32

INTERFERENCE AND DIFFRACTION

EXERCISES

Section 32.2 Double-Slit Interference

10. **INTERPRET** This problem involves interference from a double-slit arrangement. Given the parameters of the arrangement, we are to find the wavelength of the illuminating light.

DEVELOP Because $d \ll L$, we can apply Equation 32.2a with $d = 15$ µm, $L = 2.2$ m, $m = 1$, and $y = 7.1$ cm to find the wavelength λ. We may also approximate $\sin\theta$ by y/L because $y \ll L$ (i.e., it's a small angle).

EVALUATE The wavelength is

$$\lambda = \frac{d\sin\theta}{m} \approx \frac{dy}{mL} = \frac{(15\ \mu\text{m})(7.1\ \text{cm})}{(1)(2.2\ \text{m})} = 484\ \text{nm} = 480\ \text{nm}$$

to two significant figures.

ASSESS The exact formula is

$$\lambda = \frac{d\sin\theta}{m} = \frac{d}{mL}\sin\left[\tan^{-1}\left(\frac{y}{L}\right)\right]$$

which differs by 0.05% from the result above (i.e., ± 0.2 nm, which is much less than the precision of the data).

11. **INTERPRET** This problem involves double-slit interference. We are to find the spacing between adjacent bright fringes, given the wavelength of the light, the slit spacing, and the slit-screen distance.

DEVELOP The geometrical parameters of the source, slits, and screen satisfy the conditions for which Equations 32.2a and 32.2b apply (i.e., $d \ll L$ and $\lambda \ll d$). The locations of bright fringes are given by

$$y_{\text{bright}} = \frac{m\lambda L}{d}$$

where m is the order number.

EVALUATE The spacing of bright fringes is

$$\Delta y = (m+1)\frac{\lambda L}{d} - m\frac{\lambda L}{d} = \frac{\lambda L}{d} = \frac{(550\ \text{nm})(74\ \text{cm})}{0.025\ \text{mm}} = 1.63\ \text{cm}$$

ASSESS Since $\lambda \ll d$, the spacing between bright fringes is much smaller than L, as it should be.

12. **INTERPRET** For a double-slit interference arrangement, we are to find the slit-to-screen distance given the wavelength, the slit spacing, and the spacing between bright fringes.

DEVELOP We will assume that the particular geometry of this type of double-slit experiment satisfies the conditions for using Equations 32.2a and 32.2b (i.e., $d \ll L$ and $\lambda \ll L$), and verify afterward.

EVALUATE **(a)** The slit-to-screen spacing on the screen is

$$\Delta y = \frac{\lambda L}{d} \quad \Rightarrow \quad L = \frac{\Delta y d}{\lambda} = \frac{(0.12\ \text{mm})(5.0\ \text{mm})}{633\ \text{nm}} = 95\ \text{cm}$$

(b) For two different wavelengths, the ratio of the spacings is $\Delta y'/\Delta y = \lambda'/\lambda$; therefore $\Delta y' = (5\ \text{mm})(480/633) = 3.8$ mm.

ASSESS Note that the conditions $d \ll L$ and $\lambda \ll L$ are indeed met, so we are justified in using Equation 32.2a. The fringe spacing for 480 nm may also be found by applying Equation 32.2a with L = 95 cm. The result is the same.

13. **INTERPRET** This problem is about double-slit interference. We are interested in the wavelength of the light source.

DEVELOP For small angles, we may approximate $\sin\theta \sim \theta$, so Equation 32.1 gives $\Delta\theta = \lambda/d$, and the interference fringes are evenly spaced.

EVALUATE Substituting the values given, we obtain

$$\lambda = d\Delta\theta = (0.37\text{ mm})(0.065°)(\pi/180°) = 420\text{ nm}$$

ASSESS The wavelength λ is much smaller than the slit spacing d, as needed for using Equation 32.1a.

14. **INTERPRET** Given the angular position of the 5th dark fringe and the wavelength of the light, we are to find the slit separation in a double-slit experiment.

DEVELOP The interference minima fall at angles given by Equation 32.1b. The ratio $y_{\text{dark}}/L = \tan\theta$, so Equation 32.1b can be written as

$$d = (m + 1/2)\frac{\lambda}{\tan\theta}$$

Notice that the order m is defined as the number of the bright fringe, with m = 0 corresponding to the central (bright) fringe. Thus, the 5th dark fringe corresponds to m = 5.

EVALUATE For m = 5, Equation 32.1b gives

$$d = \frac{\left(5 + \frac{1}{2}\right)\lambda}{\tan\theta} = \frac{(5.5)(546\text{ nm})}{\tan(0.113°)} = 1.52\text{ mm}$$

ASSESS The condition $\lambda \ll d$ is met, so we are justified in using Equation 32.2b. Note that the significant figures are determined by the wavelength and the angle—not by the order number (which are defined as integers and so have infinite accuracy).

Section 32.3 Multiple-Slit Interference and Diffraction Gratings

15. **INTERPRET** The setup is a multiple-slit interference experiment. We want to know the number of minima (destructive interferences) between two adjacent maxima.

DEVELOP In an N-slit system with slit separation d (illuminated by normally incident plane waves), the main maxima occur for angles (see Equation 32.1a) $\sin\theta = m\lambda/d$, and minima for angles (see Equation 32.4) $\sin\theta = m'\lambda/(Nd)$ (excluding m' = 0 or multiples of N).

EVALUATE Between two adjacent maxima, say $m' = mN$ and $(m+1)N$, there are $N - 1$ minima. The number of integers between mN and $(m+1)N$ is

$$(m+1)N - mN - 1 = N - 1$$

because the limits are not included. Therefore, For N = 5, the number of minima is 4.

ASSESS The interference pattern resembles that shown in Fig. 32.8. Note that the number of minima is independent of the order number m. Also note that our result agrees with Fig. 32.8.

16. **INTERPRET** This problem involves a multiple-slit system. We are to find the first maximum, given the angular position of the first minimum.

DEVELOP Apply Equation 32.4 to find the ratio λd. For the first minima $(m = 1)$, this gives

$$\frac{\lambda}{d} = N\sin\theta_{\text{min}}$$

where $N = 3$ for this problem (i.e., three slits). The angular position of the first maxima can then be found using Equation 32.1a.

EVALUATE The first maximum is at

$$\sin\theta_{\text{max}} = \frac{\lambda}{d} = N\sin\theta_{\text{min}}$$

$$\theta = \sin^{-1}\left(N\sin\theta_{\text{min}}\right) = \sin^{-1}\left(3\sin\left(5.4°\right)\right) = 16.4°$$

ASSESS The two minima occur at approximately 5.4° and 10.8°.

17. **INTERPRET** In this problem, we want to locate certain maxima and minima in a multiple-slit interference experiment. We are given the necessary parameters.

DEVELOP According to Equation 32.1a, primary maxima occur at angles $\theta = \sin^{-1}\left(m\lambda/d\right)$. On the other hand, minima occur at angles (see Equation 32.4)

$$\theta_{\text{min}} = \sin^{-1}\left[m'\lambda/\left(Nd\right)\right], \quad m' = \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \ldots$$

where m' is an integer but not an integer multiple of N.

EVALUATE **(a)** Using the above equation, the first two maxima (after the central peak, $m = 0$) are for $m = 1$ and 2. The angular positions for these maxima are at

$$\theta_1 = \sin^{-1}\left(1\times 633\text{ nm}/7.5\ \mu\text{m}\right) = 4.8°$$
$$\theta_2 = \sin^{-1}\left(2\times 633\text{ nm}/7.5\ \mu\text{m}\right) = 9.7°$$

(b) With $N = 5$, excluded, the third minimum is for $m' = 3$ and the sixth for $m' = 7$ (because $m' = 5$ doesn't count). Then

$$\theta_{3,\text{min}} = \sin^{-1}\left[3\lambda/\left(5\lambda\right)\right] = 2.9°$$
$$\theta_{7,\text{min}} = \sin^{-1}\left[7\lambda/\left(5\lambda\right)\right] = 6.8°$$

ASSESS The minima would be difficult to observe because the secondary maxima between them are faint.

18. **INTERPRET** We are to find the first- and fifth-order diffraction angle, given the grating spacing and the wavelength.

DEVELOP For light normally incident on a diffraction grating, maxima occur at angles

$$\theta = \sin^{-1}\left(\frac{m\lambda}{d}\right)$$

(see Equation 32.1a) where m is the order number, and d is the grating spacing and is equal to the reciprocal of the number of lines per meter:

$$d = \frac{1}{3000\text{ cm}^{-1}} = 3.3\ \mu\text{m}$$

EVALUATE (**a**) In the first order,

$$\theta_1 = \sin^{-1}\left(\frac{522\text{ nm}}{3.3\ \mu\text{m}}\right) = 9.1°$$

(b) In the fifth order,

$$\theta_5 = \sin^{-1}\left(\frac{5\times 522\text{ nm}}{3.3\ \mu\text{m}}\right) = 52.3°$$

ASSESS One can see that the relationship between θ_1 and θ_5 is almost linear $(\theta_5 \sim 5\times\theta_1)$.

