ROTATIONAL MOTION

10

EXERCISES

Section 10.1 Angular Velocity and Acceleration

11. **INTERPRET** This problem involves calculating the angular speed of a variety of rotating objects.

DEVELOP Apply Equation 10.1, $\bar{\omega} = \Delta\theta/\Delta t$, where $\Delta\theta$ is the rotation and Δt is the time interval for the rotation.

EVALUATE (**a**) $\omega_{\rm E} = (1\ \text{rev})/(1\ \text{d}) = 2\pi/(86{,}400\ \text{s}) = 7.27\times10^{-5}\ \text{s}^{-1}$

(**b**) $\omega_{\min} = (1\ \text{rev})/(1\ \text{h}) = 2\pi/3600\ \text{s} = 1.75\times10^{-3}\ \text{s}^{-1}$

(**c**) $\omega_{\rm hr} = (1\ \text{rev})/(12\ \text{h}) = \omega_{\min}/12 = 1.45\times10^{-4}\ \text{s}^{-1}$

(**d**) $\omega = (300\ \text{rev})/\text{min} = 300\times2\pi/(60\ \text{s}) = 31.4\ \text{s}^{-1}$

ASSESS Note that radians are a dimensionless angular measure, that is, pure numbers; therefore angular speed can be expressed in units of inverse seconds.

12. **INTERPRET** We are asked to compute the linear speed at the equator and at an arbitrary location on Earth. The problem involves the rotational motion of the Earth.

DEVELOP We first calculate the angular speed of the Earth using Equation 10.1:

$$\omega_{\rm E} = \frac{\Delta\theta}{\Delta t} = \frac{1\ \text{rev}}{1\ \text{d}} = \frac{2\pi\ \text{rad}}{86{,}400\ \text{s}} = 7.27\times10^{-5}\ \text{s}^{-1}$$

The linear speed can then be computed using Equation 10.3: $v = \omega r$.

EVALUATE **(a)** At the equator,

$$v = \omega_{\rm E} R_{\rm E} = \left(7.27\times10^{-5}\ \text{s}^{-1}\right)\left(6.37\times10^{6}\ \text{m}\right) = 463\ \text{m/s}$$

(b) At latitude θ, $r = R_{\rm E}\cos\theta$, so $v = \omega_E r = (463\ \text{m/s})\cos\theta$.

ASSESS The angle $\theta = 0$ corresponds to the equator, so the result found in (b) agrees with (a). In addition, if we take $\theta = 90°$, then we are at the poles, and the linear speed is zero.

13. **INTERPRET** This problem involves converting angular speed from various units to radians/s (which is the same as s^{-1}, or frequency).

DEVELOP Use the appropriate conversion factors to convert each angular speed to units of rad/s.

EVALUATE (**a**) $(720\ \text{rev/min})(2\pi\ \text{rad/rev})(\text{min}/60\ \text{s}) = 24\pi\ \text{rad/s} = 75\ \text{rad/s}$, to two significant figures.

(**b**) $(50°/h)(\pi\ \text{rad}/180°)(\text{h}/3600\ \text{s}) = 2.4\times10^{-4}\ \text{rad/s}$, to two significant figures.

(**c**) $(1000\ \text{rev/s})(2\pi\ \text{rad/rev}) = 2000\pi\ \text{s}^{-1} = 6\times10^{3}\ \text{rad/s}$ to a single significant figure.

(**d**) $(1\ \text{rev/y}) = 2\pi\ \text{rad}/(\pi\times10^{7}\ \text{s}) = 2\times10^{-7}\ \text{rad/s}$, to a single significant figure.

ASSESS Note that radians are a dimensionless angular measure, that is, pure numbers; therefore angular speed can be expressed in units of inverse seconds. The approximate value for 1 y used in part (d) is often handy for estimates, and is fairly accurate.

14. **INTERPRET** The problem asks you to find the linear speed at the outer edge of a 32-cm-diameter circular saw.

DEVELOP We first convert the angular speed to rad/s:

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2100\ \text{rev}}{1\ \text{min}} = \frac{2\pi(2100)\text{rad}}{60\ \text{s}} = 220\ \text{rad}/\text{s}$$

The linear speed can then be computed using Equation 10.3, $v = \omega r$.

EVALUATE The radius of the circular saw is $r = d/2 = 16\text{ cm} = 0.16\text{ m}$. From Equation 10.3, the linear speed of the outer edge of the saw is

$$v = \omega r = (220\text{ rad/s})(0.16\text{ m}) = 35.2\text{ m/s}$$

ASSESS The linear speed of the saw is over 78 mi/hr!

15. **INTERPRET** For this problem, we are asked to find the average angular acceleration, given the initial and final accelerations and the time interval.

DEVELOP Apply Equation 10.4 (before the limit is taken), $\bar{\alpha} = \Delta\omega/\Delta t$. The change in the angular velocity is $\Delta\omega = \omega_f - \omega_i = 480\text{ rpm} - 190\text{ rpm} = 290\text{ rpm}$, and the time interval is $\Delta t = (71\text{ min})(60\text{ s/min}) = 4260\text{ s}$. Recall that $1\text{ rpm} = (2\pi\text{rad})(60\text{ s})$.

EVALUATE **(a)** Inserting the given quantities gives an average angular acceleration of

$$\bar{\alpha} = \frac{290\text{ rpm}}{4260\text{ s}} = 0.068\text{ rpm/s}$$

(b) In rad/s^2, we have

$$\bar{\alpha} = \frac{290\text{ rpm}}{4260\text{ s}}\left(\frac{2\pi\text{ rad}}{\text{rev}}\right)\left(\frac{\text{min}}{60\text{ s}}\right) = 7.1\times10^{-3}\text{ rad/s}^2$$

ASSESS Note that the units cancel out to leave units of frequency, as expected.

16. **INTERPRET** In this problem, we are given the angular acceleration of a turbine and asked how long it takes to reach its operating speed and the number of revolutions that occur during this start-up period. The key to this type of rotational problem is to understand the analogous situation for linear motion and apply the appropriate equation. The analogies are summarized in Table 10.1.

DEVELOP Given a constant angular acceleration α, the angular velocity and angular position at a later time t can be found using Equations 10.7 and 10.8, respectively:

$$\omega = \omega_0 + \alpha t$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}at^2$$

The initial and final angular velocities are $\omega_0 = 0$ and

$$\omega = 5400\text{ rpm} = \frac{5400\text{ rev}}{1\text{ min}} = \frac{2\pi(5400)\text{rad}}{60\text{ s}} = 565.5\text{ rad/s}$$

EVALUATE **(a)** The amount of time it takes to reach its operating speed is

$$t = \frac{\omega - \omega_0}{\alpha} = \frac{565.5\text{ rad/s} - 0}{0.6\text{ rad/s}^2} = 942.5\text{ s} = 15.7\text{ min}$$

(b) Using Equation 10.8, we find the number of turns made during this time interval to be

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2 = \frac{1}{2}\alpha t^2 = \frac{1}{2}(0.6\text{ rad/s}^2)(942.5\text{ s})^2 = 2.66\times10^5\text{ rad} = 4.2\times10^4\text{ rev}$$

ASSESS The responses are given to two significant figures to reflect the precision of the data. The turbine turns very fast. After 15.7 min, it has reached an angular speed of 565.5 rad/s, or 90 rev/s!

17. **INTERPRET** This problem is an exercise in angular kinematics. We are given an angular acceleration and the acceleration period, and are asked to find the revolutions made in this time and the average angular speed.

DEVELOP Apply the formulas in Table 10.1. To find the number of revolutions, we find the total angular displacement θ from Equation 10.8, and then divide this by 2π (= 1 revolution) to find the number of revolutions. The linear analog to this can be thought of as finding a distance, and then dividing it by a given distance (say, 10-km segments) to find the number of 10-km segments traveled. In both cases, we end up with a dimensionless number. To find the average angular speed, use Equation 10.1.

EVALUATE **(a)** Inserting $\alpha = 0.011\text{ rad/s}^2$ and $t = 14\text{ s}$ into Equation 10.8 gives a final rotational distance θ of

$$\Delta\theta = \theta - \theta_0 = \overbrace{\omega_0}^{=0} t + \frac{1}{2}\alpha t^2$$

$$\Delta\theta = \frac{1}{2}\left(0.011\ \text{rad/s}^2\right)\left(14\ \text{s}\right)^2 = 1.078\ \text{rad}\left(\frac{1\ \text{rev}}{2\pi\ \text{rad}}\right) = 0.17\ \text{rev}$$

(b) From Equation 10.7, with $\theta_0 = 0$, the final angular speed is

$$\bar{\omega} = \frac{\Delta\theta}{\Delta t} = \frac{\theta - \theta_0}{\Delta t} = \frac{1.078\ \text{rad}}{14\ \text{s}} = 0.077\ \text{rad/s}$$

ASSESS The final angular speed of the merry-go-round is, from Equation 10.7,

$$\omega = \overbrace{\omega_0}^{=0} + \alpha t = \left(0.011\ \text{rad/s}^2\right)\left(14\ \text{s}\right) = 0.154\ \text{rad/s}$$

which is twice the average speed. This is expected because we start from zero speed and accelerate at a constant rate, so the average speed is attained at half the acceleration period, at which point the object in question is rotating at half the angular speed.

Section 10.2 Torque

18. **INTERPRET** In this problem, we are asked to find the torque produced by the frictional force about a wheel's axis. We will apply the concept of torque, which is the force applied perpendicular to the radial direction from an axis of rotation.

DEVELOP The torque produced by a force is given by Equation 10.10:

$$\tau = rF\sin\theta$$

where $r_\perp = r\sin\theta$ is the perpendicular distance between the rotation axis and the line of action of the force F. The frictional force acts tangentially to the circumference of the wheel and is, thus, perpendicular to the radius at the point of contact. Thus, $\sin\theta = \sin(90^\circ) = 1$.

EVALUATE Using $r = d/2 = (1.0\ \text{m})/2 = 0.50\ \text{m}$, Equation 10.10 gives the torque as

$$\tau = rf = (0.50\ \text{m})(540\ \text{N}) = 270\ \text{N}\cdot\text{m}$$

opposite to the direction of rotation.

ASSESS Since frictional force opposes motion, the torque it produces tends to slow down the rotation.

19. **INTERPRET** In this problem we are asked to estimate and compare the torques produced by the two different types of braking systems. We will apply the concept of torque, which is the force applied perpendicular to the radial direction from an axis of rotation.

DEVELOP The torque produced by a force is given by Equation 10.10:

$$\tau = rF\sin\theta$$

where $r_\perp = r\sin\theta$ is the perpendicular distance between the rotation axis and the line of action of the force F. The braking force acts tangent to the circumference of the wheel and thus is perpendicular to the radius at the point of contact. Thus $\sin\theta = \sin(90°) = 1$.

EVALUATE Evaluating Equation 10.10 for each braking system gives

$$\tau_{\text{rim}} = r_r F_r = \left(0.35\,\text{m}\right)\left(1000\,\text{N}\right) \cong 300\,\text{N}\cdot\text{m}$$

$$\tau_{\text{disk}} = r_d F_d = \left(0.1\,\text{m}\right)\left(4000\,\text{N}\right) \cong 400\,\text{N}\cdot\text{m}$$

Where we have divided the given diameters to obtain the perpendicular distance $r_\perp$.

ASSESS From these we see that the disc brake torque is about 30% greater.

20. **INTERPRET** This problem involves a force that is applied at different angles to the lever arm. For each angle, we are asked to find the force required to produce the desired torque.

DEVELOP The torque produced by a force is given by Equation 10.10:

$$\tau = rF\sin\theta = r_\perp F = rF_\perp$$

where $r_\perp = r\sin\theta$ is the perpendicular distance (lever arm) between the rotation axis and the line of action of the force *F*. Alternatively, one can think of $F_\perp = F\sin\theta$ as the effective force.

EVALUATE Using Equation 10.10, the magnitude of the applied force may be obtained. The forces are **(a)**

$$F = \frac{\tau}{r\sin\theta} = \frac{32.0\ \text{N}\cdot\text{m}}{(0.235\ \text{m})\sin(90°)} = 136.17\ \text{N}$$

and **(b)**

$$F = \frac{\tau}{r\sin\theta} = \frac{32.0\ \text{N}\cdot\text{m}}{(0.235\ \text{m})\sin(104°)} = 140.34\ \text{N}$$

ASSESS As expected, we have to push harder if we do not push perpendicular to the lever arm. To produce a specific torque most effectively, the applied force should be at a right angle to $\vec{r}$, the position vector from the axis of rotation to the point where the force is applied. This would yield the maximum effective force, $F_\perp = F\sin(90°) = F$.

21. **INTERPRET** In this problem, we are asked to calculate the torque, given a force (gravity on the mouse), the radial distance at which it is applied, and the angle at which it is applied (from the geometry of a clock).

DEVELOP Draw a diagram of the situation (see figure below). From the geometry of a clock, we know that the angle the minute hand makes with the vertical is $\phi = 180^\circ/3 = 60^\circ$. The angle between the force and the radial position vector from the axis of rotation to the point where the force is applied is, therefore, $\theta = 180^\circ - 60^\circ = 120^\circ$. The force applied by the mouse is simply its weight, so $F = mg$, and the lever arm is $r = 22$ cm.

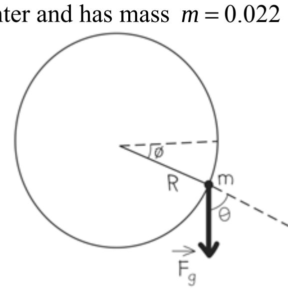
EVALUATE Inserting the given quantities into Equation 10.10 gives

$$\tau = rF\sin\theta = (0.22\ \text{m})(0.030\ \text{kg})(9.8\ \text{m/s}^2)\sin(120^\circ) = 5.6\times10^{-2}\ \text{N}\cdot\text{m}$$

ASSESS In 5 minutes, the torque applied by the mouse (assuming it doesn't move) will be $\tau = rF = (0.22\ \text{m})(0.030\ \text{kg})(9.8\ \text{m/s}^2) = 6.5\times10^{-2}\ \text{N}\cdot\text{m}$, or 16% more torque.

22. **INTERPRET** In this problem, we are asked to calculate the torque, given a force (gravity on the valve stem), the radial distance at which it is applied, and the angle at which it is applied (from the given geometry).

DEVELOP Apply Equation 10.10, $\tau = rF\sin\theta$. Make a sketch of the system, as shown in the figure below. From this, we see that the angle between the force and the position vector $\vec{r}$ is $\theta = 90° - 23° = 67°$. The valve stem is located a distance $r = 0.32$ m from the center and has mass $m = 0.022$ kg. The force applied is thus $F = mg$.



