Control Systems

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Contents

***Abstract*—The objective of this manual is to introduce control system design at an elementary level.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/ketan/codes
```

1 Compensators

1.1 lag-lead Compensator

1.1. For the unity feedback system shown in Fig. 1.1 , with

$$G(s) = \frac{K}{s(s+1)(s+4)} \tag{1.1.1}$$

Design a lag-lead compensator to yield a $K_v = 12$ as well as peak overshoot of 12% and peak time of less than or equal to 2 seconds.

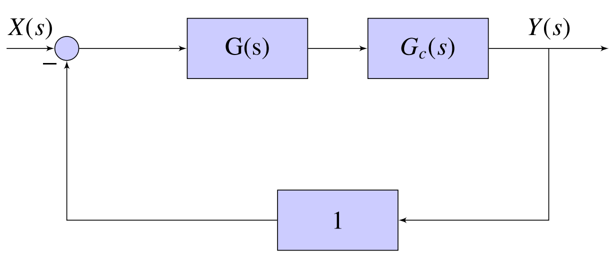

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Fig. 1.1

1.2. **Solution:**

$$K_v = \lim_{s \to 0} sG(s) = 12 \tag{1.2.1}$$
$$\implies K = 48 \tag{1.2.2}$$

The bode plot for G(s)is as follows :

$$G(s) = \frac{48}{s(s+1)(s+4)} \tag{1.2.3}$$

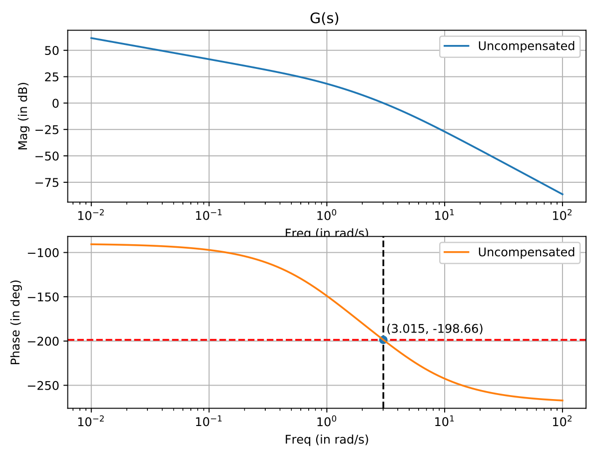

Fig. 1.2: G(s) Bode Plot

Damping Ratio ζ

$$\zeta = \frac{-\ln\left(\frac{OS\%}{100}\right)}{\sqrt{\pi^2 + \left(\ln\left(\frac{OS\%}{100}\right)\right)^2}} \tag{1.2.4}$$

Phase Margin ϕ_M

$$\phi_M = \tan^{-1}\left(\frac{2\zeta}{\sqrt{-2\zeta^2 + \sqrt{4\zeta^4 + 1}}}\right) \tag{1.2.5}$$

closed-loop Bandwidth ω_{bw}

$$\omega_{bw} = \frac{\pi}{T_p\sqrt{1-\zeta^2}}\sqrt{1 - 2\zeta^2 + \sqrt{4\zeta^4 - 4\zeta^2 + 2}} \tag{1.2.6}$$

The following code computes the above quantities.

