## ELECTRIC CIRCUITS

# 25

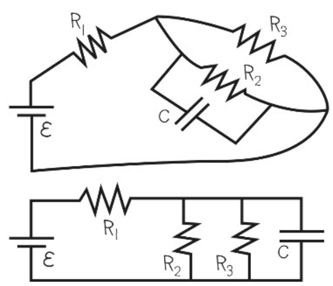
### EXERCISES

#### Section 25.1 Circuits, Symbols, and Electromotive Force

**11.** **INTERPRET** This problem is an exercise in drawing a circuit diagram, given a written description of the circuit in terms of its capacitors, resistors, and battery.

**DEVELOP** A literal reading of the circuit specifications results in connections like those in sketch **(a)**, below.

**EVALUATE** See sketch below. Because the connecting wires are assumed to have no resistance (a real wire is represented by a separate resistor), a topologically equivalent circuit diagram is shown in sketch **(b)**.

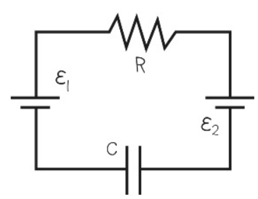



**ASSESS** There are three paths to ground (or to the negative battery terminal), two of which go through resistors and the third of which goes through the capacitor.

**12.** **INTERPRET** This problem involves connecting the various given circuit elements in series to form a closed circuit.

**DEVELOP** In a series circuit, the same current must flow through all elements. The order of the elements is not specified, so we can connect them in any order we like, provided that they are connected in series.

**EVALUATE** One possibility is shown below. The order of elements and the polarity of the battery connections are not specified.

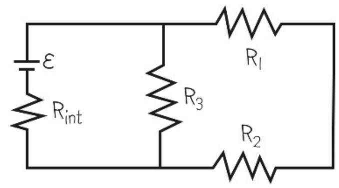



**ASSESS** An important feature about a series circuit is that the current through all the components must be the same. With two batteries, the direction of the current flow is determined by the polarity of the battery with the larger voltage.

**13.** **INTERPRET** This problem involves drawing a circuit diagram from the description given in the problem statement.

**DEVELOP** The circuit has three parallel branches: one with $R_1$ and $R_2$ in series; one with just $R_3$; and one with the battery.

**EVALUATE** See the figure below. $R_{int}$ is the internal resistance of the battery.




**ASSESS** This circuit has no capacitors, so we could replace $R_1$, $R_2$, and $R_3$ by an equivalent resistance

$$\frac{1}{R_{eq}} = \frac{1}{R_3} + \frac{1}{R_1 + R_2}$$

$$R_{eq} = \frac{R_3(R_1 + R_2)}{R_3 + R_2 + R_1}$$

**14.** **INTERPRET** This problem explores the connection between the emf of a battery and the energy it delivers. We are to find the emf of a battery given the work it does to move a given amount of charge.

**DEVELOP** Electromotive force, or emf, is defined as work per unit charge, $\mathcal{E} = W/q$ (see discussion accompanying Fig. 25.2).

**EVALUATE** Substituting the values given in the problem statement, the emf is

$$\mathcal{E} = \frac{W}{q} = \frac{27\text{ J}}{3.0\text{ C}} = 9.0\text{ V}$$

**ASSESS** For an ideal battery with zero internal resistance, the emf is equal to the terminal voltage (potential difference across the battery terminals).

**15.** **INTERPRET** This problem involves finding out for how long a battery can supply the given current while maintaining its rated voltage. We are given the total energy stored in the battery; so, if we can find the power the battery must deliver, we can find the time needed to deplete this energy reservoir.

**DEVELOP** Delivering the given current at the rated voltage results in a power expenditure of

$$\bar{P} = IV$$

(see Equation 24.7). Because the average power is defined as $\bar{P} = \Delta W / \Delta t$, we can find $\Delta t$, given that the energy we have to spend is $\Delta W = 5.0$ kJ.

**EVALUATE** Solving for the time interval $\Delta t$ and inserting the known quantities gives

$$\Delta t = \frac{\Delta W}{\bar{P}} = \frac{\Delta W}{IV} = \frac{5.0\text{ kJ}}{(1.5\text{ V})(0.60\text{ A})} = 5.6\times10^3\text{ s} = 1.5\text{ h}$$

**ASSESS** This result assumes that the battery voltage does not decrease as it depletes its store of energy, which is not a realistic assumption (although for most batteries, the departure from the ideal situation assumed in this problem is not huge).

**16.** **INTERPRET** This problem involves finding the (chemical) energy consumed in the battery for the work done.

**DEVELOP** The power delivered by an emf is $P = IV$ (see Equation 24.7). Therefore, if the voltage and current remain constant, then the energy consumed is $W = Pt = IVt.$

**EVALUATE** Substituting the values given, the energy used in

$$W = IVt = (5\text{ A})(12\text{ V})(3600\text{ s}) = 200\text{ kJ}$$

to a single significant figure.

**ASSESS** The result makes sense; the energy used up is proportional to the current drawn, the emf (i.e., voltage), and the time during which the headlights were left on.

## Section 25.2 Series and Parallel Circuits

**17.** **INTERPRET** This problem involves calculating the equivalent resistance of the given resistor combination.

**DEVELOP** Apply Equations 25.1 and 25.3b. For the parallel pair of resistors $R_1$ and $R_2$, Equation 25.3b gives

$$R_{1,2} = \frac{R_1 R_2}{R_1 + R_2}$$

Combining this in series with resistor $R_3$ (via Equation 25.1) gives

$$R_{1,2,3} = R_3 + R_{1,2} = R_3 + \frac{R_1 R_2}{R_1 + R_2}$$

**EVALUATE** Inserting the given resistances gives

$$R_{1,2,3} = 22\ \text{k}\Omega + \frac{(49\ \text{k}\Omega)(39\ \text{k}\Omega)}{49\ \text{k}\Omega + 39\ \text{k}\Omega} = 44\ \text{k}\Omega$$

**ASSESS** The final resistance is greater than the resistance of $R_3$ alone, as expected, but it is less than the resistance of $R_1$ alone. This is because $R_2$ is parallel to $R_1$ and allows some current to flow through the circuit without traversing $R_1$ (i.e., it adds another "traffic lane").

**18.** **INTERPRET** This problem is about connecting two resistors in parallel and calculating the equivalent resistance.

**DEVELOP** The equivalent resistance of two resistors connected in parallel can be found by Equation 25.3a:

$$\frac{1}{R_{\text{parallel}}} = \frac{1}{R_1} + \frac{1}{R_2}$$

The equation allows us to determine $R_2$ when $R_{\text{parallel}}$ and $R_1$ are known.

**EVALUATE** The solution for $R_2$ in Equation 25.3a is

$$R_2 = \frac{R_1 R_{\text{parallel}}}{R_1 - R_{\text{parallel}}} = \frac{(65\ \text{k}\Omega)(54\ \text{k}\Omega)}{65\ \text{k}\Omega - 54\ \text{k}\Omega} = 320\ \text{k}\Omega$$

to two significant figures.

**ASSESS** Our result shows that $R_2 > R_{\text{parallel}}$. This is consistent with the fact that the equivalent resistance $R_{\text{parallel}}$ is smaller than $R_1$ and $R_2$ because each individual resistor reduces the resistance of the parallel resistor circuit because they add new "traffic lanes" that allow charge to move through the circuit more easily.

**19.** **INTERPRET** This problem involves analyzing a real battery circuit. We are given the rated voltage of the battery and its real voltage when it powers the defective starter and are asked to find its real voltage for a proper starter.

**DEVELOP** This problem is similar to Example 25.2, which shows that the battery is connected in series with the load (i.e., the starter motor, which we treat as a resistor). To find the internal resistance of the battery, we use the macroscopic version of Ohm's law, $V = IR$:

$$V_{\text{terminals}} = 7.33\ \text{V} = \varepsilon - IR_{\text{int}} = 12.6\ \text{V} - (285\ \text{A})R_{\text{int}}$$
$$R_{\text{int}} = 0.0185\ \Omega$$

where $V_{\text{terminals}}$ is the actual voltage across the battery's terminals. Knowing the internal resistance of the battery, we can repeat this calculation to find $V_{\text{terminals}}$ when a proper starter is used.

**EVALUATE** The voltage across the terminals when a proper starter is used is

$$V_{\text{terminals}} = 12.6\ \text{V} - (112\ \text{A})(0.0185\ \Omega) = 10.5\ \text{V}$$

**ASSESS** Because the battery has an internal resistance, the voltage it can deliver is reduced by the voltage drop across the internal battery.

**20.** **INTERPRET** This problem is about the internal resistance of the battery in Problem 25.19.

**DEVELOP** The starter circuit contains all the resistances in series, as in Fig. 25.9. (We assume $R_L$ includes the resistance of the cables, connections, etc., as well as that of the motor.) With the defective starter, the terminal voltage is

$$V_{\text{term}} = \varepsilon - IR_{\text{int}}$$
$$7.33\ \text{V} = 12.6\ \text{V} - (285\ \text{A})R_{\text{int}}$$

**EVALUATE** From the equation above, we find the internal resistance to be

$$R_{\text{int}} = \frac{\varepsilon - V_{\text{term}}}{I} = \frac{(12.6\ \text{V} - 7.33\ \text{V})}{285\ \text{A}} = 0.0185\ \Omega$$

**ASSESS** The terminal voltage $V_{\text{term}} = 7.33\ \text{V}$ is substantially less than the battery's emf $\varepsilon = 12.6\ \text{V}$. The two are equal only in the ideal case where the internal resistance vanishes.

**21.** **INTERPRET** We are to find the internal resistance of a battery, given its short-circuit current and its rated voltage.
**DEVELOP** The battery contains an internal resistance (see section "Real Batteries"), which we can find using the macroscopic version of Ohm's law, $V = IR$, where $V = \varepsilon = 9V$.
**EVALUATE** Inserting the given quantities into Ohm's law gives

$$R_{\text{int}} = \varepsilon / I = (9\ \text{V})/(0.38\ \text{A}) = 24\ \Omega$$

**ASSESS** This is a rather large value for an internal resistance of a 9-V battery.

**22.** **INTERPRET** In this problem we are asked to find all possible values of equivalent resistance that could be obtained with three resistors.
**DEVELOP** Since each resistor can be placed either in parallel or in series, there are eight combinations using all three resistors. To find the equivalent resistance, use Equations 25.1 and 25.3a.
**EVALUATE** Let $R_1 = 1.0\ \Omega$, $R_2 = 2.0\ \Omega$, and $R_3 = 3.0\ \Omega$. The possible results are **(a)** one resistor in series with two resistors in parallel

$$R_3 + \frac{R_1R_2}{R_1 + R_2} = 3.0\ \Omega + \frac{(1.0\ \Omega)(2.0\ \Omega)}{1.0\ \Omega + 2.0\ \Omega} = \frac{11}{3}\ \Omega = 3.7\ \Omega$$
$$R_2 + \frac{R_1R_3}{R_1 + R_3} = 2.0\ \Omega + \frac{(1.0\ \Omega)(3\ \Omega)}{1.0\ \Omega + 3.0\ \Omega} = \frac{11}{4}\ \Omega = 8.8\ \Omega$$
$$R_1 + \frac{R_2R_3}{R_2 + R_3} = 1.0\ \Omega + \frac{(2.0\ \Omega)(3.0\ \Omega)}{2.0\ \Omega + 3.0\ \Omega} = \frac{11}{5}\ \Omega = 2.2\ \Omega$$

**(b)** one in parallel with two in series:

$$\frac{R_3(R_1 + R_2)}{R_1 + R_2 + R_3} = +\frac{(3.0\ \Omega)(1.0\ \Omega + 2.0\ \Omega)}{1.0\ \Omega + 2.0\ \Omega + 3.0\ \Omega} = \frac{9}{6}\ \Omega = \frac{3}{2}\ \Omega = 1.5\ \Omega$$
$$\frac{R_2(R_1 + R_3)}{R_1 + R_2 + R_3} = +\frac{(2.0\ \Omega)(1.0\ \Omega + 3.0\ \Omega)}{1.0\ \Omega + 2.0\ \Omega + 3.0\ \Omega} = \frac{8}{6}\ \Omega = \frac{4}{3}\ \Omega = 1.3\ \Omega$$
$$\frac{R_1(R_2 + R_3)}{R_1 + R_2 + R_3} = +\frac{(1.0\ \Omega)(2.0\ \Omega + 3.0\ \Omega)}{1.0\ \Omega + 2.0\ \Omega + 3.0\ \Omega} = \frac{5}{6}\ \Omega = 0.83\ \Omega$$

**(c)** three in series: $R_1 + R_2 + R_3 = 1.0\ \Omega + 2.0\ \Omega + 3.0\ \Omega = 6.0\ \Omega$.
**(d)** three in parallel:

$$\left(R_1^{-1} + R_2^{-1} + R_3^{-1}\right)^{-1} = \frac{R_1R_2R_3}{R_1R_2 + R_1R_3 + R_2R_3} = \frac{(1.0\ \Omega)(2.0\ \Omega)(3.0\ \Omega)}{(1.0\ \Omega)(2.0\ \Omega) + (1.0\ \Omega)(3.0\ \Omega) + (2.0\ \Omega)(3.0\ \Omega)} = \frac{6}{11}\ \Omega = 0.55\ \Omega$$

**ASSESS** The equivalent resistance is a maximum when all three are connected in series, as in **(c)**, and a minimum when all are connected in parallel, as in **(d)**.

