SYSTEMS OF PARTICLES

9

EXERCISES

Section 9.1 Center of Mass

10. **INTERPRET** This one-dimensional problem involves finding the center of mass of a system with two objects (child and father).

DEVELOP In one dimension, Equation 9.2 for the center of mass reduces to

$$x_{\text{cm}} = \frac{\sum_i m_i x_i}{M} = \frac{\sum_i m_i x_i}{\sum_i m_i}$$

Taking the origin of the coordinate system to be at the child (whom we denote with subscript 1), we have $x_1 = 0$ and $m_1 = 31$ kg. The center of the seesaw is then at $x_{\text{cm}} = 3.3/2 \text{ m} = 1.65 \text{ m}$ (where we retain an extra significant figure because this is an intermediate result). The position of the father is unknown and is labeled x_2. The mass of the father is $m_2 = 78$ kg.

EVALUATE Inserting the known quantities into the expression for center of mass gives

$$x_{\text{cm}} = \frac{\overbrace{x_1}^{=0} m_1 + x_2 m_2}{m_1 + m_2}$$

$$x_2 = \frac{x_{\text{cm}}(m_1 + m_2)}{m_2} = \frac{(1.65 \text{ m})(31 \text{ kg} + 78 \text{ kg})}{78 \text{ kg}} = 2.3 \text{ m}$$

from the child.

ASSESS The algebra was somewhat simplified by choosing the origin of the coordinate system to be at under the child's posterior. Because $x_2 < 3.3$ m, the father can sit on the seesaw and balance it with his daughter. If $m_2 \gg m_1$, then $x_2 = x_{\text{cm}}$ because it does not really matter where the child sits if the father's mass is a ton!

11. **INTERPRET** This is a two-dimensional problem about the center of mass. Our system consists of three masses located at the vertices of an equilateral triangle. Two masses are known and the location of the center of mass is given, so we can find the location of the third mass.

DEVELOP The center of mass of a system of particles is given by Equation 9.2:

$$\vec{r}_{\text{cm}} = \frac{\sum_i m_i \vec{r}_i}{\sum_i m_i} = \frac{\sum_i m_i \vec{r}_i}{M}$$

We shall choose x–y coordinates with origin (0,0) at the midpoint of the base. With this arrangement, the center of the mass is located at $x_{\text{cm}} = 0$ and $y_{\text{cm}} = y_3/2$, where y_3 is the position of the third mass (and, of course, $y_1 = y_2 = 0$ for the equal masses $m_1 = m_2 = m$ on the base).

EVALUATE Using Equation 9.2, the y-coordinate of the center of mass is

$$y_{\text{cm}} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3} = \frac{m(0) + m(0) + m_3 y_3}{m + m + m_3} = \frac{m_3(2y_{\text{cm}})}{2m + m_3}$$

Solving for m_3, we have $2m + m_3 = 2m_3$, or $m_3 = 2m$.

ASSESS From symmetry consideration, it is apparent that $x_{cm} = 0$. However, we have $m + m = 2m$ at the bottom two vertices of the triangle. Because $y_{cm} = y_3/2$ (i.e., y_{cm} is halfway to the top vertex), we expect the mass there to be $2m$ (See Example 9.2).

12. **INTERPRET** This is a one-dimensional problem in which we are asked to find the location of the center of mass of a two-body system.

DEVELOP With the origin at the center of the barbell, $x_1 = -75$ cm and $x_2 = 75$ cm. Use Equation 9.2 to find the center of mass.

EVALUATE Evaluating Equation 9.2 gives

$$x_{cm} = \frac{(50\ \text{kg})(-75\ \text{cm}) + (80\ \text{kg})(75\ \text{cm})}{50\ \text{kg} + 80\ \text{kg}} = 17\ \text{cm}$$

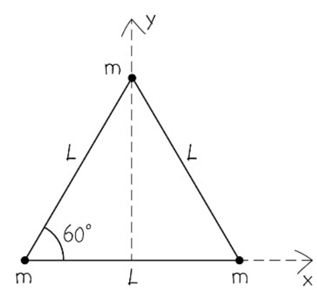
to two significant figures.

ASSESS We find that the center of mass is 17 cm from the center toward the heavier mass, or 75 cm + 17 cm = 92 cm from the light mass. This agrees with the result of Example 1.

13. **INTERPRET** This two-dimensional problem is about locating the center of mass. Our system consists of three equal masses located at the vertices of an equilateral triangle of side L.

DEVELOP We take x–y coordinates with the origin at the center of one side as shown in the figure below. The center of mass of a system of particles is given by Equation 9.2:

$$\vec{r}_{cm} = \frac{\sum_i m_i \vec{r}_i}{\sum_i m_i} = \frac{\sum_i m_i \vec{r}_i}{M}$$



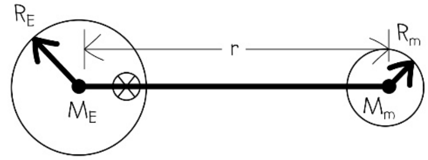
EVALUATE From the symmetry (for every mass at x, there is an equal mass at $-x$) we have $x_{cm} = 0$. As for y_{cm}, because $y = 0$ for the two masses on the x-axis, and $y_3 = L\sin(60°) = L\sqrt{3}/2$ for the third mass, Equation 9.2 gives

$$y_{cm} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3} = \frac{m(0) + m(0) + mL\sqrt{3}/2}{m + m + m} = \frac{\sqrt{3}}{6}L = 0.289L$$

ASSESS From symmetry consideration, it is apparent that $x_{cm} = 0$. On the other hand, we have $m + m = 2m$ at the bottom two vertices of the triangle, and m at the top of the vertex. Therefore, we should expect y_{cm} to be one-third of y_3. This indeed is the case, as y_{cm} can be rewritten as $y_{cm} = y_3/3$.

14. **INTERPRET** This is a one-dimensional problem for which we need to find the center of mass of a two-body system.

DEVELOP Take the origin to be at the center of the Earth (see drawing below), and apply Equation 9.2 for the center of mass. From Appendix E, we know that the Earth-Moon distance is $r = 3.85 \times 10^5$ m, $M_E = 5.97$ Mkg, and $M_M = 0.0735$ Mkg.



EVALUATE Measured from the center of the Earth, the center of mass of the Earth-Moon system is

$$x_{\text{cm}} = \frac{M_{\text{E}}(0) + M_{\text{M}} r}{M_{\text{E}} + M_{\text{M}}} = \frac{(7.35\ \text{Mkg})(3.85\times10^5\ \text{km})}{(597\ \text{Mkg} + 7.35\ \text{Mkg})} = 4680\ \text{km}$$

ASSESS Notice that we are not obliged to use SI units, provided all masses are expressed in the same units so that the units of mass cancel out.

Section 9.2 Momentum

15. **INTERPRET** This problem involves conservation of linear momentum (Equation 9.7), which we can apply to find the speed of one out of two particles that separate after an explosion, given the speed of the other particle and the masses of both particles.

DEVELOP Before the explosion, the popcorn kernel has zero momentum, $(m_1 + m_2)\vec{v} = \vec{0}$. After the explosion, the total momentum of the two particles must still sum to zero, so we have $m_1\vec{v}_1 + m_2\vec{v}_2 = \vec{0}$. Thus,

$$\vec{v}_2 = -\frac{m_1}{m_2}\vec{v}_1$$

So, we can solve for $\vec{v}_2$, given $m_1 = 86\ \text{mg}$, $m_2 = 69\ \text{mg}$, and $\vec{v}_1 = (49\ \text{cm/s})\hat{i}$.

EVALUATE Inserting the given quantities gives $\vec{v}_2 = -(86\ \text{mg}/69\ \text{mg})(49\ \text{cm/s})\hat{i} = (-61\ \text{cm/s})\hat{i}$.

ASSESS Notice that the smaller piece moves faster than the larger piece. Also, notice that the total mechanical energy is not conserved because $K = 0$ before the explosion while $K = m_1v_1^2 + m_2v_2^2 \neq 0$ after the explosion. In this case, the extra energy comes from the hot pan, which turns water in the kernel to steam, thus causing the kernel to "pop."

16. **INTERPRET** The object of interest is the skater. We want to find her velocity after she tosses a snowball in a certain direction.

DEVELOP On frictionless ice, momentum would be conserved in the process. Since the initial momentum of the skater-snowball system is zero, their final total momentum must also be zero:

$$0 = m_1\vec{v}_1 + m_2\vec{v}_2$$

where subscripts 1 and 2 refer to the snowball and skater, respectively.

EVALUATE By momentum conservation, the final velocity of the skater is

$$\vec{v}_2 = -\frac{m_1}{m_2}\vec{v}_1 = -\frac{10\ \text{kg}}{72\ \text{kg}}\left(53.0\hat{i} + 14.0\hat{j}\ \text{m/s}\right) = -7.36\hat{i} - 1.94\hat{j}\ \text{m/s}$$

ASSESS As expected, the skater moves in the direction opposite to that of the snowball. This is a consequence of momentum conservation.

17. **INTERPRET** This problem involves conservation of linear momentum (Equation 9.7), which we can apply to find the speed of one out of two particles that separate after an explosion, given the speed of the other particle and mass of both particles.

DEVELOP Before the explosion, the uranium atom has zero momentum, so $(m_1 + m_2)\vec{v} = 0$. After fission, the total momentum of the two particles must still sum to zero, so we have $m_\alpha\vec{v}_\alpha + m_{\text{U}^{235}}\vec{v}_{\text{U}^{235}} = 0$. Thus,

$$\vec{v}_{\text{U}^{235}} = \frac{m_\alpha}{m_{\text{U}^{235}}}\vec{v}_\alpha$$

The initial speed can be obtained from the kinetic energy, $\vec{v}_\alpha = \pm\sqrt{2K_\alpha/m_\alpha}\,\hat{i} \equiv \sqrt{2K_\alpha/m_\alpha}\,\hat{i}$, so we can solve for $v_{\text{U}^{235}}$ using data from Appendix D for the masses of the particles.

EVALUATE Solving for $v_{\text{U}^{235}}$ gives

$$\vec{v}_{\text{U}^{235}} = -\frac{\sqrt{2m_\alpha K_\alpha}}{m_{\text{U}^{235}}}\hat{i} = -\left[\frac{2(4\ \text{u})(5.15\ \text{MeV})(1.60\times10^{-3}\ \text{J/MeV})}{(235\ \text{u})^2 1(.66\times10^{-27}\ \text{kg/u})}\right]^{1/2}\hat{i} = \left(-2.68\times10^5\ \text{m/s}\right)\hat{i}$$

We're asked for the speed, a scalar quantity, so we report 2.68×10^5 m/s or 0.268 Mm/s.

ASSESS Because $K_\alpha = 5.15\ \text{MeV} \ll m_\alpha c^2 = 3.73\ \text{Gev},$ relativity can be ignored.

18. **INTERPRET** This problem involves using conservation of linear momentum to find the final speed of a moving toboggan after some snow drops onto it.

DEVELOP Because there is no net external horizontal force, the total momentum of the snow-toboggan system is conserved. The initial momentum of the system is $P_i = m_t v_{ti}$. Because the snow and the toboggan move together with the same speed v_f, the final momentum is $P_f = (m_t + m_s)v_f$.

EVALUATE By conservation of momentum, $P_i = P_f$, the final speed of the snow-toboggan system is

$$v_f = \frac{m_t}{m_t + m_s} v_{ti} = \frac{9.4\ \text{kg}}{9.4\ \text{kg} + 17\ \text{kg}}(17\ \text{km/h}) = 6.1\ \text{km/h}$$

ASSESS To see that our result makes sense, let's consider the following limiting cases: **(i)** m_s=0. In this situation, we have $v_f = v_{ti}$, which indicates that the toboggan continues with the same speed. **(ii)** $m_s \to \infty$. In the situation where a large quantity of snow is dumped onto the toboggan, we expect the system to slow down considerably, which is indeed what our equation gives $(v_f = 0)$.

Section 9.3 Kinetic Energy of a System

19. **INTERPRET** In this problem we are asked about the energy gained by the rocket pieces after the rocket explodes. We can apply conservation of linear momentum to solve this problem.

DEVELOP Applying conservation of linear momentum to the rocket gives

$$\vec{P}_i = \vec{P}_f \to (m_1 + m_2)\vec{v}_0 = m_1\vec{v}_1 + m_2\vec{v}_2$$

The initial kinetic energy of the system is $K_i = \frac{1}{2}(m_1 + m_2)v_0^2$, and the total final kinetic energy is $K_\text{f} = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2$. Therefore, the change in kinetic energy is

$$\Delta K = K_\text{f} - K_\text{i} = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 - \frac{1}{2}(m_1 + m_2)v_0^2 = \frac{1}{2}m_1\left(v_1^2 - v_0^2\right) + \frac{1}{2}m_2\left(v_2^2 - v_0^2\right)$$

EVALUATE Let the forward direction be positive. By conservation of momentum, the velocity of the second piece, with mass $m_2 = m - m_1 = 0.995\ \text{kg} - 0.372\ \text{kg} = 0.623\ \text{kg}$ is

$$v_2 = \frac{(m_1 + m_2)v_0 - m_1 v_1}{m_2} = \frac{(0.995\ \text{kg})(18.6\ \text{m/s}) - (0.372\ \text{kg})(31.3\ \text{m/s})}{(0.623\ \text{kg})} = 11.02\ \text{m/s}$$

Now we have all the values needed to find the difference in kinetic energy. We find the amount of energy gained by the two pieces after the rocket bursts is equal to

$$\Delta K = \Delta K_1 + \Delta K_2 = \frac{1}{2}m_1(v_1^2 - v_0^2) + \frac{1}{2}m_2(v_2^2 - v_0^2) = 47.9\ \text{J}$$

Here, we note that the expression for the total change in kinetic energy has been arranged to show the change in the individual piece's kinetic energies.

ASSESS The change in kinetic energy for the first piece (ΔK_1) is positive because $v_1 > v_0$, but negative for the second ($\Delta K_2 < 0$ because $v_2 < v_0$).

20. **INTERPRET** Before an explosion, an object has kinetic energy $K = \frac{1}{2}mv_\text{i}^2$. After the explosion, it has two pieces (m_1 and m_2) each moving at twice the initial speed, $v_\text{f} = 2v_\text{i}$. We are asked to find and compare the internal and center-of-mass energies after the explosion.

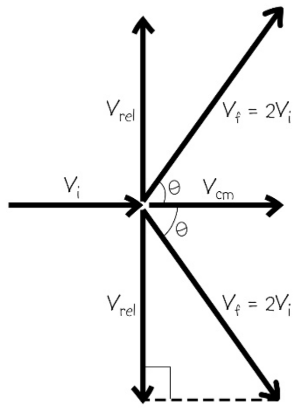
DEVELOP After the explosion, the final kinetic energy is a combination of the center-of-mass and internal energies: $K_\text{f} = K_\text{cm} + K_\text{int}$ (Equation 9.9). We assume that the explosion is like the radioactive decay in Example 9.6, in which case momentum is conserved. But we can't assume that $m_1 = m_2$, which means the direction that the two pieces fly off relative to the original direction is unknown. But we don't need to know these quantities to find the final kinetic energy:

$$K_\text{f} = \frac{1}{2}m_1 v_{1\text{f}}^2 + \frac{1}{2}m_2 v_{2\text{f}}^2 = \frac{1}{2}m_1(2v_\text{i})^2 + \frac{1}{2}m_2(2v_\text{i})^2 = 4\left(\frac{1}{2}mv_\text{i}^2\right) = 4K$$

where we have assumed that the mass is conserved: $m_1 + m_2 = m$.

EVALUATE Because momentum is conserved, the center-of-mass velocity is constant: $v_{\text{cm}} = v_{\text{i}}$, which implies that the kinetic energy of the center of mass is the same as the kinetic energy before the explosion, $K_{\text{cm}} = K$. Plugging this into Equation 9.9, we find $K_{\text{int}} = K_{\text{f}} - K_{\text{cm}} = 3K$. In other words, the internal kinetic energy is three times the center-of-mass energy.

ASSESS If one did assume that the two pieces have equal mass, then the angle, θ, between the initial and final velocities would be the same for each piece, see the figure below.



In this case, both particles would have the same speed relative to the center of mass:

$$v_{\text{rel}} = \sqrt{v_{\text{f}}^2 - v_{\text{cm}}^2} = \sqrt{(2v_{\text{i}})^2 - (v_{\text{i}})^2} = \sqrt{3}v_{\text{i}}$$

And the internal energy would be

$$K_{\text{int}} = \sum \frac{1}{2} m_i v_{i\text{rel}}^2 = \frac{1}{2} m_1 v_{\text{rel}}^2 + \frac{1}{2} m_2 v_{\text{rel}}^2 = 3\left(\frac{1}{2} m v_{\text{i}}^2\right) = 3K$$

This is exactly what we found for the general case above.

Section 9.4 Collisions

21. **INTERPRET** In this problem we are asked to estimate the impulse imparted onto a car and the mass of the car from a graph depicting the force applied onto it over time.

DEVELOP We are told in the figure that the area under the green force-time curve is equivalent to that under the grey shaded region, where the average force is considered. Knowing the extent of the Force and Time axes we can estimate the area of the rectangle to determine the impulse $\Delta\vec{p}$ since Equation 9.10a shows: $\Delta\vec{p} = \bar{\vec{F}}\Delta t$.

EVALUATE We know the full extent of the vertical and horizontal axes are 10 kN and 800 ms, respectively. Looking at the graph we can estimate the average force lies about 1/3 of the way to the end, and the duration extends about 4/5 of the way to the end. Plugging these values into Equation 9.10a gives an impulse of about

$$\Delta\vec{p} = \bar{\vec{F}}\Delta t = \left(\frac{1}{3}\right)(10\ \text{kN})\left(\frac{4}{5}\right)(0.8\ \text{s}) \cong 20\ \text{kN}\cdot\text{s}$$

Since the car came to a stop after this collision, this is essentially the amount of momentum the car had to begin with. Since we are also interested in the initial velocity of the vehicle, we can divide this initial momentum by the mass give to find that

$$v_i = \frac{p_i}{m} = \frac{(\sim 20\ \text{kN}\cdot\text{s})}{2000\ \text{kg}} \cong 10\ \text{m/s}$$

ASSESS This crash test was modeling a relatively low speed collision since 10 m/s is around 22 mph.

22. **INTERPRET** We want to determine the average force and impulse acting on a jumping flea.

DEVELOP We're given the average acceleration during the jump, so the ground must supply an average force on the flea of $\bar{F} = m\bar{a}$ is just multiplied by the flea's mass. We can then use Equation 9.9a $\left(J = \bar{F}\Delta t = \Delta p\right)$ to find the impulse imparted by the ground and the resulting momentum change for the flea. We neglected gravity because, during the jump, the average impulsive force is much larger than the gravitational force.

EVALUATE (a) The average force exerted by the ground on the flea is

$$\bar{F} = m\bar{a} = \left(220\times10^{-9}\,\text{kg}\right)\left(100\cdot 9.8\ \text{m/s}^2\right) = 2.16\times10^{-4}\,\text{N} \approx 220\ \mu\text{N}$$

(b) Multiplying the average force by the time gives the impulse:

$$J = \bar{F}\Delta t = \left(2.16\times10^{-4}\,\text{N}\right)\left(1.2\ \text{ms}\right) = 2.6\times10^{-7}\,\text{N}\cdot\text{s}$$

(c) The momentum change for the flea is equal to the impulse provided by the floor:

$$\Delta p = J = 2.6\times10^{-7}\,\text{N}\cdot\text{s} = 2.6\times10^{-7}\,\text{kg}\cdot\text{m/s}$$

Notice that we can write the momentum change in units $(\text{kg}\cdot\text{m/s})$ that might be more familiar for momentum.

ASSESS If we assume the flea starts its jump from rest, then at the end of its jump it reaches a velocity of $v = \Delta v = \Delta p / m = 1.2$ m/s. That seems reasonable.

23. **INTERPRET** You need to determine how to fire a rocket to obtain the needed impulse.

DEVELOP We're given the average thrust, so the time needed comes from Equation 9.9a: $\Delta t = J / \bar{F}$.

