Wein Bridge Oscillator

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Redesign the circuit of Fig. 0 for operation at 10KHz using same values of resistance. If at 10KHz the op amp provides an excess phase shift (lag) of 5.7°, what will be the frequency of oscillation? Assume the phase shift introduced by the op amp remains constant for frequencies around 10KHz.)To restore operation to 10KHz, what change must be made in the shunt resistor of the wien bridge? Also, to what value must R_2/R_1 be changed ? (initially $R_2 = 10k\Omega$, $R_1 = 5k\Omega$)
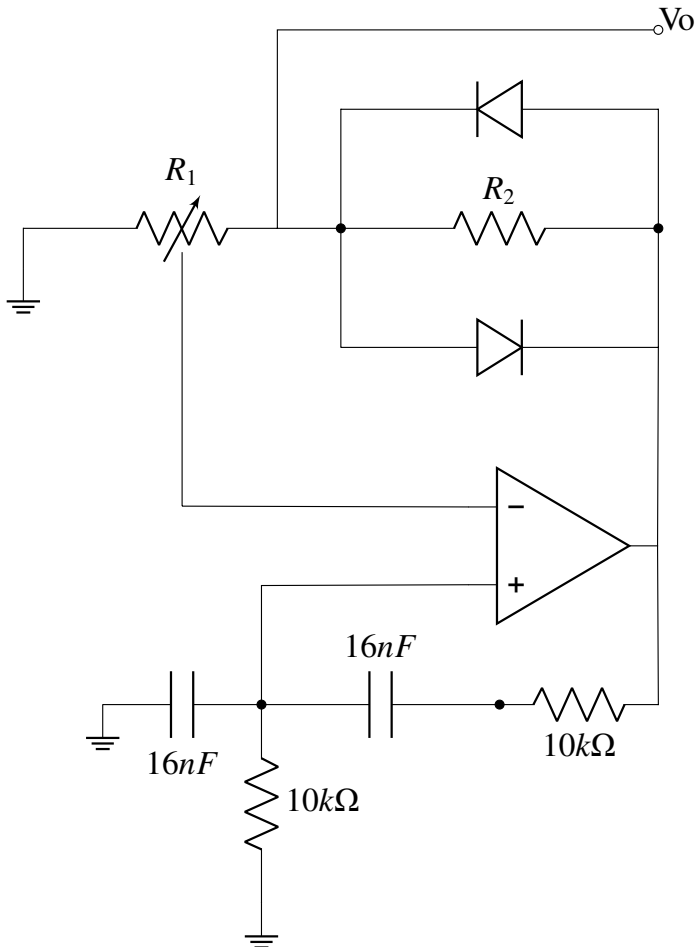

Fig. 0

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1. Draw block diagram for the above circuit .
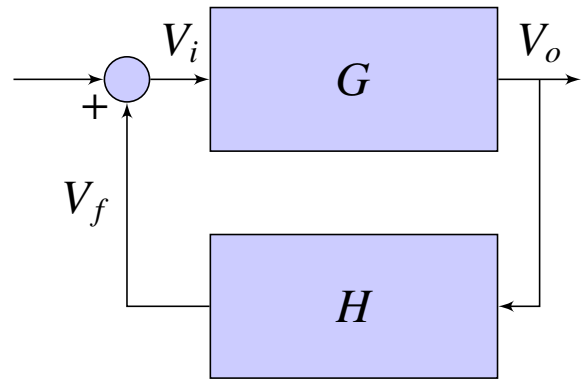   **Solution:**

Fig. 1

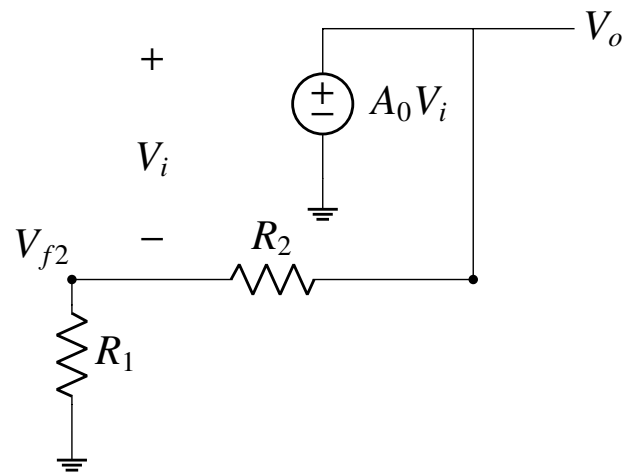2. Find G and draw circuit diagram for G .
   Solution:

Fig. 2

from fig. 2

$$V_{f2} = \left(\frac{R_1}{R_1 + R_2}\right) V_o \qquad (2.1)$$

$$G_1 = \frac{V_{f2}}{V_o} = \left(\frac{R_1}{R_1 + R_2}\right) \qquad (2.2)$$

from fig 2 , A_o is the gain of amplifier, and G_1 is placed as negative feedback factor. Therefore total G is given as

$$G = \frac{A_0}{1 + A_0 G_1} \tag{2.3}$$

$$= \frac{1}{\frac{1}{A_0} + G_1} \tag{2.4}$$

$$\implies G \approx \frac{1}{G_1}, \quad A_0 \to \infty \tag{2.5}$$

$$\text{or, } G = \frac{R_1 + R_2}{R_1} = 1 + \frac{R_2}{R_1} \tag{2.6}$$

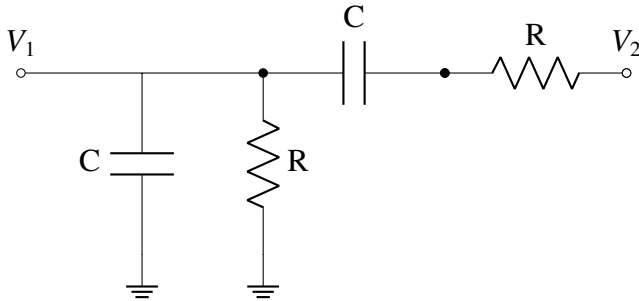3. Find H and draw circuit diagram for H
   Solution:

Fig. 3

from fig 3

$$V_1 = \frac{R \parallel \frac{1}{sC}}{\left(R \parallel \frac{1}{sC}\right) + R + \frac{1}{sC}} V_2 \tag{3.1}$$

$$H = \frac{1}{3 + j(\omega CR - 1/\omega CR)} \tag{3.2}$$

4. Design the circuit for operation at 10khz , without varying resistance .

   Solution: As we know the RC circuit (*Hblock*) determines the frequency of oscillation.

$$f = \frac{1}{2\pi RC} \tag{4.1}$$

$$\implies RC = 1.6 * 10^{-5} \tag{4.2}$$

   So, we change the Capacitor value to 1.6nF

5. Find frequency of oscillation when the excess phase shift (*lag*) of 5.7°

   Solution: The phase shift in loop can

only be caused by H block as it contains the frequency components unlike G block.
From eq.3.2 we get ,

$$\phi(\omega) = \tan^{-1}\left(\frac{\omega CR - 1/\omega CR}{3}\right) \tag{5.1}$$

$$\phi(\omega) = 5.7° \tag{5.2}$$

$$\frac{\omega CR - 1/\omega CR}{3} = 0.1 \tag{5.3}$$

From RC value in eq 4.2 we get

$$\omega = 53824.21 rad/s \tag{5.4}$$

$$\frac{1}{2\pi f} = 53824.21 \tag{5.5}$$

$$\implies f = 8.5khz \tag{5.6}$$

6. verify the above result using differential analysis.
   Solution:

$$L(j\omega) = H(j\omega) G(j\omega) \tag{6.1}$$

$$L(j\omega) = \frac{1 + R_2/R_1}{3 + j(\omega CR - 1/\omega CR)} \tag{6.2}$$

   As the change in phase is very small.

$$\phi(\omega) = \tan^{-1}\left(\frac{\omega CR - 1/\omega CR}{3}\right) \tag{6.3}$$

   Differentiating $\phi(\omega)$ with ω

$$\frac{\partial \phi(\omega)}{\partial \omega} = \frac{-1}{1 + \left(\frac{\omega CR - 1/\omega CR}{3}\right)^2} \frac{\partial(\omega CR - 1/\omega CR)}{\partial \omega} \tag{6.4}$$

   And since $\omega = 1/CR$ upon evaluating

$$\frac{\partial \phi(\omega)}{\partial \omega} = \frac{-2CR}{3} = \frac{-2}{3\omega} \tag{6.5}$$

$$\text{and } \Delta\omega = \frac{\Delta\phi}{\frac{\partial \phi(\omega)}{\partial \omega}} \tag{6.6}$$