EVALUATE Inserting the given quantities into Equation 10.10 gives the torque as

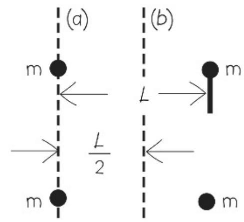
$$\tau = rF\sin\theta = rmg\sin\theta = (0.32\ \text{m})(0.022\ \text{kg})(9.8\ \text{m/s}^2)\sin(67°) = 0.064\ \text{N}\cdot\text{m}$$

ASSESS We see that the torque would be zero if the angle ϕ were 90° because $\sin(90° - 90°) = 0$. This makes sense because the valve stem would be directly below the axis of rotation, so its weight would apply no torque.

Section 10.3 Rotational Inertia and the Analog of Newton's Law

23. **INTERPRET** This problem involves rotational inertia, which includes both mass and the spatial distribution of that mass. Rotation inertia is the rotational analog of mass in linear motion. Because the spatial mass distribution enters into rotational inertia, the position of the axis of rotation is important. We are asked to find the rotational inertia of an arrangement of four masses about two different axes of rotation.

DEVELOP Draw a diagram of the situation (see figure below), and apply Equation 10.12.



EVALUATE (a) For the axis labeled (a), two masses have $r = 0$, and the other two masses have $r = L$. Inserting these quantities into Equation 10.12 gives

$$I = \sum_i m_i r_i^2 = m(0)^2 + m(0)^2 + mL^2 + mL^2 = 2mL^2$$

(b) For the axis labeled (b), each mass has $r = L/2$, so Equation 10.12 gives

$$I = \sum_i m_i r_i^2 = m\left(\frac{L}{2}\right)^2 + m\left(\frac{L}{2}\right)^2 + m\left(\frac{L}{2}\right)^2 + m\left(\frac{L}{2}\right)^2 = mL^2$$

ASSESS Thus, there is more rotational inertia when the axis of rotation is at the edge of the object than when it is at the center of the object, as expected.

24. **INTERPRET** We want to find the moment of inertia of a shaft that has the shape of a solid cylinder.

DEVELOP The rotational inertia of a solid cylinder or disk about its axis is $I = \frac{1}{2}MR^2$ (see Table 10.2). The radius is half the diameter and the mass, in more familiar units, is 6.4×10^3 kg.

EVALUATE The rotational inertia of the shaft is

$$I = \frac{1}{2}MR^2 = \frac{1}{2}\left(6.4\times10^3\ \text{kg}\right)\left(\frac{1}{2}\times0.91\ \text{m}\right)^2 = 662\ \text{kg}\cdot\text{m}^2$$

ASSESS The numerical value is reasonable, given its mass and radius, and the units ($\text{kg}\cdot\text{m}^2$) are correct.

25. **INTERPRET** This problem involves combining the rotational inertia of several objects to find the overall rotational inertia of the combined object. In addition, we are asked to find the torque needed to give the object the given angular acceleration.

DEVELOP Because both the cylinder and the end caps rotate about the same axis, we can sum the rotational inertia of each object to find the total rotational inertia: $I_{\text{tot}} = I_{\text{cyl}} + 2I_{\text{cap}}$. The rotational inertia of the individual components are given in Table 10.2, and are $I_{\text{cyl}} = M_{\text{cyl}}R^2$ and $I_{\text{cap}} = M_{\text{cap}}R^2/2$. To find the torque, apply the rotational analog of Newton's second law (for constant mass), Equation 10.11: $\tau = I\alpha$.

EVALUATE (a) The total rotational inertia of the capped cylinder is

$$I_{\text{tot}} = I_{\text{cyl}} + 2I_{\text{cap}} = M_{\text{cyl}}R^2 + 2\left(\frac{1}{2}M_{\text{cap}}R^2\right) = R^2(M_{\text{cyl}} + M_{\text{cap}})$$
$$= (0.073\ \text{m})^2(0.100\ \text{kg} + 0.024\ \text{kg}) = 6.6\times10^{-4}\ \text{kg}\cdot\text{m}^2$$

(b) The torque needed to accelerate the capped cylinder is

$$\tau = I_{\text{tot}}\alpha = (6.6\times10^{-4}\ \text{kg}\cdot\text{m}^2)(3.2\ \text{rad/s}^2) = 2.1\times10^{-3}\ \text{N}\cdot\text{m}$$

ASSESS Notice that we used more significant figures for the total rotational inertia in part (b) because it was an intermediate result in this case.

26. **INTERPRET** In this problem, we are asked to find the minimum total mass of a wheel, given its diameter and rotational inertia.

DEVELOP Every part of the wheel has a distance from the center that is less than or equal to the maximum radius. Therefore, using Equation 10.12, we obtain the following inequality:

$$I = \sum m_i r_i^2 \le (m_i) r_{\text{max}}^2$$

EVALUATE **(a)** The above equation implies that

$$M = \sum m_i \ge \frac{I}{r_{\text{max}}^2}$$

Therefore, the minimum total mass is

$$M_{\text{min}} = \frac{I}{r_{\text{max}}^2} = \frac{7.1\ \text{kg}\cdot\text{m}^2}{(0.87\ \text{m}/2)^2} = 37.52\ \text{kg}$$

(b) If not all the mass of the wheel is concentrated at the rim, the total mass is greater than this minimum.

ASSESS To have the same rotational inertia I, we can have some of the mass of the wheel concentrated near the axis of rotation. Its contribution to I would be small because $I = \sum m_i r_i^2$.

27. **INTERPRET** By assuming Earth to be a solid sphere with uniform mass distribution, we want to estimate its rotational inertia, and the torque needed to change the length of the day by 1 second every century.

DEVELOP From Table 10.2, the rotational inertia of a solid sphere of radius R and mass M is

$$I = \frac{2}{5}MR^2$$

Once I is known, the torque needed to slow down the rotation can be found by using Equation 10.11: $\tau = I\alpha$.

EVALUATE **(a)** For a uniform solid sphere with an axis through the center,

$$I_E = \frac{2}{5}M_E R_E^2 = \frac{2}{5}\left(5.97\times10^{24}\ \text{kg}\right)\left(6.37\times10^{6}\ \text{m}\right)^2 = 9.69\times10^{37}\ \text{kg}\cdot\text{m}^2$$

(b) The angular speed of rotation of Earth is $\omega = 2\pi/T$, where the period is T = 1 d = 86,400 s. If the period were to change by 1s per century,

$$\frac{dT}{dt} = \frac{1\ \text{s}}{(100\ \text{y})\left(3.16\times10^{7}\ \text{s/y}\right)} = 3.16\times10^{-10}$$

This would correspond to an angular acceleration of

$$\alpha = \frac{d\omega(T)}{dt} = \frac{d}{dt}\left(\frac{2\pi}{T}\right) = -\frac{2\pi}{T^2}\frac{dT}{dt}$$

Therefore, to change the length of a day by ±1 s would require a torque of magnitude

$$\tau = I|\alpha| = \frac{2\pi I}{T^2}\frac{dT}{dt} = \frac{2\pi\left(9.69\times10^{37}\ \text{kg}\cdot\text{m}^2\right)}{(86{,}400\ \text{s})^2}\left(3.16\times10^{-10}\right) = 2.58\times10^{19}\ \text{N}\cdot\text{m}$$

ASSESS The torque in (b) is actually generated by tidal friction between the Moon and the Earth. Note that the Earth has a core of denser material, so its actual rotational inertia is less than that obtained in (a).

28. **INTERPRET** We are asked about the rotational inertia of a Frisbee, given its mass distribution, and the torque required to generate the rotation.

DEVELOP The Frisbee rotates around an axis through its center and perpendicular to its flat surface. Its rotational inertia is the sum from a disk $\left(I_{\text{d}} = \frac{1}{2}M_{\text{d}}R^2\right)$ and a ring $\left(I_{\text{r}} = M_{\text{r}}R^2\right)$, each accounting for half the mass of the Frisbee $\left(M_{\text{d}} = M_{\text{r}} = \frac{1}{2}M_{\text{f}}\right)$. In part (b), the torque exerted by the student can be found from the rotational analog of Newton's second law: $\tau = I_{\text{f}}\alpha$ (Equation 10.11). We don't have the angular acceleration, but it can be determined from Equation 10.9, $\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$, given that the Frisbee goes from rest $(\omega_0 = 0)$ to 450 rpm after a quarter-turn given by the student $\left(\theta - \theta_0 = \frac{1}{4}\text{rev}\right)$.

EVALUATE **(a)** The Frisbee's rotational inertia is the sum of the inertias from the disk and the ring:

$$I_\text{f} = I_\text{d} + I_\text{r} = \frac{3}{4}M_\text{f}R^2 = \frac{3}{4}(0.102\text{ kg})\left(\frac{1}{2}\times 0.23\text{ m}\right)^2 = 1.01\times 10^{-3}\text{ kg}\cdot\text{m}^2$$

(b) To find the torque, we first calculate the angular acceleration:

$$\alpha = \frac{\omega^2 - \omega_0^2}{2(\theta - \theta_0)} = \frac{(450\text{ rpm})^2}{2\left(\frac{1}{4}\text{rev}\right)}\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)^2 = 706.9\text{ rad/s}^2$$

The torque given by the student is then

$$\tau = I_\text{f}\alpha = \left(1.01\times 10^{-3}\text{ kg}\cdot\text{m}^2\right)\left(707\text{ rad/s}^2\right) = 0.714\text{ N}\cdot\text{m}$$

ASSESS The units are all correct and the numerical values seem reasonable. Using Equation 10.10, we can estimate that the force exerted by the student when flicking the Frisbee is roughly: $F \sim \tau / r = 6.2\text{ N}$, which is well within the strength limits of a human wrist.

29. **INTERPRET** This problem involves calculating the torque that results from a frictional force applied about a 41-cm shaft, and the angular acceleration this engenders. We are then asked to find the time it takes the shaft (and the accompanying flywheel) to stop, given their initial rotational speed.

DEVELOP From Equation 10.10, the torque applied to the flywheel is

$$\tau = rF\sin\theta = R_\text{shaft} f_\text{k}$$

where $\theta = 90°$, $f_\text{k} = 34$ kN, and $R_\text{shaft} = (41\text{ cm})/2 = 0.205$ m. Inserting this torque into the rotational analog of Newton's second law (for constant mass), we can find the angular acceleration. We find $\alpha = -\tau/I_\text{fw}$, where the negative sign indicates that the acceleration is directed opposite to the motion. Use Table 10.2 to find the formulas for the rotational inertia of the flywheel (which we take to be a solid disk). This is

$$I_\text{fw} = \frac{1}{2}M_\text{fw}R_\text{fw}^2$$

where $M_\text{fw} = 7.7 \times 10^4$ kg and $R_\text{fw} = 2.4$ m. The time it will take the flywheel to stop is, from Equation 10.7 with $\omega = 0$,

$$0 = \omega_0 + \alpha t$$

$$t = -\frac{\omega_0}{\alpha} = \frac{\omega_0 I_\text{fw}}{\tau} = \frac{\omega_0 M_\text{fw} R_\text{fw}^2}{2R_\text{shaft} f_\text{k}}$$

EVALUATE Inserting the given quantities into the expression for the time gives

$$t = \frac{\omega_0 M_\text{fw} R_\text{fw}^2}{2 f_\text{k} R_\text{shaft}} = \frac{(360\text{ rpm})\left(7.7\times 10^4\text{ kg}\right)(2.4\text{ m})^2}{2\left(34\times 10^3\text{ N}\right)(0.205\text{ m})}\left(\frac{2\pi\text{ rad}}{\text{rev}}\right)\left(\frac{\text{min}}{60\text{ s}}\right) = 1200\text{ s} = 20\text{ min}$$

ASSESS The exact rotational inertia for the flywheel is $M\left(R_\text{shaft}^2 + R_\text{fw}^2\right)/2$, which is just 0.7% different from $MR_\text{fw}^2/2$ for the given radii.

Section 10.4 Rotational Energy

30. **INTERPRET** The problem asks about the rotational kinetic energy of the blade of a circular saw. In addition, we also want to find the power required for the saw to start from rest and reach a given angular speed, which involves the work-energy theorem (see Equation 6.14).

DEVELOP The rotational kinetic energy of the saw can be found by using Equation 10.18:

$$K = \frac{1}{2}I\omega^2$$

where $I = MR^2/2$ for a disk (see Table 10.2). The average power required is the work done on the saw divided by the time over which the work is done $(\bar{P} = W/\Delta t)$. The work done on the saw may be found by using the work-energy theorem (Equation 6.14): $W_{\text{net}} = \Delta K$.

EVALUATE **(a)** With $\omega = 2500 \text{ rpm} = 2\pi(2500 \text{ rpm})/(60 \text{ s}) = 261.8 \text{ rad/s}$, the final rotational kinetic energy is

$$K_{\text{f}} = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{1}{2}MR^2\right)\omega^2 = \frac{1}{4}(0.85 \text{ kg})(0.165 \text{ m})^2(261.8 \text{ rad/s})^2 = 396.5 \text{ J}$$

(b) The average power required is

$$\bar{P} = \frac{W_{\text{net}}}{\Delta t} = \frac{\Delta K}{\Delta t} = \frac{K_{\text{f}} - \overbrace{K_{\text{i}}}^{=0}}{\Delta t} = \frac{396.2 \text{ J}}{3.5 \text{ s}} = 113.2 \text{ W}$$

where we have used the result from part (a) to three significant figures because it is an intermediate result.

ASSESS To check the answer, we present an alternative approach to computing K. In this problem, the angular acceleration is

$$\alpha = \frac{\omega - \omega_0}{t} = \frac{261.7 \text{ rad/s}}{3.5} = 74.49 \text{ rad/s}^2$$

and the angular displacement is (using Equation 10.9)

$$\theta = \theta_0 + \frac{\omega^2 - \omega_0^2}{2\alpha} = \frac{(261.7 \text{ rad/s})^2}{2(74.49 \text{ rad/s}^2)} = 460 \text{ rad}$$

With these quantities, the rotational energy may be calculated as

$$K = W = \tau\theta = I\alpha\theta = \frac{1}{2}(0.85 \text{ kg})(0.165 \text{ m})^2(74.49 \text{ rad/s}^2)(460 \text{ rad}) = 396.5 \text{ J}$$

which is the same as before.

31. **INTERPRET** We're asked to imagine extracting energy from the Earth's rotational kinetic energy. We want to estimate how long it would take to slow the rotation rate enough for the day to increase by 1 second.

