# 5

USING NEWTON'S LAWS

EXERCISES

Section 5.1 Using Newton's Second Law

11. **INTERPRET** This problem involves applying Newton's second law in a two-dimensional situation. The object of interest is the 3.25-kg mass, and we are asked to find the second of two forces acting on the mass that would give it the given acceleration.

    DEVELOP Draw a free-body diagram of the situation (see figure below). Include the acceleration of the mass, and write the acceleration in component form. Now apply Newton's second law (for constant mass) in vector form to find the second force.
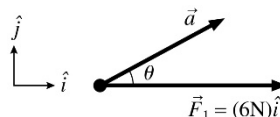
    EVALUATE The acceleration in component form is $\vec{a} = a\cos(\theta)\hat{i} + a\sin(\theta)\hat{j}$, with $a = 5.48$ m/s^2 and $\theta = 38.0°$. Thus, Newton's law gives

    $$\vec{F}_{net} = m\vec{a}$$
    $$\vec{F}_2 + \vec{F}_1 = ma\left[\cos(\theta)\hat{i} + \sin(\theta)\hat{j}\right]$$
    $$\vec{F}_2 = (3.25 \text{ kg})(5.48 \text{ m/s}^2)\left[\cos(38.0°)\hat{i} + \sin(38.0°)\hat{j}\right] - (8.63 \text{ N})\hat{i} = (5.40 \text{ N})\hat{i} + (11.0 \text{ N})\hat{j}$$

    ASSESS This force has magnitude $F_2 = \sqrt{(5.40 \text{ N})^2 + (11.0 \text{ N})^2} = 12.2$ N and points in the direction $\theta = \tan^{-1}(11.0 \text{ N}/5.40 \text{ N}) = 63.8°$ counterclockwise from the $\hat{i}$ direction.

12. **INTERPRET** This problem requires an application of Newton's second law in two dimensions. Two forces are exerted on the object of interest (i.e., the 3.1-kg mass) and produce an acceleration. With the mass of the object and one force given, we are asked to find the other force.

    DEVELOP Newton's second law for this mass says $F_{net} = \vec{F}_1 + \vec{F}_2 = m\vec{a}$, where we assume no other significant forces are acting. Thus, the second force is given by $\vec{F}_2 = m\vec{a} - \vec{F}_1$.

    EVALUATE Inserting the expressions given in the problem statement for $\vec{F}_1$ and $\vec{a}$, we obtain

    $$\vec{F}_2 = m\vec{a} - \vec{F}_1 = (3.1 \text{ kg})\left[(0.95 \text{ m/s}^2)\hat{i} - (0.32 \text{ m/s}^2)\hat{j}\right] - \left[(-0.8 \text{ N})\hat{i} - (2.1 \text{ N})\hat{j}\right] = (3.745 \text{ N})\hat{i} + (1.108 \text{ N})\hat{j}$$

    ASSESS This force has magnitude $F_2 = \sqrt{(3.745 \text{ N})^2 + (1.108 \text{ N})^2} = 3.91$ N and points in the direction $\theta = \text{atan}(1.108 \text{ N}/3.745 \text{ N}) = 16.5°$ counterclockwise from the x-axis.

13. **INTERPRET** In this problem, we are asked to find the tilt angle of an air table such that the acceleration of an object sliding on the surface of the table is the same as the gravitational acceleration near the surface of the Mars.

    DEVELOP Example 5.1 shows that the acceleration down an incline is $a = g\sin(\theta)$. By setting the acceleration equal to the acceleration due to gravity on the surface of the Mars ($a = g_M = 3.71$ m/s^2), we can solve for the tilt angle θ.

EVALUATE   The angle of tilt should be

$$\theta = \sin^{-1}\left(\frac{a}{g}\right) = \sin^{-1}\left(\frac{3.71 \text{ m/s}^2}{9.8 \text{ m/s}^2}\right) = 22.2°$$

above the horizontal.

ASSESS   Notice that the tilt angle does not depend on the mass of the object.

14.   **INTERPRET**   This problem involves Newton's second law applied to a two-dimensional situation to find the acceleration of the skier, then kinematics to find the time it takes him to reach the bottom of the slope.

DEVELOP   Draw a free-body diagram of the situation (see figure below) and apply Newton's second law to find the acceleration. The angle is $\theta = 24°$, and the weight of the skier is $w = mg$. Given the acceleration (which is constant), we can use Equation 2.10, $x = x_0 + v_0 t + at^2/2$, with $x - x_0 = 1.5 \text{ km} = 1500 \text{ m}$, to find how long it takes him to reach the bottom.

$$\vec{n} = n\hat{j}$$
$$mg\sin(\theta)\hat{i}$$
$$\theta \quad -mg\cos(\theta)\hat{j}$$
$$\vec{w} = m\vec{g}$$
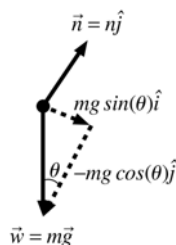
EVALUATE   Applying Newton's second law to the skier gives two equations (one for the x-direction and one for the y-direction):

$$\vec{F}_{net} = m\vec{a}$$

$$n\hat{j} + mg[\sin(\theta)\hat{i} - \cos(\theta)\hat{j}] = m\vec{a} \implies \begin{cases} n - mg\cos(\theta) = 0 \\ g\sin(\theta) = a \end{cases}$$

Solving the second scalar equation for acceleration, we find $a = (9.8 \text{ m/s}^2)\sin(24°) = 3.986 \text{ m/s}^2$, and inserting this result into Equation 2.10 gives

$$x = x_0 + \overset{=0}{v_0 t} + at^2/2$$
$$t = \pm\sqrt{\frac{2(x - x_0)}{a}} = \sqrt{\frac{2(1500 \text{ m})}{3.986 \text{ m/s}^2}} = 27.4 \text{ s}$$

where we have chosen the positive square root, and we have used $v_0 = 0$ because the skier starts from rest.

ASSESS   If the slope becomes vertical, $\theta \to 90°$ and $a = g$, which is what we expect because the skier would be in free fall.

15.   **INTERPRET**   This is a static problem in which we are looking for the force exerted on the tendon by the two muscles.

DEVELOP   In this case, there are two forces pulling on the tendon as shown below in the figure:
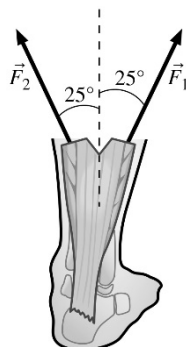
We are told that the horizontal pulls are opposite each other (meaning that the two forces are in the x–y plane), and we assume that the net horizontal force is zero: $F_1 \sin 25^\circ - F_2 \sin 25^\circ = 0$, in which case $F_1 = F_2$. The net vertical force pulls up on the tendon with a force equivalent to ten times the gymnast's weight:

$$F_1 \cos 25^\circ + F_2 \cos 25^\circ = 10 mg$$

EVALUATE   Solving for the force in each muscle gives

$$F_1 = F_2 = \frac{10(55 \text{ kg})(9.8 \text{ m/s}^2)}{2 \cos 25^\circ} = 2.97 \times 10^3 \text{ N} \approx 3.0 \text{ kN}$$

ASSESS   The Achilles tendon is the thickest and strongest tendon in the body. In simply walking, it has to withstand strains of as much as four times the body weight.

16.   **INTERPRET**   This problem involves Newton's second law applied to a two-dimensional situation to find the minimum slope angle for which the skier can safely traverse the snow bridge that can support a maximum normal force of 580 N.

   DEVELOP   Let the angle of the slope be θ, and weight of the skier be $w = mg$. The normal force the ground exerts on the skier is $n = mg \cos \theta$. In the problem, we are told that the weight of the skier is 700 N and $n_{max} = 580$ N.

   EVALUATE   Setting $n_{max} = mg \cos \theta$, and solving for θ, we obtain

$$\theta = \cos^{-1}\left(\frac{n_{max}}{mg}\right) = \cos^{-1}\left(\frac{580 \text{ N}}{700 \text{ N}}\right) = 34.0^\circ$$

   ASSESS   The normal force will exceed 580 N if the slope is smaller than 34.0°, and the snow bridge will be at risk of collapsing.

Section 5.2 Multiple Objects

17.   **INTERPRET**   This problem is similar to Example 5.4. We assume that your baby sister and the cake are tied together through the tablecloth, so their accelerations should be equal.

   DEVELOP   As the baby sister is pulling with all her weight, we'll assume that her feet are off the ground and that she is hanging from the tablecloth. In this case, the relevant forces acting on her are the tension (upward) from the tablecloth and her own weight (downward), which result in a downward acceleration:

$$\vec{T}_{sis} - \vec{w}_{sis} = m_{sis} \vec{a}_{sis} \qquad \rightarrow \qquad T = m_{sis}(g - a_y)$$

We'll assume the tablecloth is massless, so it applies the same tension to the cake (albeit in a different direction). Although the cake experiences vertical forces (its weight and the normal force), the only relevant force in this case is the tension from the tablecloth in the horizontal direction (we assume the table is frictionless):

$$\vec{T}_{cake} = m_{cake} \vec{a}_{cake} \qquad \rightarrow \qquad T = m_{cake} a_x$$

As we mentioned above, the horizontal acceleration of the cake is equal to the vertical acceleration of the sister: $a_x = a_y = a$.

   EVALUATE   (a) Equating the tension on the two objects, we can solve for the acceleration:

$$a = g \frac{m_{sis}}{m_{sis} + m_{cake}} = (9.8 \text{ m/s}^2)\frac{14 \text{ kg}}{14 \text{ kg} + 5.3 \text{ kg}} = 7.110 \text{ m/s}^2 \approx 7.1 \text{ m/s}^2$$

(b) Assuming the cake started at rest, it will reach the end of the table in a time of

$$t = \sqrt{\frac{2x}{a}} = \sqrt{\frac{2(0.80 \text{ m})}{(7.110 \text{ m/s}^2)}} = 0.47 \text{ s}$$

   ASSESS   Less than half a second is not a lot of time to save the cake, but it's likely that there is some friction between the tablecloth and the table, so you probably have more time than this.

18.   **INTERPRET**   We are dealing with two masses that are resting on slopes at unequal inclines; however, this time we are given one of the masses, and we must calculate the other mass needed so that the masses accelerate at the given rate.

DEVELOP Refer to the sketch for Problem 5.19. Let the masses on the right and on the left be denoted as m_R and m_L, respectively. Apply Newton's second law to each mass to find the right-hand mass required so that each mass accelerates at the given rate.

EVALUATE (a) Applying Newton's second law in the x-direction gives $T_L - m_L g \sin(52°) = m_L a_L$ and $m_R g \sin(25°) - T_R = m_R a_R$. If the string is massless, the tensions are the same, so $T_R = T_L$. If the string doesn't stretch, then the acceleration of each mass is the same, so $a_L = a_R = a$. Thus, we can sum the two equations given by Newton's second law to find

$$m_R g \sin(\theta_R) - m_L g \sin(\theta_L) = (m_L + m_R) a$$

$$m_R = m_L \frac{g \sin(\theta_L) + a}{g \sin(\theta_R) - a}$$

For the coordinate system we are using, a downward acceleration of the right-hand mass is positive, so $a = 0.64 \text{ m} / \text{s}^2$, which gives

$$m_R = (2.5 \text{ kg}) \frac{(9.8 \text{ m} / \text{s}^2) \sin(52°) + 0.64 \text{ m} / \text{s}^2}{(9.8 \text{ m} / \text{s}^2) \sin(25°) - 0.64 \text{ m} / \text{s}^2} = 5.97 \text{ kg}$$

(b) If the right-hand mass accelerates up the slope, $a = -0.76 \text{ m} / \text{s}^2$. This gives

$$m_R = (2.5 \text{ kg}) \frac{(9.8 \text{ m} / \text{s}^2) \sin(52°) - 0.76 \text{ m} / \text{s}^2}{(9.8 \text{ m} / \text{s}^2) \sin(25°) + 0.76 \text{ m} / \text{s}^2} = 3.55 \text{ kg}$$

ASSESS When the right-hand mass is smaller, the masses accelerate toward the left (and vice versa), which is what we would expect.

19. **INTERPRET** This is a two-dimensional problem that involves applying Newton's second law to two climbers, tied together by a rope and sliding down an icy mountainside. The physical quantities of interest are their acceleration and the force required to bring them to a complete stop.

DEVELOP We choose two coordinate systems where the x-axes are parallel to the slopes and y-axes are perpendicular to the slopes (see figure below). Assume that the icy surface is frictionless and that the climbers move together as a unit with the same magnitude of down-slope acceleration a. If the rope is not stretching, the tension forces are equal in magnitude, so $T_1 = T_2 = T$. To find the acceleration of the climbers, apply Newton's second law in the direction of the slope. To find the force F_{ax} exerted by the ax, apply Newton's second law again, but this time include F_{ax} and set the acceleration to zero; $a = 0$.
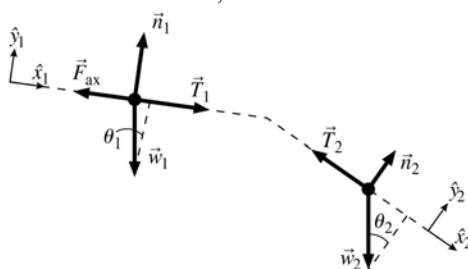
EVALUATE (a) For this part, we neglect the force due to the ax. Because we are now working in one dimension (the x-dimension), we forego vector notation, and insert the sign (±) according to the direction of the force. Of course, at the end, we must interpret the sign of the resulting force as indicating its direction (positive or negative x-direction). The magnitude of the net force in the $\hat{x}_1$ and $\hat{x}_2$ directions (downward positive) is thus

$$F_{net} = m_1 g \sin(\theta_1) + \overbrace{T_1 + T_2}^{=0} + m_2 g \sin(\theta_2)$$

$$= (80 \text{ kg})(9.8 \text{ m} / \text{s}^2) \sin(16°) + (65 \text{ kg})(9.8 \text{ m} / \text{s}^2) \sin(32°) = 553.7 \text{ N}$$

Thus, the magnitude of the acceleration of the pair is

$$a = \frac{F_{net}}{m_1 + m_2} = \frac{553.7 \text{ N}}{80 \text{ kg} + 65 \text{ kg}} = 3.8 \text{ m} / \text{s}^2$$

So the pair accelerates down the slope at 3.8 m/s^2.

(b) After they have stopped, we include the force of the ax. Thus, the magnitude of the force due to the ax is

$$F_{\text{net}} = -F_{\text{ax}} + m_1 g \sin(\theta_1) + \overbrace{T_1 - T_2}^{=0} + m_2 g \sin(\theta_2) = m\,\overset{=0}{\overset{\frown}{a}} = 0$$

$$F_{\text{ax}} = m_1 g \sin(\theta_1) + m_2 g \sin(\theta_2) = 553.7 \text{ N}$$

That is, the force exerted by the ax must be 553.7 N up the slope.