## Section 25.3 Kirchhoff's Laws and Multiloop Circuits

**23.** **INTERPRET** This problem requires us to find the currents in all parts of a multiloop circuit.
**DEVELOP** The general solution of the two loop equations and one node equation given in Example 25.4 can be found using determinants (or $I_1$ and $I_2$ can be found in terms of $I_3$, as in Example 25.4). The equations and the solution are:

$$I_1R_1 + 0 + I_3R_3 = \varepsilon_1 \qquad \text{(loop 1)}$$
$$0 - I_2R_2 + I_3R_3 = \varepsilon_2 \qquad \text{(loop 2)}$$

$$I_1 - I_2 - I_3 = 0 \qquad \text{(node A)}$$

$$\Delta \equiv \begin{vmatrix} R_1 & 0 & R_3 \\ 0 & -R_2 & R_3 \\ 1 & -1 & -1 \end{vmatrix} = R_1R_2 + R_2R_3 + R_3R_1, \quad I_1 = \frac{1}{\Delta}\begin{vmatrix} \varepsilon_1 & 0 & R_3 \\ \varepsilon_2 & -R_2 & R_3 \\ 0 & -1 & -1 \end{vmatrix} = \frac{\varepsilon_1(R_2+R_3) - \varepsilon_2 R_3}{\Delta}$$

$$I_2 = \frac{1}{\Delta}\begin{vmatrix} R_1 & \varepsilon_1 & R_3 \\ 0 & \varepsilon_2 & R_3 \\ 1 & 0 & -1 \end{vmatrix} = \frac{\varepsilon_1 R_3 - \varepsilon_2(R_1+R_3)}{\Delta}, \quad I_3 = \frac{1}{\Delta}\begin{vmatrix} R_1 & 0 & \varepsilon_1 \\ 0 & -R_2 & \varepsilon_2 \\ 1 & -1 & 0 \end{vmatrix} = \frac{\varepsilon_2 R_1 + \varepsilon_1 R_2}{\Delta}$$

**EVALUATE** With the particular values of emfs and resistors in this problem, we have

$$\Delta = R_1R_2 + R_2R_3 + R_3R_1 = (2\ \Omega)(4\ \Omega) + (4\ \Omega)(1\ \Omega) + (1\ \Omega)(2\ \Omega) = 14\ \Omega^2$$

and the currents are

$$I_1 = \left[(R_2+R_3)\varepsilon_1 - R_3\varepsilon_2\right]\Delta^{-1} = \frac{(4\ \Omega + 1\ \Omega)(6\ \text{V}) - (1\ \Omega)(1.1\ \text{V})}{14\ \Omega^2} = 2.06\ \text{A} = 2.1\ \text{A}$$

$$I_2 = \left[R_3\varepsilon_1 - (R_1+R_3)\varepsilon_2\right]\Delta^{-1} = \frac{(1\ \Omega)(6\ \text{V}) - (2\Omega + 1\ \Omega)(1.1\ \text{V})}{14\ \Omega^2} = 0.193\ \text{A} = 0.19\ \text{A}$$

$$I_3 = (R_2\varepsilon_1 + R_1\varepsilon_2)\Delta^{-1} = \frac{(4\ \Omega)(6\ \text{V}) + (2\Omega)(1.1\ \text{V})}{14\ \Omega^2} = 1.87\ \text{A} = 1.9\ \text{A}$$

to a single significant figure.

**ASSESS** The same results could be obtained by retracing the reasoning of Example 25.4, with $\varepsilon_2 = 1.1$ V replacing the original value in loop 2. Then, everything is the same until the equation for loop 2: $1.1 + 4I_2 - I_3 = 0$.

**24.** **INTERPRET** This problem asks us to find the current through a resistor in a circuit.

**DEVELOP** The right-hand side of this circuit is irrelevant for this problem because the emf source is directly connected to the resistor without any intervening resistances, so the voltage drop across the resistor is simply the emf voltage. Thus, we can apply the macroscopic version of Ohm's law to find the current.

**EVALUATE** The current is $I_{3\Omega} = V_{3\Omega}/R_{3\Omega} = (6\,\text{V})/(3\Omega\,) = 2$ A.

**ASSESS** Note that if the 6-V battery had an internal resistance, an argument like that used in Example 25.4 must be applied.

**25.** **INTERPRET** We are to find the current through a resistor in a given circuit, for which we can use Kirchhoff's laws. We will use the loops and nodes drawn in Example 25.4.

**DEVELOP** The circuit is given to us in Fig. 25.14, with one change: $\varepsilon_2 = 2.0\text{V}$. We will use node A and loops 1 and 2. These will give us three equations, which we will use to solve the three unknown currents. At node A, $-I_1 + I_2 + I_3 = 0$. For loop 1, $\varepsilon_1 - I_1R_1 - I_3R_3 = 0$. For loop 2, $\varepsilon_2 + I_2R_2 - I_3R_3 = 0$.

**EVALUATE** Because we are interested in the current $I_2$, we eliminate the other two currents. The node equation gives us $I_1 = I_2 + I_3$. Substitute this into the equation for loop 1 and solve for $I_3$

$$\varepsilon_1 - (I_2 + I_3)R_1 - I_3R_3 = 0$$

$$I_3 = \frac{\varepsilon_1 - I_2R_1}{R_1 + R_3}$$

Now substitute this value into the equation for loop 2 and solve for $I_2$:

$$\varepsilon_2 + I_2R_2 - \frac{\varepsilon_1 - I_2R_1}{R_1 + R_3}R_3 = 0$$

$$\varepsilon_2(R_1+R_3) + I_2R_2(R_1+R_3) - \varepsilon_1R_3 + I_2R_1R_3 = 0$$

$$\varepsilon_2(R_1+R_3) + I_2(R_1R_2 + R_2R_3 + R_1R_3) - \varepsilon_1R_3 = 0$$

$$I_2 = \frac{\varepsilon_1R_3 - \varepsilon_2(R_1+R_3)}{R_1R_2 + R_2R_3 + R_1R_3} = \frac{(6\text{V})(1\Omega) - (2\text{V})(3\Omega)}{8\Omega^2 + 4\Omega^2 + 2\Omega^2} = 0\ \text{V}/\Omega = 0\ \text{A}$$

**ASSESS** The current through resistor $R_2$ is zero! Looking back at the original diagram, we can see that this would mean that battery 2 is supplying no current and the voltage drops through resistors 1 and 3 equal the voltage supplied by battery 1. This is a somewhat unexpected solution, but it is consistent.

## Section 25.4 Electrical Measurements

**26.** **INTERPRET** This problem requires us to determine the error in a voltage measurement that results from the internal resistance of the voltmeter.

**DEVELOP** The voltage across the 10-kΩ resistor in Fig. 25.27 is $(150\text{ V})(10)/(10+5) = 100\text{ V}$ (the circuit is just a voltage divider, as described by Equations 25.2a and 25.2b), as would be measured by an ideal voltmeter with infinite resistance. With the real voltmeter connected in parallel across the 10-kW resistor, the effective resistance is changed to $R_{\|} = (10\text{ k}\Omega)(280\text{ k}\Omega)/(290\text{ k}\Omega) = 9.66\text{ k}\Omega,$ so we can find the measurement-error ratio.

**EVALUATE** The measured voltage is

$$\frac{(150\text{ V})(9.66)}{9.66+5} = 98.8\text{ V}$$

which is about 1.2% lower than the true voltage.

**ASSESS** The measured voltage is slightly lower than the real voltage because the voltmeter allows some of the current to bleed through it, thus reducing the current that has to traverse the 10-kΩ resistor.

**27.** **INTERPRET** This problem involves finding the measurement error caused by the nonzero resistance of the ammeter used to measure the current.

**DEVELOP** The current in the circuit of Fig. 25.27 is

$$I = \frac{V}{R_{\text{tot}}} = \frac{V}{R_1+R_2} = \frac{150\text{ V}}{5\text{ k}\Omega + 10\text{ k}\Omega} = 10\text{ mA}$$

With the ammeter inserted (in series with the resistors), the resistance $R_{\text{tot}}$ is increased by $R_{\text{A}} = 100\ \Omega$.

**EVALUATE** The resulting current after including $R_{\text{A}}$ is

$$I' = \frac{V}{R_1+R_2+R_A} = \frac{150\text{ V}}{5\text{ k}\Omega+10\text{ k}\Omega+0.10\text{ k}\Omega} = 9.93\text{ mA}$$

which is about 0.66% lower than $I$.

**ASSESS** The current reading by the ammeter is lower due to its internal resistance.

**28.** **INTERPRET** We are to find the power dissipated in a circuit where the given voltage is discharged through the resistance that is the series resistance of the internal resistances of the battery and the ammeter.

**DEVELOP** The series resistance through which the voltage is discharged is

$$R_{\text{series}} = R_{\text{bat}} + R_{\text{meter}}$$

The current flowing through this circuit is $I = V/R_{\text{tot}} = V/(R_{\text{bat}} + R_{\text{meter}}),$ and the power dissipated in the meter is (Equation 24.8a) $P = I^2 R_{\text{meter}}$.

**EVALUATE** Inserting the given values, the power dissipated is

$$P = I^2 R_{\text{meter}} = \frac{\varepsilon^2 R_{\text{meter}}}{(R_{\text{bat}} + R_{\text{meter}})^2} = \frac{(12\text{ V})^2 (0.3\ \Omega)}{(0.31\ \Omega)^2} = 0.45\text{ kW}$$

**ASSESS** This power is comparable to that consumed by a small toaster oven. The ammeter would be destroyed quickly.

## Section 25.5 Capacitors in Circuits

**29.** **INTERPRET** In this problem we are asked to show that the quantity $RC$, the product of resistance and capacitance, has units of time.

**DEVELOP** The SI units for $R$ and $C$ are Ω and F, respectively. The units can be rewritten as

$$1\,\Omega = 1\frac{\text{V}}{\text{A}} = 1\frac{\text{V}}{\text{C/s}} = 1\frac{\text{V}\cdot\text{s}}{\text{C}},\quad 1\text{ F} = 1\frac{\text{C}}{\text{V}}$$

**EVALUATE** From the expressions above, the SI units for the time constant, $RC$, are

$$\Omega \cdot \mathrm{F} = \left(\frac{\mathrm{V \cdot s}}{\mathrm{C}}\right)\left(\frac{\mathrm{C}}{\mathrm{V}}\right) = \mathrm{s}$$

as stated.

**ASSESS** The quantity $RC$ is the characteristic time for changes to occur in an $RC$ circuit.

**30.** **INTERPRET** This problem requires us to find the time units for the $RC$ time constant when the resistance $R$ is given in various units.

**DEVELOP** From the results of the previous problem, we know that an $\Omega \cdot \mathrm{F} = \mathrm{s}$, so any prefactors applied to Ω or F are simply applied to s.

**EVALUATE** **(a)** $(\Omega)(\mu\mathrm{F}) = \mu\mathrm{s}$, **(b)** $(\mathrm{k}\Omega)(\mu\mathrm{F}) = 10^3 \times 10^{-6}\ \mathrm{s} = \mathrm{ms}$, **(c)** $(\mathrm{M}\Omega)(\mu\mathrm{F}) = 10^6 \times 10^{-6}\mathrm{s} = \mathrm{s}$ .

**ASSESS** The prefactors μ, M, k, and so on are simply multiples of 10, so they can be treated mathematically as for scalar factor.

**31.** **INTERPRET** This problem involves the time dependence of the capacitor voltage in a charging $RC$ circuit. We are to find the charging ratio of a capacitor after 5 $RC$ time constants.

**DEVELOP** The capacitor voltage as a function of time is given by Equation 25.6:

$$V_{\mathrm{cap}} = \varepsilon(1 - e^{-t/RC})$$

**EVALUATE** When $t = 5RC$, the equation above gives a voltage of

$$\frac{V_{\mathrm{cap}}}{\varepsilon} = 1 - e^{-5} = 1 - 6.74 \times 10^{-3} \approx 99.3\%$$

of the applied voltage.

**ASSESS** As time goes on and after many more time constants, we find essentially no current flowing to the capacitor, and the capacitor could be considered as being fully charged for all practical purposes.

**32.** **INTERPRET** This problem involves an $RC$ circuit. We are to find the time required for the capacitor to charge, given the voltage, resistance, and capacitance of the circuit.

**DEVELOP** The capacitor voltage as a function of time is given by Equation 25.6:

$$V_{\mathrm{cap}} = \varepsilon\left(1 - e^{-t/RC}\right)$$

**EVALUATE** Solving the expression above for time and inserting the given quantities gives

$$t = RC \ln\left(\frac{\varepsilon}{\varepsilon - V_C}\right) = (10\ \mu\mathrm{F})(470\ \mathrm{k}\Omega)\ln\left(\frac{250}{250\ \mathrm{V} - 140\ \mathrm{V}}\right) = 3.86\ \mathrm{s}$$

**ASSESS** Because the circuit capacitance takes time to discharge, this explains why, to start afresh, we need to turn devices such as computers off for several seconds before turning them back on.

**33.** **INTERPRET** We are to find the voltage across the capacitor in Fig. 25.23a when it is fully charged, which implies that the current through the capacitor is zero.

**DEVELOP** Use the results of Example 25.6 and Ohm's law to find the voltage required. If the capacitor is fully charged, then no current flows through it and the circuit is equivalent to the circuit shown in 25.23c. So we find the current through resistor $R_2$ in Fig. 25.23c and then determine the voltage across resistor $R_2$, which will be the same as the voltage across the capacitor.

**EVALUATE** The current through resistor $R_2$ is given in Example 25.6 as $I = \varepsilon/(R_1 + R_2)$. The voltage is given by Ohm's law as

$$V = IR = \left(\frac{\varepsilon}{R_1 + R_2}\right)R_2 = \varepsilon\left(\frac{R_2}{R_1 + R_2}\right)$$

**ASSESS** In the limit of long charging times, this circuit behaves like a voltage divider.

## EXAMPLE VARIATIONS

**34.** **INTERPRET** This problem asks for the current in one resistor that's part of a more complex circuit. So, it's about analyzing a circuit with series and parallel components.

**DEVELOP** Approaching this problem like the original example, we will follow the steps in Tactics 25.1, but will replace the 4.0-Ω resistor with a 2.0-Ω resistor. This means that the equivalent resistance obtained from the two 2.0-Ω resistor in parallel becomes: $R_{\text{parallel}} = (2\Omega)(2\Omega)/(2\Omega + 2\Omega) = 1\Omega$ . Adding the remaining three resistances in series results in: $R_{\text{series}} = 1\Omega + 1\Omega + 3\Omega = 5\Omega$ . To obtain the current which flows through these three resistances, and in particular the 2.0-Ω resistor, we apply Ohm's law.