DEVELOP We imagine that rotational kinetic energy is extracted from the Earth at a rate of $P = 1.8\times10^{13}\,W$. The rotational kinetic energy will correspondingly decrease, manifesting itself as a slowdown in the rotational speed. Over sufficient time, t, the rotational speed will decrease from its current value of $\omega_0 = 2\pi/1\text{d}$ to a value for which the day is 1 second longer: $\omega_f = 2\pi/(1\text{d} + 1\text{s})$. Equating the change in rotational kinetic energy to the energy extracted gives:

$$\frac{1}{2}I_E\left(\omega_0{}^2 - \omega_f{}^2\right) = Pt$$

EVALUATE From Problem 27, the rotational inertia of the Earth can be estimated as $I_E = 9.69\times10^{37} \text{ kg}\cdot\text{m}^2$. Because $1s \ll 1d$, we can approximate the change in the rotational velocity squared as:

$$\left(\omega_0{}^2 - \omega_f{}^2\right) = \left(\frac{2\pi}{1 \text{ d}}\right)^2\left[1 - \left(1 + \frac{1 \text{ s}}{1 \text{ d}}\right)^{-2}\right] \cong \left(\frac{2\pi}{1 \text{ d}}\right)^2\left[2\left(\frac{1 \text{ s}}{1 \text{ d}}\right)\right]$$

Plugging this into the above energy equation and solving for the time gives:

$$t = \frac{\frac{1}{2}I_E\left(\omega_0^2 - \omega_f^2\right)}{P} = \frac{\left(9.69\times10^{37} \text{ kg}\cdot\text{m}^2\right)(2\pi)^2(1 \text{ s})}{\left(1.8\times10^{12} \text{ W}\right)(24\cdot60\cdot60 \text{ s})^3} = 3.30\times10^{11} \text{ s} \cong 10^4 \text{ y}$$

ASSESS This is around 10,000 years, which simply shows how much kinetic energy there is in the Earth's rotation. Of course, the computed time may be an underestimate, since humankind is continuously increasing its power consumption.

32. **INTERPRET** The kinetic energy of the baseball consists of two parts: the kinetic energy of the center of mass, K_{cm}, and the rotational kinetic energy, K_{rot}. We want to find the fraction of the total kinetic energy that is due to K_{rot}.

DEVELOP The total kinetic energy has center-of-mass energy and internal rotational energy associated with spin about the center of mass (see Equation 10.20):

$$K_{\text{tot}} = K_{\text{cm}} + K_{\text{rot}} = \frac{1}{2}Mv^2 + \frac{1}{2}I_{\text{cm}}\omega^2$$

EVALUATE For a solid sphere, $I_{\text{cm}} = 2\ \text{MR}^2/5$ (see Table 10.2). Therefore, the rotational fraction of the total kinetic energy is

$$\frac{K_{\text{rot}}}{K_{\text{tot}}} = \frac{I_{\text{cm}}\omega^2}{Mv^2 + I_{\text{cm}}\omega^2} = \frac{(2MR^2/5)\omega^2}{Mv^2 + (2MR^2/5)\omega^2} = \frac{2R^2\omega^2}{5v^2 + 2R^2\omega^2}$$

$$= \frac{2(0.03\ \text{m/s})^2(45\ \text{rad/s})^2}{5(26\ \text{m/s})^2 + 2(0.03\ \text{m/s})^2(45\,\text{rad/s})^2}$$

$$= 1.08\times10^{-3} = 0.11\%$$

to two significant figures.

ASSESS Rotational kinetic energy constitutes a very small fraction of the total kinetic energy, which is reasonable because the linear speed at a point on the surface of the baseball due to rotation is only $\omega R = (45\ \text{rad/s})(0.30\ \text{m}) = 1.35\ \text{m/s},$ which is much less than the linear velocity of $26\ \text{m/s}$.

33. **INTERPRET** We are asked to find the energy stored in the flywheel of Problem 10.33, so we will use the concepts of rotational inertia and kinetic energy of rotation. We also need to find the power output of a generator if the speed of the flywheel changes a given amount in a given time.

DEVELOP Apply Equation 10.18, $K = I\omega^2/2$, to calculate the kinetic energy stored in the flywheel. We will need to convert the angular speed in rpm to rad/s, and calculate the rotational inertia of the flywheel using $I = mR^2/2$ (from Table 10.2). From the work-energy theorem (see Equation 10.19) and using $\bar{P} = W/\Delta t$, we have

$$\bar{P} = \frac{W}{\Delta t} = \frac{\Delta K}{\Delta t}$$

where ω_i = 360 rpm, ω_f = 300 rpm, and Δt = 3s.

The mass of the flywheel is m = 7.7×10^4 kg, the radius is R = 2.4 m, and the initial rotation rate is 360 rpm.

EVALUATE

(a) The energy stored in the flywheel is

$$K = \frac{1}{2}I\omega^2 = \frac{1}{4}mR^2\omega^2$$

$$= \frac{1}{4}\left(7.7\times10^4\ \text{kg}\right)(2.4\ \text{m})^2\left(360\ \frac{\text{rev}}{\text{min}}\right)^2\left(\frac{2\pi\ \text{rad}}{\text{rev}}\right)^2\left(\frac{1\ \text{min}}{60\ \text{s}}\right)^2 = 1.6\times10^8\ \text{J}$$

(b) The average power output during the deceleration of the flywheel is

$$\bar{P} = \frac{\Delta K}{\Delta t} = \frac{mR^2}{4\Delta t}\left(\omega_f^2 - \omega_i^2\right)$$

$$= \frac{\left(7.7\times10^4\ \text{kg}\right)(2.4\ \text{m})^2}{4(3\ \text{s})}\left[\left(300\ \frac{\text{rev}}{\text{min}}\right)^2 - \left(360\ \frac{\text{rev}}{\text{min}}\right)^2\right]\left(\frac{2\pi\ \text{rad}}{\text{rev}}\right)^2\left(\frac{1\ \text{min}}{60\ \text{s}}\right)^2 = 16\ \text{MW}$$

ASSESS This is a good way of generating enormous power pulses.

Section 10.5 Rolling Motion

34. **INTERPRET** We are asked to find the translational and rotational kinetic energy of a rolling sphere, given its translational speed and the mass of the sphere.

DEVELOP Translational kinetic energy is $K_{\text{t}} = mv^2/2,$ and rotational kinetic energy is $K_{\text{r}} = I\omega^2/2.$ From Table 10.2, the rotational inertia of a sphere is $I = 2mR^2/5.$ The velocity is $v = r\ \omega = 7.1\ \text{m/s}.$ The mass of the sphere is $m = 2.6$ kg.

EVALUATE

(a) $K_t = \frac{1}{2}mv^2 = \frac{1}{2}(2.6\text{ kg})(7.1\text{ m/s})^2 = 65.5\text{ J}$

(b) $K_r = \frac{1}{2}I\omega^2 = \frac{1}{\cancel{2}}\left(\frac{\cancel{2}}{5}mR^{\cancel{2}}\right)\left(\frac{v}{\cancel{R}}\right)^2 = \frac{mv^2}{5} = \frac{2}{5}K_t = \frac{2}{5}(65.5\text{ J}) = 26.2\text{ J}.$

ASSESS Note that the rotational kinetic energy is $2/5$ the translational kinetic energy in this case. Does the $2/5$ look familiar? How would the two answers be related if, instead of a solid sphere, it was a hollow sphere?

35. **INTERPRET** This problem involves comparing the rotational and translational kinetic energy, so we will use the relationship between ω and v ($v = r\omega$).

DEVELOP The total kinetic energy is

$$K = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$$

where the first term is the translational kinetic energy and the second term is the rotational kinetic energy. From Table 10.2, we find that the rotational inertia of a solid disk is $I = mr^2/2$. Recalling that $v = r\omega$, we can calculate the ratio $f = K_{rot}/K$.

EVALUATE The ratio of rotational kinetic energy to total kinetic energy is

$$f = \frac{\cancel{1/2}I\omega^2}{\cancel{1/2}Mv^2 + \cancel{1/2}I\omega^2} = \frac{\left(\frac{1}{2}MR^2\right)\omega^2}{M(\omega R)^2 + \left(\frac{1}{2}MR^2\right)\omega^2}$$
$$= \frac{\frac{1}{2}}{1+\frac{1}{2}} = \frac{1}{3}$$

ASSESS This is consistent with what we noted in the previous problem: the rotational inertia is $mR^2/2$ so the rotational kinetic energy is ½ the translational kinetic energy when it rolls without slipping.

36. **INTERPRET** This problem involves rotational kinetic energy and rotational inertia. Knowing the fraction of kinetic energy due to rotation, we are to determine whether the ball is solid or hollow.

DEVELOP The given fraction of kinetic energy due to rotation is

$$f = \frac{K_{rot}}{K_{total}} = \frac{40}{100} = \frac{2}{5}$$

We know that $K_{tot} = mv^2/2 + K_{rot}$, and $K_{rot} = I\omega^2/2$. From Table 10.2, we find that the rotational inertia for a hollow sphere is $I = 2MR^2/3$, whereas for a solid sphere it is $I = 2MR^2/5$. Use these formulas to calculate the ratio of rotational kinetic energy to total kinetic energy to see which one corresponds to the ratio give ($f = 2/5$).

EVALUATE For the solid sphere,

$$f = \frac{\cancel{1/2}I\omega^2}{\cancel{1/2}Mv^2 + \cancel{1/2}I\omega^2} = \frac{\left(\frac{2}{5}MR^2\right)\omega^2}{M(\omega R)^2 + \left(\frac{2}{5}MR^2\right)\omega^2} = \frac{\frac{2}{5}}{1+\frac{2}{5}} = \frac{2}{7}$$

For the hollow sphere,

$$f = \frac{\cancel{1/2}I\omega^2}{\cancel{1/2}Mv^2 + \cancel{1/2}I\omega^2} = \frac{\left(\frac{2}{3}MR^2\right)\omega^2}{M(\omega R)^2 + \left(\frac{2}{3}MR^2\right)\omega^2} = \frac{\frac{2}{3}}{1+\frac{2}{3}} = \frac{2}{5}$$

Therefore, the sphere must be hollow.

ASSESS Notice that the rotation kinetic energy comprises a larger fraction of the total kinetic energy for a hollow sphere because more of its mass is concentrated away from the axis of rotation, so the rotational inertia is greater.

EXAMPLE VARIATIONS

37. **INTERPRET** In this problem we want to see how the rotational inertia of a meter stick varies if we model it as a thin rod or as a flat plate.

DEVELOP The meter stick rotates about a perpendicular axis through its center, so treating it as rod and a flat plate results in rotational inertias $\frac{1}{12}ML^2$ and $\frac{1}{12}M\left(a^2+b^2\right)$, respectively. Here $M=0.172\,\text{kg}$, $L=b=1\,\text{m}$, and $a=0.0254\,\text{m}$. We can calculate these and then determine by what percentage the simpler model (rod) is in error.

EVALUATE Evaluating the rotational inertias gives

$$I_r=\frac{1}{12}ML^2=\frac{1}{12}(0.172\,\text{kg})(1\,\text{m})^2=14.3\,\text{g}\cdot\text{m}^2$$

$$I_{fp}=\frac{1}{12}M\left(a^2+b^2\right)=\frac{1}{12}(0.172\,\text{kg})\left[(0.0254\,\text{m})^2+(1\,\text{m})^2\right]=(1.00064)\times 14.3\,\text{g}\cdot\text{m}^2$$

Thus, modeling the meter stick as a rod results in a lower value; low by about 0.064 %

ASSESS Rectangular objects like this meter stick are better represented by rotational inertias which better match their geometry.

38. **INTERPRET** In this problem we want to determine the location of the rotational axis for a thin rod knowing its rotational inertia. We will apply the parallel axis theorem.

DEVELOP When the rod rotates about an axis which is parallel to a perpendicular axis through its center, its rotational inertia is given by the parallel axis theorem as $I=\frac{1}{12}ML^2+Md^2$, where d is the distance from the center-of-mass axis to the parallel axis. We can set this equal to the given rotational axis and solve for d.

EVALUATE Equating the rotational inertias and solving for d gives

$$I=\frac{1}{12}ML^2+Md^2=\frac{1}{9}ML^2$$

$$d=\sqrt{\left(\frac{1}{9}-\frac{1}{12}\right)L^2}=\frac{L}{6}$$

Meaning the axis of rotation is located a distance $L/6$ from the center, or $L/3$ from one end of the rod.

ASSESS The rotational inertia of an object through a perpendicular axis can always be related to the rotational inertia about an axis through the center of mass using the parallel axis theorem.

39. **INTERPRET** In this problem we want to model the rotational inertia of a large centrifuge. We will use the equations from Table 10.2 to treat the tube as a thin rod and astronauts and seats as point masses.

DEVELOP From Table 10.2 we determine that the rotational inertia of the tube is given by $I_t=\frac{1}{12}ML^2$, and the rotational inertia of both occupied seats are given by $I_{s/a}=mx^2$.

EVALUATE Evaluating the sum of the three rotational inertias for the given mass and length values gives

$$I_{\text{cent}}=I_t+2I_{s/a}=\frac{1}{12}ML^2+2mx^2$$

$$I_{\text{cent}}=\frac{1}{12}(3880\,\text{kg})(18.0\,\text{m})^2+2(105\,\text{kg}+72.6\,\text{kg})(7.92\,\text{m})^2=127\,\text{Mg}\cdot\text{m}^2$$

ASSESS The rotational inertia of a composite object can be calculated by separately determining the rotational inertia of each component.

40. **INTERPRET** In this problem we want to remodel the rotational inertia of a large centrifuge calculated in the previous problem. We will use the equations from Table 10.2 to treat the tube as having a square cross section of length B and the astronauts/chair combination as cubes with side length c.

DEVELOP From Table 10.2 we determine that the remodeled rotational inertia of the tube will now be given by $I_t=\frac{1}{12}M(L^2+B^2)$. The remodeled rotational inertia of both occupied seats is now found by applying the parallel axis theorem to the rotational inertia of a flat plate, and is given by $I_{s/a}=\frac{1}{12}m\left(2c^2\right)+mx^2$.

EVALUATE Evaluating the sum of the three rotational inertias using $B=2.10\,\text{m}$, $c=1.85\,\text{m}$, and the given mass and length values from the previous problem gives

$$I_{\text{cent}} = I_t + 2I_{s/a} = \frac{1}{12}M(L^2 + B^2) + 2\left(\frac{1}{12}m\left(2c^2\right) + mx^2\right) = 129\,\text{Mg}\cdot\text{m}^2$$

ASSESS The rotational inertia of the more accurately represented system is approximately 2% larger than the one found in the previous problem.

41. **INTERPRET** In this problem we want to apply the conservation of mechanical energy to determine the speed of a marble after it has rolled down an incline.

DEVELOP The gravitational potential energy at the top of the incline turns into translational and rotational kinetic energy. At the bottom, all the potential energy has been converted into kinetic energy, and with no energy dissipation we can calculate the final speed of the marble.