Section 32.4 Interferometry

19. **INTERPRET** This problem involves interference in a thin film. We want to find the minimum film thickness that results in constructive interference for the given wavelength.

DEVELOP The condition for constructive interference from a soap film is Equation 32.7:

$$2nd = \left(m + \frac{1}{2}\right)\lambda$$

The minimum thickness corresponds to the integer $m = 0$.

EVALUATE Substituting the values given, we get

$$2nd_{\min} = \frac{1}{2}\lambda$$

$$d_{\min} = \frac{\lambda}{4n} = \frac{705 \text{ nm}}{4(1.333)} = 132.2 \text{ nm}$$

Note that Equation 32.7 applies to normal incidence on a thin film in air.

ASSESS The typical thickness of a thin film is on the order of 100 nm. Thin-film interference accounts for the bands of color seen in soap films or oil slicks.

20. **INTERPRET** This problem involves constructive interference. Given the thickness of the material, we are to find the wavelength which results in constructive interference.

DEVELOP We shall use the result from Problem 32.19, which gives the minimum thickness for constructive interference in a thin material. Thus, the minimum thickness of the wedge at which constructive interference occurs is

$$d_{\min} = \lambda/(4n).$$

Solve this for λ to find the wavelength.

EVALUATE Inserting the given quantities gives

$$\lambda = 4(1.52)(98 \text{ nm}) = 600 \text{ nm}$$

where the result is precise to two significant figures.

ASSESS This is orange light and is within the visible range.

21. **INTERPRET** The enhanced reflection is a consequence of constructive interference, so we shall look for the range of wavelengths that satisfies this condition.

DEVELOP Equation 32.7 gives the condition for constructive interference from a given thickness of glass surrounded by air. Solving this equation for λ gives

$$\lambda = \frac{4nd}{2m+1} = \frac{4(1.65)(450 \text{ nm})}{2m+1} = \frac{2970 \text{ nm}}{2m+1}$$

EVALUATE Integers giving wavelengths in the visible range (400 to 750 nm) are $m = 2$ and 3, which correspond to $\lambda = 594$ and 424 nm, respectively.

ASSESS The wavelengths correspond to orange and blue colors, respectively.

22. **INTERPRET** We are to find the wavelength for which constructive interference makes it reflect most strongly from the given thin material.

DEVELOP Apply the solution derived for Problem 32.20. Maximum constructive interference occurs for $\lambda = 4nd$.

EVALUATE Inserting the given quantities yields

$$\lambda = 4(1.43)(98 \text{ nm}) = 560.56 \text{ nm}$$

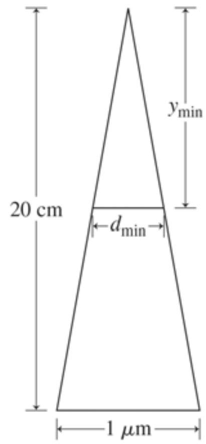
ASSESS This is yellow-green.

23. **INTERPRET** The problem asks what portion of a soap film will appear dark because it is too thin for constructive interference in reflected light.

DEVELOP The soap film is 20 cm high and goes from zero thickness at the top to 1 μm thick at the bottom. See the figure below. White light shines on the film and the reflected light from the two soap-air surfaces will constructively interfere when the thickness of the film satisfies Equation 32.7:

$$2nd = \left(m + \tfrac{1}{2}\right)\lambda,$$

where the index of refraction is that of water $(n = 1.333)$ and the integer $m = 0,1,2,3....$ We are looking for the region of the film that is too thin to support constructive interference, so we will define $d_{\min}$ as the smallest thickness for a bright band and y_{min} as the distance to this first band from the top, see figure. The region defined by y_{min} will be dark.



EVALUATE White light is a combination of wavelengths from 400 to 700 nm, so there will be bright bands of different colors coming from the soap film. Near the top of film, where it is thinnest, the bands will correspond to the zeroth order $(m = 0)$. At the top of this set of bands will be the blue band for $\lambda = 400$ nm, since this corresponds to the thinnest part of the film that still supports constructive interference:

$$d_{\text{min}} = \frac{1}{2n}\left(m + \tfrac{1}{2}\right)\lambda = \frac{1}{2(1.333)}\left(0 + \tfrac{1}{2}\right)(400 \text{ nm}) = 75 \text{ nm}$$

From the figure above, we can see that this minimum thickness occurs at

$$y_{\text{min}} = d_{\text{min}}\left(\frac{20 \text{ cm}}{1 \text{ μm}}\right) = 1.5 \text{ cm}$$

Therefore, the top 1.5 cm of the film will be dark.

ASSESS What happens to the light in this dark region? It is fully transmitted, so if we were to look at the backside of the soap film, the top portion would be bright, and we would see dark bands in transmission at the points corresponding to the bright bands in reflection.

Section 32.5 Huygens' Principle and Diffraction

24. **INTERPRET** This problem involves Huygens's principle, which we can use to find the ratio a/λ that causes the first diffraction minimum to occur at 90°.

DEVELOP Huygens principle leads to Equation 32.8 for a single slit of size a. We shall apply this equation to find the ratio a/λ that corresponds to $m = 1$ and $\theta = 90°$.

EVALUATE For the given conditions, Equation 32.8 takes the form

$$\sin\theta = \frac{\lambda}{a}$$

which leads to

$$\frac{a}{\lambda} = \csc\theta = \csc(90°) = 1$$

ASSESS This demonstrates that smaller slits will lead to wider diffraction maxima.

25. **INTERPRET** This problem involves a single-slit diffraction of light. We are interested in the angular width of the central peak.

DEVELOP The condition for destructive interference in a single-slit diffraction is given by Equation 32.8:

$$a\sin\theta = m\lambda, \quad m = \pm 1, \pm 2, \ldots$$

The first minima $(m = \pm 1)$ occur at

$$\theta_{\pm 1} = \sin^{-1}\left(\pm\frac{\lambda}{a}\right) = \sin^{-1}\left(\pm\frac{615 \text{ nm}}{2.40\ \mu\text{m}}\right) \pm 14.85°$$

EVALUATE The total angular width of the diffracted beam is $\Delta\theta = \theta_1 - \theta_{-1} = 29.7°$.

ASSESS The case $m = 0$ is excluded in Equation 32.8 because it corresponds to the central maximum in which all waves are in phase.

26. **INTERPRET** This problem involves the diffraction of electromagnetic radiation due to a slit – the window. We are to find the angular width of the beam when it emerges from the slit.

DEVELOP Take the width of the diffracted beam to be the angular separation between the first minima. Using Equation 32.8, $a\sin\theta = m\lambda$, these occur at $\theta = \pm\sin^{-1}(\lambda/a)$, so the angular width is 2θ. The wavelength is $\lambda = c/f$, where f is the given frequency.

EVALUATE Inserting the given quantities gives

$$\Delta\theta = 2|\theta| = 2\sin^{-1}\left(\frac{c}{fa}\right) = 2\sin^{-1}\left[\frac{\left(3.00\times10^{8}\text{ m/s}\right)}{\left(950\times10^{6}\text{ Hz}\right)\left(0.35\text{ m}\right)}\right] = 129°$$

ASSESS The wavelength is ~0.32 m, so for more closely spaced windows (i.e., smaller slits), the angular width would be larger.

27. **INTERPRET** We are to find the intensity of a diffraction maximum relative to the central peak. The second secondary maxima is the second small maxima next to the central peak.

DEVELOP The intensity as a function of angle in single-slit diffraction is given by Equation 32.10:

$$\bar{S} = \bar{S}_0\left[\frac{\sin(\phi/2)}{\phi/2}\right]^2$$

The second and third minima lie at angles $\sin\theta_2 = 2\lambda/a$ and $\sin\theta_3 = 3\lambda/a$.

EVALUATE If we take the mid-value to be at $\sin\theta = 5\lambda/2a$, then the intensity at this angle, relative to the central intensity, is

$$\frac{\bar{S}}{\bar{S}_0} = \left(\frac{\sin(5\pi/2)}{5\pi/2}\right)^2 = \frac{4}{25\pi^2} = 1.62\times10^{-2}$$

ASSESS The intensity at the second secondary maximum is only about 1.62% of the central-peak intensity.

Section 32.6 The Diffraction Limit

28. **INTERPRET** We are to find the minimum angular separation that can be resolved with 627-nm light through a 2.8-cm-diameter aperture. We will assume that the limiting factor is diffraction and use the Rayleigh criterion.

DEVELOP Apply the Rayleigh criterion for circular apertures (Equation 32.11b): $\theta_{\text{min}} = 1.22\lambda/\text{D}$. The wavelength is $\lambda = 627\times10^{-7}$ cm, and the diameter is $D = 2.8$ cm.

EVALUATE The minimum angular separation is

$$\theta_{\text{min}} = 1.22\left(627\times10^{-7}\text{cm}\right)/\left(2.8\text{ cm}\right) = 2.7\times10^{-5}\text{rad} = 0.0016°$$

ASSESS This angular resolution is equivalent to distinguishing two objects 2.7 millimeters apart from 100 meters away.

