# ELECTRIC CHARGE, FORCE, AND FIELD

# 20

## EXERCISES

### Section 20.1 Electric Charge

**11.** **INTERPRET** We'll estimate the charge your body would carry if the electron charge slightly differed from the proton charge.

**DEVELOP** Since the human body is about 60% water, let's assume that the number of protons/electrons per kilogram in your body is the same as that of a water molecule. Water is 2 hydrogen atoms (with one proton and one electron each) and one oxygen atom (with 8 protons and 8 electrons). If the charges on the electrons and protons are different by one part in a billion, then the net charge on a water molecule would be

$$|\Delta q| = 10|q_{\text{proton}} - q_{\text{electron}}| = 10\left(10^{-9}e\right) = 10\left(1.60\times10^{-28}\,\text{C}\right) = 1.60\times10^{-27}\,\text{C}$$

The mass of a water molecule is the mass of two hydrogens and one oxygen: $1\,\text{u} + 1\,\text{u} + 16\,\text{u} = 18\,\text{u}$, where $1\,\text{u} = 1.66\times10^{-27}\,\text{kg}$. Therefore, the net charge to mass ratio for a water molecule would be:

$$\left(\frac{|\Delta q|}{m}\right)_{\text{H}_2\text{O}} = \frac{1.60\times10^{-27}\,\text{C}}{18\left(1.66\times10^{-27}\,\text{kg}\right)} = 0.05\ \text{C/kg}$$

**EVALUATE** We'll assume your mass is 65 kg. If we approximate this mass as pure water, then the total charge on your body would be approximately

$$|\Delta q| \approx m\left(\frac{|\Delta q|}{m}\right)_{\text{H}_2\text{O}} = (65\ \text{kg})(0.05\ \text{C/kg}) \simeq 3\ \text{C}$$

**ASSESS** This is a huge amount of charge. Imagine half of the charge was in your head/chest and the other half was in your legs, about a meter away. Then the magnitude of the force of repulsion between the upper and lower parts of your body would be

$$F_{12} = \frac{kq_1q_2}{r^2} = \frac{\left(9.0\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)\left(\frac{1}{2}3\ \text{C}\right)^2}{(1\ \text{m})^2} = 2\times10^{10}\,\text{N}$$

This would rip you apart!

**12.** **INTERPRET** This problem deals with quantity of charge in a typical lightning flash. We want to express the quantity in terms of the elementary charge $e$.

**DEVELOP** Since the magnitude of elementary charge $e$ is $e = 1.6\times10^{-19}$ C, the number $N$ of electrons involved in the lightening flash is given by $N = Q/e$.

**EVALUATE** Substituting the values given in the problem statement, we find

$$N = Q/e = \frac{25\ \text{C}}{1.6\times10^{-19}\ \text{C}} = 1.6\times10^{20}$$

**ASSESS** Since 1 coulomb is about $6.25 \times 10^{18}$ elementary charges, our result has the right order of magnitude.

**13.** **INTERPRET** This problem asks us to find the combination of $u$ and $d$ quarks needed to make a proton, which has a positive unit charge $e$, and a neutron, which has zero charge.

**DEVELOP** The $u$ quark has charge $2e/3$ and the $d$ quark has charge $-e/3$, so we can combine these so the charges sum to unity (for the proton) or zero (for the neutron).

**EVALUATE** **(a)** Two *u* quarks and a *d* quark make a total charge of 2(2/3) − 1/3 = 1, so the proton can be constructed from the quark combination *uud*.

**(b)** Two *d* quarks and a *u* quark make a total charge of 2(−1/3) + 2/3 = 0, so the neutron can be constructed from the quark combination *udd*.

**ASSESS** Because of a phenomenon called color confinement, quarks can only be found in hadrons, of which protons and neutrons are the most stable examples.

**14.** **INTERPRET** This problem deals with the quantity of net charge Earth carries. We want to express the quantity in terms of the elementary charge *e*.

**DEVELOP** Since the charge carried by an electron is $q=-e=-1.6\times10^{-19}$ C, the number *N* of electrons carried by Earth is given by $N = Q/q$.

**EVALUATE** Substituting the values given in the problem statement, we find

$$N=\frac{Q}{-e}=\frac{-5\times10^{5}\ \text{C}}{-1.6\times10^{-19}\ \text{C}}=3.1\times10^{24}$$

**ASSESS** Since 1 coulomb is about $6.25 \times 10^{18}$ elementary charges, our result has the right order of magnitude.

**15.** **INTERPRET** This problem deals with the number of electrons a honeybee has to lose to acquire a net positive charge of +180 pC. We express the quantity in terms of the elementary charge *e*.

**DEVELOP** Since the charge carried by an electron is $q=-e=-1.6\times10^{-19}$ C, the number *N* of electrons the honeybee needs to lose to have a net charge of $+Q$ is given by $N = +Q/e$.

**EVALUATE** Substituting the values given in the problem statement, we find

$$N=\frac{Q}{e}=\frac{180\times10^{-12}\ \text{C}}{1.6\times10^{-19}\ \text{C}}=1.1\times10^{9}$$

**ASSESS** Being positively charged, honeybees are attracted to the negatively charged spider webs.

## Section 20.2 Coulomb's Law

**16.** **INTERPRET** The electron and proton each carry one unit of electric charge, but the sign of the charge is opposite for the two particles. Given the distance between them, we are to find the force (magnitude and direction) between these particles.

**DEVELOP** Coulomb's law (Equation 20.1) gives the force between two particles. For this problem, $|q_1|=|q_2|=e=1.6\times10^{19}$ C and $r = 52.9 \times 10^{-12}$ m. Because the charges have opposite signs, the force will be attractive.

**EVALUATE** The force between these particles is attractive and has the magnitude

$$F=\frac{ke^2}{r^2}=\frac{(9.0\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.6\times10^{-19}\ \text{C})^2}{\left(52.9\times10^{-12}\ \text{m}\right)^2}=8.2\times10^{-8}\ \text{N}$$

**ASSESS** Because the electron is much less massive than the proton, the electron does most of the accelerating in this two-particle system.

**17.** **INTERPRET** We want to know how far a proton must be from an electron to exert an electrical attraction equal to the electron's weight on Earth.

**DEVELOP** The force between a proton and electron has a magnitude given by Coulomb's law (Equation 20.1): $F=ke^2/r^2$, where $1\ e=1.60\times10^{-19}\text{C}.$ We'll solve this for the distance, *r*, at which $F=m_\text{e}g.$

**EVALUATE** The distance at which the electrical force is equal to the gravitational force is

$$r=\sqrt{\frac{ke^2}{m_\text{e}g}}=\sqrt{\frac{\left(9.0\times10^{9}\,\text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\,\text{C}\right)^2}{\left(9.11\times10^{-31}\,\text{kg}\right)\left(9.8\ \text{m/s}^2\right)}}=5.1\ \text{m}$$

**ASSESS** On the molecular scale, protons and electrons are roughly $10^{-10}\,\text{m}$ apart. Since the Coulomb attraction scales as $1/r^2,$ electric forces will clearly overwhelm any gravity effects.

**18.** **Interpret** The problem is to calculate the charge on tiny Styrofoam pieces.

**Develop** Since the charges are equal, the repulsion force at the given distance is $F = kq^2/r^2$ (Equation 20.1).

**Evaluate** Solving for the charge in Coulomb's law gives

$$q = \sqrt{\frac{Fr^2}{k}} = \sqrt{\frac{(0.02\ \text{N})(0.017\ \text{m})^2}{(9.0\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)}} = 25\ \text{nC}$$

**Assess** This seems reasonable for small charged objects.

**19.** **Interpret** This problem involves finding the unit vector associated with the electrical force that one charge exerts on another, given the coordinates of the positions of both charges.

**Develop** A unit vector from the position of charge $q$ at $\vec{r}_q = (5\ \text{m}, 0)$ to any other point $\vec{r} = (x, y)$ is

$$\hat{n} = \frac{(\vec{r} - \vec{r}_q)}{|\vec{r} - \vec{r}_q|} = \frac{(x - 5\ \text{m,y})}{\sqrt{(x - 5\ \text{m})^2 + y^2}}$$

**Evaluate** **(a)** When the other charge is at position $\vec{r} = (5\ \text{m}, 2.5\ \text{m})$, the unit vector is

$$\hat{n} = \frac{(0, 2.5\ \text{m})}{\sqrt{0 + (2.5\ \text{m})^2}} = (0,1) = \hat{j}$$

**(b)** When $\vec{r} = (0,0)$,

$$\hat{n} = \frac{(-5\ \text{m}, 0)}{\sqrt{(-5\ \text{m})^2 + 0}} = (-1,0) = -\hat{i}$$

**(c)** Finally, when $\vec{r} = (7\ \text{m}, 3.5\ \text{m})$, the unit vector is

$$\hat{n} = \frac{(2\ \text{m}, 3.5\ \text{m})}{\sqrt{(2\ \text{m})^2 + (3.5\ \text{m})^2}} = \frac{(2, 3.5)}{\sqrt{16.25}} = 0.496\hat{i} + 0.868\hat{j}$$

The sign of $q$ doesn't affect this unit vector, but the signs of both charges do determine whether the force exerted by $q$ is repulsive or attractive, that is, in the direction of $+\hat{n}$ or $-\hat{n}$.

**Assess** The unit vector always points away from the charge $q$ located at (5 m, 0).

**20.** **Interpret** This problem requires us to find the force acting on a proton due to an electron, given the positions of the two particles.

**Develop** Use the result of the previous problem to find the direction of the force acting on the proton. The relevant quantities are $\vec{r}_q = (0,0)$ for the proton and $\vec{r} = (0.41\ \text{nm}, 0.36\ \text{nm})$, so the unit vector indicating the direction of the force is

$$\hat{n} = \frac{(\vec{r} - \vec{r}_q)}{|\vec{r} - \vec{r}_q|} = \frac{(0.41\ \text{m}, 0.36\ \text{nm})}{\sqrt{(0.41\ \text{m})^2 + (0.36\ \text{nm})^2}} = 0.75\hat{i} + 0.66\hat{j}$$

Because the charges are opposite in sign, the force will act in the positive $\hat{n}$ direction. The magnitude of the force may be found using Coulomb's law (Equation 20.1).

**Evaluate** The magnitude of the force is

$$F_p = \frac{ke^2}{r^2} = \frac{(9\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.6\times 10^{-19}\ \text{C})^2}{(0.41^2 + 0.36^2)\times 10^{-18}\ \text{m}^2} = 7.7\times 10^{-10}\ \text{N}$$

so the complete (i.e., vector) force is $(5.8\hat{i} + 5.1\hat{j})\times 10^{-10}\ \text{N}$

**Assess** This force is two orders of magnitude less than the force between the electron and proton in a hydrogen atom. The direction of the force is $\theta = \tan^{-1}(0.36/0.41) = 41°$ above the $x$-axis.

**Section 20.3 The Electric Field**

**21.** **INTERPRET** This problem is about calculating the electric field strength due to a source, when the force experienced by the electron is known.

**DEVELOP** Equation 20.2a shows that the electric field strength (magnitude of the field) at a point is equal to the force per unit charge that would be experienced by a charge at that point: $E = F / q$.

**EVALUATE** With $q = |e|$, we find the field strength to be

$$E = \frac{F}{|e|} = \frac{0.61\times10^{-9}\,\text{N}}{1.60\times10^{-19}\,\text{C}} = 3.8\times10^{9}\,\text{N/C}$$

**ASSESS** Since the charge of electron is negative, the electric field will point in the opposite direction as the force.

**22.** **INTERPRET** This problem asks us to find the magnitude of the force on an electric charge that is placed in an electric field of the given strength.

**DEVELOP** The electric field is the force per unit charge, so we multiply the magnitude of the charge by that of the electric field to find the force (see Equation 20.2b).

**EVALUATE** The magnitude of the force is $F = qE = (6.0\times10^{-6}\ \text{C})(50\ \text{N/C}) = 3.0\times10^{-4}\ \text{N}$.

**ASSESS** The direction of this force will be the same as that of the electric field because the charge is positive.

**23.** **INTERPRET** This problem involves calculating the electric field needed to produce the given force on the given charge and then finding the force experienced by a second charge in the same electric field.

**DEVELOP** Equation 20.2a shows that the electric field strength (magnitude of the field) at a point is equal to the force per unit charge that would be experienced by a charge at that point: $E = F / q$. The equation allows us to calculate $E$ given that $F = 144\ \text{mN}$ and $q = 75\ \text{nC}$. For part (b), the force experienced by another charge $q'$ in the same field is given by Equation 20.2b: $F' = q'E$.

**EVALUATE** **(a)** With q = 75 nC, we find the field strength to be

$$E = \frac{F}{|e|} = \frac{144\ \text{mN}}{75\ \text{nC}} = 1.9\times10^{6}\ \text{N/C}$$

**(b)** The force experienced by a charge $q' = 35\ \mu\text{C}$ in the same field is

$$F' = q'E = (35\ \mu\text{C})\left(1.9\times10^{6}\ \text{N/C}\right) = 66.5\ \text{N}$$

**ASSESS** The force that a test charge particle experiences is proportional to the magnitude of the test charge. In our problem, since $q'(=35\ \mu\text{C}) > q\,(=75\ \text{nC})$, we find $F' > F$.

**24.** **INTERPRET** We want the force on an ion inside a cell with an internal electric field.

**DEVELOP** A singly charged ion has a charge of $q = |e|$, so the magnitude of the force will be $F = |e|E$.

**EVALUATE** The force on the ion in the cell is

$$F = |e|E = \left(1.60\times10^{-19}\text{C}\right)\left(8.0\times10^{6}\text{N/C}\right) = 1.3\ \text{pN}$$

**ASSESS** Although a pico-Newton is very small, it is a significant force at the molecular scale.

**25.** **INTERPRET** This problem is similar to Problem 20.23, in that we are given the force exerted on a given charge by an unknown electric field, and we are to find the force exerted by this field on another charge (a proton, in this case).

**DEVELOP** Apply Equation 20.2b, $\vec{E} = \vec{F}/q$, to find the electric field, with $q = -3\ \mu\text{C}$. The force on the proton $(q_p = 1.6\times10^{-19}\ \text{C})$ is given by Equation 20.2a, $\vec{F}_p = q_p\vec{E}$. Inserting the result of Equation 20.2b gives

$$\vec{F}_p = q_p\vec{E} = q_p\frac{\vec{F}}{q}$$

**EVALUATE** Inserting the given quantities into the expression above for the force on the proton gives

$$\vec{F}_p = \left(1.6\times10^{-19}\ \text{C}\right)\frac{9\hat{\imath}\ \text{N}}{-3.0\times10^{-6}\ \text{C}} = -0.48\hat{\imath}\ \text{pN}$$

**ASSESS** The force on the proton acts in the direction opposite to that of the force on the original charge because the two charges have opposite signs.