EVALUATE For the required impulse, the space probes rocket must fire for

$$\Delta t = \frac{J}{\bar{F}} = \frac{5.62\ \text{N}\cdot\text{s}}{126\times10^{-3}\,\text{N}} = 44.6\ \text{s}$$

ASSESS This might seem like a long time for such a small impulse. But the rocket exerts a tiny force on the space probe. Often, spacecraft need precision thrusters like the one here to make small adjustments in their trajectory or orientation.

Section 9.5 Totally Inelastic Collisions

24. **INTERPRET** This problem involves conservation of total linear momentum. We are to use it to find the final momentum of a two-car system. In addition, we are to find the change in kinetic energy of the two-car system after they couple.

DEVELOP If we assume the switchyard track is straight and level, the collision is one-dimensional, totally inelastic, and Equation 9.11 applies, so

$$\vec{v}_\text{f} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2} \equiv \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}\hat{i}$$

Once we have found the final velocity $\vec{v}_\text{f}$, we can insert it into the expression for the change in kinetic energy to find the fraction of kinetic energy lost.

EVALUATE (**a**) Inserting the given quantities into the expression for $\vec{v}_\text{f}$ gives

$$\vec{v}_\text{f} = \frac{(56\ \text{ton})(7.0\ \text{mi/h}) + (31\ \text{ton})(2.6\ \text{mi/h})}{56\ \text{ton} + 31\ \text{ton}}\hat{i} = (5.4\ \text{mi/h})\hat{i}$$

(**b**) The initial and final kinetic energies are

$$K_\text{i} = \frac{1}{2}\left[(56\ \text{T})(7.0\ \text{mi/h})^2 + (31\ \text{T})(2.6\ \text{mi/h})^2\right] = 1477\ \text{T}(\text{mi/h})^2;$$

$$K_\text{f} = \frac{1}{2}(56+31)\ \text{T}\ (5.43\ \text{mi/h})^2 = 1284\ \text{T}(\text{mi/h})^2$$

where we retained more significant figures than warranted by the data because these are intermediate results. The fraction of kinetic energy lost is $\left(K_f - K_i\right)/K_i = -13\%$.

ASSESS Notice that we did not need to change to SI units for part (b) because we took the ratio of initial and final kinetic energies. Thus, provided we use the same units for the initial and final kinetic energies, the answer will be correct.

25. **INTERPRET** In this problem, we are asked to show that half of the initial kinetic energy of a system is lost in a totally inelastic collision between two equal masses.

DEVELOP Suppose we have two masses m_1 and m_2 moving with velocities $\vec{v}_1$ and $\vec{v}_2$, respectively. After undergoing a totally inelastic collision, the two masses stick together and move with final velocity $\vec{v}_f$. Although the collision is totally inelastic, momentum conservation still applies, and we have (Equation 9.11):

$$m_1\vec{v}_1 + m_2\vec{v}_2 = (m_1 + m_2)\vec{v}_\mathrm{f} \;\Rightarrow\; \vec{v}_\mathrm{f} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2}$$

The initial total kinetic energy of the two-particle system is $K_i = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$, whereas the final kinetic energy of the system after collision is $K_f = \frac{1}{2}(m_1 + m_2)v_f^2$. Therefore, the change in kinetic energy is given by

$$\Delta K = K_\mathrm{f} - K_\mathrm{i} = \frac{1}{2}(m_1 + m_2)v_\mathrm{f}^2 - \frac{1}{2}m_1v_1^2 - \frac{1}{2}m_2v_2^2 = \frac{1}{2}m_1\left(v_\mathrm{f}^2 - v_1^2\right) + \frac{1}{2}m_2\left(v_\mathrm{f}^2 - v_2^2\right)$$

EVALUATE In our case, we have $m_1 = m_2 = m$, $v_1 = v$, and $v_2 = 0$. The initial kinetic energy of the system is therefore $K_i = \frac{1}{2}mv^2$. The final speed is

$$v_\mathrm{f} = \frac{m_1v_1 + m_2v_2}{m_1 + m_2} = \frac{mv}{m + m} = \frac{1}{2}v$$

Therefore, the change in total kinetic energy is

$$\Delta K = \frac{1}{2}m_1\left(v_\mathrm{f}^2 - v_1^2\right) + \frac{1}{2}m_2\left(v_\mathrm{f}^2 - v_2^2\right) = \frac{1}{2}m\left(\frac{v^2}{4} - v^2\right) + \frac{1}{2}m\left(\frac{v^2}{4} - 0\right) = -\frac{1}{4}mv^2$$

Thus, we see that half of the total initial kinetic energy is lost in the collision process.

ASSESS For a totally inelastic collision, one may show that the general expression for ΔK is

$$\Delta K = \frac{1}{2}\frac{(m_1v_1 + m_2v_2)^2}{m_1 + m_2} - \frac{1}{2}m_1v_1^2 - \frac{1}{2}m_2v_2^2 = -\frac{m_1m_2}{2(m_1 + m_2)}(v_1 - v_2)^2$$

Clearly, ΔK is always negative, and it depends on the relative speed between m_1 and m_2.

26. **INTERPRET** This is a two-dimensional problem that involves conservation of linear momentum. We are to find the initial velocity of one particle, given the initial velocity of the other, the final velocity of the combined particles, and the masses of each.

DEVELOP Apply conservation of linear momentum, Equation 9.11, for a totally inelastic collision, and solve for the velocity of the deuteron.

EVALUATE Inserting the given quantities into Equation 9.11 and solving for v_d gives

$$\vec{v}_\mathrm{d} = \frac{m_\mathrm{t}\vec{v}_\mathrm{t} - m_\mathrm{n}\vec{v}_\mathrm{n}}{m_\mathrm{d}} = \frac{3.02\text{ u}\left(15.1\hat{i} + 22.6\hat{j}\right) - 1.01\text{ u}\left(23.5\hat{i} + 14.4\hat{j}\right)}{2.01\text{ u}}\left(\frac{\text{Mm}}{\text{s}}\right) = \left(10.9\hat{i} + 26.7\hat{j}\right)\text{ Mm/s}$$

to two significant figures.

ASSESS The change in kinetic energy in the collision is

$$\begin{aligned}\Delta K &= \frac{1}{2}m_\mathrm{t}v_\mathrm{t}^2 - \frac{1}{2}mv_\mathrm{d}^2 - \frac{1}{2}mv_\mathrm{n}^2\\ &= \frac{1}{2}\left[\left(15.1^2 + 22.6^2\right)(3.02) - \left(10.9^2 + 26.7^2\right)(2.01) - \left(23.5^2 + 14.4^2\right)(1.01)\right] \approx -104\text{ u}\left(\text{Mm/s}^2\right)\end{aligned}$$

which is negative, indicating that energy is stored in the tritium (i.e., is converted to mass). We can regain this energy by splitting tritium, which is the basis of the hydrogen bomb.

27. **INTERPRET** This is a totally inelastic collision, since the trucks move together as one after the collision. You should be able to find the mass of the second truck using conservation of momentum.

DEVELOP According to Equation 9.11, conservation of momentum in the truck collision implies

$$m_1\vec{v}_1 + m_2\vec{v}_2 = (m_1 + m_2)\vec{v}_f$$

EVALUATE The first truck is at rest $(\vec{v}_1 = 0)$ which means the final velocity has to be in the same direction as $\vec{v}_2$, so we don't need to work with the full vector notation. Given that the first truck has a mass of $m_1 = 5500\text{ kg} + 3800\text{ kg} = 9300\text{ kg}$, we can solve for the mass of the second:

$$m_2 = m_1\frac{v_f}{v_2 - v_1} = (9300\text{ kg})\frac{37\text{ km/h}}{65\text{ km/h} - 37\text{ km/h}} = 12{,}300\text{ kg}$$

Subtracting the 5500-kg mass of the truck leaves a load of 6800 kg, so the second truck was 1200 kg under the legal limit of 8000 kg.

ASSESS Note that because the velocities only appear in a ratio, it isn't necessary to convert to m/s.

Section 9.6 Elastic Collisions

28. **INTERPRET** This one-dimensional problem involves an elastic collision between two particles, the Au nucleus and the alpha particle. We are to find the fraction of the alpha particle's kinetic energy that is transferred to the Au nucleus.

DEVELOP Because this problem is one-dimensional, we can apply Equation 9.15b to find the final velocity of the alpha particle. We can then use this result to find the final kinetic energy of the alpha particle in order to calculate the fraction of kinetic energy lost to the Au nucleus.

EVALUATE With m_1 =4u, m_2 = 197 u, and $v_{2i} = 0$, we find that $v_{2f} = 2(4.00\text{ u})v_{1i}/(4.00+197)\text{ u} = (8.00/201)v_{1i}$. The fraction of the initial energy transferred is

$$K_{1i}/K_{2f} = \frac{\frac{1}{2}(197\text{ u})v_{2f}^2}{\frac{1}{2}(4.00\text{ u})v_{1i}^2} = \frac{197}{4.00}\left(\frac{8.00}{201}\right)^2 = 7.80\%.$$

ASSESS We retain three significant figures in the answer because we know the data to three significant figures.

29. **INTERPRET** This problem is about head-on (i.e., one-dimensional) elastic collisions. We want to find the speed of the ball after it rebounds elastically from a moving car.

DEVELOP Both mechanical energy and linear momentum are conserved in an elastic collision. In this one-dimensional case, conservation of linear momentum gives

$$m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}$$

Conservation of energy gives

$$\frac{1}{2}m_1v_{1i}^2 + \frac{1}{2}m_2v_{2i}^2 = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2$$

Using the two conservation equations, the final speeds of m_1 and m_2 are (see Equations 9.15a and 9.15b):

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2}v_{1i} + \frac{2m_2}{m_1 + m_2}v_{2i}$$

$$v_{2f} = \frac{2m_1}{m_1 + m_2}v_{1i} + \frac{m_2 - m_1}{m_1 + m_2}v_{2i}$$

EVALUATE Let the subscripts 1 and 2 be for the car and the ball, respectively. We choose positive velocities in the direction of the car. The speed of the ball after it rebounds is

$$v_{2f} = \frac{2m_1}{m_1 + m_2}v_{1i} + \frac{m_2 - m_1}{m_1 + m_2}v_{2i} \approx 2v_{1i} - v_{2i} = 2(15\text{ m/s}) - (-18\text{ m/s}) = 48\text{ m/s}$$

where we have used $m_1 \gg m_2$.

ASSESS Similarly, the final speed of the car is

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2}v_{1f} + \frac{2m_2}{m_1 + m_2}v_{2i} \approx v_{1i} = 15\text{ m/s}$$

We do not expect the speed of the car to change much after colliding with a ball. However, the ball rebounds with a much greater speed than before. If the car were stationary with $v_{1i} = 0$, then we would find $v_{2f} = -v_{2i} = 18\text{ m/s}$.

30. **INTERPRET** This problem involves a one-dimensional elastic collision between two masses, so conservation of mechanical energy and conservation of linear momentum applies. We are asked to find how the masses are related given that the objects have the same speed after colliding.

DEVELOP Apply Equations 9.15a and 9.15b and solve for M, given that $v_{2i} = 0$. We are also told that the blocks have the same speed after the collision, so we know that $v_{2f} = -v_{1f}$, where we have inserted the negative sign because the blocks must move in opposite directions if this is an elastic collision.

EVALUATE Equations 9.15a and 9.15b give

$$v_{1f} = \frac{m-M}{m+M} v_{1i}$$
$$v_{2f} = \frac{2m}{m+M} v_{1i}$$

Using $v_{2f} = -v_{1f}$, we find

$$\frac{m-M}{m+M} v_{1i} = -\frac{2m}{m+M} v_{1i}$$
$$M = 3m$$

ASSESS After the collision, the larger block will have three times the kinetic energy of the smaller block.

31. **INTERPRET** In this problem, we are asked to find the speeds of the protons after they collide elastically head-on. The problem is thus one-dimensional and involves conservation of mechanical energy and linear momentum.
DEVELOP Consider the general situation in which two masses m_1 and m_2 moving with velocities $\vec{v}_1$ and $\vec{v}_2$, respectively, undergo elastic collision. Both momentum and energy are conserved in this process. Using the conservation equations, the final speeds of m_1 and m_2 are (see Equations 9.15a and 9.15b):

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \frac{2m_2}{m_1 + m_2} v_{2i}$$
$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} + \frac{m_2 - m_1}{m_1 + m_2} v_{2i}$$

EVALUATE We choose positive velocities to be in the direction of $\vec{v}_1$. With $m_1 = m_2 = m$, $v_{1i} = 6.1\ \text{Mm/s}$, and $v_{2i} = -9.2\ \text{Mm/s}$, the final velocities are

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \frac{2m_2}{m_1 + m_2} v_{2i} = v_{2i} = -9.2\ \text{Mm/s}$$
$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} + \frac{m_2 - m_1}{m_1 + m_2} v_{2i} = v_{1i} = 6.1\ \text{Mm/s}$$

ASSESS In this case, the protons simply exchange velocities—the final velocity of the first proton is equal to the initial velocity of the second proton, while the final velocity of the second proton is equal to the initial velocity of the first proton.

32. **INTERPRET** This one-dimensional problem involves an elastic collision, so we can apply conservation of mechanical energy and linear momentum.

DEVELOP Apply Equations 9.15a and 9.15b to find the requisite relationships. We are given that $v_{1i} = -v_{21} = v$, $v_{1f} = 0$, and $m_1 > m_2$.

EVALUATE (a) Equations 9.15a and 9.15b lead to

$$0 = \frac{m_1 - m_2}{m_1 + m_2} v - \frac{2m_2}{m_1 + m_2} v$$
$$m_1 - m_2 = 2m_2$$
$$\frac{m_1}{m_2} = 3$$

(b) The final speed of the less massive particle is

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v + \frac{m_1 - m_2}{m_1 + m_2} v = \left(\frac{6}{4} + \frac{2}{4}\right) v = 2v$$

ASSESS We know that the initial speeds must be in the opposite direction because that is the only way that they could collide head-on given that the magnitude of their speeds are the same.

EXAMPLE VARIATIONS

33. **INTERPRET** In the absence of external forces, a system's total momentum can't change. Thus, we treat this as a momentum conservation problem.

DEVELOP After the decay, we have two momenta to account for, so Equation 9.7 becomes

$$m_{\rm Li}\vec{v}_{\rm Li} = m_{\rm p}\vec{v}_{\rm p} + m_\alpha \vec{v}_\alpha$$

Choosing the x-axis along the direction of $\vec{v}_{\rm Li}$, the two components of the momentum conservation equation become

$$p_x : m_{\rm Li}v_{\rm Li} = m_{\rm p}v_{\rm px} + m_\alpha v_{\alpha x}$$

$$p_y : 0 = m_{\rm p}v_{\rm py} + m_\alpha v_{\alpha y}$$

Where: $v_{\alpha x} = v_\alpha \cos\varphi$ and $v_{\alpha y} = v_\alpha \sin\varphi$ are the velocity components of the alpha particle.

EVALUATE We can thus solve our two equations to get

$$v_{\rm px} = \frac{m_{\rm Li}v_{\rm Li} - m_\alpha v_\alpha \cos\varphi}{m_{\rm p}} = 7.47\,{\rm Mm/s}$$

$$v_{\rm py} = \frac{-m_\alpha v_\alpha \sin\varphi}{m_{\rm p}} = -1.65\,{\rm Mm/s}$$

With these we find the magnitude and direction of the proton's velocity to be, respectively

$$v_{\rm p} = \sqrt{(v_{\rm px})^2 + (v_{\rm py})^2} = 7.65\,{\rm Mm/s}$$

$$\theta = {\rm atan}\left(\frac{v_{\rm py}}{v_{\rm px}}\right) = -12.4^\circ$$

Where the proton is moving at 12.4° to the alpha particle's original direction with perpendicular component opposite that of the alpha particle.

ASSESS The two momenta after the decay event have equal but opposite vertical components, reflecting that the total momentum of the system never had a vertical component.

34. **INTERPRET** In the absence of external forces, a system's total momentum can't change. Thus, we treat this as a momentum conservation problem.

DEVELOP After the decay, we have two momenta to account for, so Equation 9.7 becomes

$$m_{\rm Li}\vec{v}_{\rm Li} = m_{\rm p}\vec{v}_{\rm p} + m_\alpha \vec{v}_\alpha$$

Choosing the x-axis along the direction of $\vec{v}_{\rm Li}$, and assuming the lithium-5 nucleus has two velocity components, the two components of the momentum conservation equation become

$$p_x : m_{\rm Li}v_{\rm Lix} = m_{\rm p}v_{\rm px} + m_\alpha v_{\alpha x}$$

$$p_y : m_{\rm Li}v_{\rm Liy} = m_{\rm p}v_{\rm py} + m_\alpha v_{\alpha y}$$

Where: $v_{\alpha x} = v_\alpha \cos\varphi$, $v_{\alpha y} = v_\alpha \sin\varphi$, $v_{\rm px} = v_{\rm p}\cos\theta$, and $v_{\rm py} = v_{\rm p}\sin\theta$ are the velocity components of the alpha particle and proton, respectively.

EVALUATE We can thus solve our two equations to get

$$v_{\rm Lix} = \frac{m_{\rm p}v_{\rm p}\cos\theta + m_\alpha v_\alpha \cos\varphi}{m_{\rm Li}} = 1.98\,{\rm Mm/s}$$

$$v_{\rm Liy} = \frac{m_{\rm p}v_{\rm p}\sin\theta - m_\alpha v_\alpha \sin\varphi}{m_{\rm Li}} = 0.867\,{\rm Mm/s}$$

Which we can express in vector notation as $\vec{v}_{\rm Li} = 1.98\hat{i} + 0.867\hat{j}\,{\rm Mm/s}$.

ASSESS The lithium-5 nucleus was moving at 23.6° above the x-axis before decaying.

35. **INTERPRET** In the absence of external forces, a system's total momentum can't change. Thus, we treat this as a momentum conservation problem.

DEVELOP After the separation, we have two momenta to account for, so Equation 9.7 becomes

$$(m_o + m_l)\vec{v}_i = m_o\vec{v}_o + m_l\vec{v}_l$$

Choosing the *x*-axis along the direction of $\vec{v}_i$, the two components of the momentum conservation equation become

$$p_x : (m_o + m_l)v_i = m_o v_{ox} + m_l v_{lx}$$

$$p_y : 0 = m_o v_{oy} + m_l v_{ly}$$

Where: $v_{ox} = v_o \cos\varphi$ and $v_{oy} = v_o \sin\varphi$ are the velocity components of the orbiter.

EVALUATE We can thus solve our two equations to get

$$v_{lx} = \frac{(m_o + m_l)v_i - m_o v_o \cos\varphi}{m_l} = 176\,\text{km/s}$$

$$v_{ly} = \frac{-m_o v_o \sin\varphi}{m_l} = -85.3\,\text{km/s}$$

With these we find the magnitude and direction of the lander's velocity to be, respectively

$$v_l = \sqrt{(v_{lx})^2 + (v_{ly})^2} = 195\,\text{km/s}$$

$$\theta = \text{atan}\left(\frac{v_{ly}}{v_{lx}}\right) = -25.9°$$

Where the lander is moving at $25.9°$ to the original direction with perpendicular component opposite that of the orbiter.

ASSESS The two momenta after the separation event have equal but opposite vertical components, reflecting that the total momentum of the system never had a vertical component.

36. **INTERPRET** In the absence of external forces, a system's total momentum can't change. Thus, we treat this as a momentum conservation problem.

DEVELOP After separation, we have two momenta to account for, so Equation 9.7 becomes

$$(m_o + m_l)\vec{v}_i = m_o\vec{v}_o + m_l\vec{v}_l$$

Choosing the *x*-axis along the direction of $\vec{v}_i$, and assuming the composite spacecraft has two velocity components, the two components of the momentum conservation equation become

$$p_x : (m_o + m_l)v_{ix} = m_o v_{ox} + m_l v_{lx}$$

$$p_y : (m_o + m_l)v_{iy} = m_o v_{oy} + m_l v_{ly}$$

EVALUATE We can thus solve our two equations to get

$$v_{ix} = \frac{m_o v_{ox} + m_l v_{lx}}{(m_o + m_l)} = 125\,\text{m/s}$$

$$v_{iy} = \frac{m_o v_{oy} + m_l v_{ly}}{(m_o + m_l)} = 0\,\text{m/s}$$

Which we can express in vector notation as $\vec{v}_i = 125\hat{i}$ m/s.

