

# MATH 355 Notes

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## 1 Topic 1 - Functions and Cardinality

**Definition** Given sets  $A$  and  $B$ , the **Cartesian Product of  $A$  and  $B$**  is the set of ordered pairs

$$A \times B = \{(a, b) | a \in A, b \in B\}$$

Given any finite list of  $N$  sets  $A_1, A_2, \dots, A_N$ , the **Cartesian Product of  $A_i$**  is

$$A_1 \times A_2 \times \dots \times A_N = \{(a_1, a_2, \dots, a_n) | a_i \in A_i; i = 1, 2, \dots, N\}$$

**Example**

$$\begin{aligned}\mathbb{R}^2 &= \mathbb{R} \times \mathbb{R} \\ \mathbb{R}^N &= \underbrace{\mathbb{R} \times \mathbb{R} \times \dots \times \mathbb{R}}_N\end{aligned}$$

When  $\mathbb{T}$  is the unit circle (i.e.  $\mathbb{T} = \{(a, b) | a, b \in \mathbb{R}, a^2 + b^2 = 1\}$ ),  $\mathbb{T}^2$  is the unit torus.

**Definition** A **function**  $f$  from  $A$  to  $B$  is a subset  $f \subseteq A \times B$  with the additional property that for all  $a \in A$ , there is a single  $b \in B$  so that  $(a, b) \in f$ . Then  $b = f(a)$ . Given a function  $f : A \rightarrow B$ ,  $A$  is the **domain** of  $f$  (denoted  $\text{dom}(f)$ ), and  $B$  is the **codomain** of  $f$  (denoted  $\text{codom}(f)$ ). The **range** of  $f$  is the set  $f(a) | a \in A$  (denoted  $\text{ran}(f)$ ).

**Example**

$$\begin{aligned}f &: \mathbb{R} \rightarrow \mathbb{R}, f(x) = x^2 \\ \text{dom}(f) &= \text{codom}(f) = \mathbb{R} \\ \text{ran}(f) &= [0, \infty)\end{aligned}$$

$$\begin{aligned}g &: \mathbb{R} \rightarrow \mathbb{R}, g(x) = x^3 \\ \text{dom}(g) &= \text{codom}(g) = \text{ran}(g) = \mathbb{R}\end{aligned}$$

$$\begin{aligned}h &: [0, \pi] \rightarrow \mathbb{R}, h(x) = \cos(x) \\ \text{dom}(h) &= [0, \pi] \\ \text{codom}(h) &= \mathbb{R} \\ \text{ran}(h) &= [-1, 1]\end{aligned}$$

**Definition** Given  $f : A \rightarrow B$ ,

- (i)  $f$  is injective  $\iff f(a_1) = f(a_2) \implies a_1 = a_2 \forall a_1, a_2 \in A$
- (ii)  $f$  is surjective  $\iff \forall b \in B, \exists a \in A : f(a) = b$
- (iii)  $f$  is bijective  $\iff f$  is injective and  $f$  is surjective

**Example**

$$k : \mathbb{Z}^+ \rightarrow \mathbb{Z}, k(n) = \begin{cases} \frac{n}{2}, & n \text{ even} \\ -\left(\frac{n-1}{2}\right), & n \text{ odd} \end{cases}$$