ASSESS   If the two climbers were not roped together, then their acceleration would have been

$$a_1 = \frac{F_1}{m_1} = g\sin(\theta_1) = \left(9.8 \text{ m/s}^2\right)\sin(16°) = 2.7 \text{ m/s}^2$$

$$a_1 = \frac{F_2}{m_2} = g\sin(\theta_2) = \left(9.8 \text{ m/s}^2\right)\sin(32°) = 5.19 \text{ m/s}^2$$

The acceleration of the pair is the mass-weighted average of the individual accelerations:

$$a = \frac{F_{\text{net}}}{m_1 + m_2} = \frac{m_1 g\sin(\theta_1) + m_2 g\sin(\theta_2)}{m_1 + m_2} = \left(\frac{m_1}{m_1 + m_2}\right) g\sin(\theta_1) + \left(\frac{m_2}{m_1 + m_2}\right) g\sin(\theta_2)$$

$$= \left(\frac{m_1}{m_1 + m_2}\right) a_1 + \left(\frac{m_2}{m_1 + m_2}\right) a_2 = 3.9 \text{ m/s}^2$$

Section 5.3 Circular Motion

20.   **INTERPRET**   This problem involves circular motion and the force required to centripetally accelerate the Moon toward the Earth.

DEVELOP   We can calculate the force required to centripetally acceleration of the Moon from the data in Appendix E and Equation 5.1.

EVALUATE   From Equation 5.1, the force required is $F_c = ma_c = mv^2/r$. The orbital speed of the Moon is given in Appendix E as $v = 1.0$ km/s $= 1000$ m/s, its orbital radius is $r = 0.385 \times 10^6$ km $= 3.85 \times 10^8$ m, and its mass is $m = 7.35 \times 10^{22}$ kg. Thus the cable tension required is

$$T = \frac{mv^2}{r} = \frac{\left(7.35 \times 10^{22} \text{ kg}\right)\left(1000 \text{ m/s}\right)^2}{3.85 \times 10^8 \text{ m}} = 1.91 \times 10^{20} \text{ N}$$

ASSESS   We could also calculate the velocity by dividing the circumference of the Moon's orbit by its period. This gives $v = 2\pi r/T = 2\pi\left(3.85 \times 10^8 \text{ m}\right)/\left(27.3 \text{ d}\right) = 1.03$ km/s. Inserting this speed into Equation 5.1 gives a tension of 2.01×10^{20} N. Which answer (2.01 or 1.91×10^{20} N) is likely to be more accurate (i.e., closer to the real answer)? For the latter calculation, we made the assumption that the Moon's orbit is circular in calculating its orbital speed, which is an approximation. In reality the orbit is slightly elliptical. The average speed reported in Appendix E is therefore a more reliable datum, so that the calculation using the value $v = 1000$ m/s is probably more accurate.

21.   **INTERPRET**   In this problem we are asked to show that the force required to keep a mass m in a circular path of radius r with period T is $4\pi^2 mr/T^2$.

DEVELOP   To derive the formula, we first note that for an object of mass m in uniform circular motion, the magnitude of the net force is given by Equation 5.1: $F = ma = mv^2/r$. Next, we make use of the fact that the period of the motion (i.e., time for one revolution) is the circumference $C = 2\pi r$ divided by the speed v. Thus, $T = 2\pi r/v$.

EVALUATE   Combining the two expressions, the force can be rewritten as

$$F = \frac{mv^2}{r} = \frac{m}{r}\left(\frac{2\pi r}{T}\right)^2 = \frac{4\pi^2 mr}{T^2}$$

ASSESS Our result indicates that for a fixed radius r, the centripetal force is inversely proportional to T^2. For example, if T is very large (i.e., it takes a very long time for the mass to complete one revolution), then the speed v is very small and the centripetal force F is also very small.

22. **INTERPRET** For the rock to whirl around in a circle, the string has to supply the centripetal force through its tension. To keep this tension below the limit, the string makes an angle with the horizontal, as shown in Figure 5.11 in the text.

    DEVELOP The situation is the same as that described in Example 5.5. The net vertical force is zero, so the weight is balanced by the vertical component of the tension, $T\sin\theta = mg$. As for the horizontal component of the tension, it is providing the needed centripetal force, $T\cos\theta = mv^2/r$. The radius of the rock's trajectory is $r = L\cos\theta$.

    EVALUATE (a) We are asked to first find the minimum angle that keeps the tension under the string's breaking limit:

    $$\theta_{min} = \sin^{-1}\left(\frac{mg}{T_{max}}\right) = \sin^{-1}\left(\frac{(0.980\text{ kg})(9.8\text{ m}/\text{s}^2)}{(150\text{ N})}\right) = 3.67°$$

    (b) At this angle, the speed of the rock is

    $$v = \sqrt{\frac{T_{max}L}{m}}\cos\theta_{min} = \sqrt{\frac{(150\text{ N})(1.80\text{ m})}{(0.980\text{ kg})}}\cos 3.67° = 16.55\text{ m}/\text{s}$$

    ASSESS The stronger the string is, the closer to the horizontal it can whirl the rock around (i.e., $\theta_{min} \to 0$ as T_{max} increases). But to maintain such a trajectory, the velocity has to increase, so that the centripetal acceleration is sufficient.

23. **INTERPRET** You're asked to find the velocity of the subway while rounding the curve. We'll assume uniform circular motion, in which case all the objects in the subway are accelerating according to $a = v^2/r$.

    DEVELOP The only information you have is that a strap is dangled at 15° to the vertical during the turn. You can assume that the strap hung straight down before the turn and that its displacement was outward with respect to the center of the curve. We can think of the strap as a mass hanging from an attachment (perhaps a chain or belt) that provides a tension, as shown in the figure below.
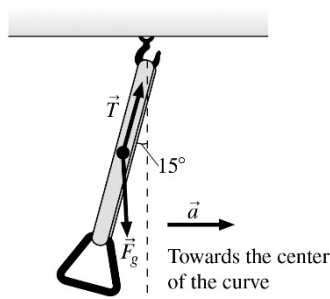
    As we said above, the strap and everything else in the subway experience an acceleration, which obeys Newton's second law: $F_{net} = F_g + T = ma$. We'll choose our coordinate system such that the acceleration is in the $+x$-direction. Therefore, in component form, the second law is

    $$T_x = T\sin 15° = ma = \frac{mv^2}{r}$$

    $$T_y - F_g = T\cos 15° - mg = 0$$

    We'll evaluate these equations to find the velocity of the subway.

    EVALUATE We see from above that $T = mg/\cos 15°$, so $a = g\tan 15°$ and

    $$v = \sqrt{ar} = \sqrt{(9.8\text{ m}/\text{s}^2)(\tan 15°)(150)} = 19.8\text{ m}/\text{s}$$

   To compare to the speed limit, we express this quantity in km/h and see that

$$v = 19.8 \frac{m}{s} \left(\frac{1\,\text{km}}{1000\,\text{m}} \right) \left(\frac{3600\,\text{s}}{h} \right) = 71 \frac{\text{km}}{\text{h}}$$

the train did exceed the 50-km/h speed limit on this curve by 21 km/h, thus provoking the derailment.

ASSESS   As part of the derivation, we found that the acceleration obeys: $a = g\tan\theta$. Does that make sense? If the strap was hanging straight down $\theta = 0$, the acceleration would be zero, as we would expect for a train that is moving straight at a constant speed.

24.   **INTERPRET**   This problem involves circular motion and Newton's second law. We are asked to find the speed at which a tetherball circles given the angle its cord makes with the horizontal.

DEVELOP   Draw a free-body diagram of the ball (see figure below). Because the vertical component of the ball's acceleration is zero, we know from Newton's second law that the vertical component of the tension force pulling the ball up must cancel the gravitation force pulling the ball down, so $w = T\sin(\theta)$. Applying Newton's second law in the horizontal direction, and using Equation 5.1 for the acceleration of the ball, gives $F_{\text{net}} = T\cos(\theta) = mv^2/r$. Note that the radius involved is the horizontal radius, so $r = (1.55\text{ m})\cos(\theta)$.
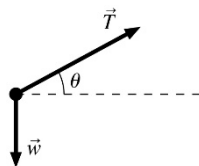
EVALUATE   Combining these two formulas, we can solve for the speed v. This gives

$$v = \pm\sqrt{\frac{Tr\cos(\theta)}{m}} = \pm\sqrt{\frac{mgr\cos(\theta)}{m\sin(\theta)}} = \pm\sqrt{\frac{gr}{\tan(\theta)}} = \pm\sqrt{\frac{(9.8\text{ m/s}^2)(1.55\text{ m})\cos(15°)}{\tan(15°)}} = \pm 8.36\text{ m/s}$$

ASSESS   This expression is the same as found in Example 5.5, step 6. The $\pm$ sign indicates that the result is the same whether the ball travels clockwise or counterclockwise.

25.   **INTERPRET**   This problem involves circular motion and Newton's second law. The object of interest is the plane. By analyzing the force acting on the plane while it travels a circular path, its speed can be determined.
DEVELOP   We follow the strategy outlined in Example 5.6. There are two forces acting on the plane: the gravitational force $\vec{w}$ and the normal force $\vec{n}$. Applying Newton's second law gives $\vec{F}_{\text{net}} = \vec{w} + \vec{n} = m\vec{a}$. Breaking up this vector equation into its components gives

$$x : n\sin(\theta) = \frac{mv^2}{r}$$

$$y : n\cos(\theta) = mg$$

EVALUATE   Dividing the two equations allows us to eliminate n and obtain

$$\frac{n\sin(\theta)}{n\cos(\theta)} = \frac{mv^2/r}{mg} \implies \tan(\theta) = \frac{v^2}{rg}$$

With $\theta = 22°$ and $r = 4.0\text{ km} = 4000\text{ m}$, the speed of the plane is

$$v = \pm\sqrt{gr\tan\theta} = \pm\sqrt{(9.8\text{ m/s}^2)(4000\text{ m})\tan(22°)} = \pm 125.8\text{ m/s} = \pm 453.1\text{ km/h}$$

ASSESS   The result shows that the speed of the plane is proportional to $\sqrt{\tan\theta}$. If we want to increase the speed v while the radius r is kept fixed, then the banking angle would also need to be increased. This is due to the fact that the horizontal component of the normal force, $n\sin(\theta)$, is what keeps the plane in circular motion. The $\pm$ sign indicates that the result is the same whether the planes travels clockwise or counterclockwise.

Section 5.4 Friction

26.   **INTERPRET**   This problem involves calculating the force due to friction that is exerted upon an object of a given mass with a given coefficient of kinetic friction. It also involves Newton's second law to find the normal force.

DEVELOP   If the floor is level, then, by Newton's second law, the normal force must exactly cancel the weight of the cabinet. Thus, $n = mg$. To solve the problem, use this result in Equation 5.3 for kinetic friction, with $m = 79$ kg and $\mu_k = 0.85$.

EVALUATE   From Equation 5.3, the force exerted on the cabinet due to friction is

$f_k = \mu_k mg = (0.85)(79 \text{ kg})(9.8 \text{ m/s}^2) = 658.07$ N. The direction of the friction force is opposite to the motion of the cabinet.

ASSESS   Note that the coefficient of friction is dimensionless, so the units work out to be N.
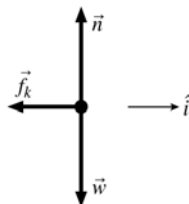
27.   **INTERPRET**   This problem involves Newton's second law, friction, and kinematics. The object of interest is the hockey puck. Three forces are involved: gravity, the normal force, and friction; friction is the force that causes the acceleration of the puck (in the direction opposing the motion of the puck). The physical quantity to be computed is the coefficient of kinetic friction.

DEVELOP   Draw a free-body diagram of the situation (see figure below). We define the positive x-direction to be the direction of the puck's initial velocity, $v_0 = 11$ m/s. Given that it travels a distance of $x - x_0 = 60$ m, we can find the distance traveled by using the kinematic Equation 2.11, $v^2 = v_0^2 + 2a(x - x_0)$.
The result is

$$a = -\frac{v_0^2}{2(x - x_0)}$$

where we have used the fact that the final velocity is $v = 0$. Notice that the acceleration is negative, meaning that the puck decelerates. By applying Newton's second law in the vertical direction, we know that the normal force must have the same magnitude as the weight because the puck does not accelerate vertically. Thus, $n = w$. The force of friction is the only horizontal force acting on the puck, so Newton's second law and Equation 5.3 tell us that $F_{net} = -f_k = -\mu_k mg = ma$, where we have inserted a minus sign because the friction force acts to oppose the motion, which we take to be in the positive x-direction.

EVALUATE   Inserting the known quantities into Newton's second law gives

$$-\mu_k mg = ma$$

$$\mu_k = -\frac{a}{g} = -\frac{-v_0^2}{2g(x - x_0)} = \frac{(11 \text{ m/s})^2}{2(9.8 \text{ m/s}^2)(60 \text{ m})} = 0.1$$

ASSESS   The result $a = \mu_k g$ shows that increasing the coefficient of friction would result in a greater acceleration. This makes sense because friction is what causes the acceleration.

28.   **INTERPRET**   This problem involves Newton's second law, the force due to kinetic friction, and kinematics. The object of interest is the skier, and we are asked to find how much longer it would take him to descend a slope with a non zero coefficient of kinetic friction as compared to if there were no friction.

DEVELOP   Start with a free-body diagram, and choose a coordinate system in which the positive x-direction is down the slope (see figure below). The forces acting on the skier are the force of gravity, $w = mg$, the normal force n exerted by the slope, and the force f_k due to kinetic friction. To find the skier's acceleration, apply Newton's second law in the $\hat{i}$ direction. This gives

$$F_{net} = ma$$
$$-f_k + w\sin(\theta) = ma$$

where $w = mg$ and we have made $f_k < 0$ because it always acts to oppose the motion, so in this case it acts in the negative x-direction. The force due to kinetic friction may be found from Equation 5.3, $f_k = \mu_k n = \mu_k mg\cos(\theta)$, where we have used Newton's second law in the $\hat{j}$ direction in the final equality:

$$F_{net} = n - mg\cos(\theta) = ma = 0 \quad \Rightarrow \quad n = mg\cos(\theta)$$

We can now calculate the acceleration a, from which we can find the time to descend the slope using kinematic Equation 2.10 for constant acceleration, $x = x_0 + v_0 t + at^2/2$, with $x - x_0 = 100$ m and $v_0 = 0$ m/s.
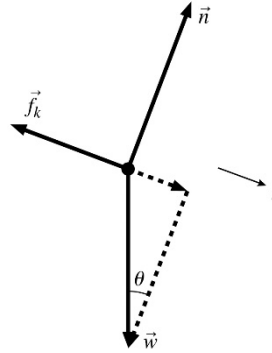
EVALUATE If there is no kinetic friction ($\mu_k = 0$ so fk = 0), then, from the first equation above, the acceleration is $a_1 = g\sin(\theta)$. From Equation 2.10, the time to descend the slope is

$$x - x_0 = \overset{0}{\cancel{v_0 t_1}} + \frac{a_1 t_1^2}{2}$$

$$t_1 = \sqrt{\frac{2(x - x_0)}{a_1}} = \sqrt{\frac{2(x - x_0)}{g\sin(\theta)}} = \sqrt{\frac{2(100\text{ m})}{(9.8\text{ m/s}^2)\sin(28°)}} = 6.59\text{ s}$$

where we have taken the positive square root. If $\mu_k = 0.17$, then the acceleration is $a = g\sin(\theta) - g\mu_k\cos(\theta)$, so the time to descend the slope is

$$t_2 = \sqrt{\frac{2(x - x_0)}{a_2}} = \sqrt{\frac{2(x - x_0)}{g\sin(\theta) - g\mu_k\cos(\theta)}} = \sqrt{\frac{2(100\text{ m})}{(9.8\text{ m/s}^2)\left[\sin(28°) - 0.17\cos(28°)\right]}} = 7.99\text{ s}$$

The difference in the time to descend the slopes is $t_2 - t_1 = 7.99$ s $-$ 6.59 s $=$ 1.4 s.

