RELATIVITY

33

EXERCISES

Section 33.2 Matter, Motion, and Ether

11. **INTERPRET** In this problem, we are asked to take wind speed into consideration to calculate the travel time of an airplane. Because the speeds involved are much, much less than *c*, we can use nonrelativistic physics.

DEVELOP Since the velocities are small compared to *c*, Equation 33.5a takes the form

$$\vec{u} = \vec{u}' + \vec{v}$$

This is the nonrelativistic Galilean transformation of velocities as given in Equation 3.7, where $\vec{u}$ is the airplane's velocity relative to the ground, $\vec{u}'$ is the velocity of the airplane relative to the air, and $\vec{v}$ is the velocity of the air relative to the ground (i.e., the wind speed). We can use this expression to find the round-trip time *t*, which is

$$t = \frac{2d}{u}$$

with $d = 1800\text{ km}$ and $u = 800\text{ km/h}$.

EVALUATE **(a)** If $\vec{v} = 0$ (no wind), then $\vec{u} = \vec{u}'$ (ground speed equals air speed), and the round-trip travel time is

$$t_a = \frac{2(1800\text{ km})}{760\text{ km/h}} = 4.74\text{ h}$$

(b) If $\vec{v}$ is perpendicular to $\vec{u}$, then $u'^2 = u^2 + v^2$, or

$$u = \sqrt{u'^2 - v^2} = \sqrt{(760\text{ km/h})^2 - (130\text{ km/h})^2} = 749\text{ km/h}$$

and the round-trip travel time is

$$t_b = \frac{2(1800\text{ km})}{749\text{ km/h}} = 4.81\text{ h}$$

(c) If $\vec{v}$ is parallel or anti-parallel to $\vec{u}$ on alternate legs of the round trip, then $u = u' \pm v$, and the travel time is (see Equation 33.2, but with *c* replaced by u')

$$t_c = \frac{d}{u'+v} + \frac{d}{u'-v} = \frac{1800\text{ km}}{760\text{ km/h} + 130\text{ km/h}} + \frac{1800\text{ km}}{760\text{ km/h-}130\text{ km/h}} = 4.88\text{ h}$$

ASSESS We find $t_a < t_b < t_c$, as mentioned in the paragraph following Equation 33.2.

12. **INTERPRET** This problem concerns the Michelson–Morley experiment, which studies the travel time of a wave (i.e., light) that propagates in a moving medium (i.e., the ether). The initial wave is split into two waves, one of which travels parallel to the velocity of the medium and the other of which travels perpendicular to the velocity of the moving medium. We are to find the difference in the travel times for each wave.

DEVELOP The round-trip travel time for light in each arm of the Michelson–Morley experiment is given by Equations 33.1 and 33.2. The difference between these times is

$$\Delta t = t_{\parallel} - t_{\perp} = \frac{2cL}{c^2 - v^2} - \frac{2L}{\sqrt{c^2 - v^2}} = \frac{2L}{c}\left(\frac{1}{1 - v^2/c^2} - \frac{1}{\sqrt{1 - v^2/c^2}}\right)$$

EVALUATE (**a**) If $v^2/c^2 \ll 1$ we can expand the denominators (Appendix A) to obtain

$$\Delta t \approx \frac{2L}{c}\left[1+\frac{v^2}{c^2}-\left(1+\frac{v^2}{2c^2}\right)\right]=\frac{Lv^2}{c^3}$$

Inserting the given values yields

$$\Delta t \approx \frac{(11\text{ m})\left(30\times10^3\text{ m/s}\right)^2}{\left(3.00\times10^8\text{ m/s}\right)^2}=3.7\times10^{-18}\text{ s}$$

(**b**) For v =0.01c, we find

$$\Delta t \approx \frac{(11\text{ m})(0.01)^2}{3.00\times10^8\text{ m/s}}=3.7\times10^{-12}\text{ s}$$

(**c**) For v = 0.5c, we use the exact expression, which gives

$$\Delta t = \frac{2(11\text{ m})}{3.00\times10^8\text{ m/s}}\left(\frac{1}{1-1/4}-\frac{1}{\sqrt{1-1/4}}\right)=13\text{ ns}$$

(**d**) For v = 0.99c, we find

$$\Delta t = \frac{2(11\text{ m})}{3.00\times10^8\text{ m/s}}\left(\frac{1}{1-0.99^{-2}}-\frac{1}{\sqrt{1-0.99^{-2}}}\right)=3.2\ \mu\text{s}$$

ASSESS The faster the ether wind, the greater is the time difference between the two paths.

Section 33.4 Space and Time in Relativity

13. **INTERPRET** This problem involves measuring a distance in two different frames of reference, the first of which is at rest with respect to the endpoints of the measurement and the second of which is not.

DEVELOP The distance between stars at rest in system S appears Lorentz-contracted in the spaceship's system S' according to Equation 33.4:

$$\Delta x' = \Delta x\sqrt{1-v^2/c^2}$$

EVALUATE With $\Delta x = 35$ ly and $v = 0.50c$, we get

$$\Delta x' = (35\text{ ly})\sqrt{1-0.5^2} = 30\text{ ly}$$

ASSESS The distance appears to be shortened or "contracted" as observed by the spaceship. Note that length contraction occurs only along the direction of motion.

14. **INTERPRET** We want to find how long a trip to Pluto takes according to a clock on Earth and on the spacecraft.

DEVELOP For the distance, we'll assume that Earth and Pluto are along the same radial line from the Sun, in which case:

$$\Delta r = r_P - r_E = 5.91\times10^9\text{ km} - 150\times10^6\text{ km} = 5.76\times10^9\text{ km}$$

where we've taken the orbital radii from Appendix E. But from the perspective of the spacecraft, the distance is contracted according to Equation 33.4: $\Delta r' = \Delta r\sqrt{1-v^2/c^2}$. The time to reach Pluto in either case is the distance divided by the velocity, which is 0.66c for both cases.

EVALUATE **(a)** According to clocks on Earth, the trip takes

$$\Delta t = \frac{\Delta r}{v} = \frac{5.76\times10^9\text{ km}}{0.66\left(3\times10^8\text{ m/s}\right)} = 8.1\text{ h}$$

(b) According to clocks on the spacecraft, the trip takes

$$\Delta t' = \frac{\Delta r'}{v} = \frac{\Delta r}{v}\sqrt{1-v^2/c^2} = (8.1\text{ h})\sqrt{1-(0.66)^2} = 6.1\text{ h}$$

Assess The same answer can be reached by arguing that time dilation (Equation 33.3) makes the clock on the spacecraft appear to run slower than the clock on Earth.

15. **Interpret** This problem involves measuring the length of an object in its rest frame and in a frame of reference that is moving with respect to the object. The concept of length contraction will apply.

Develop We are given the length $\Delta x' = 40$ m, which is the length measured in a frame moving at $v = 0.5c$. Equation 33.4, $\Delta x' = \Delta x\sqrt{1 - v^2/c^2}$, gives the length Δx measured in the rest system of the spaceship.

Evaluate Solving the equation for Δx gives

$$\Delta x = \frac{\Delta x'}{\sqrt{1 - v^2/c^2}} = \frac{40 \text{ m}}{\sqrt{1 - 1/4}} = 46.2 \text{ m}$$

Assess The spaceship is longest in its own rest frame and is shorter to observers for whom it's moving.

16. **Interpret** This problem involves measuring time in two different frames of reference. We are to find the time it takes a spacecraft to travel between two points according to an observer who is stationary with respect to these points and according to its internal clock. We shall use the principles of time dilation and proper time.

Develop According to an observer on Earth, the spacecraft moves a distance $d = 8.3$ light minutes at a speed $v = 0.80c$. From the definition of a light minute, we can obtain the time Δt the spacecraft takes according to the Earth observer. We then use the time dilation equation $\Delta t' = \Delta t\sqrt{1 - v^2/c^2}$ to find the time that a clock on the spacecraft measures.

Evaluate (**a**) Because the distance is measured in the Earth rest frame, the time is simply the distance divided by the speed of the spaceship:

$$\Delta t = \frac{8.3\, c \cdot \text{min}}{0.80\, c} = 10 \text{ min}$$

(b) An observer on the spaceship measures a time

$$\Delta t' = \Delta t\sqrt{1 - v^2/c^2} = (10.4 \text{ min})\sqrt{1 - 0.80^{-2}} = 6.2 \text{ min}$$

Assess "Moving clocks run slow." According to the clock in the spacecraft, it takes less time to cross the distance from the Earth to the Sun than it does according to the clock on the Earth.

17. **Interpret** This is a problem about length contraction. The meter stick is measured to be shorter when it appears to be moving relative to you.

Develop The distance you measure in a frame of reference S' moving (in a direction parallel to the length of the meter stick) with speed v is $\Delta x' = 97$ cm, whereas the proper length of the meter stick is $\Delta x = 100$ cm (in system S). These are related by Equation 33.4, $\Delta x' = \Delta x\sqrt{1 - v^2/c^2}$, which gives $0.97 = \sqrt{1 - v^2/c^2}$.

Evaluate Solving for v, we get $v = c\sqrt{1 - 0.97^2} = 0.24c$.

Assess For the meter stick to measure 1% shorter, you would need to be moving at about 24% the speed of light with respect to the meter stick.

18. **Interpret** This problem deals with length contraction.

Develop In the electrons' reference frame, the accelerator is moving at a speed of $0.98c$. Therefore, its length will appear contracted according to Equation 33.4: $\Delta x' = \Delta x\sqrt{1 - v^2/c^2}$.

Evaluate The accelerator length in the electron frame is

$$\Delta x' = \Delta x\sqrt{1 - v^2/c^2} = (1.6 \text{ m})\sqrt{1 - (0.98)^2} = 0.32 \text{ m}$$

Assess This is a factor of 5 smaller than the length in the rest frame of the accelerator.

Section 33.7 Energy and Momentum in Relativity

19. **INTERPRET** We want to know the change in momentum when we double the speed, both in the nonrelativistic and relativistic limits.

DEVELOP The measure of momentum valid at any speed is given by Equation 33.7:

$$\vec{p} = \frac{m\vec{u}}{\sqrt{1-u^2/c^2}} = \gamma m\vec{u}$$

Thus, doubling the speed (u becomes $2u$) increases the momentum by a factor

$$\frac{\vec{p}(2u)}{\vec{p}(u)} = \frac{2\sqrt{1-u^2/c^2}}{\sqrt{1-4u^2/c^2}}$$

EVALUATE **(a)** When $u = 28\text{ m/s},\ u/c \approx 0;$ so, the above factor is 2.0 (to two significant figures).
(b) If $u/c = 110/300,$ the factor is

$$\frac{p(2u)}{p(u)} = \frac{2\sqrt{1-(110/300)^2}}{\sqrt{1-4(110/300)^2}} = 2.74$$

ASSESS In the nonrelativistic limit, momentum $\vec{p}$ is linear in $\vec{u}$ and $\vec{p} \approx m\vec{u},$ but this no longer holds in the relativistic limit.

20. **INTERPRET** This problem involves relativistic momentum. We are to find the speed at which an object with 1 unit of mass has the same momentum as that of a 4-unit-mass particle traveling at $0.5c$.