29. **INTERPRET** We shall use the Rayleigh criterion to determine how large an aperture is needed on a telescope to resolve the given angle.

DEVELOP Apply the Rayleigh criterion for circular apertures (Equation 32.11b): $\theta_{\text{min}} = 1.22\lambda/D$. The wavelength is $\lambda = 515$ nm, and the angular resolution needed is

$$\theta_{\text{min}} = 0.41\text{ arcseconds} = \left(1.41\times10^{-6}\right)° = 1.99\times10^{-6}\text{ rad}$$

Solve for D.

EVALUATE The diameter needed is

$$D = \frac{1.22\lambda}{\theta_{\text{min}}} = \frac{1.22(515\text{ nm})}{1.99\times10^{-6}\text{ rad}} = 0.32\text{ m} = 32\text{ cm}$$

ASSESS Make sure that you always use radians for your angle measurements in this type of problem!

30. **INTERPRET** We shall use the Rayleigh criterion to determine the longest wavelength of light with which one is able to resolve the given angle through the given aperture.

DEVELOP Apply the Rayleigh criterion for circular apertures (Equation 32.11b): $\theta_{\text{min}} = 1.22\lambda / D.$ The angular resolution necessary is $\theta_{\text{min}} = 0.43$ mrad, and the aperture is $D = 1.8$ mm. Solve for the wavelength.

EVALUATE The longest wavelength that can resolve this is

$$\lambda = \frac{\theta_{\text{min}} D}{1.22} = \frac{(0.43\text{ mrad})(1.8\text{ mm})}{1.22} = 630\text{ nm}$$

to two significant figures.

ASSESS Using a short wavelength gives you a much better angular resolution. This is why an *electron* microscope gives such high resolution, as we will discover later.

31. **INTERPRET** We're asked to find the diffraction limit of the eye in bright light when the pupil has contracted.

DEVELOP The minimum angular resolution of a circular aperture is given in Equation 32.11b: $\theta_{\text{min}} = 1.22\lambda / D$, where the result is in radians.

EVALUATE For the given pupil diameter and light wavelength, the resolution is

$$\theta_{\text{min}} = \frac{1.22\lambda}{D} = \frac{1.22(550\text{ nm})}{(2\text{ mm})} = 3\times10^{-4}\text{rad}$$

In terms of degrees, this is about 0.02°, or about 1 arcminute.

ASSESS This says that on a bright day our eyes would be able to distinguish objects 3 mm apart at a distance of 10 m. This is a little unrealistic. Our eyes are not only limited by the diffraction through the pupil; their resolution is also affected by the spacing of receptors (rods and cones) on the back of the retina.

EXAMPLE VARIATIONS

32. **INTERPRET** The concept behind the grating spectrometer is multiple-slit interference, so our job is to find the angles to which the grating sends the given wavelengths. "First-order" means we have $m = 1$.

DEVELOP Like in the original example, we will use Equation 32.1a to find the angle for each wavelength using the grating slit spacing, and determine the angular separation between them

EVALUATE Applying Equation 32.1a with $m = 1$ for the gamma and delta lines gives

$$\theta_\gamma = \sin^{-1}\left(\frac{\lambda}{d}\right) = \sin^{-1}\left(\frac{434.0\text{nm}}{1.667\,\mu\text{m}}\right) = 15.10°$$

$$\theta_\delta = \sin^{-1}\left(\frac{\lambda}{d}\right) = \sin^{-1}\left(\frac{410.0\text{nm}}{1.667\,\mu\text{m}}\right) = 14.25°$$

Making the separation between these

$$\Delta\theta = 0.85°$$

ASSESS In order to resolve these lines individually, the spectrometer's detectors should have a pixel array capable of distinguishing from signals separated by this amount of less.

33. **INTERPRET** The concept behind the grating spectrometer is multiple-slit interference, so our job is to find the angles to which the grating sends the given wavelengths. "First-order" means we have $m = 1$. We are given angular separation between two of the four visible lines of atomic hydrogen and are asked to determine which ones they could be.

DEVELOP Like in the original example, we used Equation 32.1a in the previous problem to find the angle for each wavelength using the grating slit spacing, and to determine the angular separation between them. Using these four angles we can then determine which lines the given spacing of $\Delta\theta = 8.1°$ corresponds to.

EVALUATE Applying Equation 32.1a with $m = 1$ for all the lines resulted in the angles

$$\theta_{\alpha\beta\gamma\delta} = [23.2°, 17.0°, 15.10°, 14.25°]$$

We determined spacing between Hα and Hβ lines in the original example, and between Hγ and Hδ lines in the previous problem, so we can rule those out. Spacing between Hβ and Hγ, as well as Hβ and Hδ is too small to correspond to the given separation. The only two possibilities are thus between Hα and Hγ, and Hα and Hδ. The latter has a separation closer to 9° than 8°, meaning the given separation corresponds to the Hα line and the Hγ line.

ASSESS If the spectrometer resolution is not very high, differentiation between the gamma and delta lines could prove difficult, making this spacing hard to assign to specific lines.

34. **INTERPRET** The concept behind the grating spectrometer is multiple-slit interference, so our job is to find the angles to which the grating sends the given wavelengths. "Third-order" means we have $m = 3$. We will determine the spacing between two atomic sodium lines.

DEVELOP Like in the original example, we will use Equation 32.1a to find the angle for each wavelength using the grating slit spacing, and determine the angular separation between them

EVALUATE Applying Equation 32.1a with $m = 3$ for the given sodium lines gives

$$\theta_1 = \sin^{-1}\left(\frac{m\lambda}{d}\right) = \sin^{-1}\left(\frac{3\times 588.995\,\text{nm}}{4.348\,\mu\text{m}}\right) = 23.978°$$

$$\theta_2 = \sin^{-1}\left(\frac{m\lambda}{d}\right) = \sin^{-1}\left(\frac{3\times 589.592\,\text{nm}}{4.348\,\mu\text{m}}\right) = 24.004°$$

Where we have used the grating spacing to calculate $d = 1/2300\,\text{cm}^{-1}$, making the separation between these

$$\Delta\theta = 0.026°$$

ASSESS In order to resolve these lines individually, the spectrometer's detectors should have a pixel array capable of distinguishing from signals separated by this amount of less.

35. **INTERPRET** The concept behind the grating spectrometer is multiple-slit interference, and we are asked to find an expression for the grating line spacing in terms of the angular separation between two wavelengths. We will consider "first-order" meaning we have $m = 1$.

DEVELOP Like in the original example, we will use Equation 32.1a to find the angle for each wavelength using the grating slit spacing, and determine the angular separation between them in terms of the lines spacing d, which we can then solve for.

EVALUATE Applying Equation 32.1a with $m = 1$ for two wavelengths gives

$$m\lambda_i = d\sin\theta_{\lambda_i} \cong d\theta_{\lambda_i}$$
$$\theta_{\lambda_i} = \lambda_i/d$$

Making the separation between these

$$\Delta\theta = \frac{|\lambda_1 - \lambda_2|}{d}$$

And the lines spacing equal to

$$d = \frac{|\lambda_1 - \lambda_2|}{\Delta\theta}$$

ASSESS Using this equation one can determine the necessary line spacing to resolve a separation between two wavelengths, making it possible to design a spectrometer's resolution for a given spectroscopic application.

36. **INTERPRET** This is a problem about the diffraction limit with a circular aperture. We're after the minimum physical size for the asteroid at a given distance. We identify $D = 6.5$ m as the aperture size and $\lambda = 850$ nm as the wavelength of the light.

DEVELOP Like in the original example, we will use Equation 32.11b to define the minimum angular size that can be resolved. For a small angle, we can relate it to the physical size of the asteroid l and the distance to the asteroid L as $\theta_{\min} = l/L$, allowing us to relate the size l using the wavelength and mirror size from Equation 32.11b.
EVALUATE Applying Equation 32.11b we find the minimum asteroid size which can be imaged is

$$l = \frac{1.22\lambda L}{D} = \frac{1.22(850\,\text{nm})(20\times10^{6}\,\text{km})}{6.5\,\text{m}} = 3.2\,\text{km}$$

ASSESS Although the light used is of longer wavelength, making it more difficult to image a smaller target, the larger mirror diameter of the James Webb Space Telescope will allow scientists to image distant objects of smaller size than currently possible with the Hubble Space Telescope.

37. **INTERPRET** This is a problem about the diffraction limit with a circular aperture. We're after the necessary mirror size to image an asteroid of a given size, at a given distance, with a particular wavelength of light.
DEVELOP Like in the original example, we will use Equation 32.11b to define the minimum angular size that can be resolved. For a small angle, we can relate it to the physical size of the asteroid l and the distance to the asteroid L as $\theta_{\min} = l/L$, allowing us to relate the size l using the wavelength and mirror size from Equation 32.11b.
EVALUATE Applying Equation 32.11b we find the minimum mirror size is

$$D = \frac{1.22\lambda L}{l} = \frac{1.22(535\,\text{nm})(1.20\,\text{Gm})}{35\,\text{m}} = 22.4\,\text{m}$$

ASSESS Such a mirror would be twice the size of the current largest telescope mirrors being used in observatories around the world.