**26.** **INTERPRET** For this problem, we are to calculate the electric field strength due to a positive point charge—the proton.

**DEVELOP** The electric field strength at a distance $r$ from a point source charge $q$ is given by Equation 20.3:

$$\vec{E} = \frac{kq}{r^2}\hat{r}$$

The proton in a hydrogen atom behaves like a point charge, so we can apply this equation to find the electric field of the proton. The charge of the proton is $q = +e = 1.6 \times 10^{-19}$ C.

**EVALUATE** At a distance of one Bohr radius ($a_0 = 0.0529$ nm) from the proton, the electric field strength is

$$E = \frac{ke}{a_0^2} = \frac{\left(9.0\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.6\times10^{-19}\ \text{C}\right)}{\left(5.29\times10^{-11}\ \text{m}\right)^2} = 5.2\times10^{11}\ \text{N/C}$$

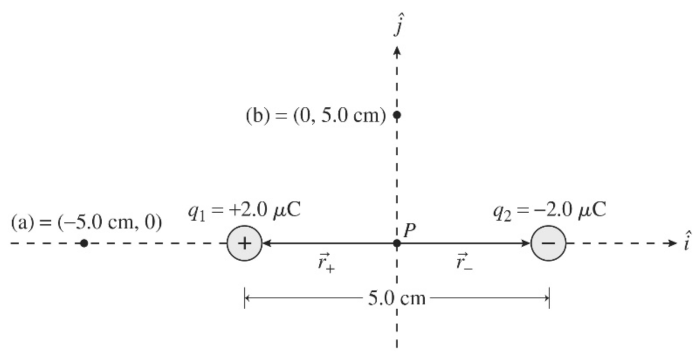
**ASSESS** The field strength at the position of the electron is enormous because of the close proximity.

## Section 20.4 Fields of Charge Distributions

**27.** **INTERPRET** For this problem, we are to find the electric field at several locations in the vicinity of two given point charges. We can apply the principle of superposition to solve this problem.

**DEVELOP** Take the origin of the $x$-$y$ coordinate system to be at the midpoint between the two charges, as indicated in the figure below, with unit vectors $\hat{i}$ pointing to the right, and $\hat{j}$ pointing up. We use Equation 20.4 to find the electric field at the given points. Let $q_1 = +2.0\ \mu$C and $q_2 = -2.0\ \mu$C. Let $\vec{r}_\pm = \pm(2.5\ \text{cm})\hat{j}$ denote the positions of the charges and $\vec{r}$ denote that of the field point (i.e., the point at which we are calculating the electric field). A unit vector from charge $i$ to the field point is $(\vec{r} - \vec{r}_\pm)/|\vec{r} - \vec{r}_\pm|$ [where the plus (minus) sign corresponds to the positive (negative) charge)]. Thus, the spatial factors in Coulomb's law are $\hat{r}_i/r_i^2 = \vec{r}_i/r_i^3 = (\vec{r} - \vec{r}_\pm)/|\vec{r} - \vec{r}_\pm|^3$. By the principle of superposition (Equation 20.4), the total electric field at any point is

$$\vec{E} = k\left(\frac{q_1\vec{r}_1}{r_1^3} + \frac{q_2\vec{r}_2}{r_2^3}\right)$$




**EVALUATE** **(a)** For the point at 5.0 cm to the left of $P$, we have

$$\vec{r} = (-5.0\ \text{cm})\hat{i}$$
$$\vec{r}_1 = \vec{r} - \vec{r}_+ = (-5.0\ \text{cm})\hat{i} - (-2.5\ \text{cm})\hat{i} = (-2.5\ \text{cm})\hat{i}$$
$$\vec{r}_2 = \vec{r} - \vec{r}_- = (-5.0\ \text{cm})\hat{i} - (2.5\ \text{cm})\hat{i} = (-7.5\ \text{cm})\hat{i}$$

so the electric field is

$$\vec{E} = k\left(\frac{q_1\vec{r}_1}{r_1^3} + \frac{q_2\vec{r}_2}{r_2^3}\right) = k\left(-\frac{q_1\hat{\imath}}{r_1^2} - \frac{q_2\hat{\imath}}{r_2^2}\right)$$
$$= \left(9.0\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left[-\frac{\left(2.0\times10^{-6}\ \text{C}\right)\hat{\imath}}{(0.025\ \text{m})^2} - \frac{\left(-2.0\times10^{-6}\ \text{C}\right)\hat{\imath}}{(0.075\ \text{m})^2}\right] = (-26\ \text{MN/C})\hat{\imath}$$

or 26 MN/C, pointing to the left.

**(b)** For the point at 5.0 cm directly above $P$, we have

$$\vec{r} = (5.0\ \text{cm})\,\hat{\jmath}$$
$$\frac{\vec{r}_1}{r_1^3} = \frac{\vec{r}-\vec{r}_+}{\left|\vec{r}-\vec{r}_+\right|^3} = \frac{-(-2.5\ \text{cm})\hat{\imath} + (5.0\ \text{cm})\,\hat{\jmath}}{\left[(5.0\ \text{cm})^2 + (-2.5\ \text{cm})^2\right]^{3/2}}$$
$$\frac{\vec{r}_2}{r_2^3} = \frac{\vec{r}-\vec{r}_-}{\left|\vec{r}-\vec{r}_-\right|^3} = \frac{-(2.5\ \text{cm})\hat{\imath} + (5.0\ \text{cm})\,\hat{\jmath}}{\left[(5.0\ \text{cm})^2 + (2.5\ \text{cm})^2\right]^{3/2}}$$

so the electric field is

$$\vec{E} = \left(9.0\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left(\frac{2.0\times10^{-6}\ \text{C}}{\text{m}^2}\right)\left\{\frac{-(-0.025)\hat{\imath} + 0.050\hat{\jmath}}{\left[(-0.025)^2 + (0.050)^2\right]^{3/2}} - \frac{-(0.025)\hat{\imath} + 0.050\hat{\jmath}}{\left[(0.025)^2 + (0.050)^2\right]^{3/2}}\right\}$$
$$= \left(9.0\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left(\frac{2.0\times10^{-6}\ \text{C}}{\left[(0.025)^2 + (0.050)^2\right]^{3/2}\ \text{m}^2}\right)\left(0.050\hat{\imath}\right) = (5.2\ \text{MN/C})\hat{\imath}$$

or 5.2 MN/C to the right.

**(c)** For $\vec{r} = 0$, we have

$$\frac{\vec{r}_1}{r_1^3} = \frac{\vec{r}-\vec{r}_+}{\left|\vec{r}-\vec{r}_+\right|^3} = \frac{-(2.5\ \text{cm})\hat{\imath}}{(2.5\ \text{cm})^3} = \frac{-\hat{\imath}}{(2.5\ \text{cm})^2}$$
$$\frac{\vec{r}_2}{r_2^3} = \frac{\vec{r}-\vec{r}_-}{\left|\vec{r}-\vec{r}_-\right|^3} = \frac{(2.5\ \text{cm})\hat{\imath}}{(2.5\ \text{cm})^3} = \frac{\hat{\imath}}{(2.5\ \text{cm})^2}$$

so the electric field is

$$\vec{E} = \left(9.0\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left(\frac{2.0\times10^{-6}\ \text{C}}{\text{m}^2}\right)\left[\frac{\hat{\imath}}{(0.025)^2} - \frac{-\hat{\imath}}{(0.025)^2}\right] = (+58\ \text{MN/C})\hat{\imath}$$

or 58 MN/C to the right.

**Assess** The electric field for part **(b)** is much weaker because the fields from the two charges largely cancel.

**28.** **Interpret** Given the magnitude of the dipole moment, we are asked to calculate the distance between the pair of opposite charges that make up the dipole.

**Develop** As shown in Equation 20.5, the electric dipole moment $p$ is the product of the charge $q$ and the separation $d$ between the two charges making up the dipole:

$$p = qd$$

**Evaluate** Using the equation above, the distance separating the charges of a dipole is

$$d = \frac{p}{q} = \frac{6.2\times10^{-30}\ \text{C}\cdot\text{m}}{1.6\times10^{-19}\ \text{C}} = 39\ \text{pm} = 0.039\ \text{nm}$$

**Assess** The distance $d$ has the same order of magnitude as the Bohr radius $(a_0 = 0.0529\ \text{nm})$.

**29.** **Interpret** We are given a long wire with a uniform charge density and are asked to find the electric field strength 38 cm from the wire. We can assume that the wire length is much, much greater than 38 cm.

**Develop** For a very long wire $(L \gg 38\text{ cm})$, Example 20.7 shows that the magnitude of the electric field falls off like $1/r$. Therefore, the electric field is simply scaled by the ratio of the distances, or

$$E_2 = E_1 \frac{r_1}{r_2}$$

**Evaluate** Inserting the given quantities into the expression above gives

$$E_2 = (1.9\text{ kN/C})\frac{22}{38} = 1.1\text{ kN/C}$$

**Assess** The electric field gets weaker the farther we are from the wire, as expected.

**30.** **Interpret** In this problem, we are asked to find the line charge density, given the field strength at a distance from a long wire. We can assume that the wire is much, much longer than the distance involved (39 cm), so the result of Example 20.7 applies.

**Develop** If the electric field points radially toward the long wire $(L >> 39\text{ cm})$, the charge on the wire must be negative. The magnitude of the field is given by the result of Example 20.7,

$$E = \frac{2k\lambda}{r}$$

**Evaluate** Using the equation above, we find the line charge density to be

$$\lambda = \frac{Er}{2k} = \frac{(-210\text{ kN/C})(0.39\text{ m})}{2\left(9.0\times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)} = -4.55\ \mu\text{C/m}$$

**Assess** The electric field strength due to a line charge density decreases as $1/r$. Compare this to the $1/r^2$ dependence of the electric field of a point charge.

**31.** **Interpret** We will use Coulomb's law and the definition of electric field to find the electric field at a point on the axis of a charged ring.

**Develop** From Example 20.6, which is done for a general distance $x$, we see that

$$E = \frac{kQx}{\left(x^2 + a^2\right)^{3/2}}$$

We want to know the field $E$ at position $x = a$.

**Evaluate** Inserting $x = a$ into the expression above gives

$$E = \frac{kQx}{\left(x^2 + a^2\right)^{3/2}} = \frac{kQa}{\left(2a^2\right)^{3/2}} = \frac{kQ}{\sqrt{8}a^2}$$

**Assess** The units are $kQ/(\text{distance})^2$, which are correct for an electric field.

## Section 20.5 Matter in Electric Fields

**32.** **Interpret** For this problem, we are to find the force generated by 10 elementary charges in the given electric field, and calculate the mass that can be suspended by this force in the Earth's gravitational field.

**Develop** By Newton's second law, the force due to gravity and the force due to the electric field must cancel each other for the oil drop to have no acceleration. This condition gives

$$\vec{F}_g = -\vec{F}_e, \text{ or}$$
$$mg = -qE$$

The equation can be used to compute the mass $m$.

**Evaluate** Using the equation above, the mass is

$$m = \frac{qE}{g} = \frac{\left(10\times 1.6\times 10^{-19}\text{ C}\right)\left(2.0\times 10^7\text{ N/C}\right)}{\left(9.8\text{ m/s}^2\right)} = 3.3\times 10^{-12}\text{ kg}$$

**Assess** Because this mass is so small, the size of such a drop may be better appreciated in terms of its radius, $R = (3m/4\pi\rho_{\text{oil}})^{1/3}$. Millikan used oil of density $\rho_{\text{oil}} = 0.9199\text{ g/cm}^3$, so $R = 9.46\ \mu\text{m}$ for this drop.

**33.** **INTERPRET** This problem involves kinematics, Newton's second law, and Coulomb's law. We can use these concepts to find the electric field strength necessary to accelerate an electron from rest to $c/10$ within 4.7 cm.

**DEVELOP** If the electric field is constant in space, the force applied to the charge will be constant, so we can use Equation 2.11, which applies for constant acceleration, to find the necessary acceleration for the electron. This gives

$$v^2 = \overbrace{v_0^2}^{=0} + 2a(x - x_0)$$

where $v = c/10$, $x - x_0 = 4.7$ cm, and $v_0 = 0$ because the electron starts from rest. Thus, the acceleration is

$$a = \frac{v^2}{2(x - x_0)} = \frac{c^2}{200(4.7\text{ cm})}$$

The force needed to provide this acceleration is given by Newton's second law, which, combined with Coulomb's law, gives

$$F = Eq = ma = \frac{mc^2}{9.4\text{ m}}$$

which we can solve for $E$.

**EVALUATE** Inserting $|q| = 1.6\times10^{-19}$ C for the electron's charge (since we are only concerned with the strength of the electric field, not its direction), we find

$$E = \frac{mc^2}{q(9.4\text{ m})} = \frac{(9.11\times10^{-31}\text{ kg})(3.0\times10^{8}\text{ m/s})^2}{(1.6\times10^{-19}\text{ C})(9.4\text{ m})} = 5.45\times10^{4}\text{ N/C}$$

**ASSESS** Because the electron has a negative charge, it would move in a direction opposite to that of this electric field.

**34.** **INTERPRET** This problem involves the motion of a proton, which has charge $+e$, in an electric field that points to the left. The proton enters the field region with the given velocity, and we are to find how far it travels in the field before it reverses direction. We are also to describe its subsequent motion. To address this problem, we will use Newton's second law and Coulomb's law, and some kinematics from Chapter 2.

**DEVELOP** Choose the $x$-axis to the right, in the direction of the proton's initial velocity, so that the electric field is oriented to the left. If only the Coulomb force (Equation 20.2b) acts on the proton, the acceleration can be found from Newton's second law (for constant mass: Equation 4.3):

$$\vec{F} = q\vec{E} = qE(-\hat{i}) = m\vec{a} \Rightarrow a_x = -\frac{eE}{m}$$

where $a_x$ is the magnitude of the proton's acceleration along the $x$-axis. The negative sign means that the proton decelerates as it enters the electric field. Because the acceleration is constant, we can apply Equation 2.11, $v^2 = v_0^2 + 2a(x - x_0)$. When the proton reverses direction, its velocity $v = 0$ momentarily, so we insert this into Equation 2.11 to find the distance traveled, $x - x_0$.