**EVALUATE** From Ohm's law we find: $I_{5\Omega} = 12V/5\Omega = 2.4\Omega$ . Since this current flows across the equivalent resistance $R_{\text{parallel}}$ made up of two identical 2.0-Ω resistors, half of that current will flow through each one, making the current through the resistor $I_{2\Omega} = 1.2\,\text{A}$ .

**ASSESS** Note how, in solving this problem, we used Ohm's law to find, alternately, voltage and then current in different resistances. We also find that this current is smaller since the 2.0-Ω resistor is now connected in parallel with a resistance which is smaller than in the original example.

**35.** **INTERPRET** This problem asks for the possible voltage across a circuit, if the power dissipated across one resistor is known. So, it's about analyzing a circuit with series and parallel components.

**DEVELOP** Approaching this problem like the original example, we will follow the steps in Tactics 25.1, but will treat the voltage as $V$ unknown. This means that the equivalent resistance obtained from the two resistors in parallel is again: $R_{\text{parallel}} = (2\Omega)(4\Omega)/(2\Omega + 4\Omega) = 1.33\Omega$ . Adding the remaining three resistances in series results in: $R_{\text{series}} = 1\Omega + 1.33\Omega + 3\Omega = 5.33\Omega$ . We are given the power dissipated across one of the two resistors connected in parallel, so we can determine both the voltage and current across them. We can then use these currents to find the remaining resistor's current and voltage, which can then be used to determine the overall voltage output by the battery

**EVALUATE** From the power dissipated by the 4.0-Ω resistor we find the voltage across the parallel resistance: $P_{4\Omega} = V_{\text{parallel}}^{\;2}/R_{4\Omega} \rightarrow V_{\text{parallel}} = \sqrt{P_{4\Omega}R_{4\Omega}} = 0.8\,\text{V}$ . Applying Ohm's law for both resistors connected in parallel we find the currents: $I_{2\Omega} = V_{\text{parallel}}/R_{2\Omega} = 0.4\,\text{A}$ and $I_{4\Omega} = V_{\text{parallel}}/R_{4\Omega} = 0.2\,\text{A}$ . The sum of these currents is equal to the current which passes through both the 1.0-Ω and 3.0-Ω resistors, meaning we can find the voltage drop across both resistors: $V_{1\Omega} = I_{1\Omega}R_{1\Omega} = 0.6\,\text{V}$ and $V_{3\Omega} = I_{3\Omega}R_{3\Omega} = 1.8\,\text{V}$ . The voltage of the battery is found by adding up voltage drops across these resistors and the equivalent resistance from the two parallel resistors:

$$V = V_{1\Omega} + V_{\text{parallel}} + V_{3\Omega} = 3.2\,\text{V}$$

**ASSESS** Note how, in solving this problem, we used Ohm's law to find, alternately, voltage and then current in different resistances.

**36.** **INTERPRET** This problem asks for the voltage across a resistor that's part of a more complex circuit. We are told the resistance of all components, including that of the battery connected.

**DEVELOP** Approaching this problem like the original example, we will follow the steps in Tactics 25.1. We begin by finding the equivalent resistance of the two resistors connected in parallel:

$$R_{\text{parallel}} = (33.0\Omega)(47.0\Omega)/(33.0\Omega + 47.0\Omega) = 19.4\Omega$$

Adding the remaining three resistances (which includes $R_1$ and the internal resistance of the battery) in series results in:

$$R_{\text{series}} = 12.5\Omega + 33.0\Omega + 19.4\Omega = 64.9\Omega$$

We can apply Ohm's law to find the current which flows through this circuit and determine the voltage drop across $R_{\text{parallel}}$ .

**EVALUATE** Applying Ohm's law for the effective resistance contributed from all components we find the current which flows through the circuit is: $I = V/R_{\text{series}} = 92.4\,\text{mA}$ . Since this is the current that flows through all components in series, we find that the voltage which is found across the parallel resistors is equal to:

$$V_{\text{parallel}} = V - V_{\text{battery}} - V_1 = 6.00\,\text{V} - 1.16\,\text{V} - 3.05\,\text{V} = 1.79\,\text{V}$$

Where we have subtracted the voltage drops across the battery's internal resistance and $R_1$.

**ASSESS** Note how, in solving this problem, we used Ohm's law to find, alternately, voltage and then current in different resistances.

**37.** **INTERPRET** This problem asks for the power dissipated by a resistor that's part of a more complex circuit. We are told the resistance of all components and are treating the battery as ideal.

**DEVELOP** Approaching this problem like the original example, we will follow the steps in Tactics 25.1. We begin by finding the equivalent resistance of the two resistors connected in parallel:

$$R_{\text{parallel}} = (180\,\Omega)(68\,\Omega)/(180\,\Omega + 68\,\Omega) = 49.4\,\Omega$$

Adding the remaining three resistances in series results in:

$$R_{\text{series}} = 220\,\Omega + 49.4\,\Omega = 269.4\,\Omega$$

We can apply Ohm's law to find the current which flows through this circuit and use it to find the voltage, and ultimately the power dissipated, across $R_3$.

**EVALUATE** Applying Ohm's law for the effective resistance contributed from all components we find the current which flows through the circuit is: $I = V/R_{\text{series}} = 33.4\,\text{mA}$ . Since this is the current that flows through all components in series, we find that the voltage which is found across the parallel resistors is equal to:

$$V_{\text{parallel}} = V - V_1 = 9.0\,\text{V} - 7.35\,\text{V} = 1.65\,\text{V}$$

Where we have subtracted the voltage drops across $R_1$. Since this is the voltage found across $R_3$, we find the power dissipated by $R_3$ is equal to: $P_3 = V_{\text{parallel}}{}^2/R_3 = 40.0\,\text{mW}$ .

**ASSESS** Note how, in solving this problem, we used Ohm's law to find, alternately, voltage and then current in different resistances.

**38.** **INTERPRET** This is a problem about a charging capacitor, and we want to find the time to reach a given voltage.

**DEVELOP** Equation 25.6, $V_{\text{C}} = \mathcal{E}\left(1 - e^{-t/RC}\right)$ , gives the voltage across a charging capacitor, so our plan is to solve this equation for the time $t$.

**EVALUATE** First we solve for the exponential term that contains the time: $e^{-t/RC} = 1 - \frac{V_{\text{C}}}{\mathcal{E}}$ , Then we take the natural logarithm of both sides: $-\frac{t}{RC} = \ln\left(1 - \frac{V_{\text{C}}}{\mathcal{E}}\right)$. Solving for $t$ and plugging in the given numerical values gives:

$$t = -RC\ln\left(1 - \frac{V_{\text{C}}}{\mathcal{E}}\right) = 8.4\,\text{s}$$

**ASSESS** The high capacitance of this camera's circuit will require a longer charging time than the one discussed in the original example.

**39.** **INTERPRET** This is a problem about a charging capacitor, and we want to find the amount of charging resistance that will reduce the wait time of the camera flash from the preceding problem.

**DEVELOP** Equation 25.6, $V_{\text{C}} = \mathcal{E}\left(1 - e^{-t/RC}\right)$ , gives the voltage across a charging capacitor, so our plan is to solve this equation for the resistance $R$.

**EVALUATE** First we solve for the exponential term that contains the resistance: $e^{-t/RC} = 1 - \frac{V_{\text{C}}}{\mathcal{E}}$ , Then we take the natural logarithm of both sides: $-\frac{t}{RC} = \ln\left(1 - \frac{V_{\text{C}}}{\mathcal{E}}\right)$. Solving for $R$ and plugging in the given numerical values gives:

$$R = \frac{-t}{C\ln\left(1 - \frac{V_{\text{C}}}{\mathcal{E}}\right)} = 1.5\,\text{k}\Omega$$

**ASSESS** The high capacitance of this camera's circuit will require a larger voltage supplied and less charging resistance to decrease the time between flashes.

**40.** **INTERPRET** This is a problem about a discharging capacitor, and we want to find the time to reach a given voltage.

**DEVELOP** Equation 25.8, $V = V_0 e^{-t/RC}$, gives the voltage across a discharging capacitor, so our plan is to solve this equation for the time *t*.

**EVALUATE** **(a)** First we solve for the exponential term that contains the time and then we take the natural logarithm of both sides: $-\frac{t}{RC} = \ln\left(\frac{V}{V_0}\right)$. Solving for *t* and plugging in the given numerical values gives:

$$t = -RC\ln(0.1) = 14\,\text{ms}$$

where we have solved for the time when the voltage reaches 10% of the given 2.5 kV for $V_0$.

**(b)** To find the energy delivered during this time we calculate the initial energy stored in the fully charged capacitor and subtract from it the energy left over after it has depleted 90% of its voltage.

$$\Delta U = \frac{1}{2}C\left(V_0^2 - (0.1V_0)^2\right) = \frac{1}{2}C(0.99)V_0^2 = 464\,\text{J}$$

**ASSESS** Although 90% of the voltage is used, 99% of the available energy is delivered upon discharging.

**41.** **INTERPRET** This is a problem about a discharging capacitor like the one described in the preceding problem, and we want to find an expression for the capacitance which will lead a to certain fraction of the voltage to discharge in a given time.

**DEVELOP** Equation 25.8, $V = V_0 e^{-t/RC}$, gives the voltage across a discharging capacitor. We can relate this expression to the fraction *f* of energy stored in the capacitor which is discharged by performing the same calculation as in part (b) of the preceding problem.

$$\Delta U = \frac{1}{2}C\left(1 - \phi^2\right)V_0^2 = f\,U_\text{i}$$

Here we have calculated the amount of energy discharged by subtracting the amount of energy left over after a time $\Delta t$, where the remaining voltage at time *t* is equal to $V(t) = \phi V_0$, meaning: $\phi = \sqrt{1-f}$. Using this, we can express the voltage after a given time $\Delta t$ and solve for the capacitance which would result in the desired fractional energy discharge.

**EVALUATE** Solving for the exponential term in Equation 25.8 and plug in the voltage found after a time $\Delta t$ we find:

$$e^{-\Delta t/RC} = \frac{V}{V_0} = \phi = \sqrt{1-f}$$

$$C = \frac{-2\Delta t}{R\ln(1-f)}$$

**ASSESS** We can check the validity of this expression by plugging in the 99% of energy discharged from the preceding problem to recover the given capacitance of the defibrillator.

## PROBLEMS

**42.** **INTERPRET** This problem involves a multiloop circuit for which we are to find the resistance between the different points given.

**DEVELOP** The resistance between *A* and *B* is equivalent to two resistors of value *R* in series with the parallel combination of resistors of values *R* and 2*R*. Thus, the equivalent resistance may be found by combining Equations 25.1 and 25.3b. $R_\text{AC}$ is equivalent to just one resistor of value *R* in series with the parallel combination of *R* and 2*R* (since the resistor at point *B* carries no current, that is, its branch is an open circuit).

**EVALUATE** **(a)** $R_\text{AB} = R + R + R(2R)/(R+2R) = 8R/3$. **(b)** $R_\text{AC} = R + R(2R)/(3R) = 5R/3$.

**ASSESS** $R_\text{AB} > R_\text{AC}$ because the stem B carries no current.

**43.** **INTERPRET** This problem asks for the current in a resistor that is part of a more complex multiloop circuit. We will find the voltage drop over this resistor, which is part of a parallel combination of resistors, to find the current passing through it.

**DEVELOP** The circuit in Fig. 25.28, with a battery connected across points *A* and *B*, is similar to the circuit analyzed in Example 25.3. In this case, we have one 2.0-Ω resistor in parallel with two 2.0-Ω resistors in series. Thus, combining Equations 25.1 and 25.3b, we find

$$\frac{1}{R_{||}}=\frac{1}{2.0\ \Omega}+\frac{1}{2.0\ \Omega+2.0\ \Omega}=\frac{3}{4\ \Omega}\ \rightarrow\ R_{||}=\frac{4}{3}\ \Omega$$

and the total resistance is $R_{||}$ in series with two 2.0-Ω resistors: $R_{\text{tot}}=2.0\ \Omega+2.0\ \Omega+\frac{4}{3}\ \Omega=\frac{16}{3}\ \Omega$ The total current through the battery is

$$I_{\text{tot}}=\frac{\varepsilon}{R_{\text{tot}}}=\frac{6.0\ \text{V}}{16/3\ \Omega}=1.125\ \text{A}$$

**EVALUATE** Using the macroscopic version of Ohm's law, the voltage across the parallel combination is

$$V_{||}=I_{\text{tot}}R_{||}=(1.125\ \text{A})\left(\frac{4}{3}\ \Omega\right)=1.5\ \text{V}$$

which is the voltage across the vertical $R_{\text{v}}=2.0\text{-}\Omega$ resistor. Thus, the current through this resistor is then

$$I_{\text{v}}=\frac{V_{||}}{R_{\text{v}}}=\frac{1.5\ \text{V}}{2.0\ \Omega}=0.75\ \text{A}$$

**ASSESS** We have a total of 1.125 A of current flowing around the circuit. At the vertex of the triangular loop, it is split into $I_{\text{v}}=0.75\ \text{A}$ and $I'=I_{\text{tot}}-I_{\text{v}}=0.375\ \text{A}.$ The voltage drop across the vertical resistor $\left(V_{||}=1.5\ \text{V}\right)$ is the same as that going through point *C* and the two 2.0-Ω resistors: $V'=(0.375\ \text{A})(2.0\ \Omega+2.0\ \Omega)=1.5\ \text{V}.$ Thus, the result is consistent.

**44.** **INTERPRET** We are to find (to three significant figures) the voltage across the terminals of a battery for three different internal resistances, and with a 2-Ω resistor connected between the terminals.

**DEVELOP** The circuit diagram is like Fig. 25.9, and the voltage across the load (from Kirchhoff's voltage law) is $V_{\text{L}}=\varepsilon-IR_{\text{int}}$. Since $I=\varepsilon/(R_{\text{L}}+R_{\text{int}})$, we have $V_{\text{L}}=\varepsilon R_{\text{L}}/(R_{\text{L}}+R_{\text{int}})$ (as for a voltage divider).