DEVELOP Using Equation 33.7, we see that the momenta of the proton and alpha particle are equal when

$$\frac{m_p u_p}{\sqrt{1-u_p^2/c^2}} = \frac{4m_p u_\alpha}{\sqrt{1-u_\alpha^2/c^2}}$$

EVALUATE Square and solve for u_p to obtain

$$\frac{u_p}{c} = \left[1+\left(\frac{m_p}{m_\alpha}\right)^2\left(\frac{c^2}{u_\alpha^2}-1\right)\right]^{-1/2} = \left[1+\left(\frac{1}{4}\right)^2\left(\frac{1}{0.5^2}-1\right)\right]^{-1/2} = \frac{1}{\sqrt{1-3/16}} = 0.92$$

ASSESS The speed of the proton would need to be over 90% the speed of light.

21. **INTERPRET** The Newtonian expression $\vec{p} = m\vec{u}$ is valid only when $u \ll c$ and for constant mass. We want to know the speed at which the difference between this and the relativistic expression is 1%.

DEVELOP From Equation 33.7, we find that the error in the Newtonian expression of momentum is

$$\frac{\Delta p}{p} = \frac{\gamma mu - mu}{\gamma mu} = 1 - \frac{1}{\gamma} = 1 - \sqrt{1-u^2/c^2}$$

EVALUATE When this factor is equal to 0.04, the speed is

$$u = c\sqrt{2\Delta p/p - \Delta p^2/p^2} = c\sqrt{2(0.04)-(0.04)^2} = 0.28c$$

ASSESS Although $\vec{p} = m\vec{u}$ is valid at low velocity, in the relativistic limit where v/c is not negligible, Equation 33.7 should be used instead.

22. **INTERPRET** This problem concerns the relationship between relativistic momentum and velocity. We are to find by how much the momentum increases for a 10% increase in the given velocity.

DEVELOP Apply Equation 33.7, $p = \gamma mu$. The initial momentum is

$$p(u) = \gamma(u)mu = \frac{(0.90)mc}{\sqrt{1-(0.90)^2}}$$

and the final momentum is

$$p(1.1u) = 1.1\gamma(1.1u)mu = \frac{(1.1)(0.90)mc}{\sqrt{1-(1.1)^2(0.90)^2}}$$

Take the ratio to find the increase in momentum.

EVALUATE The increase in momentum is

$$\frac{p(1.1u)}{p(u)} = \frac{(1.1)\sqrt{1-(0.90)^2}}{\sqrt{1-(1.1)^2(0.90)^2}} = 3.4$$

ASSESS This results seems reasonable in light of Fig. 33.17, which shows that the momentum increases superlinearly for $u/c \approx 0.9$.

23. **INTERPRET** The electron is moving at a relativistic speed, and we want to know its total energy and kinetic energy.

DEVELOP The total energy of the electron is given by Equation 33.9:

$$E = \gamma mc^2 = \frac{mc^2}{\sqrt{1-u^2/c^2}}$$

Its kinetic energy is given by Equation 33.8:

$$K = \gamma mc^2 - mc^2 = (\gamma - 1)mc^2$$

For this problem, the electron's speed is $v = 0.92c$, so

$$\gamma = \frac{1}{\sqrt{1-v^2/c^2}} = \frac{1}{\sqrt{1-(0.92)^2}} = 2.55$$

and $mc^2 = 0.511$ MeV.

EVALUATE **(a)** From the information above, the total energy is

$$E = \gamma mc^2 = (2.55)(0.511\text{ MeV}) = 1.3\text{ MeV}$$

(b) The kinetic energy is

$$K = (\gamma - 1)mc^2 = (2.55 - 1.00)(0.511\text{ MeV}) = 0.79\text{ MeV}$$

ASSESS The total energy and kinetic energy are related by $E = K + mc^2$, where mc^2 is the rest energy of the particle. The expression demonstrates the equivalence between mass and energy. The results are reported to two significant figures to reflect the precision of the input data.

24. **INTERPRET** This problem is similar to Problem 33.22, except that we are now to find the speed at which the relativistic and Newtonian kinetic energy differ by 10%.

DEVELOP The Newtonian kinetic energy is $E_N(u) = mu^2/2$ and the relativistic kinetic energy is (Equation 33.8) $E(u) = \gamma(u)mc^2 - mc^2$. A difference of 10% translates to

$$0.10 = \frac{E(u) - E_N(u)}{E(u)} = \frac{(\gamma-1)mc^2 - mu^2/2}{(\gamma-1)mc^2} \quad \Rightarrow \quad \frac{u^2}{c^2} = 1.8(\gamma - 1)$$

$$1 - \frac{1}{\gamma^2} = \frac{(\gamma-1)(\gamma+1)}{\gamma^2} \quad \Rightarrow \quad 0 = 1.8\gamma^2 - \lambda - 1$$

which we can solve for γ and thus find the speed u.

EVALUATE The solution to the quadratic in γ is

$$\gamma = \frac{1+\sqrt{1+7.2}}{3.6} = 1.07$$

so the speed u is

$$u = c\sqrt{1 - 1/\gamma^2} = c\sqrt{1-(1.07)^{-2}} = 0.36c$$

ASSESS The result is reported to two significant figures to reflect the precision of the data. From Fig. 33.17, we see that a 10% error in kinetic energy occurs for speeds much lower than for momentum.

EXAMPLE VARIATIONS

25. **INTERPRET** This is a problem about time dilation. Since the events of departure from Earth and arrival at the star nebula occur at the same place in the ship's reference frame, we identify the ship time as the proper time $\Delta t'$ and the Earth time as Δt in our discussion of time dilation. We want to find both times knowing the distance traveled and speed of the spacecraft.

DEVELOP We will calculate the time as seen from observers on Earth using the speed and distance given, and then following the same approach as the original example, we use the given speed and Equation 33.3 to determine the proper time (i.e., the time experienced by those on board the ship).

EVALUATE **(a)** We begin by calculating the value of the relativistic factor

$$\gamma = \frac{1}{\sqrt{1-(0.995)^2}} = 31.6$$

Using this, we find the time for this trip measured from the spacecraft's frame is equal to

$$\Delta t' = \frac{\Delta x}{\gamma v} = \frac{(1344\,\text{ly})(c\times 1\,\text{y})}{(31.6)(0.995c)} = 42.52\,\text{y}$$

(b) The time for this trip measured from the Earth's frame is equal to s

$$\Delta t = \frac{\Delta x}{v} = \frac{(1344\,\text{ly})(c\times 1\,\text{y})}{(0.995c)} = 1345\,\text{y}$$

ASSESS The time experienced by those on the ship is considerably shorter than what those on Earth measure, confirming our statement that the time between events is shortest as measured in a reference frame where the events occur at the same place.

26. **INTERPRET** This is a problem about time dilation. Since the events of departure from Earth and arrival at the star nebula occur at the same place in the ship's reference frame, we identify the ship time as the proper time $\Delta t'$ and the Earth time as Δt in our discussion of time dilation. We want to determine the speed a space probe would need to travel at to make the trip of the preceding problem in a given amount of time.

DEVELOP The relevant time here is that measured by the probe, $\Delta t'$, which we can use to obtain the speed of the spacecraft from the expression

$$\Delta t' = \frac{\Delta x}{\gamma v} \rightarrow v\frac{\Delta t'}{\Delta x} = \sqrt{1-\frac{v^2}{c^2}}$$
$$v = \sqrt{\frac{\Delta x}{\Delta x + c^2\Delta t'^2}}\,c$$

EVALUATE Evaluating the speed for the distance given in the previous problem and the proper time desired we obtain a required speed of

$$v = \frac{\Delta x}{\sqrt{\Delta x^2 + c^2\Delta t'^2}}c = \frac{(1344\,\text{ly})(c\times 1\,\text{y})}{\sqrt{(1344\,\text{ly})^2(c\times 1\,\text{y})^2 + c^2(675\,\text{y})^2}}c = 0.8936c$$

ASSESS Such a trip would be measured to take approximately 1500 years by those on Earth.

27. **INTERPRET** This is a problem about time dilation. Since the events of departure from Earth and arrival at the asteroid occur at the same place in the sample's reference frame, we identify the sample time as the proper time $\Delta t'$ and the Earth time as Δt in our discussion of time dilation. We want to find the number of half-lives which pass by the time the O-15 sample arrives at the asteroid given the travel distance and speed.

DEVELOP We will use the given distance and speed, along with Equation 33.3, to determine the proper time (i.e., the time experienced by the sample of O-15). We will then determine the number of half-lives transpired in the journey to the astronauts on the asteroid.

EVALUATE We begin by calculating the value of the relativistic factor

$$\gamma = \frac{1}{\sqrt{1-(0.842)^2}} = 1.854$$

Using this, we find the time for this trip measured from the sample's frame is equal to

$$\Delta t' = \frac{\Delta x}{\gamma v} = \frac{(6.38\text{l-min})(c\times 1\,\text{min})}{(1.854)(0.842c)} = 4.08\,\text{min}$$

With a half-life of 2.04 minutes, the O-15 sample experiences 2.00 lifetimes during its trip to the asteroid.

ASSESS The time experienced by the sample is considerably shorter than what those on Earth measure, confirming our statement that the time between events is shortest as measured in a reference frame where the events occur at the same place.

28. **INTERPRET** This is a problem about time dilation. Since the events of departure from Earth and arrival at the Mar's base occur at the same place in the sample's reference frame, we identify the sample time as the proper time $\Delta t'$ and the Earth time as Δt in our discussion of time dilation. We want to determine the speed a ship carrying the sample from the previous problem would need to travel at to make the trip to Mars within two half-lives.

DEVELOP The relevant time here is the two half-lives experienced by the sample, $\Delta t'$, which we can use to obtain the speed of the spacecraft from the expression

$$\Delta t' = \frac{\Delta x}{\gamma v} \;\rightarrow\; v\frac{\Delta t'}{\Delta x} = \sqrt{1-\frac{v^2}{c^2}}$$
$$v = \sqrt{\frac{\Delta x}{\Delta x + c^2\Delta t'^2}}\,c$$

EVALUATE Evaluating the speed for the 22.8 light-minute trip to Mars and the proper time desired we obtain a required speed of

$$v = \frac{\Delta x}{\sqrt{\Delta x^2 + c^2\Delta t'^2}}c = \frac{(22.8\text{l-min})(c\times 1\,\text{min})}{\sqrt{(22.8\text{l-min})^2(c\times 1\,\text{min})^2 + c^2(2.00\,\text{min})^2}}c = 0.984c$$

ASSESS Such a trip would be relativistically doable but technically impossible since no current man-made probe can travel at such high speeds.

29. **INTERPRET** We're given two distant events—both supernova explosions—that are simultaneous in a particular reference frame. We're asked to find the time between them in a different reference frame. So, this problem is about using the Lorentz transformations for time coordinates.

DEVELOP Following the same strategy taken in the original example, we will set the coordinates of the civilizations A and B in reference frame S as: $x_A = 0$, $t_A = 0$, $x_B = 95.0\times 10^3$ ly, and $t_B = 0$. The traveling ship's coordinate system S' travels along a line passing from civilization A to B, so we set $t = 0$ to coincide with $t_A' = 0$. We can use the Lorentz transformation to determine the temporal coordinate t_B' as seen by the traveling ship.

