Modelling orbital decay of Hot Jupiters

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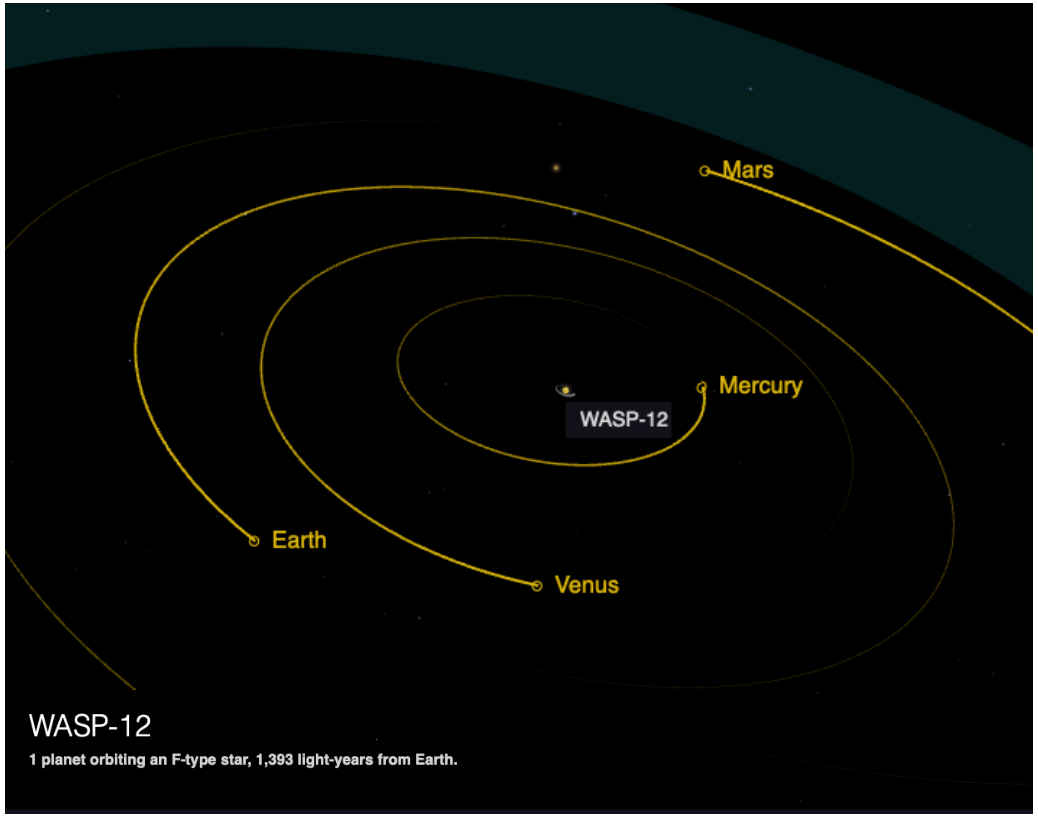
[Nasa]

Introduction



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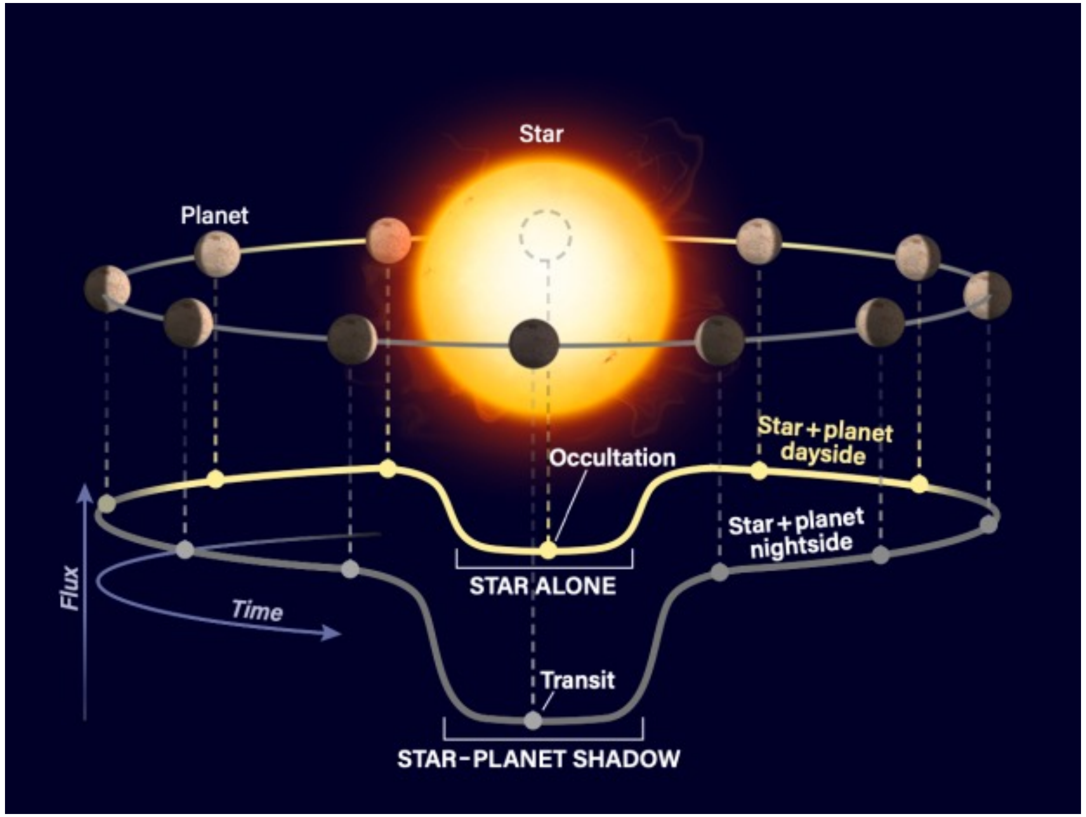


[Astronomy Magazine]

- What are **hot Jupiters**?
- What is the **transit method**?
- What are **transit timing variations**?

Motivations + Goals

- **Motivations** (why are TTVs interesting?):
  - TTVs can be a sign of **orbital decay**. Understanding how and why planets decay helps inform and test planetary formation theories.
  - **Tidal decay** is a consequence of stellar and planetary composition and structure. Decay rates could tell you something about the star and planet's structure.
- **Goals** of my project:
  - Identify hot Jupiters with **statistically significant** transit timing variations (TTVs).
    - Which are good candidates for *follow up study*
    - What are the *physical causes* of TTV in those systems
  - For those systems undergoing tidal decay, measure and classify **the strength of tidal dissipation** by stellar age and position on the main sequence.
    - **Test** and **inform** models of stellar structure

Last time

Planet Name	Orbit Semi-Major Axis [au]	Planet Radius [Earth Radius]	Planet Mass or Mass*sin(i) [Earth Mass]	Stellar Mass [Solar mass]	Stellar Radius [Solar Radius]	Data show Transit Timing Variations	Eccentricity	-dP/dt [ms/year]	-dP/dE [ms/epoch]
OGLE-TR-56 b	0.02383	15.278	441.77000	1.17	1.74	0	0.6700	178.005454	0.591036
KELT-1 b	0.02466	12.442	8654.15000	1.32	1.46	0	0.0099	131.921014	0.440043
Kepler-718 b	0.03220	16.560	41600.00000	1.18	1.30	0	0.0000	104.449640	0.587307
BD+20 2457 b	1.05000	11.600	17668.16970	10.83	71.02	0	0.1500	84.247629	87.624458
WASP-18 b	0.02024	13.899	3241.84975	1.29	1.32	1	0.0051	81.929558	0.211323
NGC 2682 Sand 364 b	0.53000	12.700	2126.28270	9.06	39.59	0	0.3500	41.394288	13.803010
Kepler-493 b	0.04390	15.130	37500.00000	1.36	1.54	0	0.0000	40.460531	0.332982
HATS-70 b	0.03632	15.513	4100.00700	1.78	1.88	0	0.1800	34.585177	0.178918
K2-22 b	0.00900	2.300	444.96200	0.60	0.58	0	0.1900	34.490373	0.036009
WASP-103 b	0.01985	17.127	473.54700	1.22	1.44	0	0.1500	32.298133	0.081899
Kepler-91 b	0.07310	15.323	257.44230	1.31	6.30	0	0.0519	26.801132	0.458672
KELT-16 b	0.02044	15.861	874.03250	1.21	1.36	0	0.0000	26.018995	0.069075
GPX-1 b	0.03380	16.477	6261.21962	1.68	1.56	0	0.0000	21.559931	0.103049
HATS-18 b	0.01740	14.897	629.14449	1.04	1.02	0	0.1660	19.019742	0.043659
WASP-12 b	0.02320	21.712	465.62095	1.43	1.66	1	0.0447	17.959839	0.053703
TYC 4282-00605-1 b	0.42200	12.400	3426.20740	0.97	16.21	0	0.2800	17.274872	4.805727
Kepler-1658 b	0.05440	11.994	1868.84040	1.45	2.89	0	0.0628	15.911431	0.167806
KELT-9 b	0.03462	21.196	915.35040	2.52	2.36	0	0.0000	14.769635	0.059933
HD 4760 b	1.14000	12.300	4417.83700	1.05	42.40	0	0.2300	14.506742	17.249113
Kepler-471 b	0.06260	14.960	37000.00000	1.49	1.80	0	0.0000	13.482317	0.185215