ASSESS The composite aircraft did not have a vertical component to its velocity before separation since the vertical components of the orbiter and lander velocities sum to zero.

37. **INTERPRET** We need to apply momentum conservation to find the fraction of a neutron's kinetic energy that's transferred to an initially stationary carbon in a head-on elastic collision.

DEVELOP Since we have a one-dimensional elastic collision, Equations 9.15 apply. Taking the neutron to be particle 1, and noting that the carbon (particle 2) is initially stationary, we can express the final velocity of the carbon as

$$v_{2f} = \frac{2m_1 v_{1i}}{(m_1 + m_2)}$$

Which we can use to determine the kinetic-energy ratio

EVALUATE Using our equation for v_{2f} gives

$$K_2 = \frac{1}{2}m_2\left(\frac{2m_1 v_1}{(m_1 + m_2)}\right)^2 = \frac{2m_2 m_1^2 v_1^2}{(m_1 + m_2)^2}$$

Which when comparing to neutron's initial kinetic energy gives

$$\frac{K_2}{K_1} = \left(\frac{2m_2 m_1^2 v_1^2}{(m_1 + m_2)^2}\right)\left(\frac{1}{\frac{1}{2}m_1 v_1^2}\right) = \frac{4m_1 m_2}{(m_1 + m_2)^2}$$

Here we have $m_1 = 1$ u and $m_2 = 12$ u, so we find $K_2/K_1 = 28.4\%$

ASSESS This leaves the neutron with about 72% of its initial energy.

38. **INTERPRET** We need to apply momentum conservation to determine the mass of particle with which a neutron collides and transfers a known fraction of its kinetic energy.

DEVELOP Since we have a one-dimensional elastic collision, Equations 9.15 apply. Taking the neutron to be particle 1, and noting that the particle of unknown mass (particle 2) is initially stationary, we can express the final velocity of the carbon as

$$v_{2f} = \frac{2m_1 v_{1i}}{(m_1 + m_2)}$$

Which we can use to determine the kinetic-energy ratio.

EVALUATE Using our equation for v_{2f} gives

$$K_2 = \frac{1}{2}m_2\left(\frac{2m_1 v_1}{(m_1 + m_2)}\right)^2 = \frac{2m_2 m_1^2 v_1^2}{(m_1 + m_2)^2}$$

Which when comparing to neutron's initial kinetic energy gives

$$\frac{K_2}{K_1} = \left(\frac{2m_2 m_1^2 v_1^2}{(m_1 + m_2)^2}\right)\left(\frac{1}{\frac{1}{2}m_1 v_1^2}\right) = \frac{4m_1 m_2}{(m_1 + m_2)^2}$$

Since we know the value of K_2 / K_1 and m_1, we solve for m_2

$$(m_1 + m_2)^2 = 4m_1 m_2\left(\frac{K_1}{K_2}\right)$$

$$m_2^2 + \left(2 - 4\frac{K_1}{K_2}\right)m_1 m_2 + m_1^2 = 0$$

$$m_2 = \frac{1}{2}\left[-\left(2 - 4\frac{K_1}{K_2}\right)m_1 \pm \sqrt{\left(2 - 4\frac{K_1}{K_2}\right)^2 m_1^2 - 4m_1^2}\right] = m_1\left[2\frac{K_1}{K_2}\left(1 \pm \sqrt{1 - \frac{K_2}{K_1}}\right) - 1\right]$$

Plugging in our values for the ratio of kinetic energies we find that $m_2 = 6.10m_1$. So, the nucleus has a mass 6.10 times larger than the neutron.

ASSESS Here we have chosen the positive root from the solution of the quadratic equation since choosing the negative would've resulted in a mass smaller than that of the nucleus.

39. **INTERPRET** We need to apply momentum conservation to find the fraction of a block's kinetic energy that's transferred to another initially stationary block in a head-on elastic collision.

DEVELOP Since we have a one-dimensional elastic collision, Equations 9.15 apply. Taking the more massive block to be mass m_1, and noting that the lighter block (mass m_2) is initially stationary, we can express the final velocity of the lighter block as

$$v_{2f} = \frac{2m_1 v_{1i}}{(m_1 + m_2)}$$

Which we can use to determine the kinetic-energy ratio.

EVALUATE Using our equation for v_{2f} gives

$$K_2 = \frac{1}{2}m_2\left(\frac{2m_1v_1}{(m_1+m_2)}\right)^2 = \frac{2m_2m_1^2v_1^2}{(m_1+m_2)^2}$$

Which when comparing to the more massive block's initial kinetic energy gives

$$\frac{K_2}{K_1} = \left(\frac{2m_2m_1^2v_1^2}{(m_1+m_2)^2}\right)\left(\frac{1}{\frac{1}{2}m_1v_1^2}\right) = \frac{4m_1m_2}{(m_1+m_2)^2}$$

Here we have $m_1 = 0.685\,\text{kg}$ and $m_2 = 0.232\,\text{kg}$, so we find $K_2/K_1 = 75.6\%$.

ASSESS This leaves the more massive block with about 24% of its initial energy.

40. **INTERPRET** We need to apply momentum conservation to determine the relationship between the masses of two blocks which collide and for which a known transfer of kinetic energy occurs.

DEVELOP Since we have a one-dimensional elastic collision, Equations 9.15 apply. We are told mass m_1 is moving, and mass m_2 is initially stationary, so we can express the final velocity of mass m_2 as

$$v_{2f} = \frac{2m_1 v_{1i}}{(m_1 + m_2)}$$

Which we can use to determine the kinetic-energy ratio.

EVALUATE Using our equation for v_{2f} gives

$$K_2 = \frac{1}{2}m_2\left(\frac{2m_1v_1}{(m_1+m_2)}\right)^2 = \frac{2m_2m_1^2v_1^2}{(m_1+m_2)^2}$$

Which when comparing to the initial kinetic energy of the moving mass gives

$$\frac{K_2}{K_1} = \left(\frac{2m_2m_1^2v_1^2}{(m_1+m_2)^2}\right)\left(\frac{1}{\frac{1}{2}m_1v_1^2}\right) = \frac{4m_1m_2}{(m_1+m_2)^2}$$

Since we know the value of K_2/K_1 and m_1, we solve for m_2

$$(m_1+m_2)^2 = 4m_1m_2\left(\frac{K_1}{K_2}\right)$$

$$m_2^2 + \left(2 - 4\frac{K_1}{K_2}\right)m_1m_2 + m_1^2 = 0$$

$$m_2 = \frac{1}{2}\left[-\left(2-4\frac{K_1}{K_2}\right)m_1 \pm \sqrt{\left(2-4\frac{K_1}{K_2}\right)^2 m_1^2 - 4m_1^2}\right] = m_1\left[2\frac{K_1}{K_2}\left(1\pm\sqrt{1-\frac{K_2}{K_1}}\right)-1\right]$$

Plugging in our values for the ratio of kinetic energies we find that $m_2 = 3m_1$.

ASSESS Here we have chosen the positive root from the solution of the quadratic equation since choosing the negative would've resulted in a mass m_2 smaller than m_1.

PROBLEMS

41. **INTERPRET** In this problem we want to find the center of mass of a pentagon of side a with one triangular section missing.

DEVELOP We choose coordinates as shown in the figure below. If the fifth isosceles triangle (with the same uniform density as the others) were present, the center of mass of the whole pentagon would be at the origin, so

$$0=\frac{my_5+4my_{\rm cm}}{5m}=\frac{y_5+4y_{\rm cm}}{5}$$

where $y_{\rm cm}$ gives the position of the center of mass of the figure we want to find, and y_5 is the position of the center of mass of the fifth triangle. Of course, the mass of the figure is four times the mass of the triangle.

EVALUATE From symmetry, the x-coordinate of the center of mass is $x_{\rm cm}=0$. Now, to calculate $y_{\rm cm}$, we make use of the result obtained in Example 9.3 where the center of mass of an isosceles triangle is calculated. This gives $y_5=-2L/3$. In addition, from the geometry of a pentagon, we have $\tan(36°)=a/(2L)$. Therefore, the y-coordinate of the center of mass is

$$y_{\rm cm}=-\frac{1}{4}y_5=\frac{L}{6}=\frac{a}{12}\cot(36°)=0.115\,a$$

ASSESS From symmetry, the center of mass must lie along the vertical line that bisects the figure. With the missing triangle, we expect it to be located above $y=0$, which would have been the center of mass for a complete pentagon.

42. **INTERPRET** This problem concerns stopping a charging rhino with rubber bullets that lose all their momentum when they hit the animal.

DEVELOP The impulse imparted on the rhino by one bullet is equal to the rhino's change in momentum $\left(\text{Equation 9.10a: } \vec{J}_{\rm b}=\Delta\vec{p}_{\rm r}\right)$. But we don't know the mass of the rhino, so it is easier to deal with the bullets. Their change in momentum is equal and opposite to the change in momentum of the rhino: $\Delta p_{\rm r}=-\Delta p_{\rm b}$. Note that we've dropped the vector notation, since the momenta are collinear, but we'll assume that the bullets are initially moving in the positive direction. We're given the initial velocity of the bullets, and we know that they fall straight to the ground after impact, so their final velocity must be zero. Putting all this together, we have:

$$J_{\rm b}=\Delta p_{\rm r}=-\Delta p_{\rm b}=-\left(0-m_{\rm b}v_{\rm b0}\right)=m_{\rm b}v_{\rm b0}$$

We can find the mass of the rhino by calculating the total impulse supplied by all the bullets fired at the rhino: $J_{\rm tot}=N_{\rm b}J_{\rm b}$. This total impulse is what supposedly brings the rhino to rest:

$J_{\rm tot}=\Delta p_{\rm r,tot}=0-m_{\rm r}v_{\rm r0}=-m_{\rm r}v_{\rm r0}$. The minus sign is not a problem, since the rhino's initial velocity is negative compared to the positive velocity of the bullets.

EVALUATE **(a)** From the expression above, the impulse imparted by one bullet is

$$J_{\rm b}=m_{\rm b}v_{\rm b0}=(19\text{ g})(74\text{ m/s})=1.4\text{ kg}\cdot\text{m/s}\approx 1.4\text{ N}\cdot\text{s}$$

(b) To find the mass of the rhino, we first need to calculate the number of bullets, which is the rate at which the gun is fired multiplied by the time:

$$N_{\rm b}=rt=(15\text{ bullets/s})(30\text{ s})=450\text{ bullets}$$

The mass can then be found from the total impulse from all these bullets:

$$m_{\rm r}=\frac{\Delta p_{\rm r,tot}}{-v_{\rm r0}}=\frac{N_{\rm b}J_{\rm b}}{-v_{\rm r0}}=\frac{(450)(1.4\text{ kg}\cdot\text{m/s})}{-(-0.75\text{ m/s})}=840\text{ kg}$$

ASSESS The answer is reasonable for a black rhino. But note that white rhinos typically have twice this mass.

43. **INTERPRET** This is a two-dimensional problem involving a system of three objects for which we want to find the kinetic energy of the center of mass and the internal kinetic energy. We can consider the kinetic energy to be composed of the energy belonging to the center of mass and to the individual parts moving relative to the center of mass.

DEVELOP Much like the location of the center of mass can be found by using Equation 9.2, so can the velocity of the center of mass by taking

$$\vec{v}_{\text{cm}} = \frac{\sum m_i \vec{v}_i}{M}$$

Knowing the velocity of the center of mass, we can determine the relative velocities of the individual particles since $\vec{v}_i = \vec{v}_{\text{cm}} + \vec{v}_{i\text{ rel}}$. Then, we can use these values to calculate the kinetic energy of the center of mass and the internal kinetic energy since

$$K_{\text{cm}} = \frac{1}{2}Mv_{\text{cm}}^2 \qquad K_{\text{int}} = \frac{1}{2}\sum m_i v_{i\text{ rel}}^2$$

EVALUATE Using the velocities given, we find that the velocity of the center of mass is equal to

$$\begin{aligned}\vec{v}_{\text{cm}} &= \frac{\sum m_i \vec{v}_i}{M} = \frac{(0.2\text{ kg})\left[(15.0\hat{j}\text{ m/s}) + (6.7\hat{i} - 3.45\hat{j}\text{ m/s}) + (-6.7\hat{i} - 4.32\hat{j}\text{ m/s})\right]}{(3)(0.2\text{ kg})} \\ &= 2.41\hat{j}\text{ m/s}\end{aligned}$$

which makes the relative velocities equal to

$$\vec{v}_{1\text{ rel}} = 12.59\hat{j}\text{ m/s} \qquad \vec{v}_{2\text{ rel}} = 6.7\hat{i} - 5.86\hat{j}\text{ m/s} \qquad \vec{v}_{3\text{ rel}} = -6.7\hat{i} - 6.73\hat{j}\text{ m/s}$$

Plugging these into our expressions for the center of mass and internal kinetic energies, we find

$$K_{\text{cm}} = 1.74\text{ J} \qquad K_{\text{int}} = 32.8\text{ J}$$

ASSESS The total kinetic energy of the system is the sum of the kinetic energy of the center of mass and the internal kinetic energy, which, for these masses, is equal to 34.5 J.

44. **INTERPRET** We are asked about the motion of the boat, but the problem is fundamentally related to the center of mass of the system.

DEVELOP This problem is similar to Example 9.4. Take the *x*-axis to be horizontal from bow to stern, with the origin at the center of mass (CM) of the boat and people. In the absence of external horizontal forces like friction, the CM remains stationary. Thus,

$$0 = m_{\text{p}}x_{\text{pi}} + m_{\text{B}}x_{\text{Bi}} = m_{\text{p}}x_{\text{pf}} + m_{\text{B}}x_{\text{Bf}}$$

where x_{Bi} is the initial position of the CM of the boat, x_{Bf} is its final position, and x_{pi} and x_{pf} are the initial and final positions, respectively, of the people. Note that $x_{\text{pi}} < 0$, $x_{\text{Bi}} > 0$, $x_{\text{pf}} > 0$, and $x_{\text{Bf}} < 0$. This equation can be rewritten as

$$m_{\text{B}}(x_{\text{Bi}} - x_{\text{Bf}}) = m_{\text{p}}(x_{\text{pf}} - x_{\text{pi}})$$

since $x_{\text{Bi}} - x_{\text{Bf}}$ is the distance the boat moves relative to the fixed CM. The distances are related to the dimensions of the boat, since the length of the boat is equal to

$$|x_{\text{pi}}| + x_{\text{Bi}} + |x_{\text{Bf}}| + x_{\text{pf}} = x_{\text{pf}} - x_{\text{pi}} + x_{\text{Bi}} - x_{\text{Bf}} = 5.8\text{ m}$$

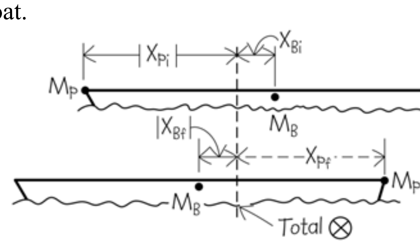
EVALUATE Substituting into the first equation, one finds

$$m_{\text{B}}(x_{\text{Bi}} - x_{\text{Bf}}) = m_{\text{p}}\left[5.8\text{ m} - (x_{\text{Bi}} - x_{\text{Bf}})\right]$$

Thus, we find

$$x_{\text{Bi}} - x_{\text{Bf}} = (5.8\text{ m})\frac{m_{\text{p}}}{m_{\text{p}} + m_{\text{B}}} = (5.8\text{ m})\frac{1200\text{ kg}}{1200\text{ kg} + 13{,}000\text{ kg}} = 49\text{ cm}$$

to two significant figures, which is the precision of the data. Note that we did not have to assume that the CM of the boat was at the center of the boat.



ASSESS The boat's displacement of 49 cm is less than the distance the people walked. This makes sense because the boat is much more massive than the people.

45. **INTERPRET** This problem involves the center of mass of a two-body system, which remains stationary in the absence of external horizontal forces.

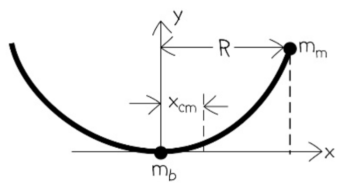
DEVELOP When the mouse starts at the rim, the center of mass of the mouse-bowl system has *x*-component:

$$x_{\rm cm} = \left(m_{\rm b}x_{\rm b} + m_{\rm m}x_{\rm m}\right)/\left(m_{\rm b} + m_{\rm m}\right) = m_{\rm m}R/\left(m_{\rm b} + m_{\rm m}\right)$$

since initially $x_{\rm b} = 0$ and $x_{\rm m} = R$. Because there is no external horizontal force (no friction), $x_{\rm cm}$ remains constant as the mouse descends. When it reaches the center of the bowl, the center of mass of the system is

$$x_{\rm cm} = \left(m_{\rm b}x'_{\rm b} + m_{\rm m}x'_{\rm m}\right)/\left(m_{\rm b} + m_{\rm m}\right) = \left(m_b d/10 + m_m d/10\right)/\left(m_{\rm b} + m_{\rm m}\right)$$

Because the center of mass does not move, we can equate these two expressions for the center of mass to find the ratio of $m_{\rm b}$ to $m_{\rm m}$.



EVALUATE Using the fact that $2R = d$, we find

$$m_{\rm m}R/\left(m_{\rm b} + m_{\rm m}\right) = \left(m_b R/5 + m_m R/5\right)/\left(m_{\rm b} + m_{\rm m}\right)$$
$$m_b = 4m_m$$

ASSESS The bowl is four times more massive than the mouse, which makes sense because the bowl has been horizontally displaced.

46. **INTERPRET** This problem involves the impulse exerted on a needle shot into the body in order to obtain a sample of internal organs.

DEVELOP The needle starts at rest and is accelerated by the force of the spring. We don't know this force or how long it is applied, but we know that the momentum gain from the spring, Δp, is lost when the needle is stopped by the skin. To say it another way, the impulse imparted by the spring is equal but opposite to the impulse imparted by the skin's stopping force. And we have the information needed to calculate the impulse from the skin: $J_{\rm skin} = F\Delta t$. For part (b), to determine the penetration distance, we take the acceleration of the needle as the force of the tissue acts on it: $a_{\rm n} = F/m_{\rm n}$. Since this acceleration is constant, we can use the formalism from Chapter 2.

EVALUATE **(a)** As explained above, the impulse imparted by the spring has the same magnitude as the impulse imparted by the skin:

$$J_{\rm spring} = J_{\rm skin} = F\Delta t = (39\ \text{mN})(86\ \text{ms}) = 3.4\ \text{mN}\cdot\text{s}$$

(b) As far as we can tell, the force and corresponding acceleration are constant. The initial speed of the needle (just before entering the skin) must have been $v_0 = a_{\rm n}\Delta t$, and the distance traveled through the body is (Equation 2.10)

$$\Delta x = v_0\Delta t - \frac{1}{2}a_{\rm n}\Delta t^2 = \frac{F}{2m_{\rm n}}\Delta t^2 = \frac{\left(39\times10^{-3}\ \text{N}\right)}{2\left(8.6\times10^{-3}\ \text{kg}\right)}\left(86\times10^{-3}\ \text{s}\right)^2 = 1.7\ \text{cm}$$

where we have been careful to treat the acceleration as a deceleration.

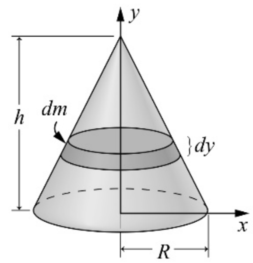
ASSESS A distance of 1.7 cm is a reasonable depth to sample internal organs.

47. **INTERPRET** In this problem we are asked to find the center of mass of a uniform solid cone. We will need to integrate thin slices of the cone to find the answer.