ASSESS Notice that the units for the formulas giving the time are seconds. Considering that the fastest runners can cover 100 m in slightly less than 10 s, we see that our skier travels considerably faster than an extremely fast runner.

29. **INTERPRET** This problem involves Newton's second law, friction, and kinematics. Knowing the recommended speed to traverse a curve, you are tasked with finding whether it is safe to do so when the presence of snow has lowered the frictional coefficient.

    DEVELOP As the vehicle traverses the curve it experiences an inward linear acceleration given by $a = v^2/r$, which in the case of an unbanked curve needs to be provided by frictional force $f = \mu n$ in order for the vehicle not to slip. We can use Newton's second law to solve for the coefficient of friction μ_{min} necessary for the vehicle to traverse the curve at the recommended speed of $v = 60\text{km/h} = 16.7\text{m/s}$, and compare with the frictional coefficient $\mu = 0.20$ present when the road is covered in snow.

    EVALUATE Applying Newton's second law gives

    $$F_{net} = f = \mu_{min} n = \frac{mv^2}{r}$$

    $$\mu_{min} mg = \frac{mv^2}{r} \rightarrow \mu_{min} = \frac{v^2}{gr} = \frac{(16.7\text{m/s})^2}{(9.8\text{m/s}^2)(115\text{m})} = 0.25$$

    Here we have taken the normal force to be equal to the weight of the vehicle since it is traveling on a flat road. We see that the frictional coefficient in the presence of snow is less than the minimum frictional coefficient, making it unsafe to traverse the curve at the posted speed of 60 km/h.

    ASSESS Plugging in the frictional coefficient in the presence of snow and solving for the speed gives us the maximum safe speed to traverse the curve is equal to 54 km/h.

EXAMPLE VARIATIONS

30.   **INTERPRET**   We need to find the climber's acceleration, and from that we can get the time before the rock goes over the edge. We identify two objects of interest, the climber and the rock, and we note that the rope connects them. There are two forces on the climber: gravity and the upward rope tension. There are three forces on the rock: gravity, the normal force from the surface, and the rightward-pointing rope tension.

DEVELOP   Like in the original example, we can deduce the forces acting on each object from the free-body diagrams in Figure 5.8 of the textbook. Newton's law applies to each, so we write two vector equations:

$$\text{Climber: } \vec{T}_c - \vec{F}_{gc} = m_c \vec{a}_c$$

$$\text{Rock: } \vec{T}_r - \vec{F}_{gr} = m_r \vec{a}_r$$

where the subscripts c and r stand for climber and rock, respectively.

EVALUATE   Like in the original example we can recognize that the climber only experiences forces along the vertical direction, the vertical forces on the rock do not contribute, the tension is the same for both the rock and the climber, and choosing the same coordinate axes as the one shown in Figure 5.8 we can write Newton's laws for the climber and rock as

$$\text{Climber, } y: \quad T - m_c g = -m_c a$$

$$\text{Rock, } x: \quad T = m_r a$$

Using the value for the tension T from the rock's x-equation, and plugging into the climber's y-equation we find the acceleration is equal to

$$a = \frac{m_c g}{m_c + m_r} = \frac{(63.2\,\text{kg})(9.8\,\text{m/s}^2)}{(63.2\,\text{kg} + 1220\,\text{kg})} = 0.483\,\text{m/s}^2$$

To find the time it takes the rock to go over the edge we recognize that this is a problem of an object under constant acceleration and we can solve for t in kinematic Equation 2.10, $x = x_0 + v_0 + \frac{1}{2}at^2$, to get

$$t = \sqrt{\frac{2x}{a}} = \sqrt{\frac{(2)(48.6\,\text{m})}{(0.483\,\text{m/s}^2)}} = 14.2\,\text{s}$$

ASSESS   Just like in the original example, the climber accelerates down at a much slower rate due to the rock. This could potentially decrease further if we were to consider the rock resisting the tension force in the presence of a frictional force.

31.   **INTERPRET**   We need to find the mass of the rock that will withstand the tension force exerted by the climber's gravitational acceleration long enough for help to arrive. We identify two objects of interest, the climber and the rock, and we note that the rope connects them. There are two forces on the climber: gravity and the upward rope tension. There are three forces on the rock: gravity, the normal force from the surface, and the rightward-pointing rope tension.

DEVELOP   Like in the original example, we can deduce the forces acting on each object from the free-body diagrams in Figure 5.8 of the textbook. Newton's law applies to each, so we write two vector equations:

$$\text{Climber: } \vec{T}_c - \vec{F}_{gc} = m_c \vec{a}_c$$

$$\text{Rock: } \vec{T}_r - \vec{F}_{gr} = m_r \vec{a}_r$$

where the subscripts c and r stand for climber and rock, respectively.

EVALUATE   Like in the original example we can recognize that the climber only experiences forces along the vertical direction, the vertical forces on the rock do not contribute, the tension is the same for both the rock and the climber, and choosing the same coordinate axes as the one shown in Figure 5.8 we can write Newton's laws for the climber and rock as

$$\text{Climber, } y: \quad T - m_c g = -m_c a$$

$$\text{Rock, } x: \quad T = m_r a$$

Using the value for the tension T from the rock's x-equation, and plugging into the climber's y-equation we find the mass of the rock can be expressed as

$$m_r = m_c \left(\frac{g}{a} - 1 \right)$$

From kinematic Equation 2.10, $x = x_0 + v_0 + \frac{1}{2} at^2$, we find $a = 2x / t^2$. Plugging this into our expression for the rock's mass we obtain

$$m_r = m_c \left(\frac{gt^2}{2x} - 1 \right) = (63.2\,\text{kg}) \left(\frac{(9.8\,\text{m/s}^2)(36.5\,\text{s})^2}{(2)(48.6\,\text{m})} - 1 \right) = 8430\,\text{kg}$$

ASSESS   Since the climber had to be held by the rock three times longer than in the original example, the rock had to be about nine times more massive.

32.   **INTERPRET**   Much like in the original example we have two objects connected by a rope, where one is free to fall and the other is accelerated in the horizontal direction due to the tension on the rope they share. We are to find the time it takes the latter to reach the end of the track.

   DEVELOP   Like in the original example, we can deduce the forces acting on each object from the free-body diagrams in Figure 5.8 of the textbook. Newton's law applies to each, so we write two vector equations:

$$1: \quad \vec{T_1} - \vec{F}_{g1} = m_1 \vec{a}_1$$
$$2: \quad \vec{T_2} - \vec{F}_{g2} = m_2 \vec{a}_2$$

where the subscripts 1 and 2 note the two masses.

   EVALUATE   Like in the original example we can recognize that the suspended mass only experiences forces along the vertical direction, the vertical forces on mass number 2 do not contribute, the tension is the same for both masses, and choosing the same coordinate axes as the one shown in Figure 5.8 we can write Newton's laws for the masses as

$$m_1, y: \quad T - m_1 g = -m_1 a$$
$$m_2, x: \quad T = m_2 a$$

Using the value for the tension T from the x-equation of m_2, and plugging into the y-equation of m_1 we find the acceleration is equal to

$$a = \frac{m_1 g}{m_1 + m_2}$$

From kinematic Equation 2.10, $x = x_0 + v_0 + \frac{1}{2} at^2$, we find $t = \sqrt{2x / a}$. Plugging our acceleration into this expression we find the time it takes for m_2 to reach the end of the track is equal to

$$t = \sqrt{\frac{2x(m_1 + m_2)}{m_1 g}} = \sqrt{\frac{(2)(0.672\,\text{m})(0.0149\,\text{kg} + 0.326\,\text{kg})}{(0.0149\,\text{kg})(9.8\,\text{m/s}^2)}} = 1.77\,\text{s}$$

   ASSESS   From this expression we can see that the time it takes m_2 to reach the end of the track grows the more massive m_2, and shrinks the more massive m_1 is since it is responsible for the accelerating force.

33.   **INTERPRET**   Much like in the original example we have two objects connected by a rope, where one is free to fall and the other is accelerated in the horizontal direction due to the tension on the rope they share. Knowing the mass of one object, we are to find the mass of the other, so that they accelerate at the same rate they would experience being dropped just above the surface of Jupiter's moon Callisto.

   DEVELOP   Like in the original example, we can deduce the forces acting on each object from the free-body diagrams in Figure 5.8 of the textbook. Newton's law applies to each, so we write two vector equations:

$$1: \vec{T}_1 - \vec{F}_{g1} = m_1\vec{a}_1$$

$$2: \vec{T}_2 - \vec{F}_{g2} = m_2\vec{a}_2$$

where the subscripts 1 and 2 note the two masses.

EVALUATE   Like in the original example we can recognize that the suspended mass only experiences forces along the vertical direction, the vertical forces on mass number 2 do not contribute, the tension is the same for both masses, and choosing the same coordinate axes as the one shown in Figure 5.8 we can write Newton's laws for the masses as

$$m_1 , y: \quad T - m_1 g = -m_1 a_c$$

$$m_2 , x: \quad T = m_2 a_c$$

Using the value for the tension T from the x-equation of m_2 , and plugging into the y-equation of m_1 we can find an expression for the mass of hanging object. Plugging in the value for the acceleration on the surface of Callisto from Appendix E, $a_c = 1.24\,\text{m/s}^2$, along with the given value for m_2 and Earth's gravitational constant we get

$$m_1 = \frac{m_2 a_c}{(g - a_c)} = \frac{(0.326\,\text{kg})(1.24\,\text{m/s}^2)}{(9.8\,\text{m/s}^2 - 1.24\,\text{m/s}^2)} = 47.2\,\text{g}$$

ASSESS   From this expression we can see what m_1 needs to be for the system to accelerate at the desired rate in this simplified scenario.

34.   **INTERPRET**   Like in the original example, we have circular motion described by Newton's second law. We are tasked with finding the minimum speed at the top of the loop for the car to stay on the track.

DEVELOP   In order for the car to stay on track there must be an inward accelerating force. Using the same free-body diagram from the original example, and orienting our coordinate axes so that the positive y-axis points downward, we obtain from Newton's second law that: $\vec{n} - \vec{F}_g = m\vec{a}$.

EVALUATE   Since all forces are directed along the y-direction both forces only have y-components. These two contribute to the inward radial acceleration $a_y = v^2/r$, and we find that the velocity at the top of the loop is given by

$$n + mg = \frac{mv^2}{r}$$

$$v = \sqrt{(nr/m) + gr}$$

Which is minimized when gravity alone provides the force that keeps the object in its circular path and n is arbitrarily small. Thus, the minimum velocity obtained by setting $n = 0$ is

$$v_{\text{min}} = \sqrt{gr} = \sqrt{(9.8\,\text{m/s}^2)(19.5\,\text{m})} = 13.8\,\text{m/s}$$

ASSESS   Just like in the original example, mass doesn't affect the minimal speed needed to stay on track since the normal force is not contributing to the accelerating force.

35.   **INTERPRET**   Like in the original example, we have circular motion described by Newton's second law. We are tasked with finding the direction and magnitude of the normal force exerted by the car's seat on passengers at the top of the loop.

DEVELOP   Passengers on the car feel a normal force equal to that between the car and track. Using the same free-body diagram from the original example, and orienting our coordinate axes so that the positive y-axis points downward, we obtain from Newton's second law that: $\vec{n} - \vec{F}_g = m\vec{a}$.

EVALUATE   Since all forces are directed along the y-direction both forces only have y-components. These two contribute to the inward radial acceleration $a_y = v^2/r$, and we find that the normal force is given by

$$n + mg = \frac{mv^2}{r}$$

$$n = m\left(\frac{v^2}{r} - g\right) = (72.1\,\text{kg})\left(\frac{(17.7\,\text{m/s})^2}{(19.5\,\text{m})} - (9.8\,\text{m/s}^2)\right) = 451\,\text{N}$$

Which is directed downward since we chose a coordinate system where the positive y-axis points downward.

ASSESS   From this expression we can see that the faster the passengers are traveling around the loop, the larger the magnitude of the normal force they feel acting on them from the car's seats.

36.   **INTERPRET**   Like in the original example, we have circular motion described by Newton's second law. We are tasked with finding the minimum speed at the top of the circle that will keep water in the bucket.
DEVELOP   In order for water to stay in the bucket there must be an inward accelerating force. Using the same free-body diagram from the original example, and orienting our coordinate axes so that the positive y-axis points downward, we obtain from Newton's second law that: $\vec{n} - \vec{F}_g = m\vec{a}$.
EVALUATE   Since all forces are directed along the y-direction both forces only have y-components. These two contribute to the inward radial acceleration $a_y = v^2/r$, and we find that the velocity at the top of the loop is given by

$$n + mg = \frac{mv^2}{r}$$

$$v = \sqrt{(nr/m) + gr}$$

Which is minimized when gravity alone provides the force that keeps the object in its circular path and n is arbitrarily small. Thus the minimum velocity obtained by setting $n = 0$ is

$$v_{\text{min}} = \sqrt{gr} = \sqrt{(9.8\,\text{m/s}^2)(1.22\,\text{m})} = 3.46\,\text{m/s}$$

ASSESS   Just like in the original example, mass doesn't affect the minimal speed needed for water to stay in the bucket since the normal force is not contributing to the accelerating force.

37.   **INTERPRET**   Like in the original example, we have circular motion described by Newton's second law. We want to determine whether the bucket from the preceding problem can withstand the reaction force originating from the normal force exerted on the water as it travels around a circle.
DEVELOP   The bucket feels a force equal to that exerted on the water by the bucket as it goes around a circle. Using the same free-body diagram from the original example, and orienting our coordinate axes so that the positive y-axis points downward, we obtain from Newton's second law that: $\vec{n} - \vec{F}_g = m\vec{a}$.
EVALUATE   Since all forces are directed along the y-direction both forces only have y-components. These two contribute to the inward radial acceleration $a_y = v^2/r$, and we find that the normal force is given by

$$n + mg = \frac{mv^2}{r}$$

$$n = m\left(\frac{v^2}{r} - g\right)$$

Which is maximized when the bucket is on either side of the circle and gravity does not contribute to the accelerating force. Thus the maximum normal force on the water, and the maximum force exerted on the bucket, are obtained by removing the gravitational force from the expression above.

$$n_{max} = \frac{mv^2}{r} = (3.25\,\text{kg})\left(\frac{(5.64\,\text{m/s})^2}{(1.22\,\text{m})}\right) = 84.7\,\text{N}$$

Which is greater than the maximum force of 65 N the bucket can tolerate, meaning it will not survive the whirl.

ASSESS   Unless the bucket begins the whirl at a lower speed, it won't make it to the top since the greatest force exerted on the bucket takes place when the acceleration is directed along the horizontal direction.

PROBLEMS

38. **INTERPRET** We wish to find the acceleration of a skier on a frictionless slope, which we can do using Newton's second law (for constant mass). This problem was done for us in Example 5.1, but this time we do it with a coordinate system that has horizontal and vertical axes.