EVALUATE First we evaluate the relativistic factor and obtain

$$\gamma = \frac{1}{\sqrt{1-(0.774)^2}} = 1.58$$

We then evaluate the perceived time of civilization B's satelite launch as seen by the traveling ship to be

$$t_B' = \gamma\left(t_B - \frac{vx_B}{c^2}\right) = 1.58\left(0 - \frac{(0.774\,\text{ly/y})(95.0\times 10^3\,\text{ly})}{(1\,\text{ly/y})^2}\right) = -116{,}000\,\text{y}$$

Meaning, observers in the spacecraft see civilization B's satellite launching first by 116,000 years.

ASSESS You can easily show that a spacecraft observer going the other way would judge the civilization B's satellite launch to occur 116,000 years later.

30. **INTERPRET** We're given two distant events—both supernova explosions—that are simultaneous in a particular reference frame. We're asked to find the time between them in a different reference frame. So, this problem is about using the Lorentz transformations for time coordinates.

DEVELOP Following the same strategy taken in the original example, we will set the coordinates of the civilizations A and B in reference frame S as: $x_A = 0$, $t_A = 0$, $x_B = 95.0\times10^3$ ly, and $t_B = 0$. The traveling ship's coordinate system S' travels along a line passing from civilization A to B, so we set $t = 0$ to coincide with $t_A' = 0$. We can use the Lorentz transformation to determine the temporal coordinate t_B' as seen by the traveling ship. Since we are told what this time difference is, we solve for the speed in the expression for the coordinate change.

EVALUATE Solving for the speed in the expression for temporal coordinate transformation we obtain

$$-|t'_B| = \frac{\left(0 - \frac{vx_B}{c^2}\right)}{\sqrt{1-\frac{v^2}{c^2}}} \rightarrow t'^2_B\left(1-\frac{v^2}{c^2}\right) = \frac{v^2 x_B^2}{c^4}$$

$$v = \frac{c^2 t'_B}{\sqrt{x_B^2 + c^2 t'^2_B}}$$

We then evaluate the necessary speed of civilization D's ship to see the events separated by 52,300 years to be

$$v = \frac{c^2 t'_B}{\sqrt{x_B^2 + c^2 t'^2_B}} = \frac{(1\,\text{ly/y})(52{,}300\,\text{y})}{\sqrt{(95.0\times10^3\,\text{ly})^2 + (1\,\text{ly/y})^2(52{,}300\,\text{y})^2}}c = 0.482c$$

ASSESS Since this ship is moving slower than the one discussed in the previous problem, it sees these events occurring closer in time, perceiving them "closer" to simultaneously than the faster moving ship.

31. **INTERPRET** We're given two distant events—both supernova explosions—that are not simultaneous in a particular reference frame. We're asked to find the time between them in a different reference frame. So, this problem is about using the Lorentz transformations for time coordinates.

DEVELOP Following the same strategy taken in the original example, we will set the coordinates of the civilizations A and B in reference frame S as: $x_A = 0$, $t_A = 0$, $x_B = 95.0\times10^3$ ly, and $t_B = 24{,}700$ y. The traveling ship's coordinate system S' travels along a line passing from civilization A to B, so we set $t = 0$ to coincide with $t_A' = 0$. We can use the Lorentz transformation to determine the temporal coordinate t_B' as seen by the traveling ship. We can then solve for the speed an observer would need to travel at, along the same direction, to perceive both events occurring simultaneously.

EVALUATE **(a)** Using the relativistic factor we obtained in Example Variation 29 we can evaluate the perceived time of civilization B's satellite launch as seen by the traveling ship to be

$$t'_B = \gamma\left(t_B - \frac{vx_B}{c^2}\right) = 1.58\left((24{,}700\,\text{y}) - \frac{(0.774\,\text{ly/y})(95.0\times10^3\,\text{ly})}{(1\,\text{ly/y})^2}\right) = -77{,}100\,\text{y}$$

Meaning, observers in the spacecraft see civilization B's satellite launching first by 77,100 years.

(b) Using the expression for the velocity we found in the previous problem, we find the required speed to perceive both events as simultaneous is equal to

$$-|t'_B| = \frac{\left(t_B - \frac{vx_B}{c^2}\right)}{\sqrt{1-\frac{v^2}{c^2}}} = 0 \;\rightarrow v = \frac{c^2 t_B}{x_B}$$

$$v = \frac{c^2 t_B}{x_B} = \frac{(1\,\text{ly/y})(24{,}700\,\text{y})}{(95.0\times10^3\,\text{ly})}c = 0.260c$$

Such an observer is moving on the same path as civilization C's spacecraft.

ASSESS Could an observer moving in the opposite direction (i.e., from civilization B to civilization A) perceive these events as simultaneous? If so, at what speed would they need to travel?

32. **INTERPRET** We're given two distant events—both supernova explosions—that are not simultaneous in a particular reference frame. We're asked to find the time between them in a different reference frame. So, this problem is about using the Lorentz transformations for time coordinates.

DEVELOP Following the same strategy taken in the original example, we will set the coordinates of the civilizations A and B in reference frame S as: $x_A = 0$, $t_A = 0$, $x_B = 95.0\times10^3$ ly, and $t_B = 114{,}000$ y. The traveling ship's coordinate system S' travels along a line passing from civilization A to B, so we set $t = 0$ to coincide with $t_A' = 0$. We can use the Lorentz transformation to determine the temporal coordinate t_B' as seen by the traveling ship. We can then solve for the speed an observer would need to travel at, along the same direction, to perceive both events occurring simultaneously.

EVALUATE **(a)** Using the relativistic factor we obtained in Example Variation 29 we can evaluate the perceived time of civilization B's satellite launch as seen by the traveling ship to be

$$t_B' = \gamma\left(t_B - \frac{vx_B}{c^2}\right) = 1.58\left((114{,}000\,\text{y}) - \frac{(0.774\,\text{ly/y})(95.0\times10^3\,\text{ly})}{(1\,\text{ly/y})^2}\right) = 63{,}900\,\text{y}$$

Meaning, observers in the spacecraft see civilization A's satellite launching first by 63,900 years.

(b) Using the expression for the velocity we found in the previous problem, we find the required speed to perceive both events as simultaneous is equal to

$$-|t_B'| = \frac{\left(t_B - \dfrac{vx_B}{c^2}\right)}{\sqrt{1-\dfrac{v^2}{c^2}}} = 0 \quad \rightarrow v = \frac{c^2 t_B}{x_B}$$

$$v = \frac{c^2 t_B}{x_B} = \frac{(1\,\text{ly/y})(114{,}000\,\text{y})}{(95.0\times10^3\,\text{ly})}c = 1.2c$$

This means no observer can judge the two events to be simultaneous because they're far enough apart in time that they could be casually related.

ASSESS What is the largest time difference between the two events judged by the galaxy's frame that would keep the events from being casual in another observer's reference frame?

PROBLEMS

33. **INTERPRET** We are to compare the round-trip times in each branch of a Michelson–Morley interferometer. One branch is taken to be parallel to the hypothesized ether wind and the other branch is perpendicular to the ether wind. In particular, we are to show that $t_{\text{parallel}} > t_{\text{perpendicular}}$ for $0 < v < c$.

DEVELOP The ratio of Equations 33.2 to 33.1 is

$$\frac{t_{\text{parallel}}}{t_{\text{perpendicular}}} = \frac{2cL\big/\left(c^2 - v^2\right)}{2L\big/\sqrt{c^2 - v^2}} = \frac{1}{\sqrt{1 - v^2/c^2}}$$

EVALUATE For $0 < v < c$, the denominator of the ratio above ranges from unity to zero, so the ratio ranges from unity to infinity. Thus, we have

$$1 < \frac{t_{\text{parallel}}}{t_{\text{perpendicular}}} < \infty$$

$$t_{\text{perpendicular}} < t_{\text{parallel}}\infty$$

as claimed in the problem statement.

ASSESS Since $t_{\text{perpendicular}} < t_{\text{parallel}}$, we conclude that the trip parallel to the ether wind always takes longer.

34. INTERPRET You want to design a Michelson interferometer that would be sensitive to speed of light variations as small as $100\ \text{m/s}$.

DEVELOP Let's define c_x and c_y as the speed of light in the perpendicular x- and y-directions, respectively. You're told to assume that $c_x - c_y = 100\ \text{m/s}$. Therefore, it will take longer for light to make the round trip in the interferometer arm aligned with the y-direction (see Fig. 33.2):

$$t_y = \frac{2L}{c_y} > \frac{2L}{c_x} = t_x$$

This time difference is supposed to cause a bright fringe in 560-nm light where the adjacent dark fringe would be in the absence of a speed difference. In other words, the speed difference should cause a 180° phase difference between the two paths:

$$\Delta\phi = \omega t_y - \omega t_x = \pi$$

EVALUATE Substituting the time values in the phase difference equation gives the length of the interferometer:

$$L = \frac{\pi}{2\omega}\left(\frac{c_x c_y}{c_x - c_y}\right)$$

Since the speed difference is much smaller than the measured value of c, the numerator in the above fraction can be approximated as $c_x c_y \approx c^2$. Therefore, the length equation becomes

$$L \approx \frac{\lambda}{4}\left(\frac{c}{c_x - c_y}\right) = \frac{560\ \text{nm}}{4}\left(\frac{3\times 10^8\ \text{m/s}}{100\ \text{m/s}}\right) = 42\ \text{cm}$$

ASSESS The length is inversely proportional to the speed difference. This means that the interferometer would have to be 100 times longer in order to measure a speed difference of $1\ \text{m/s}$.

35. INTERPRET This is a problem involving travel time measured in different reference frames—one reference frame is at rest with respect to the end points of the trip, whereas the other reference frame is not. Time dilation is involved.

DEVELOP Note that the distance is given in the system S, where Earth and the Sun are essentially at rest (the orbital speed of Earth is very small compared to the speed of light or the speed of the spacecraft). However, the time interval is given in system S', where the spacecraft is at rest. In other words, $\Delta x = 8.4c\cdot\text{min}$ and $\Delta t' = 4.6\ \text{min}$. Equations 33.3 and 33.4

$$\Delta t' = \Delta t\sqrt{1 - v^2/c^2} = \frac{\Delta t}{\gamma}$$

$$\Delta x' = v\Delta t' = \Delta x\sqrt{1 - v^2/c^2}$$

express time dilation and Lorentz contraction, respectively. The speed of the spacecraft can be found by using the second expression, which gives

$$v = \frac{\Delta x}{\Delta t'}\sqrt{1 - v^2/c^2}$$

$$= \frac{c}{\sqrt{1 + \left(c\Delta t'/\Delta x\right)^2}}$$

Once we know v, we can evaluate $\gamma = \left(1 - v^2/c^2\right)^{-1/2}$ and find the time in the rest frame using the expression above for time dilation, which gives $\Delta t = \gamma\Delta t'$.

EVALUATE **(a)** Inserting the given values into the expression for velocity gives

$$v=\frac{c}{\sqrt{1+\left[\frac{c(4.6\text{ min})}{8.4c\cdot\text{min}}\right]^2}}=0.88c$$

(b) The time in the rest frame is

$$\Delta t=\frac{\Delta t'}{\sqrt{1-v^2/c^2}}=9.6\text{ min}$$

ASSESS The result demonstrates that clocks (inside the spacecraft) moving relative to the Earth-Sun frame appear to run more slowly (4.6 min) compared to the clocks at rest (9.6 min).

36. **INTERPRET** You need to determine the time it will take to for an intergalactic message to be received.