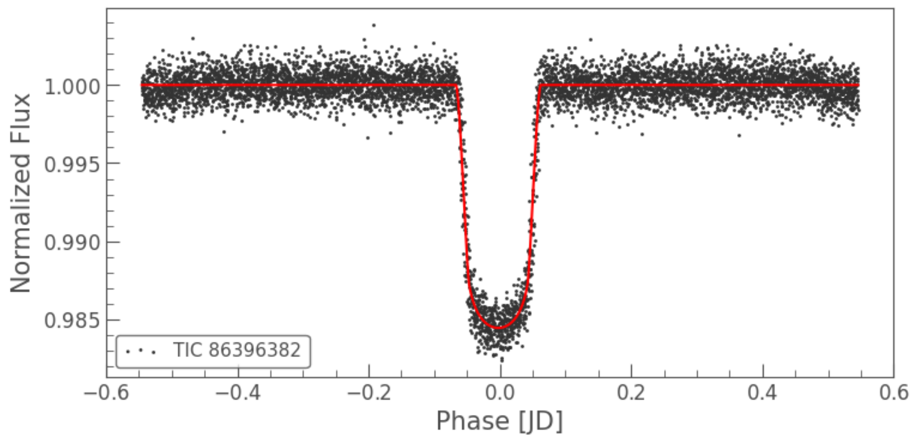
- Used **tidal decay theory** to rank planets by predicted orbital decay rate
- Fitted an orbital decay model to TESS light-curve data for WASP-12b
- The fitted decay rate was similar to the predicted!
  - `(0.08 vs 0.05 ms/epoch)`

This time…

- I'm going to focus on the **observations** and try and fit decay rates for many planets.
  - Most importantly, I'm also going to infer the **uncertainty** in fitted decay rates.
  - We need this if we want to make some assessment of **statistical significance**
- Last time I used light curves and fit my own transits, but to save time I've assembled a dataset of already-fitted transits:
  - **Exoplanet Transit Database**
  - ExoClock
  - Kepler/TESS
  - My own measurements?
- Built a rigorous statistical pipeline for modelling decay rates en masse.

Why is this a hard problem?

- Uncertainty in the transit time measurements is **large**
  - Fundamental lower limit ~ sampling rate
  - Even if you could measure individual photons there's a lower limit described by photon statistics! [Holman & Murray, 2004]
  - Lots of amateur data
  - Fitted with different modelling pipelines
  - Reported uncertainties may be optimistic
  - Some observations may not even be real transits!
- Often **of the same order** as the timing variation we're trying to measure
- Easier for orbital decay
  - Longer baseline = larger timing variation
  - Change in period per time isn't even linear



We're talking about things that vary **milliseconds** per epoch and this scale is in **days**!

But decay compounds with time $TTV \approx \frac{1}{2}\frac{dP}{dE}\frac{t^2}{P_0^2}{\sim}300s$ for WASP-12b over 10 years

Recap: Bayesian statistics + MCMC

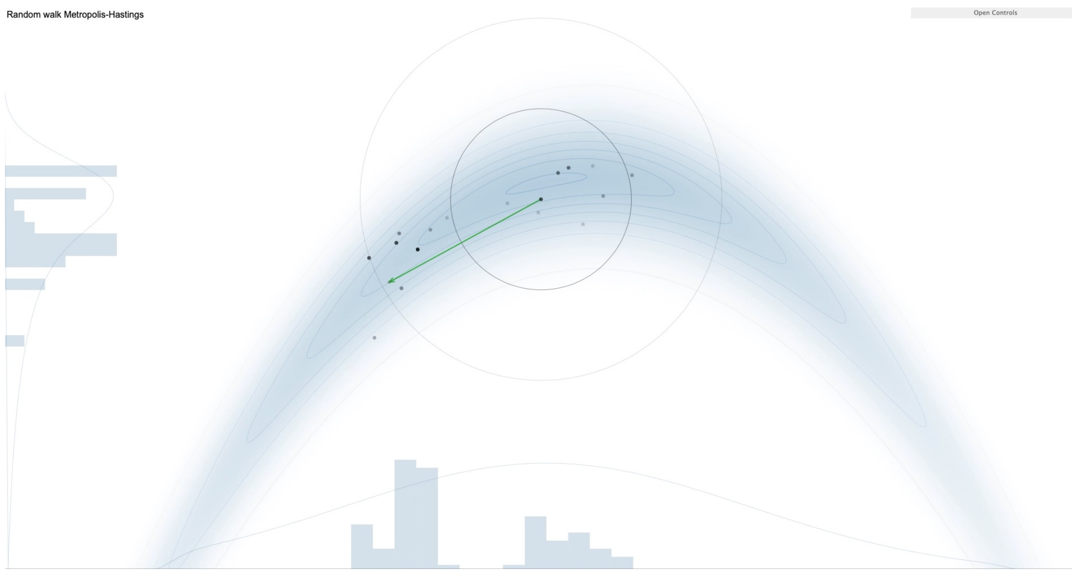
“Quadratic ephemeris” decay rate model:

$$y_i = T_0 + P_0 E + \frac{1}{2}\frac{dP}{dE}E^2 + \epsilon_i$$

- y_i = observed transit time
- T_0 = reference transit time
- P_0 = reference period at first transit
- $\frac{dP}{dE}$ = decay rate per orbit (epoch)
- ϵ_i = measurement error term

Bayes’ rule:

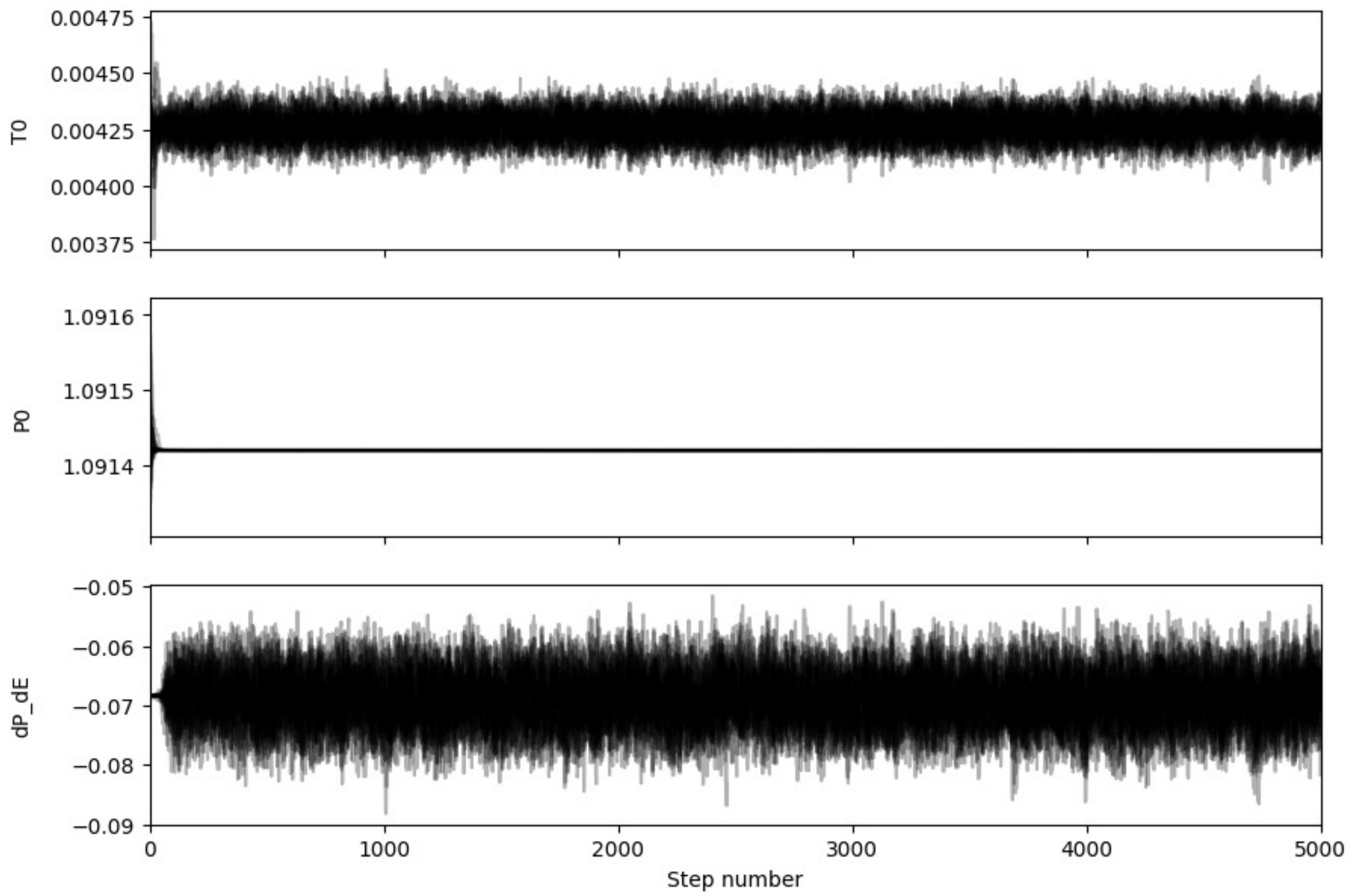
Prior	$P(\boldsymbol{\beta})$	$N(\mathbf{0}, \infty\boldsymbol{I})$
Likelihood	$P(\boldsymbol{y}\vert\boldsymbol{\beta})$	$N\left(\left\{T_0+P_0E_i+\frac{1}{2}\frac{dP}{dE}E_i^2\right\}_{i=1}^{N}, \sigma_i^2\boldsymbol{I}\right)$
Posterior	$P(\boldsymbol{\beta}\vert\boldsymbol{y}) \propto P(\boldsymbol{y}\vert\boldsymbol{\beta})P(\boldsymbol{\beta})$	**?**



