27

ELECTROMAGNETIC INDUCTION

EXERCISES

Sections 27.2 Faraday's Law and 27.3 Induction and Energy

11. **INTERPRET** Given a constant magnetic field, we are to find the magnetic flux that passes through the given loop.
DEVELOP For a stationary plane loop in a uniform magnetic field, the integral for the flux in Equation 27.1a is just $\Phi_B = \vec{B}\cdot\vec{A}$.
EVALUATE Evaluating the dot product gives

$$BA\cos\theta = B\pi r^2\cos\theta = (65\text{ mT})\pi(3.0\text{ cm})^2\cos(60°) = 9.19\times10^{-5}\text{ Wb} \approx 0.092\text{ mWb}$$

ASSESS The SI unit of flux, T·m^2, is also called a weber (Wb).

12. **INTERPRET** This problem is about the rate of change of magnetic flux through a loop due to a changing magnetic field.
DEVELOP For a stationary plane loop in a uniform magnetic field, the magnetic flux is given by Equation 27.1b, $\Phi_B = \vec{B}\cdot\vec{A} = BA\cos\theta$. Note that the SI unit of flux, $\text{T}\cdot\text{m}^2$, is also called a weber, Wb. The rate of change of magnetic flux is $d\Phi_B/dt = \Delta\Phi_B/\Delta t$.
EVALUATE **(a)** The magnetic field at the beginning $(t_1 = 0)$ is

$$\Phi_1 = B_1 A = \frac{1}{4}\pi d^2 B_1 = \frac{1}{4}\pi(45\text{ cm})^2(5.0\text{ mT}) = 7.95\times10^{-4}\text{ Wb} \approx 0.80\text{ mWb}$$

(b) The magnetic field at $t_2 = 25\text{ ms}$ is

$$\Phi_2 = B_2 A = \frac{1}{4}\pi d^2 B_2 = \frac{1}{4}\pi(45\text{ cm})^2(55\text{ mT}) = 8.747\times10^{-3}\text{ Wb} \approx 8.7\text{ mWb}$$

(c) Since the field increases linearly, the rate of change of magnetic flux is

$$\frac{d\Phi_B}{dt} = \frac{\Delta\Phi_B}{\Delta t} = \frac{\Phi_2 - \Phi_1}{t_2 - t_1} = \frac{8.747\times10^{-3}\text{ Wb} - 0.795\times10^{-3}\text{ Wb}}{25\text{ ms}} = 0.318\text{ V}$$

From Faraday's law, this is equal to the magnitude of the induced emf, which causes a current

$$I = \frac{|\varepsilon|}{R} = \frac{0.318\text{ V}}{120\ \Omega} = 2.65\text{ mA}$$

in the loop.
(d) The direction must oppose the increase of the external field downward, hence the induced field is upward and I is counterclockwise when viewed from above the loop.
ASSESS Since $\Delta\Phi_B/\Delta t = (\Delta B/\Delta t)A$ with the area of the loop kept fixed, the induced emf and hence the current scale linearly with $\Delta B/\Delta t$.

13. **INTERPRET** This problem involves Faraday's law, which we can use to find the rate at which the magnetic field is changing, given the current in the loop.
DEVELOP The flux through a stationary loop perpendicular to a magnetic field is $\Phi_B = BA$, so Faraday's law (Equation 27.2) and Ohm's law (Equation 24.5) relate this to the magnitude of the induced current:

$$I=\left|\frac{\varepsilon}{R}\right|=\left|\frac{-d\Phi_B/\mathrm{dt}}{R}\right|=\left|\frac{-d(BA)/dt}{R}\right|=A\left|\frac{-dB/\mathrm{dt}}{R}\right|$$

Evaluate Solving this expression for the rate of change of the magnetic field gives

$$\left|\frac{dB}{dt}\right|=\frac{IR}{A}=\frac{(0.35\,\mathrm{A})(14\,\Omega)}{230\times10^{-4}\ \mathrm{m}^2}=213\ \mathrm{T/s}\approx 210\ \mathrm{T/s}$$

Assess Whether the magnetic field is increasing or decreasing depends on the direction in which the current is circulating with respect to the magnetic field**.**

14. **Interpret** The problem asks for the number of turns the coil must have in order to produce a given emf when it is placed in a time-varying magnetic field, which we can find using Faraday's law.

Develop When the coil is wrapped around the solenoid, all of the flux in the solenoid ($B_{\mathrm{sol}}\,A_{\mathrm{sol}}$, for a long thin solenoid) goes through each of the N_{coil} turns of the coil. Using Faraday's law (Equation 27.2), the induced emf in the coil is

$$|\varepsilon|=\left|\frac{d\Phi_B}{dt}\right|=\frac{d}{dt}\left(N_{\mathrm{coil}}\,B_{\mathrm{sol}}\,A_{\mathrm{sol}}\right)=N_{\mathrm{coil}}\,A_{\mathrm{sol}}\left|\frac{dB_{\mathrm{sol}}}{dt}\right|$$

Evaluate Substituting the values given in the problem statement, the number of turns in the coil is

$$N_{\mathrm{coil}}=\frac{|\varepsilon|}{|dB_{\mathrm{sol}}/dt|\,A_{\mathrm{sol}}}=\frac{15\ \mathrm{V}}{(2.4\ \mathrm{T/s})\pi(0.115\ \mathrm{m})^2}=150\ \mathrm{turns}$$

Assess The number of turns is proportional to the induced emf, but inversely proportional to the rate of change of magnetic field.

Section 27.4 Inductance

15. **Interpret** We are to find the self-inductance of the given solenoid.

Develop This problem is treated in Example 27.6. Apply Equation 27.4.

Evaluate Equation 27.4 gives

$$L=\frac{\mu_0 N^2 A}{l}=\frac{\left(1.26\times10^{-6}\,\mathrm{N/A^2}\right)(1\,500)^2\,\pi\,(2.0\,\mathrm{cm})^2}{55\ \mathrm{cm}}=6.5\ \mathrm{mH}$$

Assess Note that Equation 27.4 makes use of the assumption that the solenoid length is much greater than its diameter, which holds for this problem.

16. **Interpret** This problem asks for the self-inductance of an inductor, given its emf and the rate of change of its current.

Develop The induced emf in an inductor is given by Equation 27.5: $\varepsilon_L=-L\,dI/dt$. With ε_L and dI/dt given, we can use this equation to compute the self-inductance L.

Evaluate From Equation 27.5, we find the self-inductance to be

$$L=\left|-\frac{\varepsilon_L}{dI/dt}\right|=\frac{45\ \mathrm{V}}{110\ \mathrm{A/s}}=0.41\ \mathrm{H}$$

Assess Our value of self-inductance is reasonable; inductances in common electronic circuits usually range from micro-henrys to several henrys.

17. **Interpret** We are to find the induced emf in a circuit, given the rate of change of the current and the circuit's inductance.

Develop Assume that the current changes uniformly from 2.4 A to zero in 1.7 ms (or consider average values). Then, $\dfrac{dI}{dt}=\dfrac{\Delta I}{\Delta t}=\dfrac{-2.4\ \mathrm{A}}{1.7\ \mathrm{ms}}=1.4\times10^3\ \mathrm{A/s}$, and we can apply Equation 27.5 to find the emf.

$$\frac{dI}{dt}=\frac{\Delta I}{\Delta t}=\frac{-2.4A}{1.7ms}=-1.4\times10^3\,A/s$$

Evaluate The emf is

$$\varepsilon = -L\frac{dI}{dt} = -(20\text{ H})\left(-1.4\times10^{3}\text{ A/s}\right) = 28\text{ kV}$$

ASSESS The negative sign indicates that the emf opposes the change in the current.

18. **INTERPRET** Your sister is constructing a solenoid that will work as an inductor. She asks you the number of turns it will need to have the desired inductance.

DEVELOP The inductance of a solenoid was derived in Example 27.6: $L = \mu_0 n^2 A l$, where n is the number of turns per unit length, A is the cross-sectional area, and l is the length. Your sister is asking for the total number of turns: $N = nl$.

EVALUATE Using the given values, the number of turns is

$$N = nl = \sqrt{\frac{Ll}{\mu_0 \pi r^2}} = \sqrt{\frac{(450\ \mu\text{H})(12\text{ cm})}{\left(1.26\times10^{-6}\text{ N/A}^2\right)\pi(2.0\text{ cm})^2}} = 185$$

ASSESS This seems like a reasonable number of turns for your sister to do. The units work out since $1\text{ H} = 1\text{ J/A}^2$.

19. **INTERPRET** We are to find the time constant of a circuit, given its resistance and its inductance.

DEVELOP From Equations 27.6 and 27.7, we see that the time constant is $\tau = L/R$, which we can solve for the inductance given the time constant and the resistance.

EVALUATE Inserting the given quantities gives

$$\tau_L = \frac{L}{R}$$
$$L = \tau_L R = (2.2\text{ ms})(150\,\Omega) = 330\text{ mH}$$

ASSESS To verify that the units work out correctly, note that a henry is a $\text{T}\cdot\text{m}^2/\text{A}$ and an ohm is $\text{m}^2\cdot\text{kg}\cdot\text{s}^{-3}\cdot\text{A}^{-2}$. Expressing teslas in terms of SI base units gives ($\text{kg}\cdot\text{s}^{-2}\cdot\text{A}^{-2}$)

$$\text{s}\cdot\Omega = \text{s}\cdot\text{m}^2\cdot\text{kg}\cdot\text{s}^{-3}\cdot\text{A}^{-2} = \text{m}^2\cdot\text{kg}\cdot\text{s}^{-2}\cdot\text{A}^{-2} = \text{T}$$

20. **INTERPRET** This problem is about the resistance in a series *RL* circuit. We are given the current at a given time and are asked to find the resistance of the *RL* circuit, given its time constant and its inductance.

DEVELOP The buildup of current in an *RL* circuit with a battery is given by Equation 27.7:

$$I(t) = I_\infty\left(1 - e^{-Rt/L}\right)$$

where $I_\infty = \varepsilon_0/\text{R}$ is the final current. We are given that $I(t = 3.0\ \mu\text{s}) = (0.23)I_\infty$, so we can solve for R.

EVALUATE Solving the expression above for R and inserting the values given, one finds the resistance to be

$$R = -\frac{L}{t}\ln\left(1 - \frac{I(t)}{I_\infty}\right) = -\frac{2.1\text{ mH}}{3.0\ \mu\text{s}}\ln(1-0.23) = 183\ \Omega = 180$$

ASSESS We find R to be inversely proportional to t. This means that the greater the value of R, the shorter the time it takes for the current to increase to 23% of its final value.

Section 27.5 Magnetic Energy

21. **INTERPRET** We are to find the energy stored in the given inductor through which flows the given current.

DEVELOP Apply Equation 27.9.

EVALUATE Inserting the given quantities into Equation 27.9 gives

$$U = \tfrac{1}{2}LI^2 = \tfrac{1}{2}(5.0\text{ H})(35\text{ A})^2 = 3.1\text{ kJ}$$

ASSESS This is the energy it would take to lift one liter of water a height h of

$$U = mgh$$
$$h = \frac{U}{mg} = \frac{3.1\text{ kJ}}{(1.0\text{ kg})\left(9.8\text{ m/s}^2\right)} = 320\text{ m}$$

22. **Interpret** This problem is about magnetic energy stored in an inductor. Given the inductance and the energy stored, we are to find the current.

Develop The amount of energy stored in an inductor is given by Equation 27.9: $U = LI^2/2$. This equation allows us to determine the current *I*.

Evaluate From the equation above, we find the current to be

$$I = \sqrt{\frac{2U}{L}} = \sqrt{\frac{2(25\times10^{-6}\ \text{J})}{24\times10^{-3}\text{H}}} = 0.079\ \text{A}$$

Assess Since the energy stored in the inductor is small, we expect the current (which is proportional to $\sqrt{U}$) to be small as well.

23. **Interpret** This problem is about the energy stored in the magnetic field of a solenoid. Given the solenoid parameters (number of turns, current, diameter), we are to find the energy stored in the magnetic field of the solenoid.