DEVELOP Choose the *z*-axis along the axis of the cone, with the origin at the center of the base (see figure below). Because the cone is symmetric about the *z*-axis, the center of mass is on the *z*-axis [for each mass element at position (*x*, *y*, *z*) there is an equal-mass element at position (–*x*, –*y*, –*z*), so the integral over *x* and *y* gives zero]. Thus, we only need to find the *z*-coordinate of the center of mass, so Equation 9.4 reduces to

$$z_{\rm cm} = \frac{\int z dm}{M}$$

For the mass element *dm*, take a disk at height *z* and of radius $r = R(1 - z/h)$ that is parallel to the base. Then $dm = \rho\pi r^2 dz = \rho\pi R^2 (1 - z/h)^2 dz$ where ρ is the density of the cone, and $M = \frac{1}{3}\rho\pi R^2 h$ is the total mass of the cone.



EVALUATE For the *z*-coordinate of the center of mass, the integral above gives

$$z_{\rm cm} = \frac{1}{M}\int_0^h z\ dm = \frac{3}{\rho\pi R^2 h}\int_0^h z\rho\pi R^2 (1 - z/h)^2 dz$$
$$= \frac{3}{h}\int_0^h \left(z - \frac{2z^2}{h} + \frac{z^3}{h^2} \right) dz = \frac{3}{h}\left(\frac{h^2}{2} - \frac{2h^2}{3} + \frac{h^2}{4} \right) = \frac{1}{4}h$$

so the complete center of mass coordinate is (0, 0, *h*/4).

ASSESS The result makes sense because we expect $z_{\rm cm}$ to be closer to the bottom of the cone because more mass is distributed in this region.

48. **INTERPRET** This problem involves conservation of linear momentum, which we can use to find the mass and the direction of motion of the second firecracker fragment.

DEVELOP Apply conservation of linear momentum. Because the firecracker is initially at rest, the initial momentum of the system is zero. After the explosion (ignoring air resistance and relativistic effects), the total linear momentum is still zero and is expressed as

$$0 = m_1\vec{v}_1 + m_2\vec{v}_2$$

where $m_1 = 14$ g, $\vec{v}_1 = (43 \text{ m/s})\hat{i}$, and $v_2 = 30$ m/s.

EVALUATE Because mass is always positive, we know that the direction of $\vec{v}_2$ is opposite to that of $\vec{v}_1$, so $\vec{v}_2 = (-30 \text{ m/s})\hat{i}$, so the direction of motion is $-\hat{i}$ or opposite to the direction of $\vec{v}_1$. Evaluating the expression for total linear momentum gives $m_2 = -m_1 v_1 / v_2 = -(14 \text{ g})(43 \text{ m/s})/(-30 \text{ m/s}) = 20.1$ g.

ASSESS We find that the slower-moving mass is greater than the faster-moving mass, as expected.

49. **INTERPRET** We are asked about the compression of the spring due to a totally inelastic collision.

DEVELOP Since the total momentum of the system is conserved in the process, we have

$$P_{\rm i} = P_{\rm f} \quad \Rightarrow \quad m_1 v_1 = (m_1 + m_2) v_{\rm f}$$

The potential energy of the spring at maximum compression equals the kinetic energy of the two-car system prior to contact with the spring: $\frac{1}{2}kx_{\max}^2 = \frac{1}{2}(m_1 + m_2)v_{\rm f}^2$.

For (b), we note that when the cars rebound, they are coupled together and both have the same velocity. Since the spring is ideal (by assumption), its maximum potential energy, $\frac{1}{2}kx_{\max}^2$, is transformed back into kinetic energy of the cars.

EVALUATE (a) The second car is initially at rest so $v_2 = 0$. By momentum conservation, the speed of the cars after collision is

$$v_{\rm f} = \frac{m_1 v_1}{m_1 + m_2} = \frac{(9400 \text{ kg})(8.5 \text{ m/s})}{11{,}000 \text{ kg} + 9400 \text{ kg}} = 3.92 \text{ m/s}$$

which leads to

$$x_{\max} = v_{\mathrm{f}}\sqrt{\frac{m_1+m_2}{k}} = (3.92\ \mathrm{m/s})\sqrt{\frac{11{,}000\ \mathrm{kg}+9400\ \mathrm{kg}}{0.32\times10^6\ \mathrm{N/m}}} = 0.99\ \mathrm{m}$$

(b) The spring's potential energy is converted back into the kinetic energy of the cars, so the rebound speed should be the same (only in the opposite direction) as the speed prior to the spring being compressed:

$$v_{\mathrm{reb}} = v_{\mathrm{f}} = 3.9\ \mathrm{m/s}$$

where we only keep the significant figures.

ASSESS During the collision in the first part of the motion, the momentum is conserved but energy is not. However, during the spring compression and release in the second part, energy is conserved. Therefore, the cars rebound with the same speed as that before coming into contact with the spring.

50. **INTERPRET** This one-dimensional problem involves an inelastic collision on a frictionless surface, so kinetic energy is not conserved, but total linear momentum is conserved. We can use this to find the speed of the three-vehicle wreckage.

DEVELOP Assume that the road is horizontal and the velocities are collinear. By conservation of linear momentum, we can equate the total linear momentum before and after the collision. Before the collision, the total momentum of the three vehicles is $p = m_1v_1 + m_2v_2 + m_3v_3$. After the accident, we have $p = (m_1 + m_2 + m_3)\,v$.

EVALUATE Equating the two expressions for total linear momentum, we find

$$\begin{aligned} v &= \frac{m_1v_1+m_2v_2+m_3v_3}{m_1+m_2+m_3} \\ &= \frac{(1200\ \mathrm{kg})(50\ \mathrm{km/h})+(4400\ \mathrm{kg})(35\ \mathrm{km/h})+(1500\ \mathrm{kg})(65\ \mathrm{km/h})}{1200\ \mathrm{kg}+4400\ \mathrm{kg}+1500\ \mathrm{kg}} = 44\ \mathrm{km/h} \end{aligned}$$

ASSESS Notice that the truck has increased its speed, whereas the cars have reduced their speed, as expected.

51. **INTERPRET** In this problem, we want to estimate the amount of force imparted onto a window by snowballs. We can do this by relating the change in momentum of the balls to the average force they deal as they collide.

DEVELOP We are told the mass and velocity of each snowball, so we can determine the momentum with which they arrive at the window. Since they drop vertically, we can deduce that all their horizontal momentum is imparted onto the window. We are also told that two arrive every second, so we can calculate the average force using Equation 9.10a: $\Delta\vec{p} = \bar{\vec{F}}\Delta t$.

EVALUATE We know that the momentum we are considering is only along the horizontal direction, so the average force imparted onto the window by the snowballs is given by $\bar{F} = \Delta p/\Delta t$. Thus, since $p = mv$, and we know that, on average, two snowballs arrive every second, we find that the average force is about

$$\bar{F} = \frac{\Delta p}{\Delta t} = \frac{(2)(0.4\ \mathrm{kg})(12\ \mathrm{m/s})}{(1\ \mathrm{s})} = 9.6\ \mathrm{N}$$

ASSESS This number will vary from the average if there are changes in the snowball mass, velocity, and rate of impact.

52. **INTERPRET** This two-dimensional problem involves the principle of conservation of linear momentum in an inelastic two-body collision. We can apply this principle to find the final velocity of the two-body system after the collision.

DEVELOP If there are no external horizontal forces acting on the car-wagon system, momentum (in the x–y plane) is conserved, so

$$\begin{aligned} \vec{p}_{\mathrm{i}} &= \vec{p}_{\mathrm{f}} \\ m_1\vec{v}_1 + m_2\vec{v}_2 &= (m_1+m_2)\vec{v}. \end{aligned}$$

which we can solve for the final velocity $\vec{v}$.

EVALUATE Inserting the given quantities into the expression above gives

$$\vec{v} = \frac{(1250\text{ kg})\left(36.2\hat{i} + 12.7\hat{j}\right)\text{m/s} + (450\text{ kg})\left(13.8\hat{i} + 10.2\hat{j}\right)\text{ m/s}}{(1250 + 450)\text{ kg}} = \left(30.3\hat{i} + 12.0\hat{j}\right)\text{ m/s}$$

ASSESS The magnitude of this velocity is $v = \sqrt{v_x^2 + v_y^2} = 32.6\text{ m/s}$, and its direction is $\theta = \tan^{-1}\left(v_y / v_x\right) = 21.6°$ with respect to the x-axis.

53. **INTERPRET** We're asked to find the speeds of two objects following their head-on elastic collision.
DEVELOP The collision is one-dimensional, so Equations 9.15a and 9.15b are relevant. The information that we're given is that $m_1 = m$, $v_{1i} = v$, $m_2 = 2m$, and $v_{2i} = -2v$. Notice that both objects are initially moving in opposite directions.
EVALUATE Plugging the parameters into Equations 9.15,

$$v_{1f} = \frac{m - 2m}{m + 2m}(v) + \frac{2(2m)}{m + 2m}(-2v) = -3v$$

$$v_{2f} = \frac{2m}{m + 2m}(v) + \frac{2m - m}{m + 2m}(-2v) = 0$$

ASSESS Do the minus signs make sense in the derivation? Let's assume the first object approaches from the left with positive velocity, and it bounces off to the left in the negative direction. The second object approaches from the right with negative velocity and stops.

54. **INTERPRET** This is a two-dimensional problem that involves conservation of linear momentum. The quantity of interest is the recoil velocity of the thorium nucleus, produced from the decay of the ^{238}U nucleus.
DEVELOP Because no external forces are acting on the system (ignoring gravity), linear momentum is conserved, as in Example 9.6. Equating the initial and final momenta gives

$$m_U \vec{v}_U = m_{He}\vec{v}_{He} + m_{Th}\vec{v}_{Th}$$

In terms of the x- and y-components, this vector equation gives the following two scalar equations:

$$m_U v_U = m_{He} v_{He,\,x} + m_{Th} v_{Th,\,x} = m_{He} v_{He}\cos\phi + m_{Th} v_{Th}\cos\theta \qquad (x\text{-component})$$

$$0 = m_{He} v_{He,y} + m_{Th} v_{Th,y} = m_{He} v_{He}\sin\phi + m_{Th} v_{Th}\sin\theta \qquad (y\text{-component})$$

These equations can be used to solve for the magnitude and direction of $\vec{v}_{Th}$.

EVALUATE Solving the two equations, we obtain

$$v_{Th,x} = v_{Th}\cos\theta = \frac{m_U v_U - m_{He} v_{He}\cos\phi}{m_{Th}} = \frac{(238\text{ u})\left(4.1\times10^5\text{ m/s}\right) - (4\text{ u})\left(1.5\times10^7\text{ m/s}\right)\cos 36°}{234\text{ u}} = 2.1\times10^5\text{ m/s}$$

and

$$v_{Th,y} = v_{Th}\sin\theta = -\frac{m_{He} v_{He}\sin\phi}{m_{Th}} = -\frac{(4\text{ u})\left(1.5\times10^7\text{ m/s}\right)\sin 36°}{234\text{ u}} = -1.5\times10^4\text{ m/s}$$

to two significant figures. Thus, the recoil velocity of the thorium atom is
$\vec{v}_{Th} = \left(2.1\times10^5\text{ m/s}\right)\hat{i} - \left(1.5\times10^5\text{ m/s}\right)\hat{j}$, or $v_{Th} = \sqrt{v_{Th,x}^2 + v_{Th,y}^2} = 2.6\times10^5\text{ m/s}$, and the direction is $\theta = \text{a}\tan\left(v_{Th,y} / v_{Th,x}\right) = -36°$.

ASSESS The fact that θ is negative tells us that the velocity of the thorium atom is downward, as expected, to compensate for the upward velocity of the alpha particle.

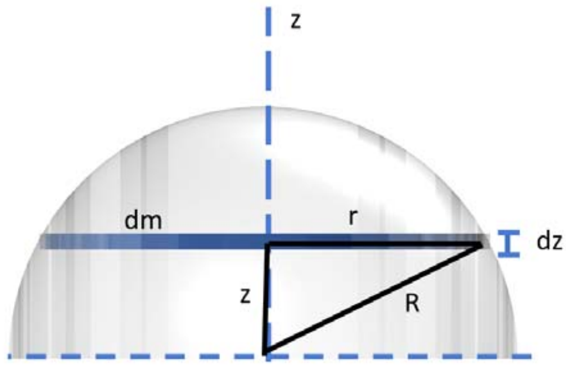
55. **INTERPRET** In this problem we want to find the center of mass of a solid hemisphere. We treat this as a problem in finding the center of mass of a continuous distribution of matter.
DEVELOP Equation 9.4 shows that the center of mass of a continuous distribution of matter is given by

$$\vec{r}_{cm} = \frac{\int \vec{r}\,dm}{M}$$

To do this integral we must find an expression for dm, and from Tactics 9.1 we deduce that we can find it from the expression

$$\frac{dm}{M}=\frac{dV}{V}$$

Which means we must first define an infinitesimal volume element dV. Looking at the figure below we can see that this can be done by considering a slice of radius $r=\sqrt{R^2-z^2}$ and thickness dz.



Thus, we find that our infinitesimal volume element is given by $dV=\pi\left(R^2-z^2\right)dz$, and our infinitesimal mass element is given by

$$dm=\frac{M}{V}dV=\frac{3M}{2R^3}\left(R^2-z^2\right)dz$$

Where, $V=2\pi R^3/3$, is the volume of the hemisphere. Plugging this back into Equation 9.4, and integrating along the z-direction will then give us the center of mass. This is because we have symmetry along the *x*–*y* plane, and our center of mass will thus be located along the *z*-direction at (0,0, z_{cm}).

EVALUATE After plugging in our expression for dm our integral becomes

$$\vec{z}_{\text{cm}}=\frac{\int \vec{z}\,dm}{M}=\int_0^R\frac{3}{2R^3}\left(R^2-z^2\right)z\,dz$$

Evaluating this integral shows the *z*-coordinate for the center of mass is located at

$$z_{\text{cm}}=\frac{3}{2R^3}\int_0^R\left(R^2-z^2\right)z\,dz=\frac{3}{2R^3}\left(\frac{1}{2}R^4-\frac{1}{4}R^4\right)=\frac{3R}{8}$$

ASSESS We can always verify that we have chosen the correct infinitesimal volume element by integrating it over the region our object occupies. In this case that would mean integrating $\int_0^R \pi(R^2-z^2)dz$, which is equal to $2\pi R^3/3$, the volume of a hemisphere.

56. **INTERPRET** This problem involves conservation of linear momentum. The object of interest is the firecracker that has exploded into three pieces. With the mass and velocity of two pieces given, we can use conservation of linear momentum to find the velocity of the third piece.

DEVELOP At the instant after the explosion (before any external forces have had any time to act appreciably), the total momentum of the three-body system (i.e., the three firecracker fragments) is still zero. Expressed mathematically, this is

$$\vec{P}_{\text{tot}}=0=m_1\vec{v}_1+m_2\vec{v}_2+m_3\vec{v}_3$$

which we can solve to find $\vec{v}_3$.

EVALUATE The mass of the third piece is $m_3=m-m_1-m_2=55\text{ g}-7\text{ g}-15\text{ g}=33\text{ g}$. Its velocity is

$$\vec{v}_3=-\frac{m_1\vec{v}_1+m_2\vec{v}_2}{m_3}=-\frac{(7\text{ g})(33\text{ m/s})\hat{i}+(15\text{ g})(26\text{ m/s})\hat{j}}{33\text{ g}}=-(7\text{ m/s})\hat{i}-(11.8\text{ m/s})\hat{j}$$

ASSESS Since the initial momentum of the firecracker is zero, we expect the momentum of the third piece to completely cancel the momentum of the first two pieces. Thus, $\vec{v}_3$ has components that are opposite to $\vec{v}_1$ and $\vec{v}_2$. Since m_3 is larger than m_1 and m_2, we expect the magnitude of $\vec{v}_3$ to be smaller than the magnitudes of $\vec{v}_1$ and $\vec{v}_2$.

57. **INTERPRET** No external forces act on the three-body system, so total linear momentum is conserved. We can use this to find the velocity of the camera discarded by the astronaut.

DEVELOP In the rest frame of the astronaut (i.e., in the inertial frame of reference in which the astronaut is at rest), the total momentum of the three-body system is zero. After the astronaut discards the two items, the total momentum must still be zero, so

$$m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 = 0$$

where the subscripts 1, 2, and 3 refer to the astronaut, the air canister, and the camera, respectively. Decomposing this vector equation into two scalar equations gives

$$m_1 v_1 \cos(200°) + m_2 v_2 \cos(0°) + m_3 v_{3,x} = 0$$
$$m_1 v_1 \sin(200°) + m_2 v_2 \sin(0°) + m_3 v_{3,y} = 0$$

which we can solve for $\vec{v}_3$.

EVALUATE Solving first for the x-component of the camera's velocity, we find

$$v_{3x} = -\frac{(67\text{ kg})(0.66\text{ m/s})\cos(200°) + (18\text{ kg})(1.1\text{ m/s})}{5.4\text{ kg}} = 4.0\text{ m/s}$$

Similarly, the y-component is

$$v_{3y} = \frac{-(67\text{ kg})(0.66\text{ m/s})\sin(200°)}{5.4\text{ kg}} = 2.8\text{ m/s}$$

So the velocity of the camera is $\vec{v}_3 = (4.0\text{ m/s})\hat{i} + (2.8\text{ m/s})\hat{j}$.

ASSESS Alternatively, we can express the result in terms of the magnitude and direction of the velocity. This gives $v_3 = \sqrt{4.0^2 + 2.8^2}\text{ m/s} = 4.9\text{ m/s}$ and $\theta_3 = \text{atan}(2.8\text{ ms}/4.0\text{ ms}) = 35°$ counterclockwise from the x-axis.

58. **INTERPRET** Before an explosion, an object has kinetic energy $K = \frac{1}{2}mv_i^2$. After the explosion, it has two pieces: each with mass of $\frac{1}{2}m$, and each moving at twice the initial speed, $v_f = 2v_i$. We are asked to find the angles at which two pieces of an object fly off from the explosion.

DEVELOP Let's assume that the original object was moving in the x-direction, with no momentum in the y-direction. After the explosion, conservation of momentum implies that the y-momentum of the two pieces sums to zero:

$$y\text{-component:}\quad 0 = \left(\tfrac{1}{2}m\right)(2v_i)\sin\theta_1 + \left(\tfrac{1}{2}m\right)(2v_i)\sin\theta_2$$

For the y-components to cancel, the angles that each piece makes with the x-axis are equal and opposite: $\theta_1 = -\theta_2$.

EVALUATE To find the value of these angles, we consider the momentum in the x-direction:

$$x\text{-component:}\quad mv_i = \left(\tfrac{1}{2}m\right)(2v_i)\cos\theta_1 + \left(\tfrac{1}{2}m\right)(2v_i)\cos\theta_2$$

This reduces to $\cos\theta_1 = \frac{1}{2}$, which means the angles are 60° and –60°.

ASSESS In this case, the speed relative to the center of mass is just the y-component of their velocity: $v_{rel} = (2v_i)\sin 60° = \sqrt{3}v_i$. This implies that the internal energy would be

$$K_{int} = \sum \tfrac{1}{2}m_i v_{irel}^2 = \tfrac{1}{2}\left(\tfrac{1}{2}m\right)\left(\sqrt{3}v_i\right)^2 + \tfrac{1}{2}\left(\tfrac{1}{2}m\right)\left(\sqrt{3}v_i\right)^2 = 3\left(\tfrac{1}{2}mv_i^2\right) = 3K$$

This agrees with the result in Problem 9.22.

59. **INTERPRET** This one-dimensional problem involves conservation of linear momentum and relative motion. We can use the former to find the speed of the sprinter with respect to the cart and the latter to find her speed relative to the ground.

