

The “Boxes and Bulbs” Problem

Mark Andrews

The problem

The “Boxes and Bulbs” problem¹ is as follows:

Box A has 10 lightbulbs, of which 4 are defective. Box B has 6 lightbulbs, of which 1 is defective. Box C has 8 lightbulbs, of which 3 are defective.

I) If we randomly choose a box, and then randomly choose a lightbulb from that box, what is the probability that we will choose a non-defective bulb? II) If we do choose a nondefective bulb, what is the probability it came from Box C?

¹This problem is taken from *Schaum's Outlines: Probability* (2nd ed., 2000), pages 87-88.

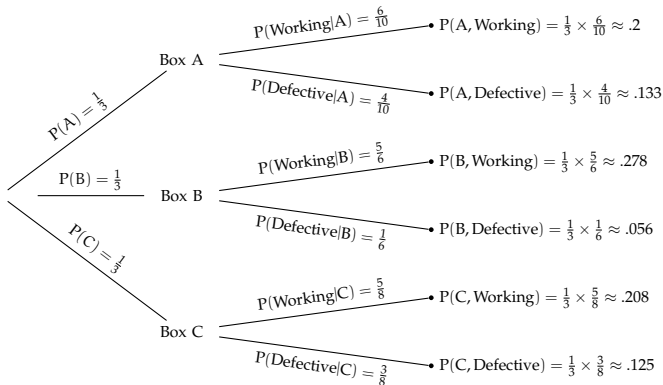
The solution

First, note that the solution requires the calculation of two probabilities:

$$P(\text{Bulb} = \text{Working}) \quad \text{and} \quad P(\text{Box} = C | \text{Bulb} = \text{Working}).$$

These are the answers to Part I and Part II, respectively. Second, if you correctly work out the first probability, i.e. $P(\text{Bulb} = \text{Working})$, then will have all the information necessary to obtain the second probability, i.e. $P(\text{Box} = C | \text{Bulb} = \text{Working})$.

The generative model



- This tree structure gives you all the information you need to answer the questions. Note that on the right are given all the joint probabilities, which can be arranged to a joint probability table as follows:

	Working	Defective
Box A	.2	.133
Box B	.278	.056
Box C	.208	.125

- Now it is simple to see the overall probability of choosing a working bulb. It is probability of choosing Box A and choosing a working bulb *or* choosing Box B and choosing a working bulb *or* choosing Box C and choosing a working bulb. This is the sum of the first column, which is $.2 + .278 + .208 \approx .686$.
- Likewise, to get the probability of choosing Box C *given* that we have chosen a working bulb, we see what proportion of the total probability of choosing a working bulb, i.e. .686 is from when we choose Box C and a working bulb, i.e. .208. In other words, it is $.208/.686 \approx .304$.

Using probability rules

- ▶ The first question asks us to calculate $P(\text{Bulb} = \text{Working})$. We know that this is equal to

$$\begin{aligned} P(\text{Working}) &= P(\text{Working}, A) + P(\text{Working}, B) + P(\text{Working}, C), \\ &= .2 + .278 + .208. \\ &\approx .686. \end{aligned}$$

- ▶ When asked for $P(C|\text{Working})$ we use the rule of conditional probability being the joint probability divided by the marginal probability, i.e.

$$\begin{aligned} P(C|\text{Working}) &= \frac{P(\text{Working}, C)}{P(\text{Working})}, \\ &= \frac{.208}{.686}, \\ &\approx .304. \end{aligned}$$