A Theory of Price Caps on Non-Renewable Resources*

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Abstract

In December 2022, following Russia's invasion of Ukraine, a G7-led coalition of countries imposed a $60 per barrel price cap on the sales of Russian oil that use western services. This paper provides a theoretical and quantitative analysis of this new tool. We build a tractable equilibrium model in which the financially constrained exporter of a non-renewable resource optimally exerts market power, and the price of the resource varies stochastically. An important insight from this framework is that the supply curve is inelastic and can even be downward sloping, rationalizing the patterns we observe in the data. Contrary to the fears that an introduction of the price cap will cause a damaging oil supply shock, the exporter may have strong incentives to increase extraction following the introduction of a binding price cap. In fact, when the producer is large and has market power, a price cap that applies to all or most sales significantly limits the degree to which market power is used in equilibrium and stabilizes world oil prices. But if the cap is expected to be temporary, is poorly enforced, or if the sanctioned state has access to a non-compliant "shadow" fleet, the cap is less effective at stabilizing world prices.

JEL: F51, L13, L71, Q41.

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1 Introduction

On December 5, 2022, the Price Cap Coalition, consisting of the G7, the European Union (EU), and Australia, responded to the continuing Russian invasion of Ukraine by imposing a cap on the price of seaborne Russian oil sold into global markets. Companies based in coalition countries are currently allowed to provide services that support Russian oil sales, including shipping, insurance and trade finance, but only if the price paid to Russia does not exceed $60 per barrel. The Coalition's goal is to reduce Russian revenue from oil sales, while also ensuring the uninterrupted flow of Russian oil to global markets, hence preventing a negative supply shock that could have adverse short-term consequences for the rest of the world.[1]

The implementation of the price cap on Russian oil is a significant development in the realm of international economic policy. It represents a novel approach to sanctions in an era of globalization when some markets are dominated by few large autocratic producers. Russia is one of the top three oil producers, every day exporting about 12% of the daily global supply of crude oil and oil products combined. Given its importance to global markets, conventional approaches to sanctioning oil producers, such as outright bans or embargoes, would have had major impacts on world prices and, by extension, the global economy. An effective price cap means that no country is too large to escape the consequences of sanctions. The price cap on Russian oil could become a blueprint for future sanctions, as well as international economic and trade policy more generally.

However, to date there has been little formal economic analysis of how the policy might impact both the sanctioned country and the global oil market. Most of the policy discussion continues to focus on static demand and supply analysis, and thus ignores several crucial aspects, such as dynamic considerations, uncertainty induced by fluctuations in oil prices, the non-renewable nature of the relevant commodities, financial frictions and market power. As a result, policymakers lack both a qualitative framework to think through the various economic forces at play, and a quantitative framework that would help inform choices such

[1] The price policy was developed in the context of the EU's "6th Sanctions Package", which was adopted in early June 2022. These measures included an embargo on the purchase of Russian oil from December 5, 2022, along with a ban on EU countries providing services in support of Russian oil exports. A similar ban on Russian oil products and services was slated for February 5, 2023. Since western services were used for a large share of Russian exports – over 70% by most accounts in the case of Russian seaborne crude trade (see Craig Kennedy, forthcoming) – there were concerns that this EU policy package could keep large volumes of Russian crude and product out of the market, effectively squeezing global supply and sharply raising oil prices everywhere. The price cap mechanism was designed to maintain the supply of Russian oil to world markets while squeezing Russian government revenue and sustaining the EU embargo.

as whether to maintain the original $60 per barrel limit on crude or lower it, or whether and how to tighten the enforcement on the cap.

This paper fills these important gaps. Its main contribution is a dynamic, continuous-time analysis of the economic incentives that drive the behavior of a large (i.e., endowed with market power), financially constrained producer of a commodity. The framework we develop is general and can be used to study a range of markets and issues, from the impact of financial constraints on commodity supply to the effect of climate change mitigation on energy markets. In the present paper our main application – motivated by the events of the past year – is the implementation of a cap on the price at which the producer sells the commodity.

Our dynamic model builds on Hotelling (1931) and focuses on the decision problem of a state exporter of a non-renewable resource. Sales of this resource fund a part of producer's consumption, and financial frictions mean that the volatility in the path of its income matters for the time-path of the producer's consumption.[2] Furthermore, the price of the commodity varies stochastically over time, reflecting a complex pattern of demand, supply or sentiment shocks. The final key element of our framework is that the producer has market power.[3] We develop a tractable approach that integrates the market power of the producer with a stochastic process for the price of oil in an equilibrium model. The producer's market power is endogenous in this setting, since it depends on its market share, which evolves over time driven by extraction decisions.

This approach contributes several novel findings, both in terms of the behavior of the producer per se, and about the effects of the price cap.

We first characterize optimal extraction at different prices – i.e. we pin down the supply curve of the producer. The supply curve in our model is inelastic and *downward-sloping* – that is, the producer optimally increases production when the price of the commodity falls – unless prices are just above the cost of extraction.[4] This surprising shape of the supply

[2]These frictions are driven, in part, by both the (ex-ante) anticipated possibility of future sanctions and by the imposition of sanctions. Russia has substantial official foreign reserves, but these were frozen by the G7 immediately after the February 2022 Russian invasion of Ukraine. Since that initial freeze, Russia has been allowed to sell oil, and some other commodities, accumulating foreign assets in Gazprombank and other "private" entities. Russian authorities may be concerned about potential future freezes of those assets.

[3]This is realistic in the current context, given that Russia is one of the the world's leading oil producers, oil prices spiked immediately after the 2022 invasion began, and a principal rationale for implementing the price cap policy was that a complete EU embargo – refusing to buy Russian oil and effectively blocking sales to third countries – could lead to a contraction in world oil supply and a spike in world prices of oil.

[4]We view this finding as consistent with the evidence of a negative correlation between the price of oil and Russian extraction that we present in Section 2, and with the observation that Russian production has changed little in the face of large fluctuations in the oil price over the past few years.

schedule emerges as a confluence of two forces: the smoothing effect and the option value effect. The smoothing effect arises due to the incentives of the producer to smooth the path of income when faced with financial frictions. This calls for reducing quantities when prices are high, generating a downward slope. The option value effect counteracts the smoothing effect at low prices. The intuition is that the producer tries to avoid selling its reserves too cheaply, and waits for terms of trade improve.

The second finding is that in this dynamic setting the use of market power *lowers* contemporaneous profits. As usual, exercising market power leads to lower quantities and higher equilibrium prices. But because the degree of market power is endogenous and diminishes as producer's oil reserves are depleted, in our dynamic setting the producer has an additional incentive to cut production below the level which maximizes contemporaneous profits, in order to save up future market power.

In terms of the economics of a price cap, our analysis points to three important insights. We start by analyzing a perfect price cap – one that applies to all of the sanctioned producer's sales and is permanent – and then turn to study the effects of a leaky cap.

Imposing a perfect price cap effectively changes the stochastic process for the price that the producer receives: namely, it eliminates the upside during periods of high prices. Thus, the price cap effectively reduces uncertainty and makes the stock of reserves less valuable to the producer. We show that, because the commodity effectively becomes a less important source of income, the producer uses it up more rapidly (as long as some alternative source of income, such as tax revenues unrelated to the commodity trade, is available). Thus, the supply curve shifts outwards, and becomes close-to-vertical at prices above the cap.

Furthermore, if the producer has market power, implementing a binding, "perfect" price cap significantly diminishes the incentives to exercise market power in equilibrium. The logic behind this is simple: when the price cap is binding, curbing supply leads to lower volumes but unaltered prices, thereby rendering the use of market power ineffective.

This finding has important implications for the impact of the price cap. Most notably, and against the concern that is pervasive among policymakers, a binding price cap can actually *drive down* world oil prices and act as a *stabilizer* of the global oil market. Such positive effects are stronger the greater is the degree of market power of the producer. This is because the cap eradicates the use of market power in equilibrium, so the greater the influence of market power at the outset, the more significant the favorable impact of a cap. Overall, our study demonstrates that a price cap can be a potent tool and suggests that its benefits might actually be greater if it is applied to the exports of a producer with significant power in a market for a given commodity. But these results hold when the producer has no ability to bypass the price cap regime.

Our final set of results concerns the effects of an imperfect price cap, i.e., a cap that applies to only a share of a country's sales of a commodity and/or is expected to be temporary. This analysis is important because monitoring and enforcement of any cap is likely to be imperfect and the sanctioning coalition is likely to be able to impact only a certain part of exporter's sales. Moreover, if the cap is expected to be temporary, the producer might respond very differently to when it is expected to be essentially permanent. Our findings show that if world prices are high, so that sufficient revenues can be generated through sales outside of the price cap regime, the producer may have strong incentives to "shut-in" production and instead sell the reduced quantities only outside of the regime. This is because, with an imperfect cap, the incentives to exercise market power remain. This dampens the stabilizing effect of the policy. However, our simulations also suggest that shutting in production can be costly in terms of contemporaneous profits, and that the impact on world prices is manageable. We also find that the expectation that the price cap is lifted at some point in the future increase the incentive to shut-in today in response to the price cap. These results have important policy implications. They emphasize the importance of effective enforcement of the price cap and highlight the benefits of credible commitment to keep the policy in place for a long time.

Literature and contribution. The price cap on a non-renewable resource such as oil is a new and live policy and there is little direct literature on this topic, which motivates this project. Early analysis of the economics of the price cap on Russian oil appears in Wolfram et al. (2022) and in Johnson et al. (2023) (forthcoming). In a recent paper that complements ours with the empirical analysis of the cap, Babina et al. (2023) use customs data to provide evidence on the effectiveness of the cap imposed by the G7 on Russia. They find that sanctions have led to a fragmentation of the oil market, with the oil that was destined to Europe trading at steep discounts and below the cap, while the oil sold elsewhere trading at close to global prices. In a complementary theoretical contribution, Salant (2023) studies the effects of pre-announcing the price cap. Sappington and Turner (2023) investigate the impact of a price cap in a static two producer Cournot model. Wachtmeister et al. (2023) consider what different price cap levels imply for net losses of Russia. Baumeister (2023) provides a broader overview of the developments in the oil market over since the Covid-19 pandemic. Price caps have also been examined in other contexts, in the industrial organization or urban economics literatures – see e.g. Bulow and Klemperer (2012) and Leautier (2018) and references within.

The framework we employ in the analysis builds on the classic work by Hotelling (1931), but appends it with the stochastic price of oil as studied in the finance literature (see Cox et al. (1985), Longstaff and Schwartz (1992), Chen and Scott (1993), Duffie and Kan (1996) for models of interest rates, and Schwartz and Smith (2000) and Pindyck (1999) for models of commodity prices) and develops a tractable way to think about market power. The Hotelling framework has been studied and extended in numerous studies.[5] A notable contribution is that of Anderson et al. (2018) who study the role of capacity constraints and drilling decisions – both margins which we abstract from – and an earlier work by Salant (1976) who studies the extraction problem in a framework with realistic industrial organization structure of the world market. Our paper analyzes the impact of the price cap on the extraction decisions and world oil prices, stopping short of analyzing the general equilibrium impact on the global economy. A complementary paper by Bornstein et al. (2023) develops a quantitative general equilibrium macroeconomic model with oil production sector, and uses it to study the advent of fracking. For broader overview of forces that drive oil prices, see Hamilton (2009).

This paper makes two main contributions. First, it provides a dynamic framework for analysis of the economics of the price cap policy with prices that vary stochastically, and demonstrates how to incorporate market power considerations into such framework. Second, it applies the model to study the economic incentives of Russia, and the policy options of the Price Cap Coalition, in the current context, thus explicitly addressing one of the timely and pressing policy questions.

Finally, it is useful to highlight what we are not doing in this paper. The decision to go to war is economically costly and naturally increases the pressure on the state budget, raising the marginal value of funds today, relative to the future. This presents an additional incentive for Russia to increase extraction at given prices – a straightforward way of capturing this in a model would be to raise the discount rate of the producer, making it more impatient. We abstract from this important effect in our analysis in order to focus on how the price cap policy itself affects the producer's incentives.

[5] Classic references include Solow (1974), Stiglitz (1976), Dasgupta and Heal (1974), Pindyck (1980), Arrow and Chang (1982). For an overview of work in the 50 years after the publication of Hotelling's article, see Devarajan and Fisher (1981). Recent work includes van der Ploeg and Withagen (2012), Newell and Prest (2017), Salant (2012) and Gaudet (2013) and most recently Harstad (2023), who considers the dynamic game between successive governments controlling an exhaustible resource.

Structure. The rest of the paper is structured as follows. Section 2 describes Russia's oil sector, including its costs, the prices it faces and typical export volumes and routes, and provides some institutional context on the price cap that is relevant to our model. Section 3 presents the baseline model without market power, and Section 4 studies the effects of the price cap in such a setting. In Section 5 we construct our equilibrium model with market power, and study the effects of the price cap on the degree of market power exercised in equilibrium. Section 7 considers the case where the producer can partially bypass the price cap. Finally, Section 8 concludes with a discussion of policy implications and directions for future research.

2 The facts

2.1 Russian extraction historically

In the 1970s, Russia was the world's largest oil producer, but with the fall of the Soviet regime, oil production dropped to as low as 6 million barrels of oil per day compared to a high of over 11 million barrels of oil per day in the late 1980s. Major investment beginning in the mid-1990s, along with access to western oil field services, helped to restore production to more than 10 millions barrels per day by 2019, which made Russia the third largest oil producer in the the world (after the US and Saudi Arabia). In recent years, most Russian production has been exported (7.5-8 million barrels per day, out of production of 10-10.5 million barrels per day), making Russia the world's top exporter of combined crude oil and product. The left panel of Figure 1 plots annual Russian oil production since 1970 and the right panel plots monthly production in the last 5 years, highlighting the major disruptions around the pandemic and the war.

2.2 Russian oil production and export markets prior to the war

In 2021, Russia exported 7.5 million barrels of oil per day, of which crude was 4.7 million barrels and refined products were 2.8 million barrels.[6] A single barrel of crude oil can be processed to produce multiple refined products such as gasoline, diesel, jet fuel, and other derivatives of oil. Refineries can be designed to produce different mixes of refined products, although the scope to change this is limited, especially in the short run. As of

[6] https://iea.blob.core.windows.net/assets/9aea25c1-5450-49db-8e1f-a67c0212720c/-16MAR2022_OilMarketReport.pdf

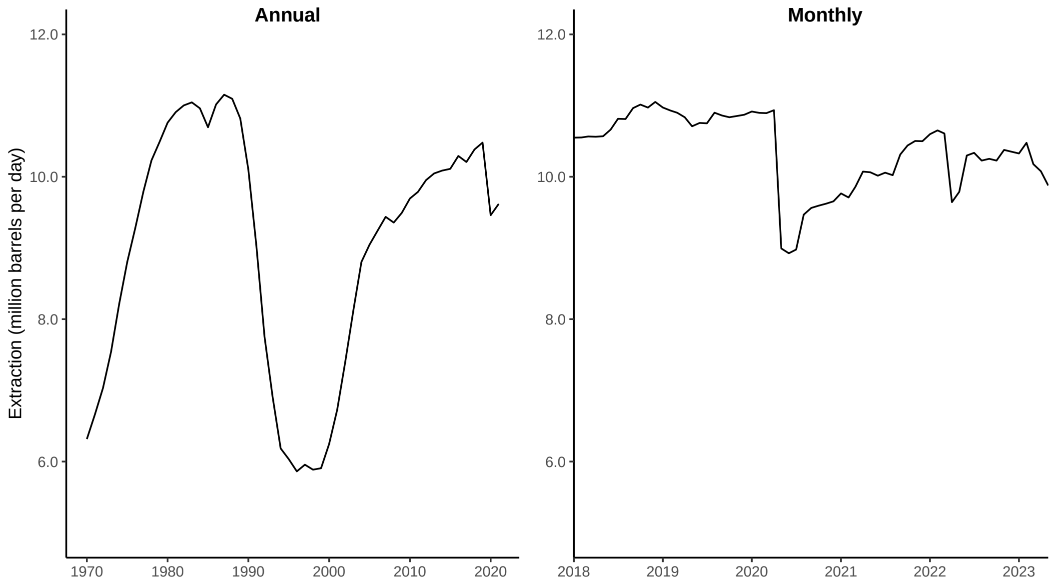


Figure 1: Russia's oil extraction historically: annual, 1970-2020 (left panel) and monthly, January 2018-March 2023 (right panel). Source: CEIC (left) and U.S Energy Information Administration (right).

2021, Russia's refining industry had the capacity to serve domestic gasoline demand and the country exported the remaining products. Substituting between exporting crude and exporting refined products is possible to some degree, but the infrastructure differs and there are pipeline and port constraints on each.

Most Russian oil is produced in Western Siberia and transported by pipeline to refineries and shipping facilities in Russia's Western ports. Before the war, Russia's largest oil customer was the European Union, which received 0.7 million barrels of crude oil per day by pipeline and 1.5 million barrels by sea in 2021. The EU also bought 1.2 million barrels of oil product, almost all of which arrived by sea. Overall, the EU imported almost half of Russia's total oil exports. Most of the tankers carrying these fossil fuels to the EU departed from three sets of ports: in the Black Sea, the Baltic Sea, and Murmansk in the far north.

China was also an important customer, primarily supplied from Russian ports in the far East. In 2021, China received 1.6 million barrels of crude per day, half by pipeline and half by sea. China did not previously buy a significant quantity of Russia's refined product.

Figure 2 plots Russia's seaborne crude oil exports by destination from January 2022 to September 2023. It does not reflect the approximately 1.5 million barrels of crude oil per day exported via pipeline, roughly half of which used to go to the EU and half to China.

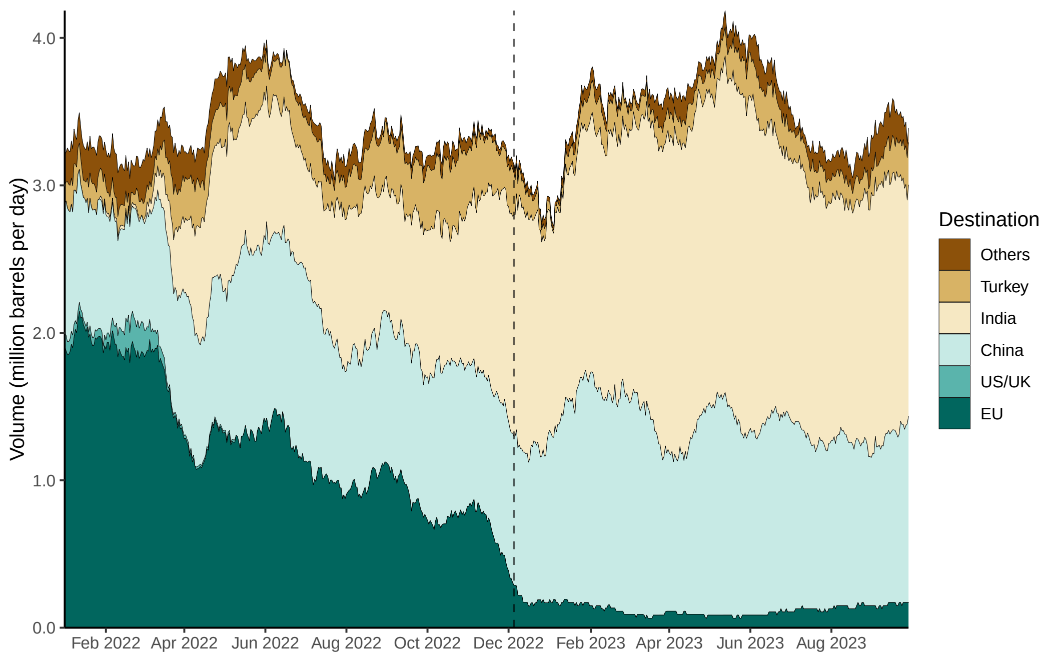


Figure 2: Russia's seaborne crude oil exports by destination, January 2022 - September 2023. Dashed line indicates the start of the price cap policy for crude oil on December 5, 2022. Source: CREA.

