho\ dV$. For volume elements, take concentric shells of radius r and thickness dr, so $dV = 4\pi r^2 dr$.

EVALUATE **(a)** Integrating over r, the charge enclosed by a Gaussian sphere of radius r is

$$q(r) = 4\pi\int_0^r \rho r^2 dr = \frac{4\pi\rho_0}{a}\int_0^r r^3 dr = \frac{\pi\rho_0 r^4}{a}$$

For $r = a$, the total charge is $Q = \pi\rho_0 a^3$.

(b) Inside the sphere, Gauss's law and Equation 21.5 give

$$\oint_{\text{surface}} \vec{E}\cdot d\vec{A} = \frac{q_{\text{enclosed}}}{\epsilon_0} 4\pi r^2$$

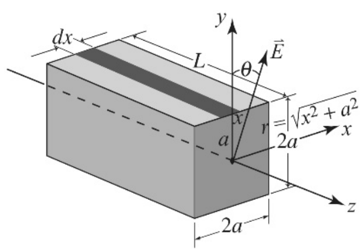
$$E(r) = \frac{q(r)}{\epsilon_0} = \frac{\pi\rho_0 r^4}{\epsilon_0 a} \quad \Rightarrow \quad E(r) = \frac{1}{4\pi\epsilon_0}\frac{\pi}{a} = \frac{\rho_0 r^2}{4\epsilon_0 a}$$

ASSESS The r^2 dependence of E inside the sphere can be contrasted with the r dependence in the case (see Example 21.1) where the charge distribution is uniform.

68. **INTERPRET** We are given a charge density with line symmetry, which we will not exploit. Instead, we are to integrate over a planar surface of a rectangular cube transpierced on its axis by the wire.

DEVELOP Draw a coordinate system on the box, as shown in the figure below. From Equation 21.6, we know that the electric field on the top surface of the box has magnitude $\lambda/2\pi\varepsilon_0 r$ and is direction radially away from the line of charge. The flux through a strip of width dx at position x is $d\Phi = EL\cos\theta\ dx = \left[\lambda/(2\pi\varepsilon_0 r)\right]L(a/r)\ dx$, where

$r = \sqrt{x^2 + a^2}$,



EVALUATE Integrating from $-a$ (the left edge of the box) to a (the right edge of the box) gives

$$\Phi_{\text{top}} = \int_{-a}^{a}\left(\frac{\lambda a L}{2\pi\epsilon_0}\right)\frac{dx}{(a^2+x^2)} = \left(\frac{\lambda a L}{2\pi\epsilon_0}\right)\left|\frac{1}{a}\text{atan}\left(\frac{x}{a}\right)\right|_{-a}^{a} = \left(\frac{\lambda a L}{2\pi\epsilon_0}\right)\left(\frac{\pi}{2a}\right) = \frac{\lambda L}{4\epsilon_0}$$

By symmetry, the flux through the entire box is four times this, or $\lambda L/\epsilon_0$, which agrees with Gauss's law because $\lambda L = q_{\text{enclosed}}$.

ASSESS Notice that this result is independent of the length L of the box, provided the box does not approach the ends of the wire.

69. **INTERPRET** We are given a charge distribution with spherical symmetry, which we can exploit with Gauss's law. Also, since the charge distribution is nonuniform, integration is needed to find the condition which gives zero electric field outside the sphere.

DEVELOP Gauss's law tells us that the field outside the sphere will be zero if the total charge within the Gaussian surface is zero. Thus, using concentric shells of thickness dr and volume $dV = 4\pi r^2 dr$ as our charge elements, we require that

$$q_{\text{enclosed}} = \int_{\text{sphere}} \rho(r)dV = 0$$

EVALUATE Inserting the expression for $\rho(r)$ into the integrand and integrating gives

$$0=\int_0^R\left(\rho_0-ar^2\right)4\pi r^2dr=4\pi\left(\frac{1}{3}\rho_0R^3-\frac{1}{5}aR^5\right)$$
$$a=\frac{5\rho_0}{3R^2}$$

ASSESS The charge density starts out from the center of the sphere as ρ_0 and decreases as r^2. At $r=R,$ the density is

$$\rho(R)=\rho_0\left(1-\frac{5}{3}\right)=-2\rho_0/3$$

Note that $\rho(r)$ must change sign (from positive to negative) in order for q_{enclosed} to be zero at $r=R$.

70. **INTERPRET** We are to calculate by integration the electric field inside and outside a uniformly charged spherical shell of radius R and total charge Q.