DEVELOP Start with a free-body diagram, as shown in the figure below. To find the acceleration, apply Newton's second law, which gives

$$\vec{F}_{net} = m\vec{a}$$
$$m\vec{a} = n\sin(\theta)\hat{i} + \left[n\cos(\theta) - w\right]\hat{j}$$

We know that the skier does not accelerate in the direction of the normal force, so the forces in this direction must cancel. This gives $n = w\cos(\theta)$, with $w = mg$. This allows us to find the acceleration and the normal force, which is the force the snow exerts on the skier.
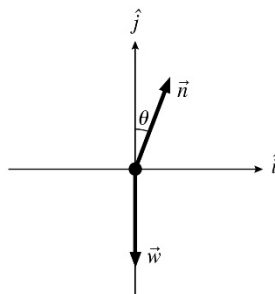
EVALUATE **(a)** Inserting the expression for the normal force into Newton's second law gives an acceleration of

$$\vec{a} = g\sin(\theta)\cos(\theta)\hat{i} + \left[g\cos^2(\theta) - g\right]\hat{j}$$
$$= \left(9.8 \text{ m/s}^2\right)\left\{\sin(32°)\cos(32°)\hat{i} + \left[\cos^2(32°) - 1\right]\hat{j}\right\} = \left(4.40 \text{ m/s}^2\right)\hat{i} - \left(2.75 \text{ m/s}^2\right)\hat{j}$$

The magnitude of this acceleration is $a = \sqrt{\left(4.40 \text{ m/s}^2\right)^2 + \left(2.75 \text{ m/s}^2\right)^2} = 5.2 \text{ m/s}^2$ and the direction is $\theta = \text{atan}(2.75/4.40) = -32°$, which is the same result as found in Example 5.1.

(b) The force exerted by the snow on the skier is

$$n = mg\cos(\theta) = \left(65 \text{ kg}\right)\left(9.8 \text{ m/s}^2\right)\cos(32°) = 540 \text{ N}$$

ASSESS The answers we get are independent of the coordinate system used. However, choosing the best coordinate system can make the problem easier to understand and to solve.

39. **INTERPRET** This problem involves a block with an initial velocity sliding up a frictionless ramp. The quantity of interest is the distance it travels before coming to a complete stop. We will apply Newton's second law and kinematics to solve this problem.

DEVELOP Draw a free-body diagram of the situation (see figure below). Applying Newton's second law in the $\hat{i}$-direction gives $-w\sin(\theta) = ma$, or $a = -g\sin(\theta)$. The stopping distance can be calculated by solving the kinematic Equation 2.11, $v^2 = v_0^2 + 2a\Delta x$.
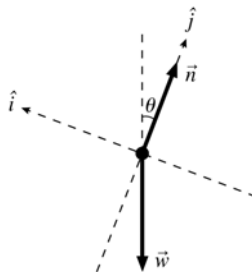
EVALUATE Inserting the acceleration into Equation 2.11, the distance travelled by the block is

$$\Delta x = \frac{v^2 - v_0^2}{2a} = \frac{v^2 - v_0^2}{2\left[-g\sin(\theta)\right]} = \frac{(0 \text{ m/s})^2 - (2.5 \text{ m/s})^2}{-2(9.8 \text{ m/s}^2)\sin(30°)} = 0.64 \text{ m}$$

ASSESS   The result shows that the distance traveled is inversely proportional to $\sin(\theta)$. To see that this makes sense, consider the limit where $\theta \to 0$. This situation would correspond to a frictionless horizontal surface. In this case, we expect the block to travel indefinitely, in agreement with our expression for Δx.

40.   **INTERPRET**   We are asked to find the sum of the forces provided by two motor proteins.

DEVELOP   The net force on the spindle pole is $\vec{F}_{net} = \vec{F}_1 + \vec{F}_2$. We're asked to find only the magnitude of this sum, so let's choose a coordinate system that makes our life easy. If the +x-axis splits the middle between the two forces, then $\theta = \pm\frac{1}{2}70° = \pm 35°$, and the y-components of the forces $\left(F_y = F\sin\theta\right)$ cancel each other out. Therefore, the net force points completely in the x-direction.

EVALUATE   The magnitude is just the sum of the x-components:

$$F_{net} = F_{1x} + F_{2x} = (6.9 \text{ pN})\left[\cos(35°) + \cos(-35°)\right] = -11.3 \text{ pN}$$

ASSESS   The answer is within the limits of what the sum could be. If the forces were aligned $(\theta = 0)$, then the net force would be $2F = 13.8 \text{ pN}$, whereas if the forces were completely opposite $(\theta = 180°)$, then the net force would be zero.

41.   **INTERPRET**   This problem involves Newton's second law. We are asked to find the tension in a rope needed to support an object of a given mass.

DEVELOP   Draw a diagram of the situation (see figure below). Apply Newton's second law in the y-direction and solve for the tension of the rope. Note that the tension of the rope is the same everywhere (for a massless rope), so $T_1 = T_2 = T$,
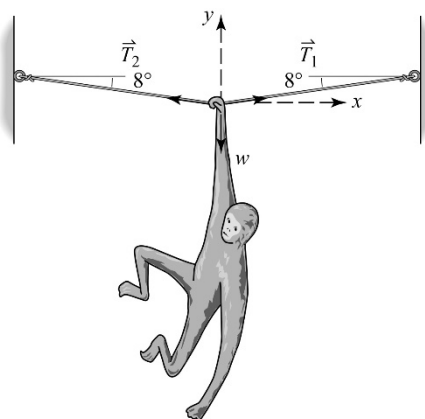
EVALUATE   Applied in the y-direction, Newton's second law gives

$$T_1 \sin(\theta) + T_2 \sin(\theta) = w$$

$$T = \frac{mg}{2\sin(\theta)} = \frac{(14.6 \text{ kg})(9.8 \text{ m/s}^2)}{2\sin(11.0°)} = 375 \text{ N}$$

The monkey's weight is $w = mg = (14.6 \text{ kg})(9.8 \text{ m/s}^2) = 143 \text{ N}$ (to two significant figures). This is about 2.62 times less than the tension force in the rope.

ASSESS   Notice that the $T \to \infty$ as $\theta \to 0$ because there is a vanishingly small component of the tension acting in the vertical direction. The majority of the tension simply serves to pull the two support points together.

42.   **INTERPRET**   This problem is similar to Problem 39, except that the angles at each end of the rope are different. We will apply Newton's second law to solve this problem.

DEVELOP   Draw a free-body diagram for the pack (see below). The angles are $\theta_1 = 28°$ and $\theta_2 = 71°$, and the weight of the pack is $w = mg$. Because the pack does not move, its acceleration in both the horizontal and vertical directions is zero. Thus, applying Newton's second law in the vertical direction gives
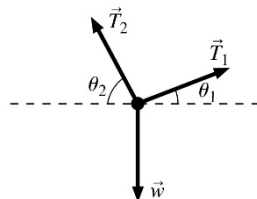
$$F_{net} = ma = 0$$
$$T_2 \sin(\theta_2) + T_1 \sin(\theta_1) - w = 0$$

and in the horizontal direction it gives

$$F_{net} = ma = 0$$
$$-T_2 \cos(\theta_2) + T_1 \cos(\theta_1) = 0$$

Using these two equations, we can solve for T_1 and T_2.

EVALUATE Using the second equation above to express T_2 in terms of T_1 gives $T_1 = T_2 \cos(\theta_2)/\cos(\theta_1)$, which when inserted in the first equation gives

$$T_2 \sin(\theta_2) + T_2 \cos(\theta_2) \tan(\theta_1) = mg$$

$$T_2 = \frac{mg}{\sin(\theta_2) + \cos(\theta_2)\tan(\theta_1)} = \frac{(26\,\text{kg})(9.8\,\text{m/s}^2)}{\sin(71°) + \cos(71°)\tan(28°)}$$

$$= 230\,\text{N}$$

Inserting this value into the expression for T_1 gives

$$T_1 = T_2 \cos(\theta_2)/\cos(\theta_1) = (228\,\text{N})\cos(71°)/\cos(28°) = 84\,\text{N}$$

ASSESS Notice that the tension in the rope on each side of the pack is unequal because the angle made by the rope is different.

43. **INTERPRET** The key concepts in this problem are circular motion and Newton's second law. The object of interest is the mass m_1 that travels in a circular path. By analyzing the force acting on m_1 and m_2, the tension in the string and the period of the circular motion of m_1 can be determined.

    DEVELOP Apply Newton's second law to each mass (see Figure 5.31). Because the table is frictionless, the only force acting on m_1 in the horizontal plane is the tension. Assuming the massless rope does not encounter any friction when it goes through the hole in the table, the tension T acting on each mass is of the same magnitude. By Newton's second law, and because m_2 does not accelerate, this tension must cancel the force due to gravity acting on m_2. Thus, we have

    $$m_1: \quad F_{net} = T = m_1 a_1 = \frac{m_1 v^2}{R} \quad \Rightarrow \quad T = \frac{m_1 v^2}{R}$$
    $$m_2: \quad F_{net} = T - m_2 g = m_2 a_2 = 0 \quad \Rightarrow \quad T = m_2 g$$

    EVALUATE (a) Newton's second law applied to the stationary mass m_2 yields $T = m_2 g$.
    (b) The tension in the string also provides the centripetal force for m_1. Let $\tau = 2\pi R/v$ be the period of the circular motion. The above equation for m_1 then gives

    $$T = \frac{m_1 v^2}{R} = \frac{m_1}{R}\left(\frac{2\pi R}{\tau}\right)^2 = \frac{4\pi^2 m_1 R}{\tau^2}$$

    But from (a), we also have $T = m_2 g$. By combining the two equations, we obtain

    $$\frac{4\pi^2 m_1 R}{\tau^2} = m_2 g$$

    $$\tau = 2\pi \sqrt{\frac{m_1 R}{m_2 g}}$$

ASSESS From the expression for τ, we can draw the following conclusions: (i) Because $\tau \propto \sqrt{R}$, the larger the radius; the longer the period. (ii) With the tension (or m_2) kept fixed, increasing m_1 also leads to a longer period. Note that the derivation above only applies for uniform circular motion. If m_1 is pulled in toward the hole (i.e., R is reduced), then the motion is no longer circular (although the acceleration is still centripetal), and the formula $a = v^2/R$ does not apply.

44. **INTERPRET** We're asked to calculate the horizontal traction force supplied by a mass and a set of massless/frictionless pulleys.

DEVELOP Because the pulleys are massless and frictionless, the tension T in the cord will be the same throughout the system. This tension has to support the mass from falling, so $T = mg$. The horizontal force on the leg is the sum from the cord above and below the pulley attached to the foot:

$$F_y = T\cos\theta_1 + T\cos\theta_2$$

EVALUATE Using the values given, the traction force on the leg is:

$$F_y = (4.6 \text{ kg})(9.8 \text{ m/s})(\cos70° + \cos20°) = 58 \text{ N}$$

ASSESS This force is about 10% of the weight of a 60-kg person, so this seems reasonable for the amount of force needed to counter some of the forces exerted by muscles in the leg.

45. **INTERPRET** This problem involves Newton's second and third laws and uniform circular motion. The objects of interest are the roller-coaster seat, the seat belt, and the rider. We are asked to find the force exerted on a rider at the top of the turn by the roller-coaster seat and by the seat belt, and to determine what would happen should the rider unbuckle his seat belt.

DEVELOP The free-body diagram for the rider at the top of the track will be the same as that for the roller coaster (see Figure 5.16). Thus, the rider has two forces acting on him—the normal force due to the seat and the force due to gravity—both of which are pushing him down. By Newton's second law, the sum of these forces must be proportional to the acceleration. Expressed mathematically, we have

$$F_{net} = n + mg = ma = \frac{mv^2}{R}$$

where $m = 55$ kg, $v = 9.5$ m/s, and $R = 6.7$ m. Solve this equation for n, which is the force exerted by the roller-coaster seat on the rider. To find the force exerted by the seat belt on the rider, make a free-body diagram of the seat belt to find the force exerted by the rider on the seat belt, and then use Newton's third law to find the force exerted by the seat belt on the rider.

EVALUATE **(a)** From Newton's second law, the seat exerts a force

$$n = \frac{mv^2}{R} - mg = (55 \text{ kg})\left[\frac{(9.5 \text{ m/s})^2}{6.7 \text{ m}} - (9.8 \text{ m/s}^2)\right] = 201.9 \text{ N}$$

on the rider. As indicated in Figure 5.16, the normal force is oriented downward, accelerating the rider toward Earth.

(b) A free-body diagram of the seat belt is exactly like that for the rider (see Figure 5.16), but the mass of the seat belt is different. Therefore, Newton's second law applied to the seat belt gives

$$F_{net} = n_{R-SB} + m_{SB}g = m_{SB}a = \frac{m_{SB}v^2}{R}$$

where $n_{R\text{-}SB}$ is now the force exerted by the rider on the seat belt and m_{SB} is the (unknown) mass of the seat belt. From Newton's third law, we know that the force exerted by the seat belt on the rider has the same magnitude as $n_{R\text{-}SB}$, but is oriented in the opposite direction. Therefore,

$$n_{SB-R} = -\frac{m_{SB}v^2}{R}$$

The negative sign means that this force is oriented upward, as expected. Without knowledge of the mass of the seat belt, we cannot find the force n_{SB-R}, but we can see that this force is simply the force needed to give the seat belt the same centripetal acceleration $a = v^2 / R$ as the rider.

(c) If the rider unbuckles at this point, nothing would happen because the seatbelt does not enter into Newton's second law in part (a). Thus, the rider would remain in his seat.

ASSESS   Because the seat pushes down on the rider with the normal force, the rider's centripetal acceleration a toward Earth is greater than that due to gravity; $a > g$. This can be verified by calculating the centripetal acceleration, which gives $a = v^2 / R = (9.7 \text{ m/s})^2 / (6.7 \text{ m}) = 14 \text{ m/s}^2 > g$. If the normal force were to go to zero, the rider's acceleration due to gravity g would be greater than the centripetal acceleration of the roller coaster, and the rider would fall out of his seat (unless the seat belt were there to restrain him).

46.   **INTERPRET**   This problem involves uniform circular motion and Newton's second law. The object of interest is the skater, and the three forces involved are gravity $w = mg$, the normal force n, and static friction f_s. The physical quantities of interest are the vertical and horizontal components of the force exerted on the blade as the skater makes a circular turn.

DEVELOP   Draw a free-body diagram for the skater (see figure below). By applying Newton's second law $\vec{F}_{net} = m\vec{a}$, we find the (vector) equation of motion for the skater: $\vec{w} + \vec{n} + \vec{f}_k = m\vec{a}$, which can be decomposed into one equation for the vertical direction and one for the horizontal direction. Assuming the skater executes uniform circular motion, her acceleration will be given by Equation 5.1, $a = mv^2 / r$. By analyzing the force acting on the skater, we can find the maximum angle at which she can lean without falling over.
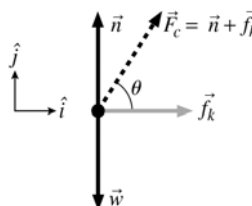
EVALUATE   **(a)** By equating the x- and y-components of the vector equation of motion, we have

$$x: f_s = ma_x = \frac{mv^2}{r}$$
$$y: n - mg = ma_y = 0$$

Thus, the vertical force exerted on her skate is upward and is equal to her weight,

$$\vec{n} = (mg)\hat{j} = (46 \text{ kg})(9.8 \text{ m/s}^2)\hat{j} = (450.8 \text{ N})\hat{j}$$

The horizontal force on her skate, which causes the centripetal acceleration, is

$$f_s = \frac{mv^2}{r}\hat{i} = \frac{(46 \text{ kg})(6.1 \text{ m/s})^2}{5.0 \text{ m}}\hat{i} = (342 \text{ N})\hat{i}$$

(b) Stability requires that the center of gravity of the skater be along the line of action of the contact force $\vec{F}_C$. Therefore, she should lean at an angle

$$\theta = \text{atan}\left(\frac{n}{f_k}\right) = \text{atan}\left(\frac{450.8 \text{ N}}{342 \text{ N}}\right) = 52.8°$$

above the horizontal.