Develop We first note that the inductance of a solenoid is given by Equation 27.4: $L = \mu_0 n^2 Al = \mu_0 N^2 A/l$. Equation 27.9, $U = LI^2/2$ can then be used to find the energy stored in the solenoid.

Evaluate Combining Equations 27.4 and 27.9, we find the stored energy to be

$$U = \frac{1}{2}LI^2 = \frac{1}{2}\frac{\mu_0 N^2 AI^2}{l} = \frac{\left(1.26\times10^{-6}\text{N/A}^2\right)\left(1250\right)^2 \pi\left(0.0158\ \text{m}/2\right)^2\left(0.165\ \text{A}\right)^2}{2\left(0.232\ \text{m}\right)} = 2.26\times10^{-5}\ \text{J} = 22.6\ \mu\text{J}$$

Assess This inductance of the solenoid is about $1.66\ \text{mH}$. The stored energy is typical for a small inductor with a small current.

24. **Interpret** This problem is an exercise in dimensional analysis. We are to show that the given expression has units of energy density (i.e., J/m^3).

Develop The permeability constant μ_0 has units of $\text{N/A}^2 = \text{N}\cdot\text{C}^{-2}\cdot\text{s}^2$ (see discussion accompanying Equation 26.7) and the magnetic field has units of $\text{N}\cdot\text{s}/(\text{C}\cdot\text{m})$ (see discussion accompanying Equation 26.1). Combine these factors in the indicated fashion to find the units of B^2/μ_0.

Evaluate The units of B^2/μ_0 are

$$\left(\frac{\text{N}\cdot\text{s}}{\text{C}\cdot\text{m}}\right)^2\left(\frac{\text{C}^2}{\text{N}\cdot\text{s}^2}\right) = \frac{\text{N}}{\text{m}^2} = \frac{\text{N}\cdot\text{m}}{\text{m}^3} = \frac{\text{J}}{\text{m}^3}$$

Assess The factor 2 in the denominator does not affect the result.

25. **Interpret** The problem concerns the energy stored in the world's largest sustained magnetic field.

Develop The energy density of a magnetic field is given in Equation 27.10: $u_B = B^2/2\mu_0$.

Evaluate A 45-T field stores energy in the amount of

$$u_B = \frac{B^2}{2\mu_0} = \frac{\left(45\ \text{T}\right)^2}{2\left(1.26\times10^{-6}\text{N/A}^2\right)} = 0.81\ \text{GJ/m}^3$$

Assess This is a lot of energy stored in something we can't see or touch. To put some perspective on this, the energy density of gasoline is only about 40 times larger.

26. **Interpret** We are to find the magnetic field strength in a region with the given magnetic energy density.

Develop Apply Equation 27.10.

Evaluate Solving Equation 27.10 for the magnetic field strength *B* gives

$$B = \sqrt{2\mu_0 u_B} = \sqrt{2\left(1.26\times10^{-6}\text{N/A}^2\right)\left(7.8\ \text{J/cm}^3\right)} = 4.4\ \text{T}$$

Assess This result is for free space (i.e., empty space). If a material occupies the space, Equation 27.10 is not valid.

Section 27.6 Induced Electric Fields

27. **Interpret** We are given the induced electric field and asked to find the rate of change of the magnetic field.

Develop The connection between the induced electric field and the rate of change of the magnetic field is given by the integral form of Faraday's law (Equation 27.11):

$$\oint \vec{E}\cdot d\vec{r} = -\frac{d\Phi_B}{dt}$$

The geometry of the induced electric field from the solenoid is described in Example 27.11, where it is shown that Faraday's law leads to

$$2\pi r|E| = \left|-\frac{d\left(\pi R^2 B\right)}{dt}\right| = \pi R^2\left|\frac{dB}{dt}\right|$$

EVALUATE Solving the expression above for the rate of change of the magnetic field gives

$$\left|\frac{dB}{dt}\right| = \frac{2r|E|}{R^2} = \frac{2(12\text{ cm})(45\text{ V/m})}{(10\text{ cm})^2} = 1.1\times 10^3\text{ T/s} = 1.1\text{ T/ms}$$

ASSESS The magnetic field is changing at a very fast rate. Note that the sign of dB/dt and the direction of the induced electric field are related by Lenz's law.

28. **INTERPRET** This problem involves a solenoid in which the current is changing, so it has a time-varying magnetic field and thus an electric field as well. We can use Faraday's law to find the electric field strength as a function of *r* inside a solenoid.

DEVELOP We'll use a circular Ampérian loop, of radius *r*, centered inside the solenoid. The flux through this loop is $\Phi = BA = \pi r^2 B.$ We are told that the field in the solenoid is $B = bt$. Faraday's law, integrated around this loop, gives us

$$\oint \vec{E}\cdot d\vec{r} = -\frac{d\Phi}{dt}$$

By symmetry, the electric field is constant around any loop of a given radius, which makes the integration easy.

EVALUATE

$$\oint \vec{E}\cdot d\vec{r} = -\frac{d\Phi}{dt} \quad\Rightarrow\quad 2\pi r|E| = \left|-\frac{d}{dt}\left[\pi r^2(bt)\right]\right| = \pi r^2 b$$

$$E = \frac{rb}{2}$$

ASSESS Just as in previous problems that use Gauss's and Ampère's laws, it is important to choose our symmetry to make things easy on ourselves.

EXAMPLE VARIATIONS

29. **INTERPRET** Here the circuit is formed by the rails, the resistance, and the conducting bar. The circuit area increases as the bar slides along the rails, so we've got a case of induction caused by a changing magnetic flux resulting from a changing area.

DEVELOP In this case of a uniform field perpendicular to the circuit, the flux is the product $\Phi_B = BA$. We can express this flux in terms of the changing position *x* of the sliding bar; since we're given the bar's speed, we'll be able to evaluate the rate of change of flux. If we take $x = 0$ at the left end of the rails, then the circuit area is $A = lx$, so the flux is $\Phi_B = BA = Blx$.

EVALUATE Differentiating the flux with respect to time gives

$$\frac{d\Phi_B}{dt} = Blv$$

Faraday's law says that *Blv* is the magnitude of the induced emf $\mathcal{E}$, so the current in the circuit becomes

$$I = \frac{\mathcal{E}}{R} = \frac{Blv}{R}$$

Which we evaluate for the given parameters to obtain

$$I = \frac{Blv}{R} = \frac{(375\,\text{mT})(45\,\text{cm})(4.87\,\text{m/s})}{(18.8\,\Omega)} = 43.7\,\text{mA}$$

Assess The faster the bar moves, the greater the rate of change of flux, and so the greater the induced emf and current.

30. **Interpret** In this problem we want to determine the bar's speed necessary to produce a given current in the preceding problem's circuit from the induced emf.

Develop In this case of a uniform field perpendicular to the circuit, the flux is the product $\Phi_B = BA$. We can express this flux in terms of the changing position *x* of the sliding bar; since we're looking for the bar's speed, we'll solve for it from the rate of change of flux. If we take $x = 0$ at the left end of the rails, then the circuit area is $A = lx$, so the flux is $\Phi_B = BA = Blx$. Once we have found the expression for the current, we can then determine the power, or rate of work done by the agent pulling the bar.

Evaluate Differentiating the flux with respect to time gives

$$\frac{d\Phi_B}{dt} = Blv$$

(a) Faraday's law says that *Blv* is the magnitude of the induced emf ε, so using the desired current in the circuit, we find a necessary speed of

$$I = \frac{\mathcal{E}}{R} = \frac{Blv}{R}$$

$$v = \frac{IR}{Bl} = \frac{(0.150\,\text{A})(18.8\,\Omega)}{(375\,\text{mT})(45.0\,\text{cm})} = 16.7\,\text{m/s}$$

(b) This means the rate of work done onto the rod is equal to

$$P = I^2R = \frac{B^2l^2v^2}{R} = \frac{(375\,\text{mT})^2(45\,\text{cm})^2(4.87\,\text{m/s})^2}{(18.8\,\Omega)} = 0.423\text{ W}$$

Assess The induced current is proportional to the applied velocity, so the rate of work needed to generate a particular current will depend equally on the applied velocity.

31. **Interpret** Here the circuit is formed by the rails, the resistance, and the conducting bar, and an additional battery in series with the resistor. The circuit area increases as the bar slides along the rails, so we've got a case of induction caused by a changing magnetic flux resulting from a changing area.

Develop In this case of a uniform field perpendicular to the circuit, the flux is the product $\Phi_B = BA$. We can express this flux in terms of the changing position *x* of the sliding bar; since we're looking for the bar's speed, we'll solve for it in the rate of change of flux. If we take $x = 0$ at the left end of the rails, then the circuit area is $A = lx$, so the flux is $\Phi_B = BA = Blx$. However, in this problem the rod begins at rest and there is an initial current in the circuit from the battery. This current then feels a force from the magnetic field and begins to accelerate the stationary rod, inducing its own emf from the changing magnetic flux.

Evaluate **(a)** At first the rod is stationary, and the only current in the circuit is due to the battery of emf $\mathcal{E}$ in series with the resistance, resulting in: $I = \mathcal{E}/R$.

(b) Once the circuit is closed, the bar will begin to move, and as it does it will start to induce a current to counteract the changing magnetic flux. This current will grow until the bar speeds up to a value that induces the same magnitude in the opposite direction. From the preceding problem we know the velocity of the bar is proportional to the current it induces, meaning the value of the constant speed reached is: $v = IR/Bl = \mathcal{E}/Bl$.

(c) Therefore, once the induced current matches the initial current as the rod reaches this speed, the current in the circuit is zero.

Assess The steady state for this circuit accelerates the rod to a speed which generates a current equal in magnitude but opposite in direction to the initial current supplied by the battery.

32. **Interpret** In this problem we evaluate the necessary emf to accelerate a rod in a circuit like the one described in the preceding problem.

Develop In the circuit described in preceding problem we found that an applied emf can accelerate a rod to a speed given by: $v = \mathcal{E}/Bl$, where $\mathcal{E}$ is the battery emf applied, *B* is the magnitude of the magnetic field, and *l* is the separating rail distance. We can evaluate this to find the emf needed to achieve a speed that's 1.5 times the lunar escape speed of: 2.38×10^3 m/s.

EVALUATE The necessary emf to accelerate the rod to this speed is equal to

$$\mathcal{E} = Blv = (0.877\,\text{T})(1.00\,\text{m})\left(1.5\cdot 2.38\times10^3\,\text{m/s}\right) = 3.13\,\text{kV}$$

ASSESS The necessary emf could be reduced by decreasing the spacing between rails in the circuit design.

33. **INTERPRET** This is a problem about the buildup of current in an *RL* circuit.

DEVELOP Equation 27.7, $I = (\mathcal{E}_0/R)\left(1-e^{-Rt/L}\right)$, determines the current; here $\mathcal{E}_0/R$ is the final current, and we want to solve for the time *t* when *I* is 0.700 *A*. That is, we want $0.8 = \left(1-e^{-Rt/L}\right)$.

EVALUATE Rearranging this expression like in the original example, we find

$$t = -\frac{L}{R}\ln(0.2) = -\frac{72\,\text{mH}}{6.86\,\Omega}\ln(0.2) = 16.9\,\text{ms}$$

ASSESS This is a little more than one time constant (L/R = 11 ms) — not surprising because we found with capacitors that we reach approximately two-thirds of the full charge in one time constant. Analogously, with inductors, we reach about two-thirds of the final current in one time constant.

34. **INTERPRET** This is a problem about the buildup of current in an *RL* circuit, in which we want to determine the maximum inductance a wire may produce to ensure current rise is maintained below a certain duration.

DEVELOP Equation 27.7, $I = (\mathcal{E}_0/R)\left(1-e^{-Rt/L}\right)$, determines the current; here $\mathcal{E}_0/R$ is the final current, and we want to solve for the inductance *L* when $I(t = 10\text{ ns})$ is $0.7(\mathcal{E}_0/R)$. That is, we want $0.7 = \left(1-e^{-Rt/L}\right)$.

