

# Control Systems

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			<p><i>Abstract</i>—This manual is an introduction to control systems based on GATE problems. Links to sample Python codes are available in the text.</p> <p>Download python codes using</p> <div style="border: 1px solid black; padding: 5px; margin-top: 10px;"> <pre>svn co https://github.com/gadepall/school/trunk/control/codes</pre> </div>

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## 1 SIGNAL FLOW GRAPH

### 1.1 Mason's Gain Formula

### 1.2 Matrix Formula

## 2 BODE PLOT

### 2.1 Introduction

### 2.2 Phase

## 3 SECOND ORDER SYSTEM

### 3.1 Damping

### 3.2 Peak Overshoot

### 3.3 Settling Time

## 4 ROUTH HURWITZ CRITERION

### 4.1 Routh Array

### 4.2 Marginal Stability

### 4.3 Stability

## 5 STATE-SPACE MODEL

### 5.1 Controllability and Observability

### 5.2 Second Order System

## 6 NYQUIST PLOT

### 6.1 Introduction

## 7 COMPENSATORS

### 7.1 Phase Lead

### 7.2 Lag Lead

## 8 GAIN MARGIN

### 8.1 Introduction

### 8.2 Example

8.1. Plot the Bode magnitude and phase plots for the following system

$$G(s) = \frac{75(1 + 0.2s)}{s(s^2 + 16s + 100)} \quad (8.1.1)$$

Also compute gain margin and phase margin .

**Solution:** From (8.1.1), we have

$$G(j\omega) = \frac{75(1 + 0.2j\omega)}{j\omega((j\omega)^2 + 16j\omega + 100)} \quad (8.1.2)$$

poles = 0 , -8-6j , -8+6j

zeros = -5

Gain and phase plots are shown in 8.1:a and 8.1:b The following code plots Fig 8.1:a and 8.1:b

```
codes/ee18btech11049.py
```

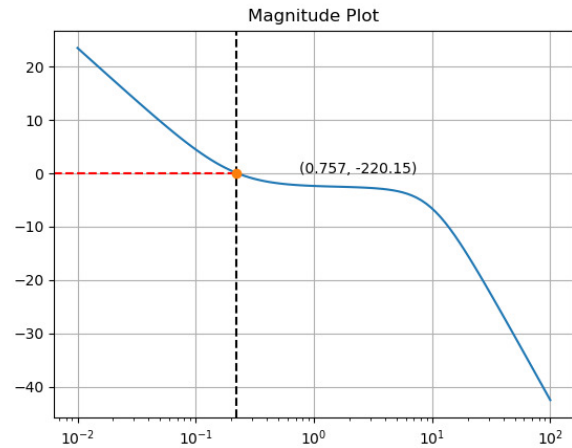


Fig. 8.1: a

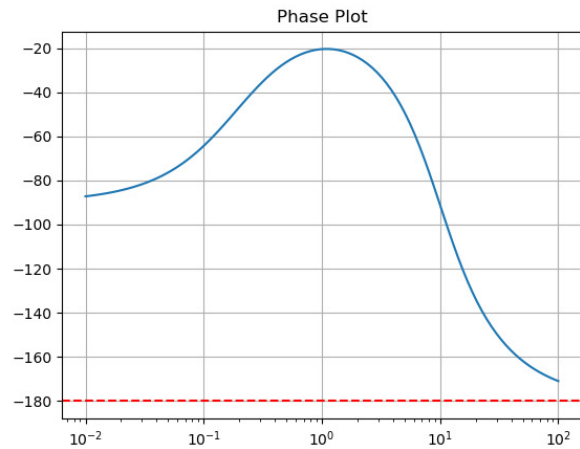


Fig. 8.1: b

8.2. Find  $\angle G(j\omega) + 180^\circ$  , where  $\omega$  is frequency when gain = 1 . This is known as *phase margin (PM)*

**Solution:**

$$\frac{75 \sqrt{\omega^2 + 25}}{\omega \sqrt{(\omega + 6)^2 + 64} \sqrt{(\omega - 6)^2 + 64}} = 1 \quad (8.2.1)$$

Solving (8.2.1) (or)

from Fig 8.1:a frequency at which gain = 1 ,is gain crossover frequency  $\omega_{gc}$  .

$$\Rightarrow \omega_{gc} = 0.757 \quad (8.2.2)$$

$$\angle G(j\omega_{gc}) = -88.3 \quad (8.2.3)$$

$$\Rightarrow PM = 91.7 \quad (8.2.4)$$

8.3. Find  $-G(j\omega)$  db , where  $\omega$  is frequency when phase =  $-180^\circ$  . This is known as *gain margin* (GM)

**Solution:** From Fig 8.1:b ,we can say that phase never crosses  $-180^\circ$  . So , the gain margin is *infinite*. Which means we can add any gain , and the equivalent closed loop system never goes unstable.

## 9 PHASE MARGIN

### 9.1 Introduction

## 10 OSCILLATOR

### 10.1 Introduction

## 11 ROOT LOCUS