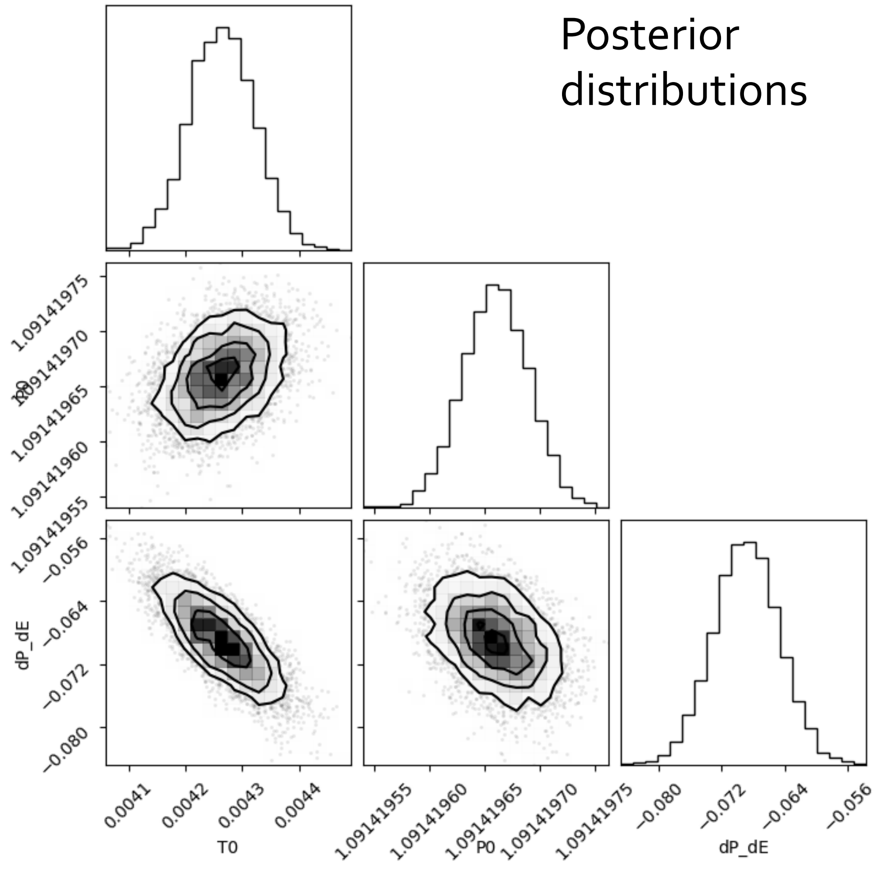
Demo of an MCMC using Metropolis-Hastings sampling