ASSESS   We see that friction provides the centripetal force for circular motion. Without friction, the skater would never be able to turn. Note that the skate experiences significant friction perpendicular to the skate, and very little friction parallel to the skate.

47.   **INTERPRET**   This problem involves applying Newton's second law to an object that is executing uniform circular motion. The object of interest is the plane, and we are to find the minimum turning radius it can make, given the maximum angle at which it can bank.

DEVELOP Apply Newton's second law $\vec{F}_{net} = m\vec{a}$ to the plane, as sketched in Figure 5.33. In component form, this gives

$$F_w \cos(\theta) - mg = ma_y = 0$$

$$F_w \sin(\theta) = ma_x = \frac{mv^2}{r}$$

where the subscripts x and y indicate the horizontal (to the left) and upward directions, respectively. Note that the acceleration in the horizontal direction is centripetal acceleration given by Equation 5.1, because the plane is executing uniform circular motion. Given the maximum bank angle is $\theta = 35°$ and the speed is $v = 960$ km / h $= 266.67$ m / s, solve for the turning radius r.

EVALUATE The minimum turning radius r is

$$F_w \cos(\theta) = mg \implies F_w = \frac{mg}{\cos\theta}$$

$$F_w \sin(\theta) = \frac{mv^2}{r}$$

$$r = \frac{mv^2 \cos(\theta)}{mg\sin(\theta)} = \frac{v^2}{g\tan(\theta)} = \frac{(266.67 \text{ m / s})^2}{(9.8 \text{ m / s}^2)\tan(35°)} = 10363.23 \text{ m} = 10.4 \text{ km}$$

where we report the result to two significant figures.

ASSESS This may seem like a rather wide turn—but the speed of the plane is high and the bank angle is low. These are actually reasonable values for a passenger jet. Will the passengers' coffee spill from their cups during such a turn?

48. **INTERPRET** This problem involves uniform circular motion and Newton's second law. The object of interest is the water in the bucket. The forces acting on the water are gravity w and the normal force n of the bucket. We are asked to find the minimum speed at which the water will remain in the bucket.

DEVELOP We will consider the bucket at the top of its trajectory, because this is the point at which the normal force has a zero upward component to counteract the downward pull of gravity; so it is the point at which the water would be most susceptible to falling out. Draw a free-body diagram of the bucket at the top of the trajectory (see Figure 5.16 for a suitable diagram). Applying Newton's second law (for constant mass) to the water gives $n + Fg = ma = mv^2/r$. Solve this equation for the speed; then, consider that if the bucket loses contact with the water, it cannot exert a normal force on the water, so $n = 0$ (see Problem 39 for a discussion of a similar question).

EVALUATE Solving the above equation for speed, we obtain $v = \pm\sqrt{gr + nr/m}$. The positive and negative speeds correspond to the two directions in which the bucket may turn. Without loss of generality, we choose to work with the positive speed. The minimum possible speed corresponds to the situation in which the water just loses contact with the bucket, so $n = 0$. Inserting $n = 0$ into the expression for the speed gives

$$v_{min} = \sqrt{gr} = \sqrt{(9.8 \text{ m / s}^2)(0.60 \text{ m})} = 2.4 \text{ m / s}$$

ASSESS With this minimum speed, the normal force vanishes at the top of the trajectory, and gravity alone provides the centripetal force that keeps the water moving in its circular path. If the bucket's speed is too slow (less than v_{min}), then the acceleration of the water due to gravity will exceed the centripetal acceleration of the bucket, and the water will accelerate downward faster than the bucket (i.e., the water will spill out).

49. **INTERPRET** This problem involves Newton's second law. The objects of interest are the child and sled, and we are to find the frictional force needed for them to have zero acceleration.

DEVELOP Draw a free-body diagram of the situation (see figure below). Note that the frictional force is drawn so that it opposes the motion (although this is not necessary—were we to draw it in the opposite direction, we would find a negative friction force). If the child does not accelerate, Newton's second law gives $\vec{F}_{net} = 0$. In component form, this is

$$n - mg \cos(\theta) = 0$$
$$mg \sin(\theta) - f_k = 0$$

where we have used $w = mg$. From Equation 5.3, we know that $f_k = \mu_k n$, so we can solve this system of equations for the coefficient of kinetic friction μ_k.
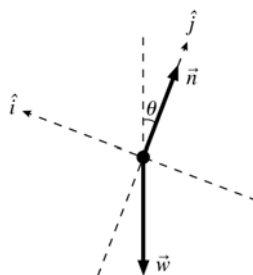
EVALUATE   Inserting the expression for f_k and using the first equation to eliminate the normal force gives

$$mg\sin(\theta) - \mu_k mg\cos(\theta) = 0$$
$$\mu_k = \tan(\theta) = \tan(6.3°) = 0.11$$

ASSESS   If the coefficient of kinetic friction does not depend on the speed of the sled, then the child will slide down the hill at her initial speed. Thus, if she takes a running start, she will continue at the speed at which she runs. If she sits on her sled with no running start, she will remain stationary. In both situations, she experiences no acceleration.

50.   **INTERPRET**   This problem involves Newton's second law in a situation with zero acceleration. The object of interest is the mower, and the forces involved are the weight w due to gravity, the normal force n, the force f_k due to friction, and the force F applied by the gardener at an angle of 37° with the horizontal. We are asked to find the force F required to produce a (nonzero) constant velocity.

DEVELOP   Draw a free-body diagram of the mower (see figure below), assuming the ground is horizontal. Newton's second law applied to the mower gives

$$\vec{F}_{net} = \vec{F} + m\vec{g} + \vec{n} + \vec{f}_k = m\vec{a} = 0$$

because the acceleration is zero. In component form, this vector equation yields two scalar equations, one for the vertical direction and one for the horizontal direction. These are

$$x : F\cos(\theta) - f_k = 0$$
$$y : n - F\sin(\theta) - mg = 0$$

which we can solve for F, given that $f_k = \mu_k n$ (Equation 5.3).
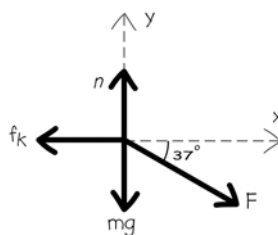
EVALUATE   Substituting the expression for f_k into the x-equation and solving for F gives

$$F = \frac{\mu_k mg}{\cos(\theta) - \mu_k\sin(\theta)} = \frac{(0.68)(26\ \text{kg})(9.8\ \text{m/s}^2)}{\cos(37°) - (0.68)\sin(37°)} = 444.9\ \text{N} \approx 445\ \text{N}$$

to two significant figures. Compared with the weight of the mower, we find

$$\frac{F}{mg} = \frac{\mu_k}{\cos(\theta) - \mu_k\sin(\theta)} = \frac{0.68}{\cos(37°) - (0.68)\sin(37°)} = 1.7$$

So, the force applied is 70% more than the weight of the mower.

ASSESS   To show that our result makes sense, let's check some limiting cases. First, if $\mu_k = 0$, our expression gives $F = 0$. This is reasonable because without friction, no force needs to be applied to keep the mower moving at constant speed. However, if $\theta = 0$, we get $F = \mu_k mg$. In this limit, the normal force is simply $n = mg$, and the

friction force is $f_k = \mu_k n = \mu_k mg$. Thus, the force applied has the same magnitude as friction, but points in the opposite direction: $\vec{F} = -\vec{f}_k$.

51.   **INTERPRET**   The interpretation of the problem is the same as that given in Example 5.4, except that the rock now has a fourth force acting on it, which is the force due to kinetic friction.

DEVELOP   Because friction always acts to oppose motion, the force f_k due to kinetic friction acting on the rock will be oriented in the negative x-direction. With the addition of the friction force, Newton's second law applied to the rock gives $\vec{T}_r + \vec{F}_{gr} + \vec{n} + \vec{f}_k = m_r \vec{a}_r$. Writing this vector equation in component form gives

$$x: \quad T - \mu_k n = m_r a$$
$$y: \quad n - m_r g = 0$$

where we have inserted a negative sign for the friction force and have used the fact that the rock does not accelerate in the y-direction and that the force due to gravity on the rock is $F_{gr} = m_r g$. Applying Newton's law to the climber gives the same result as in Example 5.4:

$$T - m_c g = -m_c a$$

Note that the accelerations of the climber and the rock have the same magnitude, although they act in different directions. This is so because the rope is considered to not stretch, so both objects must move at the same rate.

EVALUATE   Solve the three equations above for the acceleration, which gives

$$m_c g - m_r a - \mu_k m_r g = m_c a$$

$$a = g\left(\frac{m_c - \mu_k m_r}{m_c + m_r}\right) = (9.8 \text{ m}/\text{s}^2)\left[\frac{73 \text{ kg} - (0.049)(940 \text{ kg})}{73 \text{ kg} + 940 \text{ kg}}\right] = 0.26 \text{ m}/\text{s}^2$$

Inserting this result into the kinematic Equation 2.10 gives $t = \sqrt{2(51\text{ m})/(0.26 \text{ m/s}^2)} = 20 \text{ s}$.

ASSESS   Notice that our expression for the acceleration reverts to that found in Example 5.4 if we let $\mu_k \to 0$.

52.   **INTERPRET**   This problem involves Newton's second law. The object of interest is the bat, and the forces involved are gravity $\vec{F}_g$, the normal force $\vec{n}$, and the force due to static friction $\vec{f}_s$. We are asked to find the minimum acceleration of the train that prevents the bat from sliding down the window.

DEVELOP   Draw a free-body diagram for the situation (see figure below). Using Newton's second law, the force equation for the bat is $\vec{F}_{net} = \vec{F}_g + \vec{n} + \vec{f}_s = m\vec{a}$. This can be decomposed into two equations, one for the y-direction and one for the x-direction:

$$x: \quad n = ma$$
$$y: \quad f_s - mg = 0$$

The force due to static friction is $f_s \le \mu_s n$, which we can use in the equations above to find the minimum acceleration.
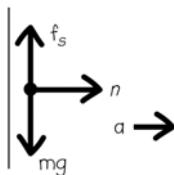
EVALUATE   Inserting the inequality for static friction into Newton's second law gives

$$mg = f_s \le \mu_s n$$

$$mg \le \mu_s ma$$

$$a \ge \frac{g}{\mu_s} = \frac{9.8 \text{ m}/\text{s}^2}{0.84} = 11.67 \text{ m}/\text{s}^2$$

ASSESS   The minimum acceleration is inversely proportional to μ_s, which is the coefficient of static friction between the bat and the train. The smaller the value of μ_s, the greater is the acceleration needed to keep the bat in

place. In the limit $\mu_s \to 0$ (frictionless surface), the acceleration would have to be infinitely large. However, for $\mu_s \to \infty$ (infinitely sticky surface), we get $a \to 0$.

53. **INTERPRET**   This problem involves Newton's second law and kinematics. The object of interest is the train, and we are asked to find if the train can stop within 150 m if the wheels maintain static contact with the rails (i.e., the wheels do not skid on the rails).

    DEVELOP   Considering all the wheels as single point of contact, make a free-body diagram for the train (see figure below). Applying Newton's law to the train wheels gives $\vec{F}_{net} = \vec{f}_s + \vec{n} + \vec{F}_g = m\vec{a}$, and writing this in component form gives

    $$x: \quad F_g + n = 0$$
    $$y: \quad -f_s = -ma$$

    where we have used the fact that there is zero acceleration in the x-direction and we have explicitly noted the sign of the friction force and the acceleration to emphasize that they are in the same direction (negative x-direction). The force due to static friction is $f_s \leq \mu_s n$ and the force due to gravity is $F_g = -mg$ (because gravity acts in the downward direction). Insert these values into the above equations to find the maximum acceleration possible without having the wheels slip on the rails, then use the kinematic Equation 2.11 $v^2 = v_0^2 + 2a(x - x_0)$ to find the stopping distance.
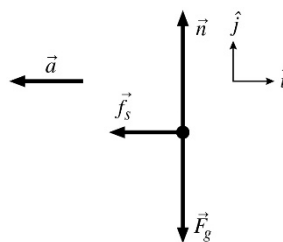
    EVALUATE   Newton's second law thus gives

    $$ma = f_s \leq \mu_s n = \mu_s mg$$
    $$a \leq \mu_s g$$

    so the maximum acceleration possible is $\mu_s g$. Inserting this result for the acceleration into Equation 2.11 gives a stopping distance of

    $$\overset{=0}{v^2} = v_0^2 + 2a(x - x_0)$$

    $$x - x_0 = \frac{v_0^2}{2a} = \frac{v_0^2}{2\mu_s g} = \frac{(140 \text{ km/h})^2}{2(0.58)(9.8 \text{ m/s}^2)}\left(\frac{10^3 \text{ m}}{\text{km}}\right)^2\left(\frac{\text{h}}{3600 \text{ s}}\right)^2 = 130 \text{ m}$$

    so the train will stop before hitting the car.

    ASSESS   The stopping time for the train is

    $$x - x_0 = (v_0 + v)t/2$$
    $$t = \frac{2(x - x_0)}{v_0} = \frac{2(133 \text{ m})}{38.9 \text{ m/s}} = 6.8 \text{ s}$$

   which should be just enough time for the passengers to get out of the car.

54. **INTERPRET**   This problem involves uniform circular motion (assuming the bug walks at a speed that is much, much less than the tangential speed of the CD at the bug's position) and Newton's second law. The object of interest is the bug, and the forces acting on it are gravity $\vec{F}_g$, the normal force $\vec{n}$, and static friction $\vec{f}_s$. We want to find out how far the bug gets from the center before it begins to slip.

    DEVELOP   Draw a free-body diagram of the bug as seen from the side (see figure below). We have chosen the x-direction to be away from the center of the CD, so the centripetal acceleration a is oriented in the negative x-direction. Using Newton's second law, the force equation for the bug is $\vec{F}_{net} = \vec{F}_g + \vec{n} + \vec{f}_s = m\vec{a}$. Assume that the

disc is level. Assuming uniform circular motion (see comment in Interpret), we can use Equation 5.1, $a = v^2/r$. Writing Newton's second law in component form thus gives:

$$x: -f_s = -\frac{mv^2(r)}{r}$$
$$y: n = mg$$

The force due to static friction is $f_s \le \mu_s n$, and the speed of the disc as a function of the bug's radial position and the frequency of revolution $(\phi = 200 \text{ s}^{-1})$ is $v(r) = 2\pi r \phi$; so we can find the distance r at which static friction can no longer supply the necessary acceleration (i.e., the bug starts to slip).
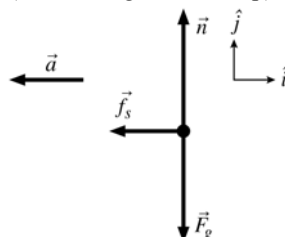
**EVALUATE**   Inserting the given quantities into Newton's second law gives

$$\frac{mv^2}{r} = f_s \le \mu_s n$$

$$\frac{m(2\pi r \phi)^2}{r} \le \mu_s mg$$

$$r \le \frac{\mu_s g}{4\pi^2 \phi^2} = \frac{1.3(9.8 \text{ m/s}^2)}{4\pi^2 (200 \text{ min}^{-1})^2} \left(\frac{60 \text{ s}}{1 \text{ min}}\right)^2 = 0.029 \text{ m}$$

Thus, the radial distance traveled by the bug before slipping occurs is 2.9 cm.