EVALUATE Rearranging this expression like in the original example, we find

$$L = -\frac{tR}{\ln(0.3)} = -\frac{(10\,\text{ns})(184\,\mu\Omega)}{\ln(0.3)} = 1.53\,\text{pH}$$

ASSESS This inductance results in a time constant of approximately 8 ns, allowing the current to build up to the necessary amount in the desired time.

35. **INTERPRET** This is a problem about the decay of current in an *RL* circuit.

DEVELOP Equation 27.8, $I = I_0 e^{-Rt/L}$, determines the current; here I_0 is the initial current, and we want to solve for the time *t* when *I* drops below 250 A. That is, we want $7.4 = e^{-Rt/L}$.

EVALUATE Rearranging this expression like in the original example, we find

$$t = -\frac{L}{R}\ln(0.135) = -\frac{342\,\text{mH}}{21.6\,\text{m}\Omega}\ln(0.135) = 31.7\,\text{s}$$

ASSESS It takes about two time constants (L/R = 15.8 ms) for the current in this circuit to reach below 250 A.

36. **INTERPRET** This is a problem about the decay of current in an *RL* circuit, where we want to ensure the fail-safe resistance present allows current decay to certain amount in a given time.

DEVELOP Equation 27.8, $I = I_0 e^{-Rt/L}$, determines the current; here I_0 is the initial current, and we want to solve for the resistance *R* that will ensure $I(t = 10\text{s})$ reaches 6.25 kA. That is, we want $0.5 = e^{-Rt/L}$. We also want to then calculate the power dissipated in copper, which we will calculate using the inductor emf, Equation 27.5.

EVALUATE **(a)** Rearranging this expression like in the original example, we find

$$R = -\frac{L}{t}\ln(0.5) = -\frac{22.0\,\text{mH}}{10\,\text{s}}\ln(0.5) = 1.52\,\text{m}\Omega$$

(b) To obtain the power dissipated we can take a look at the induced emf from Equation 27.5: $\mathcal{E}_L = -L\dfrac{dI}{dt}$,

which if we apply to the expression for the decaying current, we arrive at a power dissipation of
$P = I\mathcal{E}_L = I\left(-L\left(-\dfrac{R}{L}I\right)\right) = I^2R$, as we would expect. Plugging in the given values we find the power dissipated in the copper is equal to: $P = I^2R = (12.5\,\text{kA})^2(1.52\,\text{m}\Omega) = 238\,\text{kW}$.

ASSESS To calculate the power dissipated we used the initial current since that is the current present in the circuit as superconductivity is lost and the copper begins to dissipate the power.

PROBLEMS

37. **INTERPRET** This problem involves Faraday's law, which we can use to find current in the loop under the given conditions.

DEVELOP Apply Faraday's law (Equation 27.2) and Ohm's law (Equation 27.5) to the circuit to find

$$\left.\begin{aligned}\varepsilon &= -\frac{d\Phi_B}{dt}\\ \varepsilon &= IR\end{aligned}\right\} I = -\frac{1}{R}\frac{d\Phi_B}{dt} = -\frac{1}{R}\frac{d}{dt}\left(\vec{A}\cdot\vec{B}\right) - \frac{A}{R}\frac{dB_z}{dt}$$

Inserting $B_z = at^2 - b$ gives

$$I(t) = -\frac{A}{R}(2at)$$

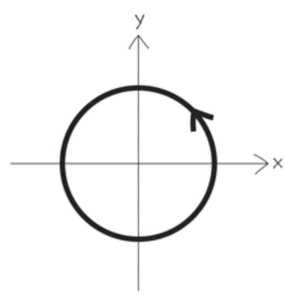
EVALUATE **(a)** Inserting $t = 1.5$ s gives

$$I(t = 1.5\text{ s}) = -\frac{0.14\text{ m}^2}{9.0\ \Omega}(2.0)\left(2.1\text{ T/s}^2\right)(1.5\text{ s}) = -0.098\text{ A}$$

(b) $B_z = 0$ implies $at^2 = b$, or $t = \pm\sqrt{b/a}$. At this time, the current is

$$I\left(t = \sqrt{b/a}\right) = -\frac{0.14\text{ m}^2}{9.0\ \Omega}(2.0)\left(2.1\text{ T/s}^2\right)\left(\sqrt{(8.2\text{ T})/\left(2.1\text{ T/s}^2\right)}\right) = -0.13\text{ A}$$

ASSESS The negative sign indicates the direction of the current with respect to the direction of the magnetic field. If the *x*- and *y*-axis are as shown below and the *z*-axis is out of the page, then $\vec{B}$ is in the same direction as $\vec{A}$ (out of the page). Using the right-hand rule, positive currents run counterclockwise and negative currents run clockwise around the loop.



38. **INTERPRET** This problem is about the work done by an external agent to move a loop at a constant speed across a region with uniform magnetic field.

DEVELOP The loop can be treated analogously to the situation analyzed in Section 27.3, under the heading "Motional EMF and Lenz's Law"; but instead of exiting the field region at constant velocity, the loop is entering. All quantities have the same magnitudes, except the current in the loop is CCW instead of CW, as in Fig. 27.13.

EVALUATE Since the applied force acts over a displacement equal to the side length of the loop, the work done can be calculated directly:

$$W_{\text{app}} = \vec{F}_{\text{app}}\cdot\vec{l} = (IlB)l = Il^2B$$

Since the induced current is

$$I = \frac{|\varepsilon|}{R} = \frac{|d\Phi_B/dt|}{R} = \frac{1}{R}\frac{d}{dt}(Blx) = \frac{Blv}{R}$$

the work applied by the external agent is

$$W_{\text{app}} = Il^2B = \left(\frac{Blv}{R}\right)l^2B = \frac{B^2l^3v}{R}$$

ASSESS Alternatively, the work can be calculated from conservation of energy:

$$P_{\text{diss}} = I^2R = \frac{(Blv)^2}{R} \quad \rightarrow \quad W_{\text{app}} = P_{\text{diss}}t = \frac{(Blv)^2}{R}\frac{l}{v} = \frac{B^2l^3v}{R}$$

39. **INTERPRET** This problem is similar to Example 27.5; it involves a conducting loop rotating in a uniform, static magnetic field, so the change in the magnetic field flux through the loop results from the rotation.

DEVELOP Use the result of Example 27.5, which shows that

$$\varepsilon = NB\pi r^2\left[2\pi f\sin(2\pi t)\right]$$

EVALUATE The maximum emf occurs at

$$\varepsilon = NB\pi r^2(2\pi f)$$

$$B = \frac{\varepsilon}{N\pi r^2(2\pi f)} = \frac{360\ \mu\text{V}}{2\cdot 5\pi^2(0.050\ \text{m})(10\ \text{s}^{-1})} = 15\ \text{mT}$$

ASSESS Because the maximum of the sine function is unity, we do not need to know at what time the maximum occurs.

40. **INTERPRET** This problem involves a time-varying magnetic field that is spatially uniform. This causes a changing magnetic flux through a conducting loop, so Faraday's law will lead to an induced emf.

DEVELOP Faraday's and Ohm's laws give the emf and the current in the loop:

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

$$I = \frac{\mathcal{E}}{R}$$

From the sign of the induced current we can then determine the direction of its flow.

EVALUATE **(a)** Inserting the given dimensions of the loop, along with rate of change in the magnetic field, into the expression for emf we find

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -A\frac{dB}{dt} = (16.4\,\text{cm})^2(1.65\,\text{T/ms}) = -44.4\,\text{V}$$

(b) Inserting the given resistance of the square loop we then obtain a current of

$$I = \frac{\mathcal{E}}{R} = -\frac{44.4\,\text{V}}{19.3\ \Omega} = -2.30\ \text{A}$$

Meaning the magnitudes of the induced emf and current are equal to 44.4 V and 2.30 A, respectively.

(c) Since the sign of the induced current is negative, it must flow in a clockwise direction.

ASSESS The time-varying magnetic field causes a current to pass through the conducting loop. We verify its direction by noting that the increasing flux would induce a current that works to counteract that change by generating a current that would result in an opposite magnetic field.

41. **INTERPRET** This problem involves a magnetic field that is changing in time, so the flux through a loop in this field changes. Thus, we can apply Faraday's law to find the induced emf.

DEVELOP To shine at full brightness, the potential drop across the bulb must be 6 V. This is equal to the induced emf if we neglect the resistance of the rest of the loop circuit. From Faraday's law (Equation 27.2),

$$|\varepsilon| = \left|-\frac{d\Phi_B}{dt}\right| = \left|-\frac{d(BA)}{dt}\right| = \left|\frac{A\Delta B}{\Delta t}\right|$$

EVALUATE **(a)** Inserting the given quantities gives

$$\Delta t = \frac{A|\Delta B|}{|\varepsilon|} = \frac{(2.6\text{ m})^2(2.5\text{ T})}{7\text{V}} = 2.4\text{ s}$$

(b) The direction of the current opposes the decrease of $\vec{B}$ into the page, and thus, it must act to increase $\vec{B}$ into the page. From the right-hand rule, this corresponds to a clockwise current in Fig. 27.38.

ASSESS The units of the expression work out to be units of time:

$$\frac{(\text{m})^2(\text{T})}{\text{V}} = \frac{(\text{m})^2(\text{kg}\cdot\text{s}^2\cdot\text{A}^{-1})}{\text{m}^2\cdot\text{kg}\cdot\text{s}^{-3}\cdot\text{A}^{-1}} = \text{s}$$

42. **INTERPRET** This problem involves a current induced in a rectangular loop due to a nearby time-varying current source. The time-varying current source creates a time-varying magnetic field, so Faraday's law will be involved.

DEVELOP The normal to the loop in Fig. 27.7 is taken to be in the direction of the magnetic field of the wire, or into the page, so the positive sense of circulation around the loop is clockwise (from the right-hand rule). Faraday's and Ohm's laws (Equations 27.2 and 26.5) give an induced current in the loop of

$$I = \frac{|\varepsilon|}{R} = \frac{|-d\Phi_B/dt|}{R}$$

The magnetic flux has been calculated in Example 27.2:

$$\Phi_B = \frac{\mu_0 Il}{2\pi}\ln\left(\frac{a+w}{a}\right)$$

EVALUATE Combining the above expressions gives

$$\begin{aligned} I &= \frac{|\varepsilon|}{R} = \frac{|d\Phi_B/dt|}{R} = \frac{\mu_0 l(dI/dt)}{2\pi R}\ln\left(\frac{a+w}{a}\right) = \frac{(1.26\times10^{-6}\text{N/A}^2)(6.0\text{ cm})(25\text{ A/s})}{2\pi(50\text{ m}\Omega)}\ln\left(\frac{4.5\text{ cm}}{1.0\text{ cm}}\right) \\ &= 9.0\ \mu\text{A} \end{aligned}$$

By Lenz's law, the induced current is counterclockwise in the loop.

ASSESS As the inward flux increases (because the current in the long wire is increasing), by Lenz's law, the induced current must flow in the counterclockwise direction to produce an outward flux to oppose the change.

43. **INTERPRET** The solenoid current is varying in time, so the magnetic field in the solenoid varies in time and Faraday's law will be involved in finding the current induced through the wire loop in the solenoid.

DEVELOP The magnetic field inside the solenoid is $B = \mu_0 nI$, so the flux through the loop is $\Phi_B = BA_{\text{loop}} = \mu_0 n\frac{1}{4}\pi D_{\text{loop}}^2 I$. From Faraday's and Ohm's laws (Equations 27.2 and 26.5), the magnitude of the induced current is

$$I_{\text{loop}} = \frac{|\varepsilon|}{R} = \frac{1}{R}\left|\frac{d\Phi_B}{dt}\right| = \mu_0\left(\frac{N}{L}\right)\frac{1}{4}\pi D_{\text{loop}}^2\frac{1}{R}\left(\frac{dI}{dt}\right)$$

EVALUATE **(a)** Inserting the given quantities gives

$$I_{\text{loop}} = (1.26\times10^{-6}\text{N/A}^2)\left(\frac{2000}{2.0\text{ m}}\right)\left(\frac{\pi}{4}\right)(0.10\text{ m})^2\frac{1.0\text{ kA/s}}{5.0\ \Omega} = 2.0\text{ mA}$$

(b) If the loop encloses the solenoid, then $\Phi_B = BA_{\text{solenoid}}$, and the induced current would increase to $A_{\text{solenoid}}/A_{\text{loop}} = (1.5)^2$ times the value in part **(a)**, or 4.4 mA.