Source: http://chi-feng.github.io/mcmc-demo/

MCMC example: WASP-12b

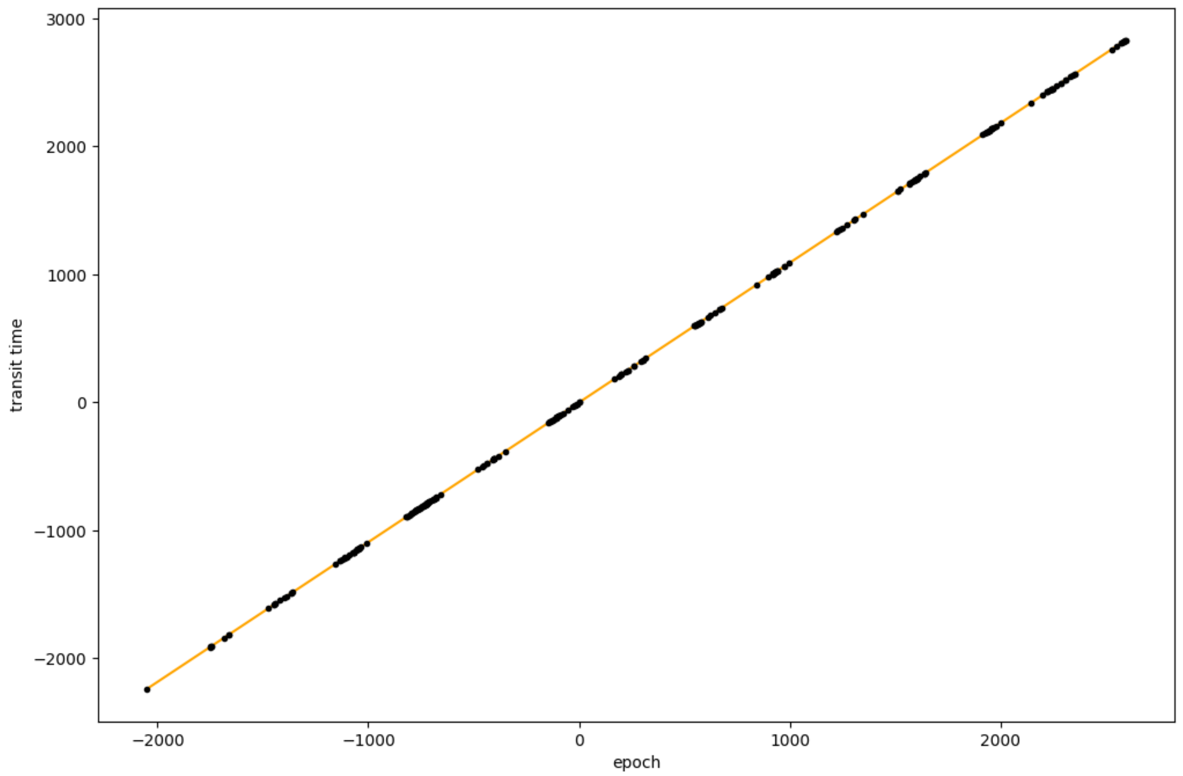


Parameter samples

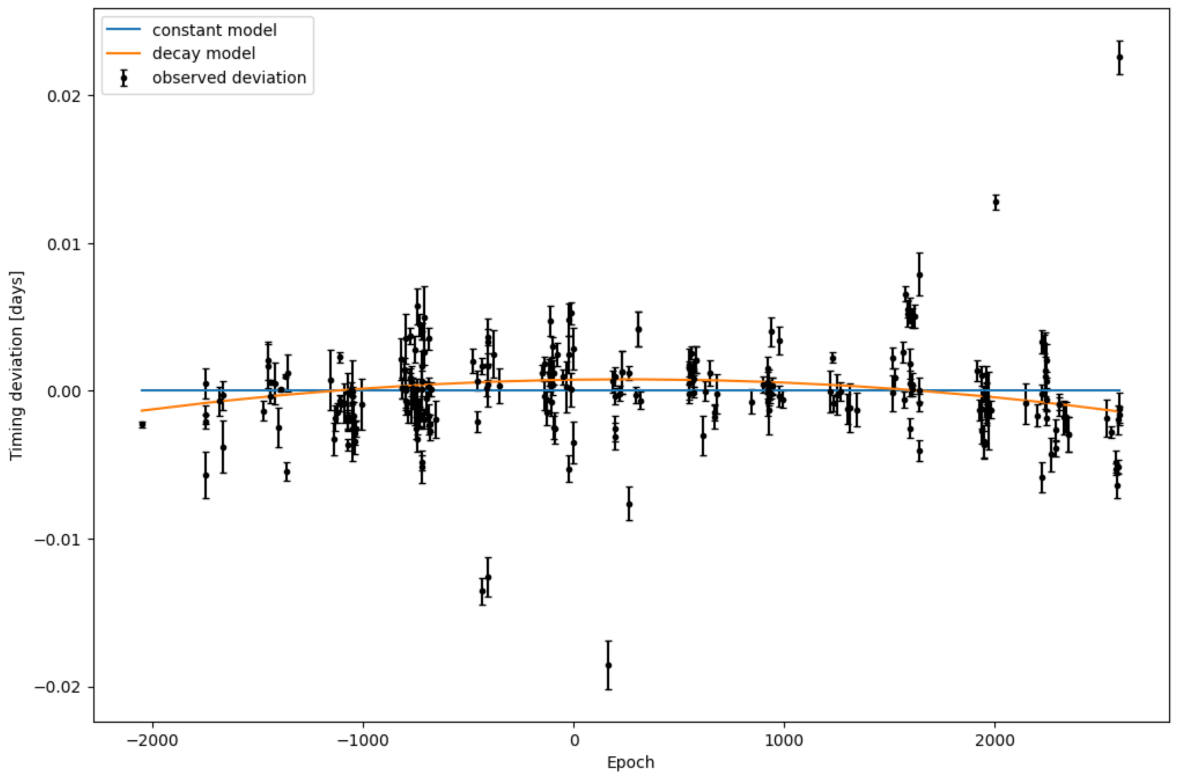


Posterior distributions

MCMC example: WASP-12b

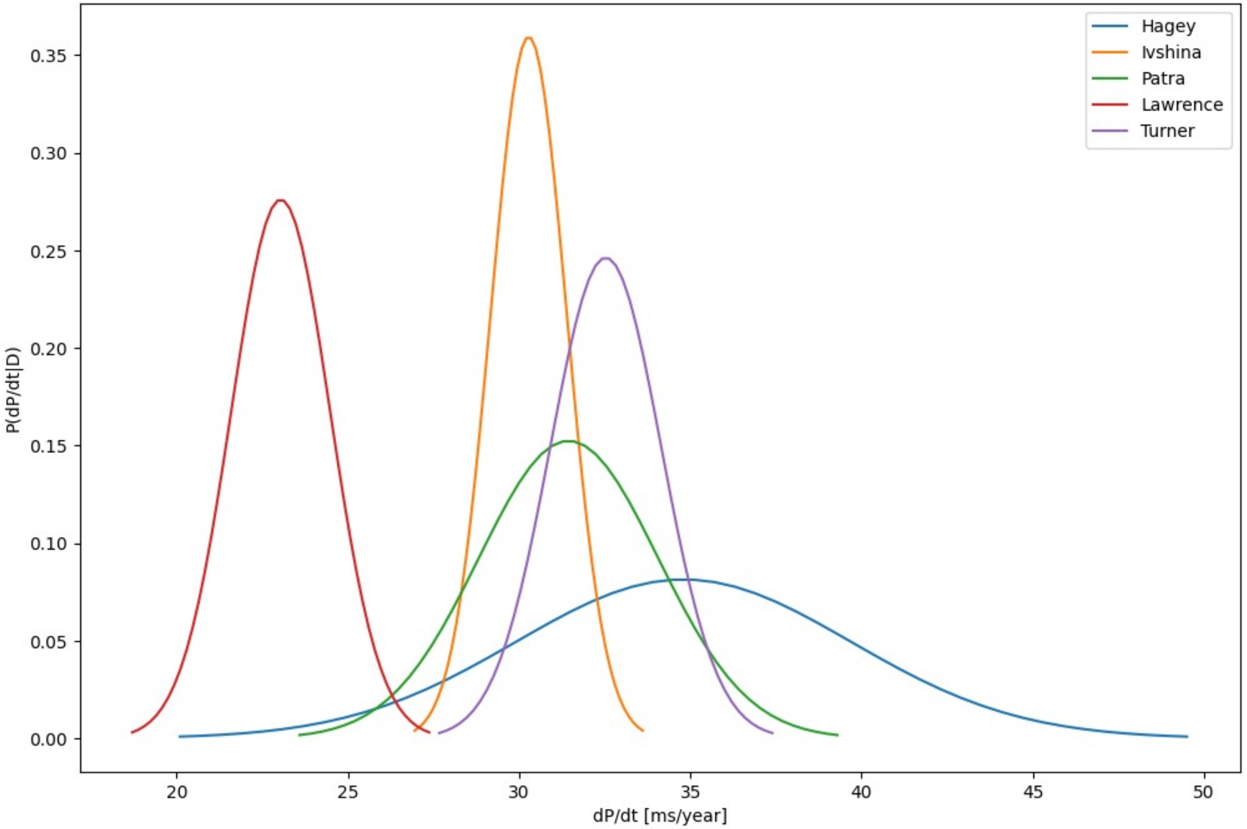


Resulting model fit



It's easier to see the non-linearity when we subtract a constant period model aka *observed-computed* aka **TTV**

What have others done?



Fairly different results 🤔

Paper	Outlier detection method	Fitting method
Ivshina & Winn 2022	• Fit a constant period model and discard datapoints more than >130s away • Added a variable σ_0 term to describe systematic errors	MCMC, quadratic ephemeris model + systematic error
Hagey 2022	• Fit a constant period model and compute the residuals • Fit a decay model, then discard datapoints more than $>3\sigma_{residuals}$ away	MCMC, quadratic ephemeris model
Patra 2022	• Visual inspection only	MCMC, quadratic ephemeris model
Turner 2020	• None mentioned	MCMC, quadratic ephemeris model

How can we improve this?

Problem

Choice of data affects results

A larger dataset should only improve your uncertainty! In fact, if you know the measurement errors exactly, uncertainty only depends on the number of measurements you make.

Parametric outlier detection can introduce a false signal

But we still want to be able to exclude fake transits…

Excluding data-points because they don't agree with our physical assumptions results in a better fit

Some transits aren't fake but appear to be "wrong"

Reported measurement errors may be wrong

Outlier detection is very binary - what if the datapoint does correctly identify a transit, but its reported error is inaccurate. A naïve model might exclude it for fear of putting too much weight on it in the resulting fit. In this case outlier detection is a bit of a blunt tool.

Solution

Keep as much data as possible. Only exclude datapoints which aren't real transits or are labelled with an incorrect epoch. Everything else contains information.

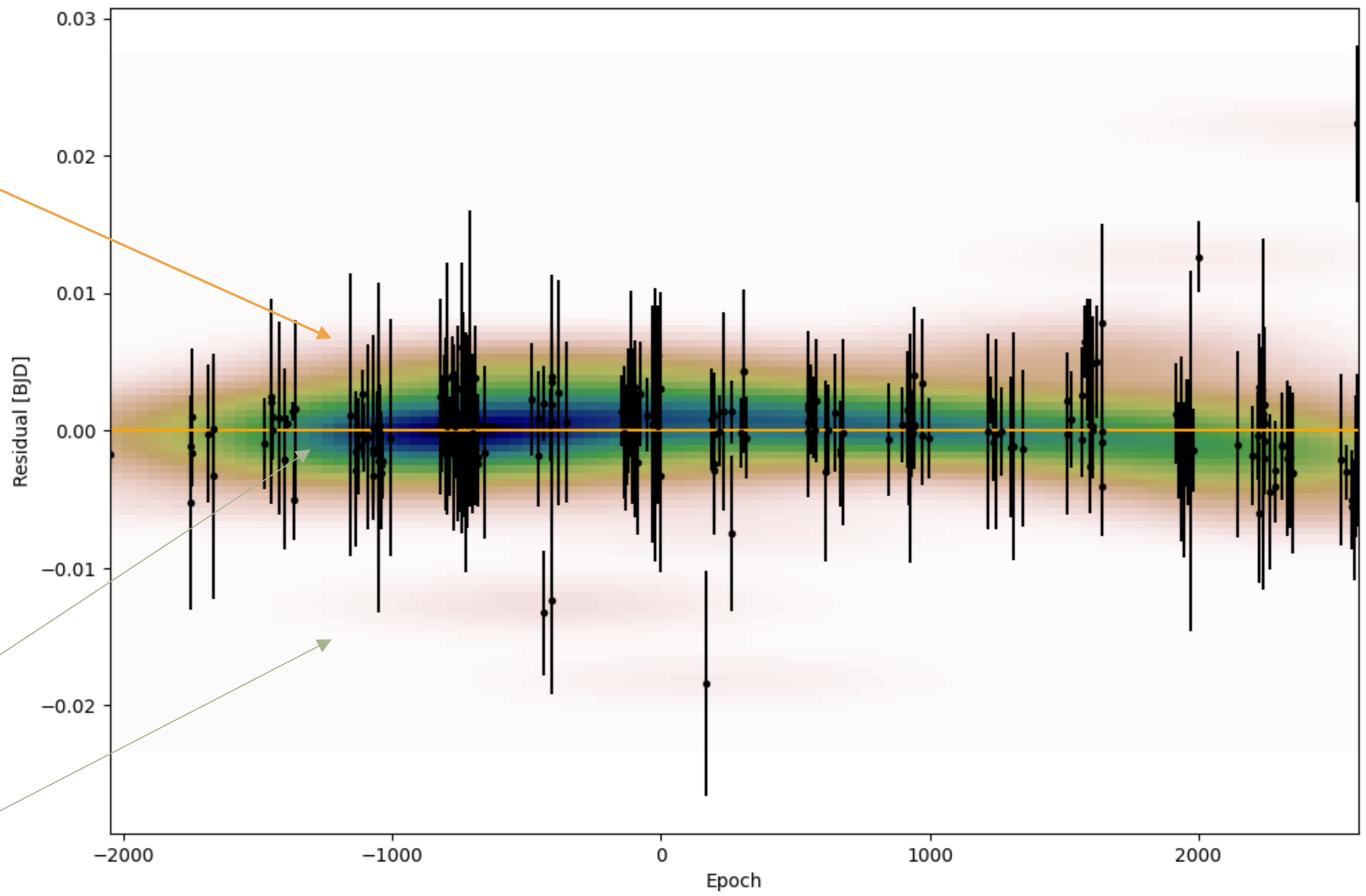
Non-parametric outlier detection.

If you have a strong belief in a particular class of models being correct, that view is better reflected as a **prior**, not by simply excluding them.

Instead, we should **let the model learn what the true measurement uncertainties are**.

Non-parametric outlier detection

"Smudging" of datapoints produces an empirical probability distribution



Higher probability

Lower probability

Bayesian multivariate regression

Rewrite the model as a linear regression:

$$y_i = T_0 + P_0 E_i + \frac{1}{2}\frac{dP}{dE}E_i^2 + \epsilon_i$$

$$y_i = \left(1, E_i, \frac{1}{2}E_i^2\right)\begin{pmatrix} T_0 \\ P_0 \\ \frac{dP}{dE} \end{pmatrix} + \varepsilon_i$$

$$= x_i^\top \beta + \varepsilon_i$$

$$\Rightarrow y = X\beta + \underline{\varepsilon}$$

Errors are not necessarily IID!

$$\text{where } \sum \sim N(0, \Sigma)$$

Likelihood becomes:

$$P(y \mid \beta, \Sigma) = N(X\beta, \Sigma)$$

$$\propto |\Sigma|^{-\frac{1}{2}} \exp\left(-\frac{1}{2}(y - X\beta)^\top \Sigma^{-1}(y - X\beta)\right)$$

$$= \underbrace{|\Sigma|^{-\frac{1}{2}} \exp\left(-\frac{1}{2}\mathrm{tr}\left((y - X\hat{\beta})(y - X\hat{\beta})^\top \Sigma^{-1}\right)\right)}_{\mathrm{IW}(\dots)} \underbrace{\exp\left(-\frac{1}{2}(\beta - \hat{\beta})^\tau X^\top \Sigma^{-1} X(\beta - \hat{\beta})\right)}_{N(\dots)}$$

Conjugate prior:

$$P(\Sigma) = IW(S_0, V_0)$$

$$P(B \mid \Sigma) = N\left(\beta_0, K_0^{-1}\left(X^\top \Sigma^{-1} X\right)^{-1}\right)$$

$$P(\beta, \Sigma) = NIW(\beta_0, K_0, S_0, V_0)$$

Prior parameters

Controls our belief in the initial "prior" parameters

Reported measurement errors

Bayesian multivariate regression: posterior

$$P(\beta,\Sigma \mid y) \propto P(y \mid \beta,\Sigma)P(\beta,\Sigma)$$
$$= NIW(\beta_y, K_y, S_y, V_y)$$
$$P(\Sigma \mid y) = IW(S_y, V_y)$$
$$P(\beta \mid y) = T\left(\beta_y, \frac{K_y^{-1}\left(X^\top S_y^{-1} X\right)^{-1}}{\nu+1-P}, v+1-P\right)$$