EVALUATE Equating the initial and final mechanical energies and solving for the final velocity gives

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$2gh = v^2 + \frac{2}{5}R^2\left(\frac{v}{R}\right)^2$$

$$v = \sqrt{\frac{10gh}{7}} = \sqrt{\frac{10\left(9.8\,\text{m/s}^2\right)(0.242\,\text{m})}{7}} = 1.84\,\text{m/s}$$

ASSESS The object is slower than if it were to have slipped since some of the potential energy went into making the object rotate.

42. **INTERPRET** In this problem we want to apply the conservation of mechanical energy to determine the speed of a marble after it has rolled down an incline.

DEVELOP The gravitational potential energy at the top of the incline turns into translational and rotational kinetic energy. We know its velocity halfway down, so we can equate that to the potential energy that remains at that height. Once we have the starting height we can find the speed at the bottom like we did in the previous problem.

EVALUATE Expressing the mechanical energy of the marble at halfway down the incline, and solving for the height gives

$$mg\left(\frac{h}{2}\right) = \frac{1}{2}mv_{h/2}{}^2 + \frac{1}{2}I\omega_{h/2}{}^2$$

$$gh = v_{h/2}{}^2 + \frac{2}{5}R^2\left(\frac{v_{h/2}}{R}\right)^2$$

$$h = \frac{7v_{h/2}{}^2}{5g} = \frac{7(1.12\,\text{m/s})^2}{5\left(9.8\,\text{m/s}^2\right)} = 17.9\,\text{cm}$$

Using the expression found in the previous problem by equating the initial and final mechanical energies we find the speed of the marble at the bottom is equal to

$$v = \sqrt{\frac{10gh}{7}} = \sqrt{\frac{10\left(9.8\,\text{m/s}^2\right)(0.179\,\text{m})}{7}} = 1.58\,\text{m/s}$$

ASSESS If there are no dissipative forces acting on the system, the mechanical energy of the system will be conserved, even while there is conversion between potential and kinetic energy.

43. **INTERPRET** In this problem we want to apply the conservation of mechanical energy to determine the speed of a wheel at the bottom of an incline, and to determine the characteristics of its mass distribution.

DEVELOP The gravitational potential energy at the top of the incline turns into translational and rotational kinetic energy. At the bottom, all the potential energy has been converted into kinetic energy, and with no energy dissipation we can calculate the final speed of the wheel.

EVALUATE Equating the initial and final mechanical energies and solving for the final velocity gives

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$2gh = v^2 + \frac{I}{m}\left(\frac{v}{R}\right)^2$$

$$v = \sqrt{\frac{2gh}{1+\frac{I}{mR^2}}} = \sqrt{\frac{2\left(9.8\,\mathrm{m/s^2}\right)(12.6\,\mathrm{m})}{1+\frac{\left(3.58\,\mathrm{kg\cdot m^2}\right)}{(29.5\,\mathrm{kg})(0.406\,\mathrm{m})^2}}} = 11.9\,\mathrm{m/s}$$

To determine whether the wheel is uniformly solid, or if its mass is concentrated at the rim, we look at the term $\frac{I}{mR^2} = 0.736$. This is the coefficient that multiplied by mR^2 gives the rotational inertia of the wheel. From Table 10.2 we see that a uniformly solid disk has a coefficient of $\frac{1}{2}$, while a ring has a coefficient of 1, meaning that wheel is structured somewhere in between.

ASSESS The simple shape of this object allows us to easily compare its rotational inertia to those introduced in the text and to determine roughly what its mass distribution resembles.

44. INTERPRET In this problem we want to apply the conservation of mechanical energy to determine the masses of two objects making up the wheel which rolls down an incline.

DEVELOP The gravitational potential energy at the top of the incline turns into translational and rotational kinetic energy. Since we are considering the wheel to be a composite object consisting of a disk and a ring, we will express separately the two rotational inertias. Then we will solve for one of the masses considering that the total mass $M = m_{\text{disk}} + m_{\text{rim}}$.

EVALUATE Equating the initial and final mechanical energies we get

$$Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}\left(I_{\text{disk}} + I_{\text{rim}}\right)\omega^2$$

$$2Mgh = Mv^2 + \left(\frac{1}{2}m_{\text{disk}} + m_{\text{rim}}\right)R^2\left(\frac{v}{R}\right)^2$$

$$\left(2gh - v^2\right)M = \left(\frac{1}{2}m_{\text{disk}} + M - m_{\text{disk}}\right)v^2$$

$$m_{\text{disk}} = 4M\left(1 - \frac{gh}{v^2}\right) = 8.31\,\mathrm{kg}$$

$$m_{\text{rim}} = M - m_{\text{disk}} = 6.39\,\mathrm{kg}$$

ASSESS We could've solved for the mass of the rim first and obtained a similar expression by instead replacing m_{disk} with $M - m_{\text{rim}}$.

PROBLEMS

45. INTERPRET The problem is related to the rotational motion of the wheel. By identifying the analogous situation for linear motion (see Table 10.1), we can apply the correct formula.

DEVELOP We are given the angular displacement, the angular acceleration, and the initial angular speed $(=0)$. To find the final angular speed, we can apply Equation 10.9, which relates all these quantities:

$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$

To find the time it takes for the wheel to make 3 turns, apply Equation 10.8:

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}at^2$$

For the calculation, we will convert the angular acceleration to s^{-2}:

$$\alpha = 17\left(\frac{\text{rev}}{\text{min}\cdot\text{s}}\right)\left(\frac{2\pi\ \text{rad}}{\text{rev}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right) = \frac{17\pi}{30}\text{rad}\cdot\text{s}^{-2}$$

EVALUATE **(a)** Inserting the angular acceleration and the angular displacement, $\theta - \theta_0 = 6\pi$, into Equation 10.9, we find the final angular velocity is

$$\omega = \pm\sqrt{\omega_0^2 + 2\alpha(\theta - \theta_0)}$$

$$= \sqrt{0 + 2\left(\frac{17\pi}{30}\right)(6\pi\ \text{rad})} = \pi\sqrt{6.8} = 8.2\ \text{rad/s}$$

where the two signs indicate that the wheel could turn either clockwise or counterclockwise (we arbitrarily chose the positive sign).

(b) Inserting the acceleration and the angular displacement, $\theta - \theta_0 = 6\pi$ rad, into Equation 10.8 gives

$$\theta - \theta_0 - \overbrace{\omega_0 t}^{=0} = \frac{1}{2}\alpha t^2$$

$$t = \pm\sqrt{\frac{2(\theta - \theta_0)}{\alpha}} = \sqrt{\frac{2(6\pi\,\text{rad})}{17\pi/30\ \text{rad/s}^2}} = \sqrt{360/17}\ \text{s} = 4.6\ \text{s}$$

ASSESS Another way to answer (b) is to use Equation 10.7:

$$\omega = \omega_0 + \alpha t \Rightarrow\ t = \frac{\omega - \overbrace{\omega_0}^{=0}}{\alpha} = \frac{\pi\sqrt{6.8}\ \text{rad/s}}{17\pi/30\ \text{rad/s}^2} = 4.6\ \text{s}$$

where we have used $\omega = \pi\sqrt{6.8}$ rad/s because it is more precise than 8.2 rad/s, which is only precise to two significant figures.

46. **INTERPRET** You need to determine how many revolutions (i.e., the angular displacement) the blender makes while accelerating between the two speeds. You can assume that the angular acceleration is constant.

DEVELOP You're not given the angular acceleration, but this is not a problem since you can combine Equations 10.1 and 10.6 to find the angular displacement: $\Delta\theta = \frac{1}{2}(\omega_0 + \omega)\Delta t.$

EVALUATE Plugging in the known values gives:

$$\Delta\theta = \frac{1}{2}(3800\ \text{rpm} + 2000\ \text{rpm})\left(\frac{1\ \text{min}}{60\ \text{s}}\right)(2.0\ \text{s}) = 96.6\ \text{rev}$$

The blender does not meet the specs, since it makes 36.6 revolutions too many.

ASSESS A blender that does meet the specs would need an angular acceleration of at least: $\alpha = (\omega^2 - \omega_0^2)/2\Delta\theta = 24.2\ \text{rev/s}^2$. Therefore, the maximum time to switch between speeds would be: $t = (\omega - \omega_0)/\alpha = 1.2$ s, which explains why the blender above is unable to meet the specs.

47. **INTERPRET** This problem involves angular acceleration, which we shall assume is constant. We are provided the initial and final angular speed of the motor and the time interval over which the motor accelerates and are asked to find several characteristics of the rotational kinematics of the engine.

DEVELOP Because we have no information about the variation in time of the acceleration, we can only calculate the average acceleration over the given time interval. This is given by Equation 10.4 in the form

$$\alpha = \frac{\Delta\omega}{\Delta t}$$

To find the tangential (i.e., linear) acceleration, differentiate Equation 10.3 with respect to time to find

$$a = \frac{dv}{dt} = \frac{d}{dt}(\omega r) = \frac{d\omega}{dt}r = \alpha r$$

(note that this result holds only for constant radius). Finally, knowing the angular acceleration and the initial and final angular velocities, we can apply Equation 10.9 to find the number of revolutions made during the given time interval.

EVALUATE **(a)** The average angular acceleration is

$$\bar{\alpha} = \frac{\Delta\omega}{\Delta t} = \frac{(5500 - 1200)\text{ rpm}}{(2.7\text{ s})}\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = 167\text{ s}^{-2} \approx 170\text{ s}^{-2}$$

to two significant figures.

(b) With d = 3.75 cm, we find an average linear acceleration of

$$\bar{a} = \bar{\alpha}r = \left(167\text{ s}^{-2}\right)\left(\frac{3.5\text{ cm}}{2}\right) = 2.9\text{ m/s}^2$$

(c) The engine makes

$$\Delta\theta = \frac{\omega^2 - \omega_0^2}{2\alpha} = \frac{(5500\text{ rpm})^2 - (1200\text{ rpm})^2}{2\left(167\text{ s}^{-2}\right)(60\text{ s/min})^2} = 150\text{ revolutions}$$

during this 2.7-s time interval (to two significant figures).

ASSESS Note that dimensional analysis would lead us to the proper formula for part (b), where we needed to multiply the angular acceleration by a length to recover linear acceleration.

48. **INTERPRET** You have to determine if a saw stops within the required time limit. You assume the saw has constant angular acceleration, so you can use the equations in Table 10.1.

DEVELOP You know the initial angular velocity, ω_0, and the number of revolutions, $\Delta\theta$, before the saw stops. What you're looking for is how long this takes. Combining Equations 10.1 and 10.6, you have an expression for the stopping time: $\Delta t = \Delta\theta / \frac{1}{2}(\omega_0 + \omega)$.

EVALUATE The time to stop the saw $(\omega = 0)$ is

$$\Delta t = \frac{2\Delta\theta}{\omega_0} = \frac{2(75\text{ rev})}{(5400\text{ rpm})} = 1.7\text{ s}$$

The saw, therefore, meets its specs with 0.3 s to spare.

ASSESS The expression that we derived for the time can be arrived at in other ways. For example, Equation 10.7 says the angular acceleration during stopping is $\alpha = -\omega_0 / \Delta t$. Plugging this into Equation 10.8 gives $\Delta\theta = \frac{1}{2}\omega_0\Delta t$, which works out to the same expression as above.

49. **INTERPRET** The problem concerns the *E. coli* bacteria, whose linear motion is related directly to the rotational motion of its flagellum.

DEVELOP The time that it takes for the bacteria to cross the microscope's field of view is simply the linear distance divided by the linear velocity: $t = \Delta x / v$. Over the same time, the flagellum completes a number of revolutions given by: $\Delta\theta = \omega t$.

EVALUATE Combining the two equations from above gives

$$\Delta\theta = \frac{\omega\Delta x}{v} = \frac{(600\text{ rad/s})(150\ \mu\text{m})}{(25\ \mu\text{m/s})}\left(\frac{1\text{ rev}}{2\pi\text{ rad}}\right) = 570\text{ rev}$$

ASSESS The units all work out, and the answer seems reasonable. Note that the v we use here is not necessarily the same as the v in Equation 10.3 $(v = \omega r)$, which is the tangential speed of the rotating object.

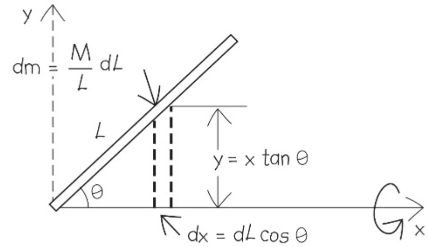
50. **INTERPRET** This problem involves finding the rotational inertia of an object about several different axes. For some axes, the object can be decomposed into objects for which the rotational inertia is given in Table 10.2, whereas for the other axes we must apply Equation 10.13 to find the rotational inertia for the various axes.

DEVELOP For part (a), the square frame can be decomposed into two rods parallel and two rods perpendicular to the axis. For the parallel rods, we can treat them as if all the mass were concentrated at the center of mass, so $I_{par} = Mr^2 = M(L/2)^2 = ML^2/4$. The rotational inertia of the perpendicular rods can be found from Table 10.2, and is

$I_{\text{per}} = ML^2/12$. For part (b), we apply Equation 10.13 first to a single rod. Using the coordinate system drawn in the figure below, the integral of Equation 10.13 becomes

$$I = \int_0^{L\cos\theta} y^2 dm = \frac{M}{L}\frac{\tan^2\theta}{\cos\theta}\int_0^{L\cos\theta} x^2 dx = \frac{M}{L}\frac{\tan^2\theta}{\cos\theta}\left(\frac{x^3}{3}\right)_0^{L\cos\theta} = \frac{ML^2}{3}\sin^2\theta$$

Because all four rods are symmetric, the total rotational inertial will be four times this result.



For part (c), apply the parallel axis theorem. From Table 10.2 we find the rotational inertia of a rod rotating about an axis through its center of mass is $I_{\text{cm}} = ML^2/12$. The parallel axis theorem tells us the rotational inertia about a parallel axis a distance L/2 from the center-of-mass axis is

$$I = I_{\text{cm}} + M\left(\frac{L}{2}\right)^2 = ML^2\left(\frac{1}{12}+\frac{1}{4}\right) = \frac{1}{3}ML^2$$

EVALUATE (a) Because we have two rods parallel to the axis and two rods perpendicular to the axis, the total rotational inertia is

$$I_{\text{a}} = 2I_{\text{par}} + 2I_{\text{per}} = 2\frac{ML^2}{4} + 2\frac{ML^2}{12} = \frac{2}{3}ML^2$$

(b) Given that we have four rods, each with the rotational inertia given by the expression above, we can sum them to find the total rotational inertia. The result is

$$I_{\text{b}} = 4I_{\text{rod}} = \frac{4}{3}ML^2\sin^2\left(\frac{\pi}{4}\right) = \frac{4}{3}ML^2\left(\frac{1}{\sqrt{2}}\right)^2 = \frac{2}{3}ML^2$$

(c) Again, we have four rods, each with the rotational inertial derived above. Therefore, the total rotational inertia is

$$I_c = \frac{4}{3}ML^2$$

ASSESS Notice how we used symmetry to simplify the calculations.

