Homework 3
MATH 191 Topics in Data Science

1a.

$$\begin{aligned}
\mathbb{E}[\mu_n] &= \mathbb{E}[\frac{1}{n}\sum_{k=1}^{n} x_k] \\
&= \mathbb{E}[\frac{1}{n}]\mathbb{E}[x_1 + x_2 + \cdots + x_n] \\
&= \frac{1}{n}(\mathbb{E}[x_1] + \mathbb{E}[x_2] + \cdots + \mathbb{E}[x_n])
\end{aligned}$$

Since $x_1, x_2, \ldots, x_n$ are independent identically distributed, they have have the same expected value μ, thus:

$$\begin{aligned}
\mathbb{E}[\mu_n] &= \frac{1}{n}n\mu \\
\mathbb{E}[\mu_n] &= \mu
\end{aligned}$$

1b.

$$\begin{aligned}
\mathbb{E}[H] &= \mathbb{E}[\frac{1}{n-1}\sum_{k=1}^{n}(x_k - \mu_n)(x_k - \mu_n)^T] \\
&= \frac{1}{n-1}\sum_{k=1}^{n}\mathbb{E}[(x_k - \mu_n)(x_k - \mu_n)^T] \\
&= \frac{1}{n-1}\sum_{k=1}^{n}\sum_{j=1}^{n-1}(\mathbb{E}\mu_n^2 - \mu^2) \\
&= \frac{1}{n-1}n(n-1)Var(\mu_n) \\
&= \frac{1}{n-1}(n-1)C \\
&= C
\end{aligned}$$

2. a. The code for run_PCA is as follows:

```
run_PCA = function(X_train) {

# Training X matrix: X_ train of size n by p

myPCA = prcomp(X_train, center = TRUE, scale. = TRUE, retx = TRUE);
print(summary(myPCA));
t5load = myPCA$x[,1:5];
return(t5load);
}

# This is the updated main_linear_regression() function
PCARet = run_PCA(RET);
X_train = PCARet[1:(nrDays-1),];
X_test = PCARet;
STATS = NULL;
#CM = NULL;

for (i in 1:nrTickers) {
  print('##############################################');
  print( paste('Stock = ',tickers[i] ) );
  y_train = RET[ 2 : nrDays , tickers[i], drop=FALSE]; # "drop=FALSE" ensurs it remains a
     2-D array..

  y_hat = compute_linear_regression(X_train, y_train, X_test);
  # compute the various performance statistics:

  daily_pnl = matrix(, nrow = nrDays - 1, ncol = 1);
  for (j in 1:(nrDays - 1)) {
    if (y_hat[j] < 0) {
      daily_pnl[j] = -RET[j+1,i]
    }
    else {
      daily_pnl[j] = RET[j+1,i];
    }
  }
  cum_pnl = cumsum(daily_pnl);
  #CM = cbind(CM,cum_pnl);
  mean_pnl = mean(daily_pnl, na.rm=TRUE);
  yearly_pnl = mean(daily_pnl, na.rm=TRUE) * 252;
  total_pnl = sum(daily_pnl, na.rm = TRUE);
  sharpe = compute_Sharpe_Ratio(daily_pnl);

  stats_this_stock = c(sharpe, mean_pnl, yearly_pnl, total_pnl)
  STATS = rbind(STATS,stats_this_stock);

  readline('Press key to continue...');
}
```

The top five components captured variance is about 90%:

```
Importance of components:
                      PC1    PC2    PC3     PC4     PC5
Proportion of Variance 0.491 0.1736 0.1002 0.07558 0.05788 = 0.89826
```

b. Using PCA and in-sample linear regression:

```
        sharpe    mean_pnl yearly_pnl total_pnl
SPY   0.5266759 0.0003336390 0.08407702 0.4210524
^VIX  0.8622320 0.0040718819 1.02611424 5.1387149
^TNX  0.3908225 0.0005656769 0.14255057 0.7138842
OIL   0.9089427 0.0010815811 0.27255843 1.3649553
GOLD  1.0403505 0.0015117883 0.38097065 1.9078768
^N225 5.8604248 0.0047542839 1.19807954 5.9999063
^FTSE 3.1129761 0.0019194213 0.48369416 2.4223096
GLD   0.6345453 0.0004399835 0.11087584 0.5552592
SPXS  1.2072723 0.0023141396 0.58316318 2.9204442
SPXL  0.4458274 0.0008583874 0.21631362 1.0832849
=================================================
AVG   1.4990069 0.0017850783 0.44983972 2.2527688
```