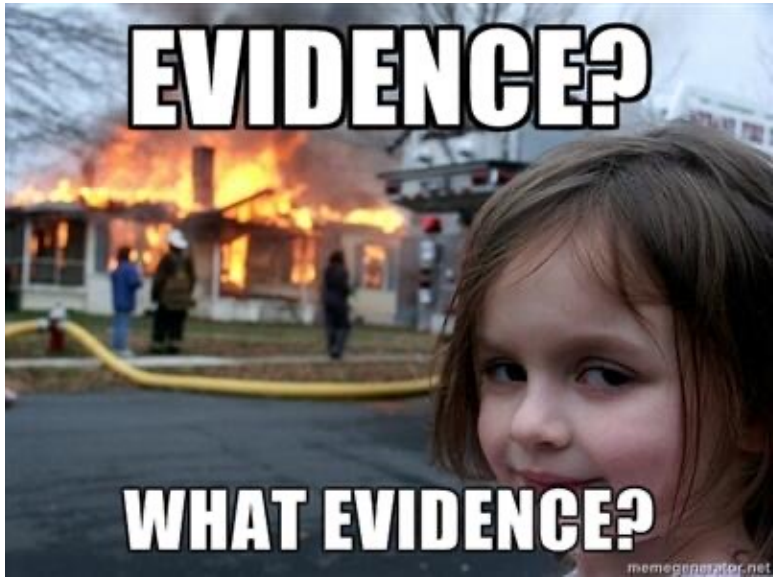
Uncertainty in the parameters is constrained by what we now think the measurement errors were

If we strongly believe the prior, we prefer to add error to the measurements to compensate for a bad fit

$$\beta_y = (k_0 + I)^{-1}\left(k_0\beta_0 + \hat{\beta}\right)$$

$$S_y = \underbrace{S_0}_{\text{Reported errors}} + \underbrace{(y - X\hat{\beta})(y - X\hat{\beta})^\top}_{\text{Empirical scatter matrix}} + \underbrace{X\left(k_0^{-1} + I\right)^{-1}\left(\hat{\beta}-\beta_0\right)\left(\hat{\beta}-\beta_0\right)^T X^T}_{\text{Virtual scatter matrix}}$$

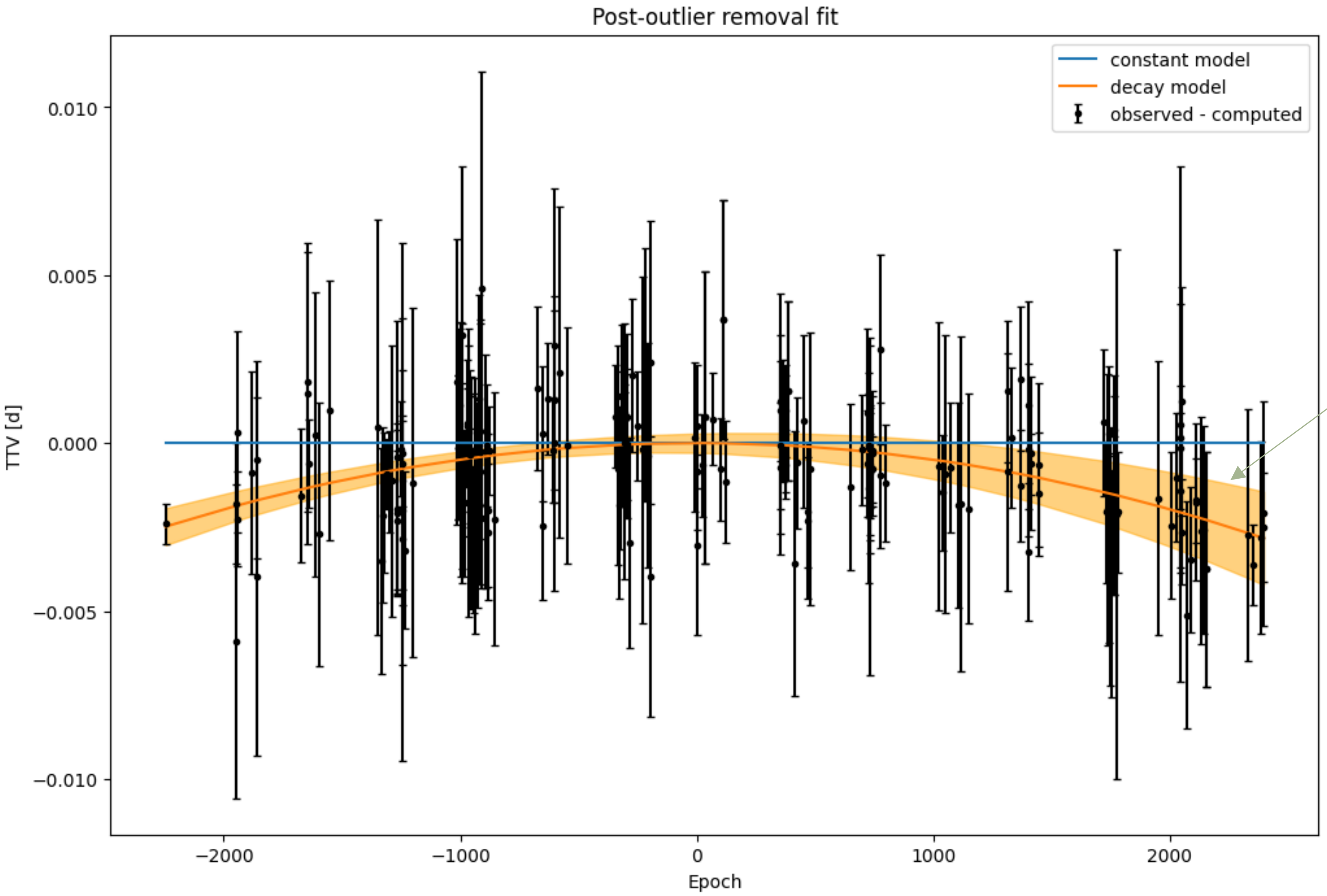
Some other issues with MCMC



The Bayesian model evidence describes the probability that the data was generated by your model (regardless of the values of the parameters of that model)

Bayesian multivariate regression: WASP-12b

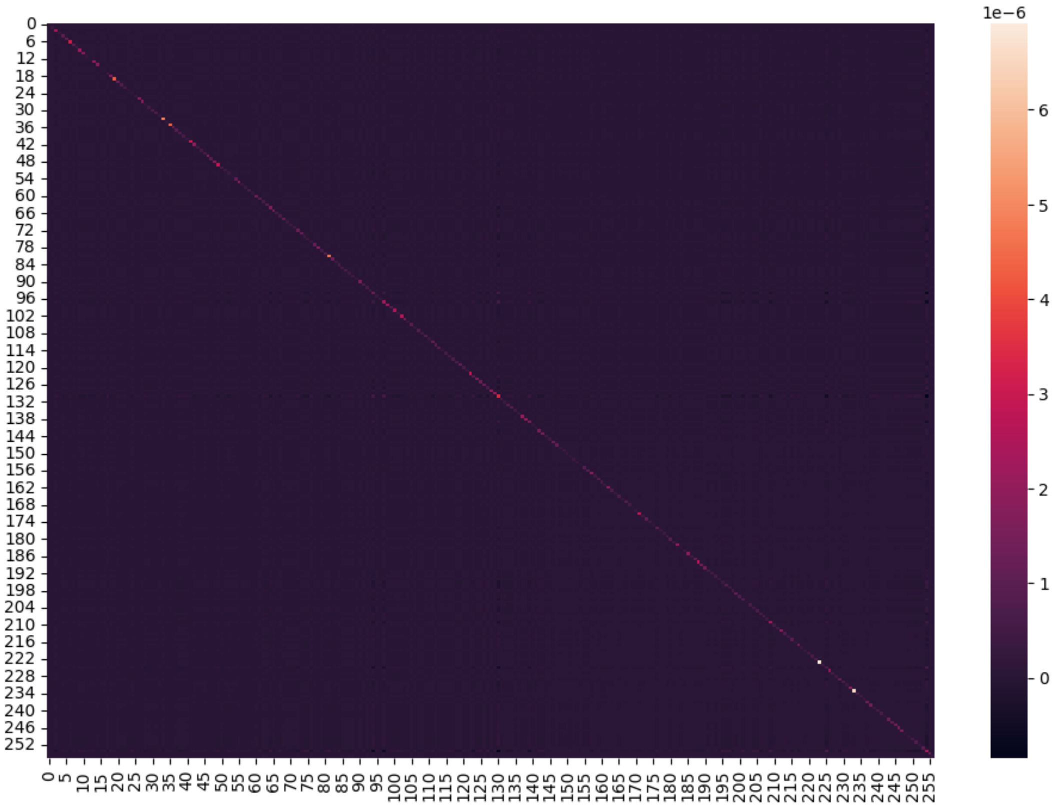
A curve fits better than a straight line!