38. **INTERPRET** This is a problem about the diffraction limit with a circular aperture. We're after the minimum physical size separating two bees observed through binoculars to be able to distinguish them as individuals. We are given the size of the lenses, the distance to the bees, and the wavelength to consider for imaging
DEVELOP Like in the original example, we will use Equation 32.11b to define the minimum angular size that can be resolved. For a small angle, we can relate it to the physical size of the asteroid l and the distance to the asteroid L as $\theta_{\min} = l/L$, allowing us to relate the size l using the wavelength and aperture size from Equation 32.11b.
EVALUATE Applying Equation 32.11b we find the minimum separation distance to be

$$l = \frac{1.22\lambda L}{D} = \frac{1.22(550\,\text{nm})(840\,\text{m})}{28\,\text{mm}} = 2.0\,\text{cm}$$

ASSESS Honey bees are approximately 1.5 cm in size, meaning they need to be separated by about 1.3 times their size to be distinguishable from this distance with these binoculars.

39. **INTERPRET** This is a problem about the diffraction limit with a circular aperture. We're after the diameter of the mirror used by the *WorldView-4* satellite. We are given its distance from earth, and its resolution at a particular wavelength of light.
DEVELOP Like in the original example, we will use Equation 32.11b to define the minimum angular size that can be resolved. For a small angle, we can relate it to the physical size of the asteroid l and the distance to the asteroid L as $\theta_{\min} = l/L$, allowing us to relate the size l using the wavelength and aperture size from Equation 32.11b.
EVALUATE Applying Equation 32.11b we find diameter of the mirror is equal to

$$D = \frac{1.22\lambda L}{l} = \frac{1.22(460\,\text{nm})(610\,\text{km})}{31\,\text{cm}} = 1.1\,\text{m}$$

ASSESS This result makes sense since we are dealing with a commercial satellite operating at short wavelengths and imaging objects while orbiting a few hundred kilometers above Earth.

PROBLEMS

40. **INTERPRET** We are to find the angle at which the second-order (i.e., $m = 2$) bright fringe occurs in a double-slit experiment with the given parameters.
DEVELOP Use Equation 32.1a to find the angle θ for the different wavelengths λ, with $d = 1.5$ μm.
EVALUATE (**a**) For $\lambda = 640$ nm, the angle is

$$\theta = \sin^{-1}\left(\frac{m\lambda}{d}\right) = \sin^{-1}\left(\frac{2\times 640\text{ nm}}{1.5\ \mu\text{m}}\right) = 59°$$

(**b**) For λ = 580 nm,

$$\theta = \sin^{-1}\left(\frac{2\times 580\text{ nm}}{1.5\ \mu\text{m}}\right) = 51°$$

(**c**) For λ = 410 nm,

$$\theta = \sin^{-1}\left(\frac{2\times 410\text{ nm}}{1.5\ \mu\text{m}}\right) = 33°$$

ASSESS Notice the nonlinear behavior in the relation between θ and λ.

41. **INTERPRET** The concept behind this problem is double-slit interference. The object of interest is the phase difference between the waves emanating from the different slits.

DEVELOP The path length difference for waves arriving from the two different slits is

$$\Delta r = d\sin\theta \approx d\tan\theta = d\frac{y}{L}$$

since $\lambda \ll d$ and the small-angle approximation can be used (see derivation of Equations 32.2a and 32.2b). The phase difference is

$$\Delta\phi = \left(\frac{2\pi}{\lambda}\right)\Delta r = \frac{2\pi}{\lambda}\frac{yd}{L}$$

EVALUATE The phase difference is

$$\Delta\phi = \frac{2\pi(0.0056\text{ m})(0.035\times 10^{-3}\text{m})}{(490\times 10^{-9}\text{ m})(1.5\text{ m})} = 1.6755\text{ rad} = 96°$$

ASSESS Constructive interference corresponds to $\Delta\phi = 2\pi m$, or $yd/L = m$, where m is an integer.

42. **INTERPRET** Our light source for the double-slit experiment has two wavelengths. For the lowest-order bright fringe, we are to find the angular position where interference is constructive for one wavelength and destructive for the other.

DEVELOP In a double-slit apparatus of the type described in the text, for a bright fringe of order m_1 from wavelength λ_1 to have the same angular position as a dark fringe of order m_2 from wavelength λ_2, we must have (see Equations 32.1a and 32.1b)

$$m_1\lambda_1 = (m_2 + 1/2)\lambda_2$$

For λ_1 = 550 nm and λ_2 = 400 nm, one finds

$$11m_1 = 8m_2 + 4$$

EVALUATE By inspection, the smallest integer values satisfying this condition are $m_1 = 4$ and $m_2 = 5$; that is, the fourth bright fringe of wavelength 550 nm coincides with the sixth dark fringe of wavelength 400 nm (recall that the first bright fringe has m = 1, whereas the first dark fringe has m = 0).

ASSESS This problem demonstrates the role played by the wavelength in determining the nature of the interference at an angular position.

43. **INTERPRET** This problem involves a multiple-slit apparatus. We are given the number of dark fringes between two adjacent major maxima and are asked to find the number of slits in the apparatus. We are also to find the slit separation given.

DEVELOP From Fig. 32.8, we see that an N-slit system has $N - 1$ minima between the major maxima. The position of the maxima is governed by Equation 32.1a, which, for small angles, takes the form

$$d\theta = m\lambda$$

The angular separation between adjacent maxima (i.e., between $m = n$ and $m = n+1$) is

$$d\Delta\theta = \lambda$$

The angular separation is $\Delta\theta = \pi(0.80°)/180° = 0.014$ rad.

EVALUATE **(a)** For seven minima, we have $N-1=7;$ we have $N=8$ slits.

(b) The slit separation is

$$d = \frac{\lambda}{\Delta\theta} = \frac{656.3 \text{ nm}}{0.014 \text{ rad}} = 46.9 \ \mu\text{m} = 47 \ \mu\text{m}$$

ASSESS The result is given to two significant figures because the angular separation $\Delta\theta$ is given to that precision.

44. **INTERPRET** You're determining the slit (line) spacing needed for a new spectrometer.

DEVELOP You need at least an angular separation of $\theta_{\text{min}} = 5°$ between the third-order $(m=3)$ lines of hydrogen and sodium, specifically: $\lambda_{\text{H}\alpha} = 656$ nm and $\lambda_{\text{Na}} = 589$ nm. The angle of each these lines will satisfy Equation 32.1a:

$$d\sin\theta_{\text{H}\alpha} = m\lambda_{\text{H}\alpha}; \quad d\sin\theta_{\text{Na}} = m\lambda_{\text{Na}}$$

We will use these equations and the fact that $\theta_{\text{H}\alpha} \geq \theta_{\text{Na}} + \theta_{\text{min}}$ to solve for the slit spacing, *d*. The number of slits (or lines) per cm is just the inverse of this, $1/d$.

EVALUATE For the given pupil diameter and light wavelength, the resolution is

$$\theta_{\text{min}} = \frac{1.22\lambda}{D} = \frac{1.22(550 \text{ nm})}{(2 \text{ mm})} = 3\times 10^{-4} \text{rad}$$

In terms of degrees, this is about 0.02°, or about 1 arcminute.

ASSESS We first eliminate one of the unknown angles with a trig identity from Appendix A:

$$\sin\theta_{\text{H}\alpha} = \sin(\theta_{\text{Na}} + \theta_{\text{min}}) = \sin\theta_{\text{Na}}\cos\theta_{\text{min}} + \cos\theta_{\text{Na}}\sin\theta_{\text{min}}$$

We can plug in $\sin\theta_{\text{H}\alpha} = m\lambda_{\text{H}\alpha}/d$ and $\sin\theta_{\text{Na}} = m\lambda_{\text{Na}}/d$, and use the fact that $\cos\theta_{\text{Na}} = \sqrt{1-\sin^2\theta_{\text{Na}}}$. With some algebra, we arrive at:

$$d = \frac{m}{\sin\theta_{\text{min}}}(\lambda_{\text{H}\alpha} - \lambda_{\text{Na}})\sqrt{1 + \frac{2\lambda_{\text{H}\alpha}\lambda_{\text{Na}}(1-\cos\theta_{\text{min}})}{(\lambda_{\text{H}\alpha} - \lambda_{\text{Na}})^2}}$$

$$= \frac{3}{\sin 5°}(67 \text{ nm})\sqrt{1 + \frac{2(656 \text{ nm})(589 \text{ nm})(1-\cos 5°)}{(67 \text{ nm})^2}} = 2.97 \ \mu\text{m}$$

The inverse of this is 3370 lines/cm, so the coarsest grating you could use is the one with 3500 lines/cm.

ASSESS Notice that for a small minimum angle, the spacing equation simplifies to $d\sin\theta_{\text{min}} \approx m\Delta\lambda$, where $\Delta\lambda = \lambda_{\text{H}\alpha} - \lambda_{\text{Na}}$. This makes it clear that in order to increase the angular separation of two nearby spectral lines, one must use a spectrometer with more slits or gratings per cm.

45. **INTERPRET** We are to find the diffraction order necessary to resolve (i.e., separate) two closely spaced spectral lines.

