

Q.no. 19 in GATE ECE 2015

Control Systems

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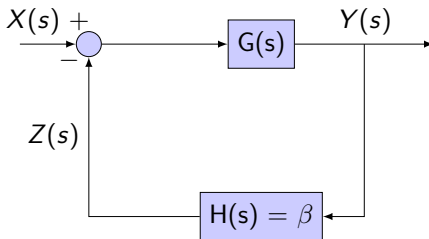
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Question

Negative Feedback in a closed-loop control system *DOES NOT*?

- a.) Reduce overall gain
- b.) Reduce sensitivity to parameter variation
- c.) Improve disturbance rejection
- d.) Reduce bandwidth

Solution : Option-a



$$\text{Closed loop gain } Y(s)/X(s) = \frac{|G(s)|}{|1 + \beta G(s)|} \quad (1)$$

$$\text{Open loop gain} = |G(s)| \quad (2)$$

$$\text{Comparing (1) and (2)} \quad (3)$$

Option-a

$$|1 + \beta H(s)| > 1 \quad (4)$$

$$\frac{1}{|1 + \beta H(s)|} < 1 \quad (5)$$

$$\frac{|G(s)|}{|1 + \beta H(s)|} < |G(s)| \quad (6)$$

from (6) we can say - closed loop gain < open loop gain

Option-b

Sensitivity to parameter variation :

$$\text{sensitivity} = \frac{\text{change in overall gain}}{\text{change in system parameter}}$$

– Lets take system parameter be G

$$\text{And closed loop Sensitivity} = \frac{(dT/T * 100)}{(dG/G * 100)} = \frac{1}{1 + GH} \quad (7)$$

$$\text{open loop Sensitivity} = 1 \quad (8)$$

Therefore, reduced sensitivity.

Option -c

The use of feedback is to remove disturbance. As the output gets changed subsequently the input will be altered through feedback terminal. Thus, reducing the disturbance. Therefore, It improves disturbance Rejection.

Option d bandwidth

With 3db(Half power) frequency we can measure bandwidth of the system.

$$\text{Power gain} = |H(jw)|^2 = \frac{|Y(jw)|^2}{|X(jw)|^2} \quad (9)$$

evaluating the magnitude squared at the "center" of passband .
Taking $w = 0$ as center of passband.

$$\text{Open loop 3db} \implies |H(jw)|^2 = \frac{1}{2} |H(0)|^2 \quad (10)$$

$$\text{Closed loop 3db} \implies \frac{|H(jw)|^2}{|1 + H(jw)|^2} = \frac{1}{2} \frac{|H(0)|^2}{|1 + H(0)|^2} \quad (11)$$

Option d

considering positive half(ω) and calculating values of $H(j\omega)$:

$$\text{Open loop 3db} \implies H(j\omega) = \frac{1}{\sqrt{2}}H(0) \quad (12)$$

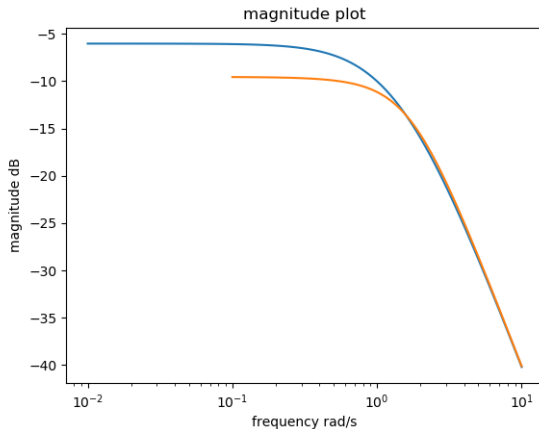
$$\text{Closed loop 3db} \implies H(j\omega) = \frac{H(0)}{\sqrt{2} + H(0)(\sqrt{2} - 1)} \quad (13)$$

$$H_o(j\omega) = H_c(j\omega) \quad (14)$$

$$\omega_o < \omega_c \text{ (As } H(j\omega) \text{ is decreasing in positive half)} \quad (15)$$

Therefore bandwidth Increases.

Proof : Plot for a system and its closed loop unity system.



From plot we can say bandwidth increases