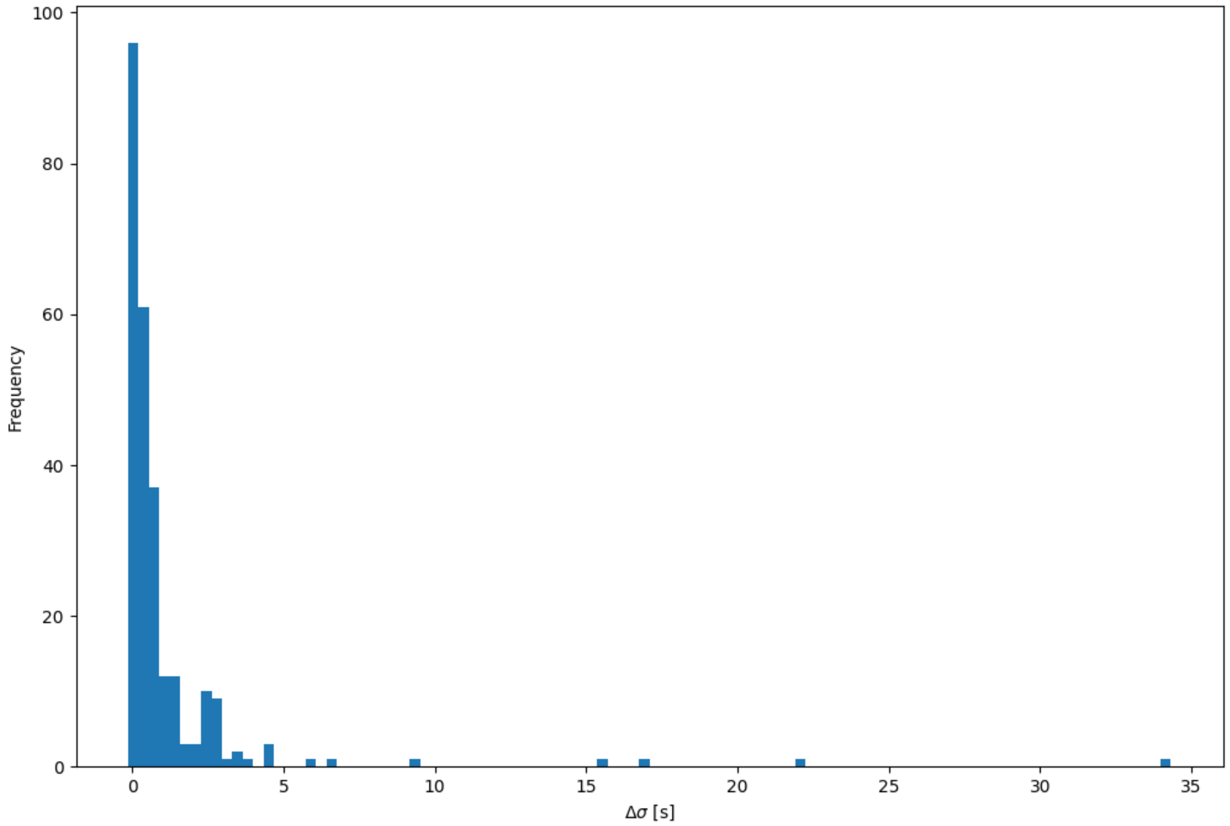
3σ posterior model error

Bayesian multivariate regression: WASP-12b

What did we learn the measurement errors to be?



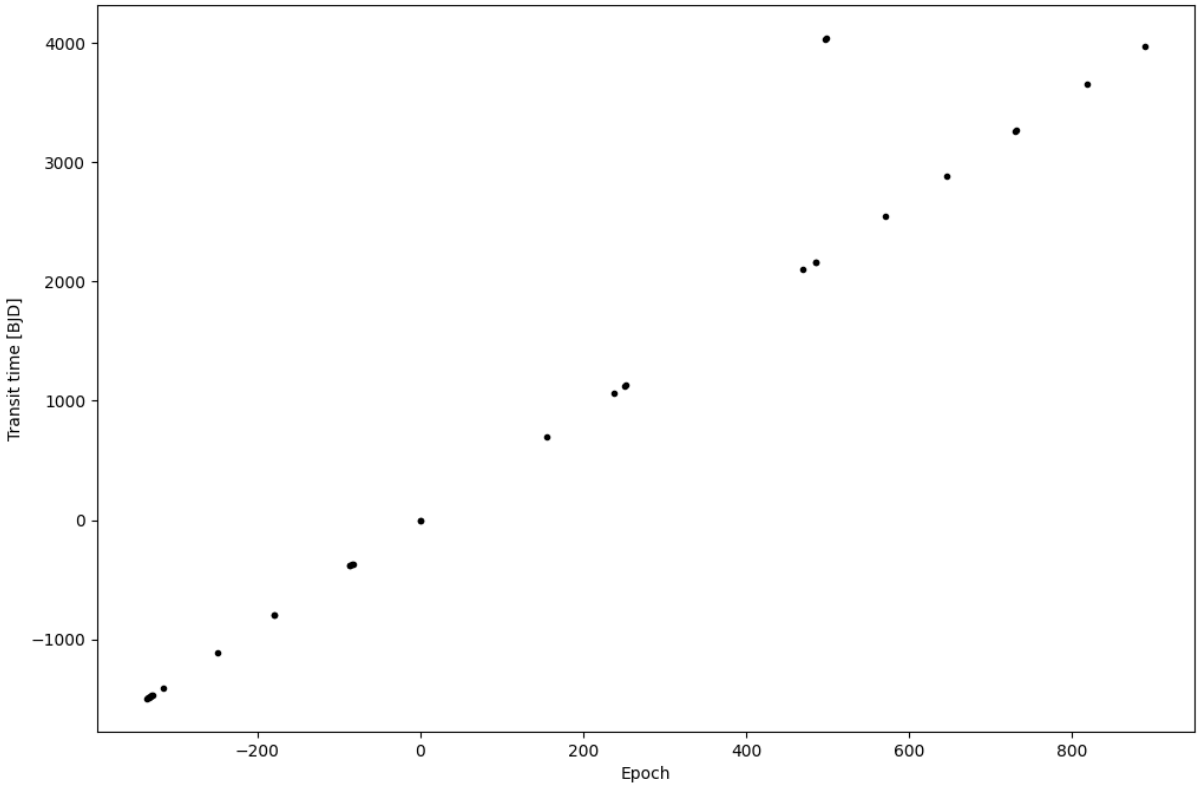
Measurements largely IID, no obvious systematic errors

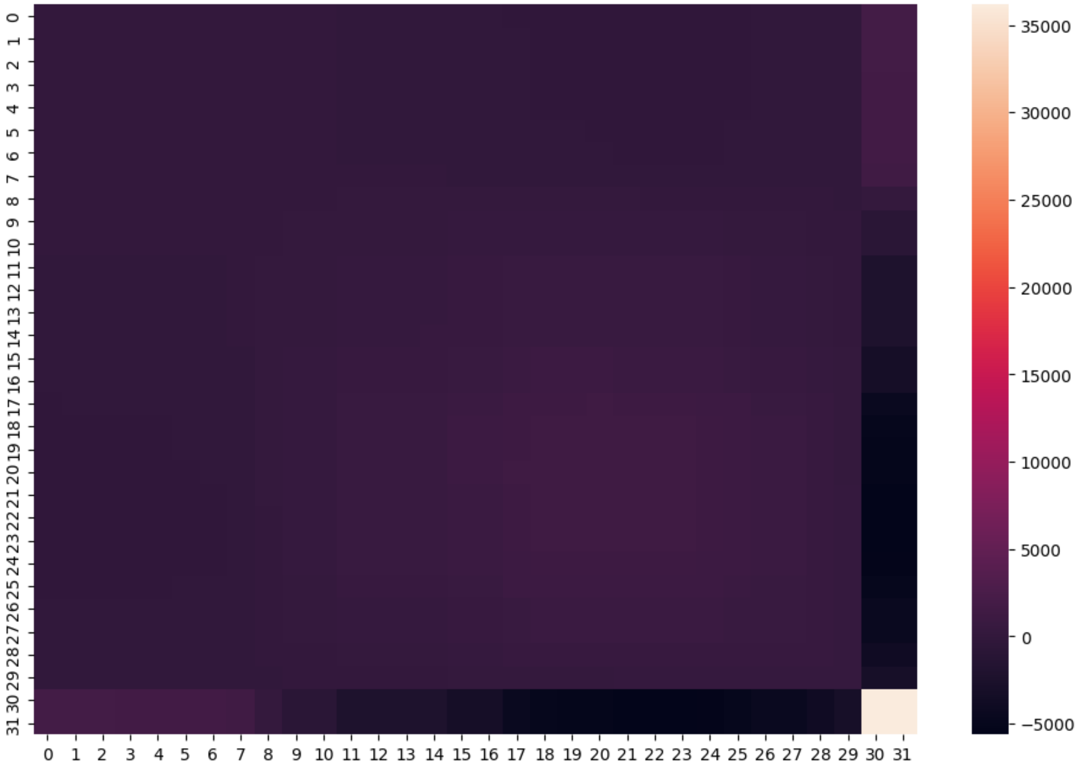


Most stayed the same, some increased ~seconds

Bayesian multivariate regression: HAT-P-1

What did we learn the measurement errors to be?