DEVELOP We will divide the spherical surface up into rings around an axis that includes the field point. Each ring has radius $a=R\sin\theta$ and width $Rd\theta,$ and holds a charge $dq=\left[Q/\left(4\pi R^2\right)\right](2\pi a)Rd\theta.$ The electric field due to each ring is

$$dE=\frac{kdqx}{\left(x^2+a^2\right)^{3/2}},$$

where x is the distance from the field point to the ring, $x=r-R\cos\theta.$ We will integrate from $\theta=0$ to π.

EVALUATE

$$E=\int_0^\pi dE=\int_0^\pi\frac{k\left(r-R\cos\theta\right)}{\left[\left(r-R\cos\theta\right)^2+\left(R\sin\theta\right)^2\right]^{3/2}}\left(\frac{Q}{2}\right)\sin\theta d\theta$$

Use of an integral table or computational package shows that for $r<R, E=0.$ For $r<R$, the integral becomes $E=kQ/r^2$.

ASSESS In both cases, this somewhat complicated integral gives us the same results as Gauss's law.

71. **INTERPRET** Given a spherically symmetric charge distribution in a sphere, we are to find the total charge within the sphere. We will use Gauss's law, and integrate the charge density to find the charge enclosed, and solve for the value of n that satisfies the given conditions.

DEVELOP The charge density is given by $\rho=\rho_0\left(r/R\right)^n$, where r is the distance from the center, R is the radius of the sphere, ρ_0 is a constant, and n is an integer greater than –3. Gauss's law tells us that

$$\oint_{\text{surface}}\vec{E}\cdot d\vec{A}=\frac{q_{\text{enclosed}}}{\epsilon_0}$$

Because the charge density is spherically symmetric, we know that the electric field will not be angularly dependent. In other words, it will be constant over a spherical surface centered at the center of the charged sphere. Furthermore, the same reasoning tells us that the direction of the electric field must parallel to $d\vec{A}$ (i.e., radial) so the dot product gives a factor of unity. We first find the charge enclosed by integrating the charge distribution in the sphere.

EVALUATE **(a)** Performing the integration for the charge enclosed as a function of the distance r gives

$$q_{\text{enc}}(r)=\int\rho(r)dV=\int_0^{2\pi}\int_0^{\pi}\int_0^{r}\rho_0\left(r'/R\right)^n r'^2\sin\theta dr'd\theta d\phi$$
$$q_{\text{enc}}(r)=4\pi\rho_0\int_0^r r'^2\left(r'/R\right)^n dr'=\frac{4\pi\rho_0}{R^n}\left[\frac{r'^{n+3}}{n+3}\right]_0^r=\frac{4\pi r^3\rho_0}{(n+3)}\left(\frac{r}{R}\right)^n$$

The total charge enclosed within the sphere is found when $r=R$, and it's given by

$$q_{\text{tot}}=\frac{4\pi R^3\rho_0}{(n+3)}$$

(b) To find the value of n we integrate over the surface area dA and express the electric field as a function of the distance r, in the region where $r<R$

$$E_{\text{in}}(r)\left[4\pi r^2\right]=\frac{q_{\text{enc}}(r)}{\epsilon_0}$$

$$E_{\text{in}}(r)=\frac{1}{4\pi\epsilon_0 r^2}\frac{4\pi r^3\rho_0}{(n+3)}\left(\frac{r}{R}\right)^n$$

$$E_{\text{in}}(r)=\frac{r\rho_0}{\epsilon_0(n+3)}\left(\frac{r}{R}\right)^n$$

We can evaluate the electric field at distance larger than the radius R by considering the total charge enclosed

$$E_{\text{out}}(r)\left[4\pi r^2\right]=\frac{q_{\text{tot}}}{\epsilon_0}$$

$$E_{\text{out}}(r)=\frac{1}{4\pi\epsilon_0 r^2}\frac{4\pi R^3\rho_0}{(n+3)}$$

$$E_{\text{out}}(r)=\frac{R^3\rho_0}{\epsilon_0(n+3)r^2}$$

We consider the case when $E_{\text{in}}(r=R/2)=E_{\text{out}}(r=4R)$, which gives that

$$\frac{(R/2)\rho_0}{\epsilon_0(n+3)}\left(\frac{(R/2)}{R}\right)^n=\frac{R^3\rho_0}{\epsilon_0(n+3)(4R)^2}$$

$$\frac{R}{2}\left(\frac{1}{2}\right)^n=\frac{R}{16}$$

$$2^n=8 \rightarrow n=3$$

ASSESS Both expressions for the electric field will yield the same value at the surface (setting $r=R$ in both equations), which shows the continuity of the electric field.