ASSESS The current is greater in the outer loop because the loop encircles greater flux.

44. **INTERPRET** We're asked at what rate an applied magnetic field should be changed to heat a stent inside a person's body. The changing magnetic field will induce an emf according to Faraday's law, and the stent's inherent resistance will dissipate the heat.

DEVELOP Faraday's law says that the induced emf in the stent will be equal to the change in the magnetic flux: $\varepsilon = -d\Phi_B/dt$ (Equation 27.2). We won't worry about the minus sign, since we're interested in only the magnitude of the current generated, not its direction. If we treat the stent as a loop, the magnetic flux through it is given in Equation 27.1b: $\Phi_B = BA\cos\theta$. The magnetic field is the only thing that is changing (dB/dt); the area and the orientation are constant. We're told the stent is oriented optimally, which means its area is perpendicular to the field $(\cos\theta = 1)$.

EVALUATE We're given the desired power output of the circuit, which, in terms of the induced emf, is $P = \varepsilon^2 / \mathrm{R}$ (Equation 24.8b). Solving for the rate of magnetic field change gives

$$\frac{dB}{dt} = \frac{\sqrt{PR}}{A} = \frac{\sqrt{(240\ \text{mW})(49\ \text{m}\Omega)}}{\pi(5.0\ \text{mm})^2/4} = 5.5\ \text{kT/s}$$

ASSESS This might seem surprisingly large, but remember that this is not the magnitude of the field but the rate at which the field changes. One way to obtain the necessary emf is by turning on and off a 5.5-T field every millisecond, or by turning on and off a 5.5-mT field every microsecond.

45. **INTERPRET** This problem involves a time-varying magnetic field that is spatially uniform. This causes a changing magnetic flux through a conducting loop, so Faraday's law will lead to an induced emf.

DEVELOP Faraday's and Ohm's laws give the current in the loop:

$$I = \frac{\varepsilon}{R} = -\frac{(d\Phi_B/\text{dt})}{R} = -\frac{A}{R}\frac{dB}{dt} = -\frac{Ab}{R}$$

EVALUATE (a) Inserting the given values leads to

$$I = -\frac{(200\ \text{cm}^2)(0.36\ \text{T/s})}{0.30\ \Omega} = -24\ \text{mA}$$

A normal to the loop is parallel to the *z*-axis and corresponds to counterclockwise positive circulation (via the right-hand rule) when viewed from above. The minus sign thus indicates a clockwise circulation when viewed from the positive *z*-axis.

ASSESS The time-varying magnetic field causes a current to pass through the conducting loop.

46. **INTERPRET** An alternator is basically an electric generator running off a fraction of the power supplied by a car's engine. You need to determine the magnetic field for a new alternator design, given the desired peak voltage and the coil size and turning rate. The magnetic field is needed to induce an emf in the rotating coil, according to Faraday's law.

DEVELOP The magnetic flux, in this case, is changing due to the changing orientation of the coil. As in Example 27.5 for a generator, you can write the flux through the multi-turn coil as

$$\Phi_B = NBA\cos\theta = NB\pi r^2\cos(2\pi ft)$$

where the frequency *f* is the number of revolutions per second.

EVALUATE When the coil rotates, the induced emf is

$$\varepsilon = -\frac{d\Phi_B}{dt} = NB\pi r^2(2\pi f)\sin(2\pi ft)$$

The magnitude of the emf will vary with time, but you need the peak voltage (when the sine term is 1) to be 14 V. From this, you can solve for the magnetic field strength:

$$B = \frac{\varepsilon_{\text{peak}}}{2\pi^2 Nr^2 f} = \frac{(14\ \text{V})}{2\pi^2(250)(5.0\ \text{cm})^2(1800\ \text{rpm})}\left[\frac{60\ \text{s}}{1\ \text{min}}\right] = 38\ \text{mT}$$

ASSESS This is a reasonable magnetic field for such an application.

47. **INTERPRET** The aim here is to find the number of turns for the rectangular coil in a generator in order to produce an alternating emf: $\mathcal{E} = \mathcal{E}_{\text{peak}}\sin(2\pi ft)$.

DEVELOP In Example 27.5, the expression for the emf from a generator was derived. The only difference in this case is that the coil is rectangular, not circular:

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = NBlw(2\pi f)\sin(2\pi ft)$$

EVALUATE Solving for N, the number of turns, gives

$$N = \frac{\mathcal{E}_{\text{peak}}}{2\pi Blwf} = \frac{(6.7\text{ kV})}{2\pi(0.17\text{ T})(0.75\text{ m})(1.2\text{ m})(50\text{ Hz})} = 140$$

ASSESS Notice that the alternating emf frequency is simply set by the rotation rate of the coil.

48. **INTERPRET** This problem involves a time-varying flux, with a constant magnetic field that is spatially uniform passing through a loop whose cross-sectional area is changing. This causes a changing magnetic flux through a conducting loop, so Faraday's law will lead to an induced emf.

DEVELOP Faraday's and Ohm's laws give the emf and the current in the loop:

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

$$I = \frac{\mathcal{E}}{R}$$

EVALUATE We calculate the induced current by calculating the magnetic flux caused by the changing dimensions of the loop, and dividing by the resistance

$$I = \frac{\mathcal{E}}{R} = -\frac{B(dA/dt)}{R} = -\frac{B}{R}\frac{d}{dt}\frac{\pi}{4}(d_0 + bt)^2 = -\frac{\pi b B(d_0 + bt)}{2R}$$

ASSESS The induced current will theoretically continue to increase, linearly with time, as long as the loop's dimensions continue to increase.

49. **INTERPRET** This problem is a continuation of Problem 27.46. We are given values for the circuit elements and are asked to quantitatively characterize the circuit's response to the agent that moves the conducting bar.

DEVELOP The situation is like that described in Example 27.4 and the solution to Problem 27.46.

EVALUATE **(a)** The current is

$$I = |\varepsilon|/R = Blv/R = \frac{(0.50\ \text{T})(0.10\ \text{m})(2.0\ \text{m/s})}{4.0\ \Omega} = 25\ \text{mA}$$

(b) The magnetic force on the conducting bar is

$$F_{\text{mag}} = IlB = (25\ \text{mA})(0.10\ \text{m})(0.50\ \text{T}) = 1.3\times10^{-3}\ \text{N}$$

to the left.

(c) The power dissipated in the resistor is

$$P = I^2R = (25\ \text{mA})^2(4.0\ \Omega) = 2.5\ \text{mW}.$$

(d) The agent pulling the bar must exert a force equal in magnitude to F_{mag} and parallel to v. Therefore, it does work at a rate

$$Fv = (1.25\times10^{-3}\ \text{N})(2.0\ \text{m/s}) = 2.5\ \text{mW}$$

Conservation of energy requires the answers to parts **(c)** and **(d)** to be the same.

ASSESS Note that the answer to part **(d)** uses the result of part **(c)** to three significant figures because the result of part **(c)** serves as an intermediate result in part **(d)**.

50. **INTERPRET** The problem involves a changing magnetic field that induces an electric field. Both fields exert a force on a proton.

DEVELOP The induced electric field inside a long, thin, cylindrical solenoid, whose magnetic field is increasing with time as $\vec{B} = bt\hat{k}$ (take $\hat{k}$ into the page, as in Fig. 27.30), can be found by modifying the argument in Example 27.11 for radius $r < R.$ The magnitude of $\vec{E}$ is

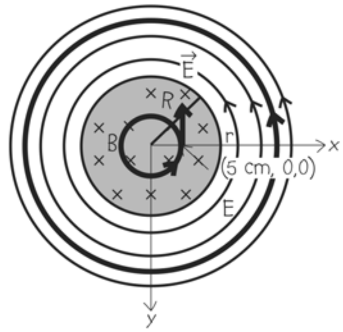
$$E = \left|-\frac{1}{2}r\frac{dB}{dt}\right| = \frac{rb}{2}$$

The induced electric field circulates CCW around the direction of $\vec{B},$ or $\vec{E} = -(bx/2)\hat{j}$ for the given point on the x-axis ($x = 5.2$ cm, $y = z = 0$) inside the solenoid (whose axis, we assume, is the z-axis). At $t = 0.3$ μs, the uniform magnetic field is $\vec{B}, = b(0.3\ \mu\text{s})\hat{k}.$ The electromagnetic force on a proton with the given velocity is $\vec{F} = e(\vec{E} + \vec{v}\times\vec{B}).$

EVALUATE Substituting the values given, we find the net force on the proton to be

$$\vec{F} = e(\vec{E} + \vec{v}\times\vec{B}) = e\left[\frac{1}{2}(3.0\ \text{T/ms})(5.2\ \text{cm})(-\hat{j}) + (4.7\times10^6\ \text{m/s})\,\hat{j}\times(3.0\ \text{T/ms})(0.3\ \mu\text{s})\hat{k}\right]$$

$$= (1.6\times10^{-19}\ \text{C})\left[-78\hat{j} + 4230\hat{i}\right]\text{N/C} = (0.068\hat{i} - 0.0125\hat{j})\text{fN}$$



ASSESS In the region $r < R$, the induced electric field rises linearly with r. Since the proton is moving, the net force on the proton is a vector sum of electric and magnetic forces (i.e., the electromagnetic force).

51. **INTERPRET** This problem involves a changing magnetic field that induces an electric field. Thus, Faraday's law (Equation 27.11) applies. We can use this to find the rate at which the magnetic field changes given the electric field strength. In using Faraday's law, we can make use of the line symmetry of the problem, which tells us that the electric field will be constant along circles concentric with the solenoid axis.

DEVELOP The magnitude of the electric field is $E = F/e$. Faraday's law tells us that

$$\oint \vec{E}\cdot d\vec{r} = -\frac{d\Phi_B}{dt}$$
$$2\pi rE = -\frac{d\Phi_B}{dt} = -A\frac{dB}{dt} = -\pi r^2\frac{dB}{dt}$$

EVALUATE Solving for the rate of change of the magnetic field gives

$$\left|\frac{dB}{dt}\right| = \frac{2E}{r} = \frac{2F}{re} = \frac{2\left(1.3\times10^{-15}\text{ N}\right)}{(0.28\text{ m})\left(1.6\times10^{-19}\text{ C}\right)} = 58\text{ T/ms}$$

ASSESS We cannot tell if the magnetic field is increasing or decreasing because we do not have information about the direction of the force.

52. **INTERPRET** You want to show that the total amount of charge that an induced emf causes to move is independent of how the magnetic field changes.

DEVELOP The flux through a stationary loop perpendicular to a magnetic field is $\Phi_B = BA$, so Faraday's law (Equation 27.2) and Ohm's law (Equation 24.5) relate this to the magnitude of the induced current:

$$I = \frac{|\mathcal{E}|}{R} = \frac{|d\Phi_B/dt|}{R} = \frac{A|dB/dt|}{R} = \frac{\pi a^2}{R}\left|\frac{dB}{dt}\right|$$

Over time, this current will result in the displacement of charge equal to the integral: $Q = \int Idt.$

EVALUATE Substituting the current expression into the charge integral gives

$$Q = \int Idt = \frac{\pi a^2}{R}\int\frac{dB}{dt}dt = \frac{\pi a^2}{R}\int dB = \frac{\pi a^2}{R}\left(B_2 - B_1\right)$$

The absolute value symbols were dropped, since you don't have enough information to calculate the sign of the charge. But note that this expression is independent of how the field is varied, just as you expected.

ASSESS This makes sense, since if you vary the magnetic field more slowly, the induced current will be smaller, but it will flow a longer time, as you still have to change the field by the given amount. Interestingly, if you varied the magnetic field but then returned it to its initial value, $B_2 = B_1$, then the net charge moved around the loop would be zero.

53. **INTERPRET** This problem involves a coil that is moved through a magnetic field, so Faraday's law can be used to relate the changing magnetic flux through the coil to the electric field and thus to the current.