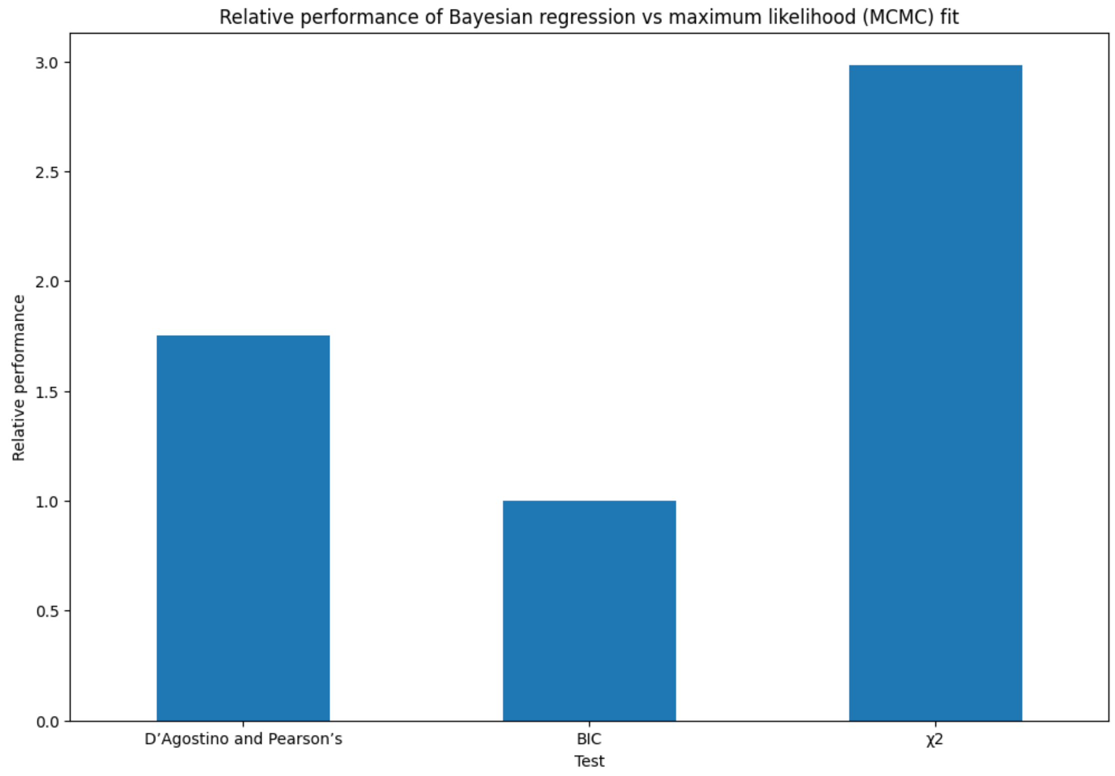


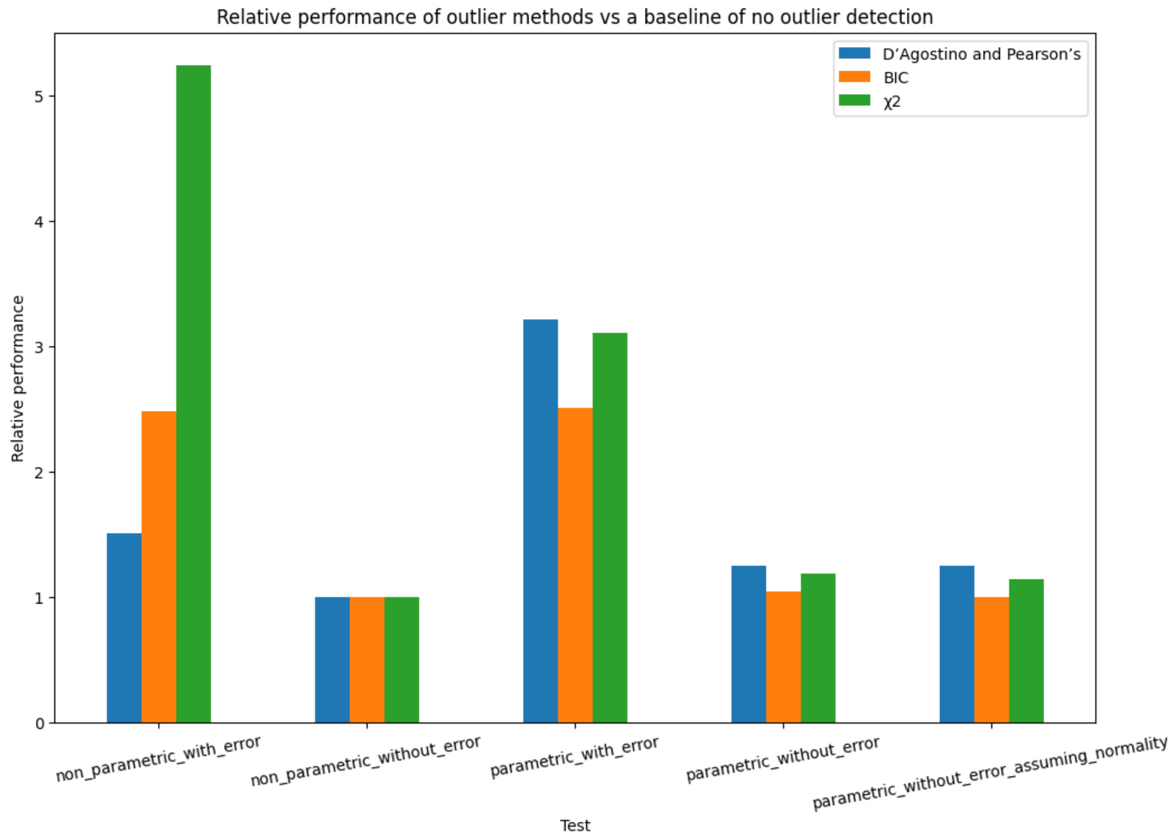


This data has two obvious outliers

The model learns that these measurements have MUCH larger error than initially reported and identifies that they are covariant

Comparison vs existing models

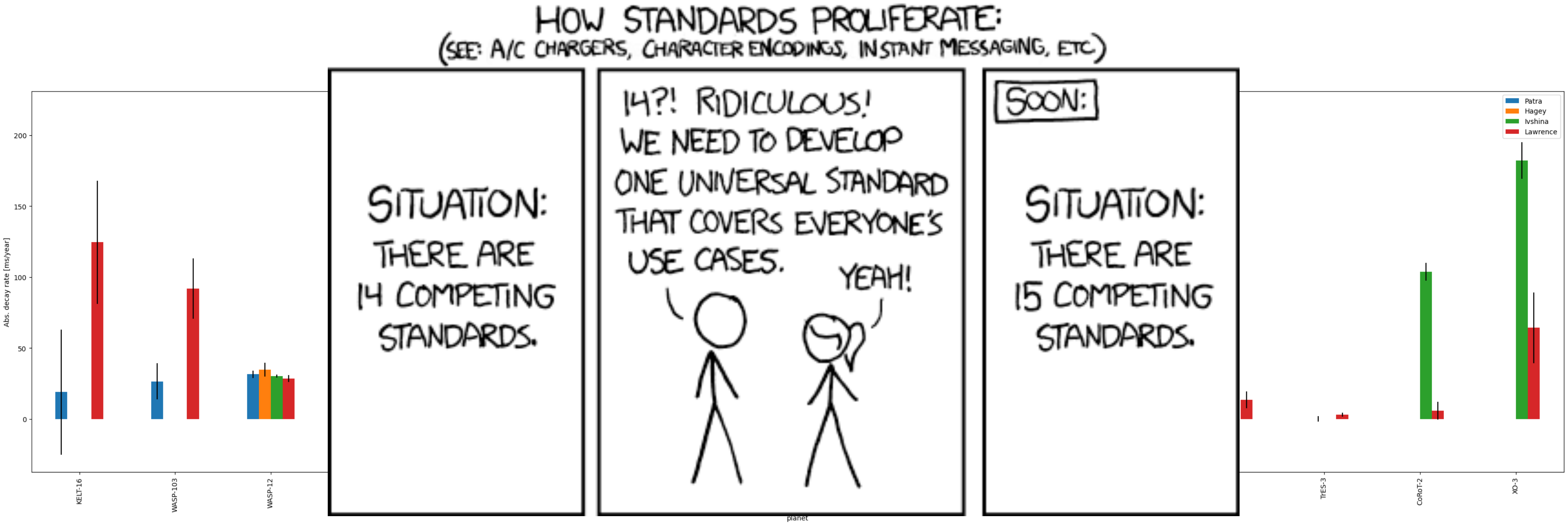




Bayesian multivariate regression wins in D'Agostino's and χ^2, BIC similar despite the regression having MANY more parameters which BIC penalises

Non-parametric outlier detection works almost as well (if not better) than parametric outlier detection