72. **INTERPRET** We are asked to derive what the gravitational field would be in a hypothetical hole drilled to the center of the Earth.

DEVELOP To apply the gravitational version of Gauss's law, we need to choose a suitable Gaussian surface. The gravitational field will point radially inward, so clearly a sphere centered on the Earth is the right choice (see Example 21.1 for a uniformly charged sphere). We will assume that the Earth has a uniform mass density, $\rho=M_{\text{E}}/\left(\frac{4\pi}{3}R_{\text{E}}^3\right)$.

EVALUATE The mass enclosed by a Gaussian sphere of radius r (for $r<R_{\text{E}}$) is

$$M_{\text{enclosed}}=\rho V_{\text{enclosed}}=\rho\left(\frac{4\pi}{3}r^3\right)$$

Thus, the gravitational version of Gauss's law gives

$$\Phi=\int\vec{g}\cdot d\vec{A}=g\left(4\pi r^2\right)=-4\pi GM_{\text{enclosed}} \quad\rightarrow\quad g=-\frac{4\pi}{3}G\rho r$$

We can simplify this by noting that for $r=R_{\text{E}}$, the formula gives the gravitational field at the Earth's surface: $g_0=-\frac{4\pi}{3}G\rho R_{\text{E}}$. Therefore,

$$g=g_0\frac{r}{R_{\text{E}}}$$

ASSESS The field is negative, corresponding to the attractive nature of gravity. Note that for $r>R_{\text{E}}$, the mass enclosed is the total Earth mass, M_{E}. In this case, Gauss's law gives: $g=-GM_{\text{E}}/r^2$, which is what we get from Newton's law of gravity (Equation 8.10).

73. **INTERPRET** We are given a long solid cylinder of radius R with radially dependent nonuniform charge density. We are asked to calculate the electric field strength inside the cylinder as a function of r, the distance from the axis.

DEVELOP The charge distribution still possesses line symmetry in this case, and we can apply Gauss's law to calculate the electric field. From Equation 21.6, we have $E=\lambda_{\text{enclosed}}/2\pi\epsilon_0 r$, where

$$q_{\text{enclosed}}=\lambda_{\text{enclosed}}l=\int_0^r\rho\, dV$$

is the charge within a length l of coaxial cylindrical surface of radius r, and $dV = 2\pi rl dr$ is the volume element of a thin shell with this surface.

EVALUATE For $r < R$, the charge enclosed is

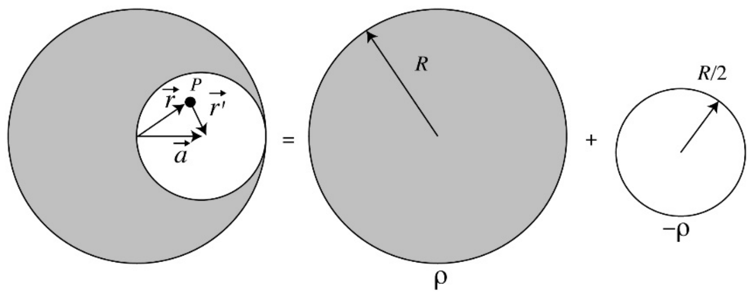
$$q_{\text{enclosed}} = \int_0^r (2\pi \rho_0 l/R) r^2 dr = \frac{2\pi \rho_0 l r^3}{3R}$$

From $E(2\pi rl) = q_{\text{enclosed}}/\epsilon_0$, we find the electric field to be $E = \dfrac{\rho_0 r^2}{3\epsilon_0 R}$.

ASSESS Similarly, one may show that the field strength outside the cylinder is given by $E = \dfrac{\rho_0 R^2}{3\epsilon_0 r}$.

74. **INTERPRET** We have a uniformly charged solid sphere of radius R with a hole of radius $R/2$ inside, and we are asked to prove that the electric field inside the hole has a magnitude of $E = \rho R/6\epsilon_0$.

DEVELOP The solid sphere can be considered to be the superposition of the sphere with a cavity plus a small solid sphere filling the cavity, one with uniform charge density ρ, and the other $-\rho$, as shown in the figure below.



Using Gauss's law, the electric field at a point P due to the large sphere is given by

$$E(4\pi r^2) = \frac{q_{\text{enclosed}}}{\epsilon_0} = \frac{1}{\epsilon_0}\frac{4\pi r^3 \rho}{3} \quad \Rightarrow \quad E = \frac{\rho r}{3\epsilon_0}$$

Similarly, the field strength due to the small sphere is

$$E'(4\pi r'^2) = \frac{q'_{\text{enclosed}}}{\epsilon_0} = \frac{1}{\epsilon_0}\frac{4\pi r'^3(-\rho)}{3} \quad \Rightarrow \quad E' = -\frac{\rho r'}{3\epsilon_0}$$

The negative sign indicates that the field points radially inward. The net electric field is the vectorial sum of the two contributions.