DEVELOP Initially, the flux through the flip coil is $\Phi_B = NBA$, but is reversed to $-NBA$ when the coil is rotated 180°, so $\Delta\Phi_B = -2NBA$. The total charge that flows is $\Delta Q = I_{\text{av}}\Delta t$, where I_{av} is the average induced current and Δt is the time for the rotation. Use Faraday's law and Ohm's law to relate the charge to the magnetic field strength.

EVALUATE From Faraday's and Ohm's laws,

$$I_{\text{av}} = -\frac{\Delta\Phi_B/\Delta t}{R} = \frac{2NBA}{\Delta tR}$$

so

$$\Delta Q = \frac{2NBA}{R}$$
$$B = \frac{R\ \Delta Q}{2NA}$$

ASSESS This result agrees with that given in the problem statement.

54. **INTERPRET** We are to find the inductance of a series *RL* circuit, given the time it takes for the current in the circuit to rise to ¾ its final value.

DEVELOP In a series *RL* circuit, the current as a function of time is given by Equation 27.7:

$$I(t) = I_\infty\left(1 - e^{-Rt/L}\right)$$

where $I_\infty = \mathcal{E}_0/R$ is the final current.

EVALUATE Using Equation 27.7, and the given values for time and resistance, we find the inductance is equal to

$$3/4 = \left(1 - e^{-Rt/L}\right)$$
$$\ln(1/4) = -Rt/L$$
$$L = Rt/\ln(4) = 3.64\,\text{H}$$

ASSESS The time constant for this circuit, $\tau_L = L/R$, is equal to 1.35 s.

55. **INTERPRET** This problem involves an *RL* circuit for which we are to find the time for which the circuit has been completed (i.e., switch closed).

DEVELOP When the switch is closed, the current starts to increase, as shown in Fig. 27.24. The current rise is given by Equation 27.7:

$$I(t) = \frac{\mathcal{E}_0}{R}\left(1 - e^{-Rt/L}\right)$$

which we can solve for the time *t*.

EVALUATE Solving for the time *t* gives

$$t = \frac{L}{R}\ln\left(\frac{1}{1 - RI(t)/\mathcal{E}_0}\right) = \frac{2.1\text{ H}}{3.3\ \Omega}\ln\left(\frac{1}{1-(3.3\ \Omega)(9.5\text{ A})/(45\text{ V})}\right) = 0.76\text{ s}$$

ASSESS The time constant for this circuit is *RL* = 6.9 s, so the current in this circuit will continue to grow for about 21 s (about three time constants).

56. **INTERPRET** In this problem we are asked to find the inductance and the long-time behavior of a series *RL* circuit.

DEVELOP In a series *RL* circuit, the rising current as a function of time is given by Equation 27.7:

$$I(t) = I_\infty\left(1 - e^{-Rt/L}\right)$$

where $I_\infty = \varepsilon_0/R$ is the final current. Solve this for the inductance.

EVALUATE **(a)** The current is 10 mA at 30 μs [i.e., *I*(30 μs) = 10 mA]. Thus, the inductance is

$$\frac{I(t)}{I_\infty} = \frac{\varepsilon_0 I(t)}{R} = 1 - e^{-Rt/L}$$

$$L = -\frac{Rt}{\ln\left[1 - \varepsilon_0 I(t)/R\right]} = -\frac{\left(30\times10^{-6}\text{ s}\right)\left(2.5\times10^3\ \Omega\right)}{\ln\left[1-(50\text{ V})\left(10\times10^{-3}\text{ A}\right)/\left(2.5\times10^3\ \Omega\right)\right]} = 380\text{ H}$$

(b) After a long time $(t \to \infty)$, the exponential term in Equation 27.7 is negligible, and the current is

$$I_\infty = \frac{\varepsilon_0}{R} = \frac{50\text{ V}}{2.5\text{ k}\Omega} = 20\text{ mA}$$

ASSESS After many time constants, there is no induced emf in the inductor and we can simply think of the inductor is a conducting wire connecting the different parts of the circuit. Note that $I_\infty = \varepsilon_0/R$ is what you would get by neglecting the inductance and using Ohm's law.

57. INTERPRET This problem involves the current decay in an *RL* circuit. We use the equation for current decay in an inductor, and energy stored in an inductor, to find the time it takes to lose 90% of the energy stored in an inductor when the circuit becomes resistive.

DEVELOP The energy initially stored in the inductor is $U_0 = \frac{1}{2}LI_0^2$ (Equation 27.9). The decaying current through an *RL* circuit is given by $I = I_0 e^{-Rt/L}$ (Equation 27.8). For this problem, the initial current is $I_0 = 2.4$ kA, the inductance is $L = 0.53$ H, and the resistance is $R = 21$ mΩ. We want to calculate the time required to dissipate 90% of the initial energy.

EVALUATE The time-dependent energy stored in the inductor is

$$U(t) = \frac{1}{2}LI^2 = \frac{1}{2}LI_0^2 e^{-2Rt/L} = U_0 e^{-2Rt/L}$$

so the time we're looking for is

$$e^{-2Rt/L} = 100\% - 90\% = 0.10$$
$$-\frac{2Rt}{L} = \ln(0.10)$$
$$t = -\frac{L}{2R}\ln(0.10) = 20 \text{ s}$$

ASSESS The initial energy stored is 1.5 MJ, so the average power loss is nearly 69 kW! Note also that the initial current was not needed in this calculation.

58. INTERPRET This problem involves finding the rate of change of current in a series *RL* circuit at different instants.

DEVELOP In a series *RL* circuit, the rising current as a function of time is given by Equation 27.7:

$$I(t) = I_\infty\left(1 - e^{-Rt/L}\right)$$

where $I_\infty = \varepsilon_0/R$ is the final current. The rate of change of current is

$$\frac{dI}{dt} = \frac{\varepsilon_0}{L}e^{-Rt/L}$$

EVALUATE **(a)** For $t = 0$,

$$\frac{dI}{dt} = \frac{\varepsilon_0}{L} = \frac{60 \text{ V}}{1.5 \text{ H}} = 40 \text{ A/s}$$

(b) Similarly, for $t = 100$ s, the rate is

$$\frac{dI}{dt} = \frac{\varepsilon_0}{L}e^{-Rt/L} = \left(\frac{dI}{dt}\right)_0 e^{-Rt/L} = (40 \text{ A/s})e^{-(22\ \Omega)(0.1\text{ s})/(1.5\text{ H})} = 9.2 \text{ A/s}$$

ASSESS The rate of change of current decreases exponentially with time. After many time constants, the current is approximately equal to $I_\infty = \varepsilon_0/R$, and the rate of change goes to zero.

59. INTERPRET You want to limit the voltage across elevator motors when the supplied current is suddenly switched off. Because the motors have a high inductance, they will try to keep the same current flowing through them even when the circuit is opened. A resistor placed in parallel with the motor will give a safe path for this current to flow out.

DEVELOP In Conceptual Example 27.1, a description is given of the behavior of a circuit with a power supply connected to an inductor and resistor in parallel. You can imagine that the inductor in this example is an elevator motor and the parallel resistor is the safety element you want to install. When the elevator is in operation, a current of $I_0 = 20$ A flows through the motor. If a switch is suddenly opened, the motor's inductance will respond by driving the same current through the mini-circuit defined by the inductor and resistor (see Fig. 27.26d). The voltage across the resistor, $V = I_0 R$, will be equal to the voltage across the motor (inductor).

EVALUATE **(a)** To limit the voltage across the motor to less than 100 V, you'll need resistors of

$$R = \frac{V}{I_0} = \frac{100\ \text{V}}{20\ \text{A}} = 5\ \Omega$$

(b) The current does not stay at the initial value. It decays exponentially according to Equation 27.8: $I = I_0 e^{-Rt/L}$. To find how much energy the resistor dissipates, you can integrate the power, $P = I^2 R$, over the time it will take for all the current to theoretically disappear (i.e., $t = \infty$).

$$\Delta U = \int_0^\infty P dt = \int_0^\infty I_0^2 R e^{-2Rt/L} dt = \tfrac{1}{2} L I_0^2 = \tfrac{1}{2}(2.5\ \text{H})(20\ \text{A})^2 = 500\ \text{J}$$

ASSESS Just as you might expect, the energy dissipated by the resistor is just the energy that was initially stored in the inductor (Equation 27.9).

60. **INTERPRET** This problem is about the buildup and decay of current in a series *RL* circuit. For the first 10 s, the current will grow toward its steady-state value. After 10 s, the switch is opened, and the current decays exponentially.

DEVELOP In a series *RL* circuit, the rising current as a function of time is given by Equation 27.7:

$$I(t) = I_\infty \left(1 - e^{-Rt/L}\right)$$

where $I_\infty = \varepsilon_0 / R$ is the final current. When the switch is thrown back to position *B*, the battery is removed from the circuit, and the current decays according to Equation 27.8:

$$I(t) = I_0 e^{-Rt/L}$$

EVALUATE **(a)** When the current is building up from zero, Equation 27.7 gives

$$I(t = 7.5\ \text{s}) = \frac{\varepsilon_0}{R}\left(1 - e^{-Rt/L}\right) = \frac{11\ \text{V}}{3.0\ \Omega}\left[1 - e^{-(3.0\ \Omega)(7.5\ \text{s})(18\ \text{H})}\right] = 2.6\ \text{A}$$

(b) At $t = 15$ s, the current has built up to

$$I(t = 15\ \text{s}) = \frac{\varepsilon_0}{R}\left(1 - e^{-Rt/L}\right) = \frac{11\ \text{V}}{3.0\ \Omega}\left[1 - e^{-(3.0\ \Omega)(15\ \text{s})(18\ \text{H})}\right] = 3.4\ \text{A}$$

This current decays when the switch is thrown back to *B*. Equation 27.8 (where *t* is the time since 15 s) gives

$$I(t = 22.5\ \text{s}) = I(t = 15\ \text{s})\left(1 - e^{-Rt/L}\right) = (3.37\ \text{A})\left[1 - e^{-(3.0\ \Omega)(22.5\ \text{s} - 15\ \text{s})(18\ \text{H})}\right] = 2.4\ \text{A}$$

where we have used the result from above to three significant figures because it is an intermediate result.

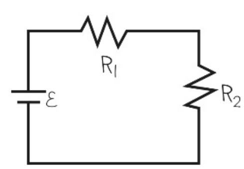
ASSESS The time constant of the circuit is $\tau_L = L / R = (18\ \text{H})/(3.0\ \Omega) = 6\ \text{s}$, so $t = 15$ s corresponds to only $2.5\tau_L$. At this instant, the current is only 92% of its steady-state value, $I_\infty = \varepsilon_0 / R = (11\ \text{V})/(3.0\ \Omega) = 3.7\ \text{A}$. One needs to wait at least three time constants for the current to approach I_∞.

61. **INTERPRET** We're asked to find the current in an *RL* circuit at different time points.

DEVELOP We are considering the short-term and long-term behavior of a circuit with an inductor, as was done in Conceptual Example 27.1.

EVALUATE **(a)** Just after the switch is closed, the inductor current is zero. We can consider this branch of the circuit as being opened $(I_3 = 0)$. Current will instead flow through R_2, which is in series with R_1 (see the figure below). The current I_2 will be

$$I_2 = I_1 = \frac{\mathcal{E}_0}{R_1 + R_2} = \frac{12\ \text{V}}{4.0\ \Omega + 8.0\ \Omega} = 1.0\ \text{A}$$



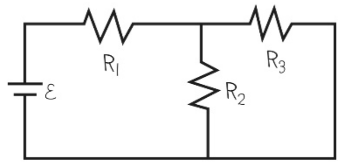
(b) After the currents have been flowing a long time, they reach steady values $(dI/dt = 0)$. This means the voltage across the inductor is zero, and we can treat it like a short-circuit. Now, R_2 and R_3 are in parallel with each other and in series with R_1 (see the figure below). This implies that the current I_1 is

$$I_1 = \frac{\mathcal{E}_0}{R_1 + R_2R_3/(R_2+R_3)} = \frac{12\text{ A}}{\left[4.0 + 8.0\times 2.0/(10)\right]\Omega} = 2.14\text{ A}$$

By Kirchhoff's rules, $I_1 = I_2 + I_3$, and $I_2R_2 = I_3R_3$. Solving for the current I_2 gives

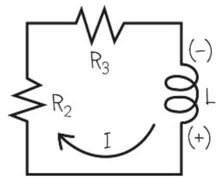
$$I_2 = \frac{R_3}{R_2+R_3}I_1 = \frac{2.0}{8.0+2.0}(2.14\text{ A}) = 0.43\text{ A}$$

The current I_3 makes up for the difference: $I_3 = I_1 - I_2 = 1.71\text{ A}$.



