Assignment-1

BANDI SAI LAXMAN - EE18BTECH11049

1 Software Installation

Run the following commands

```
sudo apt−get update
sudo apt−get install libffi−dev libsndfile1 python3
    −scipy python3−numpy python3−matplotlib
sudo pip install cffi pysoundfile
```

2 Digital Filter

2.1 Download the sound file from

```
wget https://raw.githubusercontent.com/
    gadepall/
EE1310/master/filter/codes/Sound_Noise.wav
```

2.2 You will find a spectrogram at https://academo.org/demos/spectrum-analyzer. Upload the sound file that you downloaded in Problem 2.1 in the spectrogram and play. Observe the spectrogram. What do you find?
Solution: There are a lot of yellow lines between 440 Hz to 5.1 KHz. These represent the synthesizer key tones. Also, the key strokes are audible along with background noise.

2.3 Write the python code for removal of out of band noise and execute the code.
Solution:

```
import soundfile as sf
from scipy import signal

#read .wav file
input_signal,fs = sf.read('Sound_Noise.wav
    ')

#sampling frequency of Input signal
sampl_freq = fs

#order of the filter
order = 4

#cutoff frquency 4kHz
cutoff_freq = 4000.0

#digital frequency
Wn = 2*cutoff_freq/sampl_freq

# b and a are numerator and denominator
    polynomials respectively
b,a = signal.butter(order,Wn,'low')

#filter the input signal with butterworth filter
output_signal = signal.filtfilt(b,a,input_signal
    )

#output signal = signal.lfilter(b, a,input signal
    )
print("Coeffients are as follows")
print(b)
print(a)

#write the output signal into .wav file
sf.write('Sound_with_ReducedNoise.wav',
    output_signal,sampl_freq)
```

2.4 The output of the python script in Problem 2.3 is the audio file Sound_With_ReducedNoise.wav. Play the file in the spectrogram in Problem 2.2. What do you observe?
Solution: The key strokes as well as background noise is subdued in the audio. Also, the signal is blank for frequencies above 5.1 kHz.

3 Difference equation

3.1 Write the difference equation of the above Digital filter obtained in problem 2.3.
Solution:

$$\sum_{m=0}^{M} a\left(m\right) y\left(n-m\right) = \sum_{k=0}^{N} b\left(k\right) x\left(n-k\right) \tag{3.0.1}$$

$$\begin{aligned} y(n) - 2.52y(n-1) + 2.56y(n-2) - 1.206y(n-3) \\ +0.22013y(n-4) = 0.00345x(n) + 0.0138x(n-1) \\ +0.020725x(n-2) + 0.0138x(n-3) + 0.00345x(n-4) \end{aligned} \tag{3.0.2}$$

3.2 Sketch x(n) and y(n).
Solution: The following code yields Fig. 3.2

```
codes/plot_xy.py
```

The filtered sound signal obtained through difference equation is found in

```
codes/Sound_diffEq.wav
```

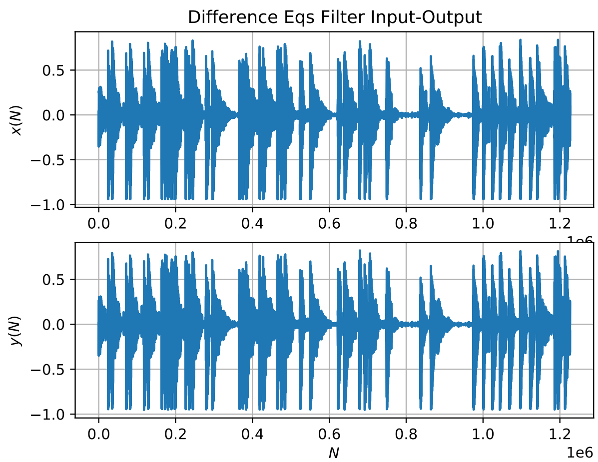

Fig. 3.2

4 Z-transform

4.1 The Z-transform of x(n) is defined as

$$X(z) = \mathcal{Z}\{x(n)\} = \sum_{n=-\infty}^{\infty} x(n)z^{-n} \tag{4.0.1}$$