DEVELOP From Equation 32.5, wavelengths can be resolved if

$$\Delta\lambda > \lambda/mN, \text{ or } m > \lambda/N\Delta\lambda = (648 \text{ nm})/(4500)(0.09 \text{ nm})$$

where $\Delta\lambda = 648.07 - 647.98$ nm $= 0.090$ nm and $N = 4500$.

EVALUATE The requisite order is

$$m > \frac{647.98 \text{ nm}}{(4500)(0.090 \text{ nm})} = 1.6$$

So the second or higher order is required to resolve these spectral lines.

ASSESS Note that *N* is a dimensionless number, so the dimensions work out in the expression for the minimum order number.

46. **INTERPRET** This problem is about thin-film interference. Three media involved are toluene, water, and air. We are to find the film thickness of toluene on water that results in the maximum reflectance of the given wavelength.

DEVELOP Since $n_{\text{toluene}} > n_{\text{water}} > n_{\text{air}}$ there is a 180° phase change for reflection at the air–toluene interface and no phase change at the toluene–water interface (see Fig. 32.12). Equation 32.7 thus applies for constructive interference (of normally incident rays):

$$2nd = \left(m + \frac{1}{2}\right)\lambda$$

EVALUATE Solving for the thickness d, we get

$$d = \left(m + \frac{1}{2}\right)\frac{\lambda}{2n} = \left(m + \frac{1}{2}\right)\frac{460\text{ nm}}{2(1.49)} = (2m+1)(77.2\text{ nm})$$

The minimum thickness is for $m = 0$, so $d_{\min} = 77.2$ nm.

ASSESS The typical thickness of a thin film is on the order of 100 nm. Thin-film interference accounts for the bands of color seen in a soap film or oil slick. Note that odd multiples of our result will also give the desired maximum reflectance at 460 nm.

47. **INTERPRET** You're assessing the feasibility of resolving Earth-sized planets with a single space telescope.

DEVELOP You can assume that resolving the planet means roughly that its angular extent in the sky is at least equal to the diffraction limit of the proposed telescope. The angular extent of an Earth-sized planet at a distance of L is $2R_{\text{E}} / L.$ Equating this to Rayleigh criterion in Equation 32.11b gives for the minimum telescope diameter:

$$D_{\min} = \frac{1.22\lambda L}{2R_{\text{E}}}$$

EVALUATE Our equation says that the smaller the wavelength we use, the smaller the telescope has to be. So you might as well choose the lower limit of the optical wavelengths: $\lambda = 400$ nm. As such, the telescope diameter needed would be

$$D_{\min} = \frac{1.22(400\text{ nm})(5\text{ ly})}{2(6.37\times10^{6}\text{m})}\left[\frac{9.46\times10^{15}\text{m}}{1\text{ ly}}\right] \approx 2\text{ km}$$

A 2-km-wide telescope in space, or even on the ground, is not feasible.

ASSESS NASA is considering ways to detect Earth-sized planets with a space telescope. However, the goal is not to resolve the planet, but merely separate its light signal from that of its host star. In this case, the angle is not set by the planet's diameter but by its orbital radius. For a planet orbiting its star at the same distance as Earth is from the Sun $\left(r_{\text{E}} = 1.50\times10^{11}\text{m}\right),$ the minimum telescope diameter is less than 20 cm. However, this is largely irrelevant. The real challenge in getting a direct image of a distant planet is not the angular resolution, but the fact that the star is so much brighter than the planet. The starlight completely overwhelms the planet's signal, so astronomers are looking for ways to filter out the light coming from the star.

48. **INTERPRET** In this problem, we are asked to calculate the wavelength of the violet light and its order of diffraction, given that it overlaps yellow light of order $m = 2$.

DEVELOP In a diffraction grating, for a bright fringe of order m_1 from wavelength λ_1 to have the same angular position as a bright fringe of order m_2 from wavelength λ_2, we must have (see Equations 32.1a and 32.1b)

$$m_1\lambda_1 = m_2\lambda_2$$

EVALUATE For $\lambda_1 = 588$ nm (yellow light) and $m_1 = 2$, one finds

$$m_1\lambda_1 = 2(588\text{ nm}) = 1176\text{ nm} = m_2\lambda_2$$

Given that m_2 is an integer and λ_2 ranges between 390 nm and 450 nm, the only combination that satisfies the condition is $m_2 = 3$, and $\lambda_2 = 392$ nm.

ASSESS The second and third orders overlap, whereas the only order where the visible spectrum does not overlap itself is the first order.

49. **INTERPRET** This problem is about X-ray diffraction in a crystal. We are interested in the spacing between the crystal planes, which we can find using Bragg's law.

DEVELOP Constructive interference in X-ray diffraction is given by the Bragg condition (Equation 32.6):

$$2d\sin\theta = m\lambda, \quad m = 1,2,3,\ldots$$

Solve this for d to find the spacing between crystal planes.

EVALUATE From the Bragg condition, one finds

$$d = \frac{m\lambda}{2\sin\theta} = \frac{(1)\left(97\times10^{-12}\text{ m}\right)}{2\sin(8.5°)} = 3.3\ \text{Å}$$

ASSESS The spacing between crystal planes is typically a few angstroms, so this result seems reasonable.

50. **INTERPRET** This problem involves interference from a thin film. In particular, we are to analyze a film to find the minimum thickness for which a reflectance of light in the visible regime still occurs.

DEVELOP Apply Equation 32.7, which gives the condition for constructive interference from a thin film with index n higher than that of its environment. The minimum thickness d_{min} occurs for $m = 0$, which gives

$$d_{\text{min}} = \frac{\lambda_{\text{min}}}{4n}$$

EVALUATE The minimum wavelength that is visible to humans is normally taken to be 400 nm. Using this result and $n = 1.33$ and in this expression gives a minimum thickness of

$$d_{\text{min}} = \frac{\lambda_{\text{min}}}{4n} = \frac{400\text{ nm}}{4(1.33)} = 75.2\text{ nm}$$

ASSESS Film thicknesses are typically around 100 nm, so the result seems reasonable because it is slightly less than 100 nm.

51. **INTERPRET** This problem involves constructive interference from a thin film. We are to find the number of times the condition for constructive interference is met for 630-nm light in a thin film that varies in thickness within the given range.

DEVELOP In a thin film of oil between air and water ($n_{\text{air}} < n_{\text{oil}} < n_{\text{water}}$), there are 180° phase changes for reflection at both boundaries (i.e., for both rays 1 and 2 in Fig. 32.7). These phase changes cancel each other, leaving only the film thickness to give the difference in path length. Therefore, for normally incident light, the term ½ in Equation 32.7 cancels due to a similar term on the left-hand side, leaving

$$2nd = m\lambda$$

The thickness d varies in the range $0.80\ \mu\text{m} \le d \le 2.1\ \mu\text{m}$, so we can find the integers m that satisfy this range for $\lambda = 630$ nm.

EVALUATE The thickness range implies

$$0.80\ \mu\text{m} \le \frac{m\lambda}{2n} \le 2.1\ \mu\text{m}$$
$$\frac{2n(0.80\ \mu\text{m})}{\lambda} \le m \le \frac{2n(2.1\ \mu\text{m})}{\lambda}$$
$$3.17 \le m \le 8.33$$

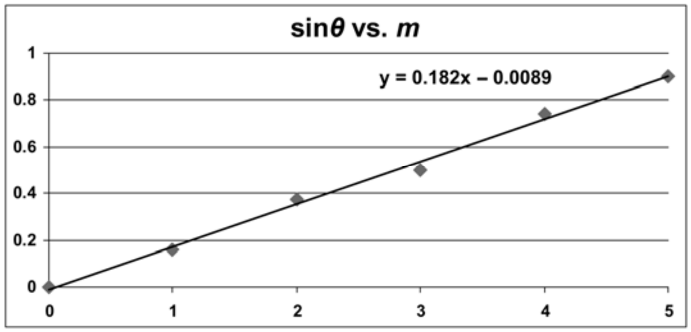
Since m is an integer, it can range from 4 to 8, inclusive.

ASSESS For 630 nm, this film will exhibit 5 bright fringes; for $m = 4$, 5, 6, 7, and 8.

52. **INTERPRET** This problem is about diffraction gratings. We are given the data of the angular positions of bright fringes as a function of order m, and asked to find the wavelength of the light.

DEVELOP The grating condition is $\sin\theta = m(\lambda/d)$. Therefore, plotting $\sin\theta$ vs. m will result in a straight line with slope equal to λ/d. Knowing the spacing d allows us to deduce λ.

EVALUATE The plot is shown below.



The slope of the best-fit line is 0.182. With $d = 3.2$ μm = 3200 nm, we find the wavelength to be

$$\lambda = 0.182(3200\text{ nm}) = 582.4\text{ nm}.$$

ASSESS The wavelength corresponds to yellow light.

53. **INTERPRET** This problem involves constructive interference from a thin film—in this case, a film of air between two glass plates. The index of air is less than that of glass, so there is a 180° phase change at the bottom interface (air–glass interface) instead of at the top interface (glass–air).