(c) When the switch is reopened, no current flows through the battery's branch, $I_1 = 0$, so we can remove it from the circuit (see the figure below). As explained in Conceptual Example 27.1, the induced emf acts to keep the current flowing through the inductor as it was before the switch was opened, that is, $I_3 = 1.71\text{ A}$ from part **(b)**. The current in R_2 will be the same as in the inductor, but it will be flowing in the opposite direction as before:

$$I_2 = -I_3 = -1.7\text{ A}$$



ASSESS Notice that the value of the inductance in L was not needed, since we are only considering the short- and long-term behavior of the circuit. If we wanted to calculate the currents at some intermediate time, then we would need the inductance to plug into Equation 27.7 or 27.8.

62. **INTERPRET** This problem is about the magnetic energy stored in a series *RL* circuit as a function of time, which we are to deduce from the given dynamics of the circuit.

DEVELOP The magnetic energy stored in an inductor is given by Equation 27.9, $U(t) = LI(t)^2/2$, where the rising current is given by Equation 27.7; $I(t) = I_\infty\left(1 - e^{-Rt/L}\right)$. Combining the two equations, the stored magnetic energy as a function of time is

$$U(t) = \frac{1}{2}LI_\infty^2\left(1 - e^{-Rt/L}\right)^2 = U_\infty\left(1 - e^{-Rt/L}\right)^2$$

where $U_\infty = LI_\infty^2/2$ is the steady-state value of the magnetic energy. When the stored energy is half its steady-state value, $U(t_U)/U_\infty = 1/2$, we have

$$\frac{1}{2} = \frac{U(t_U)}{U_\infty} = \left(1 - e^{-Rt_U/L}\right)^2$$

$$\frac{1}{\sqrt{2}} = 1 - e^{-Rt_U/L}$$

EVALUATE Solving the above equation for t_U yields

$$t_U = \frac{L}{R}\ln\left(\frac{\sqrt{2}}{\sqrt{2}-1}\right) = 1.28\frac{L}{R}$$

On the other hand, the current is half its steady-state value when $t_I = (L/R)\ln 2 = 1.0\text{ ms}$. Dividing these results, we find

$$t_U = \frac{1.28}{\ln 2} t_1 = \frac{1.28}{\ln 2}(1.0 \text{ ms}) = 1.8 \text{ ms}$$

ASSESS Since $t_U > t_1$, the magnetic energy reaches half its steady-state value after the current has already surpassed half its steady-state value. In fact, the current at $t = t_U$ is

$$I(t_U) = I_\infty\left(1 - e^{-Rt_U/L}\right) = \frac{I_\infty}{\sqrt{2}} = 0.707 I_\infty$$

63. **INTERPRET** This problem involves an *RL* circuit with a given inductance. The energy in the inductor drops by 75% in the given time, and we are to find the resistance.

DEVELOP From Equation 27.9, $U = LI^2/2,$ we find

$$U(t = 3.3\ s) = \frac{U_0}{4} = \frac{LI(t = 3.3\text{ s})^2}{2} = \frac{1}{4}\overbrace{\left(\frac{LI_0^2}{2}\right)}^{U_0} = \frac{L}{2}\left(\frac{I_0}{2}\right)^2$$

from which we deduce that $I(t = 3.3\text{ s}) = I_0/2.$ Insert this into Equation 27.8, $I(t) = I_0 e^{-Rt/L},$ to find the resistance *R*.

EVALUATE Solving Equation 27.8 for the resistance *R* and inserting the given quantities gives

$$1/2 = e^{-Rt/L}$$
$$R = (L/t)\ln(2.0) = (1.5\text{ H})/(3.3\text{ s})\ln(2.0) = 320\text{ m}\Omega$$

to two significant figures.

ASSESS The current and the resistance do not have the same time constant. Because the current is squared in the expression for energy, the time constant for the energy in the inductor is twice that for the current. Thus, the energy grows and decays at twice the rate compared to the current.

64. **INTERPRET** You need to specify the maximum resistance needed in the wires of an MRI scanner in the eventuality that superconductivity is lost suddenly.

DEVELOP If so-called "quenching" occurs, the solenoid wire will suddenly acquire a resistance. But since the solenoid is an inductor, the current will not immediately change. It will initially be the same as it was before quenching, so the power in the resistor immediately after the loss of superconductivity will be $P = I_0^2 R.$ Since the inductor and the resistance are effectively in series, the current will drop according to Equation 27.8, $I = I_0 e^{-Rt/L}.$ From this, we can solve for the time it will take for the power in the resistor to drop to 50 kW.

EVALUATE **(a)** Right after quenching, the maximum resistance needed to keep the power dissipated below 100 kW is

$$R_{\max} = \frac{P}{I_0^2} = \frac{100\text{ kW}}{(1.6\text{ kA})^2} = 39\text{ m}\Omega$$

(b) If we assume the maximum resistance from part **(a)**, and we assume that there are no other elements in the circuit except the inductance and the resistance, then the power will drop to half its initial value in a time of

$$t = -\frac{1}{2}\ln(P/P_0)\frac{L}{R} = \left(\frac{\ln 2}{2}\right)\frac{4.0\text{ H}}{39\text{ m}\Omega} = 36\text{ s}$$

ASSESS Notice that you could put in a smaller resistance, but then it will take longer for the power to drop by half. This is because the resistance must dissipate all the energy that was initially stored in the inductance: $U = \frac{1}{2}I_0^2 L.$ Either you use a relatively large resistance that dissipates the energy quickly over a short time, or you use a relatively small resistance that dissipates the energy slowly over a long time.

65. **INTERPRET** This problem involves finding the energy density in the magnetic field of a neutron star and comparing that density with other sources of energy.

DEVELOP Apply Equation 27.10,

$$u_B = \frac{1}{2\mu_0}B^2$$

to find the energy density in the magnetic field of the neutron star.

EVALUATE The energy density in a magnetic field of the neutron star is

$$u_B = \frac{1}{2\mu_0}B^2 = \frac{\left(1.0\times10^8\ \text{T}\right)^2}{2\left(4\pi\times10^{-7}\ \text{H/m}\right)} = 3.4\times10^{21}\ \text{J/m}^3$$

This is about **(a)** 1.1×10^{11} times the energy density content of gasoline $(44\ \text{MJ/kg}\times800\ \text{kg/m}^3 = 3.52\times10^{10}\ \text{J/m}^3)$, and **(b)** 2600 times that of pure U^{235} $(8\times10^{13}\ \text{J/kg}\times19\times10^3\ \text{kg/m}^3 = 1.52\times10^{18}\ \text{J/m}^3)$.

ASSESS The energy density in the magnetic field of the neutron star is very high compared to that of common energy sources found on Earth.

66. **INTERPRET** In this problem, we are asked to compare the magnetic energy density between a current-carrying loop and a solenoid of the same radius.

DEVELOP The magnetic field at the center of the loop is $B_{\text{loop}} = \mu_0 I/(2R)$ (see Equation 26.9 with $x = 0$ and $a = R$). On the other hand, the magnetic field inside a solenoid is $B_{\text{solenoid}} = \mu_0 nI$ (Equation 26.21). The energy density can be found by using Equation 27.10: $u_B = B^2/(2\mu_0)$.

EVALUATE Using the equations above, we find the energy density at the center of the loop to be

$$u_B^{(\text{loop})} = \frac{B_{\text{loop}}^2}{2\mu_0} = \frac{1}{2\mu_0}\left(\frac{\mu_0 I}{2R}\right)^2 = \frac{\mu_0 I^2}{8R^2}$$

In a long thin solenoid of the same radius,

$$u_B^{(\text{solenoid})} = \frac{B_{\text{solenoid}}^2}{2\mu_0} = \frac{1}{2\mu_0}(\mu_0 nI)^2 = \frac{\mu_0 n^2 I^2}{2}$$

so the ratio is

$$\frac{u_B^{(\text{loop})}}{u_B^{(\text{solenoid})}} = \left(\frac{\mu_0 I^2}{8R^2}\right)\left(\frac{2}{\mu_0 n^2 I^2}\right) = \frac{1}{4n^2R^2}$$

ASSESS As expected, the ratio is dimensionless (recall that n represents the number of turns per unit length of a solenoid). The result shows that the energy density inside a solenoid is greater than that at the center of a loop by a factor of $4n^2R^2$.

67. **INTERPRET** We are to find the energy per unit length within a wire that carries a given current distributed uniformly throughout the wire.

DEVELOP Equation 26.19 $B = (\mu_0 Ir)/(2\pi R^2)$ gives the magnetic field strength inside a wire at radius r. The energy density power unit length is (Equation 27.10) $u_B = B^2/(2\mu_0)$. Combine these equations and integrate to find the energy density per unit length.

EVALUATE Using $dV = 2\pi rL\,dr$, the energy density per unit length is

$$\frac{U}{L} = \int\frac{U_B dV}{L} = \int\frac{B^2}{2\mu_0 L}dV = \int_0^R \frac{\mu_0 I^2 r^2}{8\pi^2 R^4}2\pi r dr = \frac{\mu_0 I^2}{4\pi}\left|\frac{r^4}{4R^4}\right|_0^R = \frac{\mu_0 I^2}{16\pi}$$

ASSESS The energy density, like the energy, is proportional to the current squared.

68. **INTERPRET** In this problem we want to verify that the time integral of the power dissipated through the resistor in a series *RL* circuit is equal to the energy initially stored in the inductor.

DEVELOP Equation 27.8, $I(t) = I_0 e^{-Rt/L}$, gives the current decaying through a resistor connected to an inductor carrying an initial current I_0. The instantaneous power dissipated in the resistor is $P_R = I^2R$.

EVALUATE **(a)** Using the two equations above, the power dissipated in the resistor as a function of time is

$$P_R = I^2R = I_0^2 R\, e^{-2Rt/L}$$

(b) In a time interval dt, the energy dissipated is $dU = P_R dt$, so the total energy dissipated is

$$U = \int_0^\infty I_0^2 R e^{-2Rt/L}dt = I_0^2 R\left.\frac{e^{-2Rt/L}}{(-2R/L)}\right|_0^\infty = \frac{I_0^2 RL}{2R} = \frac{1}{2}LI_0^2$$

ASSESS This is precisely the energy initially stored in the inductor. The result must hold true by energy conservation.

69. **INTERPRET** We are to compare the ratio of the electric to magnetic fields given that they have the same energy density.

DEVELOP The energy density due to the electric field is (Equation 23.7) is $u_E = \frac{1}{2}\varepsilon_0 E^2$ and that due to the magnetic field is (Equation 27.10) $u_B = B^2/2\mu_0$.

EVALUATE When these two energy densities are equal, their ratio is unity, which gives

$$E/B = 1/\sqrt{\mu_0\varepsilon_0}.$$

Numerically, $\mu_0 = 1.26\times10^{-6}\,\text{N/A}^2$ and $(1/4\pi\varepsilon_0) \approx 9\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2$, so

$$1/\sqrt{\mu_0\varepsilon_0} \approx \sqrt{\left(9\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(10^{-7}\ \text{N/A}^2\right)} = 3\times10^8\ \text{m/s},$$

which is, in fact, the speed of light (see Section 29.5).

ASSESS The speed of light is a fundamental constant of nature that can be derived from measurements of the constants of electricity and magnetism. Because these latter are the same in all inertial frames of reference, the speed of light must also be the same in all inertial frames of reference.

70. **INTERPRET** The magnetic flux in the loop is changing due to the increase in the area exposed to the field. The induced current will experience a force from the magnetic field.