**EVALUATE** With the given numerical values,

$$V_{\text{L}}=(1.5\ \text{V})(2\ \Omega)/(2\ \Omega+R_{\text{int}})=1.49\ \text{V}, 1.43\ \text{V},\ \text{and}\ 1.0\ \text{V}$$

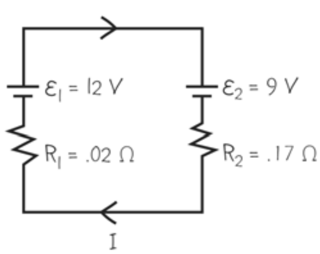
for $R_{\text{int}}$ = 0.01 Ω, 0.1 Ω, and 1 Ω, respectively.

**ASSESS** Normally, because the data is given to a single significant figure, we should retain only a single significant figure in the result.

**45.** **INTERPRET** The circuit has two batteries connected in series. We will apply Kirchhoff's law to find the current that flows through the discharged battery.

**DEVELOP** Terminals of like polarity are connected with jumpers of negligible resistance, giving a circuit as shown below. Kirchhoff's voltage law gives

$$\varepsilon_1-\varepsilon_2-IR_1-IR_2=0$$




**EVALUATE** Solving the equation above for *I*, we obtain

$$I=\frac{\varepsilon_1-\varepsilon_2}{R_1+R_2}=\frac{12\ \text{V}-9\ \text{V}}{0.02\ \Omega+0.17\ \Omega}=15.8\ \text{A}$$

**ASSESS** When you try to jump-start a car, you connect positive to positive and negative to negative terminals. The current is quite significant, which is why you want to have the charged car running to prevent the battery from being drained.

**46.** **INTERPRET** To check the safety of a battery, you must determine if a lethal dose of current could potentially flow through a person who is damp or sweaty.

**DEVELOP** The battery is not ideal. It has an internal resistance that will reduce the terminal voltage when current is flowing out of the battery. This internal resistance will be in series with the human body's resistance.

**EVALUATE** The total resistance will be the sum of the internal resistance and the human body's resistance. Therefore, the current that could potentially flow through a person with wet skin touching the battery terminals is

$$I = \frac{V}{R_{\rm int} + R_{\rm human}} = \frac{72\text{ V}}{100\ \Omega + 500\ \Omega} = 120\text{ mA}$$

Yes, this current could be fatal.

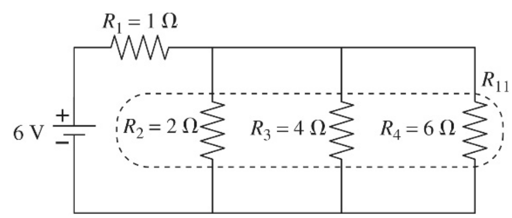
**ASSESS** You'll likely need to introduce a safety feature, such as a fuse, that can prevent such a high current from flowing out of the battery.

**47.** **INTERPRET** We are given a purely resistive circuit consisting of three resistors in parallel combined in series with a single resistor and are to find the current through the battery and the current through the 6-Ω resistor.

**DEVELOP** Label the resistors as shown below. The current supplied by the battery may be found using Ohm's law, $V = IR_{\rm tot}$, where $R_{\rm tot}$ is

$$R_{\rm tot} = R_1 + R_{\|} = R_1 + \frac{R_2R_3R_4}{R_2R_3 + R_3R_4 + R_4R_2} = 1\ \Omega + \frac{(2\ \Omega)(4\ \Omega)(6\ \Omega)}{(2\ \Omega)(4\ \Omega)+(4\ \Omega)(6\ \Omega)+(6\ \Omega)(2\ \Omega)} = 2.09\ \Omega$$

where we have used Equation 25.1 and 25.3a to find the total resistance. The voltage drop across all three resistors in parallel is $V_{\|} = \varepsilon - IR_1 = IR_{\|}$, so the current through the 6-Ω resistor can be found using Ohm's law.




**EVALUATE** **(a)** The current through the battery is

$$I = \frac{V}{R_{\rm tot}} = \frac{6\text{ V}}{2.09\ \Omega} = 2.9\text{ A}$$

to a single significant figure.

(**b**) To a single significant figure, the current through the 6-Ω resistor is

$$V_{\|} = V - IR_1 = I_4R_4$$

$$I_4 = \frac{V - IR_1}{R_4} = \frac{6\text{ V} - (2.87\text{ A})(1\ \Omega)}{6\ \Omega} = 0.52\text{ A}$$

**ASSESS** The currents passing through $R_2$ and $R_3$ are

$$I_2 = \frac{V - IR_1}{R_2} = 1.57$$

$$I_3 = \frac{V - IR_1}{R_3} = 0.78$$

which, when summed with $I_4$, give $I$, as expected. Notice that the smallest current runs through $R_4$ because it is the largest of the three parallel resistors.

**48.** **INTERPRET** We are given access to three different types of resistors which all dissipate the same maximum power. We are to come up with two configurations that will result in an effective resistance of 6.0 Ω capable of safely connecting to a 3.0-V battery.

**DEVELOP** We have 1.5-Ω, 3.0-Ω, and 12-Ω resistors available, and we can choose to assemble these in any combination of series and parallel configurations. However, neither of the configurations can have any resistors exceed a power dissipation of ½ W. Right away we can see that neither the 1.5-Ω or 3.0-Ω resistors can be safely connected in parallel with the battery while the 12-Ω resistors can. For series configurations, the order and total number of resistors will determine the amount dissipated in each component. Thus, we can first determine different configuration that will result in an effective resistance of 6.0 Ω, and then assess whether each resistor will dissipate less than or equal to ½ W when connected across the battery.

**EVALUATE** The first possible configuration has four 1.5-Ω resistors in series summing to 6.0 Ω, where each resistor has ½ A flowing through it, and experiences a voltage drop of ¾ V. This means that the power dissipated is equal to $P = IV = 3/8$ W, making it a safe configuration. A second possible configuration uses four of the 12-Ω resistors in parallel to obtain an effective resistance of $R_{\|1} = 3.0\,\Omega$, which is then placed in series with an identical set to obtain a final resistance of 6.0 Ω

$$R_{\text{tot}} = R_{\|1} + R_{\|2} = \left(\frac{4}{12\,\Omega}\right)^{-1} + \left(\frac{4}{12\,\Omega}\right)^{-1} = 6.0\,\Omega$$

In this configuration, the current flowing through both $R_{\|1}$ and $R_{\|2}$ is equal to ½ A, meaning each resistor has 1/8 A flowing through it. Since the voltage drop across both $R_{\|1}$ and $R_{\|2}$ is 3/2 V, the power dissipated in each resistor is $P = IV = 3/16$ W, again making it a safe configuration.

**ASSESS** Is it possible to come up with a third different configuration which utilizes at least one of each type? Why or why not?

**49.** **INTERPRET** This problem asks for the power dissipated in a resistor that is part of a multiloop circuit.

**DEVELOP** The three resistors in parallel have an effective resistance of

$$\frac{1}{R_\|} = \frac{1}{2\,\Omega} + \frac{1}{4\,\Omega} + \frac{1}{6\,\Omega} = \frac{11}{12\,\Omega} \;\Rightarrow\; R_\| = \frac{12}{11}\,\Omega$$

The equivalent resistance of the circuit is $R_{\text{tot}} = R_1 + R_\| = 1\,\Omega + \frac{12}{11}\,\Omega = \frac{23}{11}\,\Omega.$ Equation 25.2 gives the voltage across them as

$$V_\| = \frac{\varepsilon R_\|}{R_{\text{tot}}} = \frac{(6\text{ V})(12/11\,\Omega)}{23/11\,\Omega} = \frac{72}{23}\text{ V}$$

**EVALUATE** Using Equation 24.8b, the power dissipated in the 2-Ω resistor is

$$P_2 = \frac{V_\|^2}{R_2} = \frac{(72\text{ V}/23)^2}{2\,\Omega} = 4.9\text{ W}$$

**ASSESS** With $\varepsilon$ held fixed at 6 V, we see that the power dissipated is inversely proportional to the resistance.

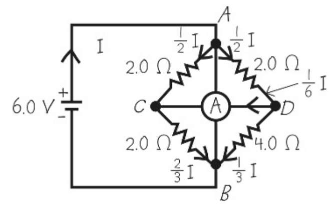
**50.** **INTERPRET** We are to find the ammeter reading when the ammeter is connected between the points of the multiloop circuit shown in the figure below.

**DEVELOP** Make a circuit diagram and label the currents and nodes as shown below. If the ammeter has zero resistance, the potential difference across it is zero, or nodes $C$ and $D$ are at equal potentials. If $I$ is the current through the battery, $\frac{1}{2}I$ must go through each of the 2-Ω resistors connected at node $A$ (Kirchhoff's current law), because the potential drop across them is the same. At node $B$, the 2-Ω resistor accepts twice the current of the 4-Ω resistor, or $\frac{2}{3}I$ and $\frac{1}{3}I$, respectively (the total current coming out of node B must be $I$, by Kirchhoff's current law). We thus know the currents $I_{\text{AC}}$ and $I_{\text{CD}}$. By Kirchhoff's current law, the current going through the ammeter must be the difference of $I_{\text{AC}}$ and $I_{\text{CD}}$, or

$$I_{\text{ammeter}} + I_{\text{AC}} - I_{\text{CD}} = 0$$
$$I_{\text{ammeter}} = -\frac{I}{2} + \frac{2I}{3} = \frac{I}{6}$$

To find the value of $I$, note that the upper pair of resistors are effectively in parallel because $V_C = V_D$, as is the lower pair. The effective resistance between $A$ and $B$ is therefore

$$R_{\text{eff}} = \frac{(2\ \Omega)(2\ \Omega)}{2\ \Omega + 2\ \Omega} + \frac{(2\ \Omega)(4\ \Omega)}{2\ \Omega + 4\ \Omega} = 1\ \Omega + \frac{4}{3}\ \Omega = \frac{7}{3}\ \Omega$$




**EVALUATE** Using Ohm's law, we find that the ammeter reads

$$V = IR_{\text{eff}} = 6I_{\text{ammeter}}R_{eff}$$

$$I_{\text{ammeter}} = \frac{V}{6R_{\text{eff}}} = \frac{6\text{ V}}{6(7\ \Omega/3)} = \frac{3}{7}\text{ A}$$

**ASSESS** To a single significant figure, this is 0.43 A.

**51.** **INTERPRET** We are to find the currents through each of the three resistors in the given circuit. We will use the circuit diagram given in Example 25.4.

**DEVELOP** The general solution of the two loop equations and the one node equation given in Example 25.4 can be found using determinants (or $I_1$ and $I_2$ can be found in terms of $I_3$, as in Example 25.4). The equations and the solution are:

$$\begin{aligned} I_1R_1 + 0 + I_3R_3 &= \varepsilon_1 && \text{(loop 1)} \\ 0 - I_2R_2 + I_3R_3 &= \varepsilon_2 && \text{(loop 2)} \\ I_1 - I_2 - I_3 &= 0 && \text{(node A)} \end{aligned}$$

$$\Delta \equiv \begin{vmatrix} R_1 & 0 & R_3 \\ 0 & -R_2 & R_3 \\ 1 & -1 & -1 \end{vmatrix} = R_1R_2 + R_2R_3 + R_3R_1 = (2\ \Omega)(4\ \Omega) + (2\ \Omega)(1\ \Omega) + (1\ \Omega)(4\ \Omega) = 14\ \Omega^2$$

$$I_1 = \frac{1}{\Delta}\begin{vmatrix} \varepsilon_1 & 0 & R_3 \\ \varepsilon_2 & -R_2 & R_3 \\ 0 & -1 & -1 \end{vmatrix} = \frac{\varepsilon_1(R_2 + R_3) - \varepsilon_2R_3}{\Delta}$$

$$I_2 = \frac{1}{\Delta}\begin{vmatrix} R_1 & \varepsilon_1 & R_3 \\ 0 & \varepsilon_2 & R_3 \\ 1 & 0 & -1 \end{vmatrix} = \frac{\varepsilon_1R_3 - \varepsilon_2(R_1 + R_3)}{\Delta}$$

$$I_3 = \frac{1}{\Delta}\begin{vmatrix} R_1 & 0 & \varepsilon_1 \\ 0 & -R_2 & \varepsilon_2 \\ 1 & -1 & 0 \end{vmatrix} = \frac{\varepsilon_2R_1 + \varepsilon_1R_2}{\Delta}$$

**EVALUATE** Using $\varepsilon_1 = 6\text{ V}$ and $\varepsilon_2 = -9\text{ V}$, we find

$$I_1 = \frac{(6\text{ V})(4\ \Omega + 1\ \Omega)}{14\ \Omega^2} = 2.8\text{ A}$$

$$I_2 = \frac{(6\text{ V})(1\ \Omega) - (-9\text{ V})(2\ \Omega + 1\ \Omega)}{14\ \Omega^2} = 2.4\text{ A}$$

$$I_3 = \frac{(-9\text{ V})(2\ \Omega) + (6\text{ V})(4\ \Omega)}{14\ \Omega^2} = 0.43\text{ A}$$

**ASSESS** From the signs of the currents, we know that $I_1$ flows down and $I_2$ and $I_3$ flow up. This is expected because the polarity of $\varepsilon_2$ is reversed with respect to Example 25.4, so the positive terminal of $\varepsilon_1$ is 15 V above the negative terminal of $\varepsilon_2$, and the central node (above $R_3$) is at 9 V with respect to the negative terminal of $\varepsilon_2$.

**52.** **INTERPRET** This problem asks us to reconsider the circuit discussed in Example 25.4 with a reversed battery. We are to determine an expression for the current running through a resistor and the voltage necessary in the reversed battery to make this current zero.

**DEVELOP** We can treat this problem similarly to Example 25.4, applying Kirchhoff's node law to each loop, and treating each quantity generally, to obtain the two equations:

$$\mathcal{E}_1 - I_1R_1 - I_1R_3 = 0$$

$$\mathcal{E}_2 - I_3R_3 + I_2R_2 = 0$$

And again recognizing that the currents satisfy the equation: $-I_1 + I_3 + I_2 = 0$ . We will use these to obtain an expression for the current $I_3$ in terms of the voltage $\mathcal{E}_2$ .