Figure 3 plots Russia's oil product exports by destination from January 2022 to September 2023, almost all of which travels by ship.

Russia's ability to shift supply across customers faces several constraints. First, crude oil and refined products travel in different types of ships. Crude oil is generally moved in very large tankers, capable of carrying 1 million barrels or more. Refined product travels in smaller tankers, not ideally suited to long-distance voyages. In addition, while Russia has some ability to route oil to different transshipment points within the country, there are pipeline constraints – including out to the far East. In 2021, Russia's pipeline to the far East was already roughly at capacity (1.6 million barrels per day).

Russia has limited on-shore storage available, and most of this was already full when the 2022 invasion of Ukraine started. Storage "on the sea" is available, but this requires chartering and insuring ships for the duration – an expensive proposition.

Oil producers can also "shut-in" production, meaning they can close down wells. However, this process is costly and creates a risk that it will be expensive to restart production later. This is a particular concern for Russia, as some of its oil fields are old and access to advanced western technologies – which would likely be required to re-open closed wells or open up new

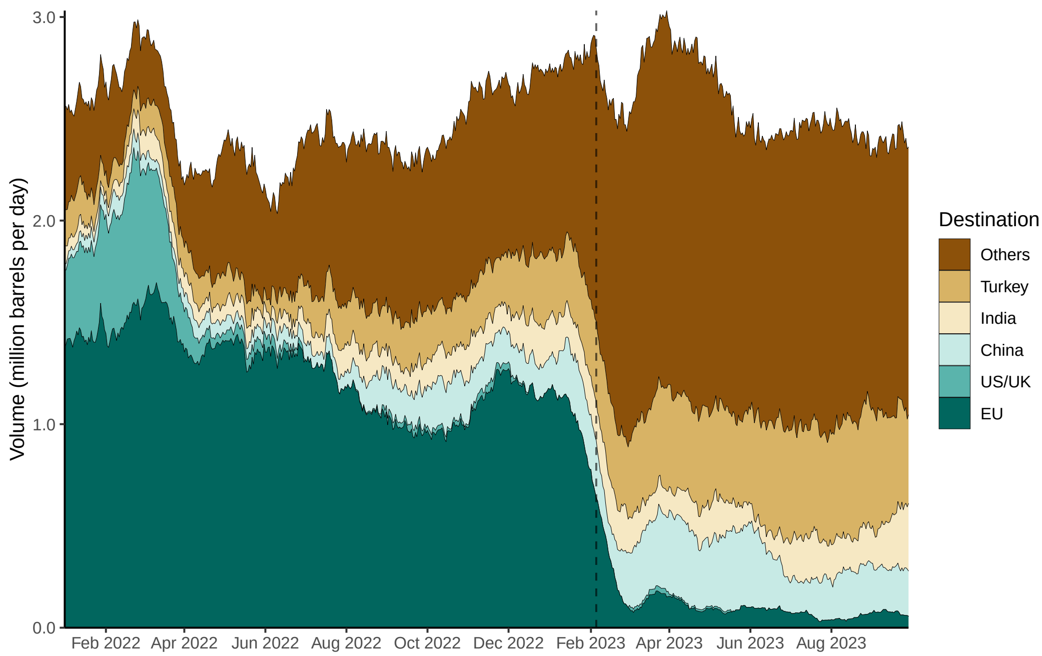


Figure 3: Russia's oil product exports by destination, January 2022 - September 2023. Dashed line indicates the onset of the price cap policy for oil products on February 5, 2023. Source: CREA.

ones – are curtailed due to sanctions. However, some observers assess that Russia could shut in several million barrels per day and face low costs to restarting production (temporarily reducing production to approximately 8 million barrels per day and exports to approximately 6 million barrels per day).

2.3 Russian extraction costs and sensitivity to oil price fluctuations

This paper aims to answer a central question regarding the responsiveness of Russia's production to the price it faces. One element of the production decision will be marginal costs. Various estimates peg marginal costs at most Russian fields at \$10 to \$40 per barrel, with the high end generally reflecting longer run marginal costs of developing new fields. At its low point at the beginning of the COVID pandemic, the price of oil was around \$20-\$25 per barrel. A presentation to investors by Rosneft, Russia's largest state-owned oil company, indicated that this price still covered short-run marginal cost, which appears to have been around \$15. According to careful analysis by CREA (a Finnish research institute) of fiscal regime operation in 2022, it appears that the Russian fiscal authorities adjust tax rates so that producers received (post-tax) around $25 per barrel. Given the power of the Russian state over its oil companies and its ability to require payment of ex-post profit taxes, it seems reasonable to model responses to the price of oil (and attempts to exert market power) as a national level decision.

The price Russia receives for its oil is heavily influenced by the world price of oil. OPEC Plus, which periodically sets production quotas, has considerable influence on world prices. Observers suggest that OPEC Plus quotas announced in early 2023 targeted world oil prices of $80 to $90 per barrel. In addition to participating in OPEC Plus actions, Russia, as one of the largest oil producers, can exert short-run influence over oil prices through its announcements and actions. The 2022 invasion of Ukraine, for example, pushed world oil prices up by nearly 40 percent, presumably because participants in the oil market were concerned about potential disruptions to Russian supply.

Immediately after the invasion, some shipping companies and customers declined to do business with Russia, resulting in a stigma that lowered the price paid for Russian oil. Consequently, Russian oil sold for a discount from the world benchmark price. Exchanges quote prices for "Urals" oil, which describes the mix typically sold by Russia, including oil from fields in the Urals, Volga and Western Siberian regions. Figure 4 plots the Urals discount (Urals price minus Brent price) since just before the invasion through September 2023. Urals prices are not based on publicly posted transactions but are collected by reporting services, like Argus Media and S&P Global, who request quotes from traders. There is some question about the accuracy and representativeness of the prices that the reporting services are able to collect, particularly after the price cap was enacted. Before the war, the Urals discount was usually small, reflecting the market value of Russia's blend of primarily heavy sour oil. The discount was largest at nearly $40 in mid-April 2022, and then declined before increasing again in early December, as the price cap was imposed. Oil sold out of Russia's Eastern ports, primarily to China, is priced relative to the benchmark "ESPO" price, referencing the Eastern Siberia-Pacific Ocean oil pipeline. ESPO prices are even less transparent than Urals prices, but historically traded close to Urals and have been discounted less than Urals since the war began.

For another perspective on Russia's production decisions, Figure 5 plots monthly production against the Urals price from January 2007 to November 2022. Months before Russia joined OPEC Plus are denoted with red circles and months after it joined OPEC Plus are denoted with blue circles. The dark fitted line suggests that, if anything, production over

Figure 4: Russian Urals price minus Brent price January 2022 - September 2023. Source: Thomson Reuters.

this period has tended to be negatively correlated with prices. While this evidence is obviously not causal, and indeed causation can run both ways, it undermines the hypothesis that Russia dramatically cuts production when faced with lower prices. Consistent with this, early 2023 reports suggest that Russia's crude oil production increased in the first four months after the price cap was imposed. For example, the International Energy Agency in April 2023 concluded that,"Russian oil exports in March soared to the highest level since April 2020 ... Total oil shipments rose by 0.6 million barrels per day to 8.1 million barrels per day, with products climbing 450 thousand barrels per day, month-on-month to 3.1 million barrels per day. Estimated oil export revenues rebounded by $1 billion to $12.7 billion but were 43% lower than a year ago."[7]

[7] https://www.iea.org/reports/oil-market-report-april-2023

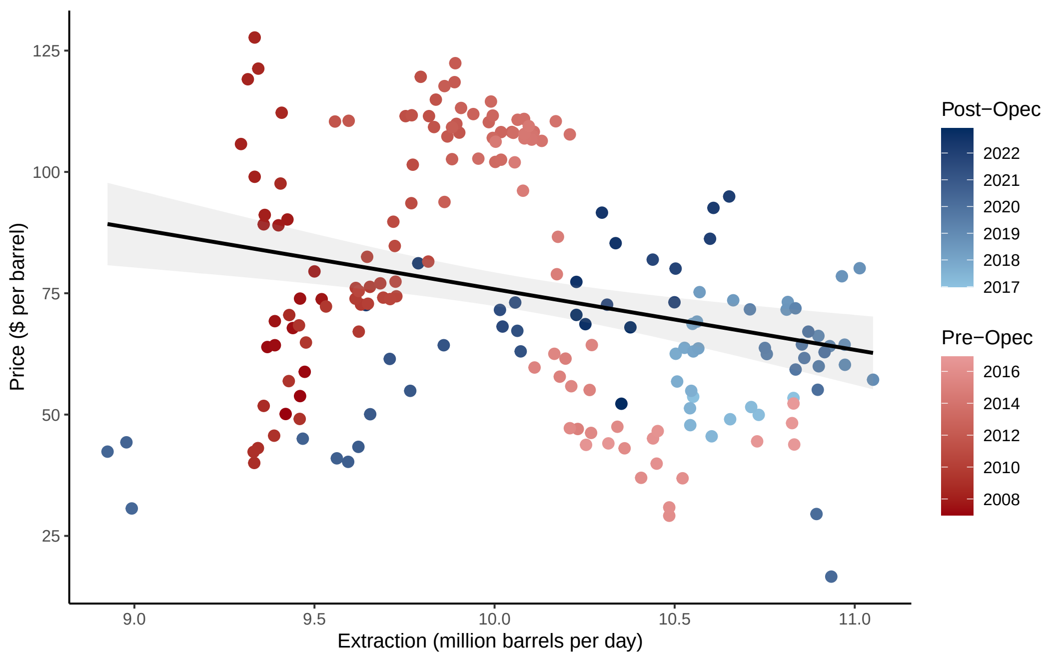


Figure 5: Russia's oil extraction versus Urals price, January 2008 - December 2022. Source: OPEC, Platts, Argus via Statista; U.S. Energy Information Administration.

2.4 Importance of oil revenues for Russia's current account and government budget

In 2021, oil (crude and product) was Russia's largest export by category, followed by natural gas and coal.[8] In total, energy accounted for over 50% of all export revenues, with oil accounting for 75% of energy exports. Reflecting the dependency on oil revenues, Russia's fiscal planning processes benchmark an oil price.

In the first nine months after the February 2022 invasion, Russia received significant revenue from the export of natural gas to Europe as it curtailed supply and prices rose to more than offset the lost revenue. Global oil prices also rose in the beginning of 2022, offsetting the growing discount so that the price for Russian oil was higher in 2022 than 2021. But European gas prices declined precipitously, by about 75%, in the six months beginning December 2022. Coal exports are also subject to sanctions, although Russia appears to have diverted some coal exports to China. Looking forward, if Russia is to earn a significant amount of foreign exchange, it is under pressure to keep oil production and exports as high as possible.

[8] https://oec.world/en/profile/country/rus

Reinforcing this pressure is the fact that Russia cannot freely access official foreign exchange reserves that were accumulated before February 2022. Russia ran a current account surplus for many years, and had a pre-invasion stock of foreign assets owned or controlled by the central bank of more than $500bn. Immediately after the invasion, the G7 forbade western counterparties from transacting with the central bank for these amounts, effectively freezing the funds. However, Russia was allowed to receive payment for ongoing oil (and other) exports, and claims on foreign entities built up during 2022 in Gazprombank and other nominally private entities. Given the continuing conflict, it is possible that those balances would be subject to an additional freeze, amounting to a form of ex post price cap.

2.5 Policies to reduce Russian oil revenues

It is important to consider the price cap relative to alternative policies that were floated at the beginning of the war, particularly the EU's 6th sanctions package. Before the price cap took shape, policy makers were stuck with two unattractive options. On the one hand, given the importance of oil revenues to Russia's fiscal well-being, indefinitely exempting oil exports from sanctions was unpopular and likely politically untenable. On the other hand, if policymakers used conventional approaches to sanctioning an oil exporting country, such as embargoes, Russia's sheer size meant that they risked driving up global oil prices, an especially unattractive option in early 2022 as the battered global economy was trying to emerge from the pandemic.

For example, in early March 2022 the U.S. Congress considered measures against Russian energy exports. The U.S. eventually banned imports of Russian energy, including oil, although this was criticized as a largely symbolic gesture since U.S. purchases did not constitute a large share of Russian energy exports. There were calls for the U.S. to take more decisive steps, including imposing secondary sanctions on Russian oil.[9] Like the approach taken to oil exports from Iran and Venezuela, secondary sanctions would have aimed to prohibit all trade in Russian oil. While there would have been some inevitable slippage and some Russian oil would have reached the market, secondary sanctions would have most likely removed millions of barrels of oil per day from global markets. The European Union's 6th sanctions package did not go as far as secondary sanctions might have but would have both banned imports of a large share of Russian oil and banned European companies from provid-

[9] See, for example, the *Wall Street Journal* editorial on March 6, 2022 https://www.wsj.com/articles/putin-sanction-russia-oil-exports-imports-invasion-ukraine-war-lng-natural-gas-energy-prices-climate-c reflink=desktopwebshare_permalink

ing services to redirect Russian oil to other markets, which could have removed substantial volumes of Russian oil from the global market.

Oil price movements in both time periods highlight the potential risks of those policies. Brent oil prices closed above $133 per barrel on March 9, 2022 (almost twice the average price of $71 per barrel in 2021), as word of potential U.S. Congressional action reached markets and again closed at $127 per barrel on June 10, 2022, after the European Union announced its 6th sanctions package. In this context, a crucial dual role of the price cap, which was publicized in an announcement by G7 leaders at the end of June 2022, was to harm Russia and prevent it from earning a war premium on its oil exports while maintaining stability in global oil markets.

Figure 6 plots monthly Brent crude oil prices, as well as prices for a basket of other commodities (both series are normalized to 100 in November 2022). The "Other Commodities" grouping summarizes world economic trends, including the Chinese economy, news about the war and other factors. As noted, oil prices peaked after the announcement of the EU 6th package in late June, later than other world commodities. Oil prices then fell more than other commodities and were flat until December 2022, when the price cap came into effect, and they fell again. Contemporaneous reports suggested that the market was unsure about how Russia would react to the cap.[10] Consistent with this, global oil inventories accumulated ahead of the December announcement and then declined in several months after the price cap was announced and it was apparent that Russia would continue to export oil.

2.6 Price cap implementation details

The price cap operates by setting terms and conditions on the provision of western financial and shipping services. Specifically, services can only be provided for shipping Russian oil by companies located in price cap coalition countries if the price paid to Russia is at or below the cap.[11] The caps were initially set at $60 per barrel for crude, $100 per barrel for high-value refined products (including diesel, gasoline and kerosene) and $45 per barrel for

[10] For example, in response to the announcement of a potential price cap regime, some western analysts predicted that Russia would reduce supply and drive up oil prices – analysts at JPMorgan Chase predicted that oil could reach $380 per barrel as a result. (See https://www.bloomberg.com/news/articles/2022-07-01/jpmorgan-sees-stratospheric-380-oil-on-worst-case-russian-cut.) In Section 5 we provide estimates of elasticities of oil demand and deduce the likely price impact of a production shut-in. For prices to go above $300 per barrel on a sustained basis, one needs to assume that Russia essentially shuts in all of its supply and that the world demand is extremely inelastic.

[11] In addition to the G7, EU and Australia, Albania, Bosnia and Herzegovina, Iceland, Liechtenstein, Montenegro, North Macedonia, Norway, Switzerland and Ukraine have all pledged to follow EU sanctions against Russia.

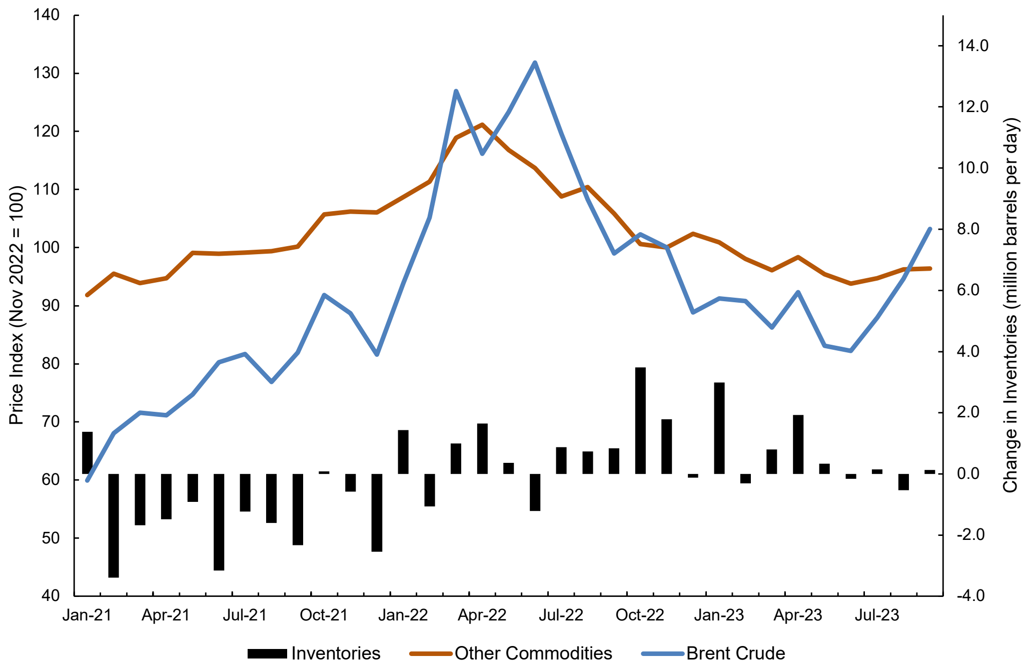


Figure 6: Commodity prices, oil prices and oil inventories, January 2021 - September 2023. Source: World Bank (commodity prices) and U.S. Energy Information Administration (oil inventories).

low-value refined products (including fuel oil and naptha).

As discussed above, the price cap was implemented in response to the EU's 6th sanctions package, which would have banned the provision of services for shipments of Russian oil altogether and could have reduced the supply of Russian oil to world markets considerably. The price cap effectively allows for an exception to that outright ban. Before the price cap was put in place, some market observers expressed concern that, if compliance with it was too complicated, western service providers would de-risk and pull back from the Russia trade completely – driving up world oil prices in much the same way as was feared under the EU's 6th sanctions package.