DEVELOP The captain assumes that clocks on Earth are running at the same rate as clocks on the spacecraft. But this is not true. To complete the 2-million-light-year trip in 50 years means the spacecraft is traveling very close to the speed of light, so you have to consider time dilation between the clocks on Earth and the spacecraft.

EVALUATE Since the ship is moving very close to the speed of light, it will take a little over 2 million years in the Earth's reference frame for the ship to complete its journey to the Andromeda Galaxy. Likewise, it will take 2 million years for the message to travel back to Earth. Thus, the message will arrive about 4 million years after the ship left Earth.

ASSESS Although its not necessary, you can calculate your ship's velocity from the time $(\Delta t'=50\text{ y})$ that it takes you to reach Andromeda. You assume there has been time dilation according to Equation 33.3: $\Delta t'=\Delta t\sqrt{1-v^2/c^2}$. You can express the time measured on Earth as $\Delta t=\Delta x/v$, where $\Delta x=2\times10^6$ ly is the distance to Andromeda measured in the Earth's reference frame. With a little algebra, you can show that the ship's velocity is

$$v=\frac{c}{\sqrt{1+(c\Delta t'/\Delta x)^2}}\approx c\left[1-\tfrac{1}{2}(c\Delta t'/\Delta x)^2\right]\approx c-10\text{ cm/s}$$

This is close enough to the speed of light that in the Earth's reference frame the ship arrives at Andromeda 2 million years after it left.

37. **INTERPRET** This is a problem about time measured in different reference frames. The first frame is the rest frame of the end points of the journey (separated by N light-years), and the second frame is the moving frame of the spaceship (in which the time spent traveling is to be N years). Time dilation is involved.

DEVELOP The distance is given in the system S, where the Earth and the distant star are essentially at rest, but the time interval is given in the moving system S', where the spacecraft is at rest. Thus, $\Delta x=N$ ly and $\Delta t'=N$ y. One ly, or light-year, is the distance light travels in 1 year, and equals c multiplied by 1 year. Equations 33.3 and 33.4,

$$\Delta t'=\Delta t\sqrt{1-v^2/c^2}=\frac{\Delta t}{\gamma}$$
$$\Delta x'=v\Delta t'=\Delta x\sqrt{1-v^2/c^2}$$

for time dilation and Lorentz contraction, relate the given quantities to $\Delta x'$ and $\Delta t'$. We use the second expression (i.e., the expression for length contraction) to find

$$v=\frac{\Delta x}{\Delta t'}\sqrt{1-v^2/c^2}$$
$$=\frac{c}{\sqrt{1+(c\Delta t'/\Delta x)^2}}$$

EVALUATE Inserting the given quantities gives

$$v = \frac{c}{\sqrt{1+\left[\frac{c(N\text{ y})}{N\,c\cdot\text{y}}\right]^2}} = \frac{c}{\sqrt{2}} = 0.71c$$

ASSESS To show that the result is consistent, we note that in the reference frame of the spacecraft the distance $\Delta x'$ is contracted:

$$\Delta x' = \Delta x\sqrt{1-v^2/c^2} = \Delta x\sqrt{1-1/2} = N/\sqrt{2}\text{ ly}$$

So it will take

$$\Delta t' = \frac{\Delta x'}{v} = \frac{N/\sqrt{2}\,c\cdot\text{y}}{c/\sqrt{2}} = N\text{ y}$$

of the traveler's life to get there.

38. **INTERPRET** This problem involves measurements of time and space in different inertial reference frames. The first frame is at rest with respect to the end points of the journey, and the second frame is the moving frame in which the spaceship is at rest. We are given the distance as measured in the first frame and are asked to find the time for this trip as measured from that frame. We also want the distance and time in the second frame.

DEVELOP The stationary frame *S* is that in which the Earth and the star are essentially at rest, where the spacecraft is traveling at a speed $v = 0.2c$, and the distance for the journey in this frame is $\Delta x = 4.24$ ly. Thus, the we can determine the time as measured from the frame of Earth simply from # speed of the spaceship is $v = \Delta x/\Delta t$. From this speed, we can also evaluate the factor γ:

$$\gamma = \frac{1}{\sqrt{1-v^2/c^2}} = \frac{1}{\sqrt{1-(0.2)^2}} = 1.021$$

and find $\Delta x'$ and $\Delta t'$ using Equations 33.4 and 33.3, which give

$$\Delta x' = \frac{\Delta x}{\gamma} \quad \text{and} \quad \Delta t' = \frac{\Delta t}{\gamma}$$

which are the distance and time for the trip as measured aboard the spaceship.

EVALUATE **(a)** We first find the time for this trip measured from the first frame is equal to

$$\Delta t = \frac{\Delta x}{v} = \frac{(4.24\text{ly})(c\times 1\text{y})}{0.2c} = 21.2\text{ y}$$

(b) Equation 33.3 for time dilation gives

$$\Delta t' = \frac{21.2\text{ y}}{1.021} = 20.8\text{ y}$$

(c) Equation 33.4 for length contraction gives

$$\Delta x' = \frac{4.24\text{ ly}}{1.021} = 4.15\text{ ly}$$

ASSESS To check that our answer is consistent, we can check that the speed of the spaceship is the same as measured by both observers (i.e., an observer in the frame *S* and one in the frame S'):

$$v = \frac{\Delta x'}{\Delta t'} = \frac{(4.15\text{ly})(c\times 1\text{y})}{20.8\text{ y}} = 0.2c$$

which is the same as found above using distance and time from the rest frame *S*.

39. **INTERPRET** This is the well-known "twin paradox" problem involving time dilation. We want to know the ages of the twins after one undergoes space travel and returns. The two reference frames are that of the Earth and the distant star, which is the rest frame, and that of the spaceship, which is the moving frame.

DEVELOP In the rest frame *S*, twin A must wait $\Delta t = \Delta x/v = 2(30\,c\cdot\text{y})/0.95c = 63.2\text{ y}$ for twin B to return. Using Equation 33.3 for time dilation, twin B (who is in the frame S') ages for

$$\Delta t' = \Delta t\sqrt{1-v^2/c^2} = (63.2 \text{ y})\sqrt{1-(0.95)^2} = 19.7 \text{ y}$$

during the trip.

EVALUATE Therefore, twin A is 83.2 y (63.2 + 20), while twin B returns at age 39.7 y (19.7 + 20).

ASSESS This is an intriguing consequence of time dilation. Note that only twin A's reference frame is inertial.

40. **INTERPRET** This problem involves a measurement of time in two reference frames: the stationary frame of the Earth and the moving frame of the spaceship. We are given the time in the stationary frame and are asked, in essence, to find the time in the moving frame.

DEVELOP For a spaceship traveling at $v = 0.80c$, the factor γ relating the two frames of reference is

$$\gamma = \frac{1}{\sqrt{1-v^2/c^2}} = \frac{1}{\sqrt{1-(0.80)^2}} = \frac{5}{3}$$

This allows us to convert Earth time Δt (i.e., frame S) to spaceship time $\Delta t'$ (i.e., frame S') using Equation 33.3 for time dilation:

$$\Delta t' = \frac{\Delta t}{\gamma}$$

EVALUATE For 6.67 minutes of Earth time, the time elapsed on the spaceship is

$$\Delta t' = \frac{6.67 \text{ min}}{5/3} = 4.00 \text{ min}$$

Four minutes is two half-lives, so one quarter of the atoms remain after this time (i.e., 250 atoms).

ASSESS The same initial number of atoms on Earth would have decayed to

$$N = N_0 \exp\left(\frac{-t\ln 2}{2 \text{ min}}\right) = 1000\exp\left(\frac{-(6.67 \text{ min})\ln 2}{2.00 \text{ min}}\right) = 99$$

Such an experiment demonstrates that time dilation is not an apparent effect, but a real physical phenomenon. Time is not absolute, but depends on the frame of reference in which it is measured.

41. **INTERPRET** This is a problem involving relativistic velocity addition.

DEVELOP Our galaxy S' is moving with speed $v = 0.75c$ relative to one of the galaxies S mentioned in the question, and the other galaxy is moving with speed $u' = 0.75c$ relative to us. All velocities are assumed to be along a common x-x'- axis. The speed of one galaxy as measured by an observer in the other galaxy can be obtained by using the relativistic velocity addition formula (Equation 33.5a):

$$u = \frac{u'+v}{1+u'v/c^2}$$

EVALUATE Substituting the values given, we get

$$u = \frac{0.75c+0.75c}{1+(0.75c)^2} = 0.96c$$

ASSESS The naïve answer, $0.75c + 0.75c = 1.5c$, is inconsistent with relativity.

42. **INTERPRET** This problem involves adding relativistic velocities. We want to find the fast ship's speed relative to the Earth, so the Earth is frame S. The slower ship is in frame S' and tells us the velocity $u' = 0.40c$ of the fast ship relative to itself. The frame S' is moving at speed $v = 0.70c$ with respect to frame S.

DEVELOP Apply Equation 33.5a to find the speed u of the fast ship in the Earth reference frame (i.e., frame S).

EVALUATE Inserting the quantities into Equation 33.5a gives

$$u = \frac{u'+v}{1+u'v/c^2} = \frac{0.40c+0.70c}{1+0.28} = 0.86c$$

ASSESS This velocity is less than the speed of light, as expected.

43. **INTERPRET** In this problem we are to show that if the speed of an object is less than c in one inertial reference frame, then the same conclusion holds in any other inertial frame.

DEVELOP The problem is equivalent to showing that, if $u' < c$ and $v < c$ in the relativistic velocity addition formula (Equation 33.5a)

$$u = \frac{u' + v}{1 + u'v/c^2}$$

then $u < c$. Note that the two initial conditions above may be written as

$$\begin{array}{ll} u' < c & v < c \\ u'/c < 1 & v/c < 1 \\ 0 < 1 - u'/c & 0 < 1 - v/c \end{array}$$

EVALUATE The conclusion follows almost immediately, because if $0 < 1 - u'/c$ and $0 < 1 - v/c$, then

$$0 < (1 - u'/c)(1 - v/c) = 1 - u'/c - v/c + u'v/c^2$$

$$u'/c + v/c < 1 + u'v/c^2$$

$$\frac{u'/c + v/c}{1 + u'v/c^2} < 1$$

$$\frac{u' + v}{1 + u'v/c^2} < c$$

but the left-hand side is just u (compare with Equation 33.5a), so we have shown that $u < c$.

ASSESS Equation 33.5a applies to the special case where all the velocities are collinear, but the conclusion is true in general.

44. **INTERPRET** This problem involves the concept of simultaneity in different reference frames. The relative speed of the Earth and the Sun is small compared to c ($v/c \approx 10^{-4}$), so we may consider the Sun to be approximately at rest at a distance $8.33\, c \cdot \text{min}$ from Earth (system S).