DEVELOP Although the phase change occurs at the second interface as opposed to the first interface as is assumed in deriving Equation 32.6, the net effect is the same—the path difference $2d$ must be an odd-integer multiple of half wavelengths. The thickness of the film varies between 0 and 0.065 mm, so we can apply Equation 32.6 to find the corresponding range of m.

EVALUATE The minimum value for m is 0. The maximum value is

$$m = \frac{2nd}{\lambda} - \frac{1}{2} = \frac{2(1.00)(0.065\text{ mm})}{535\text{ nm}} - 0.5 = 242.49$$

or $m = 242$. Thus, the observer will see 243 bright bands.

ASSESS The first bright band is at zero thickness, which corresponds to $m = 0$. This band is added to the 242 remaining bands to give the total of 243 bright bands.

54. **INTERPRET** This problem concerns constructive interference for normally incident light on the thin, wedge-shaped film of arbitrary medium between glass surfaces. We are to derive an expression giving the number of bright fringes that appear given that N bright bands are visible for air serving as the film medium.

DEVELOP With N bright bands visible for an air wedge, the maximum value for m in the condition for constructive interference is $m_{max} = N - 1$, and this corresponds to the maximum film thickness d_{max}. Inserting this into Equation 32.6 and solving for the ratio d_{max}/λ gives

$$\frac{d_{max}}{\lambda} = \frac{1}{2}\left(N - \frac{1}{2}\right)$$

When liquid of refractive index n' fills the wedge, there is still a 180° phase change at only one surface (regardless of whether or not $n' > n_{glass}$). Thus, we can apply Equation 32.6, which gives

$$2n'd_{max} = \left(m'_{max} + \frac{1}{2}\right)\lambda = \left(N' - \frac{1}{2}\right)\lambda$$

where N' is the number of fringes in the new film. We can thus solve for N' in terms of N and n'.

EVALUATE The new number of bright fringes is thus

$$\begin{aligned} N' &= 2n'\frac{d_{max}}{\lambda} + \frac{1}{2} \\ &= 2n'\left(\frac{1}{2}\right)\left(N - \frac{1}{2}\right) + \frac{1}{2} \\ &= n'N + \frac{1}{2}(1 - n') \end{aligned}$$

ASSESS This result is independent of the index of the enclosing material (the glass in this case).

55. **INTERPRET** We are asked to find the distance that corresponds to the passage of 527 bright fringes in an interferometer. Thus, as one arm of the interferometer moves, the path length in that arm changes, causing alternating constructive and destructive interference (or alternating bright and dark fringes) to occur at the output of the interferometer.

DEVELOP In each arm of the interferometer, light must travel down and back, or twice the length of the arm (see Fig. 32.16). Thus, the path-length difference corresponding to 527 bright fringes is twice the distance moved by the mirror, and each successive fringe corresponds to the distance of one wavelength. This gives

$$2\Delta L = 527\,\lambda$$

which we can solve for ΔL.

EVALUATE Inserting the given quantities gives

$$\Delta L = \frac{527\lambda}{2} = 263(486.1\text{ nm}) = 128.1\ \mu\text{m}$$

ASSESS This distance is greater than a single wavelength, which is the minimum distance this type of apparatus can detect.

56. **INTERPRET** This problem is similar to the previous one, except that we are given the distance moved by the interferometer and asked to find the wavelength of light used.

DEVELOP Applying the same logic as for the previous problem gives

$$2\Delta L = 570\lambda$$

where we have used 570 because that is the number of fringes observed for this problem. The distance moved by the mirror is ΔL = 155 μm.

EVALUATE The wavelength is

$$\lambda = \frac{2\Delta L}{570} = \frac{2(155\ \mu\text{m})}{570} = 544\text{ nm}$$

ASSESS This wavelength is in the visible spectrum and corresponds to yellow light.

57. **INTERPRET** This problem involves an interferometer, which is used to measure the refractive index of air. Initially, one arm of the interferometer contains air. This air is gradually pumped out, which reduces the index of refraction in the arm proportionally. When no air is left, 388 bright fringes have been observed at the recombination point of the interferometer. We are to calculate the index of refraction of the air.

DEVELOP When the interferometer arm contains air, its length 2L in wavelengths is $N = 2L/\lambda_{\text{air}} = 2n_{\text{air}}L/\lambda$, where $\lambda_{\text{air}} = \lambda/n_{\text{air}}$ is the wavelength in air and λ = 641.6 nm (the factor 2 arises because the light must travel down and back in the interferometer arm, so measures twice the actual length L). When the interferometer arm is in vacuum, its length in wavelengths becomes $N' = 2L/\lambda$.

EVALUATE The difference between the air length and the vacuum length, in number of wavelengths, gives the number of bright fringes observed. Therefore, we can solve for the index of refraction of air as follows:

$$N - N' = 388 = \frac{2n_{air}L}{\lambda} - \frac{2L}{\lambda}$$
$$n_{\text{air}} = \left(\frac{388\lambda}{2L} + 1\right) = \left(\frac{388(641.6\text{ nm})}{2(42.5\text{ cm})}\right) = 1 + 2.93\times 10^{-4}$$

ASSESS This result agrees with published results dating from 2003.

58. **INTERPRET** This problem is about interference between incoming waves and reflected waves, resulting in a varying signal strength for a given radio frequency. We would like to know the time it would take to drive between minimum and maximum signal strengths when traveling at a given speed.

DEVELOP If the incoming waves are roughly perpendicular to the bridge, the will reflect and have a slight offset from the incoming waves reaching the vehicle directly. The sum of these two waves will result in a signal with a frequency equal to the original signal, meaning the maxima (peak and troughs of the wave) will be separated by

$\lambda/2$. Since we are interested in the time between signal strength maximum and minimum, we want the displacement between the locations with the largest magnitudes and zero, that is $\lambda/4$. We can use the given frequency to calculate this displacement, and use the given vehicle speed to determine the time it takes to travel between them.

EVALUATE The distance between the maximum signal strength and the following minimum signal is equal to

$$\Delta x = \frac{\lambda}{4} = \frac{c}{4f} = \frac{3.00\times10^8\ \text{m/s}}{4(535\ \text{kHz})} = 140\ \text{m}$$

Making the time to travel between these

$$\Delta t = \frac{\Delta x}{v} = \frac{140\,\text{m}}{65.0\,\text{km/h}} = 7.75\,\text{s}$$

ASSESS The interference between the incoming and reflected signals is constructive when the path difference is an integer multiple of a wavelength, and destructive when it is an odd-integer multiple of a half wavelength.

59. **INTERPRET** This problem concerns the diffraction limit of an optical system. The system has circular symmetry, so we can use the Rayleigh criterion for circular apertures.

DEVELOP To resolve a spot of size l = 6.0 m in diameter at a distance d = 66 Mm away, the necessary minimum angle to resolve is very small, so we can approximate

$$\theta_{\text{min}} = l\,/\,d$$

We can insert θ_{min} into Equation 32.11b and solve for the minimum aperture diameter.

EVALUATE The minimum aperture diameter is

$$D_{\text{min}} = \frac{1.22\lambda}{\theta_{\text{min}}} = \frac{1.22\lambda d}{l} = \frac{1.22(1.06\ \mu\text{m})(66\ \text{Mm})}{6.0\,\text{m}} = 14.2\ \text{m}$$

ASSESS We find that the size of this aperture is proportional to the distance d, and inversely proportional to the resolved spot size l.

60. **INTERPRET** This is a problem about the diffraction limit with a circular aperture.

DEVELOP If one of the Keck telescopes were diffraction limited while observing with 550-nm light, its maximum resolution would be

$$\theta_{\text{min}} = \frac{1.22\ \lambda}{D} = \frac{1.22\ (550\ \text{nm})}{10\ \text{m}} = 6.71\times10^{-8}\,\text{rad}$$

At the distance of San Francisco, resolving objects requires a separation of at least

$$\Delta x = \theta_{\text{min}} r = \left(6.71\times10^{-8}\right)(3400\ \text{km}) = 22.8\ \text{cm}$$

We will assume that letters that are bigger than this can be read with the telescope.

EVALUATE **(a)** A newspaper headline might be a few centimeters high, so it would not be possible to read anything like that with a single Keck telescope.

(b) A billboard may have letters that are 50 cm tall or more, so it might be possible to read the sign.

(c) The effective aperture is 5 times wider, so the minimum angle is reduced by a factor of 5. That means letters that are about 4.6 cm tall can be resolved, so it might be possible to read very large headlines in San Francisco from Hawaii.

ASSESS We're assuming the telescope is diffraction limited, but atmospheric turbulence would reduce the resolving power.

61. **INTERPRET** We are to find the smallest spot that can be focused by the given lens system. Because the lens is circular, we shall apply the Rayleigh criterion for circular apertures.

DEVELOP The diffraction limit for a lens opening of diameter D, focusing light of wavelength λ, is given by Equation 32.11b (the Rayleigh criterion for circular apertures):

$$\theta_{\text{min}} = \frac{1.22\lambda}{D}$$

The radius of a spot at the focal length of the lens with this angular spread is $r = f\theta_{\text{min}}$ (the spot radius equals the distance between the central maximum and first minimum; see Fig. 32.7 and accompanying discussion). The minimum spot diameter is, therefore, $d = 2r = 2\theta f_{\text{min}}$.