However, in contrast to any dire predictions, setting a price cap on Russian oil at $60 per barrel seems to have had four broad effects. First, as of September 2023, the Kremlin's oil-related revenues had fallen by 49% compared to the March to November 2022 period and 23% compared to the January 2021 to January 2022 period despite the fact that global oil prices only fell by 22% in the first instance and increased by 11% in the second. Specifically, the vertical bars in Figure 7 reflect Russian government revenue from the mineral extraction and export taxes by month. The orange horizontal bars reflect averages during the pre-war, post-war and pre-price cap and post-price cap period (see also Hilgenstock et al. (2023)). Oil prices are represented on Figure 6. Second, Russia's oil exports have stayed roughly steady (see Figures 2 and 3). Third, the advent of the EU embargo (for crude in December and refined products in February) did not result in a spike in world oil prices. Fourth, most western service providers remained engaged in the Russia trade. Data from CREA suggest that about 60% of crude oil shipments and 75% of product shipments from Russia's ports in April 2023 were covered by insurers from the EU, G7 or Norway.

As a result, Russia's reported current account surplus has declined commensurately. And this has happened without a large supply shock in the world market for oil. These outcomes have been achieved through the combination of the embargo and the price cap which, relative to the 6th sanctions package originally announced, has allowed Russian oil to flow to world markets.

There are, however, serious concerns about what comes next. Russia is amassing access to a shadow fleet, which does not operate out of western countries. A major concern has already appeared with regard to trade out of Kozmino (the Russian "far east") – some of this oil is moving to China and, reportedly, to Australia despite the fact that the oil is bought at a price above the capped level and shipped using western services.

There are also questions about who exactly in the value chain is making higher than

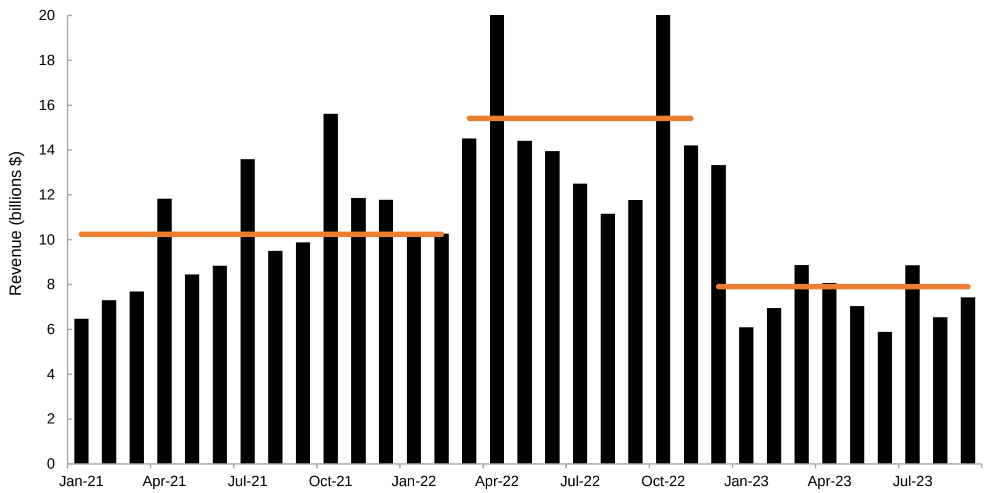


Figure 7: Russia's oil production and government tax revenue, January 2021 - September 2023. Source: ESAI Energy.

normal profits from this arrangement. If an entity, e.g., in India, buys crude at or below the cap, it is allowed to sell the refined product at world prices. This arrangement is expected to encourage the flow of Russian oil and helps explain why Russian deliveries to the world market are largely unchanged. But who exactly is profiting from the difference (world price minus capped price) remains shrouded in some mystery. As one example, a *Wall Street Journal* article in April 2023 cited evidence that Saudi Arabia and the United Arab Emirates were importing Russian oil products at low prices and earning high profits (Faucon and Said (2023)), but no systematic accounting of where the rents have gone exists.

In the remainder of this paper, we provide an analytical framework intended to help organize thinking on the price cap.

3 Model of a price-taking producer

We begin with a model of a state producer of a commodity which takes the price of the commodity as given. Thus, we model a small producer whose decisions do not affect the global commodity market. This framework offers interesting insights in its own right, and is an important input into our analysis of equilibrium with market power in the next Section.

3.1 Producer's problem

We study a dynamic problem of an agent – e.g. a government of a country – endowed with x_0 amount of natural exhaustible resource, such as oil. The ultimate goal of this agent is to maximize utility from the flow of consumption, which is financed in part by the sales of the commodity. The country is small in the sense that it takes the price of the commodity p_t as given. We normalize $x_0 = 1$ without loss of generality.

The producer uses the proceeds from the sales of the commodity to finance consumption. However, the producer might face financial frictions, and as a result might be constrained in its ability to trade financial claims intertemporally. For example, the producer might face a low, possibly negative, interest rate on its financial savings, and a high, possibly infinite, interest rate on its borrowing.

To capture the trade-offs and frictions in a tractable and transparent way, we focus on the extraction decision, while keeping the consumption-saving problem in the background. Specifically, we assume that at each instant the producer's payoff function $\tilde{u}$ takes as an argument the profits from the sales of the commodity, $\pi_t := (p_t - c)y_t$, where y_t denotes the amount of oil extracted at time t and c is marginal cost of extraction (which we assume to be constant). We first state the maximization problem, and then discuss what is captured with this formulation.

Defining the function $u(y) := \tilde{u}(\pi(y))$ such that $u : \mathbb{R}^{\geq 0} \to \mathbb{R}$ satisfies $u_y > 0$, $u_{yy} \leq 0$, the producer solves the following problem:[12]

$$\max_{y_t} \left[\int_0^\infty e^{-rt} u(y_t) dt \right] \text{ subject to } dx_t = -y_t dt,\ x_t \geq 0,\ y_t \geq 0, \tag{1}$$

taking as given the stochastic process for p_t, which we discuss momentarily. r is a discount rate. The constraints say that the stock of reserves x_t diminishes by the amount extracted y_t, and that reserves and extraction must be non-negative.

3.2 Curvature of u as the degree of financial constraints

In the state producer's optimization problem we specified in (1), profits from oil sales enter as an argument in the felicity function u. The key advantage of this formulation is that the curvature of the u function indexes the degree to which the exporter is subject to financial

[12] If $p_t = p\ \forall t$, this becomes a canonical cake-eating problem in continuous time. An agent has a cake of size $x_0 = 1$ and decides on the optimal way of eating the cake, given time-separable preferences and the instantaneous utility u over consumption of cake y_t and a discount rate ρ. x_t is the size of the cake at t; $dx_t = -y_t dt$ simply says that the size of the cake gets smaller with each bite.

frictions, i.e. the degree to which it lives hand-to-mouth. To see why this is the case, consider an underlying utility maximization problem where the consumption of real goods and services appear as an argument in the *utility* function $v(c)$ of the exporter. In this underlying problem, the profits from oil sales – as well as the financial frictions faced by the producer – determine the budget set.

Two extremes are possible in the financial environment in which the producer is operating. One is the environment of perfect capital markets. The other is financial autarky.

In the former extreme, the producer has frictionless access to borrowing and lending opportunities at the interest rate r. As a result, the time path of revenues becomes irrelevant for producer's decisions – only the expected net present value of revenues matters. This extreme therefore corresponds to no curvature in the problem we have written down – a linear u function – and the maximand is just the net present value, $\int_0^\infty e^{-rt}\pi_t dt$. In other words, the producer maximizes the net present value of profits from oil production.

At the other extreme, if the producer cannot save or borrow in financial assets at all, it must consume the proceeds from oil sales each period, implying hand-to-mouth behavior. With $c_t = \pi_t \forall t$, $\tilde{u}$ inherits the curvature from the utility function v over consumption of the underlying consumption-choice problem. The maximand is $\int_0^\infty e^{-rt} v(c_t) dt$.

In between these two extremes, the curvature of $\tilde{u}$ (and hence of u) indexes the degree of financial imperfections. We further illustrate this in the Appendix in a two-period consumption-saving problem for which a closed-form solution is attainable.[13]

3.3 Stochastic process for the price of oil

A rich and complex combination of demand, supply, and market shocks result in daily fluctuations in commodity prices. To capture this volatility, we model it using a stochastic process called Cox–Ingersoll–Ross model (also known as a Feller square root process):

$$dp_t = D(\tilde{p} - p)dt + \sigma\sqrt{p}dW_t \tag{2}$$

where W_t is the standard Wiener process and $\tilde{p}$, D, and σ are (strictly positive) parameters that satisfy $2D\tilde{p} > \sigma^2$. Parameter D determines how quickly the gap between the

[13] Beyond the financial frictions interpretation, the formulation of the problem in (1) can also be motivated by the presence of dividend smoothing motives, as in the corporate finance literature (Lintner (1956), Fama and Babiak (1968), Cui (2022)), but applied to public finance. Smoother revenues from oil sales may aid fiscal (and war) planning and might help in achieving a smoother path for taxes that finance the budget (Barro (1979)).

current price and the average price $\tilde{p}$ closes (i.e. it determines the speed of mean reversion). Parameter σ determines the volatility of the price, driven by standard Brownian motion.

The process in (2) ensures that the price always stays positive: as $p \to 0$, the importance of Brownian noise diminishes, and mean reversion drives the price away from zero. There is no upper bound to the price: we have $p_t \in (0, \infty) \forall t$. Due to mean reversion, as time becomes large, the distribution of p_∞ will approach a Gamma distribution with the probability density function

$$f(p_\infty; D, \tilde{p}, \sigma) = \frac{\beta^\alpha}{\Gamma(\alpha)} p_\infty^{\alpha-1} e^{-\beta p_\infty},$$

where $\beta := \frac{2D}{\sigma^2}$, $\alpha := \frac{2D\tilde{p}}{\sigma^2}$ and $\Gamma(\alpha)$ is the Gamma function.[14]

3.4 Optimality conditions

The Hamilton-Jacobi-Bellman equation of the problem in (1) is:

$$\rho v(x, p) = \max_{y_t} u(y) - v_x(x, p) y + v_p(x, p) D(\tilde{p} - p) + \frac{1}{2} v_{pp}(x, p) \sigma^2 p. \tag{3}$$

The first order condition is

$$u_y(y) = v_x,$$

implying the optimal rate of extraction[15]

$$y = u_y^{-1}(v_x).$$

3.5 Parametrization

Equation (3) can only be solved numerically.[16] To do so, we first parametrize the model.

[14] The variance of the limiting distribution is $\frac{2D\tilde{p}}{\sigma^2}$.

[15] The constraints $x \geq 0$ and $y \geq 0$ give rise to a state boundary condition $u_y(0) = v_x(0, p)$. This is because at $x = 0$, extraction must be zero.

[16] An analytical solution exists only when there is no uncertainty and p is a constant parameter. In this case, an interior solution satisfies the Euler equation $\frac{\dot{y}}{y} = -\mathcal{E}(y)\rho$ where $\mathcal{E}(y) := \frac{-u_y}{u_{yy} y}$ is the intertemporal elasticity of substitution (IES).

3.5.1 The process for the price, and the marginal cost of extraction

Motivated by the price cap on Russian oil, we estimate the process in (2) using daily data on real oil prices from 1987.[17] We obtain $\tilde{p} = \$72$ (in today's prices), $\sigma = 2.97$ and $D = 0.26$. With these esimated values, the limiting distribution of the oil price is skewed to the right (Figure 8). Estimated standard deviation of the price is \$35.

We set the marginal cost equal to \$15 per barrel, in line with the current estimates for a range of producers, including Russia.

3.5.2 The felicity function u

We assume that the felicity function u belongs to a HARA (hyperbolic absolute risk aversion) class:

$$u(y) = \frac{\sigma}{1-\sigma}\left(\frac{\gamma(\pi+b)}{\sigma}\right)^{1-\sigma} \tag{4}$$

with $\sigma > 0$, $\gamma > 0$ and $b \geq 0$. This broad class includes notable special cases such as linear, quadratic, exponential, and isoelastic utility functions. Parameter $b \geq 0$ can be interpreted as alternative source of funds that the state has access to in order to finance its activities (most obviously, taxes on activities other than extraction of the commodity). If $b > 0$, the level of utility at zero extraction $u(0)$, and the marginal utility at that point $u_y(0)$ are both bounded. This is useful in so far as it implies that complete shut-in of production – limiting extraction all the way to zero – is potentially a viable option for the producer facing sanctions.

Two useful special cases of the utility function are Constant Absolute Risk Aversion (CARA) utility, obtained by setting $b = \sigma/\gamma$ and taking the limit as $\sigma \to \infty$:[18]

$$u(y) = -e^{-\gamma\pi} \tag{5}$$

and power utility, obtained by setting $\sigma = \gamma$:

$$u(y) = \frac{(\pi+b)^{1-\gamma}}{1-\gamma}. \tag{6}$$

[17] We obtain our data series from the FRED database. We deflate the daily nominal Brent oil price (code DCOILBRENTEU) by US CPI index (code CPIAUCSL_NBD20230401) set to 1 in April 2023 (we extrapolate from monthly to daily data using monthly averages). We use maxiumum likelihood estimation, making use of the numerical implementation by Kladivko (2013).

[18] We have $\lim_{\sigma\to\infty} \frac{\sigma}{1-\sigma}\left(\frac{\gamma}{\sigma}\pi+1\right)^{1-\sigma} = -\lim_{\sigma\to\infty}\left(\left(1+\frac{\gamma\pi}{\sigma}\right)^{\sigma}\right)^{\frac{1-\sigma}{\sigma}}$, since $\frac{\sigma}{1-\sigma}$ goes to -1 as $\sigma \to \infty$. Using the limit definition of the exponential, this limit equals $-\lim_{\sigma\to\infty}\left(e^{\gamma\pi}\right)^{\frac{1-\sigma}{\sigma}} = -e^{-\gamma\pi}$.

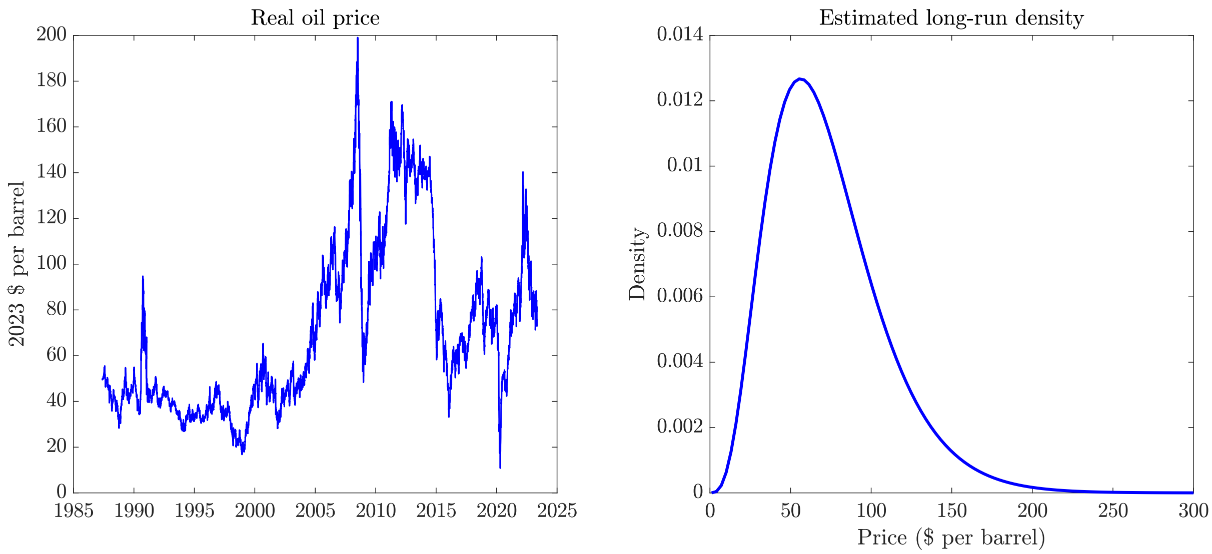


Figure 8: Data on real Brent oil prices used in estimation, and the long-run distribution of the estimated process.

Our baseline results assume that the u is a power function, as in (6). Our results are not sensitive to this choice, however. The Appendix reports the main results under the assumption that u is CARA.

We parametrize the objective function as follows. We want to set γ so as to capture the degree of financial frictions that the state faces. This parameter can lie between 0 (no frictions) and the curvature of the underlying utility function over consumption, $\frac{v''(c)c}{v'(c)}$, reflecting full financial autarky and hand-to-mouth behavior. A standard calibration would set the curvature over consumption in the $2-4$ range.[19] We set $\gamma = 2$, reflecting substantial, although probably not full, degree of financial frictions. We explore the sensitivity of our results to the assumed degree of financial frictions below.

We set the real interest rate that is used to discount future payoffs to 3%, to match the level of extraction of between 1 and 3% of the resource stock per year (when the producer has market power). Finally, we set $b = 2$, targeting a substantial share of state's income that comes from commodity sales. Our choice implies that income from commodity sales constitutes a substantial fraction – between 1/3 and 1/2 – of the overall income of the state.

[19]See e.g. Havranek et al. (2015) for a meta-study about this parameter across countries, and Best et al. (2020) for evidence using quasi-experimental variation from the UK.

3.6 Solution

Before studying the effects of the price cap policy in the next section, in this section we first analyze the solution to the problem we set out above and study the forces that are paramount in driving the producer's decisions. We solve the model using the finite difference method (we provide details of the solution method in the Appendix).

3.6.1 Policy function

The solution is a policy function $y(x, p)$ which specifies the optimal level of extraction at each price, for any level of reserves. This policy function is depicted in Figure 9. There is a flat region at the lower range of oil prices, where the producer does not extract any oil. This region gets larger as oil reserves decline: the cut-off price at which the producer extracts positive amount of the commodity increases as reserves decline. Beyond the cut-off price, as we move in the direction of higher prices, there is a steep increase in optimal extraction. But optimal extraction is non-monotonic in the price. We now explore why this is the case by zooming in on a specific part of the policy function – the contemporanous supply curve.

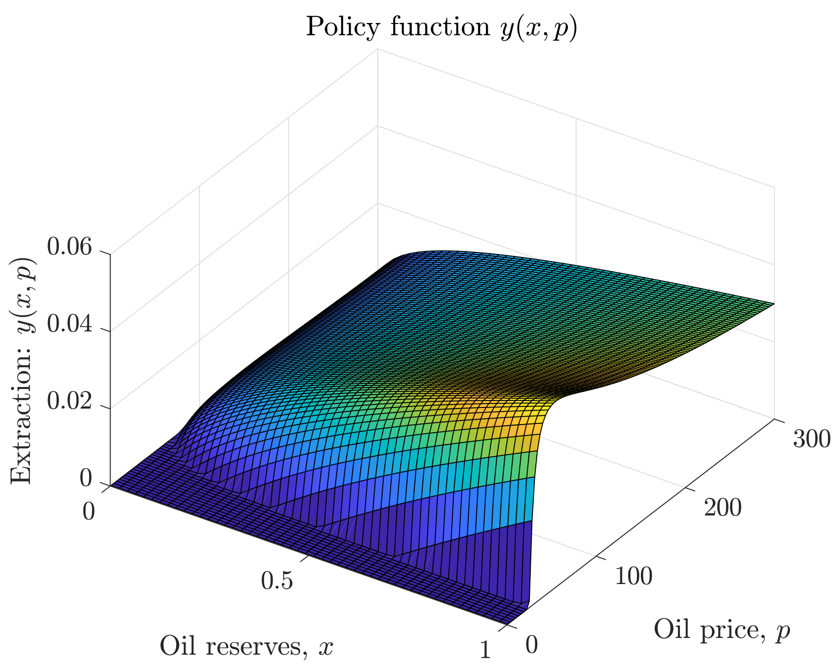


Figure 9: Optimal extraction when prices follow the estimated CIR process

3.6.2 Contemporaneous supply curve

Of particular policy interest are the contemporaneous incentives of the state: that is, we are interested in what the solution of the dynamic optimization problem implies for decisions today. To focus on this question, we concentrate the analysis on the slice of the policy function at which oil reserves are at today's levels: $y(1, p)$. This is the contemporaneous supply curve of the producer, which we depict in Figure 10. This Figure plots on the x-axis the extraction rate, and on the y-axis the ongoing price of oil. It is thus a standard supply curve linking optimal amount extracted to the ongoing price.