DEVELOP We choose a coordinate system in which the cart moves in the $-\hat{i}$ direction and the sprinter runs in the $\hat{i}$ direction. The initial momentum of the system is

$$p = (m_s + m_c)v_{cm}$$

The final momentum of the system is

$$p = m_s v_s + m_c v_c = m_c v_c$$

because she has zero velocity with respect to the ground $(v_s = 0)$. Equating these two expressions for total linear momentum (by conservation of total linear momentum), we have

$$(m_s + m_c)v_{cm} = m_s v_s + m_c v_c = m_c v_c$$

Using Equation 3.7 to express the sprinter's speed relative to the cart, we have

$$v_s = v_{rel} + v_c$$
$$v_{rel} = -v_c$$

because $v_s = 0$. We are given $v_{cm} = -6.9\ \text{m/s}$, $m_c = 180\ \text{kg}$, and $m_s = 55\ \text{kg}$.

EVALUATE Solving the equations above for v_{rel}, we find

$$(m_s + m_c)v_{cm} = m_c v_c = -m_c v_{rel}$$
$$v_{rel} = -\frac{(m_s + m_c)v_{cm}}{m_c} = -\frac{(55\ \text{kg} + 180\ \text{kg})(-6.9\ \text{m/s})}{180\ \text{kg}} = 9.0\ \text{m/s}$$

ASSESS The fact that the sprinter accelerates by pushing against the cart accelerates the cart from $-6.9\ \text{m/s}$ to $-9.0\ \text{m/s}$, which is reasonable.

60. **INTERPRET** This problem involves exerting a force on a conveyor belt to compensate for the change in momentum caused by the drops of cookie dough that drop onto the belt.

DEVELOP If the conveyor belt is horizontal and moving with speed $v = 52\ \text{cm/s}$ and the mounds of dough fall vertically, then the change in the horizontal momentum due to each mound of mass Δm is $\Delta p = (\Delta m)v$. The average horizontal force needed is equal to the rate at which mounds are dropped (a number N in time Δt, or $N/\Delta t$) times the change in momentum due to a single mound. Thus, for this problem, Equation 9.6 takes the form

$$\vec{F}_{av} = \left(\frac{N}{\Delta t}\right)\Delta\vec{p} = \left(\frac{N}{\Delta t}\right)(\Delta m)\vec{v}$$

EVALUATE Inserting the values given in the problem statement, we find that the average force the conveyor belt exerts on a cookie sheet is

$$F_{av} = \left(\frac{N}{\Delta t}\right)(\Delta m)v = \left(\frac{1}{2\ \text{s}}\right)(0.011\ \text{kg})(0.52\ \text{m/s}) = 2.86\times10^{-3}\ \text{N}$$

ASSESS The average force is just the total change in momentum, $\Delta\vec{P} = N\Delta\vec{P} = N(\Delta m)\vec{v}$, divided by the time, Δt. The greater is the change in momentum over a given time interval, the greater is the average force.

61. **INTERPRET** We're asked to find the speeds of two objects following their head-on elastic collision.

DEVELOP The collision is one-dimensional, so Equations 9.15a and 9.15b are relevant. The information that we're given is that $m_1 = m$, $v_{1i} = 2v$, $m_2 = 4m$, and $v_{2i} = v$. Notice that both objects are initially moving in the same (positive) direction.

EVALUATE Plugging the parameters into Equations 9.15,

$$v_{1f} = \frac{m-4m}{m+4m}(2v) + \frac{2(4m)}{m+4m}(v) = \left(\frac{-6}{5} + \frac{8}{5}\right)v = \frac{2}{5}v$$
$$v_{2f} = \frac{2m}{m+4m}(2v) + \frac{4m-m}{m+4m}(v) = \left(\frac{4}{5} + \frac{3}{5}\right)v = \frac{7}{5}v$$

ASSESS The first object loses some momentum from the collision $(v_{1i} < v_{1f})$, whereas the second object gets a "push" from the collision $(v_{2i} > v_{2f})$. Notice that $v_{1f} < v_{2f}$, otherwise it wouldn't make sense how the first object got ahead of the second object.

62. **INTERPRET** We're asked to verify that the final speeds that we found in the previous problem obey conservation of energy.

DEVELOP The conservation of energy in a collision is expressed in Equation 9.13.

EVALUATE The initial kinetic energy is:

$$\tfrac{1}{2}m_1v_{1i}^2+\tfrac{1}{2}m_2v_{2i}^2=\tfrac{1}{2}m(2v)^2+\tfrac{1}{2}(4m)v^2=4mv^2$$

Using the results from the previous problem we have for the final kinetic energy:

$$\tfrac{1}{2}m_1v_{1f}^2+\tfrac{1}{2}m_2v_{2f}^2=\tfrac{1}{2}m\left(\tfrac{2}{5}v\right)^2+\tfrac{1}{2}(4m)\left(\tfrac{7}{5}v\right)^2=\left(\tfrac{4}{50}+\tfrac{196}{50}\right)mv^2=4mv^2$$

So yes, the energy is conserved in this collision.

ASSESS Although both objects have the same kinetic energy initially, the second particle leaves the collision with most of the kinetic energy.

63. **INTERPRET** This problem involves conservation of momentum applied to a two-body system. The center of mass of this system does not move because of conservation of momentum. We will apply these principles to find the initial angle at which we threw the rock and the speed at which you must be moving.

DEVELOP We choose a coordinate system in which your initial position is at the origin (see figure below). Apply Equation 3.15 to find the angle θ at which you throw the rock,

$$x_1=\frac{v_0^2}{g}\sin(2\theta)$$

with x_1 = 15.2 m – x_2 and v_0 = 12.0 m/s. We can find x_2 because we know the center of mass of the two-body system does not change since there are no horizontal forces acting on it. Thus,

$$x_{cm}=0=\frac{m_1x_1+m_2x_2}{m_1+m_2}$$

$$x_2=-\frac{m_1x_1}{m_2}$$

so

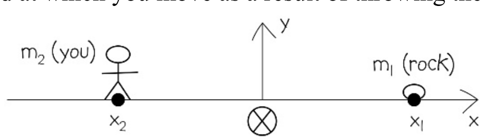
$$x_1=15.2\text{ m}-\frac{m_1x_1}{m_2}$$

$$x_1=\frac{15.2\text{ m}}{1+m_1/m_2}$$

which allows us to solve Equation 3.15 for the angle θ. To find the speed at which you move after throwing the rock, apply conservation of linear momentum. Your initial horizontal momentum is zero, so your final momentum must also be zero, or

$$m_1v_{1x}+m_2v_{2x}=0$$

where $v_{1x}=v_0\cos\theta$, with θ being the angle with respect to the horizontal at which you throw the rock. Solve this equation for v_{2x}, which is the speed at which you move as a result of throwing the rock.



EVALUATE (a) Inserting the known quantities into Equation 3.15 and solving for θ gives

$$\theta=\frac{1}{2}\text{asin}\left(\frac{x_1g}{v_0^2}\right)=\frac{1}{2}\text{asin}\left(\frac{(15.2\text{ m})g}{(1+m_1/m_2)v_0^2}\right)=\frac{1}{2}\text{asin}\left(\frac{(15.2\text{ m})(9.8\text{ m/s}^2)}{[1+(4.50\text{ kg})/(65.0\text{ kg})](12.0\text{ m/s})^2}\right)=37.7^\circ$$

(b) The speed at which you recoil after throwing the rock is

$$v_{2x}=-\frac{m_1v_{1x}}{m_2}=-\frac{m_1v_0\cos\theta}{m_2}=-\frac{(4.50\text{ kg})(12.0\text{ m/s})\cos(37.7^\circ)}{65.0\text{ kg}}=-65.8\text{ cm/s}$$

ASSESS The horizontal speed of the rock is $v_0\cos(\theta)$ = 9.50 m/s. Thus, your recoil speed is much less than the rock's horizontal speed, as expected.

64. **INTERPRET** The problem asks about the speed of the drunk driver just before a totally inelastic collision. Energy is not conserved in this process, but momentum is.

DEVELOP If the wreckage skidded on a horizontal road, the work-energy theorem requires that the work done by friction be equal to the change in the kinetic energy of both cars, or $W_{\text{nc}} = \Delta K$ (see Equation 7.5). Since

$$W_{\text{nc}} = -f_{\text{k}}x = -\mu_{\text{k}}nx = -\mu_{\text{k}}(m_1 + m_2)gx$$

and

$$\Delta K = -\frac{1}{2}(m_1 + m_2)v^2$$

where *v* is the speed of the wreckage immediately after collision, we are led to

$$\mu_{\text{k}}gx = \frac{1}{2}v^2$$

The equation can be used to solve for *v*. Once *v* is known, we can apply momentum conservation to find the initial speed of the drunk driver.

EVALUATE From the equation above, we find that the speed of the cars (wreckage) just after the collision is

$$v = \sqrt{2\mu_{\text{k}}gx}$$

Momentum is conserved at the instant of the collision; so, if v_1 is the speed of the drunk driver's car just before the collision (and $v_2 = 0$ for the parked car), then $m_1v_1 = (m_1 + m_2)v$, or

$$v_1 = \frac{m_1 + m_2}{m_1}v = \frac{m_1 + m_2}{m_1}\sqrt{2\mu_{\text{k}}gx} = \frac{2400\text{ kg} + 1700\text{ kg}}{2400\text{ kg}}\sqrt{2(0.75)(9.8\text{ m/s}^2)(25\text{ m})} = 32.7\text{ m/s}$$

This is about 73 mi/h, so the driver was speeding as well as intoxicated.

ASSESS The answer makes sense because increasing v_1 will result in a greater wreckage speed *v*, and thus, a longer skidding distance *x*.

65. **INTERPRET** This two-body problem involves kinematics (Chapters 2 and 3) and motion of the center of mass. The rocket explodes into two equal-mass fragments at its peak height, which we can calculate from the kinematic equations of Chapter 2. We are given the time it takes from the explosion for one fragment to hit the ground and are asked to find the time at which the second fragment hits the ground.

DEVELOP At the peak of the rocket's trajectory (just before the explosion), its center-of-mass *y*-velocity is zero, or $v_{\text{cm}} = 0$. The motion of the center of mass is unaffected by the explosion; so just after the explosion, the velocity $v'_{\text{cm}} = 0$. Expressing the center-of-mass velocity in terms of the velocities v_1 and v_2 of fragments 1 and 2, respectively, gives

$$v'_{\text{cm}} = 0 = \frac{mv_1 + mv_2}{m + m}$$
$$v_1 = -v_2$$

where each of the equal-mass fragments has mass *m*. We can now use the kinematic equations to find v_1. The height *h* at which the rocket explodes may be found using Equation 2.11, which gives (with a slight change in notation)

$$\overbrace{v_{\text{cm}}}^{=0} = v_0^2 - 2gh$$
$$h = \frac{v_0^2}{2g}$$

where *v* is the velocity at the peak of the trajectory and $v_0 = 40\text{ m/s}$. Knowing the height and the time t_1 for fragment 1 to hit the ground, we can find its initial velocity from Equation 3.13. This gives

$$\overbrace{y - y_0}^{=-h} = v_1t_1 - \frac{1}{2}gt_1^2$$
$$v_1 = \frac{-h + gt_1^2/2}{t_1} = \frac{-v_0^2/2g + gt_1^2/2}{t_1} = \frac{-v_0^2}{2gt_1} + \frac{gt_1}{2}$$

Knowing v_1 (and thus, v_2), we can find the time for fragment 2 to hit the ground by using the same Equation (i.e., Equation 3.13), but solving for the time instead of the velocity. This gives

$$-h = v_2 t_2 - \frac{1}{2} g t_2^2$$

$$t_2 = \frac{v_2 \pm \sqrt{v_2^2 + 2gh}}{g} = \frac{-v_1 \pm \sqrt{v_1^2 + 2gh}}{g} = \frac{-v_1 \pm \sqrt{v_1^2 + v_0^2}}{g}$$

EVALUATE Evaluating v_1 first, we find

$$v_1 = \frac{-v_0^2}{2gt_1} + \frac{gt_1}{2} = \frac{-(40 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)(2.75 \text{ s})} + \frac{(9.81 \text{ m/s}^2)(2.75 \text{ s})}{2} = -16.16 \text{ m/s}$$

where we have retained one extra significant figure because this is an intermediate result. Inserting this result into the expression for t_2 gives

$$t_2 = \frac{-v_1 \pm \sqrt{v_1^2 + v_0^2}}{g} = \frac{-(-16.16 \text{ m/s}) \pm \sqrt{(-16.16 \text{ m/s})^2 + (40 \text{ m/s})^2}}{(9.81 \text{ m/s}^2)} = 6.04 \text{ s}, -2.75 \text{ s}$$

The physically significant result is $t_2 = 6.04$ s.
ASSESS The time for fragment 2 to reach the ground will increase with increasing $|v_1|$, which is reasonable because if fragment 1 has a larger downward velocity initially, then fragment 2 has a larger initial upward velocity. Also, note that if $v_1, v_2 \to 0$, then $t_2 = t_1 = 4.4$ s, which is intermediate between 2.75 s and 6.04 s, as expected.

66. **INTERPRET** In this problem, a totally inelastic collision results in two-thirds of the kinetic energy being lost. We are asked to find the ratio of the masses.

DEVELOP The particles come at each other with equal but opposite velocities $(\vec{v}_1 = -\vec{v}_2)$. In order to obey conservation of momentum (Equation 9.11), the final velocity has to be parallel to the initial velocities. In other words, the problem is one-dimensional:

$$m_1\vec{v}_1 - m_2\vec{v}_1 = (m_1 - m_2)\vec{v}_1 = (m_1 + m_2)\vec{v}_\text{f} \qquad \to \qquad v_\text{f} = \frac{m_1 - m_2}{m_1 + m_2} v$$

EVALUATE We're told that the initial kinetic energy is three times the final kinetic energy:

$$\tfrac{1}{2} m_1 v_1^2 + \tfrac{1}{2} m_2 v_2^2 = \tfrac{1}{2}(m_1 + m_2)v^2 = 3\left[\tfrac{1}{2}(m_1 + m_2)v_\text{f}^2\right] \quad \to \quad v_\text{f} = \pm \tfrac{1}{\sqrt{3}} v$$

We'll choose the positive root, which, when plugged into the above relation, gives:

$$\frac{m_1}{m_2} = \frac{\sqrt{3}+1}{\sqrt{3}-1} = \frac{(\sqrt{3}+1)^2}{2} \approx 3.7$$

ASSESS What about the negative root? If we choose it instead, we get the inverse of our result: $m_1 / m_2 = 2/(\sqrt{3}+1)^2$. Since it wasn't specified which mass was bigger, both answers are correct.

67. **INTERPRET** In this two-body problem we are asked to find the relative speed between the satellite and the booster after the given impulse. We can apply conservation of momentum because there are no external forces acting on the system. Finally, this will be a one-dimensional problem because we are dealing with only two bodies, so their relative motion must be linear to satisfy conservation of momentum.

DEVELOP By Newton's third law, the explosion applies a force of equal magnitude to each body (satellite and booster), but in the opposite direction. We therefore have $\vec{J}_\text{s} = -\vec{J}_\text{b}$, where the subscripts s and b refer to the satellite and booster, respectively, and $J_\text{s} = J_\text{b} = 350 \text{ N}\cdot\text{s}$. As explained for Equation 9.10b, an impulse $\vec{J}$ is equal to the change of momentum: $\vec{J} = \Delta\vec{p} = m\Delta\vec{v}$, which allows us to find the speed of the satellite and the booster in the (stationary) center-of-mass frame. The relative speed of separation is $v_\text{rel} = |\vec{v}_\text{s} - \vec{v}_\text{b}|$.

EVALUATE Initially both the satellite and the booster are at rest. After explosion, their velocities are

$$\vec{v}_\text{s} = \frac{\Delta\vec{p}_\text{s}}{m_\text{s}} = \frac{\vec{J}_\text{s}}{m_\text{s}} \qquad \vec{v}_\text{b} = \frac{\Delta\vec{p}_\text{b}}{m_\text{b}} = \frac{\vec{J}_\text{b}}{m_\text{b}} = \frac{-\vec{J}_\text{s}}{m_\text{b}}$$

Thus, the relative speed of separation is

$$|\vec{v}_s - \vec{v}_b| = \left|\frac{\vec{J}_s}{m_s} + \frac{\vec{J}_s}{m_b}\right| = J_s\left(\frac{1}{m_s} + \frac{1}{m_b}\right) = (350\ \text{N}\cdot\text{s})\left(\frac{1}{950\ \text{kg}} + \frac{1}{640\ \text{kg}}\right) = 0.92\ \text{m/s}$$

ASSESS The relative speed is shown to depend on J_s, the magnitude of the impulse. The greater the impulse, the faster the satellite and the booster separate from each other.

68. **INTERPRET** You have to calculate the impulse imparted by a force that is not constant.
DEVELOP The impulse of a variable force requires integration (Equation 9.10b): $\vec{J} = \int \vec{F}(t)dt.$
EVALUATE The rocket's thrust is one-dimensional, so we can drop the vector notation. Integrating the given force equation over the burn time gives

$$J = \int_0^{\Delta t} at(t - \Delta t)dt = \tfrac{1}{3}at^3 - \tfrac{1}{2}at^2\Delta t\Big|_0^{\Delta t} = -\tfrac{1}{6}a\Delta t^3$$

Plugging in the given values:

$$J = -\frac{1}{6}\left(-4.6\ \text{N/s}^2\right)(2.8\ \text{s})^3 = 17\ \text{N}\cdot\text{s}$$

Yes, the rocket meets its specs.

ASSESS The thrust starts at zero, then rises to a peak at $t = \frac{1}{2}\Delta t$ where $F = -\frac{1}{4}a\Delta t^2$ (recall a is negative), before falling back to zero at $t = \Delta t.$ We could get a rough estimate of the impulse by assuming that the average force is approximately equal to half of the peak value $\left(\bar{F} \sim -\frac{1}{8}a\Delta t^2\right)$ Multiplying by the time the force is applied gives: $J \sim -\frac{1}{8}a\Delta t^3$, which isn't too far off from the precise result above.

69. **INTERPRET** In this problem we want to determine the possible scenarios that led to the collision described. We must energy and momentum conservation to find the final speed of the combined wreck and the individual speeds of each car, respectively.
DEVELOP The two vehicles are traveling perpendicular to each other prior to the collision, so we know that they each contribute to an individual component of the final velocity. We also know the distance traveled by the vehicles after colliding, and the coefficient of friction between the road and tires, so we can determine the speed of the combined mass by using conservation of energy. Knowing the velocity after collision we can then determine the initial velocities of the individual vehicles using momentum conservation along each direction.

EVALUATE Right after colliding the combined mass begins to slow down due to the work done by the frictional force. We equate this work to the change in kinetic energy of the combined mass to find the velocity after collision is given by

$$\Delta K = W_f \rightarrow \frac{1}{2}(m_1 + m_2)v^2 = \mu(m_1 + m_2)gL$$

$$v = \sqrt{2\mu gL} = 15.5\ \text{m / s}$$

We now use the conservation of momentum along each direction to relate the individual velocities to the final velocity as

$$p_x : m_1 v_1 = (m_1 + m_2)v\cos\theta$$
$$p_y : m_2 v_2 = (m_1 + m_2)v\sin\theta$$

Here, we have the Nissan Leaf electric car traveling along the x-direction ($m_1 = 1640\,\text{kg}$), the Toyota Land Cruiser SUV along the y-direction ($m_2 = 3220\,\text{kg}$), and the angle θ as the angle relative to the x-axis at which the combined mass moves after the collision. Now we can consider the two scenarios: If the Leaf was at the speed limit, or if the Land Cruiser was at the speed limit.
Considering the first scenario means we have $v_1 = 70\,\text{km / h} = 19.4\ \text{m / s}$, and thus

$$\theta = \text{acos}\left(\frac{m_1 v_1}{(m_1+m_2)v}\right) = 65° \rightarrow v_2 = \frac{(m_1+m_2)v\sin\theta}{m_2} = 21.2\,\text{m/s} = 76.4\,\text{km/h}$$

Considering the second scenario means we have $v_2 = 70\,\text{km/h} = 19.4\,\text{m/s}$, and thus

$$\theta = \text{asin}\left(\frac{m_2 v_2}{(m_1+m_2)v}\right) = 56° \rightarrow v_1 = \frac{(m_1+m_2)v\cos\theta}{m_1} = 25.6\,\text{m/s} = 92.2\,\text{km/h}$$

ASSESS Since both scenarios lead to the other vehicle speeding, the investigators can attempt to measure the angle at which the combined mass moved to determine which scenario is more likely to have occurred.