**EVALUATE** **(a)** We begin by rewriting the first loop equation as: $\mathcal{E}_1 - (I_2 + I_3)R_1 - I_1R_3 = 0$ , where solving for the current $I_2$ gives

$$I_2 = \frac{1}{R_1}\left(\mathcal{E}_1 - (R_1 + R_3)I_3\right)$$

which we then substitute into the second loop equation and simplify to:

$$\mathcal{E}_2 - I_3R_3 + \left[\frac{1}{R_1}\left(\mathcal{E}_1 - (R_1 + R_3)I_3\right)\right]R_2 = 0$$

$$\mathcal{E}_2 + \frac{R_2}{R_1}\mathcal{E}_1 - I_3\left[R_3 + \frac{R_2}{R_1}(R_1 + R_3)\right] = 0$$

$$I_3 = \frac{\left(\mathcal{E}_2 + \frac{R_2}{R_1}\mathcal{E}_1\right)}{\left[R_3 + \frac{R_2}{R_1}(R_1 + R_3)\right]} = \frac{(12 + \mathcal{E}_2)}{7}$$

**(b)** For there to be no current flowing through $R_3$ , the current $I_3$ would need to equal 0, which is only possible when the voltage $\mathcal{E}_2$ is equal to –12 V.

**ASSESS** We can check that this expression for $I_3$ is valid for any value and sign of $\mathcal{E}_2$ by plugging in the +9 V used in Example 25.4 to recover the 3 A current found in the original problem.

**53.** **INTERPRET** In this problem, we are to find the voltage across a given resistor as measured using a voltmeter, with the given internal resistances. Because the voltmeter is connected in parallel with the 30-kΩ resistor, the voltmeter's resistance adds in parallel to the resistor's resistance.

**DEVELOP** With a meter of resistance $R_m$ connected as indicated in the figure below, the circuit reduces to two pairs of parallel resistors in series. The total resistance is the sum of these parallel resistances (Equations 25.1 and 25.3b):

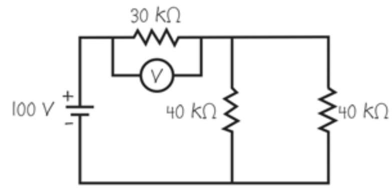
$$R_{tot} = \frac{(30\ \text{k}\Omega)R_m}{30\ \text{k}\Omega + R_m} + \frac{40\ \text{k}\Omega}{2}$$

Using Ohm's law (Chapter 24), the voltage reading is

$$V_m = R_mI_m = \frac{R_m(30\ \text{k}\Omega)I_{tot}}{30\ \text{k}\Omega + R_m}$$

where $I_{tot} = (100\ \text{V})/R_{tot}$ (the expression for $V_m$ follows from Equation 25.2, with $R_1$ and $R_2$ as the above pairs, or from $I_m$ as a fraction of $I_{tot}$).

**EVALUATE** For the three voltmeter resistances specified, $I_{tot} = 2.22$ mA, 2.10 mA, and 2.00 mA, and $V_m = 55.5$ V, 58 V, and 60 V, respectively.




**ASSESS** Of course, 60 V is the ideal voltmeter reading. This reading corresponds to an ideal voltmeter that has infinite resistance. Thus, to two significant figures, the 10-MΩ voltmeter is an ideal voltmeter.

**54.** **INTERPRET** For this problem we are to find the voltage between points *A* and *B* assuming an ideal voltmeter is used and the current from *A* to *B* assuming we connect an ideal ammeter between the two points. Recall that an ideal voltmeter has infinite resistance and an ideal ammeter has zero resistance.

**DEVELOP** An ideal voltmeter has infinite resistance, so *AB* is still an open circuit (as shown on Fig. 25.34) when such a voltmeter is connected. Thus, the meter will read the voltage across the $R_2$ = 36-kΩ resistor. From Ohm's law, the current passing through the resistors is $I = V/(R_1 + R_2)$, so the voltage across $R_2$ will be

$$V_2 = V - IR_1 = V - \frac{VR_1}{R_1 + R_2} = V\left(\frac{R_2}{R_1 + R_2}\right)$$

Because an ideal ammeter has zero resistance, it will measure the current through the points *A* and *B* when they are short-circuited (i.e., no current flows through the 36-kΩ resistor). We can find this current by applying Ohm's law to $R_1$.

**EVALUATE** (**a**) Inserting the values into the expression above, we find the voltage across $R_2$, as measured by an ideal voltmeter, is

$$V_2 = V\left(\frac{R_2}{R_1 + R_2}\right) = (48\text{ V})\left(\frac{36\text{ k}\Omega}{18\text{ k}\Omega + 36\text{ k}\Omega}\right) = 32\text{ V}$$

(**b**) The current passing through the ideal ammeter connected to points A and B is

$$I = \frac{V}{R_1} = \frac{48\text{ V}}{18\text{ k}\Omega} = 2.7\text{ mA}$$

**ASSESS** The current found in part **(b)** does not pass through $R_2$, because $R_2$ seems like an infinite resistance compared to the zero-resistance ammeter. The current passing through $R_2$ when the ammeter is not connected is

$$I_2 = \frac{V_2}{R_2} = \frac{32\text{ V}}{36\text{ k}\Omega} = 0.89\text{ mA}$$

which is less than the 2.7 mA because $R_2 > 0$.

**55.** **INTERPRET** In this problem an ammeter is used to measure the current in a circuit. The ammeter is connected in series with the resistor. We consider both the ammeter and the battery as having some internal resistance.

**DEVELOP** When the considering an ideal battery and ammeter are the internal resistances are zero, and the resistor would have a value of simply given by $R = \mathcal{E}/I$ . When considering the internal resistances of both, we must consider all three loads connected in series.

**EVALUATE** **(a)** Using the current the ammeter reads we find an unknown resistance of

$$R = \mathcal{E}/I = 18.25\Omega$$

**(b)** If the internal resistance of both battery and ammeter are known, we can consider this to be the effective resistance of all three loads. Thus, subtracting the sum of the 116.0 mΩ of the ammeter and the 59.0 mΩ of the battery from the result of part (a) we find the actual resistance of the unknown resitor is equal to: 18.07 Ω.

**(c)** To obtain the percent error between the results we calculate

$$\frac{R' - R}{R} = \frac{18.25 - 18.07}{18.25} = 1\%$$

Meaning the result in part (a) was high by 1%.

**ASSESS** This is a small error, given that the measurement device should interfere as little as possible, and the battery should provide as much of its available voltage as possible.

**56.** **INTERPRET** This problem involves an *RC* circuit, as shown in Fig. 25.18. We are to find the time required for the voltage across the capacitor to reach the given value, given the *RC* time constant. In addition, we are to find the capacitance, given the resistance of the circuit.

**DEVELOP** Equation 25.6 gives the voltage as a function of time for a charging capacitor. Given that the capacitor voltage at $t = 13$ ms is $V(t = 13\text{ ms}) = \varepsilon(1-1/e)$, we can write

$$V(t = 13\text{ ms}) = \varepsilon\left(1 - e^{-(13.0\text{ ms})/RC}\right) = \varepsilon\left(1 - e^{-1}\right)$$

which tells us that $(13.0\text{ ms})/(RC) = 1$, or $RC = 13.0$ ms. This allows us to find the time at which $V(t) = \varepsilon\left(1 - e^{-3}\right)$. To find the capacitance, we use the same result $(RC = 13.0\text{ ms})$ and insert $R = 22$ kW.

**EVALUATE** **(a)** When $V(t) = \varepsilon\left(1 - e^{-3}\right) = \varepsilon\left(1 - e^{-t/RC}\right)$, we have $t/RC = 3$, or $t = 3RC = 3(13.0\text{ ms}) = 39$ ms.

**(b)** For $R = 22\text{ k}\Omega$, $C = (13.0\text{ ms})/(22\text{ k}\Omega)$ 0.59 μF.

**ASSESS** Thus, at 13.0 ms, the capacitor is $1 - e^{-1} = 63\%$ charged, whereas at 39 ms, the capacitor is $1 - e^{-3} = 95\%$ charged.

**57.** **INTERPRET** You need to design a defibrillator that meets the desired discharge time. This is essentially an *RC* circuit, where the resistor is the human chest.

**DEVELOP** The defibrillator specs call for the capacitor to discharge to half its initial voltage in 10 ms. In terms of Equation 25.8, this implies: $e^{-t/RC} = \frac{1}{2}$. You can figure out the initial voltage using Equation 23.3: $U = \frac{1}{2}CV^2$.

**EVALUATE** Using $R = 33\ \Omega$ for the transthoracic resistance, the needed capacitance is to the nearest 10 μF:

$$C = \frac{-t}{R\ln\left(\frac{1}{2}\right)} = \frac{(10\text{ ms})}{(33\ \Omega)\ln 2} = 440\ \mu\text{F}$$

Given that the stored energy in the capacitor is 250 J, the initial voltage must be to the nearest 100 V:

$$V_0 = \sqrt{\frac{2U}{C}} = \sqrt{\frac{2(250\text{ J})}{(437\ \mu\text{F})}} = 1070\text{ V}$$

**ASSESS** The initial current going through the chest is $I_0 = V_0/R = 32.4$ A. Such a huge amount of current can sometimes cause burns (see Table 24.3). But the person will likely die if this "jolt" to the heart is not applied in time.

**58.** **INTERPRET** This problem involves an *RC* circuit, for which we are to find the resistance and then find the capacitance required to maintain the voltage across the capacitor to within 1 V for 1/60 s.

**DEVELOP** The effective resistance can be found using Ohm's law, given a voltage of $V = 35$ V and a current of $I = 1.2$ A. To find the capacitance needed to maintain the voltage above 34 V, apply Equation 25.8, which describes the rate of discharge of a capacitor. To keep the voltage within the prescribed range for the discharging capacitor, the time constant must satisfy $V/V_0 = e^{-t/RC} \geq 34/35$, which allows us to solve for $C$ using the value of $R$ from part **(a)**.

**EVALUATE** (**a**) The effective resistance of a circuit that draws 1.2 A from a constant 35-V supply is

$$R = \frac{V}{I} = \frac{35\text{ V}}{1.2\text{ A}} = 29\ \Omega$$

(**b**) Solving Equation 25.8 for the time constant, we find $RC \geq t/\ln(35/34)$. For $t = 1/60$ s and $R = 29.2$ W, the capacitance is $C \geq 20$ mF.

**ASSESS** This is a rather large capacitance, which is necessary because it must discharge a large current of 1.2 A for 1/60 s.

**59.** **INTERPRET** This problem involves energy dissipation in an *RC* circuit. Given the energy dissipated in the given time, we are to find the capacitance.

**DEVELOP** A capacitor discharging through a resistor is described by exponential decay, with time constant *RC* (Equation 25.8):

$$V(t) = V(0)e^{-t/RC}$$

The energy in the capacitor is given by Equation 23.3:

$$U_C(t) = \frac{1}{2}CV(t)^2 = \frac{1}{2}CV(0)^2 e^{-2t/RC} = U_C(0)e^{-2t/RC}$$

**EVALUATE** If 2 J is dissipated in time *t*, the energy stored in the capacitor drops from $U_C(0) = 5.0\text{ J}$ to $U_C(t) = 3.0\text{ J}$ (assuming there are no losses due to radiation, etc.). From the equation above, the capacitance is

$$C = \frac{2t}{R\ln\left[U_C(0)/U_C(t)\right]} = \frac{2(9.6\text{ ms})}{(10\text{ k}\Omega)\ln(5.0\text{ J}/3.0\text{ J})} = 3.8\ \mu\text{F}$$

**ASSESS** In this problem, the time constant is $RC = 38$ ms. Therefore, at 9.6 ms (about 0.253 *RC*), the energy decreases by a factor $e^{-2(0.253)} \approx 0.6$, which is precisely what we found (i.e., from 5.0 V to $5.0\times 0.6 = 3.0$ V).

**60.** **INTERPRET** The problem concerns what happens when a charged capacitor is connected to an uncharged capacitor. We'll worry about only the long-term behavior, that is, after the current has stopped flowing.

**DEVELOP** Initially, the charged capacitor has a charge of $Q_0 = C_2V_0$, where the subscript "2" refers to the 2-μF capacitor. When the switch is closed, charge will flow from the charged capacitor to the uncharged capacitor until the voltage across both is equal. Since the final charge on each capacitor must sum up to the initial charge: $Q_1 + Q_2 = Q_0$, the final voltage must be

$$C_1V_\text{f} + C_2V_\text{f} = C_2V_0 \quad \rightarrow \quad V_\text{f} = \frac{2\ \mu\text{F}}{1\ \mu\text{F} + 2\ \mu\text{F}}V_0 = \frac{2}{3}V_0$$

To find the total energy dissipated in the resistor, we find the difference between the stored energy in the initial and final states.

$$\Delta U = \left(\frac{1}{2}C_2V_0^2\right) - \left(\frac{1}{2}C_1V_\text{f}^2 + \frac{1}{2}C_2V_\text{f}^2\right) = \left(\frac{5}{18}C_2 - \frac{4}{18}C_1\right)V_0^2$$

**EVALUATE** Plugging in the values for the capacitors and the initial voltage, the energy dissipated in the resistor is

$$E = \Delta U = \left[\frac{5}{18}(2\ \mu\text{F}) - \frac{4}{18}(1\ \mu\text{F})\right](290\text{ V})^2 = 28\text{ mJ}$$

**ASSESS** Notice that the answer does not depend on the resistor's resistance. We might convince ourselves that this makes sense by looking at a simpler situation: a single capacitor discharging through a resistor, as in Fig. 25.22. The total energy dissipated by the resistor is the time integral of the power:

$$E = \int_0^\infty P\,dt = \int_0^\infty I^2R\,dt = \frac{V_0^2}{R}\int_0^\infty e^{-2t/RC}dt = \frac{1}{2}CV_0^2$$

This is the initial energy stored in the capacitor. We can imagine, therefore, that the two-capacitor situation is similar: the resistor dissipates the energy lost by the charged capacitor. The amount of resistance in the resistor will affect only how fast the energy is dissipated.