**EVALUATE** **(a)** Using Equation 2.11, with $v_0 = 3.3\times10^5$ m/s, we find the maximum penetration into the field region to be

$$x - x_0 = -\frac{v_0^2}{2a_x} = \frac{mv_0^2}{2eE} = \frac{(1.67\times10^{-27}\text{ kg})(3.3\times10^{5}\text{ m/s})^2}{2(1.6\times10^{-19}\text{ C})(6\times10^{4}\text{ N/C})} = 0.947\text{ cm} \approx 0.95\text{ cm}$$

**(b)** The proton subsequently moves to the left, with the same constant acceleration in the field region, until it exits with $\vec{v}_\text{f} = -\vec{v}_0$.

**ASSESS** The deceleration of the proton increases with the field strength $E$. Note that it is unrealistic to have an electric field that begins instantaneously in space or time, as we will find in later chapters.

**35.** **INTERPRET** This problem involves an electrostatic analyzer like that in Example 20.8, so we will use the results of that example. We are to find the coefficient $E_0$ in the expression for the electric field strength in the analyzer that will permit protons to exit the analyzer.

**DEVELOP** From the analysis of Example 20.8, we know that the coefficient $E_0$ of the analyzer is related to the particle mass $m$, its velocity $v$, and it charge $q$ by

$$E_0 = \frac{mv^2}{qb}$$

Given that $m = 1.67\times10^{-27}$ kg for a proton, $q = 1.6\times10^{-19}$ C, and $v$ and $b$ are given, we can find $E_0$.

**EVALUATE** Inserting the given quantities in the expression for $E_0$ gives

$$E_0 = \frac{mv^2}{eb} = \frac{\left(1.67\times10^{-27}\ \text{kg}\right)\left(84\times10^3\ \text{m/s}\right)^2}{\left(1.6\times10^{-19}\ \text{C}\right)\left(0.075\ \text{m}\right)} = 980\ \text{N/C}$$

to two significant figures.

**ASSESS** Note that the proton exits the analyzer with the same speed with which it entered that analyzer, because the force is always perpendicular to the proton's trajectory (i.e., it's a centripetal force). Thus, the force does no work on the proton, but the proton's velocity has changed direction.

## EXAMPLE VARIATIONS

**36.** **INTERPRET** Coulomb's law and the superposition principle apply, and we identify $Q$ as the charge for which we want the force. The two charges $q$ are the source charges.

**DEVELOP** Figure 20.7 depicts the charge ensemble, showing the charges, the individual force vectors, and their sum. The drawing shows that the distance $r$ in Coulomb's law is the hypotenuse $\sqrt{a^2+y^2}$. It's clear from symmetry that the net force is in the $y$-direction, so we need to find only the $y$-components of the unit vectors. The $y$-components are clearly the same for each, and the drawing shows that they're given by

$$\hat{r}_y = y\Big/\sqrt{a^2+y^2}\ \hat{j}\ .$$

**EVALUATE** From Coulomb's law, the $y$-component of the force from each $q$ is $F_y = \left(kqQ/r^2\right)\hat{r}_y$, and the net force on $Q$ becomes

$$\vec{F} = 2\left(\frac{kqQ}{a^2+y^2}\right)\left(\frac{y}{\sqrt{a^2+y^2}}\right)\hat{j} = \frac{2kqQy}{\left(a^2+y^2\right)^{3/2}}\hat{j}$$

Where the factor of 2 comes from the two charges $q$, which contribute equally to the net force. Evaluating this for the given values we obtain a net force equal to

$$\vec{F} = \frac{2(9.0\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(645\ \text{nC})(-1.87\ \mu\text{C})(12.3\ \text{mm})}{\left((2.18\ \text{mm})^2+(12.3\ \text{mm})^2\right)^{3/2}}\hat{j} = -142\hat{j}\ \text{N}$$

**ASSESS** This problem differs from the original example since the charge of $Q$ is negative, and thus the net force points in the negative $y$-direction, but the same vector calculus is applicable due the maintained symmetry of the ensemble.

**37.** **INTERPRET** Coulomb's law and the superposition principle apply, where all the charges are identical and lie at the corners of an equilateral triangle. We are asked to find the magnitude of charge and whether we can determine the sign.

**DEVELOP** Since all three charges are equivalent, they will all feel an equal repulsive force. We will choose to find the force on the top charge to remain consistent with the original example geometry. Figure 20.7 depicts the charge ensemble, showing the charges, the individual force vectors, and their sum. In this case the distance $r$ in Coulomb's law is the given length of the triangle's side. It's clear from symmetry that the net force is in the $y$-direction, so we need to find only the $y$-components of the unit vectors. The $y$-components are clearly the same for each, and for an equilateral triangle with side length $r$, they're given by

$$\hat{r}_y = \frac{\sqrt{3}}{2}\hat{j}.$$

**EVALUATE** **(a)** From Coulomb's law, the $y$-component of the force from each $q$ is $F_y = \left(kQ^2/r^2\right)\hat{r}_y$, and the net force on the top charge $Q$ becomes

$$\vec{F} = 2\left(\frac{kqQ}{r^2}\right)\left(\frac{\sqrt{3}}{2}\right)\hat{j} = \frac{\sqrt{3}kQ^2}{r^2}\hat{j}$$

Where the factor of 2 comes from the two charges $Q$, which contribute equally to the net force. Solving for the charge $Q$, and using the given values for the triangle side length and net force we obtain

$$Q = \sqrt{\frac{r^2F}{\sqrt{3}k}} = \sqrt{\frac{(3.36\,\text{mm})^2(96.2\,\text{N})}{\sqrt{3}(9.0\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)}} = \pm 264\,\text{nC}$$

**(b)** Since all the charges are equivalent, the force felt by each will be repulsive, but we can't determine if this is due to all of them being positive or negative charges. This is reflected in the square root which results in either a positive or negative value for $Q$.

**ASSESS** This problem differs from the original example since the charges and the sides of the triangle are equivalent, but the same vector calculus is applicable due the maintained symmetry of the ensemble.

**38.** **INTERPRET** Coulomb's law and the superposition principle apply, and we identify $Q$ as the charge for which we want the force. The two charges $q$ and $-q$ are the source charges.

**DEVELOP** Figure 20.7 depicts the charge ensemble, showing the charges, the individual force vectors, and their sum. The drawing shows that the distance $r$ in Coulomb's law is the hypotenuse $\sqrt{a^2+y^2}$. It's clear from symmetry that the net force is in the $x$-direction, so we need to find only the $x$-components of the unit vectors. The $x$-components are clearly the same magnitude for each, and the drawing shows that they're given by

$$\hat{r}_{x\pm} = \pm a\Big/\sqrt{a^2+y^2}\,\hat{i}.$$

**EVALUATE** **(a)** From Coulomb's law, the $x$-component of the force from each charge is $F_{x\pm} = \left(\pm kqQ/r^2\right)\hat{r}_{x\pm}$, and the net force on $Q$ becomes

$$\vec{F} = 2\left(\frac{kqQ}{a^2+y^2}\right)\left(\frac{a}{\sqrt{a^2+y^2}}\right)\hat{i} = \frac{2kqQa}{\left(a^2+y^2\right)^{3/2}}\hat{i}$$

Where the factor of 2 comes from the two charges, which contribute equally to the net force.

**(b)** In the case of $y >> a$ we can make the approximation: $\left(a^2+y^2\right)^{3/2} \cong y^3$, meaning the net force on charge $Q$ is given by $\vec{F} = \dfrac{2kqQa}{y^3}\hat{i}$, which depends on the distance $y$ as $1/y^3$.

**ASSESS** This problem differs from the original example since one of the charges $q$ is negative, and thus the net force points in the positive $x$-direction, but the same vector calculus is applicable due the maintained symmetry of the ensemble.

**39.** **INTERPRET** We are to find the location of the maximum force felt by charge $Q$ in the original charge ensemble of Example 20.2. We do this by finding the function's extrema along the $y$-direction using vector calculus.

**DEVELOP** In Example 20.2 we find that the net force on charge $Q$ is given by

$$\vec{F} = \frac{2kqQy}{\left(a^2+y^2\right)^{3/2}}\hat{j}$$

To find the location where the force is maximized, we will set the derivative along the $y$-direction equal to zero, and solve for the $y$ value for which this is true.

**EVALUATE** **(a)** Taking the first derivative of the net force along the $y$-direction, setting it equal to zero, and solving for the $y$ value we obtain

$$\frac{d}{dy}\hat{j}\cdot\vec{F} = 2kqQ\frac{d}{dy}\left(\frac{y}{\left(a^2+y^2\right)^{3/2}}\right)$$

$$\frac{dF}{dy} = 2kqQ\left(\frac{1}{\left(a^2+y^2\right)^{3/2}} - \frac{3}{2}\frac{y(2y)}{\left(a^2+y^2\right)^{5/2}}\right) = 0$$

$$\left(1-\frac{3y^2}{\left(a^2+y^2\right)^2}\right) = 0 \to y_{\max} = a/\sqrt{2}$$

**(b)** Plugging this value of $y$ back into the expression for the net force gives the magnitude of the maximum force

$$F_{\max} = \frac{2kqQ\left(a/\sqrt{2}\right)}{\left(a^2+\left(a/\sqrt{2}\right)^2\right)^{3/2}} = \frac{kqQ}{a^2}\frac{2}{\sqrt{2}}\left(\frac{2}{3}\right)^{3/2} = \frac{4kqQ}{3\sqrt{3}a^2}$$

**ASSESS** The magnitude of the maximum force is dependent on the magnitude of the charges $q$ and $Q$ as well as the distance $a$.

**40.** **INTERPRET** We identify the field point as being a distance $y$ from the wire, and the source charge as the whole wire.

**DEVELOP** Figure 20.17 is a drawing, showing a coordinate system with the field point $P$ along the $y$-axis. We divide the wire into small charge elements $dq$ and note that field contributions from two such elements $dq$ on opposite sides of the $y$-axis contribute fields $d\vec{E}$ whose $x$-components cancel. Then we need only the $y$-component of each unit vector, and Fig. 20.17 shows that's $\hat{r}_y = y/r$, where $r = \sqrt{x^2+y^2}$ .

**EVALUATE** Our wire has charge density $\lambda$ C/m, so if a charge element has length $dx$, then its charge is $dq = \lambda dx$. Putting all this together gives the $y$-component of the field from an arbitrary $dq$ anywhere on the wire:

$$dE_y = \frac{k\,dq}{r^2}\hat{r}_y = \frac{k\lambda\,dx}{r^2}\frac{y}{r} = \frac{k\lambda y}{\left(x^2+y^2\right)^{3/2}}dx$$

Since the $x$-components cancel, we can sum—that is, integrate—the $y$-components to get the net field:

$$E = E_y = k\lambda y\int_{-\infty}^{+\infty}\frac{dx}{\left(x^2+y^2\right)^{3/2}} = k\lambda y\left[\frac{x}{y^2\sqrt{x^2+y^2}}\right]_{-\infty}^{+\infty}$$

$$E = k\lambda y\left[\frac{1}{y^2}-\left(-\frac{1}{y^2}\right)\right] = \frac{2k\lambda}{y}$$

Our result is the field's magnitude; the direction is away from the line for positive $\lambda$ and toward the line for negative $\lambda$. Plugging in the given values results in a field of magnitude:

$$E = \frac{2k\lambda}{y} = \frac{2\left(9.0\times10^9\,\mathrm{N\cdot m^2/C^2}\right)(264\,\mathrm{mC})}{(54.3\,\mathrm{cm})} = 8.75\,\mathrm{MN/C}$$

**ASSESS** For an infinite line there's nothing to favor one direction along the line over another, so the only way the field can point is radially, away from or toward the line.

**41.** **INTERPRET** We identify the field point as being a distance $y$ from the wire, and the source charge as the whole wire. We want to determine the amount of charge accumulated on the wire, knowing the electric field at a given distance from it.

**DEVELOP** Figure 20.17 is a drawing, showing a coordinate system with the field point $P$ along the $y$-axis. We divide the wire into small charge elements $dq$ and note that field contributions from two such elements $dq$ on opposite sides of the $y$-axis contribute fields $d\vec{E}$ whose $x$-components cancel. Then we need only the $y$-component of each unit vector, and Fig. 20.17 shows that's $\hat{r}_y = y/r$, where $r = \sqrt{x^2 + y^2}$ .

**EVALUATE** Our wire has charge density $\lambda$ C/m, so if a charge element has length $dx$, then its charge is $dq = \lambda dx$. Putting all this together gives the $y$-component of the field from an arbitrary $dq$ anywhere on the wire:

$$dE_y = \frac{k\,dq}{r^2}\hat{r}_y = \frac{k\lambda\,dx}{r^2}\frac{y}{r} = \frac{k\lambda y}{\left(x^2+y^2\right)^{3/2}}dx$$

Since the $x$-components cancel, we can sum—that is, integrate—the $y$-components to get the net field:

$$E = E_y = k\lambda y\int_{-\infty}^{+\infty}\frac{dx}{\left(x^2+y^2\right)^{3/2}} = k\lambda y\left[\frac{x}{y^2\sqrt{x^2+y^2}}\right]_{-\infty}^{+\infty}$$

$$E = k\lambda y\left[\frac{1}{y^2} - \left(-\frac{1}{y^2}\right)\right] = \frac{2k\lambda}{y}$$

Our result is the field's magnitude, but we know that the field lines point toward the wire, meaning charge on the wire is negative. Solving the for the charge $Q$, and using the given values for the electric field and the length of the wire we obtain:

$$Q = \frac{-EyL}{2k} = -\frac{(455\text{ kN/C})(1.20\text{ cm})(2.18\text{ m})}{2\left(9.0\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)} = -661\text{ nC}$$

**ASSESS** Although the wire is not infinite, the dimensions of $y$ and $L$ vary by two orders of magnitude, making the approximation valid.

**42.** **INTERPRET** We identify the field point as being a distance $y$ from the wire, and the source charge as the whole wire.

**DEVELOP** Figure 20.29 shows a coordinate system with the field point $A$ along the $y$-axis. We divide the wire into small charge elements $dq$ and note that field contributions from two such elements $dq$ on opposite sides of the $y$-axis contribute fields $d\vec{E}$ whose $x$-components cancel. Then we need only the $y$-component of each unit vector, and Fig. 20.17 shows that's $\hat{r}_y = y/r$, where $r = \sqrt{x^2 + y^2}$ .