70. **INTERPRET** This one-dimensional problem considers an explosion on a horizontal frictionless surface, so momentum is conserved because there are no horizontal forces acting on the popcorn. Kinetic energy is not conserved because the explosion does work on the popcorn fragments, so the work-energy theorem tells us that the kinetic energy must change (see Equation 7.5).

DEVELOP By conservation of momentum, we can equate the initial and final momenta, which gives

$$2mv_0 = mv_1 + mv_2 = m(v_1 + v_2)$$
$$2v_0 = v_1 + v_2$$
$$v_2 = 2v_0$$

Where $m = 200\text{g}$, and we have arbitrarily chosen fragment 1 to be the one with zero velocity after the explosion. Knowing the initial and final speeds, we can find the initial and final kinetic energies and so calculate the change in kinetic energy $\Delta K = K_f - K_i$

$$\Delta K = \left(\frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2\right) - \frac{1}{2}(2m)v_0^2$$
$$\Delta K = \frac{1}{2}mv_2^2 - mv_0^2$$

EVALUATE Inserting the $v_2 = 2v_0$ into the expression for ΔK and evaluating the result gives

$$\Delta K = \frac{1}{2}m(2v_0)^2 - mv_0^2 = mv_0^2 = (200\times10^{-6}\,\text{kg})(0.082\,\text{m/s})^2 = 1.3\,\mu\text{J}$$

ASSESS The kinetic energy of the system increases, as expected, because the explosion of the kernel (driven by moisture turning to steam) does work on the fragments, increasing the system's kinetic energy.

71. **INTERPRET** This two-dimensional problem involves a totally inelastic collision, so momentum is conserved but kinetic energy is not conserved. We can use conservation of momentum to find the angle between the initial velocities before a the collision.

DEVELOP The collision between the two masses is totally inelastic. Conservation of momentum tells us that

$$m_1\vec{v}_1 + m_2\vec{v}_2 = (m_1+m_2)\vec{v}_\text{f}$$
$$\vec{v}_\text{f} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1+m_2}$$

We known the magnitude of all the velocities involved, but not their relative orientation. We can find this by taking the scalar product of the final velocity with itself:

$$\vec{v}_\text{f}\cdot\vec{v}_\text{f} = v_\text{f}^2 = \left(\frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1+m_2}\right)\cdot\left(\frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1+m_2}\right) = \frac{m_1^2v_1^2 + m_2^2v_2^2 + 2m_1m_2\vec{v}_1\cdot\vec{v}_2}{(m_1+m_2)^2}$$

Using the definition of a scalar product, $\vec{v}_1\cdot\vec{v}_2 = v_1v_2\cos\theta$, the angle between $\vec{v}_1$ and $\vec{v}_2$ can be found.

EVALUATE With $m_1 = m_2 = m$ and $v_1 = v_2 = v = 2v_\text{f}$, the above equation can be simplified to

$$\frac{v^2}{4} = \frac{m^2v^2 + m^2v^2 + 2m^2v^2\cos\theta}{4m^2} = \frac{1}{2}v^2(1+\cos\theta)$$

Therefore, the angle between the two initial velocities is

$$\theta = \mathrm{acos}\left(\frac{-1}{2}\right) = 120°$$

ASSESS To see that the result makes sense, suppose $\vec{v}_1$ makes an angle $-60°$ with $+x$ and $\vec{v}_2$ makes an angle $+60°$ with $+x$. The y-component of the total momentum cancels. But for the x-component, we have

$$mv\cos(-60°) + mv\cos(60°) = (m+m)v_f$$

Solving for v_f, we get $v_f = v/2$, which confirms the result obtained above.

72. **INTERPRET** This is a one-dimensional problem that involves an elastic collision between two particles, so conservation of momentum and of total mechanical energy apply. We can use these principles to find the mass and final velocity of the nonproton particle.

DEVELOP Because this is an elastic collision in one dimension, we can apply Equations 9.15a and 9.15b to find the mass and velocity of the second particle. For this problem, the pre-collision velocities are $v_{1i} = 6.90$ Mm/s and $v_{2i} = -2.80$ Mm/s, and the post-collision velocity of the proton is $v_{1f} = -8.62$ Mm/s. The known mass is $m_1 = 1$ u. We can solve Equation 9.15a for the m_2, which gives

$$m_2 = \frac{m_1(v_{1i} - v_{1f})}{v_{1f} + v_{1i} - 2v_{2i}}$$

We can then insert the result of this calculation into Equation 9.15b to find v_{2f}.

EVALUATE Inserting the given quantities into the expression above for the mass of the unknown particle gives

$$m_2 = \left[\frac{6.90\ \mathrm{Mm/s} - (-8.62\ \mathrm{Mm/s})}{-8.62\ \mathrm{Mm/s} + 6.90\ \mathrm{Mm/s} - 2(-2.80\ \mathrm{Mm/s})}\right](1\ \mathrm{u}) = 4\ \mathrm{u}$$

Inserting this result into Equation 9.15b gives

$$v_{2f} = \frac{2}{5}v_{1i} + \frac{3}{5}v_{2i} = \frac{2}{5}(6.90\ \mathrm{Mm/s}) + \frac{3}{5}(-2.80\ \mathrm{Mm/s}) = 1.08\ \mathrm{Mm/s}$$

ASSESS Because the second particle is more massive, the proton gains momentum in the collision. The alpha particle, however, loses momentum.

73. **INTERPRET** The one-dimensional collision in this problem is elastic, so both momentum and energy are conserved. We are asked to find the ratio of the two masses if the one that is initially at rest acquires, after the collision, half of the kinetic energy that the other had before the collision.

DEVELOP Momentum is conserved in this process. In this one-dimensional case, we may write

$$m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}$$

Since the collision is completely elastic, energy is conserved:

$$\frac{1}{2}m_1v_{1i}^2 + \frac{1}{2}m_2\overbrace{v_{2i}^2}^{=0} = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2$$

Using the two conservation equations, the final speeds of m_1 and m_2 are (see Equations 9.15a and 9.15b):

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2}v_{1i} + \frac{2m_2}{m_1 + m_2}v_{2i} \text{ and } v_{2f} = \frac{2m_1}{m_1 + m_2}v_{1i} + \frac{m_2 - m_1}{m_1 + m_2}v_{2i}$$

Given that $v_{2i} = 0$, the above expressions may be simplified to

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2}v_{1i} \qquad v_{2f} = \frac{2m_1}{m_1 + m_2}v_{1i}$$

Now, if half of the kinetic energy of the first object is transferred to the second, then

$$K_{2f} = \frac{1}{2}K_{1i} \quad \Rightarrow \quad \frac{1}{2}m_2\left(\frac{2m_1v_{1i}}{m_1 + m_2}\right)^2 = \frac{1}{4}m_1v_{1i}^2$$

EVALUATE The above equation can be further simplified to

$$8m_1m_2 = (m_1+m_2)^2 \;\rightarrow 8\left(\frac{m_1}{m_2}\right) = \left(\frac{m_1}{m_2}+1\right)^2$$

The resulting quadratic equation, $m_1^2 - 6m_1m_2 + m_2^2 = 0$ has two solutions:

$$m_1 = \left(3 \pm \sqrt{8}\right)m_2 = \begin{cases} 5.83m_2 \\ (5.83)^{-1}m_2 \end{cases}$$

Because the quadratic equation is symmetric in m_1 and m_2, one solution equals the other with m_1 and m_2 interchanged. Thus, one object is 5.83 times more massive than the other.

ASSESS To check that our answer is correct, let's calculate the kinetic energy of the particles after the collision. Using $m_1 = 5.83m_2$, we find

$$K_{2\text{f}} = \frac{1}{2}m_2v_{2\text{f}}^2 = \frac{1}{2}m_2\left(\frac{2m_1v_{1\text{i}}}{m_1+m_2}\right)^2 = \left(\frac{1}{2}\right)\frac{m_1}{5.83}\left(\frac{2m_1}{m_1+m_1/5.83}\right)^2 v_{1\text{i}}^2$$

$$= \frac{1}{2}\frac{m_1}{5.83}\left(\frac{2}{1+1/5.83}\right)^2 v_{1\text{i}}^2 = \frac{1}{4}m_1v_{1\text{i}}^2 = \frac{1}{2}K_{1\text{i}}$$

as expected.

74. **INTERPRET** This is a one-dimensional, three-body problem that involves elastic collisions, so both conservation of momentum and energy apply. We need to find the final velocity of each block after the collisions.

DEVELOP We can analyze separately the two collisions in this problem, and apply Equations 9.15 to each collision. For the first collision, between blocks A and B, we find (with $v_{\text{af}} \equiv v$)

$$v_{\text{Af}} = \frac{m_\text{A}-m_\text{B}}{m_\text{A}+m_\text{B}}v_{\text{Ai}} + \frac{2m_\text{B}}{m_\text{A}+m_\text{B}}\overbrace{v_{\text{Bi}}}^{=0} = \frac{m_\text{A}-m_\text{B}}{m_\text{A}+m_\text{B}}v_{\text{Ai}}$$

$$v_{\text{Bf,int}} = \frac{m_\text{B}-m_\text{A}}{m_\text{A}+m_\text{B}}\overbrace{v_{\text{Bi}}}^{=0} + \frac{2m_\text{A}}{m_\text{A}+m_\text{B}}v_{\text{Ai}} = \frac{2m_\text{A}}{m_\text{A}+m_\text{B}}v_{\text{Ai}}$$

where $v_{\text{Bf,int}}$ is the intermediate final velocity of block B. Block B then proceeds to collide with block C, and the final velocities from that collision are

$$v_{\text{Bf}} = \frac{m_\text{B}-m_\text{C}}{m_\text{B}+m_\text{C}}v_{\text{Bf,int}} + \frac{2m_\text{C}}{m_\text{B}+m_\text{C}}\overbrace{v_{\text{Ci}}}^{=0} = \frac{m_\text{B}-m_\text{C}}{m_\text{B}+m_\text{C}}v_{\text{Bf,int}}$$

$$v_{\text{Cf}} = \frac{m_\text{C}-m_\text{B}}{m_\text{C}+m_\text{B}}\overbrace{v_{\text{Ci}}}^{=0} + \frac{2m_\text{B}}{m_\text{A}+m_\text{B}}v_{\text{Bf,int}} = \frac{2m_\text{B}}{m_\text{A}+m_\text{B}}v_{\text{Bf,int}}$$

EVALUATE Inserting the masses of the blocks $m_\text{A} = m$, $m_\text{B} = 2m$, and $m_\text{C} = m$, and recalling that $v_{\text{Af}} \equiv v$, we find

$$v_{\text{Af}} = \frac{m-2m}{m+2m}v_{\text{Ai}} = -\frac{1}{3}v$$

$$v_{\text{Bf,int}} = \frac{2m}{m+2m}v_{\text{Ai}} = \frac{2}{3}v$$

$$v_{\text{Bf}} = \frac{2m-m}{2m+m}v_{\text{Bf,int}} = \left(\frac{1}{3}\right)\left(\frac{2}{3}\right)v_{\text{Ai}} = \frac{2}{9}v$$

$$v_{\text{Cf}} = \frac{2m_\text{B}}{m_\text{A}+m_\text{B}}v_{\text{Bf,int}} = \frac{4m}{m+2m}\left(\frac{2}{3}\right)v_{\text{Ai}} = \frac{8}{9}v$$

ASSESS We can verify that momentum is conserved in this process:

$$mv_{\text{Ai}} = \left(-\frac{1}{3}mv_{\text{Ai}} + \frac{2}{9}(2mv_{\text{Ai}}) + \frac{8}{9}mv_{\text{Ai}}\right) = mv_{\text{Ai}}\left(-\frac{3}{9}+\frac{4}{9}+\frac{8}{9}\right) = mv_{\text{Ai}}$$

75. **INTERPRET** We are asked to derive Equation 9.15b which we can do using conservation of momentum. In addition, since Equation 9.15b describes an elastic collision, conservation of kinetic energy also applies.

DEVELOP Use conservation of momentum,

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}.$$

Because this is an elastic collision, kinetic energy is also conserved, so

$$\frac{1}{2}m_1v_{1i}^2 + \frac{1}{2}m_2v_{2i}^2 = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2.$$

Use these two equations to solve for v_{2f}. Much of this problem is done already in Equations 9.12a through 9.14.

EVALUATE First solve Equation 9.14 for v_{1f} to get $v_{1f} = v_{2f} + v_{2i} - v_{1i}$. When we substitute this result into Equation 9.12, using the sign of v to denote the direction, we obtain $m_1v_{1i} + m_2v_{2i} = m_1(v_{2f} + v_{2i} - v_{1i}) + m_2v_{2f}$. Solving this for v_{2f} gives

$$m_1v_{1i} + m_2v_{2i} = m_1v_{2i} - m_1v_{1i} + (m_1 + m_2)v_{2f}$$
$$v_{2f} = \frac{2m_1}{m_1 + m_2}v_{1i} + \frac{m_2 - m_1}{m_1 + m_2}v_{2i}$$

ASSESS Our result agrees with Equation 9.15b, as expected.

76. **INTERPRET** This two-dimensional, two-body problem involves conservation of momentum and energy because the collision is elastic. We are to show that a collision that is not head-on will result in the final velocities being perpendicular.

DEVELOP With one of the bodies initially at rest, conservation of energy gives

$$\vec{p}_{1i} = \vec{p}_{1f} + \vec{p}_{2f}$$

and conservation of energy gives [using $K = p^2/(2m)$]

$$p_{1i}^2/2m_1 = p_{1f}^2/2m_1 + p_{2f}^2/2m_2$$
$$p_{1i}^2 = p_{1f}^2 + p_{2f}^2$$

where the second line uses the fact that the objects have equal mass. These two expressions for p_{1i}^2 can be equated to find the angle between the resulting momentum vectors.

EVALUATE Equating the two expressions above for p_{1i}^2 gives

$$p_{1f}^2 + p_{2f}^2 = |\vec{p}_{1f} + \vec{p}_{2f}|^2$$
$$p_{1f}^2 + p_{2f}^2 = p_{1f}^2 + 2\vec{p}_{1f}\cdot\vec{p}_{2f} + p_{2f}^2$$
$$0 = 2\vec{p}_{1f}\cdot\vec{p}_{2f}$$

Recalling that the scalar product is defined as $\vec{p}_{1f}\cdot\vec{p}_{2f} = p_{1f}p_{2f}\cos\theta$, we see that the angle $\theta = 90°$, unless $p_{1f} = 0$, as for a head-on collision.

ASSESS You can verify this result on a pool table.

77. **INTERPRET** The two-dimensional problem involves an elastic collision between a proton and an initially stationary deuteron. Given the angle between their final velocities, we are to find the fraction of kinetic energy transferred from the proton to the deuteron in the process.

DEVELOP Using the coordinate system shown in the sketch below (the deuteron's recoil angle θ_{2f} is negative), the components of the conservation of momentum equations for the elastic collision become

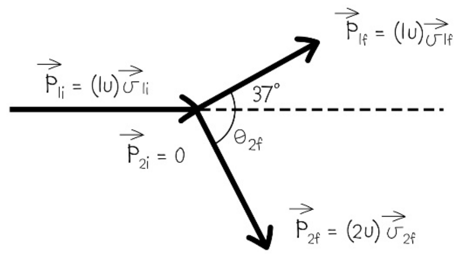
$$m_p v_{pi} = m_p v_{pf}\cos\theta_{1f} + m_d v_{df}\cos\theta_{df}$$
$$0 = m_p v_{pf}\sin\theta_{pf} + m_d v_{df}\sin\theta_{df}$$

In addition, conservation of energy gives us

$$\frac{1}{2}m_p v_{pi}^2 = \frac{1}{2}m_p v_{pf}^2 + \frac{1}{2}m_d v_{df}^2$$

so the fraction of initial kinetic energy transferred to the deuteron is

$$\frac{K_{\mathrm{df}}}{K_{\mathrm{pi}}} = 1 - \frac{K_{\mathrm{pf}}}{K_{\mathrm{pi}}} \quad \rightarrow \quad \frac{m_d}{m_p}\left(\frac{v_{\mathrm{df}}}{v_{\mathrm{pi}}}\right)^2 = 1 - \left(\frac{v_{\mathrm{pf}}}{v_{\mathrm{pi}}}\right)^2$$



EVALUATE With $m_{\mathrm{d}} = 2m_{\mathrm{p}}$, the conservation equations become

$$v_{\mathrm{pi}} = v_{\mathrm{pf}}\cos\theta_{\mathrm{pf}} + 2v_{\mathrm{pf}}\cos\theta_{\mathrm{pf}}$$
$$0 = v_{\mathrm{pf}}\sin\theta_{\mathrm{pf}} + 2v_{\mathrm{df}}\sin\theta_{\mathrm{df}}$$
$$v_{\mathrm{pi}}^2 = v_{\mathrm{pf}}^2 + 2v_{\mathrm{df}}^2$$

To find the final velocities, eliminate $\theta_{2\mathrm{f}}$ from the first and second equations and $v_{2\mathrm{f}}$ from the third to get

$$v_{\mathrm{pi}}^2 - 2v_{\mathrm{pi}}v_{\mathrm{pf}}\theta_{\mathrm{pf}} + v_{\mathrm{pf}}^2 = 4v_{\mathrm{df}}^2\left(\sin^2\theta_{\mathrm{df}} + \cos^2\theta_{\mathrm{df}}\right) = 4v_{\mathrm{df}}^2 = 2v_{\mathrm{pi}}^2 - 2v_{\mathrm{pf}}^2$$

This results in a quadratic equation for v_{pf}: $3v_{\mathrm{pf}}^2 - 2v_{\mathrm{pi}}v_{\mathrm{pf}}\cos\theta_{\mathrm{pf}} - v_{\mathrm{pi}}^2 = 0$, with positive solution

$$v_{\mathrm{pf}} = \frac{1}{3}v_{\mathrm{pi}}\left(\cos\theta_{\mathrm{pf}} + \sqrt{\cos^2\theta_{\mathrm{pf}} + 3}\right) = 0.902v_{\mathrm{pi}}$$

where we have used $\theta_{1\mathrm{f}} = 37°$. From the kinetic energy equation, we have $v_{\mathrm{df}} = \sqrt{\tfrac{1}{2}(v_{\mathrm{pi}}^2 - v_{\mathrm{pf}}^2)} = 0.305v_{\mathrm{pi}}$, and from the transverse momentum equation, we have

$$\theta_{\mathrm{df}} = \sin^{-1}\left(\frac{-v_{\mathrm{pf}}\sin 37°}{2v_{\mathrm{df}}}\right) = \sin^{-1}\left(\frac{-\left(0.902v_{\mathrm{pi}}\right)\sin 37°}{2\left(0.305v_{\mathrm{pi}}\right)}\right) = -62.7°$$

From either v_{pf} or v_{df}, the fraction of transferred kinetic energy is found to be

$$\frac{K_{\mathrm{df}}}{K_{\mathrm{pi}}} = 1 - \frac{K_{\mathrm{df}}}{K_{\mathrm{pi}}} = 1 - \left(\frac{v_{\mathrm{pf}}}{v_{\mathrm{pi}}}\right)^2 = 1 - \left(0.902\right)^2 = 18.6\%$$

ASSESS The fraction of energy transfer can also be obtained as

$$\frac{K_{\mathrm{df}}}{K_{\mathrm{pi}}} = \frac{m_{\mathrm{d}}}{m_{\mathrm{p}}}\left(\frac{v_{\mathrm{df}}}{v_{\mathrm{pi}}}\right)^2 = 2\left(0.305\right)^2 = 0.186 = 18.6\%$$

Here, one does not need both final velocities to answer this question, but a more complete analysis of this collision, including the deuteron recoil angle, is instructive.

78. **INTERPRET** This two-dimensional problem involves a three-body, totally elastic collision. We can therefore apply conservation of total linear momentum and conservation of energy to find the velocities of the three balls after the collision.