51. **INTERPRET** We are asked to find the rotational inertia of a thick ring with inner and outer radii $R_1 = R$ and $R_2 = R/2$. The mass distribution is continuous, so we need to do an integral.

DEVELOP For a thick ring, the ring-shaped mass elements used in Example 10.7 have mass

$$dm = \sigma dA = \frac{M}{\pi\left(R_2^2 - R_1^2\right)} 2\pi r dr$$

where $\sigma = M/A$ is the mass density (units: kg/m^2). Note that the ring only extends in radius from R_1 to R_2. The rotational inertia can then be obtained by integrating over

$$I = \int_{R_1}^{R_2} r^2 dm$$

EVALUATE Upon carrying out the integration, the rotational inertia about an axis perpendicular to the ring and through its center is

$$I = \int_{R_1}^{R_2} r^2 dm = M\int_{R_1}^{R_2} \frac{2\pi r^3 dr}{\pi\left(R_2^2 - R_1^2\right)} = \frac{M\left(R_2^4 - R_1^4\right)}{2\left(R_2^2 - R_1^2\right)} = \frac{M}{2}\left(R_1^2 + R_2^2\right)$$

With $R_1 = R$ and $R_2 = R/2$, the rotational inertia becomes $I = 5MR^2/8$.

ASSESS To see that the result makes sense, let's consider the following limits: **(i)** $R_1 \to 0$: In this case, we have a disk with radius R_2 and $I = MR_2^2/2$ **(ii)** $R_1 \to R_2$: In this limit, we have a thin ring with $I = MR_2^2$.

52. **INTERPRET** This problem involves applying the parallel axis theorem to find the rotational inertia of an object. We can use Table 10.2 to find the expression for the rotational inertia for an axis through the center of mass of the object.

DEVELOP The object is a flat plate that is rotating about one of its long edges (of length b). Therefore, if we displace the axis of rotation to go through the center of the plate, we have the situation depicted in the last entry of Table 10.2, so $I_{\rm cm} = Ma^2/12$. The displacement of the axis of rotation is $d = a/2$.

EVALUATE Applying the parallel axis theorem (Equation 10.17), gives

$$I = I_{\rm cm} + Md^2 = Ma^2\left(\frac{1}{12} + \frac{1}{4}\right) = \frac{1}{3}Ma^2$$

ASSESS Notice the length b of the long side does not enter into the result. This makes sense because a longer plate will simply have more mass than a shorter one, but the distribution of the mass will not have changed.

53. **INTERPRET** The problem concerns the cellular motor that drives the flagellum of the *E. coli* bacteria. We are asked to find the force exerted by this motor, given the torque and the radius at which the force is applied.

DEVELOP We're told that the force is applied tangentially, so $\theta = 90°$, and Equation 10.10 reduces to: $\tau = rF$.

EVALUATE Solving for the motor's applied force:

$$F = \frac{\tau}{r} = \frac{420\ \text{pN}\cdot\text{pm}}{15\ \text{nm}} = 28\ \text{pN}$$

ASSESS This is a very small force, but it's rather impressive that an *E. coli*, with a typical mass of about 10^{-15} kg, can exert a force that is over 1000 times its own weight.

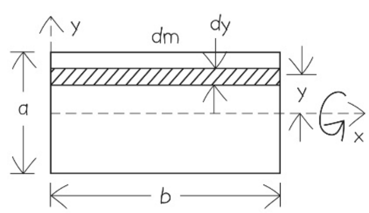
54. **INTERPRET** This problem is an exercise in finding the rotational inertia of an object. The object in question is a flat plane that rotates about a central axis (i.e., the last entry in Table 10.2).

DEVELOP Following the hint, we divide the plane up into strips parallel to the central axis (see figure below). For a uniform plate

$$dm/M = b\left[dy/(ab)\right]$$

$$dm = \frac{M}{a}dy$$

Insert this result into Equation 10.13 to and integrate from $y = -a/2$ to $y = a/2$ to find the rotational inertial.



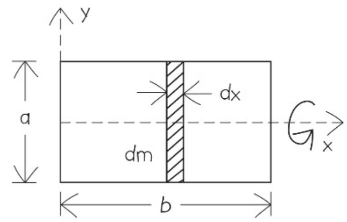
EVALUATE Evaluating the integral gives the rotational inertia as

$$I = \int y^2 dm = \frac{M}{a}\int_{-a/2}^{a/2} y^2 dy = \frac{M}{a}\left|\frac{y^3}{3}\right|_{-a/2}^{a/2} = \frac{Ma^2}{12}$$

which agrees with Table 10.2.

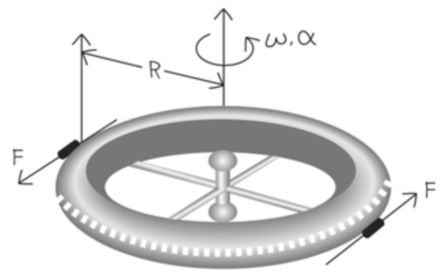
ASSESS To double check this result, we can divide the plate into thin strips perpendicular to the axis of rotation (see figure below). The rotational inertia of each strip is $dI = \left(a^2/12\right)dm$, where $dm/M = \left[a/(ab)\right]dx = dx/b$. Therefore,

$$I = \int dI = \int \frac{a^2}{12}\,dm = \frac{Ma^2}{12}\int_0^b \frac{dx}{b} = \frac{Ma^2}{12}$$



55. **INTERPRET** You are asked to find the time it takes for the space station to start from rest and reach a certain angular speed, with a given thrust.

DEVELOP The space station is essentially a ring with radius $R = 9$ m and rotational inertia $I = MR^2$ (from Table 10.1). The two rockets provide a net torque of $\tau = 2FR$, as can be seen from the figure below.



This torque causes an angular acceleration, $\alpha = \tau / I = 2F / MR$, that spins up the station from rest to an angular velocity ω. This final rotation speed is chosen such that the centripetal acceleration at the rim is equal to the gravitational acceleration on the surface of Earth:

$$a_c = \frac{v^2}{R} = \frac{(\omega R)^2}{R} = \omega^2 R = g \quad \rightarrow \quad \omega = \sqrt{\frac{g}{R}}$$

Your job is to determine how long the rockets must fire to reach this angular velocity and how many rotations the station makes during this time period.

EVALUATE **(a)** The time can be found with Equation 10.7:

$$t = \frac{\omega}{\alpha} = \frac{\sqrt{g/R}}{2F/MR} = \frac{M\sqrt{gR}}{2F} = \frac{(9\times10^5 \text{ kg})\sqrt{(9.8 \text{ m/s}^2)(9 \text{ m})}}{2(140 \text{ N})} = 3.01\times10^4 \text{ s} = 8.3 \text{ h}$$

(b) We could use Equation 10.8 to find the number of revolutions completed in this time, but Equation 10.9 provides a simple formula with the weight of the space station:

$$\Delta\theta = \frac{\omega^2}{2\alpha} = \frac{Mg}{4F} = \frac{(9.0\times10^5 \text{ kg})(9.8 \text{ m/s}^2)}{4(140 \text{ N})} = \frac{15{,}750 \text{ rad}}{2\pi \text{ rad/rev}} = 2506.7 \text{ rev}$$

ASSESS These are relatively small rockets, so it takes a fair amount of time to reach the desired rotational velocity. Since $t \sim 1/F$, a larger thrust will shorten this spin-up time.

56. **INTERPRET** In this problem, we are asked to estimate and compare the rotational inertia of an ice skater before and after they extend their arms outward. We can model the human body using simple shapes from Table 10.2 for which we know the rotational inertias.

DEVELOP We are told that we can model the ice skater's body as a cylinder when they are holding their arms tight to their torso. When their arms are extended, we can consider the addition of a thin rod rotating about its center. We will have to make estimates on the size of the radii and length, as well as the mass distribution after the arms are extended. We will estimate the height, and, thus, the arm span, of the ice skater to be ~ 1.8 m, and we can estimate the distance from the center of rotation to their shoulders to be ~ 0.2 m. This, in turn, means the cylinder radius is $R_c = 0.2$ m and the rod length is $L_r = 0.9$ m. Before arm extension, the mass located in the cylinder is $M_{ci} = 65$ kg, and after extension, we estimate that the cylinder mass becomes $M_{cf} = (0.9)M_{ci}$ and the rod mass is $M_r = (0.1)M_{ci}$. With these, we can now express the rotational inertias before and after arm extension as

$$I_{\rm i} = I_{\rm c} = \frac{1}{2}M_{\rm ci}R_{\rm c}^2$$

$$I_{\rm f} = I_{\rm c} + I_{\rm r} = \frac{1}{2}M_{\rm cf}R_{\rm c}^2 + \frac{1}{12}M_{\rm r}L_{\rm r}^2$$

EVALUATE Evaluating the expressions for rotational inertia found in Table 10.2 gives the following approximate results:

$$I_{\rm i} = \left(\frac{1}{2}\right)(65\ \text{kg})(0.2\ \text{m})^2 \cong 1\ \text{kg}\cdot\text{m}^2$$

$$I_{\rm f} = \left(\frac{1}{2}\right)(0.9\times 65\ \text{kg})(0.2\ \text{m})^2 + \left(\frac{1}{12}\right)(0.1\times 65\ \text{kg})(0.9\ \text{m})^2 \cong 2\ \text{kg}\cdot\text{m}^2$$

Thus, upon extending their arms, the ice skater experiences an approximately 100% increase in rotational inertia.

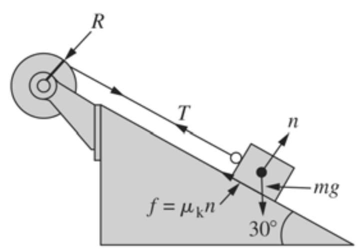
ASSESS From these estimates, we see that $I_{\rm i}$ roughly doubles, but this depends heavily on the proportions used for the skater.

57. **INTERPRET** This problem involves Newton's second law in both linear and rotational form, which we can apply to find the coefficient of friction between block and slope, given the acceleration of the block. We will also need to consider the rotational inertia of the wheel in this problem.

DEVELOP Draw a diagram of the situation (see figure below). Applying Newton's second law (Equation 4.3) to the mass gives

$$\left.\begin{aligned} mg\sin\theta - f_{\rm k} - T &= ma \\ n - mg\cos\theta &= 0 \end{aligned}\right\} mg\sin\theta - \mu_{\rm k}\, mg\cos\theta - T = ma$$

where we have used Equation 5.3 to express the force due to kinetic friction, $f_{\rm k} = \mu_{\rm k} n$.



Likewise, applying the rotational analog of Newton's second law (Equation 10.11) to the wheel gives

$$\tau_{\rm net} = I\alpha$$

$$TR = I\alpha$$

where $t_{\rm net} = TR$ because the tension is the only torsional force acting on the wheel, $I = MR^2/2$ (from Table 10.2), and $a = \alpha R$ (Equation 10.4). These equations allow us to determine $\mu_{\rm k}$.

EVALUATE Solving first for the tension from the rotational application of Newton's second law gives

$$T = \frac{I\alpha}{R} = \frac{(MR^2/2)(\text{a}/\text{R})}{R} = \frac{1}{2}Ma$$

Insert this into the equation derived from Newton's second law applied to the block, and solve for $\mu_{\rm k}$:

$$\mu_{\rm k} = \frac{mg\sin\theta - ma - Ma/2}{mg\cos\theta}$$

$$= \frac{(3\ \text{kg})(9.8\ \text{m/s}^2)\sin(30°) - (3\ \text{kg} + 0.45\ \text{kg})(1.9\ \text{m/s}^2)}{(3\ \text{kg})(9.8\ \text{m/s}^2)\cos(30°)} = -0.32$$

ASSESS To see that our expression for $\mu_{\rm k}$ makes sense, let's check some limits: **(i)** If $a = 0$, then $\mu_{\rm k} = \tan\theta$. This is precisely the equation we obtained in Chapter 5 (see Example 5.10). **(ii)** $M = 0$ and $\mu_{\rm k} = 0$. The situation corresponds to a block of mass *m* sliding down a frictionless slope with acceleration $a = g\sin\theta$.

58. **INTERPRET** This problem combines Newton's second law for rotational motion and the concept of torque. Combining these with the rotational kinematic equations (Equations 10.6–10.9), we can find the final angular speed of the wheel.

DEVELOP Assuming the wheel spins about an essentially frictionless axis, the only torsional force acting on the wheel is due to the wrench, so Newton's second law (Equation 10.11) gives

$$\tau_{\text{net}} = \tau_{\text{wrench}} = I\alpha$$

From Example 10.6, the rotational inertia of the bicycle wheel is $I = MR_2$, and from Equation 10.10, the torque applied by the wrench is $\tau_{\text{wrench}} = -f_k R = -\mu_k F_{\text{app}} R$. Note that $\theta = 90°$ in this case for Equation 10.10 because the frictional force is applied tangentially to the wheel, and we have used Equation 5.3 to express the frictional force. This gives us the angular acceleration, which we can use in Equation 10.7 to find the final angular speed ω.

EVALUATE Inserting the given quantities into the expression derived above using Newton's second law gives

$$\omega = \omega_0 + \alpha t = \omega_0 + \frac{\tau_{\text{wrench}}}{I}t = \omega_0 - \frac{\mu_k F_{\text{app}} R}{MR^2}t = \omega_0 - \frac{\mu_k F_{\text{app}} t}{MR}$$
$$= \left(230\ \frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\ \text{rad}}{\text{rev}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right) - \frac{0.46(2.7\ \text{N})(3.1\ \text{s})}{(1.9\ \text{kg})(0.33\ \text{m})} = 18\ \text{rad/s} = 170\ \text{rev/min}$$

ASSESS Notice that the greater the applied force, the smaller will be the final angular momentum. One might think the wheel will reverse direction if the applied force is great enough, but this will not happen because friction only acts to counter the motion, not to create motion. Once the wheel stops, the friction force will be static and will not create motion. (It could, however, prevent another force from turning the wheel.)

59. **INTERPRET** In this problem, we want to find the angular speed of the potter's wheel after he exerts a tangential force to the edge of the wheel. We can address this problem in several ways—either through the work-energy theorem or through Newton's second law (Equation 10.11). The force produces a torque that causes the wheel to rotate.