DEVELOP We chose the Earth as the origin and the x-axis in the direction of the Sun. In frame S, events A and B have coordinates $(x_A, t_A) = (0,0)$ and $(x_B, t_B) = (8.33\, c \cdot \text{min}, 2.45\text{ min})$. An observer in system S', moving along the x-axis with speed v sees these events separated by a time interval

$$\Delta t' = t'_B - t'_A = \gamma[t_B - x_B(v/c^2) - t_A + x_A(v/c^2)] = \gamma[2.45\text{ min} - 8.33\text{ min}(v/c)]$$

Using the Lorentz transformations from Table 33.1, we can convert these times in S' to times in S:

$$\Delta t' = \gamma\left(t_B - x_B\, v/c^2 - t_A + x_A\, v/c^2\right)$$
$$= \gamma\left[2.45\text{ min} - (8.33\text{ min})\frac{v}{c}\right]$$

EVALUATE **(a)** For $v = 0.75c$, $\gamma = 1.51$, so the time difference in frame S' is

$$\Delta t' = 1.51\left[2.45\text{ min} - (8.33\text{ min})(0.75)\right] = -5.74\text{ min}$$

so event B occurs before event A.

(b) For $v = -0.75c$, $\gamma = 1.51$, and the time difference in frame S' is

$$\Delta t' = 1.51\left[2.45\text{ min} - (8.33\text{ min})(-0.75)\right] = 13.1\text{ min}$$

so event A occurs before event B.

(c) For $v = 0.294c$, $\gamma = 1.046$, and the time difference in frame S' is

$$\Delta t' = 1.51\left[2.45\text{ min} - (8.33\text{ min})(-0.75)\right] = 0.00103\text{ min}$$

so the events are essentially simultaneous in this frame.

ASSESS The order of events reverses in frames that are moving in opposite directions with respect to the order in frames moving in the same direction.

45. **Interpret** The touchdown of *Curiosity* rover on Mars is observed in two reference frames, one on Earth and another one on a spacecraft heading toward Mars. We are asked to find out who observes the event first.

Develop Denote the Earth frame by S. The landing occurs simultaneously as the clock reads $t = 10:31\text{ PM}$. Let S' be the frame of the observer on the spacecraft. The Lorentz transformation between S and S' is summarized in Table 33.1, where $v = 0.35c$, and

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} = \frac{1}{\sqrt{1 - (0.35)^2}} = 1.06752$$

To see which event happened first in S', use the Lorentz transformation to transform $\Delta t = 0$ (relative to 10:31 PM) into S'.

Evaluate In S',

$$\Delta t' = \gamma\left(\Delta t - xv/c^2\right) = (1.06752)\left(0 - (14\ c\cdot\text{min})(0.35c)/c^2\right) = -5.2\text{ min}$$

As judged by observers in S', touchdown occurred earlier by 5.2 minutes.

Assess This problem demonstrates that two observers may see the occurrence of two events differently, depending on their frame of reference.

46. **Interpret** Given the Lorentz transformations for space (see Table 33.1), we are to derive those for time.

Develop The Lorentz transformation from S to S' for space is

$$x' = \gamma(x - vt)$$

Solving this for t gives

$$t = \frac{x}{v} - \frac{x'}{v\gamma}$$

Use the Lorentz transformation from S' to S [i.e., $x = \gamma(x' + vt')$] to eliminate x in the expression above and get an expression for t as a function of x' and t'.

Evaluate Performing the indicated substitution gives

$$t = \frac{\gamma(x' + vt')}{v} - \frac{x'}{v\gamma} = \frac{\gamma(x' + vt')}{v} - \frac{\gamma x'}{v\gamma^2} = \gamma\left[\frac{x'}{v} + t' - \frac{x'}{v}\left(1 - \frac{v^2}{c^2}\right)\right]$$
$$= \gamma\left(t' + x'v/c^2\right)$$

Reversing the process gives the Lorentz transformation from S to S'.

Assess We have verified the work of Lorentz and Fitzgerald.

47. **Interpret** In this problem we want to use the "light box" to derive the time dilation formula given in Equation 33.3.

Develop The reference frame of the box S' is moving with speed v in the x- direction relative to the frame S, which is at rest in Fig. 33.6b. Let the S coordinates of event A be t_A and x_A, and those of event B be t_B and $x_B = x_A + v(t_B - t_A)$. To find $\Delta t'$, we apply the Lorentz transformation from S to S' (see Table 33.1).

Evaluate With $t' = \gamma\left(t - xv/c^2\right)$, we get

$$\Delta t' = t'_B - t'_A = \gamma\left[t_B - t_A - (x_B - x_A)v/c^2\right] = \gamma(t_B - t_A)\left(1 - v^2/c^2\right) = \Delta t\sqrt{1 - v^2/c^2}$$

which is Equation 33.3.

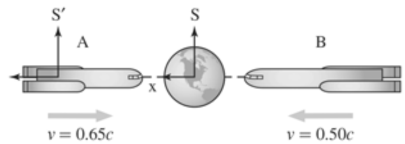
Assess The equation shows that $\Delta t' < \Delta t$. That is, the time interval measured in the spaceship frame S' is shorter than that measured in S.

48. **Interpret** This problem involves length contraction and relativistic velocity addition. Our two frames of reference are (1) that of the Earth and (2) that of ship A (see the figure on the next page). We are to find the length of ship B in both reference frames.

Develop In the Earth's reference frame S, the velocity of ship B is $v_B = 0.50c$, so we can use this in Equation 33.4, which gives the length contraction of an object due to its velocity. In this case, the distance $\Delta x'_E$ will be the distance measured in the Earth's frame, and the distance $\Delta x_B = 25$ m is the distance measured in the rest frame of ship B. The velocity of ship A (system S') relative to Earth is $v_A = -0.65c$ (since ship A is approaching along the x-axis from the opposite direction compared to ship B), so the velocity of ship B relative to ship A can be found from Equation 33.5b, which gives

$$u'_B = \frac{u_B - v_A}{1 - u_B v_A / c^2} = \frac{0.50c - (-0.65c)}{1 - (0.50c)(-0.65c)/c^2} = 0.868c$$

This is the speed we shall use in Equation 33.4 to find the length of ship B in the frame of reference of ship A.



EVALUATE **(a)** Inserting the given quantities into Equation 33.4 gives the length of ship B in the frame of reference of the Earth. The result is

$$\Delta x'_E = \Delta x_B \sqrt{1 - v_B^2/c^2} = (25\text{ m})\sqrt{1 - (0.50c)^2/c^2} = 22\text{ m}$$

(b) Substituting u'_B for v in Equation 33.4 gives the length of ship B in the frame of reference of ship A. The result is

$$\Delta x'_A = \Delta x_B \sqrt{1 - u'^2_B/c^2} = (25\text{ m})\sqrt{1 - (0.868)^2} = 12\text{ m}$$

ASSESS The length of the ship is more contracted in the reference frame of ship A than in the reference frame of the Earth. This is because ship A is moving faster than the Earth is with respect to ship B.

49. **INTERPRET** This problem involves time dilation. We are given the time limit of 65 years in the reference frame of the human who is traveling and asked how fast she should go to reach a star 170 ly away as measured in the reference frame of Earth.

DEVELOP The distance is given in the system S, where Earth and the star are essentially at rest, but the time interval is given in the system S', where the spacecraft is at rest. Thus, $\Delta x = 170$ ly and $\Delta t' = 65$ y. Equations 33.3 and 33.4,

$$\Delta t' = \Delta t \sqrt{1 - v^2/c^2} = \frac{\Delta t}{\gamma}$$

$$\Delta x' = v\Delta t' = \Delta x \sqrt{1 - v^2/c^2}$$

for time dilation and Lorentz contraction, respectively, relate the given quantities to $\Delta x'$ and $\Delta t'$. We use the second expression (i.e., the expression for length contraction) to find

$$v = \frac{\Delta x}{\Delta t'}\sqrt{1 - v^2/c^2}$$

$$= \frac{c}{\sqrt{1 + (c\Delta t'/\Delta x)^2}}$$

EVALUATE Inserting the given values into the expression above for velocity gives

$$v = \frac{c}{\sqrt{1 + (65c \cdot y/170c \cdot y)^2}} = 0.93c$$

ASSESS The time elapsed on Earth may be found from Equation 33.3 for time dilation. The result is

$$\Delta t = \frac{\Delta t'}{\sqrt{1 - v^2/c^2}} = \frac{65\text{ y}}{\sqrt{1 - (0.93)^2}} = 177\text{ y}$$

Thus, none of her colleagues would be alive when she arrives at the star (not to mention that it would take another 177 years for her to signal her arrival to those on Earth!).

50. **INTERPRET** This problem involves time dilation and length contraction. We are given the speed of the spaceship with respect to the galaxy's frame S and the distance in this frame, and are asked to find the time it takes to cross the galaxy in the ship's reference frame *S′* and the size of the galaxy in the ship's reference frame.

DEVELOP In frame *S*, the time it takes to cross the galaxy is

$$\Delta t = \frac{\Delta x}{v} = \frac{6.5\times10^4 c\cdot \text{y}}{c-\Delta} = \frac{6.5\times10^4\ \text{y}}{1-\delta} \approx \left(6.5\times10^4\ \text{y}\right)(1+\delta)$$

where $\Delta = 80\ \text{km/s}$ and $\delta = (80\ \text{km/s})/c = 2.7\times10^4$. The factor γ may also be approximated to the lowest order in δ as follows (see binomial approximation in Appendix A):

$$\gamma = \frac{1}{\sqrt{1-v^2/c^2}} = \frac{1}{\sqrt{1-(1-\delta)^2}} = \frac{1}{\sqrt{2\delta-\delta^2}} \approx \frac{1}{\sqrt{2\delta}}$$

Use Equations 33.3 and 33.4 to find the trip time and the galaxy size in the spaceship's reference frame *S′*.

EVALUATE **(a)** To the first order in δ, the time it takes to cross the galaxy in the ship's reference frame is

$$\Delta t' = \frac{\Delta t}{\gamma} \approx \left(6.5\times10^4\ \text{y}\right)(1+\delta)\sqrt{2\delta} \approx \left(6.5\times10^4\ \text{y}\right)\sqrt{2\delta}$$

$$= \left(6.5\times10^4\ \text{y}\right)\sqrt{2\left(2.7\times10^{-4}\right)} = 1.5\times10^3\ \text{y}$$

(b) To the first order in δ, the galaxy's diameter as measured in the ship's reference frame is

$$\Delta x' = \frac{\Delta x}{\gamma} = \left(6.5\times10^4 c\cdot\text{y}\right)\sqrt{2\delta} = \left(6.5\times10^4 c\cdot\text{y}\right)\sqrt{2\left(2.7\times10^{-4}\right)} = 1.5\times10^3\ l\text{y}$$

ASSESS In the ship's reference frame, the clock runs slow and the length is contracted, as expected. Any correction to these results would be second order in δ and thus, below the precision of the data.

51. **INTERPRET** This is a problem about calculating the distance and time between two events, as measured in different reference frames. The first reference frame *S* is essentially stationary with respect to the Earth and the star, whereas the second reference frame S' is that of the spaceship moving at velocity *v* with respect to *S*.

DEVELOP We shall follow the Problem-Solving Strategy 33.1 for Lorentz transformation. In the Earth-star frame (system *S*), we choose $x_A = 0$ and $t_A = 0$. In system S', events A and B both occur at the spaceship, for which we can choose $x'_A = x'_B = 0$ and $t'_A = 0$.

EVALUATE **(a)** In system S, we are given $x_B - x_A = 10\ \text{ly}$, so

$$\Delta t = t_B - t_A = \frac{x_B - x_A}{v} = \frac{10\ \text{ly}}{0.80c} = 13\ \text{y}$$

(b) In system S', we have $x'_B - x'_A = 0$. However, $\Delta t' = t'_B - t'_A = (t_B - t_A)/\gamma$ from time dilation (Equation 33.3), so $\Delta t' = (12.5\ y)\sqrt{1-(0.80)^2} = 7.5\ \text{y}$.

