CS 346: Bottom Up Parser

Resource: Textbook

Alfred V. Aho, Ravi Sethi, and Jeffrey D. Ullman, "*Compilers: Principles, Techniques, and Tools*", Addison-Wesley, 1986.

Bottom-Up Parsing

- **Bottom-up parser:**
  - parse tree created from the given input starting from leaves towards the root
  - tries to find the right-most derivation of the given input in the reverse order

    $S \Rightarrow \ldots \Rightarrow \omega$ (the right-most derivation of ω)

    $\leftarrow$ (the bottom-up parser finds the right-most derivation in the reverse order)

- **Bottom-up parsing**: also known as **shift-reduce parsing** because its two main actions are *shift* and *reduce*
  - At each shift action, the current symbol in the input string is pushed to a stack
  - At each reduction step, the symbols at the top of the stack (this symbol sequence is the right side of a production) replaced by the non-terminal at the left side of that production
  - Two more actions: *accept* and *error*

Shift-Reduce Parsing

- Shift-reduce parser tries to reduce the given input string into the starting symbol

a string $\xrightarrow[\text{reduced to}]{}$ the starting symbol

- At each reduction step, a substring of the input matching to the right side of a production rule is replaced by the non-terminal at the left side of that production rule

- If the substring is chosen correctly, the right most derivation of that string is created in the reverse order

Rightmost Derivation: $S \overset{*}{\underset{rm}{\Rightarrow}} \omega$

Shift-Reduce Parser finds: $\omega \underset{rm}{\Leftarrow} \ldots \underset{rm}{\Leftarrow} S$

Shift-Reduce Parsing -- Example

$S \rightarrow aABb$

$A \rightarrow aA \mid a$

$B \rightarrow bB \mid b$

input string: aaabb

aaAbb

aAbb $\Downarrow$ reduction

aABb

S

$$S \underset{rm}{\Rightarrow} aABb \underset{rm}{\Rightarrow} aAbb \underset{rm}{\Rightarrow} aaAbb \underset{rm}{\Rightarrow} aaabb$$

Right Sentential Forms

- How do we know which substring to be replaced at each reduction step?

Handle

- **Handle** of a string is a substring that matches the right side of a production rule
  - *not every substring that matches the right side of a production rule is handle*
- A **handle** of a right sentential form γ ($\equiv \alpha\beta\omega$) :

  a production rule $A \rightarrow \beta$ and a position of γ

  where the string β may be found and replaced by A to produce

  the previous right-sentential form in a rightmost derivation of γ

$$S \overset{rm}{\underset{}{\overset{*}{\Rightarrow}}} \alpha A \omega \overset{rm}{\Rightarrow} \alpha\beta\omega$$

ω is a string of terminals

- If the grammar is *unambiguous*, then every right-sentential form of the grammar has exactly one handle

Handle Pruning

- A right-most derivation in reverse can be obtained by **handle-pruning**

$$S=\gamma_0 \underset{rm}{\Rightarrow} \gamma_1 \underset{rm}{\Rightarrow} \gamma_2 \underset{rm}{\Rightarrow} \ldots \underset{rm}{\Rightarrow} \gamma_{n-1} \underset{rm}{\Rightarrow} \gamma_n = \omega$$

input string

- Start from γ_n, find a handle $A_n \rightarrow \beta_n$ in γ_n, and replace β_n by A_n to get γ_{n-1}
- Then find a handle $A_{n-1} \rightarrow \beta_{n-1}$ in γ_{n-1}, and replace β_{n-1} by A_{n-1} to get γ_{n-2}
- Repeat this, until we reach S

A Shift-Reduce Parser

$E \rightarrow E+T \mid T$

$T \rightarrow T*F \mid F$

$F \rightarrow (E) \mid id$

Right-Most Derivation of `id + id*id`

$$E \Rightarrow E+T \Rightarrow E+T*F \Rightarrow E+T*id \Rightarrow E+F*id$$
$$\Rightarrow E+id*id \Rightarrow T+id*id \Rightarrow F+id*id \Rightarrow id+id*id$$

Right-Most Sentential Form	Reducing Production
id+id*id	$F \rightarrow id$
F+id*id	$T \rightarrow F$
T+id*id	$E \rightarrow T$
E+id*id	$F \rightarrow id$
E+F*id	$T \rightarrow F$
E+T*id	$F \rightarrow id$
E+T*F	$T \rightarrow T*F$
E+T	$E \rightarrow E+T$
E	

Handles are red and underlined in the right-sentential forms.