Show that

$$\mathcal{Z}\{x(n-1)\} = z^{-1}X(z) \tag{4.0.2}$$

and find

$$\mathcal{Z}\{x(n-k)\} \tag{4.0.3}$$

Solution: From (4.0.1),

$$\mathcal{Z}\{x(n-k)\} = \sum_{n=-\infty}^{\infty} x(n-1)z^{-n} \tag{4.0.4}$$

$$= \sum_{n=-\infty}^{\infty} x(n)z^{-n-1} = z^{-1}\sum_{n=-\infty}^{\infty} x(n)z^{-n} \tag{4.0.5}$$

resulting in (4.0.2). Similarly, we can shown that

$$\mathcal{Z}\{x(n-k)\} = z^{-k}X(z) \tag{4.0.6}$$

4.2 Find

$$H(z) = \frac{Y(z)}{X(z)} \tag{4.0.7}$$

from (3.0.2) assuming that the Z-transform is a linear operation.
Solution: Applying (4.0.6) in (3.0.2) we get,

$$H(z) = \frac{Y(z)}{X(z)}$$

$$= \frac{b[0] + b[1]z^{-1} + b[2]z^{-2} + b[3]z^{-3} + b[4]z^{-4}}{a[0] + a[1]z^{-1} + a[2]z^{-2} + a[3]z^{-3} + a[4]z^{-4}} \tag{4.0.8}$$

4.3 Find the Z transform of

$$\delta(n) = \begin{cases} 1 & n = 0 \\ 0 & \text{otherwise} \end{cases} \tag{4.0.9}$$

and show that the Z-transform of

$$u(n) = \begin{cases} 1 & n \geq 0 \\ 0 & \text{otherwise} \end{cases} \tag{4.0.10}$$

is

$$U(z) = \frac{1}{1-z^{-1}}, \quad |z| > 1 \tag{4.0.11}$$

Solution: It is easy to show that from (4.0.10),

$$U(z) = \sum_{n=0}^{\infty} z^{-n} \tag{4.0.12}$$

$$= \frac{1}{1-z^{-1}}, \quad |z| > 1 \tag{4.0.13}$$

4.4 Let

$$H\left(e^{\jmath w}\right) = H\left(z = e^{\jmath w}\right). \tag{4.0.14}$$

Plot $|H\left(e^{\jmath w}\right)|$. Comment. $H(e^{\jmath\omega})$ is known as the *Discret Time Fourier Transform* (DTFT) of $x(n)$.
Solution: The following code plots Fig. 4.4.

```
codes/dtft.py
```

5 Impulse Response

5.1 From the difference equation eq. 3.0.2. Sketch h(n).
Solution: We know that when input is impulse function, we get Impulse response - h(n) as output.
From eq.3.0.1,
Substituting $x(n-k) = \delta(n-k)$, So
$y(n-k)$ becomes $h(n-k)$ for every k=0,1,2,3,4.

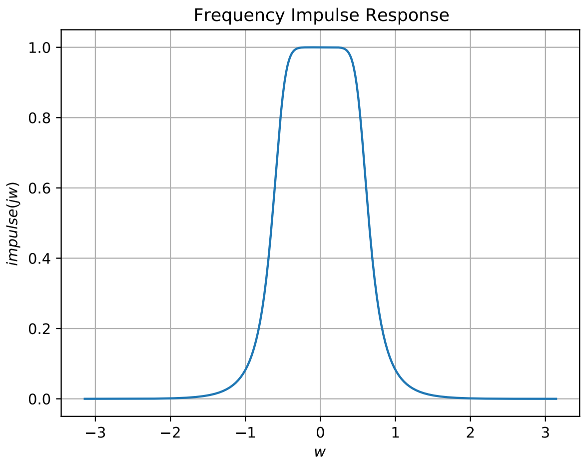

Fig. 4.4: $|H(e^{jw})|$

The following code plots Fig. 5.1

```
codes/impulse_response.py
```

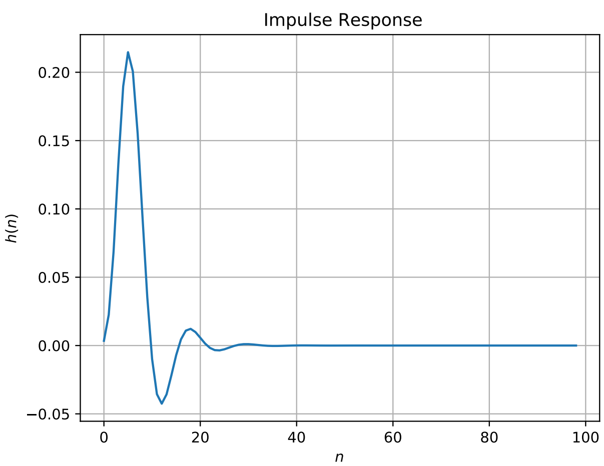

Fig. 5.1: $h(n)$

5.2 Is the above obtained h(n) stable?

Solution:

For a system to be stable, output should be bounded for every bounded input. This is called as BIBO . The system is defined by the eq. 3.0.2

we know that the audio input x(n) is bounded, let B_x be some finite value, we have

$$|x(n)| < B_x < \infty \tag{5.0.1}$$

From convolution we have,

$$|y(n)| = \left|\sum_{-\infty}^{\infty} h(k)x(n-k)\right| \tag{5.0.2}$$

$$|y(n)| \leq \sum_{-\infty}^{\infty} |h(k)|\,|x(n-k)| \tag{5.0.3}$$

Let B_x be the maximum value x(n-k) can take, So

$$|y(n)| \leq B_x \sum_{-\infty}^{\infty} |h(k)| \tag{5.0.4}$$

If

$$\sum_{-\infty}^{\infty} |h(k)| < \infty \tag{5.0.5}$$

Then

$$|y(n)| \leq B_y < \infty \tag{5.0.6}$$

Therefore we can say that y(n) is bounded if x(n) and h(n) are bounded.

$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty \tag{5.0.7}$$

The above equation can be written as,

$$\sum_{n=-\infty}^{\infty} |h(n)|\left|z^{-n}\right|_{|z|=1} < \infty \tag{5.0.8}$$

$$\sum_{n=-\infty}^{\infty} \left|h(n)z^{-n}\right|_{|z|=1} < \infty \tag{5.0.9}$$

From Triangle inequality,

$$\sum_{n=-\infty}^{\infty} \left|h(n)z^{-n}\right|_{|z|=1} < \left|\sum_{n=-\infty}^{\infty} h(n)z^{-n}\right|_{|z|=1} \tag{5.0.10}$$

$$\implies |H(n)|_{|z|=1} < \infty \tag{5.0.11}$$

Therefore, the Region of Convergence should include the unit circle for the system to be stable.

Since, h(n) is right sided the ROC is outside the outer most pole. From the equation (4.0.8) Poles of the given transfer equation is:

$$\begin{aligned} z(approx) = 0.6938 \pm 0.41i, \\ 0.56617 \pm 0.13442i \end{aligned} \tag{5.0.12}$$

From the above poles, we can observe that the ROC of the system is $|z| > \sqrt{0.694^2 + 0.41^2}$. $\implies |z| > 0.806$

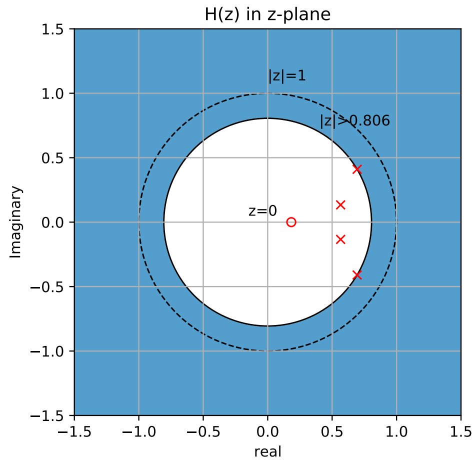

Fig. 5.2: $H(z)$ in z-plane

The code for plotting H(z) in z-plane is:

```
codes/roc.py
```

From the figure we can observe that ROC of the system includes unit circle $|z| = 1$. Which implies that the given IIR filter is stable, because h(n) is absolutely summable.
Since the audio input is bounded,and h(n) is also bounded the system is said to be stable.
Verification:
Given bounded input - audio signal x(n) , and system difference equation 3.0.2
we know that the maximum value of x(n) is 0.839 and minimum value is -0.9417.
Similarly ,the maximum value of y(n) is 0.82225 and minimum value is -0.95376 and it tends to zero as n tends to infinity.
We can say that the bounded input x(n) gives bounded output y(n). So, we can say that the system is BIBO stable i.e stable.