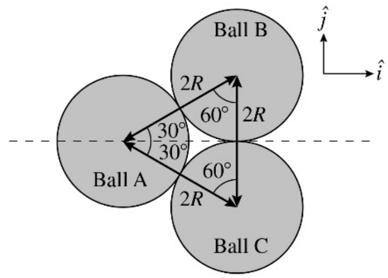
DEVELOP Because the balls are the same size, the direction of the impact force is at ±30° with respect to the horizontal (see figure below). By symmetry, balls B and C receive the same impulse, so their horizontal velocity components and the magnitude of their velocities must be equivalent. Thus, $v_{\mathrm{C}} = v_{\mathrm{B}}$ and $\vec{v}_C = v_{\mathrm{B},x}\hat{i} - v_{\mathrm{B},y}\hat{j}$. Applying conservation of momentum in the $\hat{i}$ direction therefore gives

$$mv_0 = mv_{\mathrm{A}} + mv_{\mathrm{B}}\cos\left(30°\right) + mv_{\mathrm{C}}\cos\left(30°\right)$$
$$v_0 = v_{\mathrm{A}} + v_{\mathrm{B}}\sqrt{3}$$

Applying conservation of energy and using the result from conservation of momentum and the result that $v_C = v_B$ gives

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mv_A^2 + \frac{1}{2}mv_B^2 + \frac{1}{2}mv_C^2$$
$$v_0^2 = v_A^2 + 2v_B^2$$
$$v_0^2 = v_A^2 + 2\left(\frac{v_0 - v_A}{\sqrt{3}}\right)^2$$

which we can solve for v_A. Once we know v_A, we can use the expression above from conservation of momentum to find v_B (and v_C).



EVALUATE Solving the expression above for v_A gives

$$v_0^2 - v_A^2 = \frac{2}{3}(v_0 - v_A)^2$$
$$\cancel{(v_0 - v_A)}(v_0 + v_A) = \frac{2}{3}(v_0 - v_A)^{\cancel{2}}$$
$$v_A = -\frac{v_0}{5}$$

Because v_A has only a horizontal component, we have $\vec{v} = (v_0/5)\hat{i}$. Using the result for v_A to find v_B gives

$$v_0^2 = \frac{v_0^2}{25} + 2v_B^2$$
$$v_B = \pm\frac{2v_0}{5}\sqrt{3}$$

so in component form we have

$$\vec{v}_B = (v_B\cos\theta)\hat{i} + (v_B\sin\theta)\hat{j}$$
$$= \left(\frac{2\sqrt{3}}{5}v_0\frac{\sqrt{3}}{2}\right)\hat{i} + \left(\frac{2\sqrt{3}}{5}v_0\frac{1}{2}\right)\hat{j}$$
$$= \left(\frac{3}{5}v_0\right)\hat{i} + \left(\frac{\sqrt{3}}{5}v_0\right)\hat{j}$$

From the symmetry arguments above, we now have

$$\vec{v}_C = \left(\frac{3}{5}v_0\right)\hat{i} - \left(\frac{\sqrt{3}}{5}v_0\right)\hat{j}$$

ASSESS We can check that momentum and energy are conserved. For momentum, we have

$$m\vec{v}_0 = m\vec{v}_A + m\vec{v}_B + m\vec{v}_C$$
$$v_0\hat{i} = -\frac{1}{5}v_0\hat{i} + \frac{3}{5}v_0\hat{i} + \frac{\sqrt{3}}{5}v_0\hat{j} + \frac{3}{5}v_0\hat{i} - \frac{\sqrt{3}}{5}v_0\hat{j}$$
$$= v_0\hat{i}$$

and for energy, we have

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mv_A^2 + \frac{1}{2}mv_B^2 + \frac{1}{2}mv_C^2$$
$$v_0^2 = \frac{v_0^2}{25} + \left(\frac{9}{25} + \frac{3}{25}\right)v_0^2 + \left(\frac{9}{25} + \frac{3}{25}\right)v_0^2$$
$$= v_0^2$$

79. **INTERPRET** This one-dimensional, two-body problem involves an inelastic collision, so we can apply conservation of momentum but not conservation of energy. We will also need to apply some kinematics to find the maximum height and the speed with which the combination hits the ground.

DEVELOP By conservation of momentum, we can equate the momentum of the two-body system before and after the Frisbee-mud collision. This gives

$$m_m v_i = (m_F + m_m)v_f$$

Using Equation 2.11, we find the velocity $v_{m,i}$ with which the mud hits the Frisbee to be

$$v_{m,i} = \sqrt{v^2{}_{m,0} - 2g(y - y_0)} = \sqrt{(18.7\text{ m/s})^2 - 2(9.8\text{ m/s}^2)(7.25\text{ m} - 1.23\text{ m})} = 15.2\text{ m/s}$$

Therefore, the initial velocity of the mud-Frisbee combination is

$$v_f = \frac{m_m v_i}{m_F + m_m} = \frac{(0.240\text{ kg})(15.2\text{ m/s})}{0.240\text{ kg} + 0.124\text{ kg}} = 10.44\text{ m/s}$$

upward. Use this result in the kinematic Equation 2.11 to find the maximum height and the speed on hitting the ground for the mud-Frisbee combination.

EVALUATE **(a)** The maximum height reached is

$$y = y_0 + \frac{v^2 - v_f^2}{-2g} = 7.25\text{ m} - \frac{(0.00\text{ m/s})^2 - (10.04\text{ m/s})^2}{2(9.8\text{ m/s}^2)} = 12.4\text{ m}$$

(b) An object falling from this height, unimpeded by air resistance or other obstacles, would attain a speed of

$$v = \sqrt{2gy} = \sqrt{2(9.8\text{ m/s}^2)(12.4\text{ m})} = 15.6\text{ m/s}$$

ASSESS Our answers are only approximate because air resistance significantly affects a large, light object like a Frisbee. Note that we retained additional significant figures in intermediate calculations.

80. **INTERPRET** This one-dimensional two-body problem involves an elastic collision and kinematics. We can apply conservation of energy and momentum to find the height the small ball rebounds after being dropped together with a larger ball and rebounding from the ground.

DEVELOP The balls reach the ground, after a vertical fall through a height h, with speed $v_0 = \sqrt{2gh}$ (see Equation 2.11). Assume that they undergo an elastic head-on collision, with the large ball M rebounding from the ground with initial velocity $v_{2i} = v_0$ (positive upward), and the small ball still falling downward with initial velocity $v_{1i} = -v_0$. Equation 9.15a gives the final velocity of the small ball as

$$v_f = \left(\frac{m - M}{m + M}\right)(-v_0) + \left(\frac{2M}{m + M}\right)v_0 = \left(\frac{3M - m}{m + M}\right)v_0 \approx 3v_0$$

since $M \gg m$. Once v_f is known, the height it rebounds can be readily calculated by using energy conservation (or kinematic Equation 2.11).

EVALUATE Conservation of total mechanical energy requires that $mv_f^2/2 = mgh_f$, so

$$h_f = \frac{v_f^2}{2g} = \frac{(3v_0)^2}{2g} = 9\frac{v_0^2}{2g} = 9h$$

or about nine times the original height.

ASSESS This demonstration, sometimes called a Minski cannon, is striking. Try it with a new tennis ball and properly inflated basketball.

81. **INTERPRET** This one-dimensional, two-body problem involves a collision that is neither elastic nor inelastic, so we can apply conservation of momentum. Given that we are told the amount of kinetic energy that is lost in the collision, we can apply conservation of energy as well. We can use these principles to find the velocity of the wreckage.

DEVELOP Let the initial velocity of car 1 be v, and that of cars 1 and 2 after the wreckage be v_1 and v_2, respectively. Conservation of momentum requires that

$$mv = mv_1 + mv_2$$
$$v = v_1 + v_2$$

and conservation of energy gives

$$\frac{1}{2}mv^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 + \frac{7}{32}\left(\frac{1}{2}mv^2\right)$$
$$\frac{25}{32}v^2 = v_1^2 + v_2^2$$

Solve these two equations for the final velocities.

EVALUATE Solving the expression from conservation of energy for v_1 gives

$$v_1 = \pm\sqrt{\frac{25}{32}v^2 - v_2^2}$$

Taking the positive square root (the negative solution simply corresponds to the cars moving in the opposite direction) and using the result from conservation of momentum to eliminate v_2 gives

$$v_1 = \sqrt{\frac{25}{32}v^2 - (v - v_1)^2}$$
$$v_1^2 = \frac{25}{32}v^2 - (v^2 - 2vv_1 + v_1^2)$$
$$2v_1^2 - 2vv_1 + \frac{7}{32}v^2 = 0$$
$$2\left(v_1 - \frac{1}{8}v\right)\left(v_1 - \frac{7}{8}v\right) = 0$$

which has the solutions $v_1 = 7v/8$ or $v/8$. Because $v_1 < v_2$ (because car 1 is behind car 2 after the wreck), we choose $v_1 = v/8$, which leads to $v_2 = 7v/8$.

ASSESS We can easily verify that conservation of momentum and energy (including the converted kinetic energy) are respected with this solution.

82. **INTERPRET** This problem involves kinematics, conservation of momentum, and conservation of energy. A small block slides down a frictionless incline and collides on a horizontal frictionless surface with a second, larger block in an elastic collision. The smaller block rebounds and travels back up the incline a certain distance, then slides down the incline and catches up to the larger block for a second collision. We are asked to calculate the time that elapses before the two blocks collide for the second time.

DEVELOP From conservation of energy, and assuming a smooth transition from incline to horizontal surface, the small block has speed $v_{1i} = \sqrt{2gh}$ when the first collision occurs. Use Equation 9.15a, with $m_2 = 4m_1$ and $v_{2i} = 0$ to find the speeds of the blocks immediately after the first collision (at $t = 0$):

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_{1i} = \left(\frac{1-4}{1+4}\right)v_{1i} = -\frac{3}{5}v_{1i}$$
$$v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right)v_{1i} = \frac{2}{5}v_{1i}$$

The larger block moves with constant speed of $v_{2f} = 2v_{1i}/5$ to the right; its position, relative to the bottom of the incline, is

$$x_{2,f}(t) = x_{2,i} + v_{2f}t = 1.4\text{ m} + \frac{2}{5}v_{1,i}t$$

The smaller block takes time $t_1 = (1.4\text{ m})(3v_{1,i}/5)^{-1}$ to get back to the incline, and $t_2 = 2(3v_0/5)a^{-1}$ to go up and down the incline, where $a = g\sin(30°) = g/2$. (Use Equation 2.7, with initial speed $-3v_{1,i}/5$ up the incline and final speed $-3v_{1,i}/5$ down the incline, to calculate *t*.) The small block then proceeds with constant speed in pursuit of the larger block, its position being

$$x_{1,f}(t) = \frac{3}{5}v_{1,i}(t - t_1 - t_2) \text{ for } t \ge t_1 + t_2$$

The blocks collide for the second time when $x_1(t) = x_2(t)$.

EVALUATE The condition $x_1(t) = x_2(t)$ implies

$$\frac{3}{5}v_{1,i}(t - t_1 - t_2) = 1.4\text{ m} + \frac{2}{5}v_{1,i}t$$

Solving for *t*, we find

$$t = \frac{5\left[1.4\text{ m} + (3v_{1,i}/5)(t_1 + t_2)\right]}{v_{1,i}} = (7\text{ m})/v_{1,i} + 3(t_1 + t_2)$$

Numerically, we have $v_0 = \sqrt{2(9.8\text{ m/s}^2)(0.25\text{ m})} = 2.21\text{ m/s}$, which gives

$$t_1 = \frac{1.4\text{ m}}{3(2.21\text{ m/s})/5} = 1.05\text{ s}$$

$$t_2 = \frac{6v_{1,i}}{5a} = \frac{12v_{1,i}}{5g} = \frac{12(2.21\text{ m/s})}{5(9.8\text{ m/s}^2)} = 0.542\text{ s}$$

$$t = (7\text{ m})/(2.21\text{ m/s}) + 3(1.05\text{ s} + 0.542\text{ s}) = 7.95\text{ s}$$

ASSESS This problem is rather involved. However, the validity of our result can be checked by substituting the numerical values for t_1 and t_2 back to the equations in the intermediate steps.

83. **INTERPRET** In this problem a small mass shoots backward from a rocket, and we want to verify the conservation of momentum results in the expressions given.

DEVELOP We know that the initial momentum is equal to the product of the initial mass and velocity of the rocket, and that the final momentum is comprised of the backward moving mass momentum plus the now lighter rocket's forward momentum. We are also told the change in mass dm is infinitesimal, so we can assume that the rockets final mass doesn't change significantly.

EVALUATE Applying the conservation of momentum we have

$$Mv_0 = (M - dm)v - dm\,v_{\text{ex}}$$

If we then assume that $M - dm \approx M$, and note that $dv = v - v_0$ gives the change in speed, we find that

$$Mdv = dm\,v_{\text{ex}}$$

Here, we note that the change in mass dM experienced by the rocket can be more thoroughly related to the infinitesimal mass dm as

$$dM = (M - dm) - M = -dm$$

This allows us to express the momentum conservation in terms of the rocket's change in mass and velocity as

$$Mdv = -dM\,v_{\text{ex}}$$

Now we can solve for dv and integrate both sides to show the rocket's final velocity is given by

$$dv = \frac{-dM}{M}v_{\text{ex}} \rightarrow \int_{v_i}^{v_f} dv = v_{\text{ex}}\int_{M_i}^{M_f}\frac{-dM}{M}$$

$$v\Big|_{v_i}^{v_f} = v_{\text{ex}}\ln M\Big|_{M_f}^{M_i} \rightarrow v_f = v_i + v_{\text{ex}}\ln\left(\frac{M_i}{M_f}\right)$$

ASSESS This is known as the ideal rocket equation and it is used by aerospace engineers to model rocket-powered flights.

84. **INTERPRET** The problem is about finding the fraction of the initial kinetic energy transferred from one block to the second block in the course of a collision. The fraction is related to the mass ratio.

DEVELOP With $v_{2i} = 0$, Equations 9.15a and 9.15b become

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_{1i} \qquad v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right)v_{1i}$$

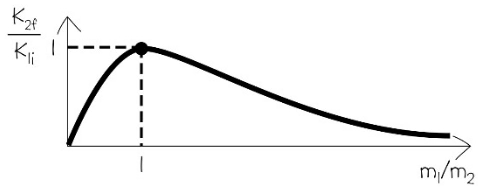
The fraction of the initial kinetic energy transferred to m_2 is

$$\frac{K_{2f}}{K_{1i}} = \frac{\frac{1}{2}m_2 v_{2f}^2}{\frac{1}{2}m_1 v_{1i}^2}$$

EVALUATE Substituting the expression for v_{1f} into the equation above, we obtain

$$\frac{K_{2f}}{K_{1i}} = \frac{m_2}{m_1}\left(\frac{2m_1}{m_1 + m_2}\right)^2 = \frac{4m_1 m_2}{(m_1 + m_2)^2} = \frac{4(m_1/m_2)}{(1 + m_1/m_2)^2}$$

We plot this ratio in the figure below.



ASSESS The fraction of energy transfer reaches a maximum of unity when the mass ratio equals one. This corresponds to Case 2 in Section 9.6 where $m_1 = m_2$. The first object stops completely and transfers all its energy to the second object, which moves on with the initial speed $(v_{2f} = v_{1i})$.

85. **INTERPRET** This problem is like the previous problem where we looked at the transfer of kinetic energy from a moving block to an initially stationary block. Here, we show that the fraction of energy transferred is independent of which block is the moving one and which is the stationary one.

DEVELOP We can use the result from the previous problem to write the fraction of the initial energy in the moving block that is transferred to the initially stationary block:

$$\frac{K_{stat}}{K_{mov}} = \frac{4(m_1/m_2)}{(1 + m_1/m_2)^2}$$

where we assume in this case that the moving block has mass m_1 and the stationary block has mass m_2.

EVALUATE If instead m_1 is the stationary block, and m_2 is the moving block, then the fraction becomes

$$\frac{K_{stat}}{K_{mov}} = \frac{4(m_2/m_1)}{(1 + m_2/m_1)^2} = \frac{4(m_2/m_1)}{(1 + m_2/m_1)^2}\frac{(m_1/m_2)^2}{(m_1/m_2)^2} = \frac{4(m_1/m_2)}{(1 + m_1/m_2)^2}$$

But this is the same as before, so the energy transfer is independent of which mass is initially stationary.

ASSESS As an example of this independence, one can imagine a light block colliding with a heavy stationary block (like the ping-pong ball and bowling ball collision in Section 9.6). After the collision, the heavy block barely moves, whereas the light block ricochets backward with essentially the same speed. In other words, very little energy is transferred $(K_{stat}/K_{mov} \approx 0)$. Then imagine the blocks switch places, with the light block initially at rest. In this case, the heavy block plows into the light block and knocks it forward. But the heavy block keeps moving with pretty much its initial speed, so again not much energy is transferred $(K_{stat}/K_{mov} \approx 0)$.

86. **INTERPRET** We are to find the center of mass of a semicircular wire with radius of curvature R. From Figure 9.6, we see that the wire is symmetric left-to-right, so the center of mass is along the centerline. We need to find the distance of the center of mass above the center of the semicircle. We would expect that it's somewhere over halfway up.

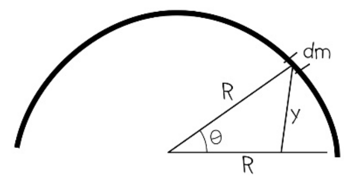
DEVELOP Use the coordinate system defined in the figure below. The equation for center of mass is (Equation 9.4)

$$\vec{r}_{\rm cm} = \frac{1}{M}\int \vec{r}dm$$

Because the system is left-right symmetric, we need only the vertical component y of $\vec{r}_{\rm cm}$, so the equation for center of mass reduces to

$$y_{\rm cm} = \frac{1}{M}\int ydm$$

The mass per unit length of the wire is $\lambda = M(C/2)^{-1} = M(2\pi R/2)^{-1} = M/\pi R$, so $dm = \lambda Rd\theta$ and $y = R\sin\theta$.



EVALUATE We integrate over the entire wire, from $\theta = 0$ to $\theta = \pi$

$$y_{\rm cm} = \frac{1}{M}\int_0^\pi ydm = \frac{1}{M}\int_0^\pi R\sin\theta(\lambda Rd\theta) = \frac{R^2\lambda}{M}\int_0^\pi \sin\theta d\theta$$
$$= \frac{R^{\cancel{2}}\left(\frac{\cancel{M}}{\pi \cancel{R}}\right)}{\cancel{M}}(-\cos\theta)_0^\pi = \frac{2R}{\pi}$$
$$= 0.637R$$

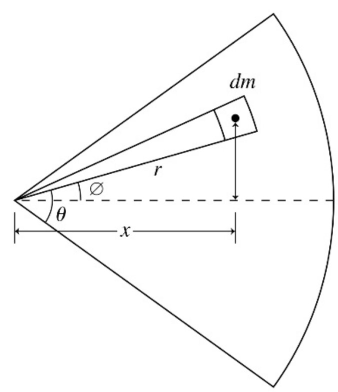
ASSESS This is a bit short of 2/3 of the way up, which is what we expected.

87. **INTERPRET** We find the center of mass of a slice of pizza with central angle θ and radius R. We would expect that it's along the center of the slice, and closer to the crust than to the tip.

DEVELOP The equation for center of mass is $\vec{r}_{\rm cm} = \frac{1}{M}\int \vec{r}\,dm$, which in two dimensions can be written out as:

$$\vec{r}_{\rm cm} = \frac{1}{M}\int\left(x\hat{i} + y\hat{j}\right)dm = \frac{1}{M}\int xdm\,\hat{i} + \frac{1}{M}\int ydm\,\hat{j} = x_{\rm cm}\hat{i} + y_{\rm cm}\hat{j}$$

We set up our coordinate system such that the slice is symmetric about the y-axis, see the figure below. In this case, $y_{\rm cm} = 0$. As for $x_{\rm cm}$, we use $x = r\cos\phi$ and integrate over r from 0 to R, and over ϕ from $-\theta/2$ to $\theta/2$, where the angles are in radians.



