

For readability, we will denote *total\_recs* as  $L$  and integer division as  $\frac{a}{b}$  instead of  $\left\lfloor \frac{a}{b} \right\rfloor$ . I will prove that my code does indeed divide the array into  $n$  roughly-equal parts. I will prove the correctness of the following:

1.  $Length = \frac{L + i}{n}$  calculates the amount the  $i^{th}$  child should sort
2.  $Start + = Length$  calculates the starting index the  $i^{th}$  child should sort.

Proof of 1: What does it mean to have  $n$  roughly equal parts? This is two conditions:

- A series of  $n$  numbers that all add up to  $L$
- The difference of these numbers is at most 1.

We will show that *Length* as defined above generates such a series.

Let's prove the second bullet point: Let  $S_i$  be the value of length at the  $i^{th}$  iteration of the loop. At the start of the loop when  $i = 0$ ,  $Length = S_0 = \frac{L}{n}$ . In the last iteration,  $i = n - 1$ . Since  $i$  does not exceed  $n$ , then  $S_0 \leq S_{n-1} + 1$ .

Clearly,  $S_i$  is an increasing sequence, and thus,  $S_b - S_a \leq 1$  for any  $b > a \in [0, n-1]$ . I.e. the greatest difference is between the first and last value of  $S_i$ . Second bullet point done.

Now we will prove the first bullet point, that is:  $\sum_{i=0}^{n-1} S_i = L$ .

If  $n|L$ , this is clear.  $i$  is never greater than  $n$  in the formula, and so  $S_i = \frac{L}{n}$  for all  $i$ , and so  $\sum_{i=0}^{n-1} S_i = (n - 1 + 1) \frac{L}{n} = L$ .

Now suppose  $n \nmid L$ . Within the loop, there is a point where the length is incremented by 1 (this is true by intermediate value theorem since if  $d = (L \bmod n) > 0$ , then  $i + d \geq n$  which is clear at the last iteration of the loop). This value  $d$  is essentially "lost" is during the floor division, but we get it back when  $i$  reaches a sufficient value.

To be clear, let's do an example. Let  $L = 107$  and  $n = 10$ . We know there exist a value  $j$  such that  $S_j = S_{j-1} + 1$ . We want to prove that this  $j = 3$  (because  $107/10$  has a remainder of 7, so we want the LAST 7 elements to be 1 greater than  $S_0$ ). In general, we want that  $j = n - (L \% n)$ . Indeed,

$$S_{n-(L\%n)} = \frac{L + n - L\%n}{n} = \frac{(L - L\%n - 1) + n}{n} + 1 = S_{n-(L\%n)-1} + 1$$

by the properties of modulo. Bullet point 1, done.

Proof of 2: This follows immediately from (1), since we sum up all the values of *Length*, we always get the next immediate index not already partitioned.