```
codes/ee18btech11049/ee18btech11049_1.py
```

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$$\zeta = 0.557 \tag{1.2.7}$$

$$\phi_M = 56.13° \tag{1.2.8}$$

$$\omega_{bw} = 2.27 \text{ rad/sec} \tag{1.2.9}$$

The required phase margin to yield a 12% OS is 56.13°
Let us select ω =1.83 rad/s as the new phase-margin frequency.
At this frequency, the uncompensated phase is -176 and would require, if we add a -6 contribution from the lag compensator, a +56 contribution from the lead compensator.

$$G_{Lead}(s)\,G_{Lag}(s) = \left(\frac{s+\frac{1}{T_1}}{s+\frac{\gamma}{T_1}}\right)\left(\frac{s+\frac{1}{T_2}}{s+\frac{1}{\gamma T_2}}\right) \tag{1.2.10}$$

Choose the lag compensator 1-decade below, so that its will have minimal effect at the new phase-margin frequency.

$$\phi_{max,lead} = 56 = \sin^{-1}\frac{1-\beta}{1+\beta} \tag{1.2.11}$$

$$\beta = 0.092 \tag{1.2.12}$$

$$\gamma = 10.86 \text{ since } \gamma = \frac{1}{\beta} \tag{1.2.13}$$

Thus with $\gamma = 10.86$

$$G_{Lag}(s) = \left(\frac{s+\frac{1}{T_2}}{s+\frac{1}{\gamma T_2}}\right) = \frac{s+0.183}{s+0.0168} \tag{1.2.14}$$

$$\omega_{max} = \frac{1}{T_1\sqrt{\beta}} \tag{1.2.15}$$

Using Values of $\omega_{max} = 1.83$ and $\beta = 0.094$

$$G_{Lag}(s) = \left(\frac{s+\frac{1}{T_1}}{s+\frac{\gamma}{T_1}}\right) = \frac{s+0.55}{s+5.69} \tag{1.2.16}$$

The lag-lead Compensated System's open loop transfer function is

$$G_{total}(s) = \frac{48\,(s+0.183)\,(s+0.55)}{s\,(s+1)\,(s+4)\,(s+0.0168)\,(s+5.69)} \tag{1.2.17}$$

1.3. **Verification :** We could observe the affect of the lag-lead compensator from these phase plots.

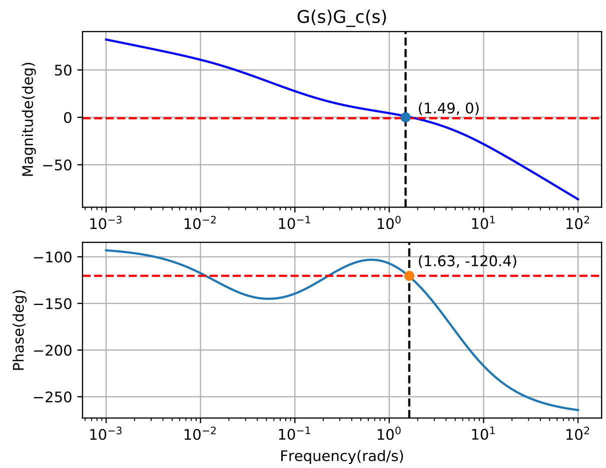

Fig. 1.2: G(s) Compensated Bode Plot

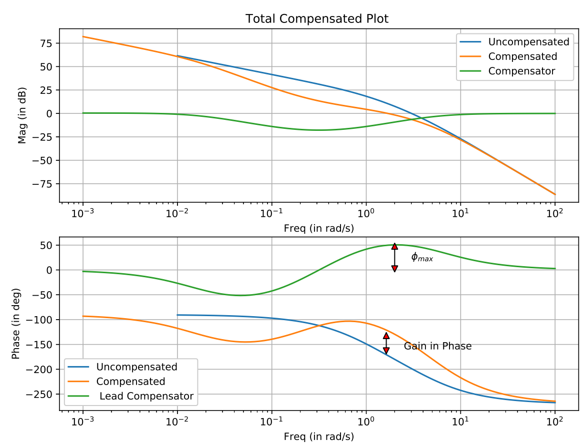

Fig. 1.3: Combined Bode Plots

These plots are generated using the below code:

```
codes/ee18btech11049/ee18btech11049_2.py
```

1.4. **Result :** The below is the summary for the designed lead-lead compensator.

Specifications	**Expected**	**Proposed**
OS%	12%	10.2%
T_p	<= 2	1.61
ϕ_M	56	59.6
ω_{bw}	>= 2.27	3
ω_{gc}	-	1.63
K_v	12	12

TABLE 1.4: Comparing the desired and obtained results