This figure shows clearly that the supply curve is highly non-monotonic. We first discuss the forces behind this, and then perform comparative statics with respect to the key parameters.

When prices are about \$24 or lower, the producer ceases to extract the commodity. Note that this cutoff price is above marginal cost, assumed to be \$15 in our analysis. Driving this is the *option value of waiting effect:* at low prices, it is optimal for the producer to save oil underground and wait until prices increase. As prices rise above the cutoff, the producer increases extraction sharply, resulting in an upward sloping and highly elastic (almost flat) supply curve in the \$30 per barrel region.

At high prices, beyond \$50 per barrel, the supply curve becomes highly inelastic and eventually bends backwards: higher prices now result in lower extraction rates. Driving this pattern is the *smoothing effect:* since the producer is financially constrained, it has incentives to smooth revenues from commodity sales (as smoother revenues will translate into smoother consumption path). At high prices revenues are substantial already. In that region the producer saves its oil reserves for more challenging times,leaving them in the ground. These reserves are then used to smooth revenues when prices fall.

3.6.3 Forces shaping the supply curve

To explore the forces shaping the supply curve in more detail, Figure 11 decomposes the supply curve, taking as a reference benchmark the supply curve of a producer who faces no uncertainty in the price of oil and has no access to alternative source of funds. It is easy to see that when p is fixed (and can be treated as a parameter, rather than as a state variable), and when $b = 0$, the maximization problem of the producer boils down to a simple

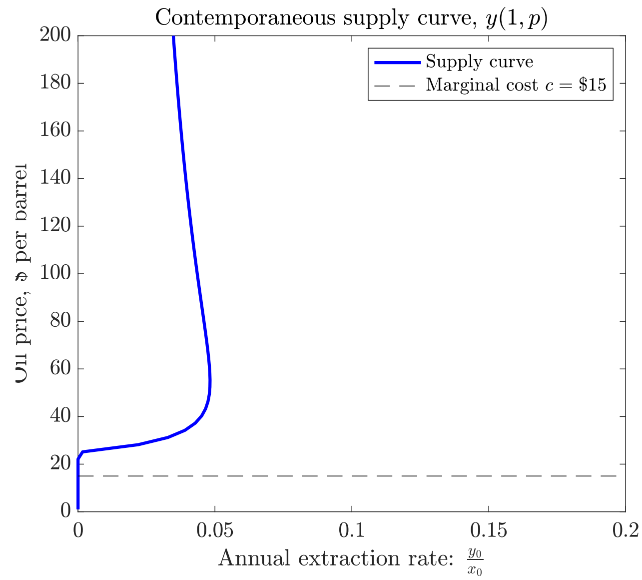


Figure 10: Supply curve when price is stochastic

Hamilton-Jacobi-Bellman equation

$$rv(x) = \max_y \frac{(y(p-c))^{1-\gamma}}{1-\gamma} - v_x(x)y.$$

As long as $p > c$, this equation has a (closed form) solution, namely, the producer extracts a constant fraction of remaining reserves:

$$y_t = \frac{r}{\gamma} x_t \ \forall t.$$

Since this policy is independent of p, the contemporaneous supply curve is a vertical line at $\frac{r}{\gamma}$ – the solid black line in the left panel of Figure 11.

Relative to this benchmark case, the fact that oil prices fluctuate induces the smoothing effect. Letting prices follow the estimated Cox-Ingersoll-Ross process, but maintaining the homotheticty of preferences assumption of $b = 0$, results in a downward sloping supply curve (the red line in Figure 11). As prices approach marginal cost, the producer extracts ever larger amounts, in order to offset the impact on profits. Note that, since utility diverges to $-\infty$ when income from oil sales diminishes towards zero, this force becomes stronger the lower the resource price. When price is high, the producer does the opposite: it reduces extraction rate, saving oil in the ground for the times when prices are low.

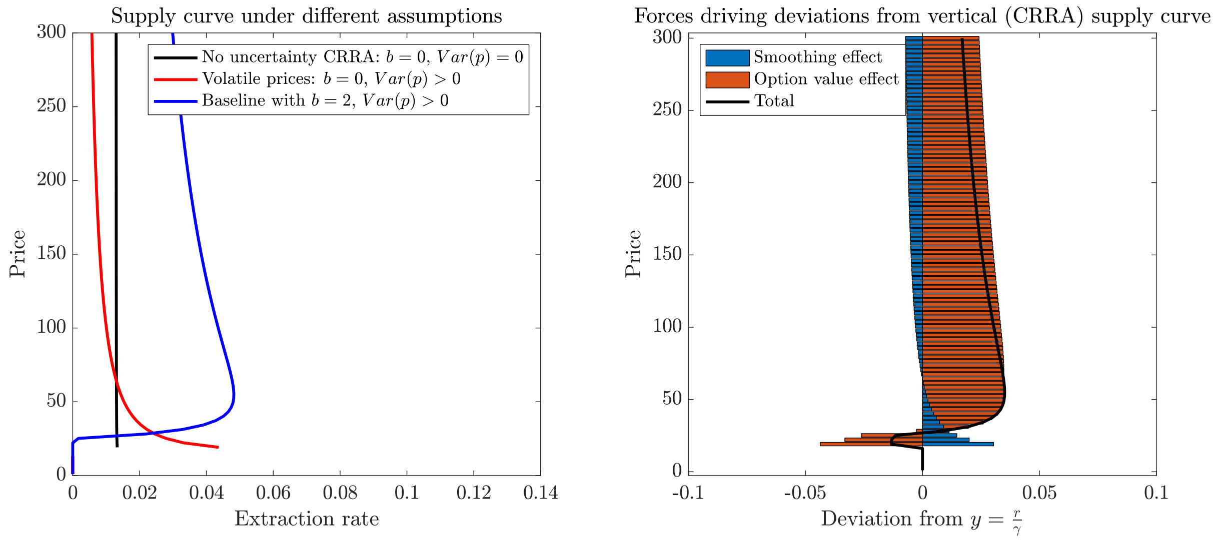


Figure 11: Forces shaping the contemporaneous supply curve

Allowing for the additional source of revenue for the producer – letting $b > 0$ – means that the producer is able and willing to cut extraction when prices are close to marginal cost (since utility no longer diverges to $-\infty$ as revenues dry up). The option value effect is allowed to operate: with a secondary source of income, the outcomes are not so dire when commodity revenues are low, and so it is optimal to withhold oil in the ground and wait for more lucrative terms of trade ahead.

The right panel of Figure 11 decomposes the difference between the supply curve in our baseline model (the blue line in the left panel) and the benchmark no-uncertainty CRRA case (the vertical black line) into the two effects. While the option value effect dominates at low prices, the smoothing effect drives the downward slope of the supply curve.

3.6.4 How the results depend on the degree of financial frictions

Recall that the producer whose decisions we are analysing operates under financial frictions, and the severity of these frictions is indexed by parameter γ. We now consider how the balance of the forces discussed above changes as we vary the degree of financial frictions.

Figure 12 shows our baseline case in the solid line, as well as three alternative calibrations. We do not view these calibrations as representing realistic alternatives – rather, they are extreme cases that illustrate the direction of the comparative statics with respect to γ well.

The pink-dashed line shows the effect of tightening the degree of financial frictions (corresponding to a high γ of 5). Since the timing of the flow of revenues matters more in this

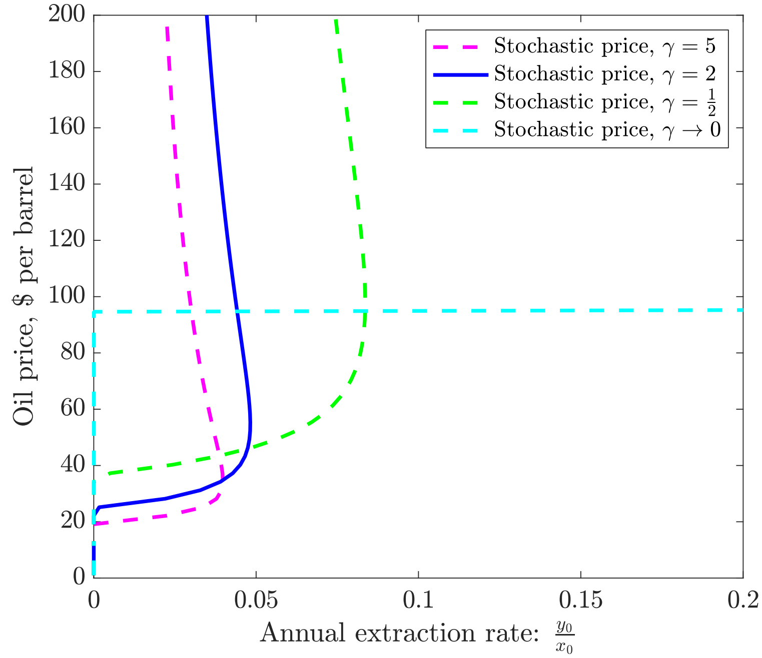


Figure 12: The supply curve and the degree of financial frictions

case, the smoothing effect is more powerful. It takes an even lower price for the producer to leave the commodity underground, and the production at high prices is significantly curtailed. Overall, the supply curve shifts in (except for prices just above marginal cost) – the producer becomes more intertemporally inelastic, and so spreads the revenues from the commodity more evenly across time, extracting less today but, given finiteness of reserves, being able to operate for longer.

The green-dashed line illustrates what happens when the degree of financial frictions is less severe (we set $\gamma = 0.5$ in this case). This strengthens the option value effect: the producer starts restricting supply at a relatively high prices (around \$80), and reduces extraction to all the way to zero as the price drops below \$40. Even with this small degree of frictions, the smoothing effect is still there with a clearly visible downward slope of the supply curve at prices above \$80 per barrel.

In the limit, as financial frictions disappear and $\gamma \to 0$ (the u function becomes linear), the supply curve becomes discontinuous, and the producer extracts all of its reserves as long as prices are high, and extracts nothing otherwise (the turquoise dashed line).

We summarise this Section in the following remark.

Remark 1. *In a model with no market power, the optimal extraction policy today is non-*

monotonic with respect to the price.

- *As long as the producer has bounded utility at zero extraction – e.g. because of alternative sources of income – it is optimal to reduce extraction to zero when prices are close to marginal cost. This is due to the* option value effect*: producer optimally saves the commodity until it can achieve better terms of trade.*

- *At higher prices, the supply curve is downward sloping, meaning that extraction decreases in price. This is due to the* smoothing effect*: with financial frictions, a smoother path of income implies a smoother path of consumption. When prices are high, desired income level can be achieved with lower extraction.*

4 Price cap

This Section incorporates a price cap policy into the framework outlined so far. The price cap we consider in this section is "perfect", in the sense that it applies to all of the exporter's sales. We also assume it is permanent. We continue to assume that the producer has no market power in the global market for oil (i.e., that it faces a perfectly elastic demand). We relax both assumptions in subsequent analysis.

4.1 Price that the producer receives under a price cap

A price cap limits upside exposure to the volatility in oil prices. Denoting with p_r the price received by the producer, we have

$$p_r = \min\{p, \bar{p}\}. \tag{7}$$

The price that Russia receives for its oil is simply the cap $\bar{p}$ whenever the price cap is binding, and the ongoing price when it is not. The resulting distribution of prices faced by Russia is depicted in Figure 13.[20]

4.2 How does a price cap affect supply?

There are two effects that a price cap has on optimal extraction behavior and thus on the supply schedule.

[20]Formally, there is a Dirac point mass at $\bar{p}$.

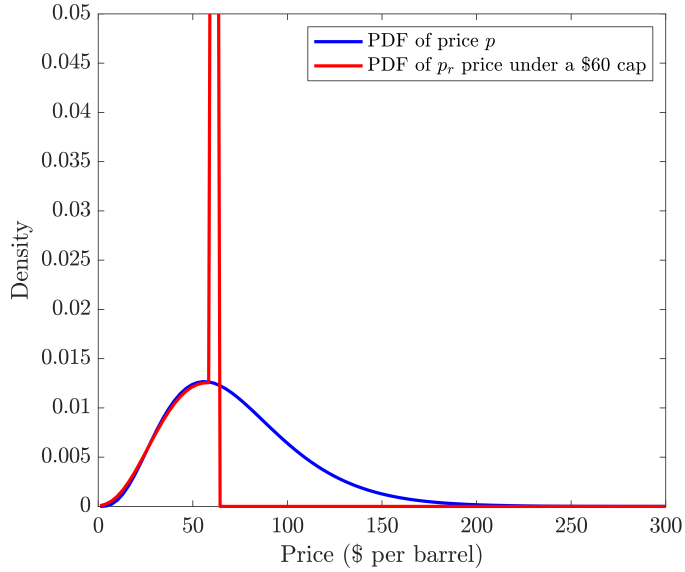


Figure 13: Distribution of the oil prices faced by Russia under the cap

First, the price cap changes the nature of the stochastic process of the price that the producer receives for its commodity. It limits the upside from the swings in the price, effectively reducing uncertainty faced by the producer. In other words, it brings the environment that the producer is operating in closer to one without uncertainty, but with a permanently lower average price. As a result of this fundamental change, the policy function – and so the contemporaneous supply curve – shifts towards the supply curve that would have been observed absent uncertainty. In our calibration, the supply curve shifts outwards for any price cap below $60 per barrel.[21] Thus, for any level of the price, the producer tends to extract more of the commodity with the cap than without.

Second, when the price cap is binding, the fluctuations in the price do not affect the exporter's revenues. As a result, the supply curve becomes insensitive to global prices at and above the price cap.[22]

[21]The precise intuition for this shift is as follows. From the producer's perspective, the price cap makes the resources buried underground less valuable. With non-homothetic u function due to alternative source of income $b > 0$, less valuable resource implies a higher extraction rate. To see the intuition why, consider for example a producer with $b > 0$ who has only one last barrel of oil in the ground. This producer will exhaust all the resource in finite (and likely very short!) time, i.e. it would have a high extraction *rate* (unlike in the homothetic case with $b = 0$, where a constant fraction of a very small pool of resources will be extracted ad infinitum, implying that reserves will diminish only asymptotically). The price cap effectively maps into less valuable resource pool, raising the extraction rate through the same mechanism. Formally, with $b > 0$, the producer becomes more intertemporally elastic at lower prices.

[22]The supply schedule is not *exactly* vertical above $\bar{p}$, because the expected duration of the price being above the cap is different at different levels of the reference price: if the price today is at $200 per barrel, it

Figure 14 shows how this intuition translates into a supply curve under a price cap. It shows the supply schedules under no cap and under three alternative caps, the \$60 cap that has been implemented, the lower \$45 cap, and the \$30 cap proposed e.g., by The International Working Group on Russian Sanctions (2023). As anticipated above, in each case the supply curve features a close to vertical segment above the price cap, as the producer's decisions become insensitive to fluctuations in p. The supply curve shifts out and to the right, more so the lower the cap. Thus, implementing a price cap can lead to an increase in the quantity of the commodity supplied to the market, and a lower price cap leads to larger increases in supply. This is an important conclusion as it goes strongly against the static intuition that limiting prices will necessarily lower the quantity supplied by the sanctioned state. Of course, the price cap set below marginal cost will result in sharp declines in quantity extracted. Since the exact level of the marginal cost is not known, policy that takes this uncertainty into account would want to be more cautious in setting the cap.

We summarize these important findings in the following proposition:

Proposition 1. *In a model with no market power a price cap can lead to an increase in the quantity of sanctioned exporter's supply to the global market.*

4.3 How much does a price cap hurt the producer?

Price caps are the new weapons of economic warfare. But how powerful are they? Back-of-the-envelope calculations can give us some sense of the revenue losses, but tell us little about the dynamic *welfare* losses. To address this, we can use our model to compute the loss of welfare that Russia suffers as a result of the price caps set at different levels. Figure 15 plots the model-based measure of welfare (the value function $v(x, p)$) under different assumptions about the price cap. The x-axis denotes the amount of reserves still in the ground, so that the right-most point corresponds to welfare from having today's level of oil reserves.

The model suggests that the impact of the \$60 cap imposed so far – recall that we assume that the cap is permanent – is to reduce the welfare from oil by about 20%, equivalent to reduction in reserves of about 35%. And lowering the price cap further would deal an even more significant blow to Russia. With a \$30 cap, the hit to oil welfare would be in the region of 50%, equivalent to wiping out 80% of Russia's reserves.

will take some time to cross the $\bar{p} = \$60$ threshold, while if it is \$65, there is a good chance it will be below the cap soon.

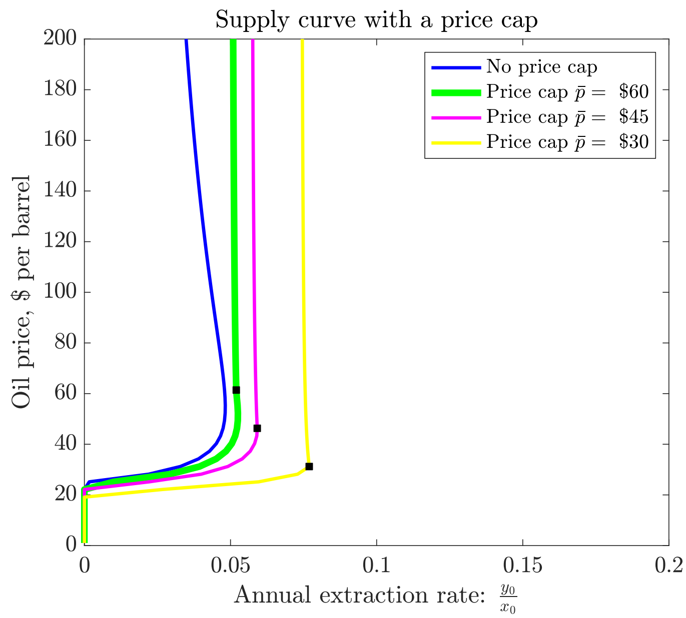


Figure 14: Russia's supply curve under three price cap regimes

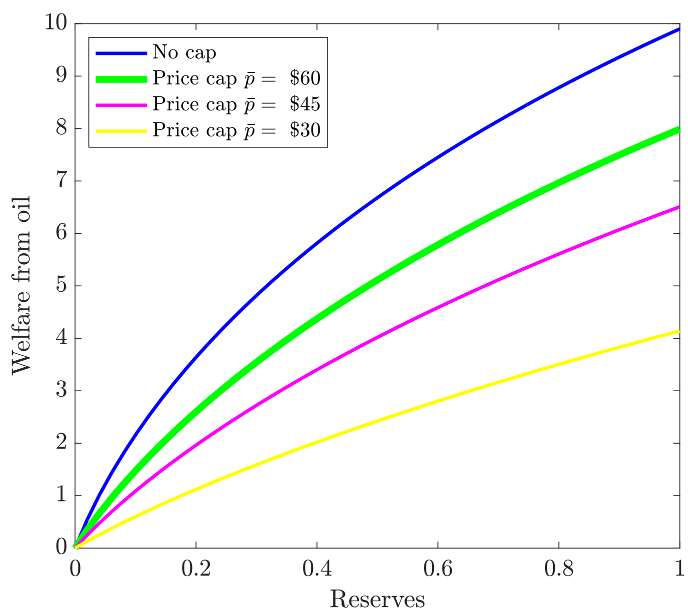


Figure 15: Value functions with and without a price cap

Note: the value function is normalized by adding a constant so that $v(0, p) = 0$. The value functions are plotted assuming that current oil price is \$90 per barrel, but the current price does not affect the results quantitatively.