DEVELOP The magnetic flux through the loop is proportional to the vertical distance y that it falls into the field region, $\Phi_B = BA = Bwy$. The rate of change of the flux is

$$\frac{d\Phi_B}{dt} = \frac{d}{dt}(Bwy) = Bwv$$

where $v = dy/dt$.

EVALUATE **(a)** The changing magnetic flux induces an emf in the loop: $\mathcal{E} = -d\Phi_B/dt$. By Lenz's law, the resulting current, $I = \mathcal{E}/R$, will move in a counterclockwise direction, so as to generate a magnetic field out of the page and thus reduce the increase in magnetic flux. The external magnetic field will exert a force on the bottom of the loop: $F = I\vec{w}\times\vec{B}$, which will point upward in the opposite direction of gravity. (There will be forces also on the left and right sides of the loop but they will be equal and opposite to each other.) So assuming the loop is long enough, it will accelerate downward until its speed is high enough that the magnetic force cancels the gravitational force. With net force of zero, the loop will have reached terminal speed.

(b) From the arguments above, the terminal speed occurs when the magnitude of the magnetic force (IwB) is equal to that of the gravitational force (mg). The induced current in this case is $I = Bwv/R$, so the terminal speed equals

$$v = \frac{IR}{Bw} = \frac{mgR}{(Bw)^2}$$

ASSESS Once the loop falls far enough that its top end enters into the magnetic field, there will be no more change in the magnetic flux. This will shut off the induced emf and the induced current. With no more magnetic force, the loop will start to accelerate again due to gravity.

71. **INTERPRET** A conductive disk is in a changing magnetic field, and we are asked to find the current density in the disk and the rate of power dissipation in the disk. We will use Faraday's law and the resistance of the individual loops that make up the disk.

DEVELOP We will treat the disk as a set of infinitesimal loops with radius r, thickness h, resistivity ρ, and width dr. The resistance of each such loop, using $R = \rho\frac{L}{A}$, is $R = \rho\frac{2\pi r}{hdr}$. The magnetic flux through each loop is the magnetic field dotted with the area normal, or

$$\Phi_B = B\pi r^2 = bt\pi r^2,$$

The induced emf around the loop is (Faraday's law, Equation 27.2)

$$\varepsilon = -\frac{d\Phi_B}{dt} = -b\pi r^2.$$

The current density is given by $J = \frac{I}{A} = \frac{\varepsilon/R}{hdr}$. To find the total power, we will integrate the power in each infinitesimal loop:

$$dP = \varepsilon dI \quad \Rightarrow \quad P = \int_0^a \varepsilon dI.$$

EVALUATE **(a)** The current density is $J = \frac{\varepsilon}{Rhdr} = \frac{-\pi br^2}{\left(\rho\frac{2\pi r}{hdr}\right)hdr} = -\frac{br}{2\rho}$

(b) The power dissipation is

$$P = -\int_0^a \pi br^2 dI; \quad dI = \frac{\varepsilon}{R} = \frac{-\pi br^2}{\rho\frac{2\pi r}{hdr}} - \frac{brhdr}{2\rho}$$

$$P = \int_0^a \pi br^2 \frac{brh}{2\rho} dr = \frac{\pi b^2 h}{2\rho}\int_0^a r^3 dr$$

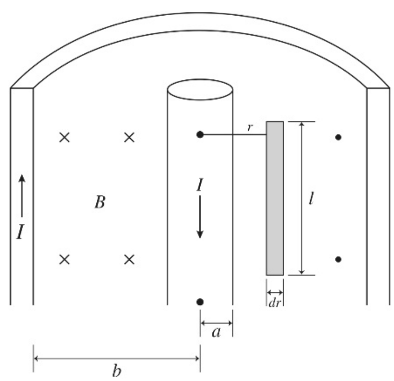
$$= \frac{\pi b^2 h a^4}{8\rho}$$

ASSESS There are several interesting aspects of this problem. First, the current density is linear with *r*, and is independent of the thickness *h*. This makes sense: a thicker disk would have more current, but the current density would be the same. Second, the power actually depends on the fourth power of disk radius *a*, so increasing the size of this disk increases the power dissipation dramatically. This phenomenon is used in metal detectors, and explains why large metal objects are easier for metal detectors to find than small ones.

72. **INTERPRET** We calculate the self-inductance per length of a coaxial cable, using the flux through the area between the two conductors.

DEVELOP The self-inductance is defined in Equation 27.3 as $L = \Phi_B / I$. To find the magnetic flux, we recall Example 26.7, where it was shown that the magnetic field lines around a single wire form concentric circles with magnitude $B = \mu_0 I / 2\pi r$. With a coaxial cable, there is an outer conductor that carries the opposite current, so the encircled current is zero outside the cable, and by Ampère's law, the field is zero as well. Therefore, we only need to concern ourselves with the magnetic flux in between the two conductors.

EVALUATE Since the magnetic field lines wrap around the inner conductor, we imagine the flux flowing through a strip of length *l* and width *dr*, at a distance *r* from the center of the cable. See the figure below.



Since by construction the field is normal is normal to the strip's area, the flux through it is

$$d\Phi_B = BdA = \frac{\mu_0 Il}{2\pi r}dr$$

We now integrate this over the region where the field is nonzero, that is, from the inner conductor's radius, *a*, to the outer conductor's radius, *b*.

$$\Phi_B = \int d\Phi_B = \int_a^b \frac{\mu_0 Il}{2\pi r} dr = \frac{\mu_0 Il}{2\pi} \ln\left(\frac{b}{a}\right)$$

The self-inductance per unit length is therefore

$$\frac{L}{l} = \frac{\mu_0}{2\pi} \ln\left(\frac{b}{a}\right)$$

ASSESS We check this result by calculating the energy stored in the magnetic field of the coaxial cable. From Equation 27.10, the magnetic energy density is $u_B = B^2/2\mu_0$. Integrating this over the volume of a section of the cable with length *l* gives

$$U = \int u_B dV = \int_a^b \int_0^{2\pi} \int_0^l \frac{1}{2\mu_0}\left(\frac{\mu_0 I}{2\pi r}\right)^2 r dr d\theta dz = \frac{\mu_0 I^2 l}{4\pi} \ln\left(\frac{b}{a}\right)$$

Comparing this to Equation 27.9, $U = \frac{1}{2}LI^2$, we get the same answer for the self-inductance per unit length, L/l.

73. **INTERPRET** We are given the data of currents as a function of time in an *RL* circuit to find the inductance in a series *RL* circuit.

DEVELOP As the switch is thrown from A to B at $t = 0$, current begins to decrease according to Equation 27.8:

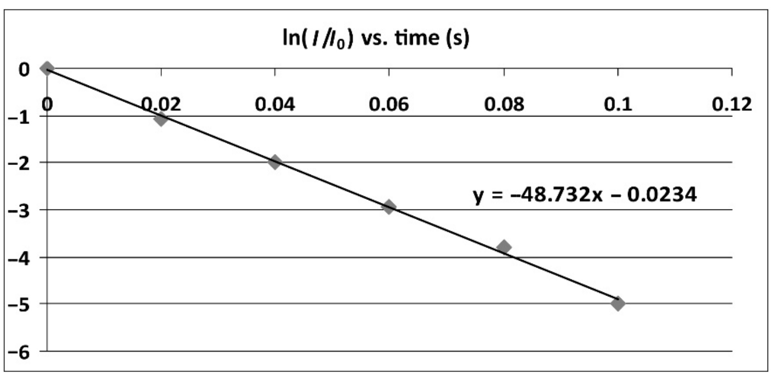
$$I(t) = I_0 e^{-Rt/L}$$

where I_0 is the initial current. Taking logarithm on both sides of the equation and rearranging, we obtain

$$\ln\left(\frac{I(t)}{I_0}\right) = -Rt/L = -(R/L)t$$

Thus, plotting $\ln(I/I_0)$ versus time will give a straight line with slope $-R/L$.

EVALUATE The plot is shown below.



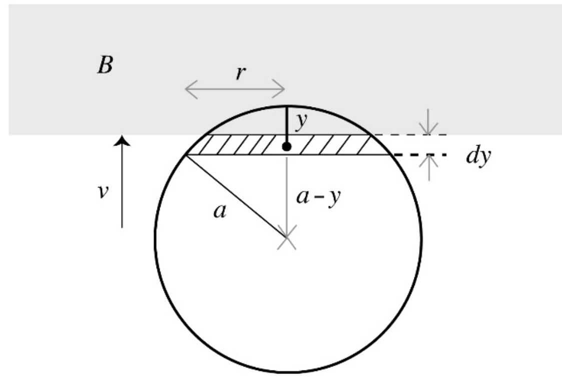
The slope of the best-fit line is –48.732/s, from which we deduce the inductance to be

$$L = \frac{R}{48.732} = \frac{180\ \Omega}{48.732\ \text{s}^{-1}} = 3.69\ \text{H}$$

ASSESS The time constant is inversely proportional to the resistance *R*. The physical meaning of the time constant $\tau_L = L/R$ is that significant changes in current cannot occur on time scales much shorter than τ_L.

74. **INTERPRET** We want to find the induced current in a circular loop as it enters a uniform magnetic field.

DEVELOP As the loop enters the region with uniform magnetic field *B*, the flux through the loop increases. The rate of change of flux is $\frac{d\Phi_B}{dt} = B\frac{dA}{dt}$.



As shown in the figure above, the change in area is given by $dA = 2r dy = 2\sqrt{2ay - y^2}\, dy$, where we have used $r^2 + (a-y)^2 = a^2$. Thus, $\frac{dA}{dt} = 2\sqrt{2ay - y^2}\frac{dy}{dt} = 2v\sqrt{2ay - y^2}$, where $v = dy/dt$. The induced current is

$$I = \frac{\mathcal{E}}{R} = \frac{1}{R}\frac{d\Phi_B}{dt} = \frac{B}{R}\frac{dA}{dt}$$

EVALUATE With $y = vt$, we find the induced current to be

$$I = \frac{B}{R}\frac{dA}{dt} = \frac{2Bv}{R}\sqrt{2ay - y^2} = \frac{2Bv}{R}\sqrt{vt(2a - vt)}$$

ASSESS The loop becomes fully immersed in the magnetic field at $t = 2a/v$. At this time, the flux through the loop becomes constant, and the induced current goes to zero.

75. **INTERPRET** In this problem we revisit the circuit described in Problem 27.31 in more detail and seek to express the time-dependent current induced in the circuit and velocity of the rod.

DEVELOP In problem 27.31 we determined the time dependent voltage in the circuit is given by the initial constant emf of the battery $\mathcal{E}$, and the induced time-dependent emf from the moving rod $\mathcal{E}(t) = Bl\,v(t)$, meaning

$$V(t) = \mathcal{E} - Bl\,v(t).$$

We can thus use this voltage to define the time-dependent current $I(t)$, and use it to determine the magnetic force felt by the bar. Finally, we will show that the equation of motion for the bar satisfies the given expression for the time-dependent velocity, and results in the constant speed found in Problem 27.31 as time goes to infinity.

EVALUATE **(a)** As the rod begins to move, it induces time-dependent voltage in the circuit given by

$$I(t) = V(t)/R = \left(\mathcal{E} - Bl\,v(t)\right)/R$$

(b) Once the bar begins to move, it will also feel a magnetic force perpendicular to both the flow of current and the magnetic field. This results in a force along the direction of motion equal to

$$F(t) = I(t)lB = lB\left(\mathcal{E} - Bl\,v(t)\right)/R$$

(c) We can use this force to obtain an equation of motion for the moving bar, whose mass *m* will dictate how it accelerates as

$$F(t) = lB\left(\mathcal{E} - Bl\,v(t)\right)/R = m\frac{dv(t)}{dt}$$

(d) We can then show the given expression for the time-dependent velocity satisfies this equation by directly substituting it and simplifying the expression.

$$\frac{lB}{R}\left(\mathcal{E} - Bl\left(\frac{\mathcal{E}}{Bl}\left(1 - e^{-B^2l^2t/mR}\right)\right)\right) = m\frac{d}{dt}\left(\frac{\mathcal{E}}{Bl}\left(1 - e^{-B^2l^2t/mR}\right)\right)$$

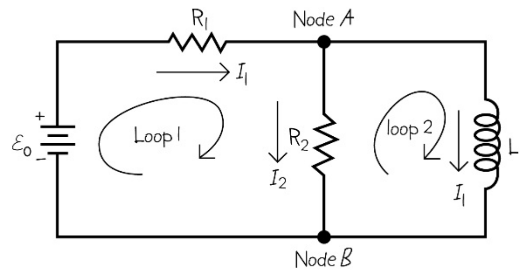
$$\frac{Bl\mathcal{E}}{R}e^{-B^2l^2t/mR} = \frac{Bl\mathcal{E}}{R}e^{-B^2l^2t/mR}$$

(e) As $t \to \infty$ the exponential approaches 0, and the time-dependent velocity simplifies to the constant speed $\mathcal{E}/Bl$ we found in Problem 27.31.