**EVALUATE** **(a)** Our wire has charge density $\lambda$ C/m, so if a charge element has length $dx$, then its charge is $dq = \lambda dx$. Putting all this together gives the $y$-component of the field from an arbitrary $dq$ anywhere on the wire:

$$dE_y = \frac{k\,dq}{r^2}\hat{r}_y = \frac{k\lambda\,dx}{r^2}\frac{y}{r} = \frac{k\lambda y}{\left(x^2+y^2\right)^{3/2}}dx$$

Since the $x$-components cancel, we can sum—that is, integrate—the $y$-components to get the net field:

$$E = E_y = \frac{kQy}{L}\int_{-L/2}^{L/2}\frac{dx}{\left(x^2+y^2\right)^{3/2}} = \frac{2kQy}{L}\left[\frac{x}{y^2\sqrt{x^2+y^2}}\right]_0^{L/2}$$

$$\vec{E}(y) = \frac{kQ}{y\sqrt{(L/2)^2 + y^2}}\,\hat{j} = \frac{2kQ}{y\sqrt{L^2+4y^2}}\,\hat{j}$$

Where we have chosen to perform the integral from 0 to $L/2$ and multiply the result by 2, since symmetry of the ensemble permits.

**(b)** In the case of $y >> L$ we can make the approximation: $\sqrt{L^2+4y^2} \cong 2y$, meaning the electric field at points far away from the rod is given by

$$\vec{E}(y >> L) \rightarrow \frac{2kQ}{y(2y)}\,\hat{j} = \frac{kQ}{y^2}\,\hat{j}$$

**ASSESS** As mentioned in the original example, far from a finite line, its field will resemble that of a point charge.

**43.** **INTERPRET** We identify the field point as being a distance $x$ from the center of the wire, and the source charge as the whole wire.

**DEVELOP** Figure 20.29 shows a coordinate system with the field point $B$ along the $x$-axis. We divide the wire into small charge elements $dq$ and note that field components along the $y$-direction do not contribute to the resultant field at point $B$. Then we need only the $x$-component of each unit vector.

**EVALUATE** **(a)** Our wire has charge density $\lambda$ C/m, so if a charge element has length $dx$, then its charge is $dq = \lambda dx$. Putting all this together gives the $x$-component of the field from an arbitrary $dq$ anywhere on the wire:

$$dE_x = \frac{k\,dq}{r^2}\hat{r}_x = \frac{k\lambda\,dx}{r^2} = \frac{k\lambda}{x^2}dx$$

We integrate along the $x$-direction to get the net field:

$$E = E_x = \frac{kQ}{L}\int_{x-L/2}^{x+L/2}\frac{dx}{x^2} = \frac{kQ}{L}\left[-\frac{1}{x}\right]_{x-L/2}^{x+L/2}$$

$$\vec{E}(x) = \frac{kQ}{L}\left[\frac{1}{(x-L/2)} - \frac{1}{(x+L/2)}\right]\hat{i} = \frac{4kQ}{\left(4x^2-L^2\right)}\hat{i}$$

**(b)** In the case of $x >> L$ we can make the approximation: $\left(4x^2-L^2\right) \cong 4x^2$ , meaning the electric field at points far away from the rod is given by

$$\vec{E}(x >> L) \to \frac{4kQ}{4x^2}\hat{i} = \frac{kQ}{x^2}\hat{i}$$

**ASSESS** As mentioned in the original example, far from a finite line, its field will resemble that of a point charge.

## PROBLEMS

**44.** **INTERPRET** This problem involves Coulomb's law, which we can use to relate the force experienced by the two particles to their charges.

**DEVELOP** Coulomb's law (Equation 20.1) gives the force between charged particles 1 and 2 as

$$F = \frac{kq_1q_2}{r^2}$$

We are given that $q_1 = 3q_2$ and $r = 14.5\text{ cm} = 0.145\text{ m}$, so we can solve for the magnitude of the larger charge $q_1$.

**EVALUATE** Substituting for $q_2$ in Coulomb's law and solving for $q_1$ gives

$$q_1q_2 = q_1\left(\frac{q_1}{3}\right) = \frac{Fr^2}{k}$$

$$q_1 = \pm r\sqrt{\frac{3F}{k}} = \pm(0.145\text{ m})\sqrt{\frac{3(156\text{ N})}{9.0\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2}} = \pm 33\ \mu\text{C}$$

**ASSESS** Because the force is attractive, the charges must have oppisite signs. However, from the information given in the problem statement, we cannot tell whether the sign is positive or negative.

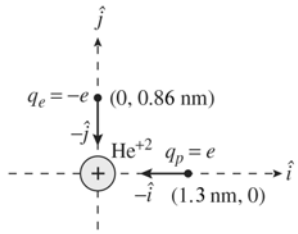
**45.** **INTERPRET** In solving this problem, we follow Problem-Solving Strategy 20.1. The source charges are the proton and the electron, and the charge on which the forces act is the helium nucleus (with charge $+2e$). Using the principle of superposition, we can sum the individual forces to find the total force.

**DEVELOP** Make a sketch of the situation that shows the charges and their positions (see figure below). The unit vector from the proton's position toward the origin is $-\hat{i}$. Using Equation 20.1, the Coulomb force exerted by the proton on the helium nucleus is

$$\vec{F}_{\text{P,He}} = \frac{kq_pq_{\text{He}}}{r_{p,\text{He}}^2}\left(-\hat{i}\right) = \frac{\left(9.0\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)(e)(2e)}{\left(1.3\times10^{-9}\text{ m}\right)^2}\left(-\hat{i}\right) = \left(-0.273\text{ nN}\right)\hat{i}$$

where we have retained an extra significant figure in the result because this is an intermediate result. Similarly, the unit vector from the electron's position to the origin is $-\hat{j}$, so the force it exerts on the helium nucleus is

$$\vec{F}_{e,\text{He}} = \frac{kq_e q_{\text{He}}}{r_{e,\text{He}}^2}(-\hat{j}) = \frac{\left(9.0\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(-e)(2e)}{\left(0.86\times10^{-9}\ \text{m}\right)^2}(-\hat{j}) = (0.623\ \text{nN})\,\hat{j}$$




**EVALUATE** To two significant figures, the net Coulomb force on the helium nucleus is the sum of these:

$$\vec{F}_{\text{net}} = \vec{F}_{p,\text{He}} + \vec{F}_{e,\text{He}} = (-0.273\ \text{nN})\,\hat{i} + (0.623\ \text{nN})\,\hat{j}$$

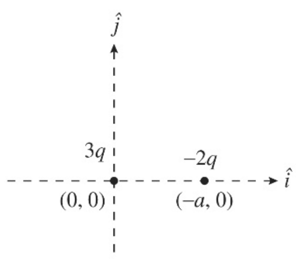
**ASSESS** In situations where the number of source charges is more than one, we apply the superposition principle and add the electric forces vector-wise. Because the electron is closer to the He nucleus than the proton in this problem $(r_{e,\text{He}} < r_{p,\text{He}})$, we expect $\left|\vec{F}_{p,\text{He}}\right| < \left|\vec{F}_{e,\text{He}}\right|$. The direction of the force is $\theta = \text{atan}(0.623/-0.273) = 114°$ with respect to the positive $x$-axis.

**46. INTERPRET** Coulomb's law applies here. Since more than one source charge is involved, we make use of the superposition principle to find the net force on the test charge.

**DEVELOP** Make a sketch of the situation (see figure below). By symmetry, the test charge must be placed on the axis; if not, it will experience a nonzero force in the $\hat{j}$ direction. If we place a charge to the left of the origin, it will experience a nonzero force because the charge $3q$ is both larger (in magnitude) and closer than the charge $-2q$, so it will always generate a greater force for all $x < 0$. If we place a positive (negative) test charge between the two charges, the net force will be to the right (left); that is, nonzero in both cases. Thus, the test charge $Q$ must be placed on the $x$-axis to the right of the $-2q$ charge; that is, at $x > a$. Using the principle of superposition (Equation 20.4) with Coulomb's law (Equation 20.1), the net Coulomb force on the test charge is

$$F_x = \frac{kQ(3q)}{x^2} + \frac{kQ(-2q)}{(x-a)^2}$$

Set $F_x = 0$ to solve for $x$.




**EVALUATE** The condition $F_x = 0$ implies that $3(x-a)^2 = 2x^2$, or $x^2 - 6xa + 3a^2 = 0$. Thus,

$$x = 3a \pm \sqrt{9a^2 - 3a^2} = \left(3 \pm \sqrt{6}\right)a$$

Only the solution $x = \left(3+\sqrt{6}\right)a = 5.45a$ is to the right of $x = a$.

**ASSESS** At $x = \left(3+\sqrt{6}\right)a$ the forces acting on $Q$ from $3q$ and $-2q$ exactly cancel each other. Notice that our result is independent of the sign and magnitude of the third charge $Q$.

**47. INTERPRET** This problem involves Coulomb's law and the principle of superposition, which we can use to find the condition such that the three given charges all experience zero net force.

**DEVELOP** By symmetry, the negative charge placed at the midpoint between the two positive charges experiences zero net force. On the other hand, the Coulomb force on $+Q$ is

$$F_x = \frac{kQ(-q)}{a^2} + \frac{kQ^2}{(2a)^2}$$

**EVALUATE** Setting $F_x = 0$, we obtain $Q = 4q$.

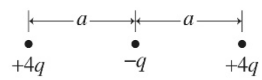
**ASSESS** To verify that we have the correct positioning, we calculate the net force on the left-hand charge. Coulomb's law and the superposition principle give

$$k\frac{4q^2}{a^2}\left(-\hat{i}\right) + k\frac{16q^2}{(2a)^2}\left(\hat{i}\right) = 0$$

The expression for the force on the right-hand charge is the same, except that the sign of the unit vectors is reversed. The force on the central charge is

$$k\frac{4q^2}{a^2}\left(-\hat{i}\right) + k\frac{4q^2}{a^2}\left(\hat{i}\right) = 0$$

Thus, all three charges experience zero net force. The situation is depicted below.




Note that the equilibrium is unstable, since if $-q$ is displaced slightly toward one charge, the net force on it will be in the direction of that charge.

**48.** **INTERPRET** More than one source charge is involved in this problem. Therefore, we use Coulomb's law and apply the superposition principle to find the net force on $q_3$.

**DEVELOP** We denote the positions of the charges by $\vec{r}_1 = (1\text{ m})\hat{j}$, $\vec{r}_2 = (2\text{ m})\hat{i}$, and $\vec{r}_3 = (2\text{ m})\hat{i} + (2\text{ m})\hat{j}$ (see figure below). The unit vector pointing from $q_1$ toward $q_3$ is

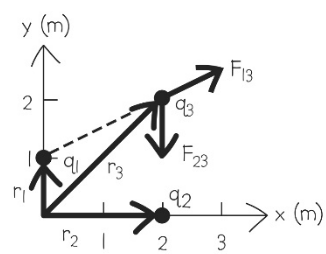
$$\hat{r}_{13} = \frac{(\vec{r}_3 - \vec{r}_1)}{|\vec{r}_3 - \vec{r}_1|}$$

Similarly, the unit vector pointing from $q_2$ toward $q_3$ is

$$\hat{r}_{23} = \frac{(\vec{r}_3 - \vec{r}_2)}{|\vec{r}_3 - \vec{r}_2|}$$

The vector form of Coulomb's law and the superposition principle give the net electric force on $q_3$ as

$$\vec{F}_3 = \vec{F}_{13} + \vec{F}_{23} = \frac{kq_1q_3(\vec{r}_3 - \vec{r}_1)}{|\vec{r}_3 - \vec{r}_1|^3} + \frac{kq_2q_3(\vec{r}_3 - \vec{r}_2)}{|\vec{r}_3 - \vec{r}_2|^3}$$




**EVALUATE** Substituting the values given in the problem statement, we find the force acting on $q_3$ to be

$$\begin{aligned}\vec{F}_3 &= \vec{F}_{13} + \vec{F}_{23} = \frac{kq_1q_3(\vec{r}_3 - \vec{r}_1)}{|\vec{r}_3 - \vec{r}_1|^3} + \frac{kq_2q_3(\vec{r}_3 - \vec{r}_2)}{|\vec{r}_3 - \vec{r}_2|^3} \\ &= \left(9.0\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(15\times10^{-6}\text{C}\right)\left[\frac{\left(68\times10^{-6}\text{C}\right)\left(2.0\hat{i} + 1.0\hat{j}\right)}{5.0\sqrt{5.0}\text{ m}^2} + \frac{\left(-34\times10^{-6}\text{C}\right)2\hat{j}}{8.0\text{ m}^2}\right] = \left(1.6\hat{i} - 0.33\hat{j}\right)\text{ N}\end{aligned}$$

or $F_3 = \sqrt{F_{3x}^2 + F_{3y}^2} = 1.7\text{ N}$ at an angle of $\theta = \tan^{-1}(F_{3y}/F_{3x}) = -11°$ to the $x$-axis.

**ASSESS** The force between $q_1$ and $q_3$ is repulsive ( $q_1q_3 > 0$ ) while the force between $q_2$ and $q_3$ is attractive ( $q_2q_3 < 0$ ). The two forces add vectorially to give the net force on $q_3$.

**49.** **INTERPRET** This problem is similar to the preceding one, only the magnitude of the charges has changed. Therefore, we can use the same strategy to solve this problem.