The welfare results must be interpreted with caution, since they miss important feedbacks that are due to the producer's potential to exercise market power and miss the fact that the cap does not apply to all exports and may not be perfectly enforced. We return to the welfare calculations below, once we introduce these important elements into our framework.

5 A model with market power

We now enrich our model by considering a state that is large enough to affect global equilibrium prices. In the current context, we want to study the implications of the fact that by limiting supply, Russia can raise the global benchmark oil prices, potentially causing a damaging energy supply shock.

To do so, we develop a new model of equilibrium price determination that captures both the stochastic behavior of commodity prices, driven by a range of exogenous shocks in the global economy, as well as the endogenous determination of equilibrium prices that results from the optimal use of market power by the state producer. To the best of our knowledge the model is new to the literature and so is one of the major contributions of this paper.

5.1 World demand for oil and producer's market power

We denote the world price of oil with $p_{w,t}$, and assume that the global demand for oil is isoelastic

$$p_{w,t} = \delta_t(\bar{y} + y_t)^{-\epsilon}, \tag{8}$$

where parameter $\epsilon \geq 0$ is the inverse of the elasticity of demand, $\bar{y}$ is average level of world supply excluding the state producer we are focusing on, and y_t, as before, is the state producer's output. The fact that global supply $\bar{y}$ is time invariant is without loss of generality, because of the time-varying δ_t term which represents shocks to global demand and supply of the commodity. These shocks induce a stochastic process for the *reference price,* p_t, which follows the Cox-Ingersoll-Ross process, as specified in equation (2).

The reference price p_t is an important concept in what follows. It is a hypothetical equilibrium price that would prevail *if the exporter did not exercise its market power.* The idea here is that while in the background there are shocks in the global economy (and the commodity market itself) that drive the fluctuations in the reference price, the state producer whose decisions we are studying can, through its use of market power, steer equilibrium prices above the reference price.

Another important concept in our model is the "no-market-power" level of extraction $y_{N,t}$. This is a hypothetical quantity of the commodity supplied to the market by the producer under the assumption that the producer exercises no market power. In other words, it answers the following question: given the state variables in the producer's problem, what would be the quantity supplied to the market by the producer who had no pricing power but was otherwise identical? Since there are two state variables of the producer's problem – the level of reserves and the price of the commodity – $y_{N,t}$ is a function $y_{N,t}(x,p)$. Importantly, we have already characterized this function: it is simply the optimal level of extraction absent market power, depicted in Figure 9.

The definitions of the variables introduced so far imply

$$p_{w,t} = \delta_t \left(\bar{y} + y_t(x,p)\right)^{-\epsilon} \tag{9}$$

$$p_t = \delta_t \left(\bar{y} + y_{N,t}(x,p)\right)^{-\epsilon}. \tag{10}$$

where $p_{w,t}$ is the equilibrium price and p_t is the reference price of the commodity. Taking the ratio, and dropping the time subscripts where it does not create confusion, and re-arranging, we get:

$$p_w = p\left(\frac{\bar{y}}{\bar{y}+y_N(x,p)} + \frac{y_N(x,p)}{\bar{y}+y_N(x,p)}\frac{y(x,p)}{y_N(x,p)}\right)^{-\epsilon}. \tag{11}$$

Clearly, producer's extraction y influences the world price p_w. The degree of producer's market power is governed by the slope of the demand curve ϵ and what we call the *reference market share* $\psi(x,p) := \frac{y_N(x,p)}{\bar{y}+y_N(x,p)}$. The greater is this share, the greater is the ability of the producer to affect global prices. In the limit, if the producer's share in global production is close to zero, such producer cannot affect world prices at all.

We denote the contemporaneous value of ψ with α:

$$\alpha := \frac{y_N(1,p)}{\bar{y}+y_N(1,p)}.$$

The reference production share $\psi(x,p)$ is then

$$\psi(x,p) = \frac{y_N(x,p)}{\frac{1-\alpha}{\alpha}y_N(1,p) + y_N(x,p)}. \tag{12}$$

Combining equations (11) and (12) we obtain the expression for the equilibrium world

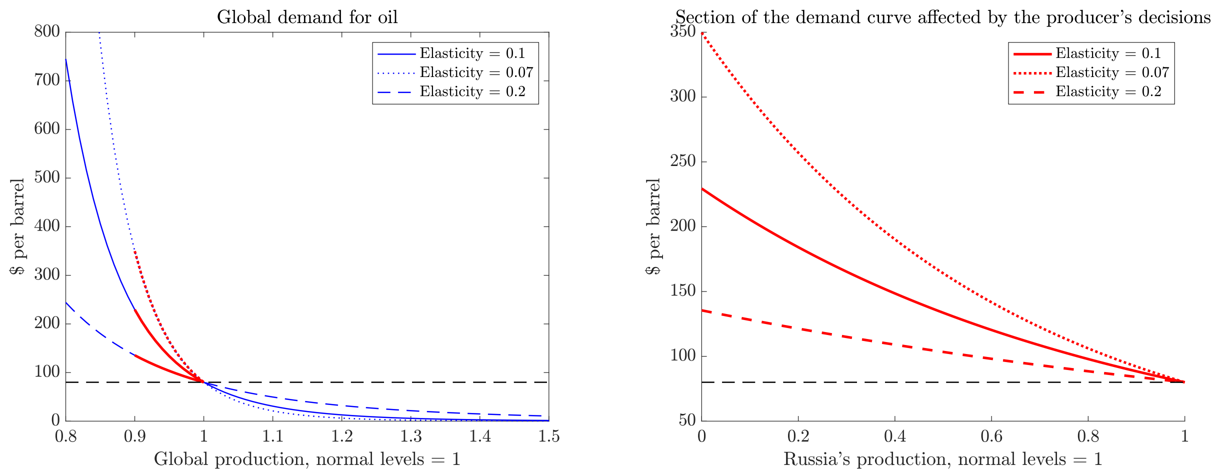


Figure 16: World demand for oil

price:

$$p_w = p\left((1-\psi(x,p)) + \psi(x,p)\cdot\frac{y(x,p)}{y_N(x,p)}\right)^{-\epsilon}. \tag{13}$$

The equilibrium price of oil is driven by two forces in this model. First, demand and supply shocks buffet the global economy and the commodity market, driving the stochastic evolution of the reference price p according to (2). Second, the producer optimally exercises its market power, potentially reducing the quantity produced below y_N and therefore raising the world price above the reference price p.

To illustrate this explicitly, Figure 16 plots the world demand curve for three different values of the demand elasticity $1/\epsilon$, and highlights the section of the world demand curve that can be reached, depending on the producer's extraction decisions, if such a producer controls 10% of the global supply of the commodity. For example, the Figure says that if today's reference price is \$80 per barrel and if the demand elasticity is 0.1, then prices would rise to \$230 per barrel if the producer withheld all of its supply. Clearly, more inelastic demand and a higher market share gives the producer more market power.

5.2 An aside: OPEC

While our framework focuses on the single commodity producer and does not explicitly model the strategic behavior of the OPEC cartel, there is a close mapping from the parameters of our framework into OPEC's behavior.

First, to the extent that the cartel targets the specific level of the price of oil in global markets, the average price parameter $\tilde{p}$ in equation (2) can be thought as reflecting that target level (plus some spread that accounts for the use of market power by the state producer).

Second, the responsiveness of the OPEC cartel to shocks in the global market is subsumed in the parameter D in the process for the reference price of oil in equation (2). This parameter determines the speed of mean-reversion of the oil price. If OPEC stands ready to adjust the production levels of oil to stabilize oil prices, this mean reversion parameter is high, and consequently oil prices will move within narrow bands around $\tilde{p}$. Conversely, if the control OPEC has over prices is weak, large and persistent deviations from the target price would occur, and parameter D would be estimated to have a lower value.

Third, the responsiveness of the OPEC cartel to shocks emanating from inside of Russia is one of the forces that ultimately shapes the value of the elasticity parameter ϵ in our model. In other words, we view this parameter as capturing both the slope of the world demand curve *and* OPEC's willingness to tolerate unilateral use of market power. Suppose that Russia restricts its supply of oil in the hope of driving up the oil price. A strong OPEC response in the opposite direction would make these efforts futile. This can be mapped in a reduced-form way into a low ϵ, which ultimately tells us how successful is the unilateral action of the producer in raising prices. Conversely, if OPEC responds to Russia's cuts in supply by holding production steady, one might expect the world price to be significantly more responsive. This can be captured by setting ϵ to a high value, making world demand that enters Russia's decision problem inelastic.

We now turn to the producer's problem to determine whether and how the producer exerts its market power in this environment. We start with an environment with no price cap in place.

5.3 Producer's problem when the producer has market power

The optimization problem of the producer becomes:

$$\max_{y_t} E_0 \left[\int_0^\infty e^{-\rho t} \tilde{u}(\pi_t) dt \right] \text{ subject to } dx_t = -y_t dt,\ x_t \geq 0,\ y_t \geq 0 \tag{14}$$

and (2), where now

$$\pi_t = (p_{w,t} - c) y_t = \left(p_t \left((1 - \psi_t) + \psi_t \frac{y_t}{y_{N,t}} \right)^{-\epsilon} - c \right) y_t \tag{15}$$

in which $y_{N,t}$ is the policy function that solves problem (3) and p_t is the reference price that follows the diffusion process specified in (2). Note that when the producer extracts the no-market-power reference level of output y_N, profits are simply equal to $(p - c)y$.

The above problem is complex, not least because the degree of market power changes dynamically and is endogenous to producer's actions. Specifically, extraction decisions at t affect future reference market share ψ_s and reference output $y_{N,s}$ for all $s > t$. The following Proposition derives the necessary conditions for a solution of this dynamic monopoly problem.

Proposition 2. *The optimal extraction path satisfies the necessary condition*

$$\tilde{u}_\pi(\pi_t) \cdot (p_{w,t} \cdot (1 - \varepsilon_{D,t}) - c) = v_x, \tag{16}$$

where

$$\varepsilon_{D,t} := -\frac{\partial p_{w,t}}{\partial y} \frac{y_t}{p_{w,t}} = \epsilon \cdot \psi_t \cdot \frac{y_t}{(1 - \psi_t) y_{N,t} + \psi_t y_t}. \tag{17}$$

is the effective elasticity of demand.

Equation (16) states that at the optimum the marginal utility of extraction is equal to the marginal value of reserves, and thus as accords with standard intuition in dynamic optimization. In turn, equation (17) shows that the marginal revenue depends upon the effective elasticity of demand ε_D, which is endogenous and consists of three terms: the elasticity of demand parameter ϵ, the shadow market share ψ_t (itself a function of (x, p)), and the rate of extraction.

To understand the intuition for why the effective elasticity takes this form, consider the extremes. That is, suppose $\psi_t = 1$, i.e. the state in question is the sole producer of the commodity in the world. Then $\varepsilon_D = \epsilon$, as in the standard monopoly problem with isoelastic demand. If on the other extreme $\psi_t \to 0$, the producer is small and its actions do not affect global prices ($\varepsilon_D \to 0$).

5.4 Equilibrium

An *equilibrium* is a policy function $y(x, p)$ that solves producer's problem (14) and the price function $p_w(y(x, p), p)$ that clears the market for oil (and thus satisfies (13)).

5.5 Parametrizing the elasticity of world demand for oil

Estimating oil demand elasticity is a subject of an extensive empirical literature. Meta-analysis in Uria-martinez et al. (2018) suggests the range for this elasticity in the short-run (around one year) is in the $[0.07, 0.14]$ range, while the long-run elasticity (after over a decade) is within the $[0.26, 0.82]$ range.[23] Intuitively, the price responds more in the short-run than in the long-run. In our analysis we want to err on endowing the producer with significant market power. At the same time, we recognize that, in the case of oil, any single country might struggle to control prices unilaterally, given the cartel structure of the market (see the discussion of OPEC above). To reflect these considerations, we set the ϵ parameter towards the upper end of the short-run estimates of the world demand elasticity, with $1/\epsilon = 0.14$ (i.e. we set $\epsilon = 7$). As we shall see momentarily, this endows the producer with very significant pricing power (in term of how much the actions of the producer impact on world prices).

5.6 Characterization

We develop a novel algorithm to solve the model with market power. In a nutshell, we first solve the model with no market power which yields the policy function $y_N(x, p)$, and then we develop a fast algorithm that incorporates the necessary conditions (16) and (17) within the search for the value function. We describe the algorithm in detail in the appendix.

The contemporaneous supply curve is plotted in Figure 17. The supply curve with market power is shifted in relative to the case we have studied previously, where the producer faces a perfectly elastic demand (and hence takes prices as given). The shape of the two curves is similar: the supply curve remains downward sloping in the range where the price is well above the marginal cost.

The right panel of the Figure shows the price impact. In relative terms the biggest impact occurs at low prices – the blue curve essentially tracks the difference between the supply curves in the left panel. The impact in absolute terms – expressed in dollars per barrel – is increasing in the shadow price in close to linear fashion. Our calibration implies a significant degree of market power – the producer optimally restricts supply by about a half, raising global prices by 40% above the reference price, on average.

Figure 18 shows the policy functions over the entire state space for the case with and without market power. The left panel shows that quantity is always restricted, but by how much depends on the ongoing price and on the remaining reserve levels. The right panel

[23] We report the absolute value of the elasticity; of course the demand curve is downward sloping.

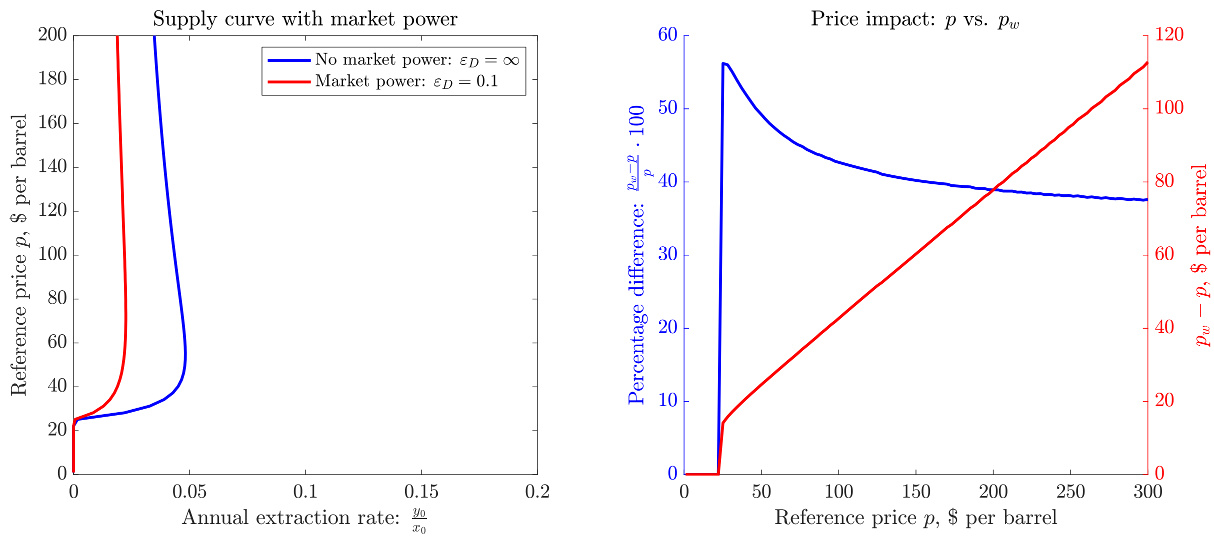


Figure 17: Contemporaneous supply curve with market power and the price impact

illustrates the price impact across the state space, making the point that as time goes by and producer's reserves are depleted, its ability to affect world prices diminishes.

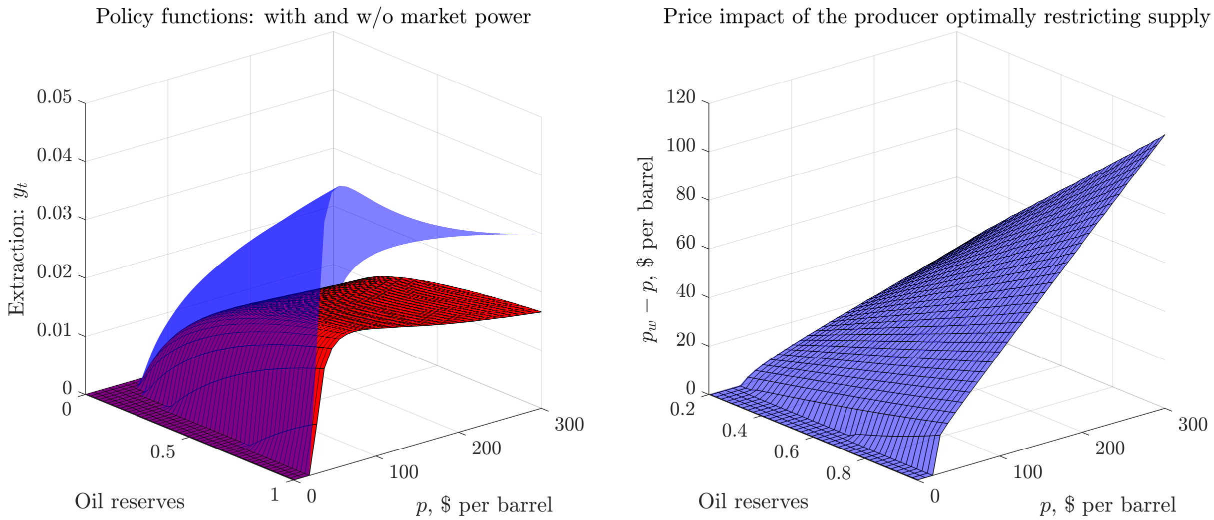
5.7 What does the producer gain from restricting supply?

The answer to this question is somewhat surprising – it turns out that producer's contemporaneous profits are *lower* when it can optimally exercise its market power, relative to the case when it cannot. That is, the effect of lower quantities can dominate over the effect of higher prices. This is surprising because, with market power, it is of course still feasible for the producer to recreate the no-market-power revenues, simply by setting $y(x,p) = y_N(x,p)$ for all x and p.