(c) For the space time interval, one has

$$(\Delta s)^2 = c^2(\Delta t)^2 - (\Delta x)^2 = (12.5\ \text{ly})^2 - (10\ \text{ly})^2 = 56.25\ \text{ly}^2$$
$$(\Delta s')^2 = c^2(\Delta t')^2 - (\Delta x')^2 = (7.5\ \text{ly})^2 - 0 = 56.25\ \text{ly}^2$$

as required by invariance.

ASSESS Our result shows that $\Delta t' < \Delta t$. In other words, the time interval measured in the spaceship frame S' is shorter than that measured in *S*. The space time interval, however, remains the same in both reference frames; $\Delta s^2 = \Delta s'^2$.

52. **INTERPRET** This problem involves finding the spacetime intervals between the two events that are the subject of Example Variations 33.29 and 33.32.

DEVELOP The square of the spacetime interval between the first launchings by civilizations A and B in Example Variation 33.29 is most easily calculated in the frame *S* of the galaxy. In this case, $\Delta t = 0\ \text{y}$ and $\Delta x = 95.0\times10^3\ c\cdot\text{y}$. Since the spacetime interval is Lorentz-invariant, the same result would be found in any frame moving with constant velocity relative to S. For events A and B in Example Variation 33.32, $\Delta t = 1.14\times10^5$ y and $\Delta x = 95.0\times10^3\ c\cdot$y

EVALUATE **(a)** The spacetime interval for the events in Example Variation 33.29 is

$$\begin{aligned}(\Delta s)^2 &= c^2(\Delta t)^2 - (\Delta x)^2 = c^2(0\text{ y})^2 - \left(95.0\times10^3\ c\cdot\text{y}\right)^2\\ &= 0 - 9.0\times10^9\text{ ly}^2 = -9.0\times10^9\text{ ly}^2\end{aligned}$$

(b) The spacetime interval for the events in Example Variation 33.32 is

$$\begin{aligned}(\Delta s)^2 &= c^2(\Delta t)^2 - (\Delta x)^2 = c^2\left(1.14\times10^5\text{ y}\right)^2 - \left(95.0\times10^3\ c\cdot\text{y}\right)^2\\ &= 1.30\times10^{10}\text{ ly}^2 - 9.0\times10^9\text{ ly}^2 = 4.0\times10^9\text{ ly}^2\end{aligned}$$

ASSESS If the square of the spacetime interval between two events is positive (called a timelike separation), then the events can be causally connected. If Δs^2 is negative (a spacelike separation), the events cannot be causally connected, and a Lorentz frame exists in which they occur simultaneously.

53. **INTERPRET** This is a problem about the spacetime interval between two events. The events are connected by a light signal.

DEVELOP Choose the x-axis along the line separating the positions of the events. Since A and B are connected by the passage of a light beam,

$$|\Delta x| = |x_B - x_A| = c|t_B - t_A| = c|\Delta t|$$

EVALUATE From Equation 33.6, one sees that the spacetime interval between them is zero:

$$(\Delta s)^2 = c^2(\Delta t)^2 - (\Delta x)^2 = 0$$

ASSESS An event with zero spacetime interval relative to A is said to lie on the light cone of A.

54. **INTERPRET** We are to find the momentum changes that correspond to the given changes in speed.

DEVELOP Relativistic momentum is given by Equation 33.7:

$$\vec{p} = \gamma m\vec{u}$$

We shall calculate the momentum for each speed and take the difference to find the momentum change needed.

EVALUATE For $u_1 = 0.1c$ and $u_2 = 0.2c$, the change in momentum is

$$\begin{aligned}\Delta p_a &= p_2 - p_1 = \gamma_2 m u_2 - \gamma_1 m u_1 = \frac{mu_2}{\sqrt{1-u_2^2/c^2}} - \frac{mu_1}{\sqrt{1-u_1^2/c^2}}\\ &= m\left(\frac{0.2c}{\sqrt{1-0.04}} - \frac{0.1c}{\sqrt{1-0.01}}\right)\\ &= 0.1mc\end{aligned}$$

For $u_1 = 0.8c$ and $u_2 = 0.9c$, the change in momentum is

$$\begin{aligned}\Delta p_b &= m\left(\frac{0.9c}{\sqrt{1-0.81}} - \frac{0.8c}{\sqrt{1-0.64}}\right)\\ &= 0.7mc\end{aligned}$$

Thus, the $\Delta p_b \approx 7\Delta p_a$.

ASSESS This result reflects the fact that ever-increasing energy is required to accelerate as the speed approaches c. For objects with nonzero rest mass, an infinite amount of energy is required to achieve the speed of light.

55. **INTERPRET** We're given the time and distance between two distant events observed in a particular reference frame S, and are to find the time and distance in another reference frame S' that is moving at the given speed with respect to S.

DEVELOP The coordinates of the events in S and S' are related by the Lorentz transformation in Table 33.1, with $v/c = 0.66$ and $\gamma = \left(1 - v^2/c^2\right)^{-1/2} = 1.33$.

EVALUATE **(a)** The distance between A and B measured by an observer in S' is

$$x'_B - x'_B = \gamma\left[x_B - x_A - v(t_B - t_A)\right]$$

$$= 1.33\left[4.1\text{ ly} - (0.66c)(1.6\text{ y})\right] = 4.05\text{ ly}$$

(b) Similarly, the time between A and B measured by an observer in S' is

$$t'_A - t'_B = \gamma\left[t_B - t_A - \left(v/c^2\right)(x_B - x_A)\right]$$

$$= 1.33\left[1.6\text{ y} - (0.66/c)(4.1\text{ ly})\right] = -1.5\text{ ly}$$

Thus, B occurs before A in S'.

ASSESS Since the travel time of light from the position of A to that of B is greater than the magnitude of the time difference (4.1 y versus 1.6 y in S, or 4.0 y versus 1.5 y in S'), the events are not causally connected.

56. **INTERPRET** We are given two speed and momentum ratios of a particle and are asked to find its original speed.

DEVELOP We are given $v_2 = 2v_1$ and $p_2 = 3p_1$. Divide these equations and use Equation 33.7 to obtain:

$$\frac{v_2}{p_2} = \frac{2v_1}{3p_1} \quad\Rightarrow\quad \frac{v_2}{\gamma_2 m v_2} = \frac{2v_1}{3\gamma_1 m v_1} \quad\Rightarrow\quad \frac{3}{\gamma_2} = \frac{2}{\gamma_1}$$

Square this and use the first condition again to solve for the original speed.

EVALUATE The original speed is

$$\frac{9}{\gamma_2^2} = \frac{4}{\gamma_1^2}$$

$$9\left(1 - 4v_1^2/c^2\right) = 4\left(1 - v_1^2/c^2\right)$$

$$v_1 = \sqrt{5/32}c = 0.395c$$

ASSESS The original speed is about 40% the speed of light.

57. **INTERPRET** We're given the kinetic energy of a proton and asked to find its speed and momentum.

DEVELOP For the proton, $mc^2 = 938\text{ MeV}$, so $K = (\gamma - 1)mc^2 = 500\text{ MeV}$ implies

$$\gamma = 1 + \frac{K}{mc^2} = 1 + \frac{500\text{ MeV}}{938\text{ MeV}} = 1.53$$

EVALUATE **(a)** Since $\gamma = \left(1 - v^2/c^2\right)^{-1/2}$, the speed of the proton is

$$v = c\sqrt{1 - 1/\gamma^2} = 0.758c$$

(b) Using Equation 33.7, we find the momentum to be

$$p = \gamma m v = (1.53)\left(938\text{ MeV}/c^2\right)(0.758c) = 1.09\text{ GeV}/c = 5.82\times10^{-19}\text{ kg}\cdot\text{m/s}$$

ASSESS Since the kinetic energy is not negligible compared to the rest energy, the Newtonian expression of momentum ($p = mv$) is not applicable.

58. **INTERPRET** We are given a proton's rest energy and are asked to find its momentum and to compare that momentum to that of a crawling insect.

DEVELOP Apply Equation 33.10

$$E^2 = p^2c^2 + \left(mc^2\right)^2$$

to find the momentum of the proton. The momentum of the insect may be found using Newtonian mechanics.

EVALUATE **(a)** The kinetic energy of the bug is

$$K_{\text{bug}} = \frac{1}{2}m_{\text{bug}}v^2 = \frac{1}{2}\left(25\times10^{-6}\text{ kg}\right)\left(2.0\times10^{-3}\text{ m/s}\right)^2 = 5.0\times10^{-11}\text{ J} = 3.13\times10^{8}\text{ eV}$$

The energy of the proton is $E = 14\times10^{12}\text{ eV}$. Thus,

$$\frac{E}{K_{\text{bug}}} = \frac{14\times10^{12}\text{ eV}}{3.13\times10^{8}\text{ eV}} = 4.5\times10^{4}$$

(b) The proton's energy is so much greater than its rest energy (mc^2 = 938 MeV) that

$$p \approx E/c = 1.4\times10^{13}\ \text{eV/c} = \left(1.4\times10^{13}\ \text{eV}/c\right)\overbrace{\left(\frac{1.6\times10^{-19}\ \text{kg}\cdot\text{m}^2/\text{s}^2}{\text{eV}}\right)}^{=1}\overbrace{\left(\frac{c}{3.0\times10^{8}\ \text{m/s}}\right)}^{=1} = 7.47\times10^{-15}\ \text{kg}\cdot\text{m/s}$$

The momentum of the insect is

$$p_{\text{bug}} = m_{\text{bug}}v = \left(25\times10^{-6}\ \text{kg}\right)\left(2.0\times10^{-3}\ \text{m/s}\right) = 5.0\times10^{-8}\ \text{kg}\cdot\text{m/s}$$

Thus,

$$\frac{p}{p_{\text{bug}}} = \frac{7.47\times10^{-15}\ \text{kg}\cdot\text{m/s}}{5.0\times10^{-8}\ \text{kg}\cdot\text{m/s}} = 1.49\times10^{-7}$$

or $p_{\text{bug}}/p = 6.7\times10^{6}$.

ASSESS This is quite amazing considering that the insect contains the equivalent of about

$$n = \frac{25\times10^{-6}\ \text{kg}}{1.7\times10^{-27}\ \text{kg}} = 15\times10^{21}$$

protons.

59. **INTERPRET** This is a problem about mass-energy conversion using $E = mc^2$.

DEVELOP The energy-equivalent of 9 g is

$$E = mc^2 = \left(9\times10^{-3}\ \text{kg}\right)\left(3\times10^{8}\ \text{m/s}\right)^2 = 8.1\times10^{14}\ \text{J}$$

EVALUATE This amount of energy could supply a large city, with a power consumption of 2×10^9 W, for a period of time

$$t = \frac{E}{P} = \frac{8.1\times10^{14}\ \text{J}}{2\times10^{9}\ \text{W}} = 4.05\times10^{5}\ \text{s} = 112.5\ \text{h}$$

ASSESS This is an enormous amount of energy harnessed from just 9 g of raisin (or of any other matter).

60. **INTERPRET** This problem involves converting energy into mass.

