Q.no. 6 in jee linalg 2d Matrix Project

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Geometric Question

The sides of a rhombus ABC are parallel to the lines

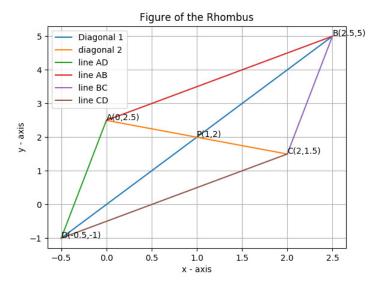
$$\begin{bmatrix} 1 & -1 \end{bmatrix} X + 2 = 0 \longrightarrow (1)$$

$$\begin{bmatrix} 7 & -1 \end{bmatrix} X + 3 = 0 \longrightarrow (2)$$

If the diagonals of the rhombus intersect at

$$P = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

and the vertex A (different) from the origin is on the y-axis, then find the ordinate of A.



Shifting the origin to $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ we get the new equations as

$$\begin{bmatrix} 1 & -1 \end{bmatrix} X + 1 = 0 \\ 7 & -1 \end{bmatrix} X + 8 = 0$$

The slope of the doesn't change if change the origin.

eq(1) can be written in general form as

$$\begin{bmatrix} t-1 \\ t \end{bmatrix}$$

By using linear transformation of matrices We can rotate $\begin{bmatrix} t-1 \\ t \end{bmatrix}$ with respect to origin by multiplying with this matrix

$$\begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$

where $\theta = acuteanglebetween original two equations$

So after rotation we get the point as
$$\begin{bmatrix} t/5 + 1/5 \\ 7t/5 + 7/5 \end{bmatrix}$$

$$[7 \ -1] X=5$$

Solving the obtained equation with eq(1)

$$\begin{bmatrix} 7 & -1 \\ 1 & -1 \end{bmatrix} X = \begin{bmatrix} 5 \\ -1 \end{bmatrix}$$

upon Solving we get

$$\begin{bmatrix} -2/3 \\ 1/3 \end{bmatrix}$$

So the direction vector of diagnol is $\begin{bmatrix} -2/3\\1/3 \end{bmatrix}$