c. Using PCA and out-sample linear regression:

```
# This is the updated main_linear_regression() function
  for (i in 1:nrTickers) {

     daily_pnl = matrix(, nrow = nrDays - 101, ncol = 1);

     for (j in 101: (nrDays - 1)) {
        y_train = RET[ (j - 99) : j , tickers[i], drop=FALSE];
        X_train = RET[ (j - 100) : (j - 1) ,]
        X_test = RET[j,];
        Q = rbind(X_train, X_test);
        Qt = run_PCA(Q);
        Xt_train = Qt[1:100,];
        Xt_test = Qt[101, ,drop = FALSE];
        y_hat = compute_linear_regression(Xt_train, y_train, Xt_test);
        if(y_hat < 0) {
           daily_pnl[j - 100] = -RET[j + 1, i];
        }
        else {
           daily_pnl[j - 100] = RET[j + 1, i];
        }
     }
     mean_pnl = mean(daily_pnl, na.rm=TRUE);
     yearly_pnl = mean(daily_pnl, na.rm=TRUE) * 252;
     total_pnl = sum(daily_pnl, na.rm = TRUE);
```

```
    sharpe = compute_Sharpe_Ratio(daily_pnl);
    stats_this_stock = c(sharpe, mean_pnl, yearly_pnl, total_pnl)
    STATS = rbind(STATS,stats_this_stock);
  }
```

```
          sharpe     mean_pnl yearly_pnl total_pnl
SPY   -0.42483859 -0.0002630994 -0.06630105 -0.3057215
^VIX  -0.27237255 -0.0012693194 -0.31986848 -1.4749491
^TNX   0.23200048 0.0003427176 0.08636484 0.3982379
OIL   -0.20493539 -0.0002444066 -0.06159045 -0.2840004
GOLD  -0.35436615 -0.0005135521 -0.12941512 -0.5967475
^N225  5.45665707 0.0044504105 1.12150345 5.1713770
^FTSE  1.67787593 0.0010291882 0.25935542 1.1959167
GLD   -0.24228651 -0.0001684264 -0.04244346 -0.1957115
SPXS  -0.09852636 -0.0001848992 -0.04659460 -0.2148529
SPXL  -0.43634300 -0.0008222165 -0.20719855 -0.9554155
======================================================
AVG    0.53328649 0.0002356397 0.05938120 0.2738133
```

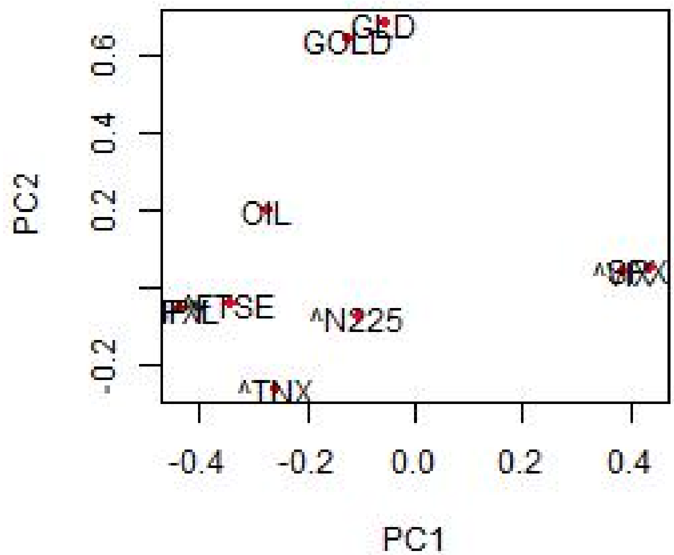
We see that the out-sample analysis using the sliding window approach gives a much lower average for our profit. Many of the instrument's Sharpe Ratios fall below 0. However, this result should fit more accurately than our in-sample analysis with its over-fit values. We would also still make a net profit with the forecast model.

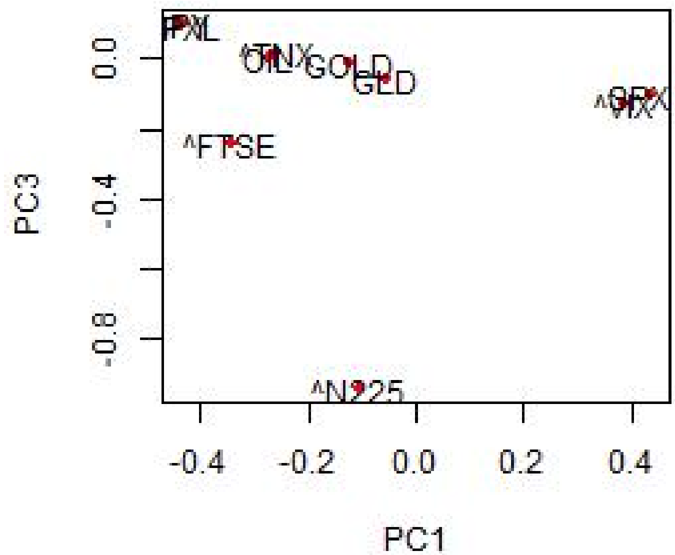
d. Using knn-regression:

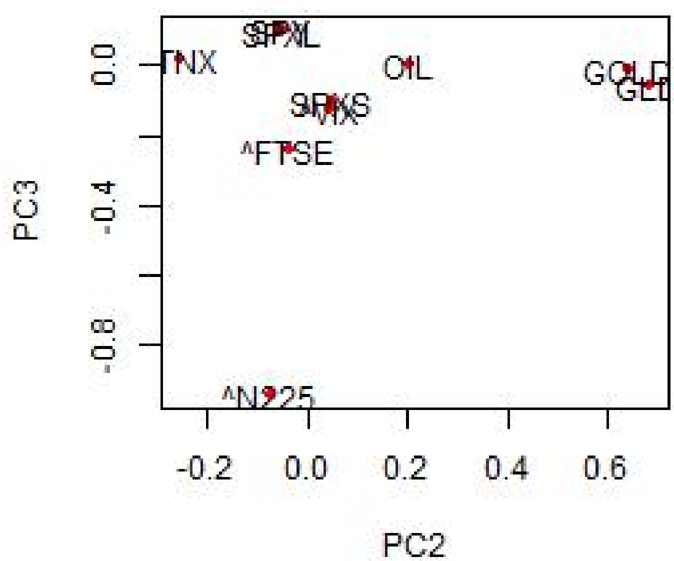
```
          sharpe     mean_pnl yearly_pnl  total_pnl
SPY    0.12984995 8.044124e-05 0.02027119 0.09347272
^VIX   0.16864502 7.859963e-04 0.19807108 0.91332774
^TNX   0.12552468 1.854426e-04 0.04673154 0.21548430
OIL   -0.08910839 -1.062781e-04 -0.02678209 -0.12349517
GOLD  -0.21211124 -3.074436e-04 -0.07747578 -0.35724943
^N225  4.75759522 3.930419e-03 0.99046558 4.56714685
^FTSE  1.00264345 6.172068e-04 0.15553612 0.71719432
GLD   -0.81183224 -5.636769e-04 -0.14204657 -0.65499253
SPXS  -0.02379274 -4.465138e-05 -0.01125215 -0.05188491
SPXL  -0.05639919 -1.063145e-04 -0.02679125 -0.12353742
=======================================================
AVG    0.49910145 4.471142e-04 0.11267277 0.51954665
```

Using k nearest neighbors regression, we have a much tighter result with lower averages across the board except for total pnl, which indicates that this forecast would have resulted in a better profit than using linear regression.

e.