**61.** **INTERPRET** This problem is about the long-term and short-term behavior of an *RC* circuit. For each extreme, we are to find the voltage and current in both resistors of the *RC* circuit of Example 25.6.

**DEVELOP** In addition to the explanation in Example 25.7, we note that when the switch is closed, Kirchhoff's voltage law applied to the loop containing both resistors yields $\varepsilon = I_1R_1 + I_2R_2$, and Kirchhoff's law applied to the loop containing just $R_2$ and $C$ is $V_\text{C} = I_2R_2$.

**EVALUATE** **(a)** If the switch is closed at $t = 0$, the circuit behaves as if it were the circuit of Fig. 25.23b, and Example 25.6 explains that $V_\text{C}(0) = 0$, $I(0) = 0$; so

$$I_1(0) = \frac{\varepsilon}{R_1} = \frac{150\text{ V}}{4.0\ \text{k}\Omega} = 37.5\text{ mA}$$

**(b)** As $t \rightarrow \infty$, the circuit behaves like the circuit of Fig. 25.23c, and Example 25.7 shows that

$$I_1(\infty) = I_2(\infty) = \frac{\varepsilon}{R_1 + R_2} = \frac{150\text{ V}}{10\ \text{k}\Omega} = 15\text{ mA}$$

and $V_C(\infty) = I_2(\infty)R_2 = (15\ \text{mA})(6.0\ \text{k}\Omega) = 90\ \text{V}.$

**(c)** Under the conditions stated, the fully charged capacitor $(V_C = 90\ \text{V})$ simply discharges through $R_2$. ($R_1$ is in an open-circuit branch, so $I_1 = 0$ for the entire discharging process.) The initial discharging current is

$$I_2 = \frac{V_C}{R_2} = \frac{90\ \text{V}}{6.0\ \text{k}\Omega} = 15\ \text{mA}$$

**(d)** After a very long time, $I_2$ and $V_C$ decay exponentially to zero.

**ASSESS** We deduced the short-term and long-term behavior of the *RC* circuit without having to solve a complicated differential equation. A long time after the circuit has been closed, the capacitor becomes fully charged and no more current can cross it, so it behaves as an open circuit. When the circuit switch is reopened, the capacitor starts to discharge and eventually loses all its stored energy. It is now capable of storing charge again and behaves like a short circuit for times much less than its *RC* time constant.

**62.** **INTERPRET** We're asked to find the short-term and long-term behavior of a complicated *RC* circuit.

**DEVELOP** Right after the switch is closed, the two capacitors will act like short-circuits, that is, like wires with zero resistance. Current will flow through them in preference to any parallel resistors. Much later, the capacitors will be nearly fully charged, in which case they will act like an open circuit. No more current will flow through them.

**EVALUATE** **(a)** When the switch is closed, the capacitor $C_1$ in Fig. 25.36 will offer an essentially zero-resistance pathway for current from the emf to flow. Therefore, no current will flow through $R_2$, or $R_3$ for that matter. If we label the currents by the resistor they go through: $I_1 = \mathcal{E}/R_1,\ I_2 = I_3 = 0.$

**(b)** Long after the switch is closed, both capacitors $C_1$ and $C_2$ will be charged, so no more current will flow into these two branches of the circuit. All of the current from the emf will now flow through $R_2$, which means $I_1 = I_2 = \mathcal{E}/2R,$ and $I_3 = 0.$

**ASSESS** One can easily guess that $I_1$ and $I_2$ respectively decrease and increase monotonically from their initial to their final values, and that $I_3$ first increases from, and then decreases to zero.

**63.** **INTERPRET** We are asked to find the voltage and internal resistance of a battery using the measured voltage values of two voltmeters with different internal resistances.

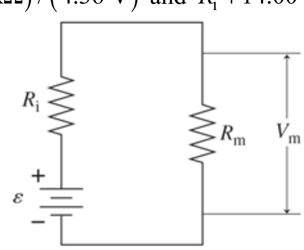
**DEVELOP** The internal resistance $R_i$ of the battery and the resistance $R_m$ of the voltmeter are in series with the battery's internal emf (see circuit below), so the current is $I = \varepsilon/(R_i + R_m)$. The potential drop across the meter (its reading) is

$$V_m = IR_m = \frac{\varepsilon R_m}{R_i + R_m}$$

From the given data, we can write

$$4.36\ \text{V} = \frac{\varepsilon(10.00\ \text{k}\Omega)}{R_i + 10.00\ \text{k}\Omega} \text{ and } 4.41\ \text{V} = \frac{\varepsilon(14.00\ \text{k}\Omega)}{R_i + 14.00\ \text{k}\Omega}$$

$$\text{or } R_i + 10.00\ \text{k}\Omega = \varepsilon(10.00\ \text{k}\Omega)/(4.36\ \text{V}) \text{ and } R_i + 14.00\ \text{k}\Omega = \varepsilon(14.00\ \text{k}\Omega)/(4.41\ \text{V})$$




**EVALUATE** Solving the simultaneous equations for $\varepsilon$ and $R_i$ gives

$$\varepsilon = 4.54\ \text{V and } R_i = 413\ \Omega$$

**ASSESS** When we let $R_i \to 0$, its reading approaches the battery's internal emf $\varepsilon$.

**64.** **INTERPRET** We are asked to find the voltage and internal resistance of a battery using the measured voltage values of two ammeters with different internal resistances.

**DEVELOP** The internal resistance $R_i$ of the battery and the resistance $R_m$ of the ammeter are in series with the battery's internal emf, so the current is $I = \mathcal{E}/(R_i + R_m)$. From the given data, we can write

$$\mathcal{E} = (9.78 \text{ A})(1.42\ \Omega + R_i) \text{ and } \mathcal{E} = (7.46 \text{ A})(2.11\ \Omega + R_i)$$

Equating the two expressions allows us to solve for $R_i$.

**EVALUATE** **(b)** Solving the equation $(9.78 \text{ A})(1.42\ \Omega + R_i) = (7.46 \text{ A})(2.11\ \Omega + R_i)$ gives

$$R_i = \frac{(7.46 \text{ A})(2.11\ \Omega) - (9.78 \text{ A})(1.42\ \Omega)}{9.78 \text{ A} - 7.46 \text{ A}} = 0.799\ \Omega$$

**(a)** The battery's internal emf is

$$\mathcal{E} = (9.78 \text{ A})(1.42\ \Omega + 0.799\ \Omega) = 21.7 \text{ V}$$

**ASSESS** An ideal ammeter has zero resistance.

**65.** **INTERPRET** This problem asks for the current through a resistor that is part of a more complex, multiloop circuit. The solution requires analyzing a circuit using Kirchhoff's laws.

**DEVELOP** Applying Kirchhoff's law to the right loop and the big loop gives

$$I_1 = I_2 + I_3$$
$$\mathcal{E}_1 - I_1 R_1 - I_3 R_3 + \mathcal{E}_3 = 0 \qquad \text{(big loop)}$$
$$\mathcal{E}_2 - I_3 R_3 + \mathcal{E}_3 + I_2 R_2 = 0 \qquad \text{(right loop)}$$

The three equations allow us to solve for the three currents.

**EVALUATE** Solving for $I_2$, we find

$$I_2 = \frac{(\mathcal{E}_1 - \mathcal{E}_2)R_3 - (\mathcal{E}_2 + \mathcal{E}_3)R_1}{R_1R_2 + R_1R_3 + R_2R_3} = \frac{(12.0 \text{ V} - 6.00 \text{ V})(4.00\ \Omega) - (6.00 \text{ V} + 3.00 \text{ V})(1.00\ \Omega)}{(1.00\ \Omega)(2.00\ \Omega) + (1.00\ \Omega)(4.00\ \Omega) + (2.00\ \Omega)(4.00\ \Omega)} = \frac{15}{14} \text{ A} = 1.07 \text{ A}$$

The direction is from left to right.

**ASSESS** The currents through $R_1$ and $R_2$ are

$$I_1 = \frac{(\mathcal{E}_1 + \mathcal{E}_3)R_2 + (\mathcal{E}_1 - \mathcal{E}_2)R_3}{R_1R_2 + R_1R_3 + R_2R_3} = \frac{27}{7} \text{ A} = 3.86 \text{ A}$$
$$I_3 = \frac{(\mathcal{E}_1 + \mathcal{E}_3)R_2 + (\mathcal{E}_2 + \mathcal{E}_3)R_1}{R_1R_2 + R_1R_3 + R_2R_3} = \frac{39}{14} \text{ A} = 2.79 \text{ A}$$

One can verify that $I_1 = I_2 + I_3$.

**66.** **INTERPRET** This problem involves a multiloop circuit. We find the condition such that no current goes through $\mathcal{E}_2$. The solution requires analyzing a circuit using Kirchhoff's laws.

**DEVELOP** Applying Kirchhoff's law to the right loop and the big loop gives

$$I_1 = I_2 + I_3$$
$$\mathcal{E}_1 - I_1 R_1 - I_3 R_3 + \mathcal{E}_3 = 0 \qquad \text{(big loop)}$$
$$\mathcal{E}_2 - I_3 R_3 + \mathcal{E}_3 + I_2 R_2 = 0 \qquad \text{(right loop)}$$

The three equations allow us to solve for the three currents. Solving for $I_2$, we find

$$I_2 = \frac{(\mathcal{E}_1 - \mathcal{E}_2)R_3 - (\mathcal{E}_2 + \mathcal{E}_3)R_1}{R_1R_2 + R_1R_3 + R_2R_3}$$

The condition that no current goes through $\mathcal{E}_2$ implies $I_2 = 0$.

**EVALUATE** **(a)** Setting $I_2 = 0$ gives

$$(\mathcal{E}_1 - \mathcal{E}_2)R_3 - (\mathcal{E}_2 + \mathcal{E}_3)R_1 = 0$$

Solving for $\mathcal{E}_2$, we obtain

$$\mathcal{E}_2 = \frac{\mathcal{E}_1 R_3 - \mathcal{E}_3 R_1}{R_1 + R_3} = \frac{(12.0 \text{ V})(4.00\ \Omega) - (3.00 \text{ V})(1.00\ \Omega)}{1.00\ \Omega + 4.00\ \Omega} = 9.00 \text{ V}$$

**(b)** Under these conditions, the currents in $R_1$ and $R_3$ are

$$I_1 = I_3 = \frac{(\mathcal{E}_1+\mathcal{E}_3)R_2 + (\mathcal{E}_1-\mathcal{E}_2)R_3}{R_1R_2+R_1R_3+R_2R_3} = \frac{(12.0\text{ V}+3.00\text{ V})(2.00\ \Omega)+(12.0\text{ V}-9.00\text{ V})(4.00\ \Omega)}{(1.00\ \Omega)(2.00\ \Omega)+(1.00\ \Omega)(4.00\ \Omega)+(2.00\ \Omega)(4.00\ \Omega)} = 3.00\text{ A}$$

**ASSESS** One can also explore conditions that give $I_1 = 0$ or $I_3 = 0$. The required conditions would be:

$$I_1 = 0 \Rightarrow (\mathcal{E}_1+\mathcal{E}_3)R_2 + (\mathcal{E}_1-\mathcal{E}_2)R_3 = 0$$
$$I_3 = 0 \Rightarrow (\mathcal{E}_1+\mathcal{E}_3)R_2 + (\mathcal{E}_2+\mathcal{E}_3)R_1 = 0$$

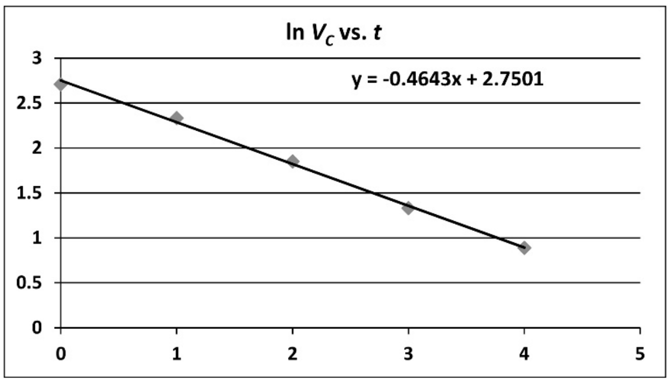
**67.** **INTERPRET** Using the data provided of the capacitor voltage as a function of time, we are to find the capacitance of the *RC* circuit.

**DEVELOP** From Equation 25.8, the voltage across the discharging capacitor is $V_C = \varepsilon e^{-t/RC}$. Thus, we see that the voltage $V_C$ across the capacitor decreases exponentially to zero as $t \to \infty$. Thus, taking logarithm on both sides of the equation gives

$$\ln V_C = \ln \varepsilon - \frac{1}{RC}t$$

Plotting $\ln V_C$ versus *t* will then give a straight line with slope $-1/RC$.

**EVALUATE** The plot is shown below.




The slope of the straight-line fit is $-0.4643 = -1/RC$, from which we find the capacitance to be

$$C = \frac{1}{0.4643R} = \frac{1}{0.4643(1.0\times10^6)} = 2.15\times10^{-6}\text{ F} = 2.15\ \mu\text{F}$$

**ASSESS** From the plot, we see that the *y*-intercept corresponds to $\ln\varepsilon$, which allows us to deduce the initial voltage across the capacitor:

$$\ln\varepsilon = 2.7501 \Rightarrow \varepsilon = 15.6\text{ V}$$

**68.** **INTERPRET** We are to find the resistance necessary in an *RC* circuit (see Fig. 25.18) to charge the given capacitor to 41% charge in 140 ms.

**DEVELOP** Apply Equation 25.6, which describes the voltage across a capacitor as a function of time.

**EVALUATE** Setting $V_C / e = 41\%$ and solving for the *RC* time constant in Equation 25.6, we find

$$RC = \frac{t}{-\ln(1.00-0.41)} = \frac{140\text{ ms}}{-\ln(0.59)} = 265\text{ ms}$$

For a 20 μF capacitor, the resistance must be

$$R = \frac{265\text{ ms}}{C} = \frac{265\text{ ms}}{20\ \mu\text{F}} = 13.25\text{ k}\Omega$$

**ASSESS** Notice that a higher resistance would increase the time constant, so it would take longer to charge the capacitor, whereas a small resistance would have the reverse effect.