EVALUATE In vector notations, we have $\vec{E} = \dfrac{\rho \vec{r}}{3\epsilon_0}$ and $\vec{E}' = \dfrac{\rho \vec{r}'}{3\epsilon_0}$. The resultant field at P is

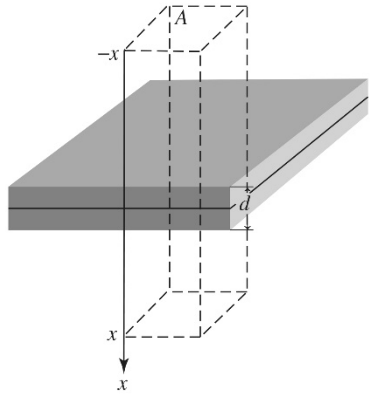
$$\vec{E}_P = \vec{E} + \vec{E}' = \frac{\rho \vec{r}}{3\epsilon_0} + \frac{\rho \vec{r}'}{3\epsilon_0} = \frac{\rho}{3\epsilon_0}\left(\vec{r} + \vec{r}'\right) = \frac{\rho}{3\epsilon_0}\vec{a}$$

where $\vec{a} = (R/2)\hat{i}$, as shown in the figure above. Thus, we find the field at P to be $\vec{E}_P = \dfrac{\rho R}{6\epsilon_0}\hat{i}$.

ASSESS The field inside the hole is uniform with a magnitude of $\rho R/6\epsilon_0$ and points in the x-direction. Note that the general expression $\rho \vec{a}/3\epsilon_0$ holds for any size of spherical hole.

75. **INTERPRET** The infinitely large slab has plane symmetry, and we can apply Gauss's law to compute the electric field.

DEVELOP When we take the slab to be infinitely large, the electric field is everywhere normal to the slab's surface and symmetrical about the center plane. We follow the approach outlined in Example 21.6 to compute the electric field. As the Gaussian surface, we choose a box that has area A on its top and bottom and that extends a distance x both up and down from the center of the slab. See figure below.



EVALUATE **(a)** For points inside the slab $|x| \le d/2$, the charge enclosed by our Gaussian box is

$$q_{\text{enclosed}} = \int_{-x}^{x} \rho dV = \int_0^x \rho_0 \frac{x}{d} A dx + \int_{-x}^{0} \rho_0 \frac{(-x)}{d} A dx = \frac{\rho_0 A}{d}\left(\frac{x^2}{2} + \frac{x^2}{2}\right) = \frac{\rho_0 A}{d} x^2$$

Thus, Gauss's law gives

$$\Phi = \int \vec{E}_{\text{in}} \cdot d\vec{A} = E(2A) = \frac{q_{\text{enclosed}}}{\epsilon_0} \quad \rightarrow \quad E_{\text{in}} = \frac{\rho x^2}{2\epsilon_0 d}$$

The direction of $\vec{E}_{\text{in}}$ is away from (toward) the central plane for positive (negative) charge density.
(b) For points outside the slab $|x| > d/2$, the enclosed charge is

$$q_{\text{enclosed}} = \int_{-d/2}^{d/2} \rho dV = \int_0^{d/2} \rho_0 \frac{x}{d} A dx + \int_{-d/2}^{0} \rho_0 \frac{(-x)}{d} A dx = \frac{\rho_0 A}{d}\left(\frac{d^2}{8} + \frac{d^2}{8}\right) = \frac{\rho_0 A d}{4}$$

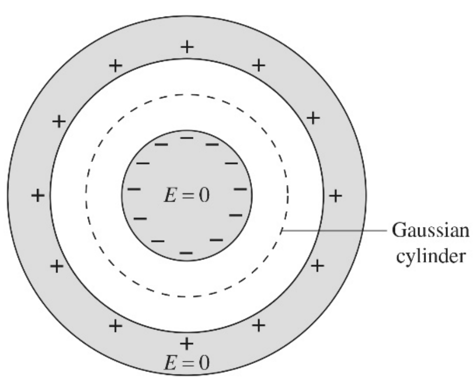
Applying Gauss's law again gives

$$\Phi = \int \vec{E}_{\text{out}} \cdot d\vec{A} = E(2A) = \frac{q_{\text{enclosed}}}{\epsilon_0} \quad \rightarrow \quad E_{\text{out}} = \frac{\rho d}{8\epsilon_0}$$

ASSESS Outside the slab, the charge density is equivalent to a sheet with $\sigma = \rho d / 4$.