DEVELOP We will apply the work-energy theorem for constant torque (Equation 10.19). This gives

$$W = \tau\Delta\theta = \Delta K = K_{\text{f}} - K_0 = \frac{1}{2}I\omega^2$$

because the wheel starts from rest. The equation allows us to determine the angular velocity ω.

EVALUATE Because the force acting on the wheel is tangential to the wheel circumference, $\theta = 90°$ in Equation 10.10, so $\tau = FR$. In addition, from Table 10.2, we know that the rotational inertia of a disk is $I = MR^2/2$. Inserting $\Delta\theta = \frac{1}{8}\text{rev} = \frac{\pi}{4}\text{rad}$, we have

$$\omega^2 = \frac{2\tau\Delta\theta}{I} = \frac{2FR\Delta\theta}{MR^2/2} = \frac{4F\Delta\theta}{MR}$$

or

$$\omega = \pm\sqrt{\frac{4F\Delta\theta}{MR}} = \pm\sqrt{\frac{4(60\ \text{N})(\pi/4\ \text{rad})}{(140\ \text{kg})(0.375\ \text{m})}} = \pm 1.89\ \text{rad/s}$$

ASSESS The two signs indicate that the potter may spin the wheel either clockwise or counterclockwise. The greater the force exerted on the wheel, the larger the angular speed. On the other hand, larger M and R result in a larger rotational inertia and smaller angular speed (if the same force is applied). If we apply Newton's second law to this problem, we find

$$\tau_{\text{net}} = FR = Ia = MR^2\alpha/2$$
$$\alpha = \frac{2F}{MR}$$

Inserting this result into Equation 10.9 and solving for the final angular velocity gives

$$\omega^2 = \overbrace{\omega_0^2}^{=0} + 2\alpha\overbrace{(\theta - \theta_0)}^{\Delta\theta}$$

$$\omega = \pm\sqrt{\frac{4F\Delta\theta}{MR}}$$

which is the same expression as that found using the work-energy theorem.

60. **INTERPRET** This problem involves conservation of total mechanical energy, which we can use to find the angular speed of the hollow drum when the anchor hits the water.

DEVELOP By conservation of total mechanical energy, we can equate the initial and final mechanical energies. The initial energy is just the gravitational potential energy of the anchor, $E_0 = U = mgh$. The final energy is the sum of the kinetic energies of the rotating hollow drum and the dropping anchor,

$$E_f = K_{\text{drum}} + K_{\text{anchor}} = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$$

From Table 10.2, the rotational inertia of the hollow drum is $I = MR^2$.

EVALUATE Equating the initial and final total mechanical energies gives

$$mgh = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2 = \frac{1}{2}MR^2\omega^2 + \frac{1}{2}m(\omega R)^2$$

$$\omega = \pm\sqrt{\frac{2mgh}{R^2(M+m)}} = \pm\sqrt{\frac{2(5000\ \text{N})(16\ \text{m})}{(1.1\ \text{m/s})^2\left[380\ \text{kg} + (5000\ \text{N})\big/\left(9.8\ \text{m/s}^2\right)\right]}} = \pm 12\ \text{rad/s}$$

ASSESS The result has two signs because we cannot tell if the hollow drum rotates clockwise or counter-clockwise.

61. **INTERPRET** This problem involves conservation of energy: gravitational potential energy is converted to center-of-mass kinetic energy and rotational kinetic energy.

DEVELOP By conservation of energy, the sum of the gravitational potential energy and the total kinetic energy (Equation 10.20) is a constant. If we assume the gravitational potential is zero where the ball is at rest, then this constant is zero, or in other words:

$$K_{\text{cm}} + K_{\text{rot}} = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 = -U$$

As it rolls down the incline, the potential decreases: $U = -Mgh$, where the height is related to the distance rolled down the incline by: $h = d\sin\theta$. The ball is hollow, so its rotational inertia is $I = \frac{2}{3}MR^2$, and we assume that it rolls without slipping, so $v = \omega R$ (Equation 10.21).

EVALUATE Plugging in the various expressions into the energy conservation equation gives:

$$\frac{1}{2}Mv^2 + \frac{1}{3}Mv^2 = Mgd\sin\theta$$

Solving for the speed,

$$v = \sqrt{\frac{6}{5}gd\sin\theta}$$

ASSESS If the ball were sliding down the incline without friction, the speed would have been $v = \sqrt{2gd\sin\theta}$. The fact that the ball is rolling means it will go slower down the incline.

62. **INTERPRET** This problem involves conservation of total mechanical energy, which we can apply to find the height to which the ball rolls up the incline.

DEVELOP If the ball rolls without slipping and we define potential energy to be zero at the incline's base, then the initial energy is only kinetic energy, and it is given by

$$E_{\text{i}} = K_{\text{cm}} + K_{\text{rot}} = \frac{1}{2}Mv_{\text{cm}}^2 + \frac{1}{2}I_{\text{cm}}\omega^2$$

The final energy is only potential energy and is $E_{\text{f}} = Mgh$. Conservation of total mechanical energy allows us to equate the initial and final mechanical energies, which we can then solve for the height h.

EVALUATE Setting the initial and final energies to be equal gives

$$Mgh = \frac{1}{2}I_{\text{cm}}\omega^2 + \frac{1}{2}Mv_{\text{cm}}^2$$

With $I_{cm} = 2MR^2/3$ (from Table 10.2) and $\omega = v_{cm}/R$, this becomes

$$Mgh = \frac{1}{2}\left(\frac{2}{3}MR^2\right)(v_{cm}/R)^2 + \frac{1}{2}Mv_{cm}^2 = \frac{5}{6}Mv_{cm}^2$$

$$h = \frac{5v_{cm}^2}{g} = \frac{5(2.5\ \text{m/s})^2}{(9.8\ \text{m/s}^2)} = 3.19\ \text{m}$$

ASSESS The height attained is proportional to the linear speed of the ball squared.

63. **INTERPRET** The kinetic energy of the wheel consists of two parts: the kinetic energy of the center of mass, K_{cm}, and the rotational kinetic energy, K_{rot}. We want to find how changing the moment of inertia and mass of the wheel affects the total kinetic energy.

DEVELOP The total kinetic energy of the wheel consists of the center-of-mass energy and the internal rotational energy associated with the spin about the center of mass (see Equation 10.20):

$$K_{tot} = K_{cm} + K_{rot} = \frac{1}{2}Mv_{cm}^2 + \frac{1}{2}I_{cm}\omega^2$$

With the condition for rolling without slipping, $v = \omega R$, the total kinetic energy can be rewritten as

$$K_{tot} = K_{cm} + K_{rot} = \frac{1}{2}Mv_{cm}^2 + \frac{1}{2}I_{cm}\left(\frac{v_{cm}}{R}\right)^2 = \frac{1}{2}Mv_{cm}^2\left(1 + \frac{I_{cm}}{MR^2}\right)$$

The initial condition is $I_{cm}/\left(MR^2\right) = 0.37 = 37\%$. After the redesign,

$$\frac{I'_{cm}}{M'R^2} = \frac{0.8I_{cm}}{(0.74M)R^2} = 1.081\frac{I_{cm}}{MR^2} = (1.081)(0.37) = 0.40$$

EVALUATE The fractional decrease in kinetic energy is

$$\frac{K-K'}{K} = 1 - \frac{K'}{K} = 1 - \frac{M'\left[1 + I'_{cm}/(M'R^2)\right]}{M\left[1 + I_{cm}/(MR^2)\right]} = 1 - \frac{0.74M}{M}\frac{(1+0.40)}{(1+0.37)} = 0.244 = 24\%$$

to two significant figures.

ASSESS Initially, K_{cm} accounts for $1/1.37 = 73\%$ of the total kinetic energy, while K_{rot} accounts for the remaining $0.37/1.37 = 27\%$. After the redesign, $M \to M' = 0.74M$, so the translational kinetic energy decreases by 37%, while the rotational kinetic energy goes down by 20% ($I_{cm} \to I'_{cm}\ 0.8I_{cm}$). Therefore, the total kinetic energy is now

$$(0.74)\frac{1}{1.37} + (0.80)\frac{0.37}{1.37} = 0.756 = 76\%$$

of the original. This is a 24% decrease.

64. **INTERPRET** This problem involves conservation of total mechanical energy, which is composed in this case of rotational kinetic, translational kinetic, and gravitational potential energies. We shall take the bottom of the trajectory to be the zero of gravitational potential energy. Using these concepts, we can find the height to which the ball rises on the frictionless trajectory. Note that Newton's second law also applies, because when the ball enters the frictionless surface, no tangential force will act on it, so its rotational speed will remain constant.

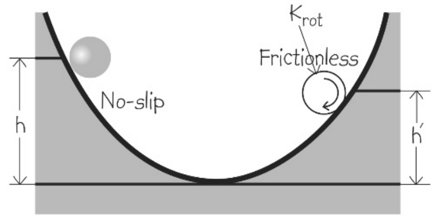
DEVELOP To apply conservation of total mechanical energy, we must express the initial and final total mechanical energies. The initial mechanical energy is $E_i = Mgh$. At the bottom of the trajectory, the total mechanical energy is purely kinetic and consists of rotational and translational kinetic energy:

$$E_b = K_{rot} + K_{cm} = \frac{1}{2}I_{cm}\omega_b^2 + \frac{1}{2}Mv_{cm}^2$$

At the right top of the trajectory, the final total mechanical energy is

$$E_f = U + K_{rot} = Mgh' + \frac{1}{2}I\omega_b^2$$

because the ball continues to spin at the constant angular speed ω_b once it enters the frictionless surface (see figure below—by Newton's second law, because no tangential forces act on the ball, its angular acceleration is zero, so its angular speed is constant). By conservation of total mechanical energies, all three of these expressions for the total mechanical energy must give the same result, which allows us to solve for h'.



EVALUATE Equating E_0 and E_b, we find

$$Mgh = \frac{1}{2}I_{cm}\omega_b^2 + \frac{1}{2}Mv_{cm}^2$$

Using $\omega_b R = v_{cm}$, and $I_{cm} = 2MR^2/5$ (from Table 10.2), and solving for ω_b gives

$$Mgh = \frac{1}{2}\left(\frac{2MR^2}{5}\right)\omega_b^2 + \frac{1}{2}M\left(\omega_b R\right)^2$$

$$\omega_b^2 = \frac{10gh}{7R^2}$$

Equating now the initial and final energies, and using this result for ω_b, we find

$$Mgh = Mgh' + \frac{1}{2}I_{cm}\omega_b^2 = Mgh' + \frac{1}{2}\left(\frac{2MR^2}{5}\right)\left(\frac{10gh}{7R^2}\right)$$

$$h' = h - \frac{2h}{7} = \frac{5}{7}h$$

ASSESS Notice that neither the size of the ball does nor its mass enters into the final result.

65. **INTERPRET** This problem involves finding the rotational inertia of a circular disk after an off-center hole has been drilled through it. The parallel axis theorem is likely to be useful here.

DEVELOP Equation 10.12 shows that the rotational inertia of an object is the sum of the rotational inertias of its pieces, so

$$I_{disk} = I_{hole} + I_{remainder}$$

The hint expresses this fact as $I_{remainder} = I_{disk} - I_{hole}$. Here $I_{disk} = MR^2/2$ is the rotational inertia of the whole disk about an axis perpendicular to the disk and through the disk center (see Example 10.7). Use the parallel axis theorem to find the rotational inertia of the hole, I_{hole}. This gives

$$I_{hole} = M_{hole}\left(\frac{R}{4}\right)^2 + I_{cm} = M_{hole}\frac{R^2}{16} + M_{hole}\frac{R^2}{32} = \frac{3}{32}M_{hole}R^2$$

where $R/4$ is the distance of the hole's center of mass from the axis of the disk, and we have $I_{cm} = M_{hole}\left(R/4\right)^2/2$ as the rotational inertia of the hole material about a parallel axis through its center of mass (see Example 10.7). With these equations, we can determine $I_{remainder}$.

EVALUATE Because the planar mass density of the disk (assumed to be uniform) is $\sigma = M/\pi R^2$, the mass of the hole material is

$$M_{hole} = \sigma A_{hole} = \frac{M}{\pi R^2}\pi\left(\frac{R}{4}\right)^2 = \frac{M}{16}$$

Therefore, the rotational inertia of the hole is

$$I_{hole} = \left(\frac{3}{32}\right)\left(\frac{M}{16}\right)R^2 = \frac{3}{512}MR^2$$

and

$$I_{\text{remainder}} = I_{\text{disk}} - I_{\text{hole}} = \frac{1}{2}MR^2 - \frac{3}{512}MR^2 = \frac{253}{512}MR^2 = 0.494MR^2$$

ASSESS If the hole drilled were concentric with the disk, we would have

$$I'_{\text{hole}} = I_{\text{cm}} = \frac{1}{2}\frac{M}{16}\left(\frac{R}{4}\right)^2 = \frac{1}{512}MR^2$$

and

$$I'_{\text{remainder}} = I_{\text{disk}} - I'_{\text{hole}} = \frac{1}{2}MR^2 - \frac{1}{512}MR^2 = \frac{255}{512}MR^2 = 0.498MR^2$$

The same result is obtained if we use the formula $M'\left(R_1^2 + R_2^2\right)/2$ derived in Problem 51, with $M' = \pi R^2 - \pi(R/4)^2 = (15/16)\pi R^2 = (15/16)M$, $R_1 = R$ and $R_2 = R/4$.

66. **INTERPRET** This problem involves Newton's second law and the rotational inertia of a solid cylinder. We are given a mass that hangs from a cylindrical drum by a massless rope that is wrapped around the drum. The mass is allowed to fall, but it is restrained by the rope that makes the drum spin (causing an angular acceleration of the drum). We are to find the tension in the rope as the mass falls and the drum's mass.

DEVELOP The situation is similar to Example 10.9. Applying Newton's second law to the falling mass gives $mg - T = ma$ (where we have taken the downward direction to be positive), which we can solve for the tension T. For part (b), apply the rotational analog of Newton's second law, Equation 10.11: $\tau_{\text{net}} = I\alpha,$ which gives $\tau_{\text{net}} = RT = I\alpha,$ where the rotational inertia is $I = MR^2/2$ (from Table 10.2) and $\alpha R = a.$

EVALUATE **(a)** Solving for the tension gives $T = mg - ma = (20\text{ kg})(9.8\text{ m/s}^2 - 5.0\text{ m/s}^2) = 96\text{ N}$ (to two significant figures).

(b) Inserting the known quantities into the expression for the net torque gives

$$RT = \frac{MR^2}{2}\frac{a}{R}$$

$$M = \frac{2T}{a} = \frac{2(96\text{ N})}{5.0\text{ m/s}^2} = 38.4\text{ kg}$$

to two significant figures.

ASSESS Notice that we retained three significant figures for the tension in part (b) because the tension is an intermediate result in this case.