**DEVELOP** The positions of the charges are again denoted by $\vec{r}_1 = (1\text{ m})\hat{j}, \vec{r}_2 = (2\text{ cm})\hat{i},$ and $\vec{r}_3 = (2\text{ m})\hat{i} + (2\text{ m})\hat{j}.$ The unit vector pointing from $q_3$ toward $q_1$ is

$$\hat{r}_{31} = \frac{(\vec{r}_1 - \vec{r}_3)}{|\vec{r}_1 - \vec{r}_3|} = \frac{(1\text{ m})\hat{j} - (2\text{ m})\hat{i} - (2\text{ m})\hat{j}}{\sqrt{(-2)^2 + (-1)^2}\text{ m}} = -\frac{2}{\sqrt{5}} - \frac{1}{\sqrt{5}}\hat{j}$$

Similarly, the unit vector pointing from $q_2$ toward $q_1$ is

$$\hat{r}_{21} = \frac{(\vec{r}_1 - \vec{r}_2)}{|\vec{r}_1 - \vec{r}_2|} = \frac{(1\text{ m})\hat{j} - (2\text{ m})\hat{i}}{\sqrt{1^2 + (-2)^2}\text{ m}} = -\frac{2}{\sqrt{5}}\hat{i} + \frac{1}{\sqrt{5}}\hat{j}$$

The vector form of Coulomb's law and the superposition principle give the net electric force on $q_3$ as

$$\vec{F}_1 = kq_1\left[\frac{q_2\hat{r}_{21}}{r_{21}^2} + \frac{q_3\hat{r}_{31}}{r_{31}^2}\right] = kq_1\left[\frac{q_2\left(-2\hat{i} + \hat{j}\right)}{5^{3/2}\text{ m}^2} + \frac{q_3\left(-2\hat{i} + \hat{j}\right)}{5^{3/2}\text{ m}^2}\right]$$

We are told that the force on $q_1$ is in the $\hat{i}$ direction, so the $\hat{j}$ component of $\vec{F}_1$ must be zero. This gives

$$q_2 - q_3 = 0$$

**EVALUATE** **(a)** From the equation above, we find that $q_3 = q_2$, or $q_3 = 18\ \mu\text{C}$.

**(b)** Inserting the result from part **(a)** into the expression for $\vec{F}_1$ gives

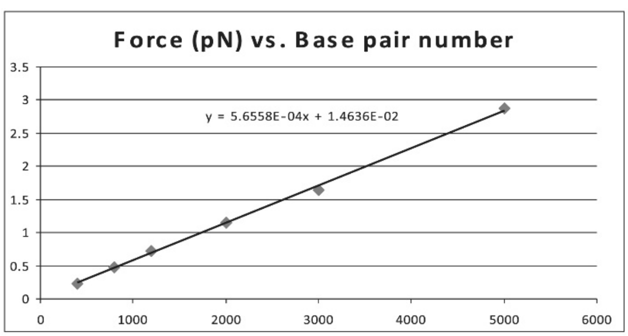
$$\vec{F}_1 = \left(9.0\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)(21\ \mu\text{C})\left(-2\hat{i}\right)(18\ \mu\text{C} + 18\ \mu\text{C})\left(5^{-3/2}\text{ m}^{-2}\right) = (-1.22)\hat{i}\text{ N}$$

**ASSESS** Because the charges $q_2$ and $q_3$ are positioned symmetrically above and below $q_1$, respectively, the result $q_2 = q_3$ is expected.

**50.** **INTERPRET** We are given the data of the electric forces on the DNA segments in an electrophoresis apparatus. We would like to determine the electric field strength of the apparatus based on the data given.

**DEVELOP** The electric charge $q$ carried by the DNA segments is directly proportional to the number of base pairs, with each base pair carrying a charge of $q = -2e$. Since $F = |q|E$, plotting $F$ versus the number of base pairs give a straight line, and the slope is equal to $2eE$, where $E$ is the field strength.

**EVALUATE** **(a)** A plot of the electric force versus the number of base pairs is given below.




The electric field is

$$E = \frac{(5.66\times10^{-4})\text{ pN}}{2(1.6\times10^{-19}\text{ C})} = \frac{5.66\times10^{-16}\text{N}}{2(1.6\times10^{-19}\text{ C})} = 1.77\times10^3\text{ N/C}$$

**ASSESS** Electrophoresis is a widely used application of electric fields for separating molecules by size and molecular weight. It's especially useful in biochemistry and molecular biology for distinguishing larger molecules like proteins and DNA fragments.

**51.** **INTERPRET** This problem involves two source charges, a proton at $x = 0$ and an ion at $x = 8.0$ nm. We can apply the principle of superposition to find the electric field at $x = -4.0$ nm.

**DEVELOP** The field at the point $x = -4$ nm due to the proton is

$$\vec{E}_p = \frac{ke}{(4\text{ nm})^2}(-\hat{i})$$

The field at the same point due to the ion is

$$\vec{E}_i = \frac{kq_i}{(12\text{ nm})^2}(-\hat{i})$$

The total electric field is the sum of these two and is zero at $x = -4.0$ nm, so we can solve for $q_i$.

**EVALUATE** Solving for the ion's charge gives

$$\frac{ke}{(4\text{ nm})^2}(-\hat{i}) + \frac{kq_i}{(12\text{ nm})^2}(-\hat{i}) = \vec{0}$$

$$q_i = -e\frac{(12\text{ nm})^2}{(4\text{ nm})^2} = -9e$$

**ASSESS** Note that the field due to the ion was defined for a positive charge, so the final charge is negative, as indicated.

**52.** **INTERPRET** The four charges $Q$ are positioned at each corner of a square of side $a$, and we can calculate the $x$- and $y$-components of the electrostatic force felt on one charge due to the others.

**DEVELOP** Since all the charges are equivalent they will each feel a repulsive force. The magnitude of this force is equal for all charges, but the directions depend on their position relative to the other charges. We pick the charge at the top right corner ($Q_1$) as our probe and calculate the forces in the $+x$ (right) and $+y$ (up) directions that it feels due to the other three charges using Equation 20.1. We designate the charges at the bottom right corner, bottom left corner, and top left corner as $Q_2$, $Q_3$, and $Q_4$, respectively.

**EVALUATE** Charges $Q_2$ and $Q_4$ lie a distance $a$ away from our test charge, and each contribute to the repulsive force along only one direction (namely the $+y$- and $+x$-directions, respectively). Charge $Q_3$ lies a distance $\sqrt{2}a$ away and contributes to the repulsive force along both the $+x$- and $+y$-directions, since its unit vector points at 45° relative to the $+x$-direction. Combining each contribution, we express the $x$- and $y$-components of the repulsive force as

$$\vec{F}_x = \vec{F}_{41} + \vec{F}_{31x} = \frac{kQ_4Q_1}{a^2}\hat{i} + \frac{kQ_3Q_1}{2a^2}\cos(45)\hat{i} = \frac{kQ^2}{a^2}\left(1 + \frac{\sqrt{2}}{4}\right)\hat{i}$$

$$\vec{F}_y = \vec{F}_{21} + \vec{F}_{31y} = \frac{kQ_2Q_1}{a^2}\hat{j} + \frac{kQ_3Q_1}{2a^2}\sin(45)\hat{j} = \frac{kQ^2}{a^2}\left(1 + \frac{\sqrt{2}}{4}\right)\hat{j}$$

We can then calculate the magnitude of the resulting repulsive force to be

$$F = \sqrt{F_x{}^2 + F_y{}^2} = \sqrt{2\frac{k^2Q^4}{a^4}\left(1 + \frac{\sqrt{2}}{4}\right)^2} = \frac{\left(1 + 2\sqrt{2}\right)kQ^2}{2a^2}$$

**ASSESS** We could have chosen any of the charges as our probe charge since the magnitude of the force felt every charge is equivalent.

**53.** **INTERPRET** This problem is that same as Example 20.5, except that it is rotated by 90°. There are two source particles: a proton at (0, 0.60 nm) and an electron at (0, −0.60 nm), so the system is dipole.

**DEVELOP** We can use the result of Example 20.5, with $y$ replaced by $x$, and $x$ by $-y$ (or, equivalently, $\hat{j}$ by $\hat{i}$, and $\hat{i}$ by $-\hat{j}$). The electric field on the $x$-axis is then

$$\vec{E}(x) = 2kqa\,\hat{j}\left(a^2 + x^2\right)^{-3/2}$$

where $q = e = 1.6\times10^{-19}$ C and $a$ = 0.60 nm (see Fig. 20.12 rotated 90° clockwise). The constant $2kq = 2\left(9.0\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.6\times10^{-19}\ \text{C}\right) = (2.88\ \text{GN/C})(\text{nm})^2$.

**EVALUATE** **(a)** Midway between the two charges (at $x$ = 0), the electric field is

$$\vec{E}(0,0) = \frac{2kq\hat{j}}{a^2} = \frac{(2.88\ \text{nm}^2\cdot\text{GN/C})\,\hat{j}}{(0.60\ \text{nm})^2} = (8.0\ \text{GN/C})\,\hat{j}$$

**(b)** for $x$ = 2 nm,

$$\vec{E}(2.0\ \text{nm},0) = (2.88\ \text{nm}^2\cdot\text{GN/C})(0.60\ \text{nm})\left(0.60^2 + 2.0^2\right)^{-3/2}(\text{nm})^{-3}\left(\hat{j}\right) = (190\ \text{MN/C})\,\hat{j}$$

**(c)** For $x$ = 20 nm,

$$\vec{E}(-20\ \text{nm},0) = \left(2.88\ \text{nm}^2\cdot\text{GN/C}\right)(0.60\ \text{nm})\left[0.60^2 + (-20)^2\right]^{-3/2}\left(\hat{j}\right) = (215.71\ \text{kN/C}) = \hat{j}(220\ \text{kN/C})\,\hat{j}.$$

to two significant figures.

**ASSESS** For part **(c)**, because $x \gg a$, we can apply Equation 20.6a, which gives

$$\vec{E}(-20\ nm,0) = -\frac{kp}{x^3}\hat{j} = -\frac{2kqa}{x^3}\hat{j} = -\frac{2\left(2.88\ \text{nm}^2\cdot\text{GN/C}\right)(-20\ \text{nm})}{(-20\ \text{nm})^3} = 216.0\ \text{kN/C}$$

which differs by only 0.1% from the more precise result of part **(c)**.

**54.** **INTERPRET** We find the electric field on the axis of a dipole, and show that Equation 20.6b is correct. To do this we will use the equation for electric field.

**DEVELOP** The spacing between the + and – charges is $2a$. We will use $E = k\dfrac{q}{r^2}$ for each charge to find the total field at a point $x \gg a$.

**EVALUATE**

$$\vec{E} = k\frac{+q}{(x-a)^2}\hat{i} + k\frac{-q}{(x+a)^2}\hat{i} = kq[(x-a)^{-2} - (x+a)^{-2}]\hat{i}$$

$$\rightarrow \vec{E} = \frac{kq}{x^2}\hat{i}\left[\left(1-\frac{a}{x}\right)^{-2} - \left(1+\frac{a}{x}\right)^{-2}\right]$$

For $x \gg a$, $\left(1 \pm \dfrac{a}{x}\right)^{-2} \approx 1 \mp 2\dfrac{a}{x}$, so

$$\vec{E} \approx \frac{kq}{x^2}\hat{i}[\left(1+2\frac{a}{x}\right) - \left(1-2\frac{a}{x}\right)] = \frac{kq}{x^2}\left(4\frac{a}{x}\right) = 2\frac{k(2qa)}{x^3}\hat{i}.$$

But $p = qd = 2qa$, so $\vec{E} = \dfrac{2kp}{x^3}\hat{i}$.

**ASSESS** We have shown what was required.

**55.** **INTERPRET** This problem involves finding the net charge or an unknown charge distribution. We are given the behavior of the electric field as a function of distance from the charge, for distances much, much greater than the size of the charge distribution.

**DEVELOP** Taking the hint, we suppose that the field strength varies with an inverse power of the distance, $E(r) \approx r^n$. Under this hypothesis, 296/87.7 = $(1.44/2.16)^n$, or

$$n = \ln(296/87.7)/\ln(1.44/2.16) = -3.0.$$

**EVALUATE** A dipole field falls off like $r^{-3}$ for $r >> a$, so the charge distribution must be a dipole, whose net charge is zero.

**ASSESS** Note that this result is only valid if the dipole separation $a \ll 1$ m. Because this is normally the case, the result is appears valid.

**56.** **INTERPRET** Coulomb's law and the principle of superposition applies here. There are three source charges, and we are to find the field strength at a point on the $y$-axis above the upper-most charge.

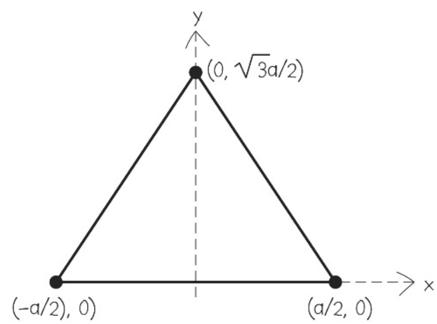
**DEVELOP** Make a sketch of the situation (see figure below). The electric field on the $y$-axis (for $y > \sqrt{3}a/2$) due to the two charges on the $x$-axis follows from Example 20.2. The only difference is that in this problem, the charges on the $x$-axis are separated by $a$ instead of $2a$. Thus,

$$\vec{E}_1 = \frac{2kqy}{\left(y^2 + a^2/4\right)^{3/2}}\hat{j}$$

On the other hand, using Equation 20.3, we find the electric field due to the charge on the $y$-axis for $y > \sqrt{3}a/2$ is

$$\vec{E}_2 = \frac{kq}{\left(y - \sqrt{3}a/2\right)^2}\hat{j}$$

Using the principle of superposition, the total field for $y > \sqrt{3}a/2$ is the sum of $\vec{E}_1$ and $\vec{E}_2$.




**EVALUATE** **(a)** For $y > \sqrt{3}a/2,$ the total field is simply the sum of both terms:

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = kq\left[\frac{2y}{\left(y^2 + a^2/4\right)^{3/2}} + \frac{1}{\left(y - \sqrt{3}a/2\right)^2}\right]\hat{j}$$

**(b)** For $y \gg a,$ the electric field may be approximated as

$$E \approx kq\left[\frac{2y}{\left(y^2\right)^{3/2}} + \frac{1}{y^2}\right]\hat{j} = \frac{3kq}{y^2}\hat{j}$$

which is like the electric field due to a point charge of magnitude $3q$.

**ASSESS** At large distances much, much greater than $a$, the charge distribution looks like a point charge located at the origin with charge $3q$.

**57.** **INTERPRET** This problem involves Coulomb's law, which generates the given forces between two charged metal spheres. The spheres initially experience an attractive force, but when the charge on the spheres is equilibrated, the force becomes repulsive.

**DEVELOP** The charges initially attract, so $q_1 = -q_2$, and, by Coulomb's law (Equation 20.1), we have

$$2.5\ \mathrm{N} = -\frac{kq_1q_2}{1\ \mathrm{m}^2}$$

When the spheres are brought together, they share the total charge equally, each acquiring $\frac{1}{2}(q_1 + q_2)$. The magnitude of their repulsion is

$$2.5\ \mathrm{N} = k\frac{(q_1 + q_2)^2}{4\ \mathrm{m}^2}$$

Because the forces have the same magnitude, we can equation them to find the original charges $q_1$ and $q_2$.

**EVALUATE** Equating these two forces, we find a quadratic equation $\frac{1}{4}(q_1+q_2)^2 = -q_1q_2$, or $q_1^2+6q_1q_2+q_2^2=0$, with solutions $q_1 = \left(-3\pm\sqrt{8}\right)q_2.$ Both solutions are possible, but since $3+\sqrt{8} = \left(3-\sqrt{8}\right)^{-1}$, they merely represent a relabeling of the charges. Since $-q_1q_2 = \left(2.5\ \text{N}\cdot\text{m}^2\right)/\left(9.0\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right) = \left(1.67\ \mu\text{C}\right)^{\mathbf{2}}$, the solutions are $q_1 = \pm\sqrt{3+\sqrt{8}}\left(16.7\ \mu\text{C}\right) = \pm40\ \mu\text{C}$ and $q_2 = \mp\left(40.2\ \mu\text{C}\right)\Big/\left(3+\sqrt{8}\right) = \mp6.9\ \mu\text{C}$, or the same values with $q_1$ and $q_2$ interchanged.