EVALUATE Inserting the given quantities gives

$$d = 2f\theta_{\text{min}} = \frac{2f(1.22)\lambda}{D} = 2(1.4)(1.22)(610 \text{ nm}) = 2.1 \text{ μm}$$

where we have used $f/D = 1.4$, as given in the problem statement.

ASSESS This resolution is good enough for most commercial cameras.

62. **INTERPRET** You want to estimate the size of a spy satellite given the smallest features it can resolve on the ground.

DEVELOP The altitude of the satellite, h, is so high that the angular separation between two objects on the ground is just: $\theta = \Delta x / h$. You can therefore estimate the minimum diameter of the camera's mirror or lens to be:

$$D > \frac{1.22\lambda}{\theta} = \frac{1.22\lambda h}{\Delta x}$$

EVALUATE Assuming $\lambda = 550$ nm, the minimum diameter is

$$D = \frac{1.22(550 \text{ nm})(100 \text{ km})}{(5 \text{ cm})} = 1.3 \text{ m}$$

Because of the uncertainties, the most that you can probably say is that the satellite's mirror or lens is slightly more than 1 m wide.

ASSESS Most optical systems this big use mirrors rather than lenses as the primary light collector. Lenses have chromatic aberration due to the wavelength dependence of the refractive index. Moreover, a meter-wide lens would weigh much more than a similar-sized mirror, which is a big consideration for anything going up into space.

63. **INTERPRET** We are to determine the largest distance at which humans can resolve a pair of automobile headlights. Because human pupils are circular, the Rayleigh criterion for circular apertures applies.

DEVELOP If we use the Rayleigh criterion (Equation 32.11b for small angles) to estimate the diffraction-limited angular resolution of the eye, at a pupil diameter of 3.1 mm and with light of wavelength 550 nm, we obtain

$$\theta_{\text{min}} = \frac{1.22\lambda}{D} = \frac{1.22(550 \text{ nm})}{3.1 \text{ mm}} = 2.1\times 10^{-4} \text{ rad}$$

EVALUATE This angle corresponds to a linear separation of $y = 1.6$ m at a distance of

$$r = \frac{y}{\theta_{\text{min}}} = \frac{1.6 \text{ m}}{2.1\times 10^{-4} \text{ rad}} = 7.4 \text{ km}$$

ASSESS Actually, the wavelength inside the eye is different $(\lambda' = \lambda / n)$ because of the average index of refraction of the eye. Even though other factors determine visual acuity, this is a reasonable ballpark estimate.

64. **INTERPRET** We are to compare the diffraction-limited resolution at 550 nm with the given limit due to atmospheric turbulence.

DEVELOP Apply the Rayleigh criterion for circular apertures (Equation 32.11b), using

$$\theta_{\text{min}} = 1'' = \frac{\pi}{180\times 3600} = 4.85\times 10^{-6} \text{ rad}$$

EVALUATE The aperture satisfying the Rayleigh criterion at the given wavelength is

$$D = \frac{1.22\lambda}{\theta_{\text{min}}} = \frac{1.22(550 \text{ nm})}{4.85\times 10^{-6} \text{ rad}} = 14 \text{ cm}$$

ASSESS The resolution of all larger-diameter ground-based telescopes is limited by atmospheric conditions at this wavelength.

65. **INTERPRET** The question is whether a microscope using ultraviolet light can resolve rhinovirus.

DEVELOP Suppose the minimum object size that your current optical microscope can resolve is

$$\Delta x = L\theta_{\min} = \frac{1.22\lambda L}{D}$$

where L is the distance between the lens and the sample, and D is the microscope aperture. You can assume that the sales rep's UV microscope has roughly the same geometry, in which case $\Delta x_{\min} \propto \lambda$.

EVALUATE You can assume your optical microscope uses the characteristic visible wavelength of $\lambda = 560$ nm. Therefore, the UV microscope using $\lambda = 280$ nm will have about a factor of 2 better resolution:

$$\Delta x_{\mathrm{UV}} = \Delta x_{\mathrm{opt}} \frac{\lambda_{\mathrm{UV}}}{\lambda_{\mathrm{opt}}} \approx \tfrac{1}{2}\Delta x_{\mathrm{opt}}$$

So, yes, the sales rep is essentially correct, but even the UV microscope won't resolve rhinoviruses.

ASSESS In general, you can only resolve objects as big as the wavelength of the light that you are using. Rhinoviruses are typically only 50 nm, so the studies would require using X-ray diffraction with wavelengths of the order 50 nanometers or smaller.

66. **INTERPRET** We are to calculate the index of refraction of air by comparing the number of fringes in a wedge-shaped film of air trapped between two glass plates with the number of fringes with the air evacuated. The fringes are caused by alternating constructive and destructive interference as the film thickness varies along the wedge-shaped film.

DEVELOP As shown in the solutions to Problems 32.53 and 32.54, the number of bright bands is the largest integer N less than or equal to $2nd/\lambda + 1/2$, where d is the maximum wedge width. For air, this gives

$$N_{\mathrm{air}} = 10{,}003 = 2n_{\mathrm{air}}/\lambda + 1/2$$

For vacuum, this gives

$$N_{\mathrm{vac}} = 10{,}000 = 2/\lambda + 1/2$$

Where we have used $n_{\mathrm{vac}} \equiv 1$.

EVALUATE Solving this system of equations for n_{air} gives

$$10{,}003 = 2n_{\mathrm{air}}(5{,}000 - 1/4) + 1/2$$
$$n_{\mathrm{air}} = \frac{10{,}003 - 1/2}{10{,}000 - 1/2} = 1.0003$$

ASSESS Within the limit imposed by the precision of the data, this result agrees with published results.

67. **INTERPRET** We are to find an expression for the refractive index of a gas that is measured using a Michelson interferometer. We are given the difference in optical path length (i.e., the difference in the number of bright fringes) between a column of gas and an equal length of vacuum.

DEVELOP The index of refraction in vacuum is defined to be unity. For light traveling through a gas, the wavelength of light depends on the gas through which it is traveling ($\lambda_{\mathrm{gas}} = \lambda/n_{\mathrm{gas}}$; λ is the vacuum wavelength). Thus, there is a difference in the number of wave cycles in the enclosed interferometer arm when the cylinder is evacuated or filled with gas. The light travels the length of the arm twice, out and back, and each cycle of difference results in one fringe shift. Thus, the number of fringes in the shift is

$$m = \frac{2L}{\lambda_{\mathrm{gas}}} - \frac{2L}{\lambda} = \frac{2L}{\lambda}(n_{\mathrm{gas}} - 1)$$

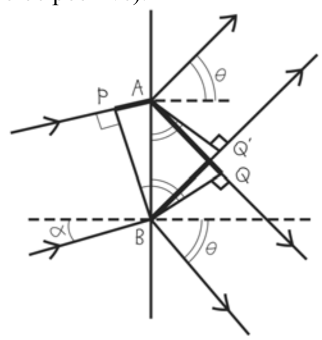
EVALUATE From the above equation, the refractive index is

$$n_{\mathrm{gas}} = 1 + \frac{m\lambda}{2L}$$

ASSESS The interferometer allows for the determination of the refractive index of a gas.

68. **INTERPRET** In this problem we want to verify the condition for maximum intensity when light is incident on a diffraction grating at an arbitrary angle.

DEVELOP Consider the sketch below, which shows light incident on a grating at an angle α with respect to normal incidence. The path difference between the two rays incident on adjacent slits of the grating (*A* and *B*, with spacing $AB = d$) at an angle α with respect to the grating normal is $PA = d\sin\alpha$. The path difference between corresponding outgoing rays making an angle θ on either side of the normal is $AQ = BQ' = d\sin\theta$. The total path difference is the sum (or difference) of these, depending on whether θ is on the same (or opposite) side of the normal as α (since we chose both angles to be positive).



EVALUATE A maximum intensity occurs when the total path difference is an integral number of wavelengths, or when

$$\left.\begin{aligned} PA + AQ &= m\lambda \\ PA - BQ' &= m\lambda \end{aligned}\right\} d\left(\sin\theta \pm \sin\alpha\right) = m\lambda$$

ASSESS When $\alpha = 0$, we recover the usual condition given in Equation 32.1a.

69. **INTERPRET** As one arm of the interferometer moves, the path length in that arm changes, causing alternating constructive and destructive interference (or alternating bright and dark fringes) to occur at the output of the interferometer. We are to calculate the effective path length change due to the presence of gravitational waves, and the shift in the interference pattern resulting from this arm length change.

DEVELOP In each arm of the interferometer, light must travel down and back multiple times. Thus, we can calculate the path length difference corresponding to 280 reflections when a displacement of 1.2 attometers increases one arm's length. We can then determine what fraction of an interference fringe spacing this shift corresponds to.