ASSESS   Our result indicates that the larger the friction coefficient, the greater the distance the bug can travel. This makes sense because friction is what produces the centripetal force. In the limit $\mu_s \to 0$ (frictionless surface), the bug would not be able to move at all $(r \to 0)$. On the other hand, if the surface of the disc is very sticky (large μ_s), we would then expect the distance traveled by the bug to be very large.

55.   **INTERPRET**   This problem involves kinematics, Newton's second law, and frictional forces. We are given information to find the acceleration of the textbook, and are asked to calculate what coefficient of static friction is necessary to keep the paperback book stuck to the textbook during the acceleration, and the maximum coefficient of static friction possible that would still let the paperback book slide off the textbook during deceleration.

DEVELOP   Draw a free-body diagram for the paperback book for both the case of acceleration a_1 and deceleration a_2 (see figure below). The accelerations can be calculated using the kinematic Equation 2.7 $v = v_0 + at$. This gives $\vec{a}_1 = \vec{v}/t = (0.96 \text{ m/s})/(0.42 \text{ s})\hat{i} = (2.286 \text{ m/s}^2)\hat{i}$ and $\vec{a}_2 = -\vec{v}_0/t = -(0.96 \text{ m/s})/(0.33 \text{ s}) = -(2.909 \text{ m/s}^2)\hat{i}$. Apply Newton's second law to the paperback book in both situations to find the coefficient of static friction.
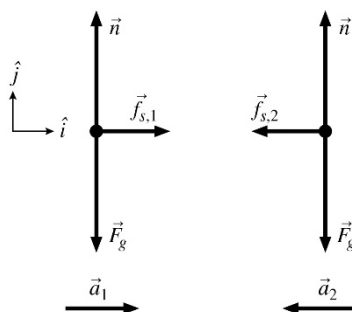
EVALUATE   When the paperback book accelerates, Newton's second law (in component form) gives

$$ma_1 = f_{s,1} \le \mu_{s,1} n \Big\} \quad \mu_{s,1} \ge \frac{a_1}{g} = \frac{2.286 \text{ m/s}^2}{9.8 \text{ m/s}^2} = 0.23$$
$$n - mg = 0$$

When the paperback book decelerates, we have

$$ma_2 > f_{s,2}^{\max} = \mu_{s,2} n \Big\} \quad \mu_{s,2} = \frac{a_2}{g} = \frac{0.33 \text{ m/s}^2}{9.8 \text{ m/s}^2} = 0.30$$
$$n - mg = 0$$

Thus, the actual coefficient of static friction must lie in the range $0.23 \le \mu_s \le 0.30$.

ASSESS   Notice that the force due to friction acts to oppose any potential velocity of the paperback book with respect to the reference book.

56.   **INTERPRET**   This problem involves Newton's second law. The object of interest is a child on a sled. Three forces act on this object: gravity $\vec{F}_g$, the normal force $\vec{n}$, and the force $\vec{f}_k$ from kinetic friction. By analyzing the forces acting on the child while he travels down the hill and using kinematics, we can determine the child's acceleration a and his speed v when he reaches level ground. Subsequently, we can solve the one-dimensional kinematics problem to determine the distance he slides before coming to a stop.

DEVELOP   Draw a free-body diagram for the sled on the slope and on level ground (see figure below), and apply Newton's second law in both cases to find the acceleration of the sled. Using Equation 5.3 $f_k = \mu_k n$, this gives

$$x: \quad mg\sin(\theta) - f_k = ma_{\text{slope}} \Big\} \quad a_{\text{slope}} = g\left[\sin(\theta) - \mu_k \cos(\theta)\right]$$
$$y: \quad n - mg\cos(\theta) = 0$$

for the acceleration on the slope. Repeating the calculation for level ground gives

$$x: \quad f_k = ma_{\text{level}} \Big\} \quad a_{\text{level}} = \mu_k g$$
$$y: \quad n - mg = 0$$

Use these results for acceleration in the kinematic Equation 2.11 $v^2 = v_0^2 + 2a(x - x_0)$ to find the speed at the bottom of the hill, then insert this result into the same equation (but as the initial speed this time) to find the distance traveled on level ground.

Slope   Level ground
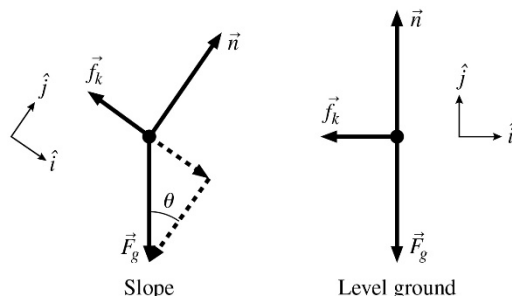
EVALUATE   Using Equation 2.11, the speed of the child at the bottom of the hill is

$$v = \pm\sqrt{v_0^2 + 2a(x - x_0)} = \pm\sqrt{0 + 2g\left[\sin(\theta) - \mu_k \cos(\theta)\right](x - x_0)}$$
$$= \pm\sqrt{2(9.8 \text{ m/s}^2)\left[\sin(25°) - 0.12\cos(25°)\right](41 \text{ m})} = 15.88 \text{ m/s}$$

where we have chosen the positive square root arbitrarily with no loss of generality (the negative answer simply corresponds to motion in the opposite direction). Now, on level ground the distance traveled before stopping is

$$x - x_0 = \frac{v^2 - v_0^2}{2a_{\text{level}}} = \frac{-v_0^2}{2(-\mu_k g)} = \frac{-(15.9 \text{ m/s})^2}{-2(0.12)(9.8 \text{ m/s}^2)} = 110 \text{ m}$$

where we have used the final speed calculated for the slope as the initial speed for the level ground, and we have used $a_{\text{level}} = -\mu_k g$ because the acceleration is in the negative x-direction.

ASSESS   After traveling 41 m down the hill, the speed of the child at the bottom of the hill is 15.9 m/s, or roughly 36 mi/h! So sliding 110 m on the level ground before coming to a complete stop sounds reasonable.

57. **INTERPRET** This problem involves Newton's second law. We are asked to find the maximum acceleration of a front-wheel-drive car, given that 70% of its mass rests on the front wheels.

   DEVELOP Draw a free-body diagram for the combined front wheels of the car, which will be the same diagram as for the sled on level ground in the preceding problem. Note that the mass resting on the front wheels is $m_f = 0.7m$, so $F_g = 0.7mg$. The mass entering into Newton's second law, however, is m, because the entire car is being accelerated. Applying Newton's second law to the combined front-wheel system gives

   $$x: \quad f_s = ma$$
   $$y: \quad n - 0.7mg = 0$$

   EVALUATE Using Equation 5.2 $f_s \leq \mu_s n$ in Newton's second law gives

   $$a = \frac{f_s}{m} \leq \frac{\mu_s n}{m} = \frac{\mu_s(0.7mg)}{m}$$
   $$a \leq 0.7\mu_s g = 0.7(0.61)(9.8 \text{ m/s}^2) = 4.2 \text{ m/s}^2$$

   Thus, the car's maximum acceleration is 4.2 m/s^2.

   ASSESS Note that as the car accelerates, the proportion of weight carried by the back wheels increases. To take advantage of this effect, some people drive backward in front-wheel-drive cars when the road is very slick.

58. **INTERPRET** This problem involves Newton's second law, the kinetic friction force, and kinematics. The object of interest is the car, and the three forces involved are gravity $\vec{F}_g$, the normal force, and the force due to kinetic friction $\vec{f}_k$. By analyzing the force acting on the car, we can solve the one-dimensional kinematics problem to determine its initial speed.

   DEVELOP On a level road, the acceleration of a skidding car is $a = -\mu_k g$ (see Example 5.8), where the negative sign indicates that the acceleration is opposite to the velocity. Given that the final speed of the car (i.e., just before hitting the stationary car) is $v = 24 \text{ km/h} = 6.67 \text{ m/s}$, we can solve Equation 2.11, $v^2 = v_0^2 + 2a(x - x_0)$, to find the car's initial speed.

   EVALUATE Using $a = -\mu_k g$ in Equation 2.11, we have

   $$v_0 = \sqrt{v^2 - 2a(x - x_0)} = \sqrt{(6.67 \text{ m/s})^2 + 2(0.71)(9.8 \text{ m/s}^2)(47 \text{ m})} = 26.4 \text{ m/s} = 95.1 \text{ km/h} = 59.1 \text{ mi/h}$$

   ASSESS Note that friction is what causes the acceleration.

59. **INTERPRET** This problem involves Newton's second law, kinematics, and the force due to kinetic friction. The object of interest is a swimmer who slides down the slide, and we are asked to find the coefficient of kinetic friction given the relative time it takes a swimmer to slide down the slide when it's wet and when it's dry.

   DEVELOP The free-body diagram for this problem is the same as that for the sled on the slope in Problem 5.50. Apply Newton's second law to a swimmer sliding down the dry slide ($\mu_k \neq 0$) to find the acceleration, and then let $\mu_k \to 0$ to find the acceleration for the wet slide. This gives

   $$\left. \begin{array}{l} x: \quad mg\sin(\theta) - f_k = ma \\ y: \quad n - mg\cos(\theta) = 0 \end{array} \right\} a = g\left[\sin(\theta) - \mu_k \cos(\theta)\right]$$

   To find the coefficient of kinetic friction, use the kinematic Equation 2.10 $x = x_0 + v_0 t + at^2/2$ to relate the acceleration to the time it takes to travel down the slide with $v_0 = 0$.

   EVALUATE The time it takes to slide down the dry slide is

   $$x = x_0 + \overset{=0}{\cancel{v_0}} t + at^2/2$$
   $$t_{dry} = \pm\sqrt{\frac{2(x - x_0)}{a}} = \pm\sqrt{\frac{2(x - x_0)}{g\left[\sin(\theta) - \mu_k \cos(\theta)\right]}}$$

   Letting $\mu_k \to 0$, we get the time it takes to descend the wet slide:

   $$t_{wet} = \pm\sqrt{\frac{2(x - x_0)}{g\sin(\theta)}} = t_{dry}\sqrt{1 - \mu_k \cot(\theta)}$$

We are told that $t_{wet}/t_{dry} = 1/3$, so

$$\frac{t_{wet}}{t_{dry}} = \frac{1}{3} = \sqrt{1 - \mu_k \cot(\theta)}$$

$$\mu_k = \frac{8 \tan(35°)}{9} = 0.62$$

ASSESS   Notice that we did not need to evaluate the acceleration, we simply needed to find that the ratio of the accelerations is $1 - \mu_k \cot(\theta)$.

60.   **INTERPRET**   This is a problem involving Newton's second law with zero acceleration. The object of interest is the trunk, and the forces involved are gravity $\vec{F}_g$, the normal force $\vec{n}$, friction $\vec{f}_s$, and the applied force $\vec{F}_a$. We need to show that if the coefficient of static friction exceeds the given value, the trunk won't move no matter how hard you push.

DEVELOP   Draw a free-body diagram of the situation (see figure below). Applying Newton's second law to the trunk gives $\vec{F}_{net} = \vec{F}_a + \vec{F}_g + \vec{n} + \vec{f}_s = m\vec{a}$, which in component form gives

$$x: \quad F_a \cos(\theta) - f_s = 0$$
$$y: \quad n - mg - F_a \sin(\theta) = 0$$

From Equation 5.2, we know $f_s \leq \mu_s n = \mu_s \left[mg - F_a \sin(\theta) \right]$, which allows us to find an expression for the applied force F_a.
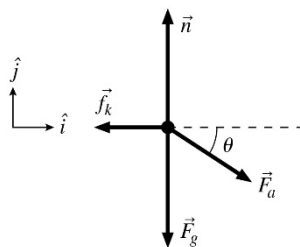
EVALUATE   Substituting the expression for f_s back to the x-component of Newton's second law and solving for F_a we obtain $F_a \cos(\theta) - \mu_s \left[F_a \sin(\theta) + mg \right] = 0$ or

$$F_a = \frac{\mu_s mg}{\cos(\theta) - \mu_s \sin(\theta)}$$

If $\cos(\theta) - \mu_s \sin(\theta) < 0$ then the applied force F_a will become negative, which means that the trunk will not budge and remain in equilibrium. Thus, the equilibrium condition will always be satisfied if

$$\mu_s > \cot(\theta) = \cot(50°) = 0.84$$

ASSESS   As the coefficient of friction increases, we must apply a greater force in order to move the trunk. However, because the applied force has a downward component, this adds to the normal force and results in a greater friction force. Obviously when the force is applied vertically downward, it's impossible to move the trunk. On the other hand, if $\theta = 0$, then we get $F_a = \mu_s mg$. In this limit the normal force is simply $n = mg$, and the friction force is $f_s = \mu_s n = \mu_s mg$. Thus, the applied force has the same magnitude as the friction, but points in the opposite direction: $F_a = -f_s$.

61.   **INTERPRET**   This problem involves Newton's second law and kinematics. The object of interest is the box, and the forces involved are gravity $\vec{F}_g$, the normal force $\vec{n}$, and static and kinetic friction $\vec{f}_k$ and $\vec{f}_s$. We are asked to find how far up a slope the box will travel given its initial speed and coefficient of kinetic friction.

DEVELOP   Draw a free-body diagram for the situation (see figure below). Use Newton's second law to find the acceleration of the block as it travels up the slope, then insert the result into the kinematic Equation 2.11 $v^2 = v_0^2 + 2a(x - x_0)$ to find the distance the box travels up the slope.
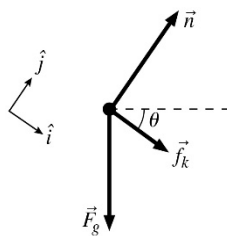
EVALUATE (a) In component form, Newton's second law gives

$$x: \quad f_k + mg\sin(\theta) = ma \atop y: \quad n - mg\cos(\theta) = 0 \Bigg\} a = g\sin(\theta) + \mu_k g\cos(\theta)$$

where we have used $F_g = mg$ and Equation 5.3 $f_k = \mu_k n$ for the force due to kinetic friction. Inserting this result into Equation 2.11 and solving for the distance $x - x_0$ gives

$$v^2 = \overset{=0}{\cancel{v_0^2}} + 2a(x - x_0)$$

$$x_0 - x = \frac{v_0^2}{2a} = \frac{v_0^2}{2\left[g\sin(\theta) + \mu_k g\cos(\theta)\right]} = \frac{(1.4 \text{ m/s})^2}{2(9.8 \text{ m/s}^2)\left[\sin(22°) + (0.70)\cos(22°)\right]} = 0.098 \text{ m} = 9.8 \text{ cm}$$

Thus, the block travels 10 cm up the slope.

(b) When the block has stopped, Newton's second law still applies, but with the force due to static friction, which will be oriented up the incline to resist motion down the incline. Thus, Newton's second law gives us

$$x: \quad -f_s + mg\sin(\theta) = ma \atop y: \quad n - mg\cos(\theta) = 0 \Bigg\} a \ge g\sin(\theta) - \mu_k g\cos(\theta)$$

where we have used Equation 5.2 $f_s \le \mu_s n$. The acceleration will be positive if $\mu_s < \tan(\theta) = \tan(22°) = 0.404$. However, this value is less than that for kinetic friction, so it is likely that μ_s exceeds this value, because $\mu_s > \mu_k$ is generally true. Thus, we conclude that the block does not slide back down the slope.

ASSESS Notice how the direction of the frictional force depends on the circumstances: to resist the motion it was oriented down the incline in part (a) and up the incline in part (b).