**ASSESS** The results are reported to two significant figures, which is the precision to which the data is known.

**58.** **INTERPRET** Two forces are involved in this problem: the Coulomb force and the spring force. The spring is stretched due to the Coulomb repulsion between the charges, and we are to find by how much the spring stretches.

**DEVELOP** The Coulomb force is given by Equation 20.1, and the spring force is given by Equation 4.9, $F_\text{s} = -kx$, where $x$ is the displacement from equilibrium of the spring. We assume that the Coulomb repulsion, $F_e$, is the only force stretching the spring. When balanced with the spring force, $F_e = F_\text{s}$, or

$$\frac{kq^2}{(L_0+x)^2} - k_\text{s}x = 0$$

where $L_0$ is the equilibrium length. This cubic equation can be solved by iteration or by Newton's method.

**EVALUATE** Substituting the values given in the problem statement gives

$$x(L_0+x)^2 = \frac{kq^2}{k_\text{s}}$$

$$x(0.493\ \text{m}+x)^2 = \frac{(9.0\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(32\ \mu\text{C})^2}{135\ \text{N/m}} = 6.83\times10^{-2}\ \text{m}^3$$

Newton's method yields $x = 16.0\ \text{cm}$, to two significant figures.

**ASSESS** Our result makes sense because the amount stretched is seen to decrease with increasing spring constant $k_\text{s}$ and increase with the magnitude of the charge $q$.

**59.** **INTERPRET** The two charges $q$ and $Q$ lie along the $x$-axis and generate an electric field which is equal to zero at another position along the $x$-axis. We can use the given charge $Q$ and the superposition principle to determine what value of $q$ would result in the scenario described in the problem.

**DEVELOP** Equation 20.3 gives the electric field of a point charge at a distance $r$ from the charge. We are told the position of charge $Q$ is the origin ($x = 0$), and that charge $q$ is located at $x = a$. The superposition principle tells us that the electric field of a distribution of charges is equal to the sum of the fields of the individual point charges, so we can express the electric field at $x = 2a$ generated from the two charges and set it equal to zero. We can then determine the relationship between the two charges $q$ and $Q$ from this equation.

**EVALUATE** Using Equation 20.4 to express the electric field at $x = 2a$ we obtain

$$E = E_Q + E_q = \frac{kQ}{(2a)^2} + \frac{kq}{a^2} = 0$$

Where we have dropped the vector notation since both charges lie along the $x$-axis, and their contribution to the electric field at the point of intersection only have an $x$-component. Solving for $q$ in the equation above we obtain

$$q = -Q/4$$

**ASSESS** In order for the electric field to equal zero at $x = 2a$, each charge needs to contribute equal and opposite amounts to the vector field. Thus, we find that the charge located twice as far has four times the charge.

**60.** **INTERPRET** The electron undergoes circular motion, where the centripetal force (Chapter 3) is provided by the Coulomb force in the form of an electric field around a long current-carrying wire.

**DEVELOP** The electric field of the wire is radial and falls off as $1/r$ ($E = 2k\lambda/r$; see Example 20.7). For an attractive force (negative electron encircling a positively charged wire), this is the same dependence as the centripetal acceleration (see Equation 3.9, $a_\text{c} = v^2/r$). For circular motion around the wire, the Coulomb force provides the electron's centripetal acceleration. Thus, applying Newton's second law gives

$$F = ma_c$$
$$-eE = m\frac{v^2}{r}$$
$$-\frac{2ke\lambda}{r} = m\frac{v^2}{r}$$

The equation can be used to deduce the tangential speed $v$ of the electron.

**EVALUATE** Substituting the values given, we find the speed to be

$$v = \sqrt{\frac{2ke\lambda}{m}} = \sqrt{\frac{2(9.0\times10^9\ \mathrm{N\cdot m^2/C^2})(1.6\times10^{-19}\ \mathrm{C})(1.4\times10^{-9}\ \mathrm{C/m})}{9.11\times10^{-31}\ \mathrm{kg}}} = 2.1\times10^6\ \mathrm{m/s}$$

**ASSESS** The speed of the electron is independent of $r$, the radial distance from the long wire. This is because both the electric field and the centripetal acceleration fall off as $1/r$, so the $r$-dependence cancels out.

**61.** **INTERPRET** The charge orbiting the wire undergoes circular motion, and the centripetal force (Chapter 3) is provided by the Coulomb force. We are asked to find the line charge density, given the particle's orbital speed. This problem is similar to Problem 20.60.

**DEVELOP** The electric field of the wire is radial and falls off as $1/r$ ($E = 2k\lambda/r$; see Example 20.7). For an attractive force (positive charge encircling a negatively charged wire), this is the same dependence as the centripetal acceleration (Equation 3.9, $a_c = v^2/r$). For circular motion around the wire, the Coulomb force provides the electron's centripetal acceleration. Thus, Newton's second law gives

$$F = qE = ma_c$$
$$a_c = \frac{v^2}{r} = \frac{qE}{m} = -\frac{2kq\lambda}{mr}$$

The equation can be used to deduce the line charge density $\lambda$, given the speed.

**EVALUATE** The above equation gives

$$\lambda = -\frac{mv^2}{2kq} = -\frac{(6.5\times10^{-9}\ \mathrm{kg})(270\ \mathrm{m/s})^2}{2(9.0\times10^9\ \mathrm{N\cdot m^2/C^2})(2.2\times10^{-9}\ \mathrm{C})} = -12\ \mu\mathrm{C/m}$$

**ASSESS** For the force to be attractive, the line charge density must be negative.

**62.** **INTERPRET** For this problem, we are to find the torque on the given dipole placed at $30^\circ$ with respect to a linear electric field (similar to what is shown in Fig. 20.20). We are also to find the work done by the electric field in orienting the dipole so that it is antiparallel to the field.

**DEVELOP** The torque on the dipole is given by Equation 20.9,

$$\tau = |\vec{p}\times\vec{E}| = pE\sin\theta$$

The work done in orienting the dipole antiparallel to the field is the change in the potential energy from the initial to the final states, where the potential energy of a dipole in an electric field is given by Equation 20.10. Thus,

$$W = \Delta U = (-\vec{p}\cdot\vec{E})_f - (-\vec{p}\cdot\vec{E})_i$$

**EVALUATE** **(a)** Evaluating the expression for torque gives

$$\tau = pE\sin\theta = (1.6\ \mathrm{nC\cdot m})(5.0\ \mathrm{MN/C})\sin(30^\circ) = 4.0\ \mathrm{mN\cdot m}$$

**(b)** Evaluating the expression for work gives

$$W = pE(\cos30^\circ - \cos180^\circ) = (1.6\ \mathrm{nC\cdot m})(5.0\ \mathrm{MN/C})(1.866) = 15\ \mathrm{mJ}$$

**ASSESS** When the dipole is oriented antiparallel to the field, the torque is also zero, but this is an unstable equilibrium because the slightest perturbation will allow the dipole to reverse its orientation in the electric field.

**63.** **INTERPRET** You want to check if a patent for an isotope separator will work. Since different isotopes have the same charge but different mass, the device can work if it discriminates between objects with different charge-to-mass ratios.

**DEVELOP** You can assume the accelerating field, $\vec{E}_1$, is constant and that the plates in the figure are separated by a distance *x*. Therefore, if an atom stripped of its electrons starts at rest at the bottom plate, it will be accelerated by the field to a final speed of

$$v = \sqrt{2ax} = \sqrt{\frac{2qE_1x}{m}}$$

So, it is true that isotopes of different charge-to-mass ratios $(q/m)$ will leave the first half of the device with different speeds. For example, an isotope with a relatively large charge-to-mass ratio will attain a higher speed in the accelerating field than another isotope with a lower charge-to-mass ratio. But the question is: can the second half of the device select just one of these speeds so that only one type of isotope emerges?

**EVALUATE** The second half of the device is an electrostatic analyzer, as described in Example 20.8. It has a curved field, $\vec{E}_2 = E_0(b/r)\hat{r}$, which points toward the center of curvature. The parameters $E_0$ and $b$ are constants with units of electric field and distance, respectively. It was shown in the text that particles entering the device from below will only emerge from the horizontal outlet if their speed satisfies:

$$v = \sqrt{\frac{qE_0b}{m}}$$

If you equate this speed with the speed from the accelerating field, you find that the charge-to-mass ratio cancels out. This means there's no discrimination between isotopes. If one type of isotope can emerge, then they all can. The device doesn't work.

**ASSESS** The problem with this device is that both the accelerating field and the curving field depend on the charge-to-mass ratio in the same way. One way to get around this is to accelerate the isotopes by heating them to high temperature. The speeds in this case won't depend on the charge. Another way is to use a magnetic field to curve the path of the isotopes. In this case, the charge-to-mass ratio doesn't cancel out, as we'll see in Example 26.2.

**64.** **INTERPRET** The problem asks us to find the electric field near a strand of DNA.

**DEVELOP** We're looking for the electric field at a distance $y = 21$ nm from a strand of DNA that has length $L = 5.0$ µm. Since the point we're considering is not near either end of the strand and since $y/L \;\; 0.004 \ll 1$, the situation is approximately the same as that for an infinite wire. In Example 20.7, it was shown that the electric field at a distance *y* from an infinite wire has magnitude $E = 2k/y$, where $\lambda$ is the charge per unit length.

**EVALUATE** We're told that the DNA carries +*e* of charge for every nm of length, so $\lambda = e/\text{nm}$. Using the above formula*:*

$$E = \frac{2k\lambda}{y} = \frac{2\left(9.0\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left(1.60\times10^{-19}\ \text{C/nm}\right)}{(21\ \text{nm})} = 0.14\ \text{GN/C}$$

**ASSESS** Since the charge per unit length is positive, the electric field will point outward from the DNA strand, as in Fig. 20.17. The actual field will deviate from this picture near either end. But we can often get away with treating a wire or a sheet as infinite, as long as we consider only points that are short distances away.

**65.** **INTERPRET** In this problem we want to determine the energy change experienced by a dipole as it aligns with an oscillating electric field.

**DEVELOP** Equation 20.10 gives the potential energy of a dipole in the presence of an electric field. We are told the dipole is initially aligned opposite the electric field, and we are interested in calculating the energy change as it swings to align with the field. We can calculate this change in energy by taking the difference between the initial and final potential energies felt by the dipole before and after it has been rotated. We can then determine the relationship between the two charges *q* and *Q* from this equation.

**EVALUATE** Express the energy change as the difference between the initial and final potential energies

$$\Delta U = \left|U_\text{f} - U_\text{i}\right| = \left|(-pE\cos(0)) - (-pE\cos(180))\right| = 2pE$$

Plugging in the given values for the dipole moment and electric field then gives an energy change equal to

$$\Delta U = 2pE = 2(6.17\times10^{-30}\ \text{C}\cdot\text{m})(2.95\text{kN/C}) = 3.64\times10^{-26}\ \text{J}$$

**ASSESS** The sign of the potential energy change is negative since the dipole is going from a state of higher potential energy to a state of lower potential energy as it swings to align with the direction of the electric field. Once the direction of the oscillating field switches, the dipole will again be in a state of higher potential energy and will again undergo a rotation to lower its potential energy. This in turn is introducing kinetic energy into the system each cycle, and results in the heat that is given to the food placed inside a microwave oven.

**66.** **INTERPRET** This problem is about the interaction between a dipole and the electric field due to a source charge.

**DEVELOP** With the $x$-axis in the direction from $Q$ to $\vec{p}$ and the $y$-axis parallel to the dipole in Fig. 20.30, we have $\vec{p} = (2qa)\hat{j}$ and $E = (kQ/x^2)\hat{i}$. In the limit $x \gg a$, the torque on the dipole is given by Equation 20.9, $\vec{\tau} = \vec{p}\times\vec{E}$, where $\vec{E}$ is the field from the point charge $Q$, at the position of the dipole.

**EVALUATE** **(a)** Using Equation 20.9, we find the torque to be

$$\vec{\tau} = \vec{p}\times\vec{E} = (2qa\hat{j})\times\left(\frac{kQ}{x^2}\hat{i}\right) = -\frac{2kQqa}{x^2}\hat{k}$$

The direction is into the page, or clockwise, to align $\vec{p}$ with $\vec{E}$.

**(b)** The Coulomb force obeys Newton's third law. The field of the dipole at the position of $Q$ is (Example 20.5 adapted to new axes)

$$\vec{E}_{\text{dipole}} = -\frac{2kqa}{x^3}\hat{j}$$

Thus, the force on $Q$ due to the dipole is

$$\vec{F}_{\text{on }Q} = Q\vec{E}_{\text{dipole}} = -\frac{2kQqa}{x^3}\hat{j}$$

The force on the dipole due to $Q$ is the opposite of this:

$$\vec{F}_{\text{on dipole}} = -\vec{F}_{\text{on }Q} = \frac{2kQqa}{x^3}\hat{j}$$

The magnitude of $\vec{F}_{\text{on dipole}}$ is $2kQqa/x^3$.

**(c)** The direction of $\vec{F}_{\text{on dipole}}$ is in $+\hat{j}$, or parallel to the dipole moment.

**ASSESS** The net force $\vec{F}_{\text{on dipole}}$ will cause the dipole to move in the $+\hat{j}$ direction. In addition, there is a torque that tends to align $p$ with $E$. So, the motion of the dipole involves both translation and rotation.

**67.** **INTERPRET** You're asked to estimate the charge distribution on two different molecules given their dipole moments.

**DEVELOP** Your given the dipole moment, $p$, for $H_2O$ and CO in debyes. A debye (D) is a unit with dimensions of "charge" times "distance." You can look in an outside reference and see that $1\ \text{D} = 3.34\times10^{-30}\text{C}\cdot\text{m}$. You can model these dipole molecules as two opposite charges, $\pm q$, separated by a distance $d$. Since the atomic separation for many molecules is about an Angstrom $\left(1\ \text{Å} = 10^{-10}\text{m}\right)$, we can estimate how the charge is distributed on each molecule: $q = p\,/\,d$.