The reason behind this finding is that the producer solves a dynamic, and not a static problem. The lower extraction rate means that oil reserves – and so the associated market power – last for a longer period of time. The producer optimally trades off lower contemporaneous profits for longer-lasting market power. Specifically, note that equation (16) implies that

$$p_{w,t} = \frac{c + \frac{v_x}{u_\pi}}{1 - \varepsilon_D},$$

meaning that the producer in a dynamic setting aims for prices that are higher than those that maximize static profits (given by $\frac{c}{1-\varepsilon_D}$). Thus, contemporaneous profits are not max-



Note: in the left panel, the red surface shows the extraction with optimal use of market power, and the blue surface shows the extraction when the use of market power is not allowed: $y_N(x, p)$. The right panel shows the price impact.

Figure 18: Policy functions with and without market power, and the price impact

imized by the optimal behavior, and they might actually be lower than profits in the no-market power case.

This is indeed what happens in our calibrated model (Figure 19). The left panel shows the quantities, prices and revenues. Profits decline when market power is utilized, as the producer goes too far in term of pushing up prices and reducing quantities. The benefit of these lower extraction rates can be seen in the right panel – producer's reserves and hence its market power are depleted much more slowly in the model where the producer has market power.

This more drawn out extraction time path raises producer's welfare even as contemporaneous profits are lower. The value of having oil in the ground is always higher when the producer has market power, as illustrated in Figure 20.

6 Price cap when the producer has market power

When the producer has market power and is subject to a perfect price cap, the price that it receives is given by

$$p_r = \min\{\bar{p}, p_w\} \tag{18}$$

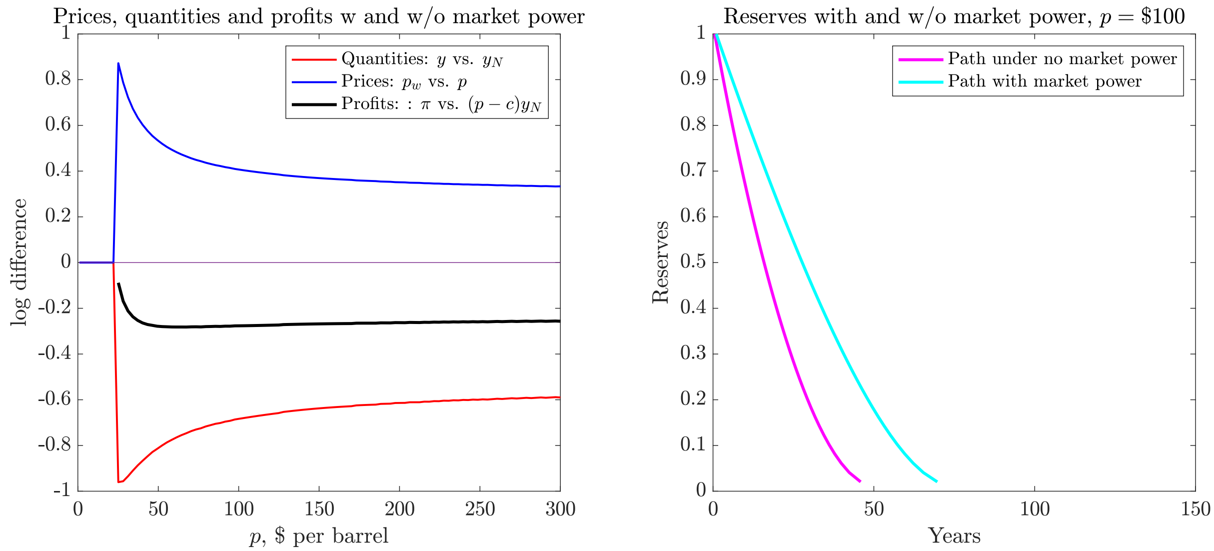


Figure 19: Contemporaneous quantity, price and revenue effect of market power, and the dynamic impact on the path of reserves

Note: the right-hand panel assumes that the reference price of oil is equal to $100 per barrel throughout.

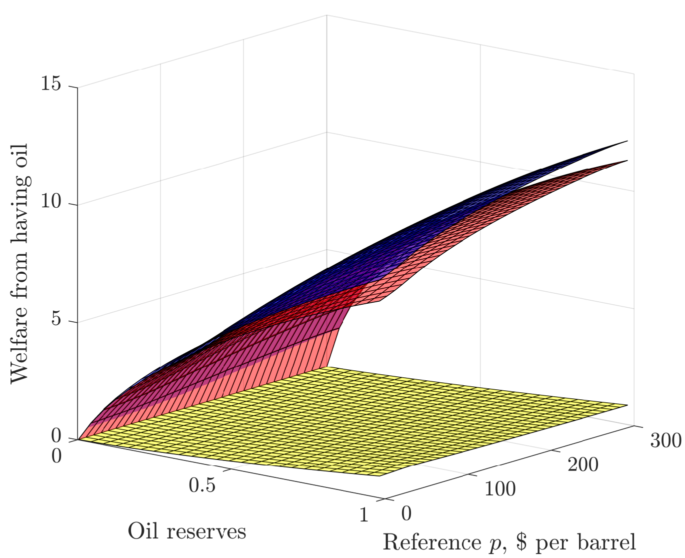


Figure 20: Value functions with and without market power

Note: The blue surface is the value function with $\epsilon = 7$. The red surface is the value function with $\epsilon = 0$. The yellow surface is the difference between the two. This difference is always positive, meaning that having market power always increases producer's welfare (even as it decreases contemporaneous profits).

where $\bar{p}$ is the level of the price cap and p_w is the equilibrium price of oil in the world market, given by equation (13). The difference to (7) is that p_w is now endogenous and determined, among other things, by the producer's decisions.

6.1 How does the price cap interact with market power?

The key insight from our model that we now describe and discuss is that the price cap limits the use of market power in equilibrium. The economics of this mechanism is straightforward: with a price cap in place, restricting quantities has a smaller effect on the sale price. The strength of this effect will naturally depend on whether and how binding the price cap is. Ultimately, if the cap is sufficiently binding – i.e. when the producer sells all of its commodity exports at a price equal to $\bar{p}$ – restricting quantity has no effect on the price at all, rendering the use of market power almost entirely ineffective.

Consequently, the supply curve with a price cap in place can be thought of as an envelope of two supply curves we have already studied in previous sections of this paper: the one with market power (and no cap) at prices well below the price cap, and the one without market power and a binding cap at prices above the price cap.

Figure 22 illustrates the main result graphically: the solid black schedule in the Figure is the supply curve with a $60 price cap in place. The other two schedules are the same as in previous sections.

The Figure shows that when the reference price is low relative to the cap, $p \ll \bar{p}$, the cap matters little for the producer's behavior. The producer exercises market power and the black solid line follows closely the supply curve we described in the previous section (the red line).

Conversely, when prices are high and the cap is binding, the producer ceases to use its market power almost completely – the supply curve is shifted to the right and close to vertical (since, as we explained before, when the cap is binding, fluctuations of reference prices do not matter much for optimal extraction decisions).

In between these two sections there is a smooth and continuous section that joins them together. In this region, as higher prices make the price cap ever more binding, the producer gradually reduces the extent to which it uses market power in equilibrium.

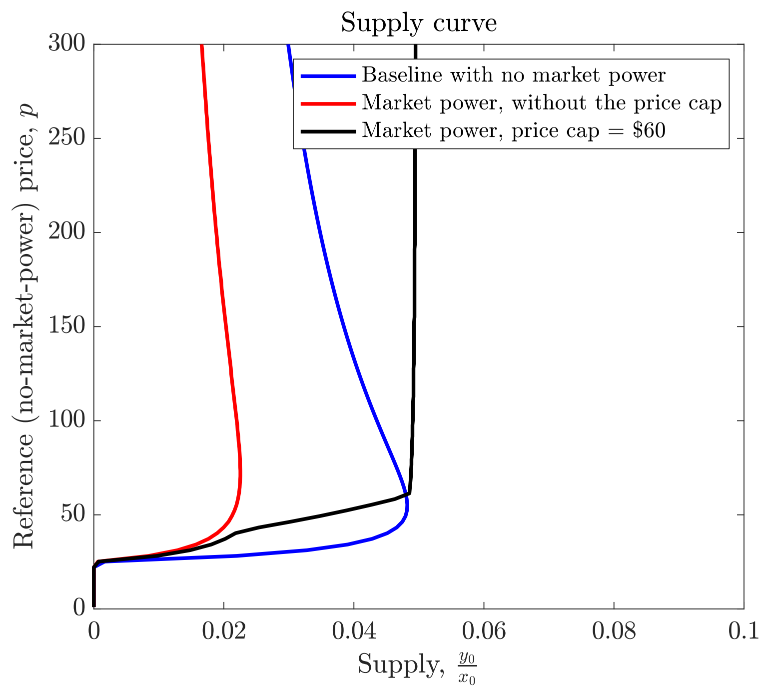


Figure 21: Equilibrium supply in a model with market power with a $60 price cap

6.2 Effect of the price cap on equilibrium prices

Given the optimal behavior of the producer we just described, what happens to equilibrium prices?

A binding price cap obviously limits the price that the sanctioned producer receives. This direct effect of the cap follows from (18) and is illustrated in the left panel of Figure 22.

A more surprising conclusion is that implementing a price cap can also lower the world equilibrium price of the commodity, especially when the reference price is high (the right panel of Figure 22). In other words, the cap has a *stabilizing effect* on world prices. This stabilization effect comes about precisely because when the cap is binding, the producer ceases to exercise market power, and instead has the incentive to supply large quantity of the commodity to the market as the smoothing effect does not operate.

It is important to note that these effects are more pronounced when the producer has substantial degree of market power. This is because the gap between production levels with and without market power naturally increases with the degree of market power, and it is this gap that the price cap eliminates.

We summarize these results in the following Proposition:

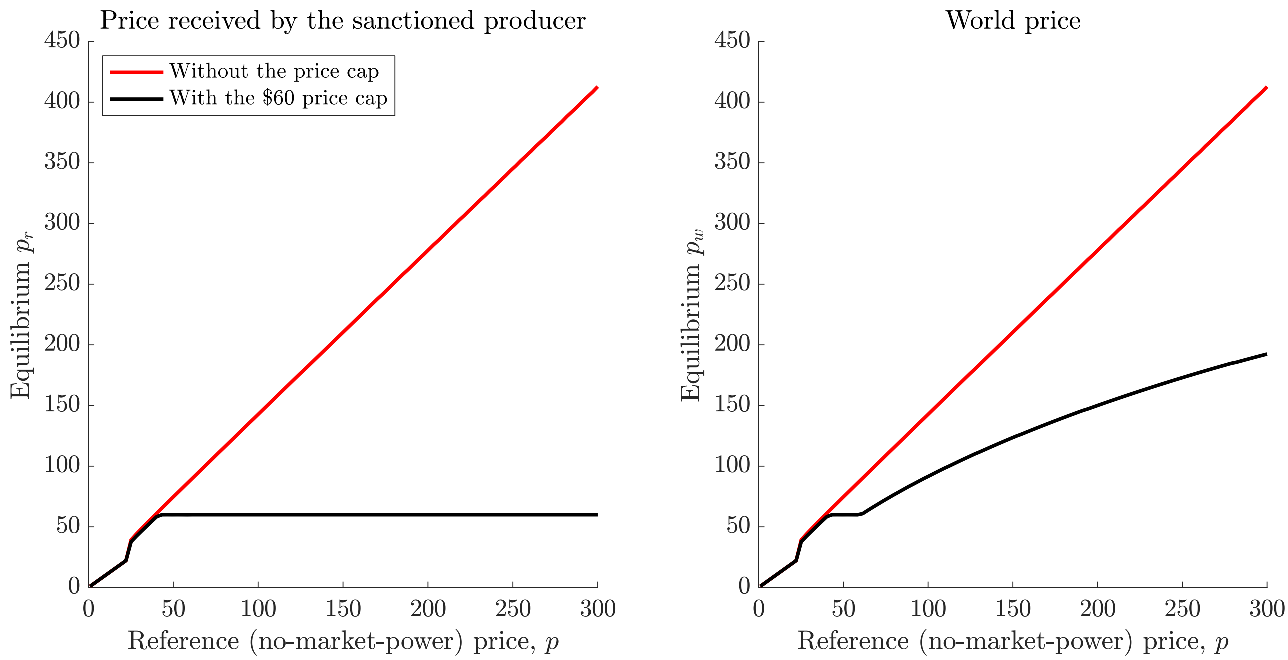


Figure 22: Equilibrium prices in the model with market power, with and without a price cap

Proposition 3. *When the sanctioned producer has market power, introducing a price cap that applies to all sales has the following effects:*

(1) the cap limits the extent to which the producer exercises market power in equilibrium;

(2) a binding cap thus reduces *equilibrium world price p_w;*

(3) the decline in p_w upon introduction of cap is larger the higher is reference price p;

(4) the cap thus stabilizes *equilibrium world price p_w;*

(5) for high reference price p, the equilibrium p_w can be below p;

(6) these effects are more powerful when the producer commands significant pricing power.

6.3 Lessons from the model and the current policy debate

One of the grave concerns of policymakers in the recent episode following Russia's aggression in Ukraine has been that implementing a price cap risks destabilizing global markets. Is this a legitimate concern? The model we have developed so far sounds a resounding "no" to this question. Instead of destabilizing the global oil market, a perfect price cap has a stabilizing effect.

Before concluding the paper, we now explore the robustness of this conclusion to the fact that the price cap policy applies to only a share of sanctioned entity's exports.

7 Imperfect price cap

In the analysis so far we have assumed that the price cap applies to all of the sales of the sanctioned producer and that the cap is expected to be in place forever. In reality, however, the price cap might only affect a specific portion of the exporter's oil sales, and it might not be in place indefinitely. In the case of Russia, the G7 price cap applies only to the price of seaborne oil and products that use Western services such as transportation and insurance.[24] Furthermore, if the price cap is not adequately enforced, a share of sales that bypass the sanctions regime could be substantial. How does such partiality of the price cap alter the analysis and the conclusions? And what if the producer expects puts a positive probability on the price cap to be abandoned sometime in the future? We consider these two possibilities in turn.

7.1 Leaky price cap (shadow fleet)

Let us represent the percentage of the producer's current oil reserves that can be exported outside of the cap with parameter $\kappa \in [0, 1]$. For instance, with $\kappa = 0.01$, the producer can export 1% of its reserves this year without being subject to the price cap. $\kappa = 0$ represents the case of a perfect price cap that applies to all of exports (meaning that the producer cannot sell outside the price cap regime), as described in previous sections. We assume that κ is fixed over time.[25]

7.2 Optimality conditions with shadow fleet

With a shadow fleet of capacity κ, the instantaneous profits from oil sales when the price cap is in place are:

$$\pi_t = \begin{cases} y \cdot (p_w(y) - c) & \text{if } y \leq \kappa \\ y \cdot (p_w(y) - c) & \text{if } y > \kappa \text{ and } p_w < \bar{p} \\ \kappa \cdot (p_w(y) - c) + (y - \kappa) \cdot (\bar{p} - c) & \text{if } y > \kappa \text{ and } p_w > \bar{p} \end{cases} \tag{19}$$

[24] There is an intense debate and speculation among experts, policymakers and the media about the ability of Russia to do without these western services, including its ability to build up such capacity – termed "shadow fleet" – over time. While there is a significant uncertainty about this, experts estimate that about 30-40% of the flow of oil exports can be legally sold outside of the cap at the moment.

[25] Future work might fruitfully revisit this and explore cases with variable κ, reflecting, for example, Russia's potential expansion of its capacity to sell oil outside of the price cap regime.

where p_w is the equilibrium oil price in equation (13). The first line shows the profits that the producer makes if extraction is smaller than κ, and hence all of the commodity is sold outside of the sanctions regime. The second line – which turns out to be the same as the first – is profits when extraction is greater than κ, but the price cap is not binding. The third line represents profits when extraction is above κ and the cap is binding. In this most interesting case, the producer receives the world equilibrium price for the quantity κ, and the price cap for the remaining sales.

The first order condition of the producer's problem becomes

$$v_x = \begin{cases} \tilde{u}_\pi \cdot \left(p_w \left(1 - \varepsilon_D\right) - c\right) & \text{if } y < \kappa \\ \tilde{u}_\pi \cdot \left(p_w \left(1 - \varepsilon_D\right) - c\right) & \text{if } y > \kappa \text{ and } p_w < \bar{p} \\ \tilde{u}_\pi \cdot \left(\bar{p} + \kappa \frac{\partial p_w}{\partial y} - c\right) & \text{if } y > \kappa \text{ and } p_w > \bar{p} \end{cases} \tag{20}$$

where ε_D is the elasticity of demand given in equation (24). Equation (20) illustrates how the price cap and the shadow fleet interact to result in endogenous degree of market power, depending on the level of production. When production is low, so that all oil can be transported outside of the cap regime (the first row in (20)), the marginal utility of extracting an additional barrel is given by the marginal utility of oil profits times the world price adjusted downwards for the impact that this extraction has on the prevailing oil price. This is also true if the marginal barrel is sold using the coalition services and so under the price cap regime, but if the price cap is not binding (the second row). Finally, when the marginal barrel is sold at a cap, the marginal benefit is just the price cap adjusted for the price impact that the sales of a marginal barrel have on the revenues from the sales of the infra-marginal κ barrels (the final row).[26]

7.2.1 The effects of the price cap in presence of own fleet

The combination of market power and the ability to bypass the price cap on some of its sales provides the producer with a potentially appealing strategy to deal with the sanctions: cut the production levels towards κ, thereby squeezing the global market and raising equilibrium prices at which the (now-lower) quantity is sold. Higher prices in part compensate for lower

[26]Note that the last line can be re-written as

$$\tilde{u}_\pi \left(p_w \left(\frac{\bar{p}}{p_w} - \frac{\kappa}{y}\varepsilon_D\right) - c\right) = v_x.$$

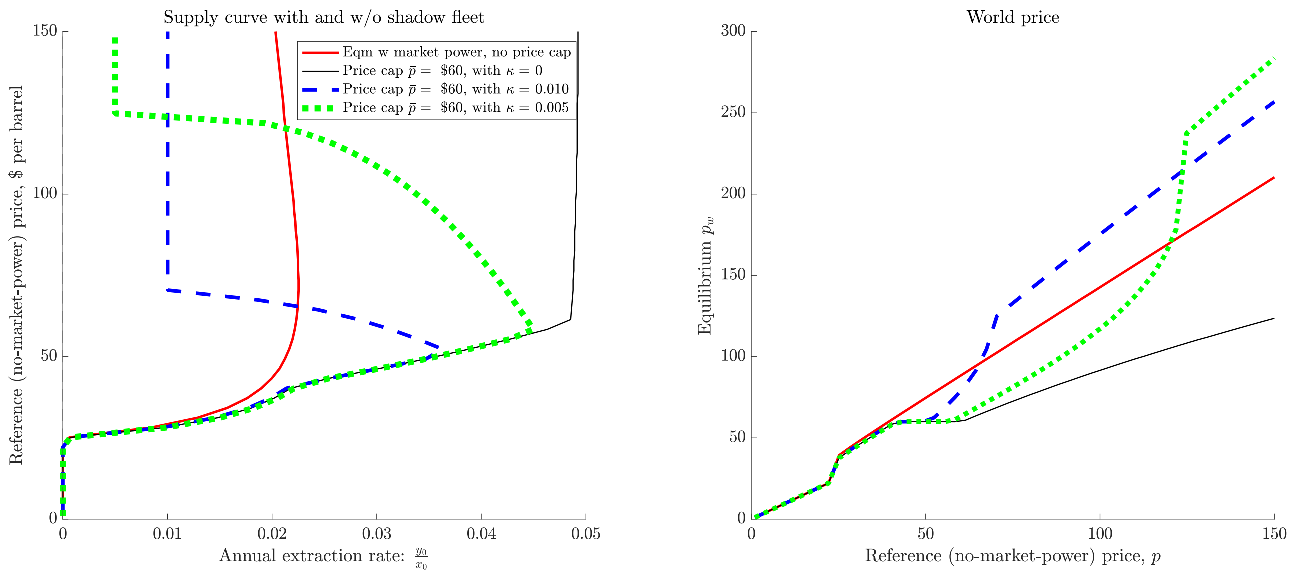


Figure 23: Equilibrium supply and prices under a price cap when Russia has access to a shadow fleet

quantity, and as an additional benefit of this strategy, the reserves (and hence the market power) deplete more slowly. We now explore whether or not this is indeed an optimal strategy of the producer, and how this depends on the level of prices and on the capacity parameter κ.