The infinitesimal mass element, dm, is equal to $\mu dA = \mu r dr d\phi$, where μ is the mass per unit area. Since the slice is uniform, the density is constant:

$$\mu = \frac{M}{A} = \frac{M}{(\theta/2\pi)(\pi R^2)} = \frac{2M}{\theta R^2}$$

One can check these values by verifying that $M = \int \mu dA$.

EVALUATE Evaluating the integral for the x-component of the center of mass gives

$$x_{\text{cm}} = \frac{1}{M}\int_{-\theta/2}^{\theta/2}\int_0^R (r\cos\phi)\left(\frac{2M}{\theta R^2}\right) r\,dr\,d\phi$$
$$= \frac{2}{\theta R^2}\left[\tfrac{1}{3}r^3\right]_0^R \left[\sin\phi\right]_{-\theta/2}^{\theta/2} = \frac{4R}{3\theta}\sin(\theta/2)$$

ASSESS We can check this answer by letting $\theta = \pi/4$, which corresponds to a 1/8th slice of pizza. In that case, $x_{\text{cm}} = 0.65R$, which matches our original prediction that the center of mass will be closer to the outer crust than to the tip. If instead $\theta = 2\pi$ (the full pizza), then the center of mass is at $x_{\text{cm}} = 0$, as we would expect.

88. **INTERPRET** We use conservation of momentum and of energy to calculate the kinetic energy of a pellet that goes *through* a ballistic pendulum. The pellet has known initial kinetic energy, and we also know the final potential energy of the pendulum, so we can find the final kinetic energy of the pellet.

DEVELOP We find the initial speed of the pellet from its initial kinetic energy, $K_{\text{i}} = 3.25$ J, and mass, $m = 0.52\times 10^{-3}$ kg. We then work backward from the maximum potential energy of the pendulum to find the speed of the pendulum. Knowing this potential energy is $U = Mgh$, where the pendulum mass is $M = 0.400$ kg and the height is $h = 5.2\times 10^{-4}$ m, we can find the initial kinetic energy, $K = Mv_{\text{p}}^2/2$, of the pendulum. This gives us the speed of the pendulum just after the collision. Next, we use conservation of momentum at the collision with the pendulum to find the pellet's speed after the collision. Finally, we answer the question using $K_{\text{f}} = mv_{\text{f}}^2/2$.

EVALUATE The initial speed of the pellet is

$$K_{\text{i}} = \frac{1}{2}mv_{\text{i}}^2$$
$$v_{\text{i}} = \sqrt{\frac{2K_{\text{i}}}{m}}$$

The speed of the pendulum after the collision is found by

$$U = \cancel{M}gh = \frac{1}{2}\cancel{M}v_{\text{p}}^2$$
$$v_{\text{p}} = \sqrt{2gh}$$

We use conservation of momentum at the collision to find the velocity of the exiting pellet:

$$mv_{\text{i}} = mv_{\text{f}} + Mv_{\text{p}}$$
$$m\sqrt{\frac{2K_{\text{i}}}{m}} = mv_{\text{f}} + M\sqrt{2gh}$$
$$v_{\text{f}} = \sqrt{\frac{2K_{\text{i}}}{m}} - \frac{M}{m}\sqrt{2gh} = \sqrt{\frac{2(3.25\text{ J})}{5.2\times 10^{-4}\text{ kg}}} - \frac{0.40\text{ kg}}{5.2\times 10^{-4}\text{ kg}}\sqrt{2(9.8\text{ m/s}^2)(0.52\times 10^{-3}\text{ m})}$$
$$= 34.1\text{ m/s}$$

So the final kinetic energy is

$$K_{\text{f}} = \frac{1}{2}mv_{\text{f}}^2 = \frac{1}{2}(5.2\times 10^{-4}\text{ kg})(34.1\text{ m/s})^2 = 0.30\text{ J}$$

ASSESS Note that we can't just use conservation of energy! Compare:

$$E_{\text{i}} = 3.25\text{ J}$$
$$E_{\text{f}} = U + K_{\text{f}} = Mgh + \frac{1}{2}mv_{\text{f}}^2 = 0.332\text{ J}$$

Most of the kinetic energy is lost in the collision because this is an inelastic collision.

89. **INTERPRET** We use conservation of momentum to find the speed of an astronaut after she throws her toolbox away, and we use this speed and the given distance to determine whether she reaches safety before her oxygen runs out.

DEVELOP The mass of the astronaut is $m_a = 75$ kg. The mass of the toolbox is $m_t = 15$ kg. The initial speed of both is zero, so the final speed of the toolbox is $v_{ta} = -5$ m/s *relative to* the astronaut. We can use conservation of momentum to find the speed of the astronaut: $0 = m_t v_t + m_a v_a$. Once we have this speed, we calculate how long it will take to travel a distance $x = 235$ m and hope that the answer is less than 5 minutes.

EVALUATE First, we find the speed of the toolbox relative to the rest frame: $v_t = v_a + v_{ta}$. Next, we plug this into the conservation of momentum equation: $0 = m_t(v_a + v_{ta}) + m_a v_a$ and solve for the astronaut's speed:

$$0 = m_t(v_a + v_{ta}) + m_a v_a$$
$$v_a(m_t + m_a) = -m_t v_{ta}$$
$$v_a = -v_{ta}\left(\frac{m_t}{m_t + m_a}\right) = 0.83 \text{ m/s}$$

The time it takes is

$$t = \frac{x}{v_a} = \frac{235 \text{ m}}{0.83 \text{ m/s}} = 282 \text{ s} = 4.7 \text{ min}$$

ASSESS She makes it with 18 seconds to spare.

90. **INTERPRET** The Sun will rotate around the center of mass of the solar system, so this problem is essentially asking how far the Sun's center is from this center of mass. For the solar system, we will consider only the Sun and Saturn and will neglect the mass contribution from the rest of the planets.

DEVELOP We can think of the Sun and Saturn as two ends of a barbell, like that in Example 9.1. The center of mass lies on the line between the two objects, so we can drop the vector notation from Equation 9.2: $r_{cm} = \frac{1}{M}\sum m_i r_i$. Let's take the center of the Sun as our origin, so that the Sun's contribution is zero $(r_S = 0)$. The only term in the sum is that for Saturn:

$$r_{cm} = \frac{1}{(m_S + m_P)}\left[m_P r_P + m_S r_S\right] = \frac{m_P}{(m_S + m_P)} r_P.$$

EVALUATE From Appendix E, the mass of Saturn is 5.68×10^{26} kg and its orbital radius is 1.43×10^{12} m. The mass of the Sun is 1.99×10^{30} kg. Plugging these values in the above expression,

$$r_{cm} = \frac{5.68\times10^{26} \text{ kg}}{(1.99\times10^{30} \text{ kg} + 5.68\times10^{26} \text{ kg})}(1.43\times10^{12} \text{ m}) = 4.08\times10^{8} \text{ m}.$$

ASSESS This is smaller than the radius of the Sun $(R_S = 6.96\times10^{8}$ m). The actual distance between the Sun and the center of mass of the solar system is constantly changing since the planets are constantly moving relative to each other due to their orbital motion.

91. **INTERPRET** We're asked to find the total mass and the center of mass for a thin rod with nonuniform density. The density increases from zero at one end to a maximum at the other end, so we'd expect the center of mass to be closer to the denser end.

DEVELOP We will have to integrate to find the total mass: $M_{tot} = \int dm = \int \mu dx.$ The limits of integration are between $x=0$ and $x=L.$ Since the mass is distributed along one dimension, the center of mass integral takes a similar from: $r_{cm} = \frac{1}{M_{tot}}\int x dm = \frac{1}{M_{tot}}\int x\mu dx.$

EVALUATE (a) The mass integral gives

$$M_{tot} = \int_0^L \mu dx = \int_0^L \frac{Mx^a}{L^{1+a}} dx = \frac{M}{L^{1+a}} \frac{x^{1+a}}{1+a}\bigg|_0^L = \frac{M}{1+a}$$

(b) Using the above result, the center of mass is

$$r_{cm} = \frac{1}{M_{tot}}\int_0^L x\mu dx = \frac{1+a}{M}\int_0^L \frac{Mx^{1+a}}{L^{1+a}} dx = \frac{1+a}{L^{1+a}} \frac{x^{2+a}}{2+a}\bigg|_0^L = \frac{1+a}{2+a} L$$

(c) If $a=0$, the density is constant: $\mu = M/L.$ The total mass is M, and the center of mass occurs at $\frac{1}{2}L,$ just as we would expect for a rod with uniform density.

ASSESS For $a=1$, the center of mass occurs at $\frac{2}{3}L$, while for $a=2$, it occurs at $\frac{3}{4}L$. This agrees with our premonition that having the density get larger toward $x=L$ will mean that the center of mass will be closer to that end.

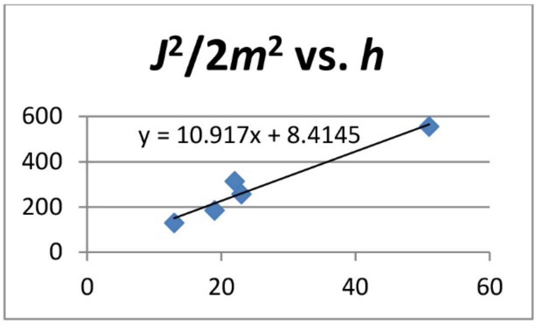
92. **INTERPRET** In this problem we analyze the data from rocket motors. From the relations between the rocket mass, impulse and maximum height reached, we are asked to deduce the value for g.

DEVELOP From impulse-momentum theorem, $J=\Delta p=m\Delta v$, the initial velocity of the rocket from the impulse is $v=J/m$ By energy conservation, $\Delta U+\Delta K=0$, the maximum height attained by the rocket is given by

$$mgh=\frac{1}{2}mv^2=\frac{J^2}{2m} \quad\Rightarrow\quad \frac{J^2}{2m^2}=gh$$

Thus, we see that plotting $J^2/2m^2$ as a function of h will yield a line with slope equal to g.

EVALUATE The plot is shown below.



The slope is 10.9 m/s^2, which is our estimate of the gravitational acceleration g.

ASSESS Our estimate of g is high compared to the accepted value of 9.8 m/s^2, because air resistance reduces the maximum height and is most significant for the higher and faster rockets.

93. **INTERPRET** The one-dimensional collision in this problem is elastic, so both momentum and energy are conserved. We are asked to explore how the masses affect the number of collisions they undergo.

DEVELOP Momentum is conserved in this process. In this one-dimensional case, we may write

$$m_1v_{1\text{i}}+m_2v_{2\text{i}}=m_1v_{1\text{f}}+m_2v_{2\text{f}}$$

Since the collision is completely elastic, energy is conserved:

$$\frac{1}{2}m_1v_{1\text{i}}^2+\frac{1}{2}m_2\overbrace{v_{2\text{i}}^2}^{=0}=\frac{1}{2}m_1v_{1\text{f}}^2+\frac{1}{2}m_2v_{2\text{f}}^2$$

Using the two conservation equations, the final speeds of m_1 and m_2 are (see Equations 9.15a and 9.15b):

$$v_{1\text{f}}=\frac{m_1-m_2}{m_1+m_2}v_{1\text{i}}+\frac{2m_2}{m_1+m_2}v_{2\text{i}} \text{ and } v_{2\text{f}}=\frac{2m_1}{m_1+m_2}v_{1\text{i}}+\frac{m_2-m_1}{m_1+m_2}v_{2\text{i}}$$

Given that $v_{1\text{i}}=v_0$, $v_{2\text{i}}=0$, $m_1=M$ and $m_2=nM$, the above expressions may be simplified to

$$v_{1\text{f}}=\frac{1-n}{1+n}v_0, \text{ (to the left)}$$

$$v_{2\text{f}}=\frac{2}{1+n}v_0, \text{(to the right)}$$

After hitting the wall, $u_{1\text{f}}=-v_{1\text{f}}=-\frac{1-n}{1+n}v_0$. So if $u_{1\text{f}}<v_{2\text{f}}$, it will never catch up, and there's only one collision between the two blocks.

EVALUATE (a) The condition $u_{1\text{f}}<v_{2\text{f}}$ implies

$$\frac{n-1}{1+n}<\frac{2}{n+1}$$

or $n<3$.

(b) If $n = 4$, then

$$v_{1\text{f}} = \frac{1-4}{1+4}v_0 = -\frac{3}{5}v_0, \text{ (to the left)}$$
$$v_{2\text{f}} = \frac{2}{1+4}v_0 = \frac{2}{5}v_0, \text{ (to the right)}$$

After hitting the wall, $u_{1\text{f}} = 3v_0/5$ which is greater than $v_{2\text{f}}$, so there will be a second collision. The speeds of the two blocks after the second collision are

$$v'_{1\text{f}} = -\frac{3}{5}\left(\frac{3}{5}v_0\right) + \frac{8}{5}\left(\frac{2}{5}v_0\right) = \frac{7}{25}v_0, \text{ (to the right)}$$
$$v'_{2\text{f}} = \frac{2}{5}\left(\frac{3}{5}v_0\right) + \frac{3}{5}\left(\frac{2}{5}v_0\right) = \frac{12}{25}v_0, \text{ (to the right)}$$

Since $v'_{1\text{f}} < v'_{2\text{f}}$, there will only be two collisions with $n = 4$.

(c) If $n = 10$, the speeds of the blocks after the first collision are

$$v_{1\text{f}} = \frac{1-10}{1+10}v_0 = -\frac{9}{11}v_0, \text{ (to the left)}$$
$$v_{2\text{f}} = \frac{2}{1+10}v_0 = \frac{2}{11}v_0, \text{ (to the right)}$$

After rebounding from the wall, $u_{1\text{f}} = 9v_0/11$, which is greater than $v_{2\text{f}}$, so the second collision ensues. The speeds of the two blocks after the second collision are

$$v'_{1\text{f}} = -\frac{9}{11}\left(\frac{9}{11}v_0\right) + \frac{20}{11}\left(\frac{2}{11}v_0\right) = -\frac{41}{121}v_0, \text{ (to the left)}$$
$$v'_{2\text{f}} = \frac{2}{11}\left(\frac{9}{11}v_0\right) + \frac{9}{11}\left(\frac{2}{11}v_0\right) = \frac{36}{121}v_0, \quad \text{(to the right)}$$

After rebounding from the wall, $u'_{1\text{f}} = 41v_0/121$, which is greater than $v'_{2\text{f}} = 36v_0/121$, so the third collision ensues. The speeds of the two blocks after the third collision are

$$v''_{1\text{f}} = -\frac{9}{11}\left(\frac{41}{121}v_0\right) + \frac{20}{11}\left(\frac{36}{121}v_0\right) = \frac{351}{1331}v_0 \approx 0.26v_0, \text{ (to the right)}$$
$$v''_{2\text{f}} = \frac{2}{11}\left(\frac{41}{121}v_0\right) + \frac{9}{11}\left(\frac{36}{121}v_0\right) = \frac{406}{1331}v_0 \approx 0.31v_0, \quad \text{(to the right)}$$

Since $v''_{1\text{f}} < v''_{2\text{f}}$, there will only be three collisions with $n = 10$.

ASSESS The number of collision depends on n. The greater the value of n, the more collisions between the two blocks.

94. **INTERPRET** We're asked to analyze the bouncing of a ball captured by a strobe camera.

DEVELOP The picture shows that the ball bounces up to a lower height after each bounce.

EVALUATE If the collisions with the floor were totally inelastic, then we'd expect the ball to stick to the floor after the first bounce, which it does not. To analyze better what does happen, let's divide up the velocity into its x- and y-components: $\vec{v} = v_x\hat{i} + v_y\hat{j}$. In terms of solely the vertical motion, the ball has a head-on collision with the ground. If this head-on collision were totally elastic, then the ball should bounce back in the opposite direction with the same speed it hit the ground $v_{y\text{f}} = -v_{y\text{i}}$. This follows from Case 1 in Section 9.6, where a small mass (the ball) collides with a much larger mass (the floor). If the ball has the same vertical speed after the collision, then by conservation of energy the ball should return to roughly the same height $\left(h \propto v_y^2\right)$ after each bounce, which it does not.

By elimination, the answer is (c).

ASSESS We would say the collision with the ground is inelastic (just not "totally"), since some of the kinetic energy is lost to internal energy (heat) of the ball and the ground.

95. **INTERPRET** We're asked to analyze the bouncing of a ball captured by a strobe camera.

DEVELOP Right before the second collision, the ball has kinetic energy $K_\mathrm{i} = \frac{1}{2}mv_{x\mathrm{i}}^2 + \frac{1}{2}mv_{y\mathrm{i}}^2$, while after the collision, it has $K_\mathrm{f} = \frac{1}{2}mv_{x\mathrm{f}}^2 + \frac{1}{2}mv_{y\mathrm{f}}^2$. We argued in the previous problem that because the ball doesn't rebound to the same height, the vertical speed at ground level must be getting smaller after each collision $\left(v_y = \sqrt{2gh}\right)$. If we just consider the motion in the vertical direction, the fraction of energy lost is:

$$\left(\frac{\Delta K}{K_\mathrm{i}}\right)_y = \frac{h_\mathrm{f} - h_\mathrm{i}}{h_\mathrm{i}}$$

EVALUATE With our fingers or with a small ruler, we can check that the peak height after the second collision is about 0.6 times the peak height before the collision. So by the equation above, the ball lost around 40% of its energy in the vertical direction. Assuming the loss in horizontal direction wasn't more than that, the fraction of the total energy lost is a little less than half.

The answer is (b).

ASSESS We've treated the components of kinetic energy separately: $K_x = \frac{1}{2}mv_x^2$ and $K_y = \frac{1}{2}mv_y^2$. It should be noted that the two are not completely separate. If the ground were flat or if the ball were spinning, a collision could transfer energy in the vertical direction to energy in the horizontal direction, or vice versa.

96. **INTERPRET** We're asked to analyze the bouncing of a ball captured by a strobe camera.

DEVELOP The vertical component of the velocity after a collision can be estimated by the height that the ball reaches at the top of the bounce: $v_y = \sqrt{2gh}$. Since there are no horizontal forces acting on the ball while it's in the air, the horizontal component of the velocity between collisions is constant. It is equal to the distance, x, the ball travels horizontally divided by the time, *t*, that it remains airborne between collisions with the floor:

$$v_x = \frac{x}{t} = \frac{x}{\left(2v_y / g\right)} = \frac{xg}{2\sqrt{2gh}} = x\sqrt{\frac{g}{8h}}$$

EVALUATE In the previous problem, we estimated that the height after a collision is 0.6 times the height before the collision, so the vertical component of the velocity is about 20% less after a collision. For the horizontal component, we roughly measure that the distance the ball travels after the second collision is about 0.8 times the distance before the collision. Therefore, the horizontal component of the velocity decreases by:

$$\frac{\Delta v_x}{v_{x\mathrm{i}}} = \frac{v_{x\mathrm{f}} - v_{x\mathrm{i}}}{v_{x\mathrm{i}}} = \frac{x_\mathrm{f} / \sqrt{h_\mathrm{f}}}{x_\mathrm{i} / \sqrt{h_\mathrm{i}}} - 1 \approx \frac{0.8}{\sqrt{0.6}} - 1 \approx 0$$

Therefore, the vertical component is more affected.

The answer is (b).

ASSESS We might have expected that the horizontal velocity remains roughly constant through the collision. The nonconservative forces that remove energy from the ball are likely to point in the vertical direction where the velocity goes through the biggest change.

97. **INTERPRET** We're asked to analyze the bouncing of a ball captured by a strobe camera.

DEVELOP The way a strobe camera works is that it takes pictures at a set interval. So we can get a rough estimate for how long the ball was between collisions or in the midst of a collision by counting how many times the camera caught the ball in either setting.

EVALUATE In the image, we count seven times that the ball's picture was taken between the first and second collisions. However, it appears that the ball's picture was taken only once during each collision. So the collision time is a tiny fraction of the time between collisions.

The answer is (a).

ASSESS This matches our experience that collisions are very short-lived events.