**EVALUATE** Let's first convert the dipole moments to SI units:

$$p_{\text{H}_2\text{O}} = (1.85\ \text{D})\left(\frac{3.34\times10^{-30}\text{C}\cdot\text{m}}{1\ \text{D}}\right) = 6.18\times10^{-30}\text{C}\cdot\text{m}$$

$$p_{\text{CO}} = (0.12\ \text{D})\left(\frac{3.34\times10^{-30}\text{C}\cdot\text{m}}{1\ \text{D}}\right) = 4.00\times10^{-31}\text{C}\cdot\text{m}$$

Assuming the atoms in the molecules are separated by about 1 Angstrom, the amount of charge on each "atom" is

$$q_{H_2O} = \frac{p}{d} = \frac{6.18\times10^{-30}\,\text{C}\cdot\text{m}}{10^{-10}\,\text{m}}\left(\frac{1\,e}{1.6\times10^{-19}\,\text{C}}\right) = 0.4\ e$$

$$q_{CO} = \frac{p}{d} = \frac{4.00\times10^{-31}\,\text{C}\cdot\text{m}}{10^{-10}\,\text{m}}\left(\frac{1\,e}{1.6\times10^{-19}\,\text{C}}\right) = 0.03\ e$$

We've written the result in terms elementary charge.

**ASSESS** In the case of water, the oxygen atom partially "steals" the electrons from the hydrogen atoms. That results in a negative fractional charge $(q \approx -0.4e)$ on the oxygen atom, and a correspondingly positive fractional charge on the hydrogen atoms. A similar situation occurs with carbon monoxide, but this time the carbon atom is the more electrophilic ("electron-loving") species in the covalent bond, so it will have the negative fractional charge and the oxygen atom will have the positive one.

**68.** **INTERPRET** This problem involves the electric field due to a charged ring. We are to find the ring's radius and the total charge on the ring, given the magnitude and direction of the electric field at two locations on the axis of the ring.

**DEVELOP** The electric field on the axis of a uniformly charged ring of radius $a$ is calculated in Example 20.6. The result is

$$E(x) = \frac{kQx}{(x^2+a^2)^{3/2}}$$

Knowing the field strengths at two different values of $x$ allows us to deduce $a$ and $Q$.

**EVALUATE** **(a)** The data given in the problem statement imply

$$E_1 = 340\ \text{kN/C} = \frac{kQ(10\ \text{cm})}{\left[(10\ \text{cm})^2 + a^2\right]^{3/2}}$$

$$E_2 = 110\ \text{kN/C} = \frac{kQ(25\ \text{cm})}{\left[(25\ \text{cm})^2 + a^2\right]^{3/2}}$$

Dividing these two equations and taking the 2/3 root gives

$$\left(\frac{340\times25}{110\times10}\right)^{2/3} = 3.91 = \frac{(25\ \text{cm})^2 + a^2}{(10\ \text{cm})^2 + a^2}$$

which, when solved for the radius $a$, gives

$$a = \sqrt{\frac{(25\ \text{cm})^2 - (3.91)(10\ \text{cm})^2}{2.91}} = 9.0\ \text{cm}$$

**(b)** To calculate $Q$, we substitute the result for $a$ into either one of the field equations above. This leads to

$$Q = \frac{(340\ \text{kN/C})\left[(10\ \text{cm})^2 + (9.0\ \text{cm})^2\right]^{3/2}}{(9.0\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(10\ \text{cm})} = 920\ \text{nC}$$

to two significant figures.

**ASSESS** To check that our results are correct, we may substitute the values obtained for $a$ and $Q$ into the field equation to calculate $E_1$ and $E_2$ at $r_1 = 10$ cm and $r_2 = 25$ cm. Note that the field strength decreases as $x$ is increased. For example, the equation for $E_2$ gives

$$E_2 = \frac{(9.0\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(916\ \text{nC})(0.25\ \text{m})}{\left[(0.25\ \text{m})^2 + (0.09\ \text{m})^2\right]^{3/2}} = 110\ \text{kN/C}$$

which agrees with the initial value.

**69.** **INTERPRET** This problem involves three source charges positioned on a line. We are to find the electric field on this line to the right of the right-most charge and show that this expression reduces to $\kappa/x^4$ for $x \gg a$, where $\kappa$ is a constant and $a$ is the characteristic size of the source-charge distribution.

**DEVELOP** The electric field of a single charge is given by Equation 20.3. Apply the principle of superposition to find the electric field due to three charges. This gives

$$\vec{E}(x) = k\hat{i}\left[\frac{q}{(x-a)^2} - \frac{2q}{x^2} + \frac{q}{(x+a)^2}\right]$$

**EVALUATE** **(a)** Simplifying the expression above for the electric field gives

$$\vec{E}(x) = 2kqa^2\frac{(3x^2-a^2)}{x^2(x^2-a^2)^2}(\hat{i})$$

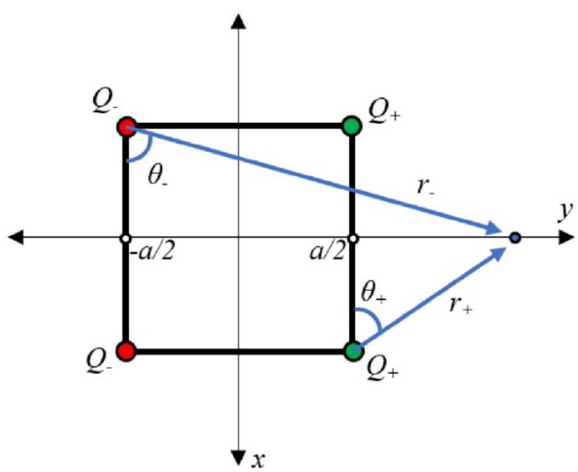
**(b)** For $x \gg a$, we can neglect the $a$ compared to $x^2$. This gives

$$\vec{E}(x) \approx \frac{6kqa^2}{x^4}(\hat{i})$$

**ASSESS** The quadrupole moment of this "linear quadrupole" is $Q_{xx} = 4qa^2$.

**70.** **INTERPRET** In this problem we want to calculate the electric field due to an ensemble of point charges. The four charges are positioned at each corner of a square of side $a$, and we can calculate the total electric field at points along the $y$-axis using the superposition principle.

**DEVELOP** Since all the charges are equivalent in magnitude, but half are positive, and half are negative, we will label quantities with ± when referring to charges with positive or negative sign. The figure below shows the locations of the positive and negative charges described in the problem statement, as well as the vectors which point from their locations to an arbitrary point located along the $y$-axis with $y > a/2$. Due to the symmetry of the ensemble, the $x$-components of the electric field generated by the positive and negative pairs will sum to zero, respectively. Only the $y$-components from each charge will contribute to the resultant electric field, which will point along the $y$-axis. To determine what this field is equal to, we use Equation 20.4 to express the value of the electric field at a point with $y > a/2$.




**EVALUATE** **(a)** Dropping the vector notation, since the resultant field contributions all point along the $y$-direction, we express the electric field at a point with $y > a/2$ as

$$E_{y>a/2} = E_+ + E_- = \frac{2kQ_+}{r_+^2}\sin\theta_+ + \frac{2kQ_-}{r_-^2}\sin\theta_-$$

Where $r_\pm = \sqrt{\left(\frac{a}{2}\right)^2 + \left(y \mp \frac{a}{2}\right)^2}$ and $\sin\theta_\pm = \frac{\left(y \mp \frac{a}{2}\right)}{r_\pm}$. Plugging these into the expression above, we obtain

$$E(y)=2kQ\left(\frac{\left(y-\frac{a}{2}\right)}{r_+^3}-\frac{\left(y+\frac{a}{2}\right)}{r_-^3}\right)=\frac{2kQ\left(y-\frac{a}{2}\right)}{\left(\left(\frac{a}{2}\right)^2+\left(y-\frac{a}{2}\right)^2\right)^{3/2}}-\frac{2kQ\left(y+\frac{a}{2}\right)}{\left(\left(\frac{a}{2}\right)^2+\left(y+\frac{a}{2}\right)^2\right)^{3/2}}$$

**(b)** In the case of $y >> a$ we can make the approximation: $\left(\left(\frac{a}{2}\right)^2+\left(y\pm\frac{a}{2}\right)^2\right)^{3/2}\cong\left(y\pm\frac{a}{2}\right)^3$, meaning the electric field becomes: $E(y>>a)=\frac{2kQ\left(y-\frac{a}{2}\right)}{\left(y-\frac{a}{2}\right)^3}-\frac{2kQ\left(y+\frac{a}{2}\right)}{\left(y+\frac{a}{2}\right)^3}=2kQ\left(\frac{1}{\left(y-\frac{a}{2}\right)^2}-\frac{1}{\left(y+\frac{a}{2}\right)^2}\right)$

$$E(y>>a)=2kQ\frac{\left(y+\frac{a}{2}\right)^2-\left(y-\frac{a}{2}\right)^2}{\left(y^2-\left(\frac{a}{2}\right)^2\right)^2}\cong 2kQ\frac{2ay}{y^4}=\frac{4kQa}{y^3}$$

Which exhibits the $1/y^3$ falloff expected for an electric dipole.

**(c)** Comparing this to Equation 20.6b, we note that the magnitude of this electric dipole is $p=2Qa$, giving an electric field on the dipole axis equal to: $\vec{E}=\frac{2k(2Qa)}{\left|y^3\right|}\hat{j}$

**Assess** The resultant dipole moment makes sense since along the $y$-axis, from far away ($y >> a$), the charge distribution appears to be made up of $2Q$ and $-2Q$ charges separated by a distance $a$.

**71.** **Interpret** This problem involves the electric field due to a 12-m-long straight wire, which is our source charge.

**Develop** For a uniformly charged wire of length $L$ and charge $Q$, the line density is $\lambda = Q/L$. Approximating the wire as infinitely long, the electric field due to the line charge can be written as (see Example 20.7)

$$E=\frac{2k\lambda}{r}$$

**Evaluate** **(a)** The charge density is

$$\lambda=\frac{Q}{L}=\frac{28\ \mu\text{C}}{12\ \text{m}}=2.3\ \mu\text{C/m}$$

**(b)** Since $r = 20$ cm << 12 m = $L$ and the field point is far from either end, we may regard the wire as approximately infinite. Then Example 20.7 gives

$$E=\frac{2k\lambda}{r}=\frac{2\left(9.0\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(2.3\ \mu\text{C/m}\right)}{0.2\ \text{m}}=207\ \text{kN/C}$$

**(c)** At $r = 450$ m and $L = 12$ m, the wire behaves approximately like a point charge, so the field strength is

$$E=\frac{kQ}{r^2}=\frac{\left(9.0\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(28\ \mu\text{C}\right)}{\left(450\ \text{m}\right)^2}=1.24\ \text{N/C}$$

**Assess** The finite-size, line-charge distribution looks like a point charge at large distances.

**72.** **Interpret** In this problem we want to calculate the electric field due to two objects with continuous charge distributions, which we can calculate using the superposition principle.

**Develop** Since the charge distributions on the rods are equivalent in magnitude, but one is positive, and the other is negative, we will label quantities with ± when referring to charge distributions with positive or negative sign. Figure 20.34 shows the locations of the positive and negative charge distributions, from which we can see that due to the symmetry of the ensemble, only the $x$-components of the electric field generated by the rods will contribute to a resultant electric field pointing along the $x$-axis. To determine what this field is equal to, we use Equation 20.7 to calculate the value of the electric field for each distribution at a point with $x > a$.

**EVALUATE** **(a)** Dropping the vector notation, since the resultant field contributions all point along the $x$-direction, we express the electric field at a point with $x > a$ as

$$E_{x>a} = E_+ + E_- = k\int_{x-a}^{x}\frac{dQ_+}{x'^2} + k\int_{x}^{x+a}\frac{dQ_-}{x'^2}$$

Where $dQ_\pm = \dfrac{\pm Q}{a}dx$. Plugging this into the expression above and evaluating the integrals results in

$$E(x) = \frac{kQ}{a}\left[\left.\frac{-1}{x}\right|_{x-a}^{x} - \left.\frac{-1}{x}\right|_{x}^{x+a}\right] = \frac{kQ}{a}\left[\left(\frac{1}{x-a}-\frac{1}{x}\right)-\left(\frac{1}{x}-\frac{1}{x+a}\right)\right]$$

$$E(x) = \frac{kQ}{a}\left[\frac{a}{x(x-a)} - \frac{a}{x(x+a)}\right] = \frac{kQ}{a}\left(\frac{2a^2}{x(x^2-a^2)}\right) = \frac{2kQa}{(x^3-a^2x)}$$

**(b)** In the case of $x >> a$ we can make the approximation: $(x^3 - a^2x) \cong x^3$, meaning the electric field becomes

$$E(x >> a) = \frac{2kQa}{x^3}$$

Which exhibits the $1/x^3$ falloff expected for an electric dipole.

**(c)** Comparing this to Equation 20.6b, we note that the magnitude of this electric dipole is $p = Qa$, giving an electric field on the dipole axis equal to: $\vec{E} = \dfrac{2k(Qa)}{|x^3|}\hat{i}$

**ASSESS** The resultant dipole moment makes sense since along the $x$-axis, from far away ($x >> a$), the charge distribution appears to be made up of a $Q$ and a $-Q$ charge separated by a distance $a$.

**73.** **INTERPRET** In this problem we want to find the electric field due to a uniformly charged disk of radius $R$.

**DEVELOP** We take the disk to consist of a large number of annuli. With uniform surface charge density $\sigma$, the amount of charge on an area element $dA$ is $dq = \sigma dA.$ Our strategy is to first calculate the electric field $dE$ due to $dq$ at a field point on the axis, simplify with symmetry argument, and then integrate over the entire disk to get $E$.

**EVALUATE** **(a)** The area of an annulus of radii $R_1 < R_2$ is just $\pi(R_2^2 - R_1^2)$. For a thin ring, $R_1 = r$ and $R_2 = r + dr$, so the area is $\pi[(r+dr)^2 - r^2] = \pi(2rdr + dr^2)$. When $dr$ is very small, the square term is negligible, and $dA = 2\pi rdr$. (This is equal to the circumference of the ring times its thickness.)