62. **INTERPRET** This problem involves Newton's second law, and similarly to Example 5.5, we have gravity and tension acting on the ball as it travels in a horizontal circle. Here we are instead given the angle the string makes with the vertical, but we can treat the problem in the same way

    DEVELOP To describe the motion we can develop a free-body diagram much like the one shown in Example 5.5, but considering that the angle θ we are given is complementary to the one shown in Figure 5.12 from the textbook. We still have the force of gravity $F_g = mg$ acting in the negative y-direction, and two components of the tension T along x and y.

    EVALUATE We can write the net forces along each direction as

    $$x: \quad T\sin\theta = ma_x = m\frac{v^2}{L\sin\theta}$$

    $$y: \quad T\cos\theta - mg = 0$$

    Where we have taken $a_x = v^2/r$, $a_y = 0$, and $r = L\sin\theta$. Solving for the tension from the y-equation, plugging into the x-equation to solve for the speed v, and evaluating at the given angle $\theta = \pi/6$ gives

$$T = \frac{mg}{\cos\theta}$$

$$\left(\frac{mg}{\cos\theta}\right)\sin\theta = m\frac{v^2}{L\sin\theta}$$

$$v = \sqrt{gL\sin\theta\tan\theta} = \sqrt{gL(1/2)(1/\sqrt{3})} = \sqrt{gL/2\sqrt{3}}$$

ASSESS   If we were to plug the complementary angle $\pi/2 - \theta$ into our expression for v and simplify the trigonometric functions using the identities found in Appendix A we will recover the expression derived in Example 5.5.

63.   **INTERPRET**   Your speed can be determined by assuming uniform circular motion through the turn. Because the curve is banked, the centripetal force is provided by the normal force, but there may also be a contribution from the friction between your tires and the road.

DEVELOP   Example 5.6 illustrates that a curve designed for a certain speed must be banked at an angle given by:

$$\tan\theta = \frac{v_d^2}{gr} = \frac{(21.11\,\text{m/s})}{(9.8\,\text{m/s}^2)(230\,\text{m})} = 0.198$$

In this case, however, we investigate what would have happened if you had gone faster than the designed speed. To stay in your lane (at the same radius), you would need to turn the steering wheel slightly, causing some friction parallel to the road and perpendicular to the car's motion. The three forces acting on the car are represented in the figure below.
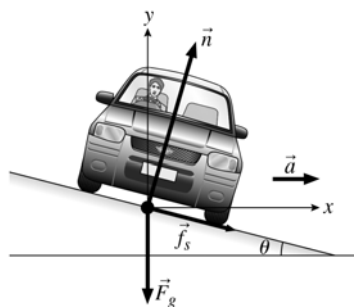
Notice that we have chosen the coordinate axes so that the centripetal acceleration, $\vec{a} = (v^2/r)\hat{r}$, points in the $+x$-direction. The sums of the forces for the two components are

$$x:\quad n\sin\theta + f_s\cos\theta = mv^2/r$$
$$y:\quad n\cos\theta - f_s\sin\theta - F_g = 0$$

The friction has a maximum limit: $f_s \le \mu_s n$. We will use this to find the maximum limit on your speed.

EVALUATE   We don't know the mass of the car, but we can combine the two component equations to eliminate the mass. With some algebra, we arrive at:

$$\frac{f_s}{n} = \frac{\dfrac{v^2}{gr}\cos\theta - \sin\theta}{\cos\theta + \dfrac{v^2}{gr}\sin\theta} \le \mu_s$$

Dividing the numerator and denominator by $\cos\theta$, and using $\tan\theta = v_d^2/gr$, we get

$$\frac{v^2}{gr} - \frac{v_d^2}{gr} \le \mu_s\left(1 + \frac{v^2 v_d^2}{(gr)^2}\right)$$

With a little more algebra and using the value of v_d^2/gr from above, we obtain

$$v \le v_d \sqrt{\frac{1 + \mu_s \frac{gr}{v_d^2}}{1 - \mu_s \frac{v_d^2}{gr}}} = \left(76 \text{ km/h}\right)\sqrt{\frac{1 + 0.20 / 0.198}{1 - 0.20 \cdot 0.198}} = \left(76 \text{ km/h}\right)\left(1.43\right) = 110 \text{ km/h}$$

So you could have been speeding around the curve by as much as 34 km/h over the posted limit.

ASSESS   We can check whether our final expression for the maximum speed makes sense. If $\mu_s = 0.2$ (characteristic of a dry road), we get $v \le 200 \text{ km/h}$. If $\mu_s = 0$, our expression becomes $v \le v_d$, which just says that we must drive at the posted speed if the road is frictionless. If the road is not banked, then $v_d \to 0$, and our expression reduces to $v \le \sqrt{\mu_s gr}$, which agrees with our derivation for the unbanked curve in chapter 3.

64.   **INTERPRET**   This problem involves uniform circular motion. The object of interest is the astronaut, and we are to find the frequency of revolution that would lead to a centripetal acceleration equal in magnitude to the acceleration due to gravity at the surface of the Earth.

DEVELOP   Because this is uniform circular motion (i.e., the space station rotates at a constant rate), we can apply Equation 5.1 $a = v^2/r$ to find the centripetal acceleration, with $r = (450 \text{ m})/2 = 225 \text{ m}$. To relate the tangential velocity v to the parameters of rotation, note that the velocity is the frequency of rotation ϕ multiplied by the circumference $C = 2\pi r$, which gives $v = 2\pi r \phi$.

EVALUATE   Equating the centripetal acceleration to the Earth's gravity gives

$$g = \frac{v^2}{r} = \frac{(2\pi r\phi)^2}{r} = 4\pi^2 r\phi^2$$

$$\phi = \pm\frac{1}{2\pi}\sqrt{\frac{g}{r}} = \pm\frac{1}{2\pi}\sqrt{\frac{(9.8 \text{ m/s}^2)}{225 \text{ m}}} = \left(0.0332 \text{ s}^{-1}\right)\left(\frac{60 \text{ s}}{1 \text{ min}}\right) = 2.0 \text{ min}^{-1}$$

so the satellite would have to rotate twice per minute.

ASSESS   The units work out to inverse time, as expected for a frequency. Notice that the larger the satellite radius, the lower the frequency of rotation.

65.   **INTERPRET**   This problem involves Newton's second law and centripetal acceleration. The forces involved are gravity $\vec{F}_g = m\vec{g}$ and the normal force $\vec{n}$ that is everywhere perpendicular to the track. The aim of this problem is to establish the condition under which a car moving too slowly as it goes around a loop-the-loop roller coaster would leave the track.

DEVELOP   Assume the roller coaster travels counter clockwise around the loop-the-loop. Draw a free-body diagram for the roller coaster at an arbitrary point on the track (see figure below), with the angle ϕ of departure from the track indicated. Applying Newton's second law to the roller coaster gives $\vec{F}_g + \vec{n} = m\vec{a}$. For the roller coaster to stay on the track, the radial component of the net force (toward the center of the track) must equate to the mass times the centripetal acceleration,

$$n + mg\cos(\phi) = \frac{mv^2}{r}$$

The tangential component of the net force is not of interest for this problem.
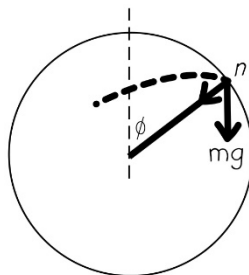
EVALUATE   The car leaves the track when the normal force becomes zero (no more contact):

$$mg\cos\phi = \frac{mv^2}{r} \quad \Rightarrow \quad \cos(\phi) = \frac{v^2}{gr}$$

which is the expression given in the problem statement.

ASSESS   The result implies that the car leaves the track when the speed is too small: $v < \sqrt{gr}$. Otherwise, the car never leaves the track, as in Example 5.7.

66.   **INTERPRET**   This problem looks at friction on a banked curve.

DEVELOP   The curve has a radius R and was engineered for a speed of v_0, which from Example 5.6 means that its banking angle satisfies the equation: $\tan\theta = v_0^2 / gR$.

Let's assume that the car is going faster than the designed speed. To maintain the same circular trajectory, there must be friction between the tires and the road. See the figure in Problem 5.57 to have an idea of the forces acting on the car. As in this previous problem, the forces in the x- and y-directions obey Newton's second law:

$$x: \quad n\sin\theta + f_s\cos\theta = mv^2 / R$$
$$y: \quad n\cos\theta - f_s\sin\theta - F_g = 0$$

We will use these two equations, and the above expression for the banking angle to derive an expression for the friction coefficient.

EVALUATE   We first divide the equation of motion in the x-direction by the equation of motion in the y-direction:

$$\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta} = \frac{v^2}{gR}$$

where we have used $f_s = \mu_s n$ (Equation 5.2). Dividing the numerator and denominator on the left-hand side by $\cos\theta$ and rearranging terms gives:

$$gR(\tan\theta + \mu_s) = v^2(1 - \mu_s\tan\theta)$$

Finally, using the expression for the banking angle with the designed speed,

$$\mu_s = \frac{v^2 - v_0^2}{gR + (v^2 v_0^2 / gR)}$$

ASSESS   Notice that if the car moves at the designed speed ($v = v_0$), the coefficient of friction is zero. This makes sense, since no friction is supposed to be necessary in a banked turn when you drive at the designed speed. Also note that if $v < v_0$, the coefficient is negative, which simply reflects the fact that driving below the designed speed requires that the friction point outward with respect to the center of the curve.

67.   **INTERPRET**   This problem involves Newton's second law, uniform circular motion, and frictional forces. The forces involved are the force due to gravity $\vec{F}_g = m\vec{g}$, the normal force $\vec{n}$ that acts perpendicular to the wall, and the force due to static friction $\vec{f}_s$. We are to find the frequency of revolution needed to prevent the book from slipping down the wall.

DEVELOP   Draw a free-body diagram of the situation (see figure below). Because this is uniform circular motion (i.e., the centrifuge rotates at a constant rate), Equation 5.1, $a = v^2/r$, applies for the centripetal acceleration. The direction of this acceleration is toward the center of the circle of rotation, as indicated in the drawing. Applying Newton's second law in the vertical and horizontal directions gives

$$x: \quad n = ma = mv^2/r$$
$$y: \quad f_s - mg = 0$$

Using Equation 5.2, $f_s = \mu_s n$, in Newton's second law leads to $g/\mu_s = v^2/r$. From this expression, we can find the frequency needed to maintain the book in place.
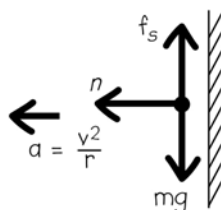
EVALUATE   The frequency of revolution ϕ is related to the tangential velocity by $v = 2\pi r\phi$. Inserting this into the expression above and solving for ϕ gives

$$\frac{g}{\mu_s} = \frac{(2\pi r\phi)^2}{r}$$

$$\phi \geq \left| \pm \sqrt{\frac{g}{4\pi^2 r\mu_s}} \right| = \sqrt{\frac{(9.8 \text{ m}/\text{s}^2)}{4\pi^2 (5.1 \text{ m})(0.72)}} = (0.26 \text{ s}^{-1})\left(\frac{60 \text{ s}}{1 \text{ min}}\right) = 16 \text{ min}^{-1}$$

Thus, the centrifuge must rotate at a frequency of $16 \text{ rev}/\text{min}$ or greater.

ASSESS   The two possible signs for the frequency correspond to clockwise and counterclockwise rotations.

68.   **INTERPRET**   This problem looks at friction on a brake pad. Knowing the force applied by the brakes we are to find the coefficient of friction for two different types of braking system.

DEVELOP   As the brakes come into contact with the disk/rim for the two different systems discussed in parts (a) and (b), they experience a reaction force equal in magnitude and opposite in direction, much like the normal force acts in reaction to the weight of an object. We can find the force of friction $f = \mu n$ using the given frictional coefficients and the magnitude of the applied force F_b since the normal force n will be equal in magnitude.

EVALUATE   For both cases we are given the applied force and the frictional coefficient, so we find the frictional forces for the disk and rim braking systems to be, respectively

$$f_d = \mu F_b = (0.51)(3.5\text{kN}) = 1.8\text{kN}$$
$$f_r = \mu F_b = (0.39)(870\text{N}) = 340\text{N}$$

ASSESS   Since the disk brakes can tolerate a much larger force from the brake pads, the resultant frictional force will be much more efficient at bringing the bicycle to a stop quickly.

69.   **INTERPRET**   This problem involves Newton's second law, uniform circular motion, and frictional forces. The object of interest is the car, and we are to find whether braking in a straight line will stop the car before it hits the truck, or whether it's better to swerve in as tight a circular turn as possible. The forces acting on the car are the force due to gravity $\vec{F}_g = m\vec{g}$ and the force due to kinetic friction $\vec{f}_k$ for the former option and the force due to static friction $\vec{f}_s$ for the latter option.

DEVELOP   For the braking option, Newton's second law applied to the car in the x- and y-directions gives

$$\left. \begin{array}{ll} x: & f_k = ma \\ y: & n - mg = 0 \end{array} \right\} \mu_s g = a ,$$

where we have used Equation 5.3 $f_s = \mu_s n$. For the swerve option, Newton's second law applied in the x- and y-directions gives

$$\left. \begin{array}{ll} x: & f_s = ma = m v^2/r \\ y: & n - mg = 0 \end{array} \right\} \mu_s g = v^2/r$$

Use the kinematic Equation 2.11 $v^2 = v_0^2 + 2a(x - x_0)$ to find the stopping distance in the braking option, and calculate the turning radius r for the swerve option. Compare these results to decide which option to take.

EVALUATE   For the braking option, the stopping distance is

$$\overset{=0}{\overbrace{v^2}} = v_0^2 + 2a(x - x_0)$$

$$x - x_0 = -\frac{v_0^2}{2a} = -\frac{v_0^2}{2(-\mu_s g)} = \frac{v_0^2}{2\mu_s g}$$

where the acceleration has a negative sign because it is oriented opposite to the velocity. For the swerving option, the turning radius is $r = v^2/\mu_s g = (x - x_0)$. Thus the turning radius is greater than the stopping distance, so you should chose to brake in a straight line rather than swerve.

ASSESS Note that if the coefficient of static friction decreases from its maximum value of μ_s, the turning radius will get larger, and the linear acceleration will decrease, as expected.

70. **INTERPRET** This problem involves Newton's second law and frictional forces. The object of interest is the block that slides up an incline and back down again. We want to show that when the coefficient of kinetic friction is $\mu_k = \frac{3}{5}\tan\theta$, the final speed of the block half its initial speed.

DEVELOP Draw a free-body diagram of the block going up and going down the incline (see figure below). Using Equation 5.3 $f_k = \mu_k n$, and applying Newton's law to the block going up the incline gives

$$\left.\begin{array}{ll} x: & mg\sin\theta + f_k = ma_{up} \\ y: & n - mg\cos\theta = 0 \end{array}\right\} a_{up} = g\sin(\theta) + \mu_k g\cos(\theta)$$

When going down, the acceleration is

$$\left.\begin{array}{ll} x: & mg\sin\theta - f_k = ma_{down} \\ y: & n - mg\cos\theta = 0 \end{array}\right\} a_{down} = g\sin(\theta) + \mu_k g\cos(\theta)$$

Knowing the acceleration, use Equation 2.11, $v^2 = v_0^2 + 2a(x - x_0)$ to express the initial upward speed and the final downward speed, then use the fact that $v_{down}/v_{up} = 1/2$ to find an expression for the coefficient of kinetic friction.
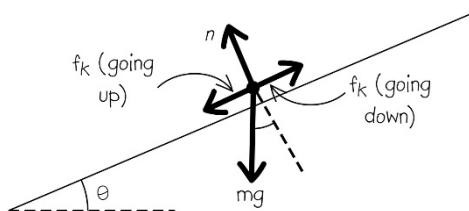
EVALUATE Suppose the block slides up a distance L. From Equation 2.11, its initial speed upward is

$$v_{up} = \sqrt{-2a_{up}L}$$

Similarly, as the block slides down the same distance, it returns to the bottom with speed

$$v_{down} = \sqrt{2a_{down}L}$$

The condition $v_{down}/v_{up} = 1/2$ implies

$$\frac{1}{4} = \left(\frac{v_{down}}{v_{up}}\right)^2 = \frac{2a_{down}L}{-2a_{up}L} = \frac{g\left[\sin(\theta) - \mu_k\cos(\theta)\right]}{g\left[\sin(\theta) + \mu_k\cos(\theta)\right]} = \frac{\tan(\theta) - \mu_k}{\tan(\theta) + \mu_k}$$

which gives $\mu_k = \frac{3}{5}\tan(\theta)$.

