Multilevel Ex-Gaussian Model for Response Times

This is a minimal example, using artificial data, of how to model human response times using a multilevel ExGaussian regression model. It is implemented in Jags and R and uses the gamlss and rjags packages. For details about how to see up R to use Jags, see a guide I wrote for a workshop that I teach that requires R to use Jags.

Details

An ExGaussian probability distribution with parameters μ, σ^2, and τ:, is a convolution of a Gaussian (or Normal) distribution with mean and variance parameters μ, σ^2 respectively, and an exponential distribution with rate parameter τ. More simply, if x is normally distributed random variable, with parameters μ, σ^2, and y is an exponentially distributed with parameters τ, then

$$z = x + y$$

is an ExGaussian random variable with parameters μ, σ^2 and τ.

The ExGaussian distribution has been used a model of human reaction times, see [Hea1991]. As such, it could be used to replace the Normal probability distribution that is the standard assumption of linear regresion models. What follows is a description of how to do this in a multilevel regression model, where the slope and intercepts for some predictor vary randomly across subjects in an experiment. Also, both μ and τ vary as (linear or transformed linear) functions of the predictor.

In detail, let us assume that our observed data are

$$(z_i, v_i, s_i)$$

for $i \in 1 \ldots n$, where z_i is the observed response time on trial i, v_i is the value of the predictor variable on trial i, and $s_i \in 1 \ldots K$ is the identity of the subject on trial i.

The main details of this model are as follows:

$$\begin{aligned} z_i &\sim \mathrm{dexgauss}(\mu_i, \tau_i, \sigma^2) \\ \mu_i &= \alpha_{0[s_i]} + \beta_{0[s_i]} v_i, \\ \log(\tau_i) &= \alpha_{1[s_i]} + \beta_{1[s_i]} v_i, \end{aligned}$$

where, for $k \in 1 \ldots K$, each of α_{0k}, β_{0k}, α_{1k}, $\beta 1k$ are normally distributed random variables. These are random slopes and intercepts for each subject.

[Hea1991] Analysis of response time distributions: An example using the Stroop task. Heathcote, Andrew; Popiel, Stephen J.; Mewhort, D. J. Psychological Bulletin, Vol 109(2), Mar 1991, 340-347.