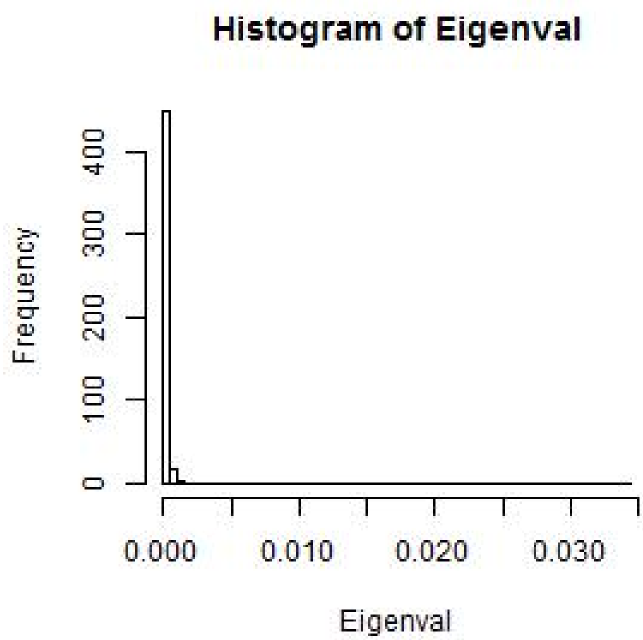

GLD
GOLD
OIL
^FTSE
^N225
^TNX
PC1
PC2

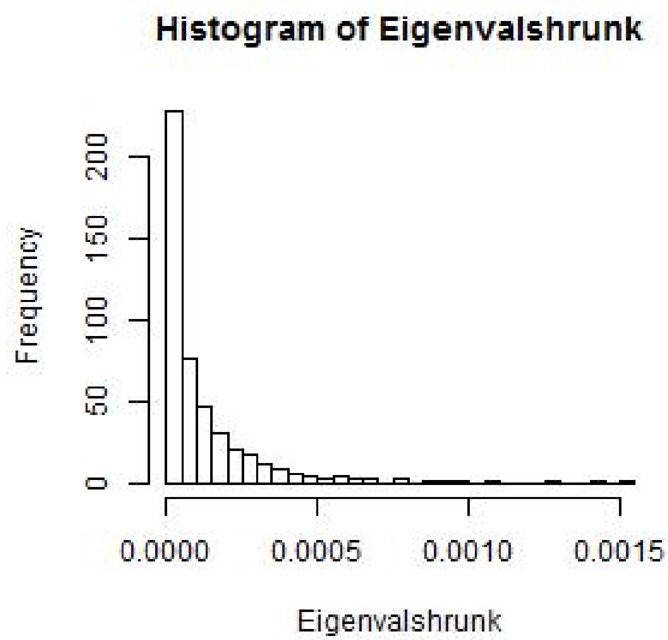

OIL
GOLD
GLD
^FTSE
^N225
PC1
PC3


OIL
GOLD
GLD
^FTSE
^N225
PC2
PC3

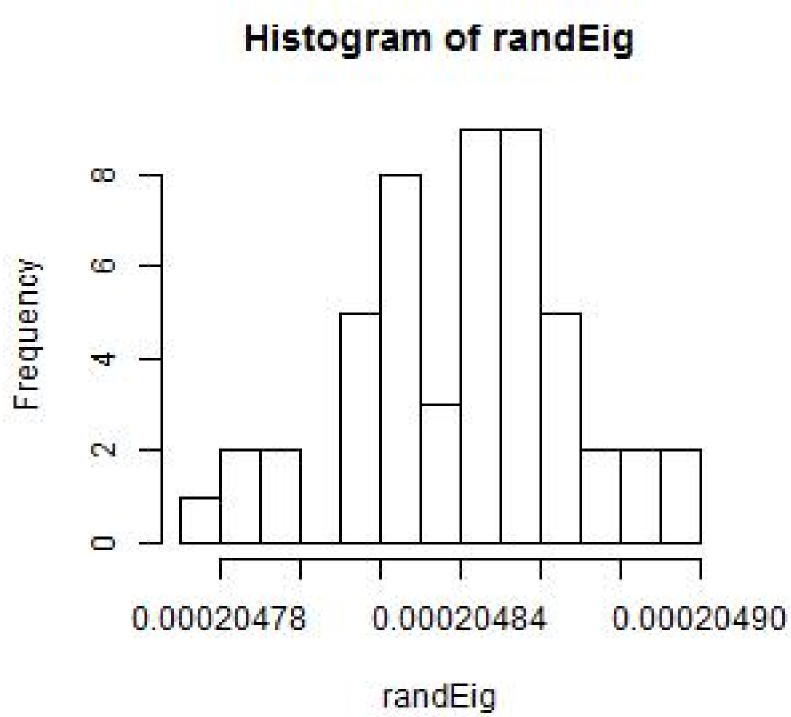
3.

a. It can be seen that most of the eigenvalues are close to 0, with a few sporadically appearing up to 0.03. Eigenvalshrunk contains all eigenvalues less than 0.002, which contains 466 of the of 472 eigenvalues.





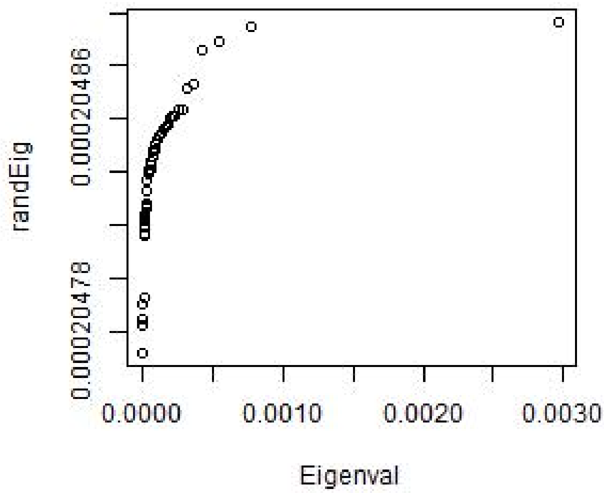
b. randEig is the average eigenvalue of the 50 iterations of randomizing RETS, giving the histogram:



c. Using $max(Eigenval)$ and $max(randEig)$, we get:

```
> max(Eigenval)
[1] 0.03429729
> max(randEig)
[1] 0.0002048967
```

d. We can see that the distributions are quite similar, with randEig being more evenly distributed in its 0.0002 range and Eigenval being a little more sparsely distributed but in the similar region.



e. The averages found in (b) do not make up the largest 20 in our actual eigenvalues. Due to a varying set of both very small and extremely large eigenvalues, are average in this case does not align with our actual data.