   Above equation 6.6 is only for small parameter changes

$$\Delta\omega = \frac{-0.1}{-2/3\omega} \quad (5.7° = 0.1rad/s) \qquad (6.7)$$

$$\Delta f = 0.15f \qquad (6.8)$$

So, the frequency of oscillation is

$$f - \Delta f = 10 - (0.15 * 10) \qquad (6.9)$$
$$\implies 8.5kHz \qquad (6.10)$$

7. What change must be made in the shunt resistor of the wien bridge, to restore the frequency of oscillation ?
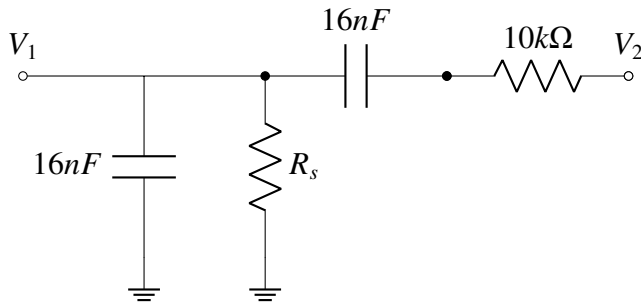   **Solution:** Assuming the parallel R = R_s as shunt shown in Fig 7

Fig. 7

Feedback loop gain $\beta(s)$

$$\beta(s) = \frac{R_s \| \frac{1}{sC}}{\left(R_s \| \frac{1}{sC}\right) + R + \frac{1}{sC}} \qquad (7.1)$$

$$\beta(s) = \frac{1}{2 + \frac{R}{R_s} + j\left(\omega CR - \frac{1}{\omega CR_s}\right)} \qquad (7.2)$$

So, phase shift $\phi(\omega)$ is

$$\phi(\omega) = -\tan^{-1}\left(\frac{\omega CR - \frac{1}{\omega CR_s}}{2 + \frac{R}{R_s}}\right) \qquad (7.3)$$

$$\omega CR - \frac{1}{\omega CR_s} = \left(2 + \frac{R}{R_s}\right)(-0.1) \qquad (7.4)$$

from 7.3 we can find value of R_s when $\omega = \frac{1}{RC}$

$$1 - \frac{R}{R_s} = -0.2 - 0.1\left(\frac{R}{R_s}\right) \qquad (7.5)$$

$$1 + 0.2 = \frac{R}{R_s} - 0.1\left(\frac{R}{R_s}\right) \qquad (7.6)$$

$$\implies 1.2 = 0.9\left(\frac{R}{R_s}\right) \qquad (7.7)$$

$$R_s = 0.75R \qquad (7.8)$$
$$R_s = 7.5k\Omega \qquad (7.9)$$

8. Also, to what value must R_2/R_1 be changed ?

   Solution: Substituting the values of R and R_s in 7.1 we get

$$\beta(j\omega) = \frac{1}{3.35} \qquad (8.1)$$

we know the loop gain $L(j\omega)$ is

$$L(j\omega) = \frac{1 + R_2/R_1}{\beta(j\omega)} \qquad (8.2)$$

Condition for oscillation is $1 - L(s) = 0$

$$1 + \frac{R_2}{R_1} = 3.35 \qquad (8.3)$$

$$\frac{R_2}{R_1} = 2.35 \qquad (8.4)$$

9. Spice simulation of the original circuit before redesign .

   Solution:
   • Refer to Fig. 9 for output, $f_r = 1khz$
   • original netlist simulated circuit here:

   > codes/ee18btech11049/ee18btech11049_1. net

   • You can find the python script for the above here:

   > codes/ee18btech11049/ee18btech11049_1. py

10. spice simulation after changing capacitance to 1.6nF
    • Refer to Fig. 9 for the spice output, $f_r = 8.5khz$
    • original netlist simulated circuit here:

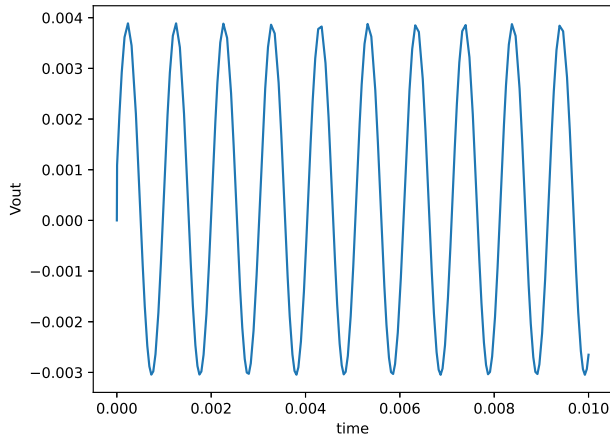   > codes/ee18btech11049/ee18btech11049_2. net

Fig. 9: Simulation result

- You can find the python script for the above, used to generate the output here:

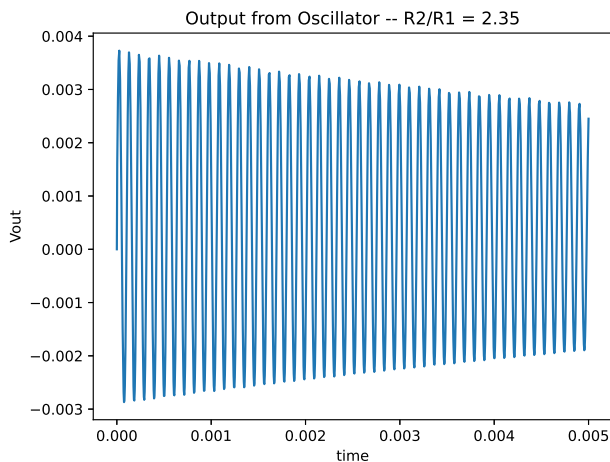codes/ee18btech11049/ee18btech11049_2. py

Fig. 10: Simulation result

11. spice simulation after changing shunt resistance to restore frequency
   - Refer to Fig. 9 for the spice output, $f_r = 10khz$
   - original netlist simulated circuit here:

   codes/ee18btech11049/ee18btech11049_3. net

   - You can find the python script for the above, used to generate the output here:
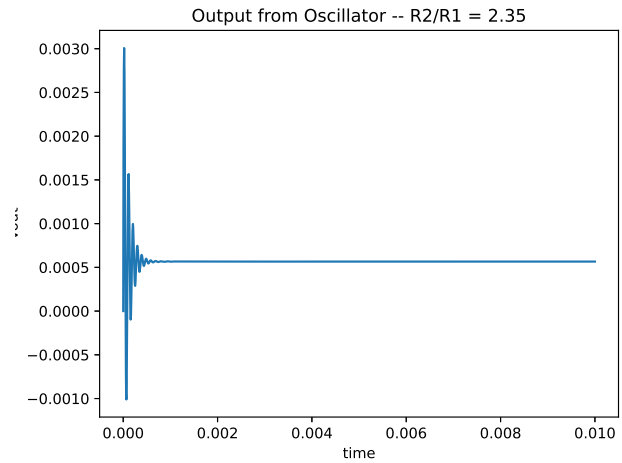
codes/ee18btech11049/ee18btech11049_3. py

Fig. 11: Simulation result

12. spice simulation after changing R_2/R_1 to restore oscillations
   - Refer to Fig. 9 for the spice output, $f_r = 10khz$
   - original netlist simulated circuit here:

   codes/ee18btech11049/ee18btech11049_4. net

   - You can find the python script for the above, used to generate the output here:
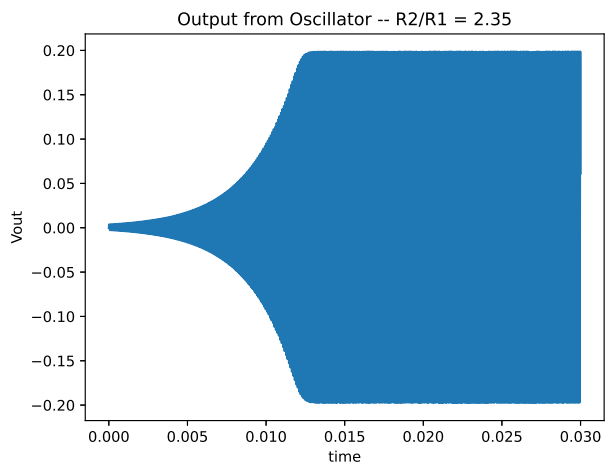
   codes/ee18btech11049/ee18btech11049_4. py

Fig. 12: Simulation result