Figure 23 illustrates optimal extraction and equilibrium world prices with a price cap that is imposed on the producer who has access to a shadow fleet capable of carrying 0.5% and 1% of its reserves, and is otherwise identical to the producer in the previous section.

The left panel displays the supply schedules. The results are striking: the supply curve features a sharp kink, meaning that for high enough prices, the producer starts restricting supply, and ultimately reduces extraction all the way to κ when world prices are high. As prices increase, the so called "shut-in" of production happens more abruptly when the shadow fleet capacity is higher (and consequently, full reduction all the way to κ happens at lower levels of the reference price: about \$70 when κ is 0.01, and about \$130 when κ is 0.005). Thus, the optimal response of the producer to an implementation of a price cap heavily depends on the producer's own capacity to sell the commodity outside of the sanctions regime, as well as on the enforcement of the cap (both of which determine κ).

The right panel of Figure 23 documents the impact on equilibrium prices. If one wants to judge the impact of an imperfect price cap, the appropriate benchmark to compare against is the red line – which is the world equilibrium price schedule absent the pricer cap. Implementing an imperfect price cap has different effects depending on the level of reference prices. When prices in the global market are low, the producer behaves as if it had no capacity to bypass the price cap regime, and so at these prices we still observe the stabilizing effect of the price cap on world price of the commodity. However, for high levels of the reference price, the shut-in becomes the optimal strategy, and the cap can increase global prices (the dashed lines are above the solid red line).

The bottom line from this analysis is that the capacity of the shadow fleet matters a lot. In particular, the presence of a fleet that can transport the flow of oil equivalent to about a half of pre-war exports ($\kappa = 1\%$) negates much of the stabilization benefits of introducing the cap.

7.2.2 The impact of a leaky price cap on profits and welfare

We have now endowed the producer with market power and we have made it possible to partially circumvent the price cap regime by exporting oil using a shadow fleet of tankers and services. We are ready to revisit the question about the effectiveness of the price cap as a tool of economic warfare. What impact does a leaky price cap have on the producer in this environment?

Figure 24 offers an answer, both in terms of contemporaneous profits from oil sales in the left panel, as well as welfare from having oil in the ground (i.e., the value function) in the right panel.

The dashed lines in the left panel show that contemporaneous profits plummet by up to 50% as the producer turns to the "shut-in" strategy, unless the market prices are already very high.[27] That is, our model suggests that even with a highly inelastic demand which we have assumed in our calibration, the sharp reduction in exports does not generate a price response that is sufficiently strong to make the shut-in a profitable strategy in the short term. In other words, higher prices in the shut-in scenario do not compensate for the lost revenues due to lower volumes. According to the model, shutting in production to κ is optimal not because it raises contemporaneous profits, but because it allows for a more spread out production profile over time (see also the relevant discussion in Section 5).

Indeeed, the static losses are more than compensated by the dynamic gains. The right-hand panel shows that welfare increases with κ, which is intuitive and unsurprising. The interesting result here is that the ability to circumvent the price cap regime significantly

[27] Note that at low prices contemporaneous profits under the cap are slightly higher than under no cap. This is the corollary of two findings discussed above: that the use of market power diminishes contemporaneous profits (see figure 19 and discussion there) and the fact that the price cap reduces the use of market power.

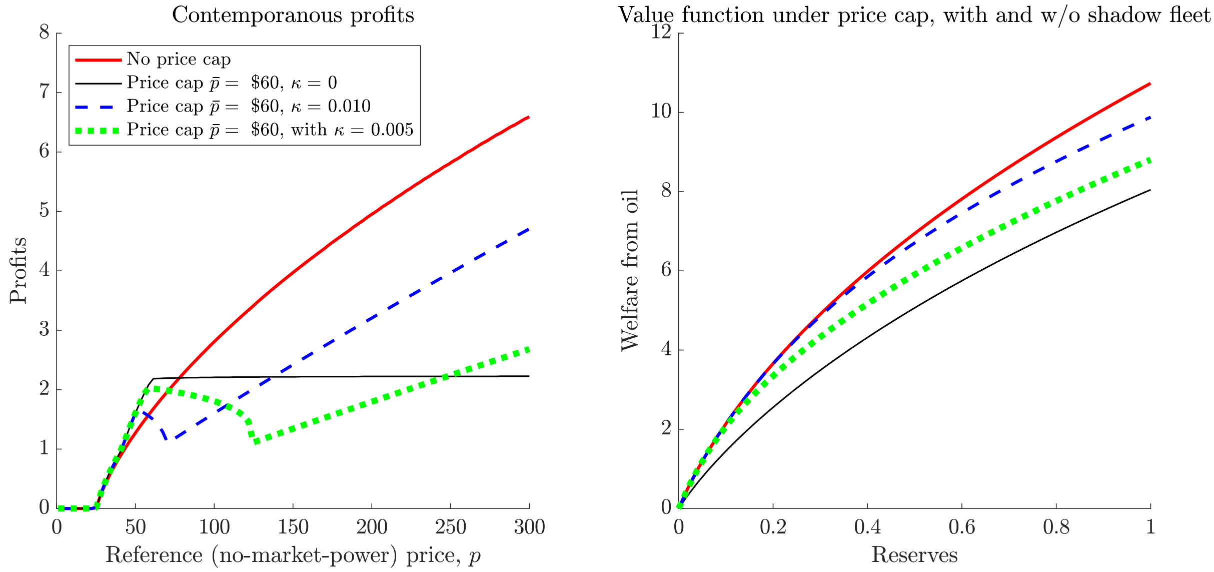


Figure 24: Effects of price caps on producer's contemporaneous revenues and welfare when Russia has access to a shadow fleet or the cap is imperfectly enforced

Note: the right-panel assumes that the (current) reference no-market-power price of oil p is \$80.

diminishes the degree to which the cap hurts the producer. Relative to a perfect cap, a leaky cap with $\kappa = 0.01$ reduces the damage in welfare terms by about $\frac{2}{3}$.

7.3 Expectation that the price cap is temporary

We now investigate how the expectation that the price cap will be lifted at some point in the future affect the producer's behavior.

Let us consider that the producer perceives the price cap to be a temporary measure. Specifically, we assume that the producer believes that the lifting of the cap is a Poisson event with intensity λ, so that the duration of the price cap is an exponentially distributed random variable and

$$Pr(\text{cap lifted before } t) = 1 - \exp(-\lambda t).$$

For concreteness, suppose that the producer perceives the probability of the cap being lifted in the first year to be 50%, implying $\lambda = 0.69$. How is the behavior of the producer subject to such a cap different to what we studied above?[28]

[28]Technically to solve the model we must introduce another state variable which takes two values, corresponding to the cap being and not being in place. We then impose a Poisson process on the switching between the cap and the no-cap state.

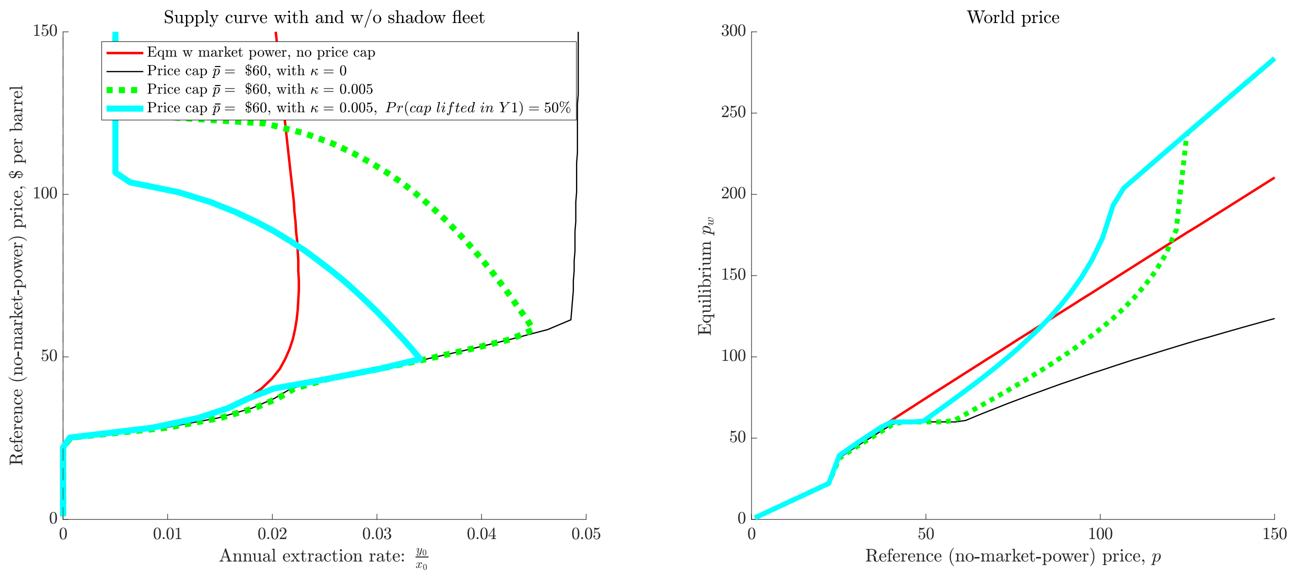


Figure 25: Equilibrium supply and prices when the producer expects the price cap to be temporary

Figure 25 illustrates how contemporaneous extraction responds to such expectations and what the consequences are for world prices. The expectation that the cap is temporary makes the producer more inclined to shut-in production, hence keeping more barrels of oil under ground and only extracting them when the price cap is lifted. Thus, as illustrated in the right panel of the Figure, such temporary characteristic of the cap reinforces the shadow fleet mechanism in further reducing the stabilization effects of the price cap.

While contemporaneous profits suffer substantially as a result of shutting in production at low prices, such profits squeeze is expected to be only temporary and thus does not weigh much on the value function (Figure 26). In other words, the price cap must be imposed for a long period of time to exert significant welfare loss on the producer.

In short, the price cap is more effective if it is expected to be in place for a long time. The policy implication coming out of this is straightforward: sanctioning countries' decision makers should make credible promises about the permanence, or persistence, of the price cap.

8 Conclusions

The main contribution of this paper is a dynamic model that helps us understand the economic incentives of a financially constrained producer of a non-renewable resource. Our

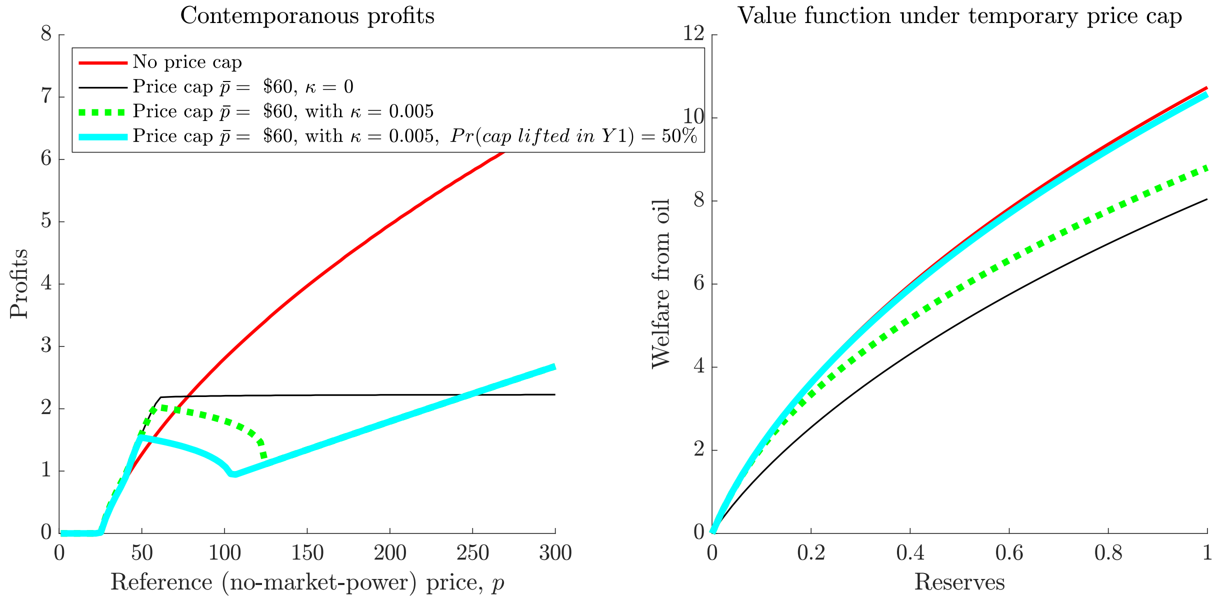


Figure 26: Effects of price caps on producer's contemporaneous revenues and welfare when price cap is expected to be temporary

particular application and focus has been the on the effects of the new instrument of international policy – a price cap.

The model takes as an input an estimated flexible diffusion process for the oil price, which we embedded in an equilibrium structure where the producer has market power which changes dynamically and endogenously to the producer's decisions. We have developed robust solution algorithms that can handle these problems highly efficiently.

The analysis uncovered interesting economic channels and forces that are at play in this setting. In particular, our model stresses the interplay between financial frictions and the dynamic optimal behavior of the producer, with an important revenue smoothing motive. It highlights the role of alternative sources of funds or other sources of non-homotheticity in producer's preferences. And it illuminates the fact that the price cap reduces the use of market power in equilibrium, which leads to a stabilizing effect of the price cap on the global commodity market.

Beyond these contributions, our analysis has important policy implications in the current context of the war in Ukraine and sanctions against the Russian Federation.

First, our economic framework supports the idea that Russia's supply curve is inelastic and may even be downward sloping, helping to explain why Russian oil production levels have remained relatively stable despite political assertions to the contrary.

Second, a binding oil price cap may increase Russia's supply to the market, stabilizing the price of oil globally. The cap may not be effective in the long run, however, if Russia can sell enough of its exports outside the price cap regime (i.e., without using western transportation and financial services). This highlights the importance of lowering the cap before Russia finds alternative ways to export its oil and the need for strict enforcement of the cap.

Third, even when a commodity producer has significant market power, this need not deter western policymakers from imposing – and lowering – the price cap. In fact, the oil price cap can effectively neutralize Russia's market power, which it already uses in equilibrium.

Finally, our simulations suggest that a lower price cap, perhaps around $45 per barrel, could significantly impact Russia's revenue flows, and depending on the capacity of the shadow fleet, potentially also its welfare.

Our paper opens up several avenues for future research. Our setting explored the use of the price cap tool in the context of non-renewable resources. But future work might want to consider a setting in which trade of products or exchange of technologies is taking place between the sanctioning and the sanctioned state. Another useful avenue for future research would be to explicitly embed the setting developed here within a general equilibrium model of a world economy, with strategic interactions across participating states. We are excited to contribute to this exciting agenda going forward.

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Appendix

A Proofs

A.1 Optimal extraction when producer has market power

We can still define $u(y) := \tilde{u}(\pi(y))$. The HJB equation is

$$\rho v(x,p) = \max_{y} u(y) - v_x(x,p)y + v_p(x,p)D(\bar{p}-p) + \frac{1}{2}v_{pp}(x,p)\sigma^2 p, \tag{21}$$

and the first order condition is

$$u_y(y) = v_x. \tag{22}$$

By the chain rule, the left hand side is

$$u_y(y) = \tilde{u}_\pi(\pi) \cdot \frac{\partial \pi}{\partial y} = \tilde{u}_\pi(\pi) \cdot p_w \cdot \left(1 - \frac{c}{p_w} + \frac{\partial p_w}{\partial y}\frac{y}{p_w}\right). \tag{23}$$

It is easy to show that

$$\varepsilon_D := -\frac{\partial p_w}{\partial y}\frac{y}{p_w} = \frac{\epsilon y}{\frac{1-\psi}{\psi}y_N + y}. \tag{24}$$

Using (13), (23) and (24), the FOC (22) thus becomes

$$\tilde{u}_\pi(\pi) \cdot \left(p \cdot \left((1-\psi) + \psi\frac{y}{y_N(x)}\right)^{-\epsilon} \cdot \left(1 - \frac{\epsilon y}{\frac{1-\psi}{\psi}y_N + y}\right) - c\right) = v_x. \tag{25}$$

The boundary condition is

$$\tilde{u}_\pi(0) \cdot (p-c) = u_y(0) = v_x(0,p), \tag{26}$$

since with no resources the producer does not have market power: $\psi(0,p) = 0 \forall p$.

We solve the problem using the finite difference method, as before. The main challenge now is that the first order condition takes a more complicated form: while for a given value function we can express y using the FOC in closed form when the demand curve is perfectly elastic, with market power the FOC in equation (22) is more complicated and cannot be solved in closed form. Specifically, with CARA utility equation (22) becomes

$$e^{-\gamma\left(p\left((1-\psi)+\psi\frac{y}{y_N}\right)^{-\epsilon}-c\right)y}\left(p\left((1-\psi)+\psi\frac{y}{y_N}\right)^{-\epsilon}\left(1-\frac{\epsilon y}{\frac{1-\psi}{\psi}y_N+\psi}\right)-c\right)=v_x,$$

where the function $\psi(x,p)$ is given by equation (12). We must solve this non-linear equation for y for every point in the $x-p$ grid when searching for the solution. With CARA utility, the boundary condition in (26) is simply

$$p-c=v_x(0,p).$$

B Curvature of function u as an index of financial constraints

Financial frictions inhibit consumption smoothing, and hence mean that the timing, volatility and uncertainty of the income flow matter for welfare. The goal of this Appendix is to show that the curvature of a felicity function over income can be used to index the degree of the income smoothing motive. It does so in a stylized two-period model with no uncertainty.