67. **INTERPRET** This problem involves conservation of total mechanical energy, which we can use to find how high up the hill the motorcyclist can go.

DEVELOP If all possible losses are neglected, the total mechanical energy of the motorcycle and rider is conserved as it coasts uphill, so the total kinetic energy at the bottom equals the total potential energy at the highest point,

$$K_{\text{trans}} + K_{\text{rot}} = M_{\text{tot}}gh$$

The translational kinetic energy of the motorcycle and rider (including the wheels) and the rotational kinetic energy of the wheels (about their center of mass) are, assuming rolling without slipping,

$$K_{\text{trans}} = \frac{1}{2}M_{\text{tot}}v^2,\ \text{K}_{\text{rot}} = 2\left(\frac{1}{2}I\omega^2\right) = I\left(\frac{v}{R}\right)^2$$

These expressions can be combined to solve for h.

EVALUATE Substituting the second equation into the first and using $v = 75\text{ km/h} = 20.8\text{ m/s},$ we find the maximum vertical height reached is

$$h = \frac{v^2}{2g}\left(1 + \frac{2I}{M_{\text{tot}}R^2}\right) = \frac{(20.8\text{ m/s})^2}{2(9.8\text{ m/s}^2)}\left(1 + \frac{2(1.5\text{ kg}\cdot\text{m}^2)}{(372\text{ kg})(0.305\text{ m})^2}\right) = 24\text{ m}$$

ASSESS If the rolling motion is ignored, the result would be $h = v^2/2g,$ which is what we expect from considering only the linear motion.

68. **INTERPRET** This problem involves conservation of total mechanical energy and Newton's second law. The former allows us to find the speed at the top of the loop, and the latter allows us to find the minimum speed necessary to stay on the track.

DEVELOP The center of mass of the marble travels in a circle of radius $R - r$ inside the loop, so at the top, $mg + N = mv^2/(R - r)$. To remain in contact with the track, Newton's second law tells us that $n \geq 0$, so that the track causes the ball to accelerate (otherwise it would be in free fall). Thus, we have

$$F_{\text{net}} = -mg - n = ma = -m\frac{v^2}{R-r}$$
$$v^2 = \frac{mg+n}{m}(R-r) \geq g(R-r)$$

where we have taken the downward direction to be negative. By conservation of total mechanical energy, we can equate the total mechanical energies at points A and B. For point A, the energy is just the gravitational potential energy, which is

$$E_A = mg(h+r)$$

For point B, the energy is the sum of the gravitational potential energy and the kinetic energies due to rotation and translation,

$$E_B = mg(2R-r) + \frac{1}{2}I_{\text{cm}}\omega^2 + \frac{1}{2}mv^2$$

For a sphere, $I_{\text{cm}} = 2MR^2/5$, and $\omega R = v$, so

$$E_B = mg(2R-r) + \frac{1}{2}\left(\frac{2mR^2}{5}\right)\left(\frac{v}{R}\right)^2 + \frac{1}{2}mv^2 = mg(2R-r) + \frac{7}{10}mv^2$$

Equating these two energies (by conservation of energy), we can find the minimum height h.
EVALUATE Equating the E_A and E_B gives

$$mg(h+r) = mg(2R-r) + \frac{7}{10}mv^2$$

Inserting the minimum value for v^2 from above gives the minimum height h:

$$mg(h+r) \geq mg(2R-r) + \frac{7}{10}mg(R-r)$$
$$h \geq 2.7(R-r)$$

ASSESS If we let $r \to 0$, then we would have $mgh' = mg(2R) + mv^2/2$ from conservation of total mechanical energy and $v^2 \geq gR$ from Newton's second law. Combing these gives $h' \geq 2.5R$, so we see that $h' < h$, even if we insert $r = 0$ in the result for h. This is because the rotational inertia of a finite-sized ball consumes some mechanical energy, so any finite-sized ball will have to start higher than a ball with zero size (a point particle).

69. **INTERPRET** In this problem we are given a disk with nonuniform mass density, and asked to find its total mass and rotational inertia. We will therefore need to use the integral expression to calculate the rotational inertia.
DEVELOP As mass elements, choose thin rings of width dr and radius r (as in Example 10.7) so that

$$dm = \rho(r)\,dV = \left(\frac{\rho_0 r}{R}\right)2\pi rw\,dr = \frac{2\pi\rho_0 w}{R}r^2 dr$$

The total mass is $M = \int_0^R dm$ and the rotational inertia about the disk axis is $I = \int_0^R r^2 dm$ (see Equation 10.13).
EVALUATE **(a)** The disk's total mass is

$$M = \int_0^R dm = \frac{2\pi\rho_0 w}{R}\int_0^R r^2 dr = \frac{2\pi\rho_0 wR^2}{3}$$

(b) The disk's rotational inertia about a perpendicular axis through its center is

$$I=\int_0^R r^2dm=\frac{2\pi\rho_0 w}{R}\int_0^R r^4dr=\frac{2\pi\rho_0 wR^4}{5}=\frac{3}{5}\left(\frac{2\pi\rho_0 wR^2}{3}\right)R^2=\frac{3}{5}MR^2$$

ASSESS Our result for I is intermediary between a disk of uniform density and a ring; $\frac{1}{2}MR^2 < I < MR^2,$ if expressed in terms of the total mass M, but is less than a disk of uniform density ρ_0; $I<\frac{1}{2}\rho_0\pi R^4 w,$ because ρ_0 is the maximum density.

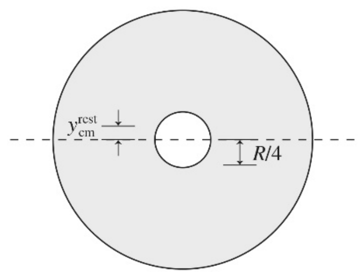
70. **INTERPRET** As the hint implies, this problem involves conservation of total mechanical energy. We can use this principle to relate the angular speed of the unbalanced disk when the void is at the bottom to that when the void is at the top.

DEVELOP Find the center of mass of the disk and compare its total mechanical energy when the void is at the top and when it is at the bottom. The vertical position of the center of mass of the complete disk (i.e., with no void, see figure below) is

$$0=\frac{m_{\rm void}y_{\rm cm}^{\rm void}+m_{\rm rem}y_{\rm cm}^{\rm rem}}{M}$$

so the center of mass of the solid piece (the "remainder") is

$$y_{\rm cm}^{\rm rem}=-\frac{m_{\rm void}y_{\rm cm}^{\rm void}}{m_{\rm rem}}$$



From Problem 10.65, we know that $m_{\rm void}/m_{\rm rem}$ = 1/15, and $y_{\rm cm}^{\rm void}=R/4$, so $y_{\rm cm}^{\rm rem}=-R/60$. Equating the total mechanical energy when the void is at the top and when it is at the bottom gives

$$K_{\rm bot}+U_{\rm bot}=K_{\rm top}+U_{\rm top}$$
$$\tfrac{1}{2}I\omega_{\rm min}^2=\tfrac{1}{2}I\omega_{\rm max}^2-(U_{\rm bot}-U_{\rm top})$$

where the minimum angular speed occurs when the void is at the bottom and the maximum angular speed occurs when the void is at the top. From Problem 10.65, we know that $I = 253MR^2/512$, where M is the mass of the complete disk ($M = m_{\rm void} + m_{\rm rem}$). The difference in height of the center of mass between positions with the hole at the bottom and at the top is $\Delta y_{\rm cm} = R/30$, so the change in potential energy is

$$U_{\rm bot}-U_{\rm top}=m_{\rm rem}g\ \Delta y_{\rm cm}=(15/16)Mg\,(R/30)=MgR/32$$

With these results we can solve for the minimum angular speed in terms of the maximum angular speed.

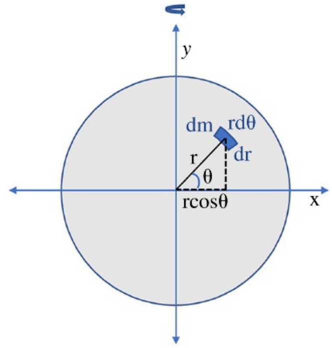
EVALUATE Solving the system of equations derived above for $\omega_{\rm min}$ gives

$$\tfrac{1}{2}I\omega_{\rm min}^2=\tfrac{1}{2}I\omega_{\rm max}^2-(U_{\rm bot}-U_{\rm top})$$
$$\omega_{\rm min}^2=\omega_{\rm max}^2-\frac{2MgR/32}{I}=\omega_{\rm max}^2-\frac{MgR}{16}\left(\frac{512}{253MR^2}\right)$$
$$\omega_{\rm min}=\pm\sqrt{\omega_{\rm max}^2-\frac{32g}{253R}}$$

ASSESS The positive and negative signs in the result reflect the fact that the expression cannot differentiate between clockwise and counter-clockwise rotation—both directions are equally valid. Notice that if $R\to\infty$, then $\omega_{\rm min} = \omega_{\rm max}$ because the void becomes negligible.

71. **INTERPRET** In this problem we are asked to calculate the rotational inertia of a disk about an axis coinciding with a diameter. We will do this by treating it as continuous matter, much like it's done in Examples 10.6 and 10.7.

DEVELOP Like in most of these problems we want to exploit the symmetry of the object to make the calculation easier. Choosing the axis of rotation to be the *y*-axis as is shown in the figure below, we note it's convenient to define the distance to the infinitesimal mass element dm as $r\cos\theta$, and integrate over both r and θ.



Furthermore, we can break the disk into four pieces, calculate the rotational inertia for the portion located in the first quadrant, and multiply by four to obtain the total rotational inertia.

EVALUATE Evaluating Equation 10.13 for the first quadrant and multiplying by four gives

$$I_{1Q} = \int r^2 dm = \int_0^{\frac{\pi}{2}}\int_0^R (r\cos\theta)^2\left(\frac{Mr}{\pi R^2}\right)dr\,d\theta$$

$$I_{1Q} = \frac{M}{\pi R^2}\int_0^{\frac{\pi}{2}}\cos^2\theta d\theta\int_0^R r^3 dr = \frac{M}{\pi R^2}\left[\frac{1}{2}\left(\theta+\frac{\sin 2\theta}{2}\right)\Bigg|_0^{\frac{\pi}{2}}\right]\left[\frac{r^4}{4}\Bigg|_0^R\right] = \frac{M}{\pi R^2}\left(\frac{\pi}{4}\right)\left(\frac{R^4}{4}\right)$$

$$I_{1Q} = \frac{MR^2}{16} \rightarrow I_T = 4I_{1Q} = \frac{MR^2}{4}$$

ASSESS We can also obtain this rotational inertia by using the perpendicular axis theorem. If a planar object has rotational symmetry such that I_x and I_y are equal, then the perpendicular axis theorem provides the useful relationship: $I_z = 2I_x = 2I_y$. For the coordinates we have chosen, that means I_z is the rotational inertia of the disk about an axis through its center and perpendicular to the disk, which is calculated in Example 10.7 ($MR^2/2$). Dividing this by two replicates our result.

72. **INTERPRET** In this problem we are asked to derive an expression for the rate of change of the total kinetic energy experienced by an accelerating car. We want to include the car's acceleration in the expression, so we should apply use force and torque to describe the power.

DEVELOP We are told to include the mass of the car M, the mass of each wheel m, the car's acceleration a, and the car's speed v. Since the wheels can be considered disks, we also know their rotational inertia is given by $I = mR^2/2$. To obtain the power, we will take the derivative of the total kinetic energy (translational and rotational). Once we have derived the expression, we can consider the scenarios described to determine the percentages by which the power requirement decreases.

EVALUATE The power applied to the car and each wheel is given by

$$P = \frac{d}{dt}\left(K_{\text{trans}} + K_{\text{rot}}\right) = \frac{d}{dt}\left(\frac{1}{2}Mv^2 + 4\frac{1}{2}I\omega^2\right)$$

$$P = Mv\frac{dv}{dt} + 4I\omega\frac{d\omega}{dt} = Mva + 4I\omega\alpha$$

$$P = Mva + 4\left(\frac{mR^2}{2}\right)\left(\frac{v}{R}\right)\left(\frac{a}{R}\right) = Mva + 2mva$$

For a car of total mass $M = 1780\,\text{kg}$, including four $15.8\,\text{kg}$ wheels, we now consider how reducing the car's total mass by $10\,\text{kg}$ affects the power required if we remove it from non-rolling parts or the wheels. We calculate this change as $\left|\dfrac{P_f - P_i}{P_i}\right|$, noting that it does not depend on v or a. The percentages by which the power requirement decreases in the two scenarios are

$$\left|\frac{P_f - P_i}{P_i}\right| = \left|\frac{\left((M-10\,kg)+2m\right)-(M+2m)}{M+2m}\right| = \left|-\frac{10\,\text{kg}}{\left(1780+2(15.8)\right)\text{kg}}\right| = 0.552\%$$

$$\left|\frac{P_f - P_i}{P_i}\right| = \left|\frac{\left((M-10\,kg)+2(m-2.5\,\text{kg})\right)-(M+2m)}{M+2m}\right| = \left|-\frac{15\,\text{kg}}{\left(1780+2(15.8)\right)\text{kg}}\right| = 0.828\%$$

In the first scenario, we have subtracted the 10 kg from the non-rolling parts, so we see that impact the total mass M . In the second scenario, we are subtracting 10 kg from the wheels, so each individual wheel's mass m is reduced by 2.5 kg and the total mass M is still decreased by 10 kg.

ASSESS Since the torque required to rotate the wheels contributes to the overall power, reducing the weight of the wheels will result in a larger decrease in power than removing that mass from non-rolling parts of the car.

73. **INTERPRET** This problem is an exercise in calculating the rotational inertia of an object (in this case, a right-circular cone). Because the mass is distributed continuously throughout the cone, we will apply the integral formula Equation 10.13 to find the rotational inertia.

DEVELOP Divide the cone into circular slices parallel to the base of the cone, and integrate over all these slices to find the total rotational inertia of the cone. The height of the cone is h and the base radius is R, so the radius of each slice is $r = Rx/h$, where x is the distance from the apex. The volume of the cone is $V = Ah/3$, where A is the area of the base, $A = \pi R^2$, so $V = \pi R^3 h/3$. The volume of each disk-shaped slice is $dV = \pi r^2 dx$. The cone has uniform mass density M/V, so each disk has mass $dm = MdV/V = 3M(\pi r^2 dx)/(\pi r^2 h)$. From Table 10.2, the rotational inertia of each disk is $dI = r^2 dm/2$.