**(b)** For surface charge density $\sigma$, $dq = \sigma dA = 2\pi\sigma rdr$.

**(c)** From Example 20.6, the electric field due to a ring of radius $r$ and charge $dq$ is

$$dE_x = k\frac{xdq}{(x^2+r^2)^{3/2}} = \frac{2\pi k\sigma xr}{(x^2+r^2)^{3/2}}dr$$

which holds for $x$ positive away from the ring's center. **(d)** Integrating from $r = 0$ to $R$, one finds

$$E_x = \int_0^R dE_x = 2\pi k\sigma x\int_0^R \frac{rdr}{(x^2+r^2)^{3/2}} = 2\pi k\sigma x\left.\frac{-1}{\sqrt{x^2+r^2}}\right|_0^R = 2\pi k\sigma\left[\frac{x}{|x|} - \frac{x}{(x^2+R^2)^{1/2}}\right]$$

For $x > 0$, $|x| = x$ and the field is

$$E_x = 2\pi k\sigma\left(1 - \frac{x}{\sqrt{x^2+R^2}}\right)$$

On the other hand, for $x < 0$, $|x| = -x$, the electric field is

$$E_x = 2\pi k\sigma\left(-1 + \frac{|x|}{\sqrt{x^2+R^2}}\right)$$

This is consistent with symmetry on the axis, since $E_x(x) = -E_x(-x)$.

**ASSESS** One may readily verify that (see Problem 71), for $x \gg R$, $E_x \approx \frac{kQ}{x^2}$. In other words, the finite-size charge distribution looks like a point charge at large distances.

**74.** **INTERPRET** We will find the electric field from an infinite sheet.

**DEVELOP** An infinite flat sheet is the same as an infinite flat disk (the shape is irrelevant when the dimensions extend to infinity). Thus, we can find the magnitude of the electric field from a uniformly charged infinite flat sheet by letting $R \to \infty$ in the result of Problem 20.73.

**EVALUATE** As shown in the previous problem, a charged disk centered at the origin and perpendicular to the $x$-axis will generate an electric field along the $x$-axis with magnitude:

$$E_x = 2\pi k\sigma\left(1 - \frac{x}{\sqrt{x^2 + R^2}}\right)$$

Where $\sigma$ is the charge density per unit area. If $R \to \infty$, then the second term in the parentheses goes to zero, and we're left with:

$$E_x = 2\pi k\sigma$$

**ASSESS** Strikingly, this result no longer depends on $x$, the distance from the sheet. The field is uniform, whether you are a nanometer or a light-year away. If the charge on the sheet is positive, the field points away from the sheet on both sides, and the opposite if the charge is negative.

**75.** **INTERPRET** In this problem we want to show that at large distances, the electric field due to a uniformly charged disk of radius $R$ reduces to that of a point charge.

**DEVELOP** The result of Problem 74 for the field on the axis of a uniformly charged disk, of radius $R$, at a distance $x > 0$ on the axis (away from the disk's center) is

$$E_x = 2\pi k\sigma\left(1 - \frac{x}{\sqrt{x^2 + R^2}}\right)$$

For $R^2/x^2 \ll 1$, we use the binomial expansion in Appendix A and write

$$\left(1 + \frac{R^2}{x^2}\right)^{-1/2} \approx 1 - \frac{1}{2}\frac{R^2}{x^2} +$$

**EVALUATE** Substituting the above expression into the first equation, we obtain

$$E_x = 2\pi k\sigma\left[1 - \left(1 + \frac{R^2}{x^2}\right)^{-1/2}\right] \approx 2\pi k\sigma\left[1 - \left(1 - \frac{1}{2}\frac{R^2}{x^2} + \cdots\right)\right] \approx \frac{2\pi k\sigma R^2}{2x^2} = \frac{kQ}{x^2}$$

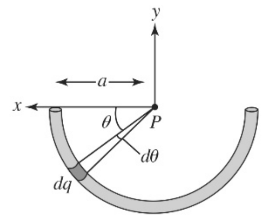
which is the field from a point charge $Q = \left(\pi R^2\right)\sigma = \sigma A$ at a distance $x$.

**ASSESS** The result once again demonstrates that any finite-size charge distribution looks like a point charge at large distances.

**76.** **INTERPRET** For this problem, we are to find the electric field at the center of a semicircular loop.

**DEVELOP** Begin by establishing a coordinate system (see figure below) with origin at point $P$, vertical $y$-axis, and horizontal $x$-axis. Then each charge element $dq$ creates an electric field of magnitude $dE = kdq/a^2$ in the direction $\hat{r} = \cos\theta\hat{i} + \sin\theta\hat{j}$. The total electric field at $P$ is then $\vec{E}(P) = \int dE\hat{r}$. The loop has a uniform charge $Q$ along its length of $\pi a$, so its linear charge density is $\lambda = Q/\pi a$ and we can write $dq = \lambda dl$. Expressing the line element as $dl = ad\theta$, we have $dq = \lambda a d\theta$. We can now formulate the integral for the total electric field in terms of $\theta$ starting with $dE = k\lambda d\theta/a$ :

$$\bar{E}(P) = \int_0^{\pi} \frac{k\lambda}{a} d\theta\hat{r}$$




**EVALUATE** Performing the integration gives

$$\bar{E}(P)=\int_0^{\pi}\frac{k\lambda}{a}d\theta\hat{r}=\frac{k\lambda}{a}\left[\int_0^{\pi}\cos\theta d\theta\hat{i}+\int_0^{\pi}\sin\theta d\theta\hat{j}\right]=\frac{2k\lambda}{a}\hat{j}=\frac{2kQ}{\pi a^2}\hat{j}$$

**ASSESS** The field decreases as $1/a^2$, and increases proportional to the total charge $Q$.

**77.** **INTERPRET** We are to find the electric field at any point $(x, y)$ due to a uniformly charged rod of length $L$ and charge $Q$. We will check our answer by showing that the result matches the special cases of Example Variation 42 and Example Variation 43. This is an electric field problem, which we will solve by direct integration.

**DEVELOP** We will find the field at point $P=(x,y)$, and for the integration variable we'll use $x'$. The infinitesimal charge $dq$ is the charge per unit length times the length $dx'$, so $dq=\frac{Q}{L}dx'$. The distance from each bit of charge $dq$ to the point $(x, y)$ is $r=\sqrt{(x-x')^2+y^2}$. The unit vector in the direction of $r$ is

$$\hat{r}=\frac{\vec{r}}{|r|}=\frac{(x-x')\hat{i}+y\hat{j}}{\sqrt{(x-x')^2+y^2}}$$

We will find the electric field by integrating $dE=\frac{k\,dq}{r^2}\hat{r}$.

**EVALUATE**

**(a)**

$$dE=\frac{k\,dq}{r^2}\hat{r}$$

$$E=\int_{-L/2}^{L/2}\frac{k\frac{Q}{L}dx'}{(x-x')^2+y^2}\left(\frac{(x-x')\hat{i}+y\hat{j}}{\sqrt{(x-x')^2+y^2}}\right)=\frac{kQ}{L}\int_{-L/2}^{L/2}\frac{(x-x')\hat{i}+y\hat{j}}{\left[(x-x')^2+y^2\right]^{3/2}}dx'$$

$$=\frac{kQ}{L}\left[\frac{1}{\sqrt{(x-x')^2+y^2}}\hat{i}-\frac{x-x'}{y\sqrt{(x-x')^2+y^2}}\hat{j}\right]_{-L/2}^{L/2}$$

$$=\frac{kQ}{L}\left[\left(\frac{1}{\sqrt{\left(x-\frac{L}{2}\right)^2+y^2}}-\frac{1}{\sqrt{\left(x+\frac{L}{2}\right)^2+y^2}}\right)\hat{i}+\left(\frac{x+\frac{L}{2}}{y\sqrt{\left(x+\frac{L}{2}\right)^2+y^2}}-\frac{x-\frac{L}{2}}{y\sqrt{\left(x-\frac{L}{2}\right)^2+y^2}}\right)\hat{j}\right]$$

When $x=0$, the electric field reduces to

$$\vec{E}=\frac{kQ}{L}\left[0\hat{i}+\frac{L}{y\sqrt{\left(\frac{L}{2}\right)^2+y^2}}\hat{j}\right]=\frac{2kQ}{y\sqrt{L^2+4y^2}}$$

**(b)** When $y=0$, the electric field reduces to

$$\vec{E}=\frac{kQ}{L}\left[\left(\frac{1}{x-\frac{L}{2}}-\frac{1}{x+\frac{L}{2}}\right)\hat{i}+\lim_{y\to 0}\left(\frac{\cancel{x+\frac{L}{2}}}{y\cancel{\left(x+\frac{L}{2}\right)}}-\frac{\cancel{x-\frac{L}{2}}}{y\cancel{\left(x-\frac{L}{2}\right)}}\right)\hat{j}\right]$$

$$=\frac{kQ}{L}\left[\left(\frac{x+\frac{L}{2}-\left(x-\frac{L}{2}\right)}{x^2-\left(\frac{L}{2}\right)^2}\right)\hat{i}+\cancel{\lim_{y\to 0}\left(\frac{1}{y}-\frac{1}{y}\right)\hat{j}}\right]=\frac{kQ}{\cancel{L}}\left[\frac{\cancel{L}}{x^2-\left(\frac{L}{2}\right)^2}\right]\hat{i}=\frac{4kQ}{4x^2-L^2}$$

**ASSESS** The electric field in part **(a)** is not simple, but we have shown that for some simple cases it reduces to simpler forms.

**78.** **INTERPRET** We are to find the electric field near a line of *nonuniform* charge density. This is an electric field calculation, and we will integrate to find the field.

**DEVELOP** The rod has charge density $\lambda = \lambda_0(\frac{x}{L})^2,$ and extends from $x = 0$ to $x = L$. We want to find the electric field at $x = -L$. We will use $d\vec{E} = \frac{k\,dq}{r^2}\hat{r},$ with $dq = \lambda\,dx$ and $r = x + L.$

**EVALUATE**

$$dE = \frac{k\,dq}{r^2}\hat{r}$$

$$\vec{E} = k\hat{i}\int_0^L \frac{\lambda_0\left(\frac{x}{L}\right)^2}{(x+L)^2}dx = \frac{k\lambda_0}{L^2}\hat{i}\left[x - \frac{L^2}{x+L} - 2L\ln(x+L)\right]_0^L$$

$$= \frac{k\lambda_0\hat{i}}{L^2}\left[L - \frac{L^2}{2L} - 2L\ln\left(\frac{2L}{L}\right) - L\right] = \frac{k\lambda_0\hat{i}}{L^2}\left[-\frac{L}{2} - 2L\ln(2)\right] = -\frac{k\lambda_0\hat{i}}{L}\left[\frac{1}{2} + 2\ln(2)\right]$$

**ASSESS** Since $\lambda_0$ is charge per length, the units are correct.

**79.** **INTERPRET** An electric field is used to deflect ink drops in an ink-jet printer. You need to find the maximum electric field for which the ink drops still can exit the deflection device. This is a problem of projectile motion where the dynamics are controlled by the electric force, not the gravitational force.

**DEVELOP** The time that it takes the ink drop to traverse the field region in the Fig. 20.37 is: $t = L/v.$ During that time it will undergo acceleration from the electric field: $a = qE/m.$ Since the field is uniform, this acceleration is constant, so the amount of vertical deflection will be: $\Delta y = v_{y0}t + \frac{1}{2}at^2.$ The drop initially has no vertical velocity, so $v_{y0} = 0.$ The maximum field is that which deflects the drop by $|\Delta y| = d/2,$ since the drop starts off in the middle between the two plates.

**EVALUATE** Solving for the maximum field magnitude,

$$E_{\max} = \frac{mdv^2}{qL^2}$$

**ASSESS** You can check that this has the right units:

$$[E_{\max}] = \left[\frac{mdv^2}{qL^2}\right] = \frac{\text{kg}\cdot\text{m}\cdot\text{m/s}^2}{\text{C}\cdot\text{m}^2} = \text{N/C}$$

Indeed, these are the right units for an electric field.

**80.** **INTERPRET** We are considering the electric fields that operate in the heart muscle.

**DEVELOP** In Equations 20.6a and 20.6b (as well as Problem 20.70), we see that the electric field from an electric dipole is proportional to one over the distance cubed, $1/r^3,$ for $r$ much larger than the dipole's length: $r \gg d.$

**EVALUATE** The heart is composed of many electric dipoles, so the electric field will be a sum of dipole fields that all are proportional to the distance, $r_i,$ separating the dipole from the point of interest:

$$\vec{E}_{\text{net}} = \sum a_i \frac{kp_i}{r_i^3}\hat{r}_i$$

where $p_i$ are the individual dipole moments, and $a_i$ are constants arising from the particular geometry. Far enough away from the heart, the individual distances will all be approximately equal to each other: $r_i \approx r,$ so the magnitude of the net field will fall off as $1/r^3.$

The answer is **(c)**.

**ASSESS** If there were a net charge on the heart, then the field might fall off as $1/r^2.$ But there apparently is a balance of positive and negative charges in the heart muscles, as shown in Fig. 20.38a.

**81.** **INTERPRET** We are considering the electric fields that operate in the heart muscle.

**DEVELOP** The magnitude of the dipole field on a line that bisects the dipole axis is $E = kp/r^3$, from Equation 20.6a. Whereas, the magnitude of the dipole field along the dipole axis is $E = 2kp/r^3$, from Equation 20.6b. So the field is twice as large along the dipole axis.

**EVALUATE** The extension of the line in Fig. 20.38c bisects the dipole axes of all the dipoles in the heart muscle. A line perpendicular to this one will approximately correspond to the axes of all the dipoles. So the field on the extension should be weaker than the field on the perpendicular.

The answer is **(a)**.

**ASSESS** At the same distance from the heart, the field on the extension of the line in Fig. 20.38c is the weakest compared to other directions. That's because the field contribution from the positive and negative charges are equal and approximately opposite.

**82.** **INTERPRET** We are considering the electric fields that operate in the heart muscle.

**DEVELOP** The figures make it clear that the net charge is zero in Figs. 20.38a and b.

**EVALUATE** To form a dipole, there only needs to be a slight shift in the relative position of positive and negative charges (see, for example, Fig. 20.23). There appears to be a slight shift in the balance of charges in Fig. 20.38b.

The answer is **(c)**.

**ASSESS** Note that a dipole is often not a stable arrangement of charge. There will be Coulomb forces between the positive and negative charges trying to pull them back into a configuration that has less of a dipole moment.

**83.** **INTERPRET** We are considering the electric fields that operate in the heart muscle.

**DEVELOP** Equations 20.6b shows that the electric field on the dipole axis is parallel to the axis and points in the same direction as the dipole moment, that is, in the direction from the negative charge to the positive charge.

**EVALUATE** Inside the heart above and below the line in Fig. 20.38c, the electric field should point in the same direction as the dipole moment.

The answer is **(a)**.

**ASSESS** If we had been asked about the internal field of the dipoles, the answer would have been opposite the direction of the dipole moment (see Fig. 20.24). But this would only comprise a small sliver of the heart area. The majority of the electric field points in the dipole moment's direction.