Results

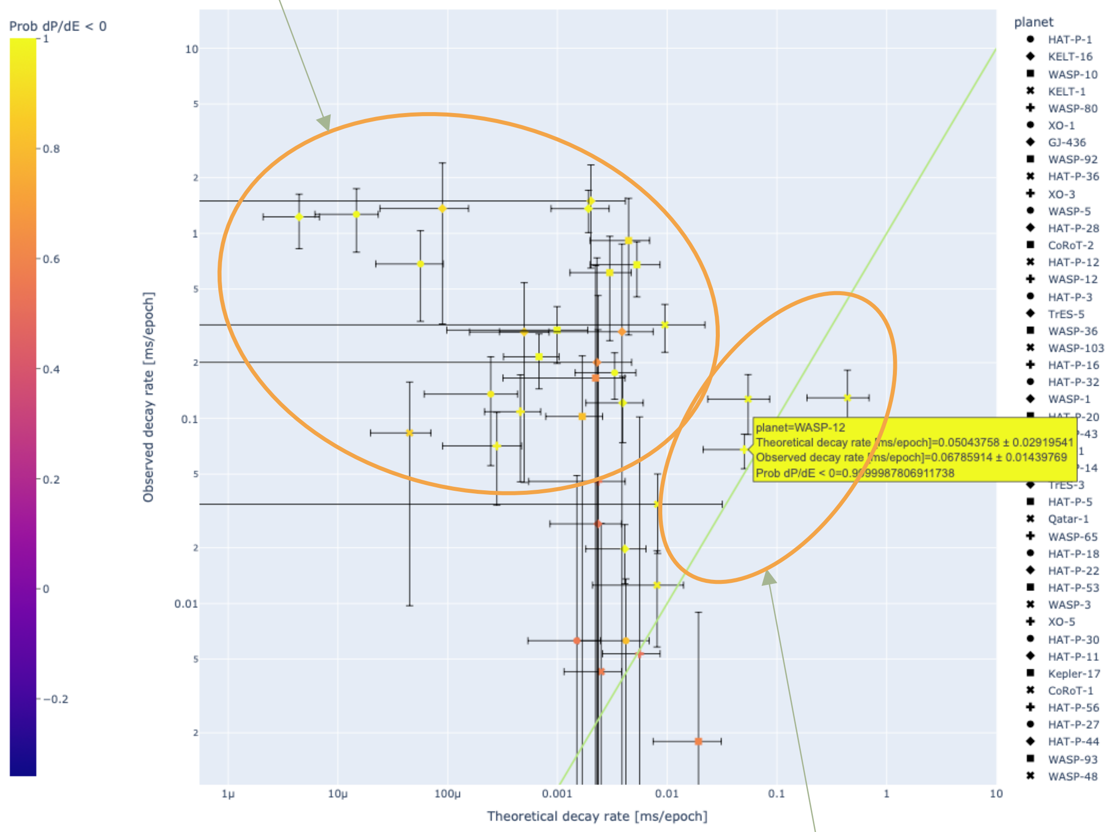


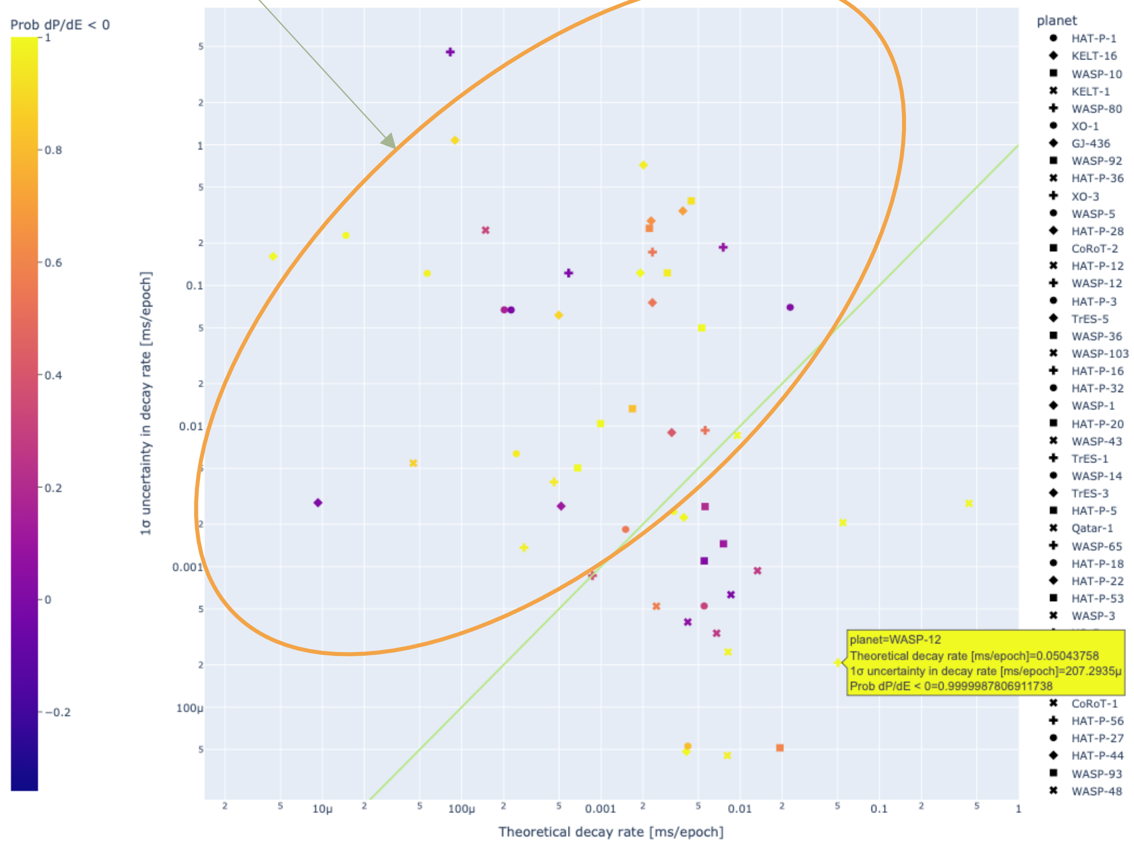
[XKCD]

Observations vs theory

Observation ≠ theory
Perhaps something else is going on?

Uncertainty > theory
These planets need further observing

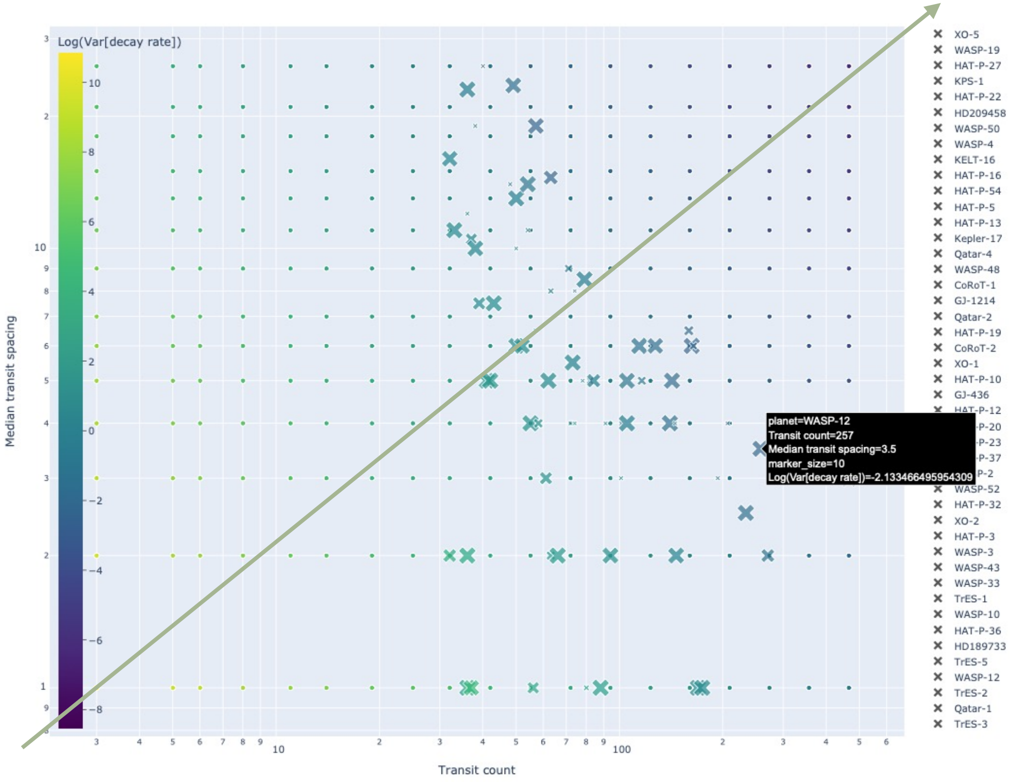


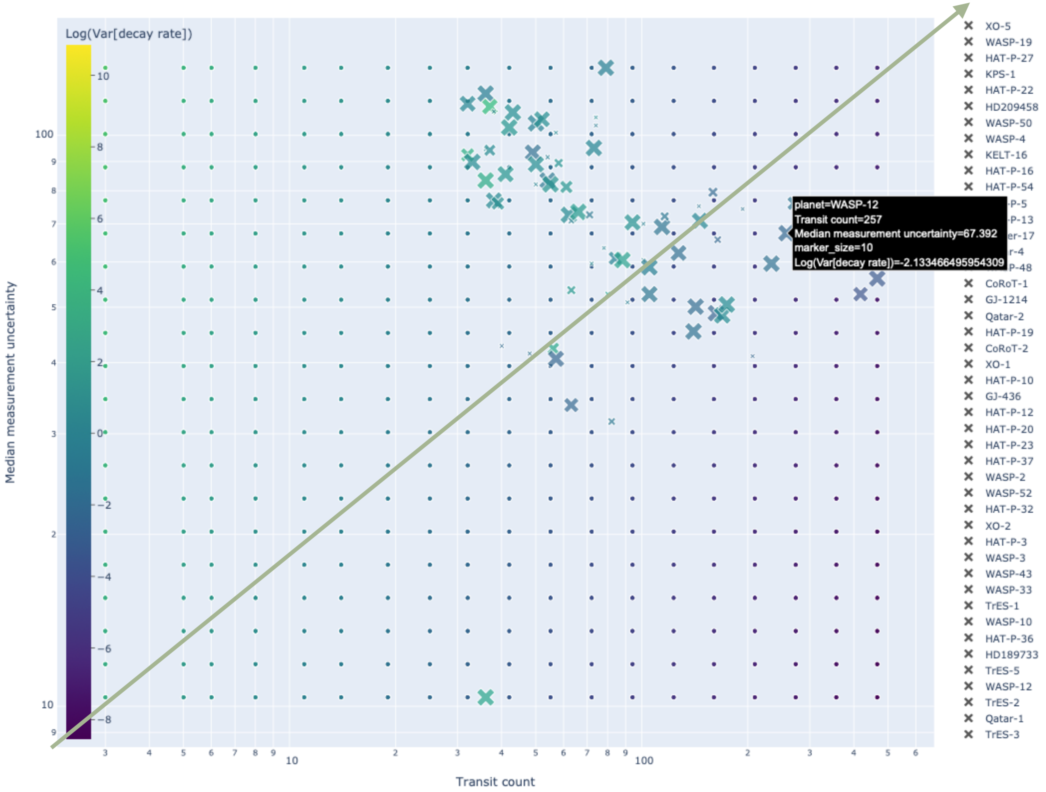


Observation ≃ theory *AND* observation ≫ 0
Good candidates for tidal decay

WASP-12, KELT-1, WASP-103

How can we improve uncertainty in decay rates?





Given a planet, we can say exactly how much a new measurement will reduce uncertainty in orbital decay. If you provide ”costs” and constraints of the use of various telescopes, we can construct an observational proposal which best reduces the uncertainty.

But is it tidal decay? + other future work…

- "Theory" is very rough - Q* could be orders of magnitude different
- Only fitted a decay model, perhaps we are missing out on other possible physical phenomena
  - e.g. we might be missing phenomena that would cause sinusoidal TTVs
- Use normality tests to a constant period model to show statistically significant variation, **then** fit different models to decide what causes that variation
  - Apsidal precession
  - Inner companions perturbing the orbit
  - etc.
- We can then compare those via the Bayesian model evidence 😃
- For those systems whose best model is stellar tidal decay, let's go one step further: fitting tidal dissipation factors and then classifying the strength of tidal dissipation by stellar parameters