76. **INTERPRET** We consider the electric fields associated with a coaxial cable.

DEVELOP The electric field has to be zero within the wire and the shield, since they are both conductors. Charges will line up on the surface of each to ensure that the net field cancels inside. To illustrate this, we draw the cross-section of the coaxial cable in the figure below, assuming for argument's sake that the inner conductor has negative charge and the outer has positive charge.



In the figure, we also include the cross section of a Gaussian cylinder. If there is a field somewhere, it will be perpendicular to the curved surface of such a cylinder.

EVALUATE In the region between the wire and shield, the enclosed charge is that of the wire. Since this is non-zero, the field in this region will be nonzero. As for the region outside the shield, the enclosed charge is both that

of the wire and the shield, which are equal and opposite. The net charge will be zero, so the field will be zero in this region. To sum up, the only place where the field is not zero is between the wire and shield.
The answer is **(a)**.

ASSESS Unlike the conductors, the insulation can have a nonzero electric field. Its charges are not free to move around until the internal field is zero. However, the insulation may be dielectric, in which case, the alignment of electric dipoles in the material will partially reduce the field (see Fig. 20.24).

77. **INTERPRET** We consider the electric fields associated with a coaxial cable.

DEVELOP As we argued in the previous problem, the electric field will be zero inside the shield. We have to think what sort of charge distribution will ensure this.

EVALUATE Let's assume as before that the wire has a negative charge, –*Q*, and the shield has positive charge, *Q*. Let the shield have inner radius r_i and outer radius r_o. If we choose a Gaussian cylinder with radius *r* such that $r_\text{i} < r < r_\text{o}$, then the electric flux will be zero on the surface (since its curved surface is inside the conductor). This implies that the enclosed charge is zero, which can only occur if all of the shield's excess charge (*Q*) lies on the shield's inner surface.
The answer is **(c)**.

ASSESS We might have guessed at this answer by imagining all of the positive charge in the shield being pulled as close as possible to the negative charge on the wire.

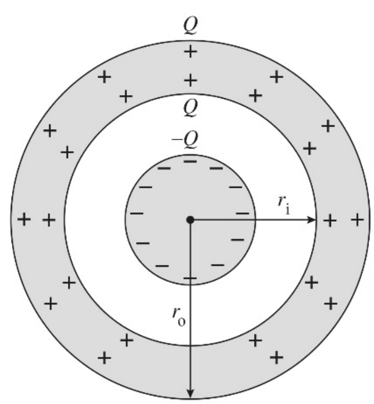
78. **INTERPRET** We consider the electric fields associated with a coaxial cable.

DEVELOP To find the field between the wire and the shield, we can use a Gaussian cylinder with length *L* and radius *r* that lies between the two conductors.

EVALUATE The cylinder will enclose the charge on the wire: $-Q = \lambda L$, where λ is the charge per unit length. The electric flux is limited to just the outer curved surface: $\Phi = 2\pi r L \cdot E$ (see Example 21.4). So the electric field in between the wire and shield will be proportional to 1/*r*.
The answer is **(b)**.

ASSESS Because of the symmetry, the wire's electric field is the same as that of an infinite line of charge. The shield only contributes to the field outside its inner radius.

79. **INTERPRET** We consider the electric fields associated with a coaxial cable.
DEVELOP Doubling the charge on the shield will create another layer of charge *Q* on the outer surface. See the figure below. This will maintain zero field inside the conductor (between r_i and r_o), but as a result the field outside the shield will no longer be zero. Note that doubling zero is still zero, so answer **(b)** is eliminated.



EVALUATE The additional charge has no effect on the field between the wire and shield, since the same amount of charge (–*Q*) is still enclosed. The field at the inner surface of the shield will be $E = (-Q/L)/2\pi\epsilon_0 r_\text{i}$, where we assume *L* extends for the length of the cable. At the outer surface, the enclosed charge is $Q_\text{enclosed} = 2Q - Q = Q$, so the field will be $E = (Q/L)/2\pi\epsilon_0 r_\text{o}$. If we assume that the shield is extremely thin, then $r_\text{i} \approx r_\text{o}$, and the fields at the inner and outer surface will have equal magnitude.
The answer is **(d)**.

ASSESS The fields have equal magnitude, but they will point in opposite directions. The field on the inner surface points inward (hence the negative sign in the derivation above), whereas the field points outward on the outer surface.