**69.** **INTERPRET** The electric field at the node increases due to charge accumulation and eventually reaches the breakdown field strength. We are to find how long this process will take, given the rate at which charge accumulates on the sphere.

**DEVELOP** The charge on the node (whether positive or negative) accumulates at a rate of $I = dq/dt = 1\ \text{A} = 1\ \mu\text{C/s},$ so $|q(t)| = (1\ \mu\text{A})t$ (where we assume that $q(0) = 0$). If the node is treated approximately as an isolated sphere, and if we assume that the charge distribution on the sphere becomes uniform at a rate much higher than the input current (so that we can treat it as a static distribution), then we can apply Gauss's law and the results of Example 21.1. Under these conditions, the electric field strength at the surface of the sphere is given as

$$E = \frac{k|q|}{r^2} = \frac{kIt}{r^2}$$

Electric breakdown occurs when $E = E_b = 3\ \text{MV/m}.$

**EVALUATE** The time when the breakdown happens is

$$t = \frac{E_b r^2}{kI} = \frac{(3\ \text{MV/m})(1.0\ \text{mm})^2}{(9\times 10^9\ \text{m/F})(1\ \mu\text{A})} = 0.3\ \text{ms}$$

to a single significant figure.

**ASSESS** This problem shows that Kirchhoff's node law must hold, or else there would be a charge buildup at the node that quickly leads to an electric breakdown.

**70.** **INTERPRET** We want to find what load resistance connected to a battery will result in the greatest power output.

**DEVELOP** A real battery has an internal resistance, as shown in Fig. 25.8. When an external load is connected to the battery, the current that flows out will be $I = \mathcal{E}/(R_{\text{int}} + R_{\text{L}}).$ We want to find what $R_{\text{L}}$ will give the maximum power: $P = I^2 R_{\text{L}}.$

**EVALUATE** The power will be a maximum when its derivative with respect to $R_{\text{L}}$ is zero:

$$\frac{dP}{dR_{\text{L}}} = \frac{d}{dR_{\text{L}}}\left[\frac{\mathcal{E}^2 R_{\text{L}}}{(R_{\text{int}} + R_{\text{L}})^2}\right] = \mathcal{E}^2\left[\frac{1}{(R_{\text{int}} + R_{\text{L}})^2} - \frac{2R_{\text{L}}}{(R_{\text{int}} + R_{\text{L}})^3}\right] = 0$$

The equation is solved when $2R_L = R_{\text{int}} + R_{\text{L}},$ or $R_L = R_{\text{int}}.$

**ASSESS** The internal resistance of a battery is typically pretty low, so connecting a load with the same resistance would be essentially short-circuiting the battery. This could cause the battery to heat up and explode.

**71.** **INTERPRET** You need to specify what loudspeaker resistance is needed to get the maximum power output from an amplifier.

**DEVELOP** The loudspeaker resistance will be in series with the amplifier's internal resistance. This is similar to the previous problem, where it was shown that the maximum power in the load (the loudspeaker, in this case) occurs when its resistance matches the internal resistance of the power supply.

**EVALUATE** From the above arguments, the optimum resistance for the loudspeaker is 7 Ω. Since this is the same as the internal resistance of the amplifier, $R_{\text{int}}$, the power output will be:

$$P_{\text{max}} = \frac{\varepsilon^2 R_{\text{int}}}{(R_{\text{int}} + R_{\text{int}})^2} = \frac{\varepsilon^2}{4R_{\text{int}}}$$

If a loudspeaker with 3.5 Ω of resistance is connected instead, the power is reduced by

$$P = \frac{\varepsilon^2\left(\frac{1}{2}R_{\text{int}}\right)}{\left(R_{\text{int}} + \frac{1}{2}R_{\text{int}}\right)^2} = \frac{2\varepsilon^2}{9R_{\text{int}}} = \frac{8}{9}P_{\text{max}}$$

The maximum power is specified as 110 W, so a 3.5-Ω loudspeaker will output 97.8 W.

**ASSESS** A loudspeaker with half the optimum resistance still produces almost 90% of the maximum power. This shows that it's not necessary to exactly match the load to the amplifier.

**72.** **INTERPRET** This problem explores the energy stored in the capacitor of an *RC* circuit. We are asked to show that the capacitor stores only half the energy supplied by the battery.

**DEVELOP** The power supplied by the battery in charging an initially uncharged capacitor in an *RC* circuit is (Equation 24.7)

$$P = I\mathcal{E} = \frac{\mathcal{E}^2}{R} e^{-t/(RC)}$$

where the current is given by Equation 25.5, $I = (\mathcal{E}/R)e^{-t/(RC)}$. The total energy supplied by the battery is thus

$$U_{\text{battery}} = \int_0^\infty P dt = \frac{\mathcal{E}^2}{R}\int_0^\infty e^{-t/RC} dt = C\mathcal{E}^2\left(e^0 - e^{-\infty}\right) = C\mathcal{E}^2$$

which we can compare with the energy stored in the capacitor (Equation 2.3.3), $U = CV^2/2$, where $V$ is the final voltage across the capacitor.

**EVALUATE** The energy stored in the fully charged capacitor is

$$U_C(\infty) = \frac{1}{2}CV(t=\infty)^2 = \frac{1}{2}C\mathcal{E}^2 = \frac{1}{2}U_{\text{battery}}$$

Thus, we see that the energy stored in the capacitor is only half of that supplied by the battery.

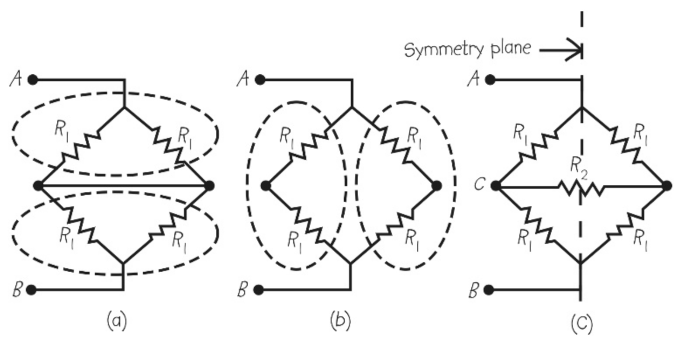
**ASSESS** The other half of the energy supplied by the battery is dissipated in the resistor:

$$U_{\text{R}} = \int_0^\infty I^2 R\ dt = \frac{\mathcal{E}^2}{R}\int_0^\infty e^{-2t/RC} dt = \frac{1}{2}C\mathcal{E}^2$$

Notice that this result is independent of the value of the resistance and capacitance of the circuit.

**73.** **INTERPRET** We're asked to determine the equivalent resistance for several complex systems of resistors.

**DEVELOP** The circuit in **(a)** can be seen as two resistors in parallel followed in series by another pair of resistors in parallel. See the figure below. The circuit in **(b)** can be seen as two parallel branches, each with two resistors in series. The circuit in **(c)** is symmetric across a plane through the middle, so the same amount of current should flow through each side.




**EVALUATE** **(a)** Each pair of parallel resistors has an equivalent resistance of $R_{||} = \frac{1}{2}R_1$. Added together in series, the total resistance is (Equation 25.1):

$$R_{\text{eq}} = R_{||} + R_{||} = R_1$$

**(b)** Each branch of resistors in series has an equivalent resistance of $R_{\text{S}} = 2R_1$. Added together in parallel, the total resistance is (Equation 25.3b):

$$R_{\text{eq}} = \frac{R_{\text{S}} \cdot R_{\text{S}}}{R_{\text{S}} + R_{\text{S}}} = R_1$$

**(c)** Due to the symmetry, the potential will be the same on both sides of $R_2$, therefore no current will flow through this resistor. If there's no current through this branch, then the circuit is identical to the one in part **(b)**, which means $R_{\text{eq}} = R_1$.

**ASSESS** Note that the reasoning in parts **(a)** and **(b)** is easily generalized to resistances of different values; the generalization in part **(c)** requires the equality of ratios of resistances that are mirror images in the plane of symmetry.

**74.** **INTERPRET** This problem involves finding the voltage and internal resistance of a battery. We are given the current values when the battery is connected to two resistors of known resistance.

**DEVELOP** The circuit diagram is like Fig. 25.8, and Kirchhoff's voltage law gives

$$\varepsilon - IR_{\text{int}} - IR_{\text{L}} = 0$$

For the two different resistors given, this may be written as

$$\varepsilon - (26\text{ mA})R_{\text{int}} = (26\text{ mA})(50\ \Omega) = 1.3\text{ V}$$

$$\varepsilon - (43\text{ mA})R_{\text{int}} = (43\text{ mA})(21\ \Omega) = 0.9\text{ V}$$

**EVALUATE** **(a)** and **(b)** Solving for $\varepsilon$ and $R_{\text{int}}$, we find

$$R_{\text{int}} = 23.5\ \Omega$$

$$\varepsilon = 1.91\text{ V}$$

**ASSESS** The terminal voltage of the battery is $V = \varepsilon - IR_{\text{int}} = 1.91\text{ V} - I(23.5\ \Omega)$, which is lower than $\varepsilon$. When the battery is connected to a resistor of resistance $R$, the current in the circuit is $I = \varepsilon/(R + R_{\text{int}})$.

**75.** **INTERPRET** This problem explores the rate of increase in voltage across the capacitor of an $RC$ circuit. We are to show that if the capacitor were to charge at its initial rate of charging (i.e., the rate at $t = 0$), then it would charge completely in a single time constant $\tau = RC$.

**DEVELOP** Kirchhoff's loop law for a battery charging a capacitor through a resistor is

$$\varepsilon - IR - V_{\text{C}} = 0$$

Differentiate this and use Equation 25.4 to obtain

$$\frac{dV_{\text{C}}(t)}{dt} = \frac{d\left[\varepsilon - I(t)R\right]}{dt} = -R\left[\frac{dI(t)}{dt}\right]$$

Using $I(t) = (\varepsilon/R)e^{-t/(RC)}$ for a charging capacitor (Equation 25.5), we find

$$\frac{dV_{\text{C}}(t)}{dt} = -R\frac{-I(t)}{RC} = \frac{I(t)}{C}$$

For an initially uncharged capacitor, $I(t=0) = \varepsilon/R \equiv I_0$, because an uncharged capacitor acts like a short-circuit. Thus, the initial rate of increase in voltage across the capacitor is

$$\frac{dV_{\text{C}}(t=0)}{dt} = \frac{\varepsilon}{RC} = \frac{\varepsilon}{\tau}$$

so we find how long it takes at this rate for the capacitor to be fully charged [i.e., to reach $V(t=\infty)$ ].

**EVALUATE** From Equation 25.6, we see that $V(t=\infty) = \varepsilon$, so charging at the above rate, the time $t$ it would take to reach this voltage is

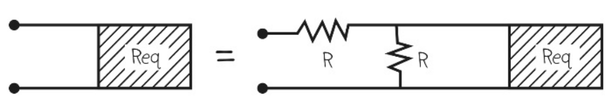
$$t\left[\frac{dV_{\text{C}}(t=0)}{dt}\right] = t\left(\frac{\varepsilon}{\tau}\right) = V(t=\infty) = \varepsilon$$

$$t\left(\frac{\varepsilon}{\tau}\right) = \varepsilon \quad \Rightarrow \quad t = \tau$$

**ASSESS** The real time it takes to reach full charge is longer than one time constant because the rate of change in the voltage is not constant.

**76.** **INTERPRET** Our circuit consists of an array of resistors of infinite extent, and we're asked to find the equivalent resistance.

**DEVELOP** Since the circuit line is infinite, the addition or deletion of one more element leaves the equivalent resistance unchanged. This can be represented diagrammatically as



The right-hand picture represents $R$ in series with the parallel combination $R$ and $R_{\text{eq}}$. Thus,

$$R_{\text{eq}} = R + \frac{RR_{\text{eq}}}{R + R_{\text{eq}}}$$

**EVALUATE** Solving for $R_{\text{eq}}$, one finds $R_{\text{eq}}^2 - RR_{\text{eq}} - R^2 = 0$, or

$$R_{\text{eq}} = \left(1+\sqrt{5}\right)\frac{R}{2} = 1.62R$$

Note that only the positive root is physically meaningful for a resistance.

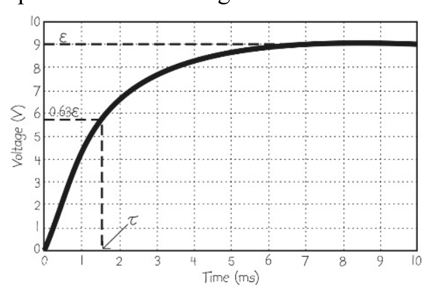
**ASSESS** Let's see how this limiting value is reached. With only two resistors, the equivalent resistance is $R_1 = R + R = 2R$. Next, consider four resistors (the four on the left of Fig. 25.41). The equivalent resistance is

$$R_2 = R + \frac{1}{1/R + 1/2R} = R + \frac{2R}{3} = 1.67R$$

Continuing the same line of reasoning leads to the quadratic equation that we solved to obtain $R_{\text{eq}} = \left(1+\sqrt{5}\right)R/2 = 1.62R.$

**77.** **INTERPRET** Using the plot provided of the capacitor voltage as a function of time, we are to find the battery voltage, time constant, and capacitance of the $RC$ circuit.

**DEVELOP** From Equation 25.6, $V_{\text{C}} = \varepsilon\left(1 - e^{-t/(RC)}\right)$, we see that the voltage $V_C$ across the capacitor asymptotically approaches the battery voltage $\varepsilon$ as $t \to \infty$. Thus, we can read the battery voltage off the graph by finding the asymptotic limit of the capacitor voltage (see the figure below). The time constant is the time it takes the capacitor voltage to reach $1 - e^{-1} = 63\%$ of its asymptotic value, as marked on the graph. From this estimate of the time constant $\tau$, we can find the capacitance from using $\tau = RC$.