EVALUATE The effective shift is equal to

$$\Delta L = (280)\left(1.2\times10^{-18}\,\text{m}\right) = 0.34\times10^{-15}\,\text{m} = 0.34\,\text{fm}$$

Using the given wavelength, we find the fraction of the fringe spacing this corresponds to

$$F = \frac{\Delta L}{\lambda/2} = \frac{0.34\,\text{fm}}{(1064\,\text{nm})/2} = 6.3\times10^{-10}\,\text{m}$$

ASSESS This shift corresponds to a distance on the order of a single proton diameter, showing how impressive the detection of these signals has been.

70. **INTERPRET** We are to derive Equation 32.10 using calculus and geometry. We shall base our derivation on Fig. 32.21.

DEVELOP The electric field due to the light coming from a section of slit of width *dy* at position *y* in the slit will be the electric field at that portion of the slit multiplied by the phase factor. There are two contributors to the phase factor: the oscillation of the field in time and the distance the light has traveled from the section of slit. So

$$dE = E_p \frac{dy}{\lambda} \sin\left[\omega t + \phi(y)\right]$$

where the term $E_p\, dy/\lambda$ is the field originating at point *y*, and $\phi(y)$ is the phase angle due to the distance. We will find $\phi(y)$ and integrate over the entire slit to derive Equation 32.10.

EVALUATE **(a)** The path length difference δ for different rays in Fig. 32.21, as a function of y, is $\delta = y\sin\theta$. This path length difference is to the wavelength as the phase difference ϕ is to 2π, so

$$\frac{\delta}{\lambda} = \frac{\phi}{2\pi} \quad \Rightarrow \quad \phi(y) = \frac{2\pi y}{\lambda}\sin\theta$$

which is what we were to show.

(b) We integrate dE from $-a/2$ to $a/2$:

$$\begin{aligned}
E &= \frac{E_p}{a}\int_{-a/2}^{a/2}\sin\left(\omega t + \frac{2\pi y}{\lambda}\sin\theta\right)dy \\
&= \frac{E_p}{a}\int_{-a/2}^{a/2}\left[\sin\omega t\cos\left(\frac{2\pi y}{\lambda}\sin\theta\right) + \sin\left(\frac{2\pi y}{\lambda}\sin\theta\right)\cos\omega t\right]dy \\
&= E_p\left[\sin\omega t\left(-\frac{\lambda}{2\pi a\sin\theta}\right)\sin\left(\frac{2\pi y}{\lambda}\sin\theta\right) + \cos\omega t\left(\frac{\lambda}{2\pi a\sin\theta}\right)\cos\left(\frac{2\pi y}{\lambda}\sin\theta\right)\right]_{-a/2}^{a/2} \\
&= E_p\left[\sin\omega t\left(-\frac{1}{\phi}\right)\sin\left(\frac{\phi}{2}\right) + \cos\omega t\left(\frac{1}{\phi}\right)\cos\left(\frac{\phi}{2}\right) - \sin\omega t\left(-\frac{1}{\phi}\right)\sin\left(-\frac{\phi}{2}\right) - \cos\omega t\left(\frac{1}{\phi}\right)\cos\left(-\frac{\phi}{2}\right)\right] \\
&= \frac{E_p}{\phi}\left[-2\sin\omega t\sin\left(\frac{\phi}{2}\right)\right] = \left(-E_p\sin\omega t\right)\frac{\sin\left(\frac{\phi}{2}\right)}{\frac{\phi}{2}}
\end{aligned}$$

Where we have defined $\phi \equiv 2\pi a/(\lambda\sin\theta)$. The light intensity is proportional to the square of the electric field, so

$$\bar{S} = \bar{S}_0\left[\frac{\sin(\phi/2)}{\phi/2}\right]^2.$$

ASSESS The function $(\sin x)/x$ is also called the "sinc" function: $\text{sinc} \equiv (\sin x)/x$. You will frequently see things written in this form in more advanced optics texts, should you have opportunity to study this field further.

71. **INTERPRET** In this problem, we will use the Rayleigh criterion to determine what angular spacing can be allowed between communications satellites. With this value of the angle, we can find the number of satellites allowed in geosynchronous orbit before their signals begin to overlap.

DEVELOP The Rayleigh criterion for circular apertures (Equation 32.11b) is $\theta_{\min} = 1.22\lambda/D$, where the wavelength is

$$\lambda = \frac{c}{f} = \frac{3.00\times10^8\ \text{m/s}}{12\ \text{GHz}} = 2.5\ \text{cm}$$

and the diameter of the satellite receiver is $D = 47$ cm. The number of satellites that can fit in a circle with this angular spacing between satellites is $N = 2\pi/\theta_{\min}$.

EVALUATE The maximum number of satellites is

$$N = \frac{2\pi}{\theta_{\min}} = \frac{2\pi D}{1.22\lambda} = \frac{2\pi(47\ \text{cm})}{1.22(2.5\ \text{cm})} = 96$$

when rounded down to the nearest integer.

ASSESS This seems rather low, but calculating $\theta_{\min}$ directly gives us an angle of 3.7°, which is consistent. We can pack more in by using shorter wavelengths or larger antennae.

72. **INTERPRET** You are trying to estimate the thickness of an oil spill from the interference pattern that you measure in reflected light.

DEVELOP Some of the light reflects off the air-oil interface, where it experiences a 180° phase change, since the index of refraction is going from lower $(n_{\text{air}}\ 1)$ to higher $(n_{\text{oil}} = 1.39)$. Some of the light also reflects off the oil-water interface, but in this case, there is no 180° phase change, since the index of refraction is going from higher

$(n_{\text{oil}}\ 1.39)$ to lower $(n_{\text{water}}\ 1.333)$. Therefore, the situation is the same as in Fig. 32.12, and constructive interference will occur when $2nd = \left(m+\frac{1}{2}\right)\lambda$ (Equation 32.7).

EVALUATE If the brightest wavelength in reflection is $\lambda = 555$ nm, and you assume this corresponds to $m=1$, then the oil film's thickness must be

$$d = \frac{\left(m+\frac{1}{2}\right)\lambda}{2n} = \frac{3(555\text{ nm})}{4(1.39)} = 299.5\text{ nm}$$

ASSESS This might seem too thin, but it's a reasonable thickness for an oil spill. At this thickness, one can estimate how much area a gallon of oil would cover:

$$A = \frac{V}{d} = \frac{1\text{ gal}}{299.5\text{ nm}}\left[\frac{3.8\times10^{-3}\text{m}^3}{1\text{ gal}}\right] = 12{,}688\ \text{m}^2$$

This is about three football fields!

73. **INTERPRET** We explore how interferometry can increase angular resolution in astronomy.

DEVELOP We are told that interfering the signal of two telescopes will give the resolution of a single telescope with aperture equal to the distance between the two telescopes, that is, $\theta_{\text{min}} = 1.22\lambda/\Delta x$, where Δx is the telescope separation.

EVALUATE Doubling the distance between the two telescopes should reduce by half the minimum angular separation that can be resolved.

The answer is **(c)**.

ASSESS Astronomers use arrays of radio telescopes with individual elements separated by 10s of meters to 1000s of kilometers. The largest arrays can obtain milliarcsecond angular resolution, which is less than a millionth of a degree.

74. **INTERPRET** We explore how interferometry can increase angular resolution in astronomy.

DEVELOP The amount of light collected will be proportional to the sum of the areas of the individual telescopes.

EVALUATE Doubling the distance between the two telescopes does not have any effect on the areas of the telescopes, so there will be no change in the light-collecting power.

The answer is **(a)**.

ASSESS Interferometry does not generally allow astronomers to see objects fainter than can be observed with a single telescope. It only gives them better angular resolution of relatively bright objects in the sky. To see deeper into space, the telescope collecting area has to be increased.

75. **INTERPRET** We explore how interferometry can increase angular resolution in astronomy.

DEVELOP For a point source directly above an interferometer, the light path to each telescope will be the same.

EVALUATE The phase difference is proportional to the path length difference, which in this case is zero. Therefore, the electromagnetic waves will be in phase.

The answer is **(a)**.

ASSESS This answer is independent of the telescope separation or the wavelength being observed. The signals from a source on the bisector between two telescopes will always be in phase. It's a bit like the fact that the zeroth $(m=0)$ order of a double-slit is always a bright fringe, since the path lengths from the two slits are equal along the bisector between them.

76. **INTERPRET** We explore how interferometry can increase angular resolution in astronomy.

DEVELOP In this case, the path length difference to each telescope will be $\Delta x \sin 45°$, where Δx is the telescope separation. This leads to a phase difference in the electromagnetic wave signals received by each telescope:

$$\phi = k(\Delta x \sin 45°) = \frac{\sqrt{2}\pi\Delta x}{\lambda}$$

where we have used $k = 2\pi/\lambda$ and $\sin 45° = 1/\sqrt{2}$.

EVALUATE Without knowing the separation and the wavelength, we can't say what the phase difference is. The answer is **(d)**.

ASSESS If the radio telescopes can receive multiple wavelengths, then it is likely that the two signals will be in phase (constructively interfere) for some wavelengths and 180° out of phase (destructively interfere) for others. Note too, that as the Earth rotates, the angle at which the source is located will change, so even at a fixed wavelength, the relative phase difference (and resultant interference pattern) will be changing with time.