A clean way of dividing an array into n-roughly equal parts.

Given an array L, the following algorithim divides the list equally into n roughly equal parts by taking advantage of integer division.

Algorithm 1: divideEqually(L, n)

```
1 partitions   ← [ ]
2 start_index  ← 0
3 length       ← 0

4 for i in 0 to n - 1 do
5 |   start_index  += length
6 |   length       ← (|L| + i)/n
7 |   partitions.add(L[start_index, start_index + length])

8 return partitions
```

For readability, I will denote $|L|$ as L and integer division as $\frac{a}{b}$ instead of $\left\lfloor \frac{a}{b} \right\rfloor$
I will prove that my code does indeed divide the array into n roughly-equal parts. I will prove the correctness of the following:

1. length $= \frac{L+i}{n}$ calculates the size of the i^{th} partition
2. start_index += length calculates the starting index the i^{th} partition.

Proof of 1: What does it mean to have n roughly equal parts? This is two conditions:

- A series of n numbers that all add up to L
- The difference of these numbers is at most 1.

We will show that length as defined above generates such a series.

Let's prove the second bullet point: Let S_i be the value of length at the i^{th} iteration of the loop. At the start of the loop when $i = 0$, length $= S_0 = \frac{L}{n}$. In the last iteration, $i = n - 1$. Since i does not exceed n, then $S_0 \leq S_{n-1} + 1$.

Clearly, S_i is an increasing sequence, and thus, $S_b - S_a \leq 1$ for any $b > a \in [0, n-1]$. I.e. the greatest difference is between the first and last value of S_i. Second bullet point done.

Now we will prove the first bullet point, that is: $\sum_{i=0}^{n-1} S_i = L$.

If $n|L$, this is clear. i is never greater than n in the formula, and so $S_i = \frac{L}{n}$ for all i, and so $\sum_{i=0}^{n-1} S_i = (n - 1 + 1)\frac{L}{n} = L$.

Now suppose $n \nmid L$. Within the loop, there is a point where the length is incremented by 1 (this is true by intermediate value theorem since if $d = (L \mod n) > 0$, then $i + d \geq n$ which is clear at the last iteration of the loop). This value d is essentially "lost" is during the floor division, but we get it back when i reaches a sufficient value.

To be clear, lets do an example. Let $L = 107$ and $n = 10$. We know there exist a value j such that $S_j = S_{j-1} + 1$. We want to prove that this $j = 3$ (because 107/10 has a remainder of 7, so we want the LAST 7 elements to be 1 greater than S_0). In general, we want that $j = n - (L\%n)$. Indeed,

$$S_{n-(L\%n)} = \frac{L + n - L\%n}{n} = \frac{(L - L\%n - 1) + n}{n} + 1 = S_{n-(L\%n)-1} + 1$$

by the properties of modulo. Bullet point 1, done.

Proof of 2: This follows immediately from (1), since we sum up all the values of length, we always get the next immediate index not already partitioned.