DEVELOP If $K = 0$ then $\gamma = 1$ and Equation 33.9 reduces to $E = mc^2$. Thus, the mass equivalent of the released energy is $\Delta m = \Delta E/c^2 = 3.3\ \text{MeV}/c^2$. Using the conversion factors from Appendix C, we can find the equivalent mass in kg.

EVALUATE In conventional SI units,

$$\Delta m = \left(3.3\times10^{6}\ \text{eV}/c^2\right)\overbrace{\left(\frac{1.6\times10^{-19}\ \text{kg}\cdot\text{m}^2/\text{s}^2}{\text{eV}}\right)}^{=1}\overbrace{\left(\frac{c^2}{9\times10^{16}\ \text{m}^2/\text{s}^2}\right)}^{=1} = 5.9\times10^{-30}\ \text{kg}$$

ASSESS The fraction f of mass converted to energy is approximately

$$f = \frac{6\times10^{-30}\ \text{kg}}{4\times1.7\times10^{-27}\ \text{kg}} \approx 10^{-3}$$

61. **INTERPRET** In this problem we are to show that the kinetic energy in Equation 33.8 reduces to the Newtonian result $K = m_e c^2/2$ when $u \ll c$.

DEVELOP The binomial expansion valid for $|x| < 1$ is

$$(1+x)^p = 1 + px + \frac{p(p-1)}{2!}x^2 + \cdots$$

EVALUATE For $u/c \ll 1$, Equation 33.8 can be expanded to yield

$$K = mc^2\left[\frac{1}{\sqrt{1-u^2/c^2}} - 1\right] = mc^2\left[1 + \frac{1}{2}\frac{u^2}{c^2} + \frac{3}{8}\left(\frac{u^2}{c^2}\right)^2 + \cdots - 1\right] = \frac{1}{2}mu^2\left(1 + \frac{3u^2}{4c^2} + \cdots\right) \approx \frac{1}{2}mu^2$$

ASSESS We indeed recover the Newtonian expression for kinetic energy when $u/c \ll 1$.

62. **INTERPRET** We are to derive the relativistic energy-momentum expression (Equation 33.10) from the expressions for kinetic energy and total energy (Equations 33.8 and 33.9).

DEVELOP Equations 33.7 and 33.9 are $p = \gamma mu$ and $E = \gamma mc^2$. Dividing the former by the latter gives

$$\frac{p}{E} = \frac{\gamma mu}{\gamma mc^2} \quad \Rightarrow \quad \frac{pc}{E} = \frac{u}{c}$$

From Equation 33.9, we find

$$E = \gamma mc^2$$
$$\frac{1}{\gamma^2} = \frac{m^2c^4}{E}$$

from which we can show the desired expression.

EVALUATE Inserting $\gamma = \left(1 - u^2/c^2\right)^{-1/2}$ into the expression above and using the result $u/c = pc/E$, we find

$$1 - u^2/c^2 = \frac{m^2c^4}{E^2}$$
$$1 - \frac{p^2c^2}{E^2} = \frac{m^2c^4}{E^2}$$
$$E^2 = p^2c^2 + m^2c^4$$

ASSESS We have derived the relativistic energy-momentum relationship.

63. **INTERPRET** In this problem we are asked to find the speed at which $K = mc^2$.

DEVELOP The kinetic energy of a particle is given by Equation 33.8:

$$K = \frac{mc^2}{\sqrt{1 - u^2/c^2}} - mc^2 = mc^2\left(\frac{1}{\sqrt{1 - u^2/c^2}} - 1\right) = mc^2(\gamma - 1)$$

When the kinetic energy is equal to the rest energy, $(\gamma - 1)mc^2 = mc^2$, or $\gamma = 2$.

EVALUATE From $\gamma = \left(1 - v^2/c^2\right)^{-1/2}$, the speed of the particle is

$$v = c\sqrt{1 - 1/\gamma^2} = c\sqrt{1 - 1/4} = c\frac{\sqrt{3}}{2} = 0.866c$$

ASSESS The speed of the particle is about $0.866c$ when $K = mc^2$. This is in the relativistic regime.

64. **INTERPRET** We are to show that if a light signal cannot travel from event A to event B, then there exists a frame of reference for which the two events are simultaneous. In addition, we are to show that if a light signal can travel between events then no frame of reference exists in which the events are simultaneous.

DEVELOP Let the first event in system S have coordinates $x_A = 0$ and $t_A = 0$ and the second x_B and t_B. If a light signal leaving A cannot reach B, then $ct_B < x_B$.

EVALUATE The events are simultaneous in system S' (with origin $x'_A = 0$ and $t'_A = 0$ and which moves along the x-x' axis with speed v relative to S) if

$$t'_B - t'_A = 0 = \gamma\left[(t_B - t_A) - \left(v/c^2\right)(x_B - x_A)\right] = \gamma\left(t_B - \mathrm{v}x_B/c^2\right).$$

Then $v/c = ct_B/x_B < 1$ describes a real Lorentz transformation, which confirms the possibility of the events being simultaneous in S'. On the other hand, if a light signal could reach x_B before t_B, then $x_B < ct_B$, and there is no system with $v/c = ct_B/x_B > 1$ in which the events are simultaneous.

ASSESS The frame of reference influences the simultaneity of events.

65. **INTERPRET** This problem is about the Doppler shift for light. Given a source moving toward us at a given speed, we are to derive the given expression for the Doppler-shifted frequency. For a source speed much, less than the speed of light, we are to show that this result simplifies to the classical result (Equation 14.15).

DEVELOP Consider the sketch below. Let S be the rest system of a source of light waves (with frequency and wavelength $\lambda f = c$) that moves with speed u toward an observer in S' (who measures $\lambda' f' = c$). Suppose that N

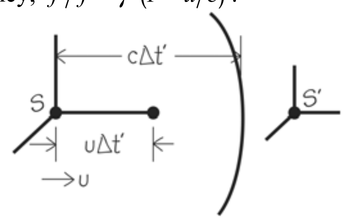
waves are emitted in S in a time interval Δt. The first wavefront has traveled a distance $c\Delta t$ in S, so the wavelength (i.e., the distance between surfaces of constant phase) is $\lambda = c\Delta t/N$. In S', however, the wavefronts are "piled up" in a smaller distance due to the motion of S so the wavelength is

$$\lambda' = \frac{c\Delta t' - u\Delta t'}{N} = \frac{(c-u)\Delta t'}{N}$$

which gives

$$\frac{\lambda'}{\lambda} = \frac{(1-u/c)\Delta t'}{\Delta t}$$

Now, Δt (the proper time interval in the source's rest system) is related to $\Delta t'$ (the time interval measured in a system where the source is moving) by time dilation, $\Delta t' = \gamma \Delta t$ (Equation 33.3 with altered notation), so $\lambda'/\lambda = \gamma(1-u/c)$ or, in terms of frequency, $f/f' = \gamma\,(1-\ u/c)$.



EVALUATE Since $\gamma = 1\big/\sqrt{1-u^2/c^2}$, this can be written as

$$\frac{f'}{f} = \frac{1-u/c}{\sqrt{1-u^2/c^2}} = \frac{\sqrt{(1-u/c)^2}}{\sqrt{(1-u/c)(1+u/c)}} = \sqrt{\frac{1+u/c}{1-u/c}}$$

which is the radial Doppler shift (i.e., along the line of sight) in special relativity, with u positive for approach and negative for recession (note the difference in signs with Equation 14.13). For $u/c \ll 1$,

$$\sqrt{\frac{1+u/c}{1-u/c}} \approx \left(1+\frac{1}{2}\frac{u}{c}\right)\left(1-\frac{1}{2}\frac{-u}{c}\right) \approx 1+\frac{u}{c}$$

which, allowing for the difference in signs, is the same as the limit of Equation 14.13 for $u/c \ll 1$.

ASSESS Note that it is more customary to write this limit as

$$\frac{\Delta f}{f} = \frac{f'-f}{f} = \frac{u}{c}$$

The relativistic Doppler effect has been used to measure the shifts in frequency of light emitted by other moving galaxies. The observation of "red shift" suggests that galaxies are receding from us and that the universe is still expanding.

66. **INTERPRET** We are to derive and expression for the speed required to make a trip to a star a distance d away, as measured from Earth, in a time $\Delta t'$ as experienced by the frame of the spaceship.

DEVELOP The speed of the spacecraft is given by the time $\Delta t'$ it takes it to travel a distance d', where $d' = d/\gamma$ is the contracted length of the trip as measured from the traveling frame of the spaceship. From this we see that:

$$v = \frac{d'}{\Delta t} = \frac{d}{\Delta t}\sqrt{1-\frac{v^2}{c^2}}$$

Which we can solve for the speed v in terms of d, $\Delta t'$, and c.

EVALUATE Simplifying the expression above and solving for the speed, we find it can be written as:

$$v^2\frac{\Delta t^2}{d^2} = \left(1-\frac{v^2}{c^2}\right)$$

$$v = \frac{cd}{\sqrt{d^2+c^2\Delta t^2}}$$

ASSESS We can check the units of our expression to verify the validity by noting the numerator is in m^2/s and the denominator is in m.

67. **INTERPRET** This is a problem involving relativistic velocity addition. We are told there are three spaceships, a large-, medium-, and small- sized one, inside of each other, traveling at 0.75*c* relative to the outer frame. We want to determine the speed of the small ship relative to Earth.

DEVELOP We will denote the speeds of the small, medium, and large ships as: v_s, v_m, and v_l, respectively. We are told each one is traveling at 0.75*c*, relative to the outer ship, with the large ship traveling at 0.75*c* relative to Earth. We can use the relativistic addition formula (Equation 33.5a) to calculate the relative speeds in each frame as we make our way inward to find the speed of the small ship relative to Earth. We will assume all ships are moving along the same direction, so that their speeds sum.

EVALUATE We begin by calculating the speed of the medium ship relative to Earth, by adding its speed to that of the large ship.

$$v_m = \frac{v'_m + v_l}{1 + v'_m v_l / c^2}$$

Here, v'_m is the speed of the medium ship relative to the large ship's frame. We repeat this process for the small ship, relative to the medium ship's frame.

$$v_s = \frac{v'_s + v_m}{1 + v'_s v_m / c^2}$$

Where again, v'_s is the speed of the small ship relative to the medium ship's frame. Thus, v_s is the speed of the small ship relative to Earth, and can be expressed as

$$v_s = \frac{(v'_s + v'_m + v_l) + (v'_s v'_m v_l)}{(1 + v'_s (v'_m + v_l)) + v'_m v_l} c$$

Plugging in the relative speeds given, we find $v_s = 0.994c$

ASSESS The naïve answer, 0.75*c* + 0.75*c* + 0.75*c* = 2.25*c*, is inconsistent with relativity.

68. **INTERPRET** This is a problem involving the energy stored in the gravitational waves emitted from the merging of two blackholes.

DEVELOP The difference in mass between the initial 36 and 29 solar mass blackholes and the final 62 solar mass black hole is released in the form of energy carried off by the gravitational waves emitted from their merging. We can calculate the amount of energy released by using the relativistic formula for the total energy of a system when stationary, resulting in the well-known expression: $E = mc^2$.

