1. Given the equation

$$(x^2 - 3x - 2)^2 - 3(x^2 - 3x - 2) - 2 - x = 0$$

prove that the roots of the equation $x^2 - 4x - 2 = 0$ are roots of the initial equation and find all real roots of the equation.

Solution 1: Let

$$f(x) = x^2 - 4x - 2$$

and r_1, r_2 be the roots of $f(x)$. Thus we can express the first equation as

$$(f(x) + x)^2 - 3(f(x) + x) - 2 - x = 0$$

Substituting r_1 into this equation, knowing that $f(r_1) = 0$ (you can also use r_2).

$$r_1^2 - 4r_1 - 2 = 0$$

This implies the first equation is true if this new equation is also true. Since

$$f(r_1) = r_1^2 - 4r_1 - 2 = 0$$

Then the first given equation must be true for $x = r_1$. This is the same for r_2.

Solution 2 (credits to n'ada): Let

$$f(x) = x^2 - 3x - 2$$

Thus if we can express the first equation as looking for an x such that

$$f(f(x)) - x = 0$$

From $x^2 - 4x - 2 = 0$ we have

$$f(x) - x = 0$$
$$f(x) = x$$

Thus

$$f(f(x)) - x = f(x) - x = 0$$

Thus the roots to the first equation are the roots to $f(x) - x = 0$ which is $x^2 - 4x - 2 = 0$.