ASSESS To see that our result makes sense, let's check some limiting cases. If the incline were frictionless with $\mu_k = 0$ then we would expect $v_{down} = v_{up}$, which is satisfied. However, if the coefficient of kinetic friction becomes too large [$\mu_k > \tan(\theta)$, see Example 5.10], the block will not slide down at all $v_{down} = 0$.

71. **INTERPRET** This problem involves Newton's second law, Hooke's law (see Equation 4.9), and uniform circular motion. The object of interest is the $m = 2.1$-kg mass, and we are to find the radius of its circular trajectory given

the spring constant and the tangential speed at which it travels. Because the table is frictionless, we only need consider the forces acting horizontally, so the only force of interest is the radial force due to the spring.

DEVELOP   In the horizontal plane, Newton's second law gives $k(r - r_0) = ma = mv^2/r$, where $r_0 = 18$ cm is the unstretched spring length and r is the stretched spring length that we are to find.

EVALUATE   Inserting the given quantities into the above expression gives

$$k(r - r_0) = mv^2/r$$

$$kr^2 - kr_0 r - mv^2 = 0$$

$$r = \frac{kr_0 \pm \sqrt{k^2 r_0^2 + 4kmv^2}}{2k} = \frac{1}{2}\left[0.18 \text{ m} \pm \sqrt{(0.18 \text{ m})^2 + 4(2.1 \text{ kg})(1.4 \text{ m/s})^2/(150 \text{ N/m})} \right] = 0.28 \text{ m}$$

ASSESS   Can you convince yourself that the units under the radical work out to be meters?

72.   **INTERPRET**   This problem involves Newton's second law, friction, and kinematics. Knowing the recommended speed and the speed with which a vehicle wishes to traverse the curve, we are tasked with finding the frictional coefficient needed if the car is to stay on the road.

DEVELOP   From Example 5.6, we know that the speed v_d, radius r, and angle θ for which banked roads are designed satisfy the equation $\tan\theta = v_d^2 / gr$. In those scenarios, the vertical component of the normal force is the only force providing the acceleration necessary to move in a circular path. If a vehicle wishes to traverse the curve at a higher speed, there needs to be enough friction provided by the road on the tires. Viewing this problem with the use of the free-body diagram drawn in Figure 5.14 of the textbook, we can decompose the frictional force f into its x- and y-components to apply Newton's second law along each direction. Then, we can find the minimum coefficient of friction needed for the car to stay on the road while traveling at the given speed.

EVALUATE   Applying Newton's second law along each direction, we find

$$x: \quad n\sin\theta + f\cos\theta = m\frac{v^2}{r}$$

$$y: \quad n\cos\theta - f\sin\theta - mg = 0$$

We are interested in the minimum coefficient of friction, which is equal to the ratio of the frictional and normal force magnitudes $\mu = f / n$, so we need to solve for that ratio from the x- and y-equations above. Since the equations of motion along each direction are not dependent on the mass of the vehicle, we can solve for m in each expression and equate them. Grouping like terms together and solving for the ratio gives

$$\mu = \frac{f}{n} = \frac{\left(\dfrac{v^2}{gr}\cos\theta - \sin\theta \right)}{\left(\dfrac{v^2}{gr}\sin\theta + \cos\theta \right)}$$

Here, we can divide the numerator and denominator by $\cos\theta$ and substitute $\tan\theta = v_d^2 / gr$ to get

$$\mu = \frac{\left(\dfrac{v^2}{gr} - \tan\theta \right)}{\left(\dfrac{v^2}{gr}\tan\theta + 1 \right)} = \frac{\left(\dfrac{v^2}{gr} - \dfrac{v_d^2}{gr} \right)}{\left(\dfrac{v^2 v_d^2}{(gr)^2} + 1 \right)} = \frac{gr(v^2 - v_d^2)}{(v^2 v_d^2 + (gr)^2)}$$

Plugging in $v = 90$ km / h $= 25$ m / s, $v_d = 60$ km / h $= 16.7$ m / s, $r = 110$ m, and $g = 9.8$ m / s^2, we find that, for the car to stay on the road, the coefficient of friction must at least be equal to $\mu = 0.28$.

ASSESS   If we set the coefficient equal to zero, we see that $|v| = |v_d|$, meaning that the vehicle must traverse the curve at the speed for which it was designed in order to stay on the road.

73.   **INTERPRET**   We need to find an equation for the speed of an object as a function of time, where instead of just a constant acceleration due to gravity there is also a drag force that depends on velocity. We also need to find the terminal velocity of the object.

DEVELOP   We choose the upward direction to be positive. Instead of $F = -mg = ma$ we have

$F = -mg - bv = ma$. Since the acceleration is $a = dv / dt$, we have a differential equation: $\frac{dv}{dt} + \frac{b}{m} v + g = 0$. The initial velocity of the object is zero. We find terminal velocity by finding the value of $v(t)$ in the limit as $t \to \infty$.

EVALUATE (a) We assume that $v(t) = Ae^{-bt/m} + B$. Substituting this into the differential equation, we find $B = -mg / b$. Using the fact that $v(0) = 0$, we find $A = mg / b$, so the object's speed as a function of time is

$$v(t) = \frac{mg}{b} \left(e^{-bt/m} - 1 \right)$$

(b) The terminal velocity is the speed of the object at large time:

$$v_{\text{term}} = \lim_{t \to \infty} v(t) = \frac{-mg}{b}$$

The minus sign is here because the velocity is downward in the negative direction.

ASSESS Our units check out for the equation in part (a), and the equation has a form consistent with what we would expect. A force of $-bv$ is sometimes called Stokes's drag. It applies for viscous fluids in which objects move relatively slowly.

74. **INTERPRET** We are asked to derive an expression for how far a block can be slid up an incline. We then use this equation to calculate the angle of the incline that would minimize this sliding distance.

DEVELOP The three forces acting on the block are gravity, the normal force, and kinetic friction. Choose a coordinate system in which the *x*-axis points up the slope, so that the block is initially moving in the +*x*-direction. The normal force and the *y*-component of the gravity balance each other out: $n = mg \cos\theta$. However, the forces in the *x*-direction do not balance out:

$$F_{\text{net},x} = -mg \sin\theta - f_{\text{k}} = ma_x$$

where the kinetic friction is $f_{\text{k}} = \mu_{\text{k}} n$ (Equation 5.3). Notice that here the frictional force points down the slope, in opposition to the direction of motion.

EVALUATE (a) We are asked to derive an expression for the distance the block moves before stopping. Using Equation 2.11,

$$d = \frac{-v_0^2}{2a_x} = \frac{v_0^2}{g(\sin\theta + \mu_{\text{k}} \cos\theta)}$$

(b) To find the angle that minimizes the distance, we first take the derivative of the above equation:

$$\frac{d}{d\theta}[d] = \frac{-v_0^2}{g(\sin\theta + \mu_{\text{k}} \cos\theta)^2}(\cos\theta - \mu_{\text{k}} \sin\theta)$$

This derivative equals zero when $\cos\theta = \mu_{\text{k}} \sin\theta$, or $\theta = \cot^{-1}(\mu_{\text{k}})$.

ASSESS One can plug the angle in part (b) into the equation of part (a) and find that the minimum distance is

$$d = \frac{v_0^2}{g\sqrt{1 + \mu_{\text{k}}^2}}$$

Furthermore, one can verify that this is a minimum (as opposed to a maximum or an inflection point) by showing that the second derivative of the function $d(\theta)$ is positive at the determined value.

75. **INTERPRET** We need to find the tension in the three strings holding up the flower arrangement. If the tension in any one segment is more than 100 N, then we need heavier string. The flowers are in equilibrium, so we use Newton's first law: The components of the forces on each pot will sum to zero.

DEVELOP We start with a free-body diagram, as shown in the figure below. After resolving each tension into the horizontal and vertical components, we set the sum of components along each axis, for each pot, equal to zero and solve for the tensions.
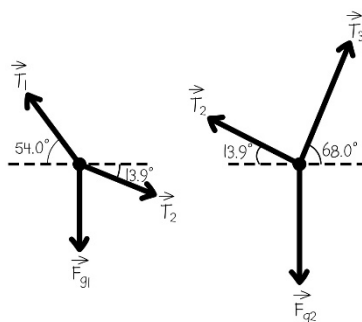
EVALUATE For the left pot,

$$T_{x1} = -T_1 \cos(54.0°)$$
$$T_{y1} = T_1 \sin(54.0°)$$
$$T_{x2} = T_2 \cos(13.9°)$$
$$T_{y2} = -T_2 \sin(13.9°)$$
$$F_{g1} = -m_1 g$$

$$-T_1 \cos(54.0°) + T_2 \cos(13.9°) = 0$$
$$T_1 \sin(54.0°) - T_2 \sin(13.9°) - m_1 g = 0$$

For the right pot,

$$T_{x3} = T_3 \cos(68.0°)$$
$$T_{y3} = T_3 \sin(68.0°)$$
$$T_{x2} = -T_2 \cos(13.9°)$$
$$T_{y2} = T_2 \sin(13.9°)$$
$$F_{g2} = -m_2 g$$

$$T_3 \cos(68.0°) - T_2 \cos(13.9°) = 0$$
$$T_3 \sin(68.0°) + T_2 \sin(13.9°) - m_3 g = 0$$

This problem is overspecified: There are three unknowns and four equations. However, we can still estimate if any of the forces are greater than 100 N.

Start with the left pot and solve the top equation for T_1 then substitute that value into the second equation to find T_2:

$$T_1 = T_2 \frac{\cos(13.9°)}{\cos(54.0°)}$$

$$T_2 \frac{\cos(13.9°)}{\cos(54.0°)} \sin(54.0°) - T_2 \sin(13.9°) = m_1 g$$

$$T_2 (1.336 - 0.240) = (3.85 \text{ kg})(9.8 \text{ m/s}^2)$$

$$T_2 = 34.4 \text{ N}$$

Now substitute this back into the equation for T_1 to find that tension: $T_1 = T_2 \cos(13.9°)/\cos(54°) = 56.9 \text{ N}$.

Do the same thing for the right pot.

$$T_3 = T_2 \frac{\cos(13.9°)}{\cos(68.0°)}$$

$$T_2 \frac{\cos(13.9°)}{\cos(68.0°)} \sin(68.0°) + T_2 \sin(13.9°) = m_3 g$$

$$T_2 (2.403 + 0.240) = (9.28 \text{ kg})(9.8 \text{ m/s}^2)$$

$$T_2 = 34.4 \text{ N}$$

Now substitute this back into the equation for T_3 to find $T_3 = T_2 \cos(13.9°)/\cos(68.0°) = 89.2 \text{ N}$. Thus, the 100 N string will suffice.

ASSESS Although 100 N string will work, we would be better off with a bigger margin of error. As it is, the far-right string will break if anyone waters the plants!

76. **INTERPRET** This problem involves analyzing the acceleration data of two objects: a hanging mass and a glider. By applying Newton's second law, we can experimentally deduce the value of the gravitational acceleration.
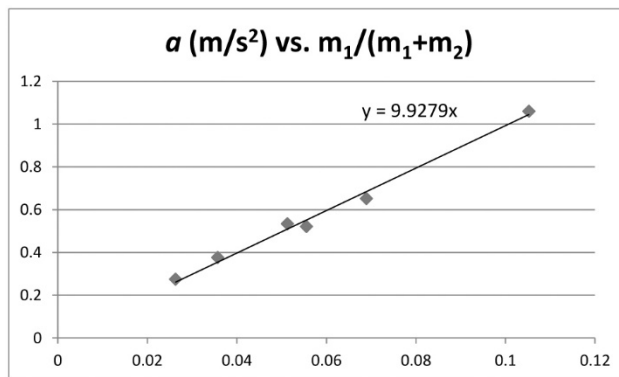
    DEVELOP Applying Newton's second law to both masses, we obtain

    $$m_1 g - T = m_1 a$$
    $$T = m_2 a$$

    The two equations can be combined to give $a = m_1 g / (m_1 + m_2)$.

    EVALUATE (a) From the expression for a, we plot a versus $m_1 / (m_1 + m_2)$ to obtain a straight line.

    (b) The slope of the straight line is about 9.9 m/s^2. This is our estimate for the gravitational acceleration g.

    ASSESS Our estimated value is about 1.3% higher than the accepted value of 9.81 m/s^2.

77. **INTERPRET** We are asked to analyze the movement of a figure skater.

    DEVELOP The skater is leaning to her left.

    EVALUATE We can imagine that her leaning will lead to a turn to her left.

    The answer is (a).

    ASSESS Although not many of us can perform figure skating maneuvers such as this, we probably can relate to leaning into a turn while running or roller skating or riding a bicycle.

78. **INTERPRET** We are asked to analyze the movement of a figure skater.

    DEVELOP The forces acting on the skater are gravity, the normal force, and friction. The friction is the only force pointing in the horizontal direction, and it must account for the centripetal acceleration that points to her left.

    EVALUATE To turn left, the net force will have to point left.

    The answer is (a).

    ASSESS If there were no friction force pointing to the left, the leaning skater's foot would slip out from underneath her and she would fall. In Chapter 12, we'll learn how to analyze a problem such as this by considering the net torque on the skater.

79. **INTERPRET** We are asked to analyze the movement of a figure skater.

    DEVELOP At higher speed but the same radius, the skater will need a higher centripetal acceleration ($a_c = v^2 / r$). This can happen only if she leans more into the turn.

    EVALUATE At higher speed, the tilt would need to be greater.

    The answer is (b).

    ASSESS Again, from our own experience we know that the faster we go through a turn, the harder we have to lean into it.

80. **INTERPRET** We are asked to analyze the movement of a figure skater.

    DEVELOP The three forces do not all act on the same part of the skater's body. The weight is applied at the center of mass, while the normal and friction forces are applied at the skate. If we imagine a line from the skate to the center of mass, there cannot be any net force perpendicular to this line, otherwise the skater will start to rotate around her center of mass (see Problem 5.40). The sum of the vertical normal force and the horizontal friction

force is a force pointing in the direction $\theta = \tan^{-1}(f_s / n)$ with respect to the vertical. The skater should be leaning at this angle to avoid having a rotating force (or torque).

EVALUATE The normal balances the downward weight, $n = mg$, and the friction is providing the centripetal force: $f_s = ma_c$. Therefore, if the skater tilt is $\theta = \tan^{-1}(0.5)$, the centripetal acceleration must be
$a_c = 0.5g \approx 5 \text{ m/s}^2$.
The answer is (c).

ASSESS As mentioned in Problem 5.75, we will later learn specific techniques for how to deal with forces applied away from the center of mass.