EVALUATE The difference in mass between the initial and final products is equal to

$$62M_S - (36M_S + 29M_S) = 3M_S$$

The rest energy associated with this mass is the amount released by the gravitational waves, and is equal to

$$E = 3M_S c^2 = 3\left(1.99\times10^{30}\,\text{kg}\right)\left(3.0\times10^{8}\,\text{m/s}\right)^2 = 5.4\times10^{47}\,\text{J}$$

ASSESS This amount of energy released is difficult to put into perspective. For context, in 2013 the estimated world energy consumption was 5.67×10^{20} joules, close to 30 order of magnitudes less than was released by this merging!

69. **INTERPRET** This is a problem involving the excursion made by a high energy proton traveling across the Milky Way. We are to consider the time it would take a cosmic ray to travel the diameter of the Milky Way, both in the galaxy's and particle's reference frames.

DEVELOP We are given the proton's total energy, which we can use to determine the value of γ using Equation 33.9, and thus the particle's speed. We can then determine the time it would take for it to travel across the diameter of the Milky way, d =100,000 ly, in both frames.

EVALUATE Using Equation 33.9 we find a relativistic factor of\

$$\gamma = \frac{E}{mc^2} = \frac{300\times10^{18}\,\text{eV}}{938\,\text{MeV}} = 3.20\times10^{11}$$

Which means the particle is traveling at a speed

$$v = \sqrt{\left(1-\gamma^{-2}\right)}c \simeq c$$

(a) In the frame of the galaxy, the time taken to travel across the Milky Way is equal to

$$\Delta t = \frac{d}{v} = \frac{100{,}000\,\text{ly}\left(c \times 1\,\text{y}\right)}{c} = 100{,}000\,\text{y}$$

(b) In the frame of the proton, the time taken to travel across the Milky Way is equal to

$$\Delta t' = \frac{d}{\gamma v} = \frac{100{,}000\,\text{ly}\left(c \times 1\,\text{y}\right)}{\left(3.20 \times 10^{11}\right)c} = \left(\frac{100{,}000\,\text{y}}{\left(3.20 \times 10^{11}\right)}\frac{\left(3.2 \times 10^{7}\,\text{s}\right)}{1\,\text{y}}\right) = 10\,\text{s}$$

ASSESS When energetic particles accumulate enough energy to travel at speeds so close to c, the degree of time dilation can be astronomical, making 100,000 years seem like a few seconds.

70. **INTERPRET** A static charge distribution in a reference frame S at rest with respect to it may be moving with respect to a moving frame S'. The former produces an electric field, while the latter gives rise to a current, and hence a magnetic field.

DEVELOP In frame S, there are just static charges, and no current. We use Gauss's law to calculate the electric field. The amount of charge, $dq = dq'$, on a length $d\ell$ in S, will appear to lie on a Lorentz-contracted length

$$d\ell' = d\ell / \gamma = d\ell\sqrt{1 - v^2 / c^2}$$

in S'. With cylindrical symmetry, we can apply Ampere's law to calculate the magnetic field due the current.

EVALUATE **(a)** Using Gauss's law, we have

$$E(2\pi r\ell) = \lambda\ell / \epsilon_0,$$

which leads to $E = \lambda / 2\pi\epsilon_0 r$. The electric field points radially away from the x-axis. There is no magnetic field since the current is zero.

(b) In S', the amount of charge stays the same:

$$dq' = dq \quad \Rightarrow \quad \lambda' d\ell' = \lambda' d\ell / \gamma = \lambda d\ell$$

Thus, $\lambda' = \gamma\lambda$.

(c) In S', using Gauss's law, we have

$$E'(2\pi r'\ell') = \lambda'\ell' / \epsilon_0 \quad \Rightarrow \quad E' = \frac{\lambda'}{2\pi\epsilon_0 r'} = \frac{\lambda'}{2\pi\epsilon_0 r} = \frac{\gamma\lambda}{2\pi\epsilon_0 r} = \gamma E$$

where $r' = r = \sqrt{y^2 + z^2}$ is measured perpendicular to the line of charge, and therefore is not affected by Lorentz transformation.

(d) In S', the charge moving with speed $v = dx' / dt'$ along the negative x'-axis, constitutes a current

$$I' = \frac{dq'}{dt'} = \frac{\lambda' dx'}{dt'} = \lambda' v = \gamma\lambda v$$

(e) Using Ampere's law, the magnetic field due to I' is

$$B'(2\pi r) = \mu_0 I' \quad \Rightarrow \quad B' = \frac{\mu_0 I'}{2\pi r} = \frac{\mu_0\gamma\lambda v}{2\pi r}$$

(f) In S, $\vec{B} = 0$, so $\vec{E} \cdot \vec{B} = 0$. In S', the electric field is radial, $\vec{E}' = E'\,\hat{r}$, while the magnetic field is in the tangential direction, $\vec{B}' = B'\,\hat{\theta}$, and

$$\vec{E}' \cdot \vec{B}' = E'B'\,(\hat{r} \cdot \hat{\theta}) = 0$$

since $\hat{r} \cdot \hat{\theta} = 0$.

(g) In S, $\vec{B} = 0$, so

$$E^2 - c^2 B^2 = E^2 = \left(\frac{\lambda}{2\pi\epsilon_0 r}\right)^2$$

Now, in S', we have

$$E'^2 - c^2B'^2 = \left(\frac{\gamma\lambda}{2\pi\epsilon_0 r}\right)^2 - c^2\left(\frac{\mu_0\gamma\lambda v}{2\pi r}\right)^2 = \left(\frac{\gamma\lambda}{2\pi\epsilon_0 r}\right)^2 - c^2{\epsilon_0}^2\mu_0^2\left(\frac{\gamma\lambda v}{2\pi\epsilon_0 r}\right)^2$$
$$= \left(\frac{\gamma\lambda}{2\pi\epsilon_0 r}\right)^2 - \frac{1}{c^2}\left(\frac{\gamma\lambda v}{2\pi\epsilon_0 r}\right)^2$$
$$= \left(\frac{\gamma\lambda}{2\pi\epsilon_0 r}\right)^2\left(1-\frac{v^2}{c^2}\right) = \left(\frac{\gamma\lambda}{2\pi\epsilon_0 r}\right)^2\frac{1}{\gamma^2} = \left(\frac{\lambda}{2\pi\epsilon_0 r}\right)^2$$
$$= E^2 - c^2B^2$$

where we have used $c^2 = 1/\mu_0\epsilon_0$.

ASSESS Our result gives a hint at how electric and magnetic fields transform, and demonstrates the fact $\vec{E}\cdot\vec{B}$ and $E^2 - c^2B^2$ always remain invariant.

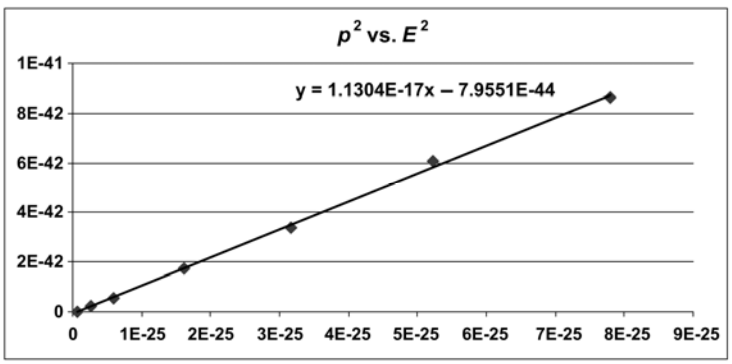
71. **INTERPRET** We are given the data of a particle's momentum as a function of its total energy. By analyzing the data, we are to find the mass of the particle and our estimate of c, the speed of light.

DEVELOP The relation between the particle's momentum p and its total energy E is given by Equation 33.10:

$$E^2 = p^2c^2 + \left(mc^2\right)^2 \;\Rightarrow\; p^2 = \frac{E^2}{c^2} - m^2c^2 = \frac{E^2}{c^2} - \frac{m^2}{1/c^2}$$

Thus, if we plot p^2 vs. E^2, we would get a straight line with slope equal to $1/c^2$, and y-intercept equal to m^2c^2.

EVALUATE **(a)** We first convert E and p to SI units using $1\text{ MeV}/c = 5.344286\times10^{-22}$ kg·m/s, and $1\text{ MeV} = 1.602\times10^{-13}$ J. The plot is shown below.



The slope is $1/c^2 = 1.1304\times10^{-17}\ \text{s}^2/\text{m}^2$, from which we deduce c to be $c = 2.974\times10^8$ m/s.

(b) The intercept is $m^2c^2 = 7.9551\times10^{-44}\ \text{J}^2/c^2$, which gives

$$m = \sqrt{(7.95541\times10^{-44}\ \text{J}^2)(1.1304\times10^{-17}\ \text{s}^2/\text{m}^2)} = 9.483\times10^{-31}\ \text{kg}$$

ASSESS The particle is an electron, but our mass is about 4% high. Note that the results are sensitive to the precision of the conversions factors used.

72. **INTERPRET** You consider possible relativistic effects on a high-speed interstellar voyage.

DEVELOP Your ship is moving fast enough that time dilation should be occurring. But this effect is only evident when you compare time as measured onboard to time measured in some other reference frame.

EVALUATE Since you are measuring your own heart rate in your own reference frame, there is no time dilation. You simply measure the pulse that you would if you were standing at rest on Earth.

The answer is **(b)**.

ASSESS If a doctor on Earth was somehow able to measure your pulse as you were moving in the spacecraft, he would observe a slower than normal heart rate.

73. **INTERPRET** You consider possible relativistic effects on a high-speed interstellar voyage.

DEVELOP From the Earth's perspective, the voyage will be completed in a time of $\Delta t = \Delta x / v$, where Δx is the distance to Proxima Centauri as measured from Earth. On the spacecraft, this time will be dilated: $\Delta t' = \Delta t\sqrt{1 - v^2 / c^2}$.

EVALUATE By journey's end, you will have aged by

$$\Delta t' = \frac{\Delta x}{v}\sqrt{1 - v^2 / c^2} = \frac{4 \text{ ly}}{0.8c}\sqrt{1 - 0.8^2} = 3 \text{ y}$$

The answer is **(a)**.

ASSESS From Earth's perspective, you will have aged 5 years.

74. **INTERPRET** You consider possible relativistic effects on a high-speed interstellar voyage.

DEVELOP "Moving clocks run slow." From the Earth's perspective, the clocks on your ship are moving, so "one second" for Earth will correspond to

$$\Delta t' = (1\text{s})\sqrt{1 - 0.8^2} = 0.6\text{s}.$$

EVALUATE However, the same argument applies to Earth from your perspective. You see Earth moving away from you at $v = 0.8c$, so you will judge that Earth's clocks are moving and that they run slow compared to the stationary clocks on your ship.

The answer is **(c)**.

ASSESS This is not a contradiction, since there is no absolute time by which to judge the "true" slowness of a clock. Each observer measures time with the clocks that are stationary in their reference frame, but there is no clock that is stationary in all reference frames.

75. **INTERPRET** You consider possible relativistic effects on a high-speed interstellar voyage.

DEVELOP The distance from Earth to the star will be length contracted in your reference frame: $\Delta x' = \Delta x\sqrt{1 - v^2 / c^2}$.

EVALUATE Plugging in the values, you find

$$\Delta x' = (4 \text{ ly})\sqrt{1 - 0.8^2} = 2.4 \text{ ly}$$

The answer is **(a)**.

ASSESS This agrees with the result from Problem 33.73 since this is the distance your ship will have traveled in 3 years' time.