ASSESS As we saw in Problem 27.31, the steady state for this circuit accelerates the rod to a speed which generates a current equal in magnitude but opposite in direction to the initial current supplied by the battery.

76. **INTERPRET** We are to use Kirchhoff's laws to find the current in a circuit element. The circuit includes an inductor, so the equations we obtain using Kirchhoff's laws will be differential equations.

DEVELOP We start by drawing our loops and nodes as shown in the figure below. Node *A* and node *B* give the same information, so we will use only loop 1, loop 2, and node *A*. From node *A* we get $I_1 - I_2 - I_3 = 0$. From loop 1, we obtain $\varepsilon - I_1R_1 - I_2R_2 = 0$, and from loop 2 we have $I_2R_2 - L\frac{dI_3}{dt} = 0$. We want to find the current I_2 as a function of time.



EVALUATE From node *A*: $I_1 = I_2 + I_3$.

Substitute this result into loop 1:

$$\varepsilon - (I_2 + I_3)R_1 - I_2R_2 = 0 \quad \Rightarrow \quad I_2 = \frac{\varepsilon - I_3R_1}{R_1 + R_2}$$

Substitute this result into loop 2:

$$\left(\frac{\varepsilon - I_3R_1}{R_1 + R_2}\right)R_2 - L\frac{dI_3}{dt} = 0$$

$$\frac{dI_3}{dt} = \left(\frac{\varepsilon - I_3R_1}{R_1 + R_2}\right)\frac{R_2}{L} = \frac{\varepsilon R_2}{L(R_1 + R_2)} - \left(\frac{R_1R_2}{L(R_1 + R_2)}\right)I_3$$

We guess at a solution of the form $I_3(t) = A + Be^{Ct}$, with initial condition $I_3(0) = 0$ since the inductor acts as an open circuit at first.

$$\frac{dI_3}{dt} = BCe^{Ct} = \frac{\varepsilon R_2}{L(R_1 + R_2)} - \left(\frac{R_1R_2}{L(R_1 + R_2)}\right)(A + Be^{Ct}) = -\frac{R_1R_2}{L(R_1 + R_2)}$$

$$A\left(\frac{R_1R_2}{L(R_1 + R_2)}\right) = \frac{\varepsilon R_2}{L(R_1 + R_2)} = \frac{\varepsilon}{R_1}$$

$$I_3(0) = 0 \quad \Rightarrow \quad \left(\frac{\varepsilon}{R_1} + B\right) = 0$$

$$B = -\frac{\varepsilon}{R_1}$$

So

$$I_3(t) = \frac{\varepsilon}{R_1}\left(1 - e^{-\left(\frac{R_1R_2}{L(R_1+R_2)}\right)t}\right)$$

ASSESS The current gradually increases, with a time constant of $R_1R_2/\left[L(R_1 + R_2)\right]$.

77. **INTERPRET** We calculate the self-inductance per length of a coaxial cable by first calculating the magnetic energy.

DEVELOP From Equation 27.10, the magnetic energy density is $u_B = B^2/2\mu_0$. Since $B = \mu_0 I/2\pi r$, and $dV = 2\pi r l dr$, we have

$$dU = u_B dV = \frac{B^2}{2\mu_0} 2\pi r l dr = \frac{1}{2\mu_0}\left(\frac{\mu_0 I}{2\pi r}\right)^2 2\pi r l dr = \frac{\mu_0 I^2 l}{4\pi r} dr$$

The magnetic energy is related to the inductance by $U = \frac{1}{2} L I^2$.

EVALUATE **(a)** Integrating over *r*, we obtain

$$U = \int_a^b \frac{\mu_0 I^2 l}{4\pi r} dr = \frac{\mu_0 I^2 l}{4\pi} \ln\left(\frac{b}{a}\right)$$

The magnetic energy per unit length is

$$\frac{U}{l} = \frac{\mu_0 I^2}{4\pi} \ln\left(\frac{b}{a}\right)$$

(b) From $U = \frac{1}{2} L I^2$, we find the inductance to be

$$L = \frac{2U}{I^2} = \frac{\mu_0 l}{2\pi} \ln\left(\frac{b}{a}\right)$$

ASSESS The result agrees with that calculated in Problem 27.72 using $\Phi_B = LI$.

78. **INTERPRET** We determine the voltage induced across the wingtips of a plane flying through a magnetic field using Faraday's law. The wings of the plane sweep out area at a certain rate, and the resulting change in flux generates a voltage. The wings are an open circuit, so the induced emf will result in a buildup of charge at the wingtips (see Fig. 27.17).

DEVELOP The induced emf comes from the change in the magnetic flux, which will be proportional to the rate at which area is swept out by the wings of the plane:

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}[BA] = -Blv$$

Here, l is the plane's wingspan and v is the plane's velocity.

EVALUATE Neglecting the sign, the induced voltage is

$$\mathcal{E} = Blv = (30\ \mu T)(60\ \text{m})(850\ \text{km/h}) = 0.4\ \text{V}$$

This is too low to charge your phone.

ASSESS In reality, this voltage cannot be used by any device on the plane. If you hooked up wires to the wingtips, those wires would have the same induced voltage, and no current would flow.

79. **INTERPRET** We're asked to derive the formula for the volume flow rate in a blood vessel being probed by an electromagnetic flowmeter.

DEVELOP The magnetic field from the flowmeter will deflect some of the moving charges in the blood, as described previously for the Hall effect. This deflection will result in an electric field across the blood vessel. Charges will continue to be deflected until the force from the electric field balances out the force from the magnetic field: $qE = qvB$, where we assume that the magnetic field is perpendicular to the blood flow. The "drift" velocity, $v = E/B$, multiplied by the cross-sectional area of the vessel, πr^2, gives the volume flow rate that we are looking for.

EVALUATE The flowmeter measures the voltage. If we assume the electric field is uniform, then the relationship between the voltage and the field is just $V = Ed$. Combining this with the electromagnetic force equation above, we get a volume flow rate of

$$\mathcal{F} = Av = \left[\pi\left(\tfrac{1}{2}d\right)^2\right]\left[\frac{V/d}{B}\right] = \frac{\pi d^2 V}{4Bd}$$

ASSESS The formula indicates that a higher voltage reading is indicative of a greater flow rate. This is because a greater flow rate results in a larger magnetic force, which requires a larger electric field to achieve equilibrium.

80. **INTERPRET** We consider how electric power might be "stolen" using electromagnetic induction.

DEVELOP The current in the power line will produce magnetic field lines that wrap around the wire in concentric circles (see Example 26.4). The magnetic flux through the rectangular wire loop, $\Phi_B = \vec{B} \cdot \vec{A}$, is changing due to the fact that the current in the power line is alternating: $d\Phi_B / dt \propto dI / dt$. This will induce an emf in the rectangular loop.

EVALUATE In Fig. 27.42, the magnetic field from the power line will be perpendicular to the vertically oriented loop, which maximizes the magnetic flux through the loop. However, if the loop were horizontally oriented, there would essentially be no more flux through it. This would result in the induced emf essentially dropping to zero. The answer is **(d)**.

ASSESS Tilting the loop horizontally would make sense if it were at the same height as the wire, in which case the magnetic field would be pointing in the vertical direction.

81. **INTERPRET** We consider how electric power might be "stolen" using electromagnetic induction.

DEVELOP The magnetic field is not uniform around the wire, so doubling the area won't necessarily double the magnetic flux. To be precise, the magnetic field is inversely proportional to the distance from the wire: $B = \mu_0 I/2\pi r.$ Let's imagine the loop has length *l* and width *a*, and that the top of the wire is a distance *y* from the power line. Then, the flux through the loop is:

$$\Phi_B = \int B dA = \frac{\mu_0 Il}{2\pi}\int_y^{y+a}\frac{dr}{r} = \frac{\mu_0 Il}{2\pi}\ln\left(\frac{y+a}{y}\right)$$

EVALUATE If the loop doubles in size by extending a distance *a* toward the wire, the flux will increase by

$$\frac{\Phi_B'}{\Phi_B} = \frac{\ln\left(\frac{y+a}{y}\right)+\ln\left(\frac{y}{y-a}\right)}{\ln\left(\frac{y+a}{y}\right)} = 1 - \frac{\ln(1-a/y)}{\ln(1+a/y)}$$

If we assume $a \ll y$, then we can use the approximation $\ln(1+x) \approx x$, in which case $\Phi_B'/\Phi_B \approx 2$. However, if *a* is nearly as big as *y*, then $\Phi_B'/\Phi_B \to \infty$. Therefore, the flux increases by some factor greater than 2. Since the induced emf is directly related to the flux: $\mathcal{E} = -d\Phi_B/dt$, it will increase by the same factor.

The answer is **(c)**.

ASSESS The magnetic field is greater near the wire $(B \propto 1/r)$, so the closer a farmer can place the loop to the wire, the more power he will be able to siphon off.

82. **INTERPRET** We consider how electric power might be "stolen" using electromagnetic induction.

DEVELOP The current in the power line is alternating at a given frequency: $I = I_0 \sin(2\pi ft)$. This causes the magnetic field to vary, which induces an emf in the loop. Using the geometry from the previous problem, we have

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = \frac{d}{dt}\left[\frac{\mu_0 Il}{2\pi}\ln\left(\frac{y+a}{y}\right)\right] = \frac{\mu_0 l}{2\pi}\ln\left(\frac{y+a}{y}\right)\left[I_0 2\pi f\cos(2\pi ft)\right]$$

EVALUATE Since $\mathcal{E} \propto f$, the induced emf in North America with $f = 60$ Hz will be slightly greater than the induced emf with $f = 50$ Hz. The answer is **(a)**.

ASSESS The relationship $\mathcal{E} \propto f$ makes sense. The higher the frequency in the current, the faster the magnetic field is changing and the greater the induced emf. Likewise, if the frequency goes to zero, the current and the magnetic field become static, and no emf is induced.

83. **INTERPRET** We consider how electric power might be "stolen" using electromagnetic induction.

DEVELOP You might think that power lines are always generating time-varying magnetic fields and the induced emfs that go with them, so the power company won't notice if a farmer uses some of this energy that is just being "lost" anyway. But in fact this is wrong. The magnetic field energy around a wire is not radiated away but only temporarily stored and then later given back to the power lines. During each cycle of the AC current, the magnetic fields will decrease in strength, thus inducing an emf back into the power line that helps to drive current in the next part of the cycle.

EVALUATE By the above logic, if the farmer's loop had no resistance, then current would slosh back and forth in the loop, but no energy would be expended. However, as soon as the farmer puts a load in the loop circuit (like a light bulb, for instance), some of the magnetic field energy is used to do work, and therefore less energy will cycle back from the field into the power line. As a result, more fuel must be consumed at the power plant supplying the line.

The answer is **(a)**.

ASSESS Another way to think about this is that the loop and the wire have a mutual inductance, L. When there's no resistance, the voltage across this inductor is just: $\mathcal{E}_L = LdI/dt$. Since this voltage is 90° out of phase with the current, the total energy lost over a full cycle is zero:

$$E = \int P dt = \int I\mathcal{E} dt \propto \int \sin(2\pi ft)\cos(2\pi ft)\,dt = 0$$

But as soon as a resistor is added to the loop, the voltage and current will no longer be out of phase, and the energy lost over a full cycle will be nonzero.