5.3 Using h(n) obtained in 5.1 compute filtered output using the below equation of convolution

$$y(n) = x(n) * h(n) = \sum_{n=-\infty}^{\infty} x(k)h(n-k) \quad (5.0.13)$$

Solution: The following code plots Fig. 5.3

```
codes/convolve.py
```

The filtered sound signal through convolution from this method is found in

```
codes/covolve_music.wav
```

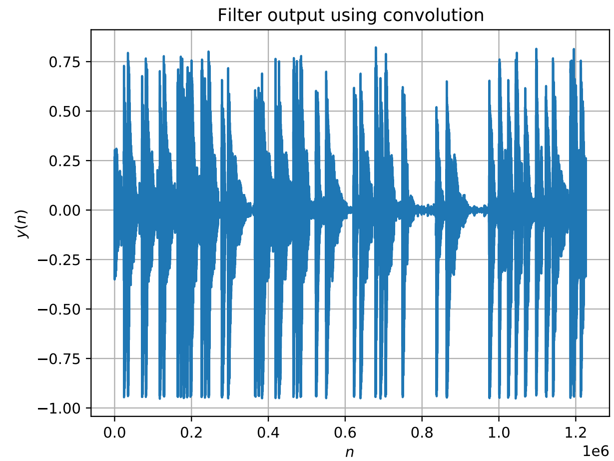

Fig. 5.3: $y(n)$ from the definition of convolution

We can observe that the output obtained is same as y(n) obtained in Fig. 3.2

6 DFT and FFT

6.1 Compute

$$X(k) \triangleq \sum_{n=0}^{N-1} x(n)e^{-j2\pi kn/N}, \quad k = 0, 1, \dots, N-1 \quad (6.0.1)$$

and $H(k)$ using h(n).
Solution: For this given IIR system with audio sample as x(n) and h(n) as impulse response h(n) obtained in 5.1
DFT of a Input Signal $x(n)$ is

$$X(k) \triangleq \sum_{n=0}^{N-1} x(n)e^{-j2\pi kn/N}, \quad k = 0, 1, \dots, N-1 \quad (6.0.2)$$

DFT of a Impulse Response $h(n)$ is

$$H(k) \triangleq \sum_{n=0}^{N-1} h(n)e^{-j2\pi kn/N}, \quad k = 0, 1, \dots, N-1 \quad (6.0.3)$$

The following code plots FFT of $x(n)$ and $h(n)$ in Fig. **??**.

```
codes/fft_x.py
```

Magnitude and Phase plots obtained through above code is

6.2 Compute

$$Y(k) = X(k)H(k) \quad (6.0.4)$$

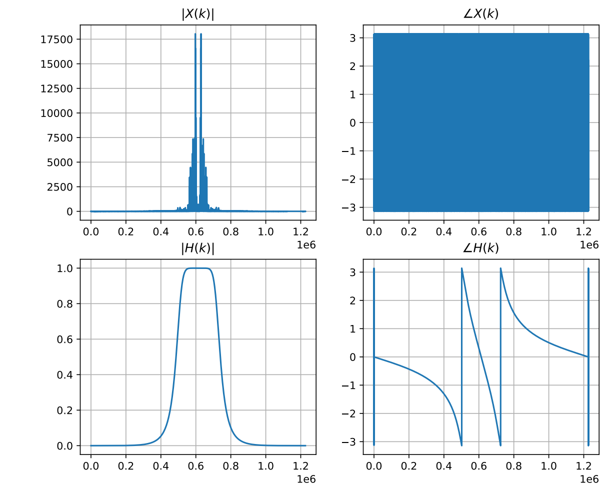

Fig. 6.1: $X(k)$ and $H(k)$

and using this find

$$y(n) \triangleq \sum_{k=0}^{N-1} Y(k) e^{j2\pi kn/N}, \quad n = 0, 1, \ldots, N-1 \tag{6.0.5}$$

Solution: The following code plots Fig.6.2

```
codes/fft_y.py
```

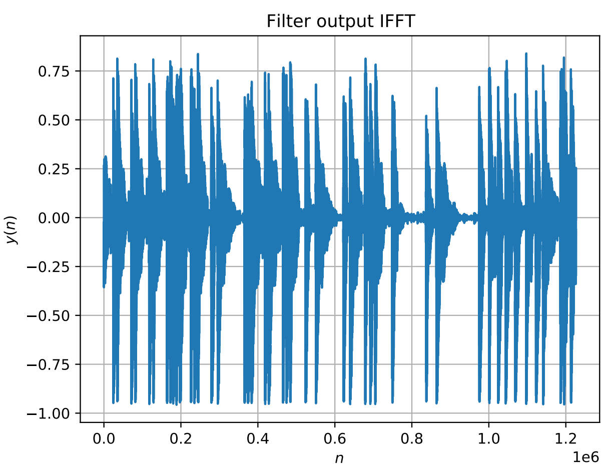

Fig. 6.2: $y(n)$ from IFFT

The filtered sound signal from this method is found in

```
codes/fft_music.wav
```

We can observe from the above plot that it is same as the y(n) observed in Fig.3.2