**EVALUATE** **(a)** From the asymptotic value of the capacitor voltage, we find that the battery voltage is $\varepsilon \sim 9$ V .
**(b)** In one time constant t, the capacitor reaches $\varepsilon\left(1 - e^{-1}\right) \approx (9\text{ V})(0.63) = 5.7$ V . From the graph, this occurs at approximately $\tau \sim 1.5$ ms .
**(c)** The time constant is $RC$, so $C = \tau/R \approx (1.5\text{ ms})/4700\ \Omega \approx 0.3\ \mu\text{F}$ .

**ASSESS** From the graph, we can also see that the rate of increase of the capacitor voltage within one time constant is approximately linear, with a rate of

$$\frac{dV_{\text{C}}}{dt} \approx \frac{\varepsilon\left(1 - e^{-1}\right)}{\tau} \approx \frac{2\varepsilon}{3\tau}$$

**78.** **INTERPRET** This problem asks for the current through an emf source that is part of a more complex, multiloop circuit. The solution requires analyzing a circuit with series and parallel components.

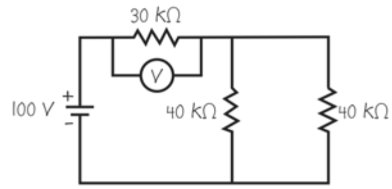
**DEVELOP** Consider the circuit diagram below, with the currents assumed as indicated. Applying Kirchhoff's law to the right loop and the big loop gives

$$I_a + I_b + I_c = 0 \quad \text{(top node)}$$
$$I_a(2R) - I_b(2R) = \varepsilon_1 - \varepsilon_3 \quad \text{(big loop)}$$
$$I_b(2R) - I_c R = \varepsilon_3 - \varepsilon_2 \quad \text{(right loop)}$$

Solve for $I_{\text{a}}$ and $I_{\text{c}}$ from the loop equations and substitute into the node equation:

$$\frac{(\varepsilon_1-\varepsilon_3)+2RI_b}{2R}+I_b+\frac{2RI_b-(\varepsilon_3-\varepsilon_2)}{R}=0$$

The current in $\varepsilon_3$ is $I_b$.




**EVALUATE** Solving for $I_b$, we find

$$I_b=\frac{(3\varepsilon_3-2\varepsilon_2-\varepsilon)}{8R}=\frac{(60\text{ mV}-90\text{ mV}-75\text{ mV})}{8(1.0\text{ M}\Omega)}=-13.1\text{ nA}$$

The negative sign means that the direction of $I_b$ is opposite to that shown in the diagram.
**ASSESS** The negative sign in $I_b$ can be easily understood by noting that $\varepsilon_3$ is smaller than $\varepsilon_1$ and $\varepsilon_2$.

**79.** **INTERPRET** This problem is an extension of the previous problem. The emf $\varepsilon_3$ changes now so that it supplies the indicated current. The rest of the circuit elements remain the same and we are to find the new value of $\varepsilon_3$.
**DEVELOP** The relation between $I_b$ and the circuit emf's and resistances, given in the solution to Problem 78, can be solved for $\varepsilon_3$ in Fig. 25.40, resulting in $\varepsilon_3=\frac{1}{3}(8RI_b+2\varepsilon_2+\varepsilon_1)$.

**EVALUATE** For $I_b=40$ nA and with the rest of the circuit elements remaining the same,

$$\varepsilon_3=\tfrac{1}{3}(8\times1.5\text{ M}\Omega\times40\text{ nA}+90\text{ mV}+75\text{ mV})=220\text{ mV}$$

**ASSESS** Thus, $\varepsilon_3$ changes by an order of magnitude from 20 mV (in Problem 25.72) to 220 mV here.

**80.** **INTERPRET** We will use Kirchhoff's laws to write a system of equations for the circuit shown in Fig. 25.27, and from the resulting equations we are to determine the possible values for the resistance $R_3$.
**DEVELOP** We are told the values of resistors $R_1$ and $R_2$, as well as the voltage across points $A$ and $B$, and the power dissipated by resistor $R_3$. This means we can determine a general expression for the current across the effective resistance $R_{123}$.

$$R_{123}=R_1+R_{23}=R_1+\frac{R_2R_3}{R_2+R_3}$$

by noting that $R_1$ is in series with the effective resistance $R_{23}$ resulting from the parallel arrangement of resistors $R_2$ and $R_3$. We will use it to find the current $I_3$ and voltage $V_3$ which are found across resistor $R_3$. From the given power dissipated, we can then solve for the possible positive values for the resistance $R_3$.
**EVALUATE** The effective resistance feels the voltage $V_{AB}$ across it, and the amount of current which flows through it is given by: $I_{AB}=V_{AB}/R_{123}$. This is the same current that flows through resistances $R_1$ and $R_{23}$, allowing us to write the expressions

$$I_1=I_2+I_3=\frac{V_{AB}(R_2+R_3)}{R_1(R_2+R_3)+R_2R_3}$$

$$V_{23}=V_{AB}-I_1R_1=V_{AB}\left(\frac{R_2R_3}{R_1(R_2+R_3)+R_2R_3}\right)$$

Since the voltage across both $R_2$ and $R_3$ is equal to $V_{23}$ we can express the current $I_2$ as $I_2=V_{23}/R_3$, and the current $I_3$ as

$$I_3=\frac{V_{AB}R_2}{R_1(R_2+R_3)+R_2R_3}$$

Multiplying this current by the voltage across $R_3$ gives the power dissipated

$$P_{23}=\left(\frac{V_{AB}R_2}{R_1(R_2+R_3)+R_2R_3}\right)^2R_3$$

We are given $P_{23}$, $V_{AB}$, $R_1$, and $R_2$, so we can expand and simplify this expression to find the possible values of $R_3$ to obtain

$$C = \frac{R_3}{\alpha R_3{}^2 + \beta R_3 + \gamma}$$

$$R_3{}^2 + \left(\frac{C\beta - 1}{\alpha C}\right) R_3 + \frac{\gamma}{\alpha} = 0$$

Where

$$C = \frac{P_{23}}{V_{AB}^2 R_2^2}$$
$$\alpha = (R_1 + R_2)^2$$
$$\beta = 2R_1R_2(R_1 + R_2)$$
$$\gamma = R_1^2 R_2^2$$

Solving the resulting quadratic equation using the given numerical values we obtain two possible values

$$R_3 = 3.88\,\Omega \text{ or } 330\,\Omega$$

**ASSESS** If the current is measured, it can used to identify which of these is the value for the unknown resistance.

**81.** **INTERPRET** We will use Kirchhoff's laws to write a system of equations for the circuit shown in Fig. 25.23a, and from the resulting equations we are to determine the time constant of the circuit.

**DEVELOP** We first sketch our loops and nodes, as shown in the figure below. We have 3 unknowns, so we will need 3 equations. Nodes *A* and *B* give us duplicate information, so we will use only one of the two: our equations must then come from loops 1 and 2, and node *A*. Node *A* gives us

$$I_1 - I_2 - I_3 = 0$$

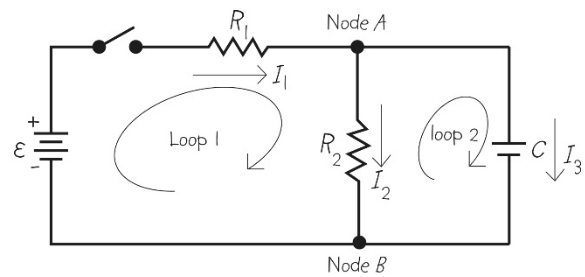
Loop 1 gives us

$$\varepsilon - I_1R_1 - I_2R_2 = 0$$

and loop 2 gives us

$$I_2R_2 - V_C = 0$$

The voltage across the capacitor is given by $V_C = Q/C$, and $I_3 = dQ/dt$. We will eliminate $I_1$ and $I_2$ in our system of equations, then rearrange the results into the form of Equation 25.4, from which we can easily identify the time constant.




**EVALUATE** From node *A*, $I_1 = I_2 + I_3$. Substitute this into the equation for loop 1:

$$\varepsilon - (I_2 + I_3)R_1 - I_2R_2 = 0$$
$$I_2 = \frac{\varepsilon - I_3R_1}{R_1 + R_2}$$

Now we substitute into the equation for loop 2:

$$\left(\frac{\varepsilon - I_3R_1}{R_1 + R_2}\right)R_2 - V_C = 0$$
$$\varepsilon R_2 - I_3R_1R_2 = \tfrac{Q}{C}(R_1 + R_2)$$

We take the time derivative of this last equation:

$$-\frac{dI_3}{dt}R_1R_2 = \frac{dQ}{dt}\frac{(R_1 + R_2)}{C}$$
$$-\frac{dI_3}{dt}R_1R_2 = I_3\frac{(R_1 + R_2)}{C}$$

Rearrange this slightly to obtain

$$\frac{dI_3}{dt} = -\frac{I_3}{R_1R_2C/(R_1+R_2)}$$

Now here's a trick: rather than solve this equation, we note that it's the *same* equation as 25.4, with a different cluster of constants in the denominator. In the solution to 25.4, we found that $\tau = RC$, so here the time constant must be

$$\tau = \frac{R_1R_2C}{R_1+R_2}$$

**ASSESS** This trick of putting the equation in a previously solved form can save us a lot of effort. Note that we can *only* do it because all the terms in the square brackets are constants: if there was a term involving $I_3$ in those brackets, then it would be a different equation and we couldn't use the same solution.

**82.** **INTERPRET** We must convert a battery energy rating (in watt-hours) at a given voltage to a charge rating of ampere-hours.

**DEVELOP** Apply Equation 24.7, $P = IV$. The battery is specified at 50 watt-hours, which means that it can supply $P = 50$ W for 1 hour. We will use $P = IV$ to find $I$, knowing that the voltage is $V = 6$ V.

**EVALUATE** $P = IV \quad \Rightarrow \quad I = P/V = (50\text{ W})/(6\text{ V}) = 8\text{ A}$ to a single significant figure.

**ASSESS** This is an 8-A·h battery, which is sufficient for our requirements.

**83.** **INTERPRET** This problem involves an *RC* circuit with a switch. We are asked to find the current supplied by the battery immediately after the switch is closed, and a long time after that.

**DEVELOP** Just after the switch is closed, the uncharged capacitor acts instantaneously like a short-circuit and the resistors act like two parallel pairs in series. The equivalent resistance of the combination is

$$R_{\text{eq}} = 2\left(\frac{1}{R}+\frac{1}{2R}\right)^{-1} = \frac{4R}{3}$$

A long time after the switch is closed, the capacitor is fully charged and acts like an open circuit. Then the resistors act like two series pairs in parallel, with an effective resistance of

$$\frac{1}{R'_{\text{eq}}} = \frac{1}{R+2R}+\frac{1}{R+2R} = \frac{2}{3R} \quad \Rightarrow \quad R'_{\text{eq}} = \frac{3R}{2}$$

**EVALUATE** **(a)** Thus, immediately after the switch is closed, the current is

$$I(t=0) = \frac{\varepsilon}{R_{\text{eq}}} = \frac{3\varepsilon}{4R}$$

**(b)** Similarly, a long time after the switch has been closed,

$$I(t\to\infty) = \frac{\varepsilon}{R'_{\text{eq}}} = \frac{\varepsilon}{3R/2} = \frac{2\varepsilon}{3R}$$

**ASSESS** Unlike a resistor that responds instantaneously to a voltage source, a capacitors exhibits "transient" behavior. In this case, it first behaves like a short-circuit, and later an open circuit after becoming fully charged.

**84.** **INTERPRET** We're asked to analyze a situation where stray voltage passes through a dairy cow.

**DEVELOP** The cow in this case completes the circuit. Its resistance is in series with the intrinsic resistance of the stray voltage.

**EVALUATE** The equivalent resistance is the sum of the cow and intrinsic resistances. The current can be found by Ohm's law:

$$I = \frac{V}{R_{\text{cow}}+R_{\text{int}}} = \frac{6\text{ V}}{500\ \Omega + 1\text{ k}\Omega} = 4\text{ mA}$$

The answer is **(b)**.

**ASSESS** We can't say for sure what a cow feels, but this is above the threshold for sensation in humans (see Table 24.3).

**85.** **INTERPRET** We're asked to analyze a situation where stray voltage passes through a dairy cow.

**DEVELOP** The voltage across the cow can be found with Ohm's law.

**EVALUATE** Given the current from the previous problem, the voltage between the cow's tongue and hoof is

$$V_{\text{cow}} = IR_{\text{cow}} = (4\text{ mA})(500\ \Omega) = 2\text{ V}$$

The answer is **(a)**.

**ASSESS** This is not a lot of voltage; it's just a little more than a D battery.

**86.** **INTERPRET** We're asked to analyze a situation where stray voltage passes through a dairy cow.

**DEVELOP** An ideal voltmeter is one with infinite resistance.

**EVALUATE** If an ideal voltmeter is attached from the water bowl to the ground, it will measure directly the emf, which in this case is 6 V.

The answer is **(c)**.

**ASSESS** The intrinsic resistance has no effect, since no current flows through the circuit with an ideal voltmeter. If we chose a more realistic case, say, a voltmeter with 10 MΩ of resistance, then a tiny current will trickle through the circuit (0.5 μA), and the voltage reading will be 4.9995 V (if indeed the voltmeter's precision is this high).

**87.** **INTERPRET** We're asked to analyze a situation where stray voltage passes through a dairy cow.

**DEVELOP** An ideal ammeter is one with zero resistance.

**EVALUATE** If an ideal ammeter is attached from the water bowl to the ground, it will close the circuit and read the current as:

$$I = \frac{V}{R_{\text{int}}} = \frac{6.0\text{ V}}{1\text{ k}\Omega} = 6\text{ mA}$$

The answer is **(b)**.

**ASSESS** This gives an idea of what the maximum current might be from the stray voltage. It also exemplifies the best way to eliminate the problem: by connecting the water bowl directly to ground. This would provide a zero-resistance pathway for current to flow, so that the cow no longer gets a shock every time it goes for a drink.