EVALUATE Evaluating the integral gives

$$I = \int_0^h dI = \frac{1}{2}\int_0^h r^2 dm = \frac{1}{2}\int_0^h \left(\frac{\cancel{R}x}{h}\right)^2 \frac{3M}{\cancel{\pi}\cancel{R^2}h}\cancel{\pi}R^2 d = \frac{3M}{2h^3}\int_0^h x^2\left(\frac{Rx}{h}\right)^2 dx$$

$$I = \frac{3MR^2}{2h^5}\int_0^h x^4 dx = \frac{3MR^2}{2\cancel{h^5}}\left[\frac{1}{5}\cancel{h^5}\right] = \frac{3}{10}MR^2$$

ASSESS The units are correct. The value of I is less than that of a cylinder, since a greater proportion of the mass is concentrated along the axis of the cone.

74. **INTERPRET** We are asked to find the rotational inertia of a thick ring with inner and outer radii R_1 and R_2 , respectively. The mass distribution is continuous, so we need to do an integral.

DEVELOP For a thick ring, the ring-shaped mass elements used in Example 10.7 have mass

$$dm = \sigma dA = \frac{M}{\pi\left(R_2^2 - R_1^2\right)}2\pi r dr$$

where $\sigma = M/A$ is the mass density (units: kg/m^2). Note that the ring only extends in radius from R_1 to R_2. The rotational inertia can then be obtained by integrating over

$$I = \int_{R_1}^{R_2} r^2 dm$$

EVALUATE Upon carrying out the integration, the rotational inertia about an axis perpendicular to the ring and through its center is

$$I = \int_{R_1}^{R_2} r^2 dm = M\int_{R_1}^{R_2}\frac{2\pi r^3 dr}{\pi\left(R_2^2 - R_1^2\right)} = \frac{M\left(R_2^4 - R_1^4\right)}{2\left(R_2^2 - R_1^2\right)} = \frac{M}{2}\left(R_1^2 + R_2^2\right)$$

ASSESS To see that the result makes sense, let's consider the following limits: **(i)** $R_1 \to 0$: In this case, we have a disk with radius R_2 and $I = MR_2^2/2$ **(ii)** $R_1 \to R_2$: In this limit, we have a thin ring with $I = MR_2^2$.

75. **INTERPRET** In this problem we are asked to derive an expression for the necessary power applied to the crank so that the bucket is lifted with an upward acceleration. We can treat the power as the rate of change of the total mechanical energy of the crank and bucket system.

DEVELOP This problem is based off a scenario investigated in Example 10.9, so we know the crank rotates a disk with rotational inertia given by $I = MR^2/2$, and lifts a bucket of mass m against the downward pull of gravity. To obtain the power we will take the derivative of the system's mechanical energy, comprised of the bucket's translational kinetic energy and gravitational potential energy, as well as the disk's rotational kinetic energy.

EVALUATE The power that needs to be applied to the crank is given by

$$P = \frac{d}{dt}\left(K_{\text{B trans}} + U_{\text{B}} + K_{\text{D rot}}\right) = \frac{d}{dt}\left(\frac{1}{2}mv^2 + mgh + \frac{1}{2}I\omega^2\right)$$

$$P = mv\frac{dv}{dt} + mg\frac{dh}{dt} + I\omega\frac{d\omega}{dt} = mva + mgv + I\omega\alpha$$

$$P = mva + mgv + \left(\frac{MR^2}{2}\right)\left(\frac{v}{R}\right)\left(\frac{a}{R}\right) = \left(m + \frac{M}{2}\right)va + mgv$$

ASSESS From this expression we can see that the power necessary is independent of the disk radius.

76. **INTERPRET** This problem involves force and torque, which are related by Equation 10.10. We can use this to find the force, given the torque, the distance from the axis at which the torque is applied, and the angle between the force and the vector from the axis of rotation to the point at which the force is applied. Use the definition of torque, and since no angular information is given, assume that the force is applied at 90°.

DEVELOP The force is applied at 90° to the vector from the axis of rotation to the point at which the force is applied, so Equation 10.10 reduces to $\tau = RF$, which we can solve for F.

EVALUATE Inserting the given quantities into the expression for torque and solving for the force F gives

$$F = \frac{\tau}{R} = \frac{12\ \text{kN}\cdot\text{m}}{0.86\ \text{m}} = 13.95\ \text{kN} = 14\ \text{kN}$$

to two significant figures.

ASSESS Since the distance is slightly smaller than 1 meter, the numerical value of the force is slightly bigger than the numerical value of the torque.

77. **INTERPRET** You want to know if a rotating flywheel has as much energy as its manufacturer claims.

DEVELOP The flywheel can be modeled as a ring with rotational inertia $I = MR^2$. Its rotational kinetic energy is $\frac{1}{2}I\omega^2$, from Equation 10.18.

EVALUATE We have to divide the given diameter by 2 to get the radius, and we have to convert the non-SI unit of rpm to rad / s. Following that, the flywheel's kinetic energy is:

$$K_{\text{rot}} = \frac{1}{2}MR^2\omega^2 = \frac{1}{2}(44\ \text{kg})\left(\frac{1}{2}0.34\ \text{cm}\right)^2(30{,}000\ \text{rpm})^2\left(\frac{\frac{2\pi}{60}\text{rad / s}}{1\ \text{rpm}}\right)^2 = 6.3\ \text{MJ}$$

The specs are incorrect. The flywheel's storage capacity is slightly more than half of what the manufacturer claims. However, at 40 kW, or 40 kJ / s, the wheel's energy would be depleted in only $(6.3\ \text{MJ})/(40\ \text{kJ / s}) = 158\ \text{s}$, or only 2.6 minutes—not the 5 minutes the manufacturer claims.

ASSESS A flywheel is like a battery that stores energy as kinetic rotational energy. It has a high rotational inertia and presumably very little friction, so it will spin freely for a long time without slowing down appreciably. When the need arises, the flywheel can be connected to an electric generator, where its rotational energy is converted to electricity.

78. **INTERPRET** We're asked to derive the parallel axis theorem for an object of arbitrary shape.

DEVELOP The law of cosines from Appendix A says that the three sides (A, B, C) of a triangle obey:

$$C^2 = A^2 + B^2 - 2AB\cos\gamma$$

where γ is the angle between line segments A and B.

EVALUATE (a) If we choose θ to be the angle between the vectors $\vec{r}_{\text{cm}}$ and $\vec{h}$, then the law of cosines stipulates

$$r^2 = r_{\text{cm}}^2 + h^2 - 2r_{\text{cm}}h\cos\theta$$

But recall that the scalar product between $\vec{h}$ and $\vec{r}_{\text{cm}}$ is equal to $\vec{h}\cdot\vec{r}_{\text{cm}} = r_{\text{cm}}h\cos\theta$. Therefore, as expected:

$$r^2 = r_{\text{cm}}^2 + h^2 - 2\vec{h}\cdot\vec{r}_{\text{cm}}$$

(b) Plugging the above relation into Equation 10.13 $\left(I = \int r^2 dm\right)$ gives us three separate integrals. The first is the integral of $r_{\text{cm}}^2 dm$, which is the rotational inertia of the mass element dm around the center of mass:

$$\int r_{\text{cm}}^2 dm = I_{\text{cm}}$$

The second term involves h^2, which is constant since it is the square of the distance between the fixed points CM and A. So the integral reduces to an integral over the mass elements, which is just the total mass:

$$\int h^2 dm = h^2 \int dm = Mh^2$$

For the third term, we can again use the fact that $\vec{h}$ is constant to rewrite the integral as

$$\int 2\vec{h}\cdot\vec{r}_{\text{cm}}\, dm = 2\vec{h}\cdot\int \vec{r}_{\text{cm}}\, dm$$

The integral $\int \vec{r}_{\text{cm}}\, dm$ is like the integral in Equation 9.4 for the center of mass. In fact, $\frac{1}{M}\int \vec{r}_{\text{cm}}\, dm$ gives the location of the center of mass in a coordinate system where the origin is already at the center of mass. Since the distance *to* the center of mass *from* the center of mass is zero, the integral $\int \vec{r}_{\text{cm}}\, dm$ must be zero.

ASSESS In summary, for an arbitrary object: $\int r^2 dm = I_{\text{cm}} + Mh^2$, which is the parallel axis theorem from Equation 10.17.

79. **INTERPRET** In this problem we analyze the data from an apparatus that measures rotational inertia of an object. We are asked to verify that the rotational inertia is $2MR^2/5$ for a solid sphere, and $2MR^2/3$ for a hollow sphere.

DEVELOP The problem involves conservation of total mechanical energy, which is composed in this case of rotational kinetic, translational kinetic, and gravitational potential energies. We shall take the bottom of the trajectory to be the zero of gravitational potential energy. The initial mechanical energy of the mass m is $E_{\text{i}} = mgh$. As it descends, both the axel and the drum with a rotational inertia of I_0, and the sphere of mass M, radius R and rotational inertia $I = \beta MR^2$, begin to rotate. Energy conservation implies

$$\begin{aligned} mgh &= \frac{1}{2}I_0\omega^2 + \frac{1}{2}mv^2 + \frac{1}{2}\left(\beta MR^2\right)\omega^2 = \frac{1}{2}I_0\left(\frac{v}{b}\right)^2 + \frac{1}{2}mv^2 + \frac{1}{2}\left(\beta MR^2\right)\left(\frac{v}{b}\right)^2 \\ &= \frac{v^2}{2b^2}\left[(I_0 + mb^2) + \beta MR^2\right] \end{aligned}$$

Suppose it takes t seconds for mass m to reach the ground. We then have $h = \frac{1}{2}at^2 = \frac{1}{2}(at)t = \frac{1}{2}vt$, or $v = 2h/t$. Substituting the expression into the equation above gives

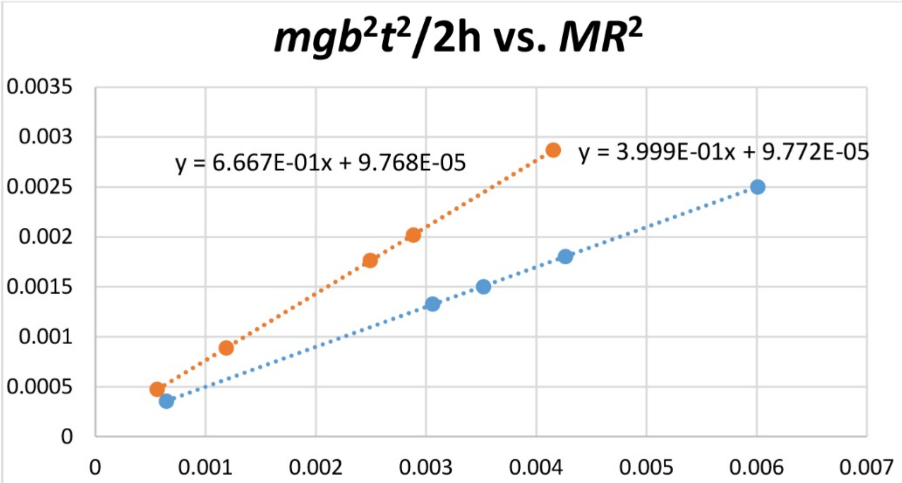
$$mgh = \frac{(2h/t)^2}{2b^2}\left[(I_0 + mb^2) + \beta MR^2\right] = \frac{2h^2}{b^2t^2}\left[(I_0 + mb^2) + \beta MR^2\right]$$

which can be rearranged as

$$\frac{mgb^2t^2}{2h} = \beta\left(MR^2\right) + \left(I_0 + mb^2\right)$$

Thus, plotting $mgb^2t^2/2h$ as a function of MR^2 will result in a straight line with slope β.

Evaluate The plot is shown below. We obtain two straight lines, one with a slope of 0.67 or 2/3 for hollow spheres, and the other with a slope of about 0.40 or 2/5 for solid spheres. The intercept is $I_0 + mb^2 = 9.77\times10^{-5}\ \text{kg}\cdot\text{m}^2$. With $mb^2 = (0.0778\ \text{kg})(0.025\ \text{m})^2 = 4.86\times10^{-5}\ \text{kg}\cdot\text{m}^2$, we find $I_0 = 4.91\times10^{-5}\ \text{kg}\cdot\text{m}^2$.



Assess The fall time is greater for a hollow sphere compared to a solid sphere of the same mass and radius.

80. Interpret We must compare two centrifuges with slightly different designs.

Develop We're told that the two centrifuges have the same mass and radius. But design A looks like a thin ring, while design B looks like a flat disk.

Evaluate Design A should have approximately a rotational inertia of $I_A \approx MR^2$, compared to the design B with $I_B \approx \frac{1}{2}MR^2$.

The answer is (a).

Assess The rotational kinetic energy is proportional to rotational inertia $\left(K_{\text{rot}} = \frac{1}{2}I\omega^2\right)$. Therefore, it will take twice the work $(W = \Delta K)$ to spin up centrifuge A to the same rotational speed as centrifuge B.

81. Interpret We must compare two centrifuges with slightly different designs.

Develop If design A were made thicker, it would start to resemble a hollow cylinder more than a thin ring. If design B were made thicker, it would start to resemble a solid cylinder more than a flat disk.

Evaluate The rotational inertia is the same for both rings and hollow cylinders, as well as for solid cylinders and disks.

The answer is (a).

Assess Solid and hollow cylinders are symmetric about their central axes, so their rotational inertias do not depend on whether they're flat or thick, as long as the mass stays the same.

82. Interpret We must compare two centrifuges with slightly different designs.

Develop The sample tubes do not rest vertically, but instead tilt outward. The bottom of the tubes are therefore at a radius greater than the radius of the centrifuges themselves.

Evaluate If the tubes are made longer, the bottom of the tubes will extend to a greater radius, so the rotational inertia will increase.

The answer is (b).

Assess Some centrifuges have a fixed angle (e.g., 45°), at which the tubes are placed. Others have a hinge that lets the tubes swing out when the device starts to turn.

83. Interpret We must compare two centrifuges with slightly different designs.

Develop When the centrifuges are spinning, the samples in the tubes are in uniform circular motion, so there must be a centripetal force acting on them.

Evaluate The centripetal force points inward.

The answer is (b).

ASSESS The walls of the sample tube provide a normal force that acts as the centripetal force on the sample. Since the sample material is "falling" toward the bottom of the tubes, the net effect is like gravity. As such, denser material will settle farther down in the tubes than less dense material. This separation is exactly what a centrifuge is designed for.

84. **INTERPRET** We must compare two centrifuges with slightly different designs.

DEVELOP For both designs, the rotational inertia is proportional to the mass times the radius squared: $I \propto MR^2$.

EVALUATE Doubling both the mass and the radius will change the rotational inertia by

$$\frac{I'}{I} = \frac{(2M)(2R)^2}{MR^2} = 8$$

The answer is (c).

ASSESS For a given rotational speed, the centripetal force is proportional to the radius: $F = m\omega^2 r$. So making the centrifuge bigger will presumably improve its ability to separate materials by their density.