B.1 Frictionless model

Consider a two period model with no discounting: $\beta=1$, and no financial frictions, and with $r=0$ so that $R:=1+r=1/\beta$. Agent has time separable preferences, and her utility over flow of consumption in each period is CRRA. The agent's total income across two periods is $Y=1$. The income is split unevenly across the two periods, with $y_1=\phi$ and $y_2=1-\phi$ (known with certainty). Agent solves

$$U=\max_{c_1,c_2}u(c_1)+u(c_2)$$

subject to

$$c_1+c_2=1.$$

Trivially, the solution is

$$c_1=c_2=\frac{1}{2}$$

and

$$\frac{dU}{d\phi} = 0$$

that is, the value function is independent of the timing of income. In other words, the timing is irrelevant for consumer's welfare, given the perfect financial markets.

Without loss of generality, in the remainder of the Appendix we focus on the agent who wants to save, i.e. we assume $\phi \in (\frac{1}{2}, 1)$, that is, more income arrives in the first period.

B.2 Financial frictions

Now suppose that the budget constraint of the agent has a kink – the interest rate on savings is negative (and the rate on borrowing is positive). In particular, let $r_S = -\chi < 0$ denote the interest rate on savings (and e.g. $r_B = \chi > 0$ the rate on borrowing). The agent solves

$$U = \max_{c_1, c_2} u(c_1) + u(c_2)$$

subject to

$$c_1 + \frac{c_2}{1-\chi} = \phi + \frac{1-\phi}{1-\chi}.$$

The first order condition is

$$u'(c_1) = u'(c_2)\,(1-\chi)$$

Thus with CRRA

$$c_2 = (1-\chi)^{1/\gamma}\, c_1$$

Combining this with the budget constraint gives optimal consumption path:

$$c_1 = \frac{\phi + \frac{1-\phi}{1-\chi}}{1 + (1-\chi)^{\frac{1-\gamma}{\gamma}}}$$

$$c_2 = \frac{(1-\chi)^{1/\gamma}\left(\phi + \frac{1-\phi}{1-\chi}\right)}{1 + (1-\chi)^{\frac{1-\gamma}{\gamma}}}.$$

Without loss of substance, simplify by assuming that underlying utility is log: $\gamma = 1$. We then have

$$c_1 = \frac{1}{2}\frac{1-\phi\chi}{1-\chi}$$

$$c_2 = \frac{1-\phi\chi}{2}.$$

Since we assumed that the consumer is saving, we must have $c_1 \leq \phi$. Thus we require

$$\frac{1}{2}\frac{1-\phi\chi}{1-\chi} < \phi$$

which is the case if

$$\chi < \bar{\chi} := 2 - \frac{1}{\phi}.$$

$\bar{\chi}$ is the threshold degree of financial frictions. If financial frictions are more severe than that, the consumer is just consuming her endowment.

The value function is:

$$U(\phi) = \log\left(\frac{1}{2}\frac{1-\phi\chi}{1-\chi}\right) + \log\left(\frac{1-\phi\chi}{2}\right)$$

which is a function of ϕ as long as $\chi \neq 0$. We have

$$\frac{\partial U}{\partial \phi} = -2\frac{\chi}{1-\phi\chi}.$$

Consider the following function: CRRA with inverse IES parameter σ, applied to the flow of income of the agent:

$$V(\phi) = \frac{\phi^{1-\sigma}}{1-\sigma} + \frac{(1-\phi)^{1-\sigma}}{1-\sigma}.$$

The question is: can we map the degree of financial frictions χ into σ so that the sensitivity of the value function to the timing of the endowment, ϕ, is the same in the underlying problem and in the "just consume the endowment" problem? In other words, we seek $\sigma(\chi)$ such that, for a given endowment process ϕ,

$$U'(\phi) = V'(\phi).$$

This implies:

$$-2\frac{\chi}{1-\phi\chi} = \phi^{-\sigma(\chi)} - (1-\phi)^{-\sigma(\chi)}$$

which implicitly pins down the $\sigma(\chi)$ function.

We can deduce some properties of this function. Note that, for $\phi > \frac{1}{2}$,

$$\sigma(0) = 0,$$

and, because $-2\frac{\bar{\chi}}{1-\phi\bar{\chi}} = \frac{1}{\phi} - \frac{1}{1-\phi}$, we have

$$\sigma(\bar{\chi}) = 1.$$

This is intuitive: with no frictions (the former case), the timing of income does not matter, so V is linear (and $\sigma = 0$). Instead, with full frictions, we have that $\sigma = \gamma$ (and recall that we set $\gamma = 1$).

Implicitly differentiating yields:

$$\frac{-2(1+\phi)}{(1-\phi\chi)^2} = -\left((\log\phi)\cdot\phi^{-\sigma(\chi)} - (\log(1-\phi))\cdot(1-\phi)^{-\sigma(\chi)}\right)\sigma'(\chi)$$

So that

$$\sigma'(\chi) = \frac{1}{((\log\phi)\cdot\phi^{-\sigma(\chi)} - (\log(1-\phi))\cdot(1-\phi)^{-\sigma(\chi)})}\frac{2(1+\phi)}{(1-\phi\chi)^2} > 0,$$

as long as $\phi > 1/2$ (which holds by assumption). Thus the $\sigma(\chi)$ function is monotonically increasing in the $[0, \bar{\chi}]$ interval, from 0 to 1.

B.3 Interpretation

Figure 27 shows the $\sigma(\chi)$ function. For any ϕ, this is a well-behaved monotonic function. Thus, to reflect a given degree of financial frictions in the decision problem of the agent, we pick a corresponding $\sigma(\chi)$, ensuring that the choices over the timing of the income stream have the impact of welfare that is consistent with the solution (and optimal behavior in) to the consumption-saving problem. We apply this principle in the main text by focusing solely on the extraction decision of the producer.

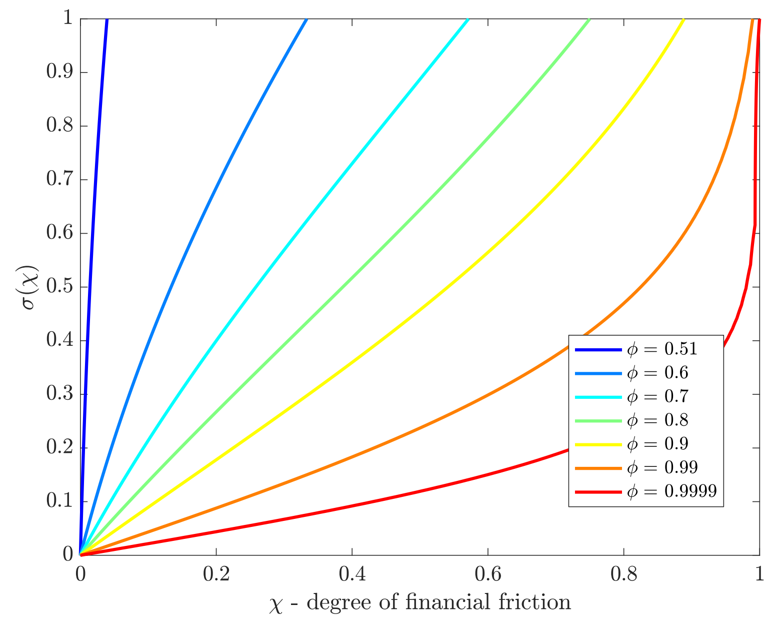

Figure 27: The $\sigma(\chi)$ function, for different levels of unevenness of the income profile (ϕ) – the agent has log utility and there is no discounting, so that the agent wants to ideally consume the same each period

C Policy functions

Figures 28 and 9 show the complete policy functions for the calibrated model under no uncertainty and with uncertain price, respectively.

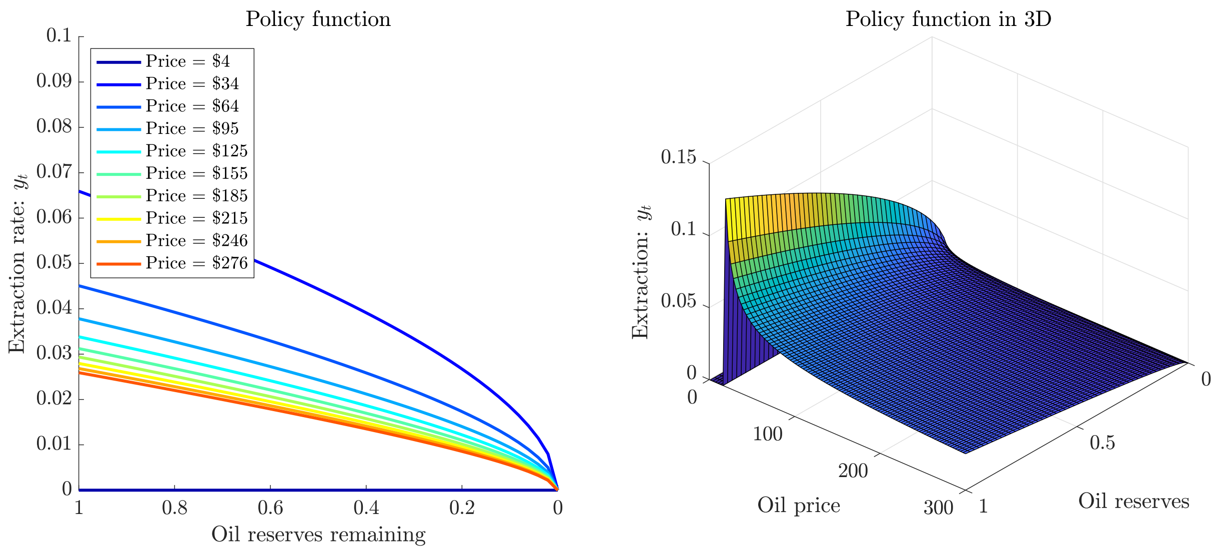

Figure 28: Policy function when p is a known parameter

D Numerical appendix

This appendix provides a summary of the numerical procedure used to solve the model.

D.1 Baseline case

We start with a model with no market power. We seek to solve the HJB equation (3), reproduced here:

$$\rho v(x,p) = \max_{y_t} u(\pi) - v_x(x,p)y + v_p(x,p)D(\tilde{p} - p) + \frac{1}{2}v_{pp}(x,p)\sigma^2 p.$$

We use a finite difference method; a useful reference in the macroeconomics literature is Achdou et al. (2017). We approximate the function v at I discrete points in the reserves grid, $x_i,\ i \in 1,...,I$ with $x_1 = 0$ and J discrete points in the price dimension, $p_j,\ j \in 1,...,J$. We use equispaced grids with Δx and Δp the distance between gridpoints. Since oil is a non-renewable resource, reserves can only stay constant or fall. Thus, the drift is always (weakly) negative. When reserves are zero, the derivative of the value function is pinned down by the boundary condition. Therefore, we approximate the derivative of the value function in the x-dimension with

$$\partial_x v_{i,j} = \begin{cases} u_y(0) & \text{if } i = 1 \\ \frac{v_{i,j} - v_{i-1,j}}{\Delta x} & \text{if } i \geq 2. \end{cases}$$

The approximations of the derivatives in the p dimensions are:

$$\begin{aligned} \partial_{p,B} v_{i,j} &= \frac{v_{i,j} - v_{i,j-1}}{\Delta p} \\ \partial_{p,F} v_{i,j} &= \frac{v_{i,j+1} - v_{i,j}}{\Delta p} \\ \partial_{pp} v_{i,j} &= \frac{v_{i,j+1} - 2v_{i,j} + v_{i,j-1}}{(\Delta p)^2}. \end{aligned}$$

We use the appropriate approximation depending on whether the price is falling or increasing. For any variable z, we use the notation $z^+ := \max\{z, 0\}$ and $z^- := \min\{z, 0\}$. The finite difference approximation to the HJB equation is then:

$$\begin{aligned} \rho v_{i,j} &= u(y_{i,j}) - \partial_x v_{i,j} y + \partial_{p,F} v_{i,j} \left[D(\tilde{p} - p)\right]^+ + \partial_{p,B} v_{i,j} \left[D(\tilde{p} - p)\right]^- + \frac{1}{2} \partial_{pp} v_{i,j} \sigma^2 p \\ y_{i,j} &= (u_y)^{-1}(\partial_x v_{i,j}). \end{aligned}$$

Algorithm The algorithm for finding the solution to the HJB eqiuation is as follows. Guess $v^0_{i,j}$, $i = 1, ..., I,\ j = 1, ..., J$ and for $n = 0, 1, 2, ...$ follow

1. Compute $\partial_x v^n_{i,j}$.
2. Compute optimal extraction y^n assuming that the marginal barrel is priced at the cap if the cap is binding. In the model without market power, compute $y^n_{i,j} = (u_y)^{-1}(\partial_x v^n_{i,j})$ where u_y^{-1} is evaluated at $\min\{p_j, \bar{p}\}$.
3. Compute extraction as if there was no price cap, $\tilde{y}^n$: $\tilde{y}^n_{i,j} = (u_y)^{-1}(\partial_x v^n_{i,j})$, where u_y^{-1} is evaluated at p_j.
4. For i, j where $y^n_{i,j} < \kappa$, set $y^n_{i,j} = \tilde{y}^n_{i,j}$.
5. Compute $\tilde{u}^n_{i,j}(\pi)$ with $\pi^n_{i,j} = \min\left\{\kappa, y^n_{i,j}\right\} \cdot p_j + \max\left\{0, y^n_{i,j} - \kappa\right\} \cdot \min\left\{p_j, \bar{p}\right\}$
6. Find v^{n+1} using the implicit method described below.
7. If v^{n+1} is close enough to v^n, stop. Otherwise go to step 1.

The implicit method for finding v^{n+1}. With the implicit method, we update the value function as follows. With a given step size Δ, v^{n+1} is defined by the following equation:

$$\frac{v_{i,j}^{n+1}-v_{i,j}^{n}}{\Delta}+\rho v_{i,j}^{n+1}=u(y_{i,j}^{n})-\partial_x v_{i,j}^{n+1}y_{i,j}^{n}+\partial_{p,F}v_{i,j}^{n+1}\left[D(\tilde{p}-p_j)\right]^{+}+\partial_{p,B}v_{i,j}^{n+1}\left[D(\tilde{p}-p_j)\right]^{-}+\frac{1}{2}\partial_{pp}v_{i,j}^{n+1}\sigma^2 p_j$$

Substituting in for the deivatives and collecting together the terms that are multiplied by the same gridpoint of v, this equation can be written as follows:

$$\frac{v_{i,j}^{n+1}-v_{i,j}^{n}}{\Delta}+\rho v_{i,j}^{n+1}=u(y_{i,j}^{n})+v_{i,j}^{n+1}z_{i,j}+v_{i,j}^{n+1}\nu_{i,j}+v_{i,j-1}^{n+1}\chi_j+v_{i,j+1}^{n+1}\zeta_j$$

where $z_{i,j}=-\frac{y_{i,j}^{n}}{\Delta x}$, $\nu_{i,j}=\left[\frac{D(\tilde{p}-p_j)}{\Delta p}\right]^{-}-\left[\frac{D(\tilde{p}-p_j)}{\Delta p}\right]^{+}-\frac{\sigma^2}{(\Delta p)^2}$, $\chi_j=-\left[\frac{D(\tilde{p}-p_j)}{\Delta p}\right]^{-}+\frac{\sigma^2}{2(\Delta p)^2}$ and $\zeta_j=\left[\frac{D(\tilde{p}-p_j)}{\Delta p}\right]^{+}+\frac{\sigma^2}{2(\Delta p)^2}$. We can now write this in a matrix form:

$$\frac{1}{\Delta}(v^{n+1}-v^{n})+\rho v^{n+1}=u^{n}+\mathbf{A}^{n}v^{n+1}$$

where v^n is a vector of length $I\cdot J$ with entries $(v_{1,1},...,v_{I,1},v_{1,2},...v_{I,2},...,v_{I,J})$ and $\mathbf{A}^n$ is a $(I\times J)\times(I\times J)$ matrix that has $z,\ \nu,\ \chi,\ \zeta$ as entries. Collecting terms, we get

$$\left(\frac{1}{\Delta}+\rho-\mathbf{A}^{n}\right)v^{n+1}=u^{n}+\frac{1}{\Delta}v^{n}.$$

This is a system $Bx=b$ which we can solve efficiently in MATLAB (note that the A matrix is sparse) using the B/b command.

D.2 Market power

The model with market power is solved in an analogous fashion. The only complication is that the first order condition cannot be solved in closed form, that is, to obtain $y_{i,j}^{n}$ in step 2 above it is necessary to solve the FOC (16) numerically for very point i,j in the grid.

We solve the FOC equation (16) by setting up the following functions:

1. The world price of oil function:

$$p_w(y_{ij})=\begin{cases}p_j\cdot\left((1-\psi_{ij})+\psi_{ij}\cdot\frac{y_{ij}}{y_{Nij}}\right)^{-\epsilon} & \text{if } y_{Nij}>0\\ p_j & \text{if } y_{Nij}=0\end{cases}$$

2. The profit function:

$$\pi(y_{ij}) = \min\{\kappa, y_{ij}\} \cdot (p_w(y_{ij}) - c) + \max\{0, y_{ij} - \kappa\} \cdot (\min\{p_w(y_{ij}), \bar{p}\} - c)$$

3. The utility function:

$$u(y_{ij}) = e^{-\gamma\pi(y_{ij})}$$

4. The elasticity function:

$$\varepsilon_D(y_{ij}) = \frac{\epsilon y_{ij}}{\frac{1-\psi_{ij}}{\psi_{ij}} y_{Nij} + y_{ij}}$$

5. The LHS (of the FOC) function:

$$LHS(y_{ij}) = \begin{cases} u(y_{ij}) \cdot \left(p_w(y_{ij}) \cdot \left(\frac{\bar{p}}{p_w(y_{ij})} - \frac{\kappa}{y_{ij}} \cdot \varepsilon_D(y_{ij})\right) - c\right) & \text{if } y_{ij} > \kappa \text{ and } p_w(y_{ij}) > \bar{p} \\ u(y_{ij}) \cdot (p_w(y_{ij}) \cdot (1 - \varepsilon_D(y_{ij})) - c) & \text{otherwise} \end{cases}$$

We then solve the non-linear equation

$$LHS(y_{ij}) - v^n_{x,ij} = 0 \tag{27}$$

for each point in the grid. The solution is an $I \times J$ matrix with entries corresponding to $y^n_{i,j}$. Note that this involves solving $I \times J$ non-linear equations at each iteration n. A robust routine that achieves this is as follows:

1. Construct a grid for candidate solutions, $z_s \in [0, 1]$, $s = 1, ..., S$. We use a power spaced grid in the $[0, 0.5]$ interval and a linear grid in the $[0.5, 1]$ interval, since any solution that is ultimately found to be optimal is well below 0.5. Then, for each point i, j in the $x - p$ grid:

2. Evaluate $LHS(y_{ij}) - v^n_{x,ij}$ at each z_s.

3. If the resulting vector is negative for all s, meaning that the value of the resource is greater than the marginal utility of extraction at that point in the grid, set $y_{ij} = 0$.

4. If the resulting vector is positive for all s, meaning that the value of the resource is lower than the marginal utility of extraction at that point in the grid, set $y_{ij} = x_i$.

5. If the resulting vector has entries of either sign, find the lowest index s at which the vector crosses zero, then set $y_{ij} = z_s$.