A Stack Implementation of A Shift-Reduce Parser

Four possible actions of a shift-parser :

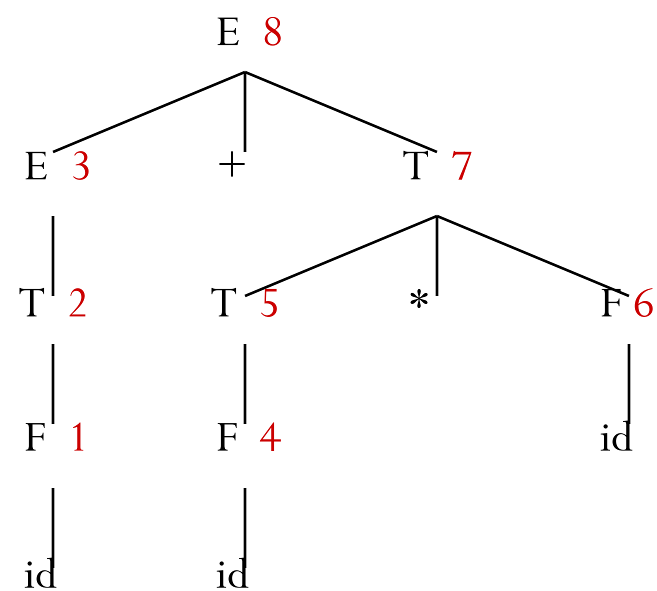
1. **Shift** : The next input symbol is shifted onto the top of the stack
2. **Reduce**: Replace the handle on the top of the stack by the non-terminal
3. **Accept**: Successful completion of parsing
4. **Error**: Parser discovers a syntax error, and calls an error recovery routine

- Initial stack: contains only the end-marker $
- End of the input string: marked by the end-marker $

A Stack Implementation of A Shift-Reduce Parser

Stack	Input	Action
$	id+id*id $	shift
$id	+id*id$	reduce by F → id
$F	+id*id$	reduce by T → F
$T	+id*id$	reduce by E → T
$E	+id*id$	shift
$E+	id*id$	shift
$E+id	*id$	reduce by F → id
$E+F	*id$	reduce by T → F
$E+T	*id$	shift
$E+T*	id$	shift
$E+T*id	$	reduce by F → id
$E+T*F	$	reduce by T → T*F
$E+T	$	reduce by E → E+T
$E	$	accept

Parse Tree



Conflicts During Shift-Reduce Parsing

- For certain class of CFGs, shift-reduce parsers cannot be used
- Stack contents and the next input symbol may not decide action:
  - **shift/reduce conflict**: Whether make a shift operation or a reduction
  - **reduce/reduce conflict**: The parser cannot decide which of several reductions to make
- If a shift-reduce parser cannot be used for a grammar, that grammar is called as non-LR(k) grammar

left to right scanning

right-most derivation

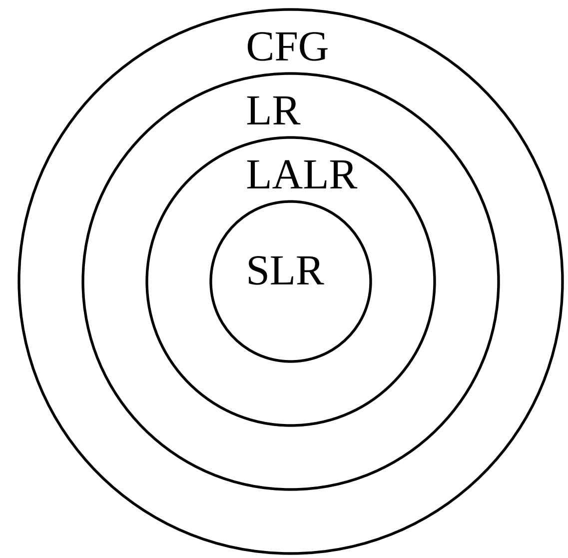
k lookhead

An ambiguous grammar can never be a LR grammar

Shift-Reduce Parsers

Two main categories of shift-reduce parsers:

1. **Operator-Precedence Parser**
   - simple, but only a small class of grammars



2. **LR-Parsers**
   - covers wide range of grammars
     - SLR – simple LR parser
     - LR – most general LR parser
     - LALR – intermediate LR parser (lookahead LR parser)
   - SLR, LR and LALR work same, only their parsing tables are different

LR Parsers

- The most powerful shift-reduce parsing (yet efficient) is:

LR(k) parsing

left to right scanning

right-most derivation

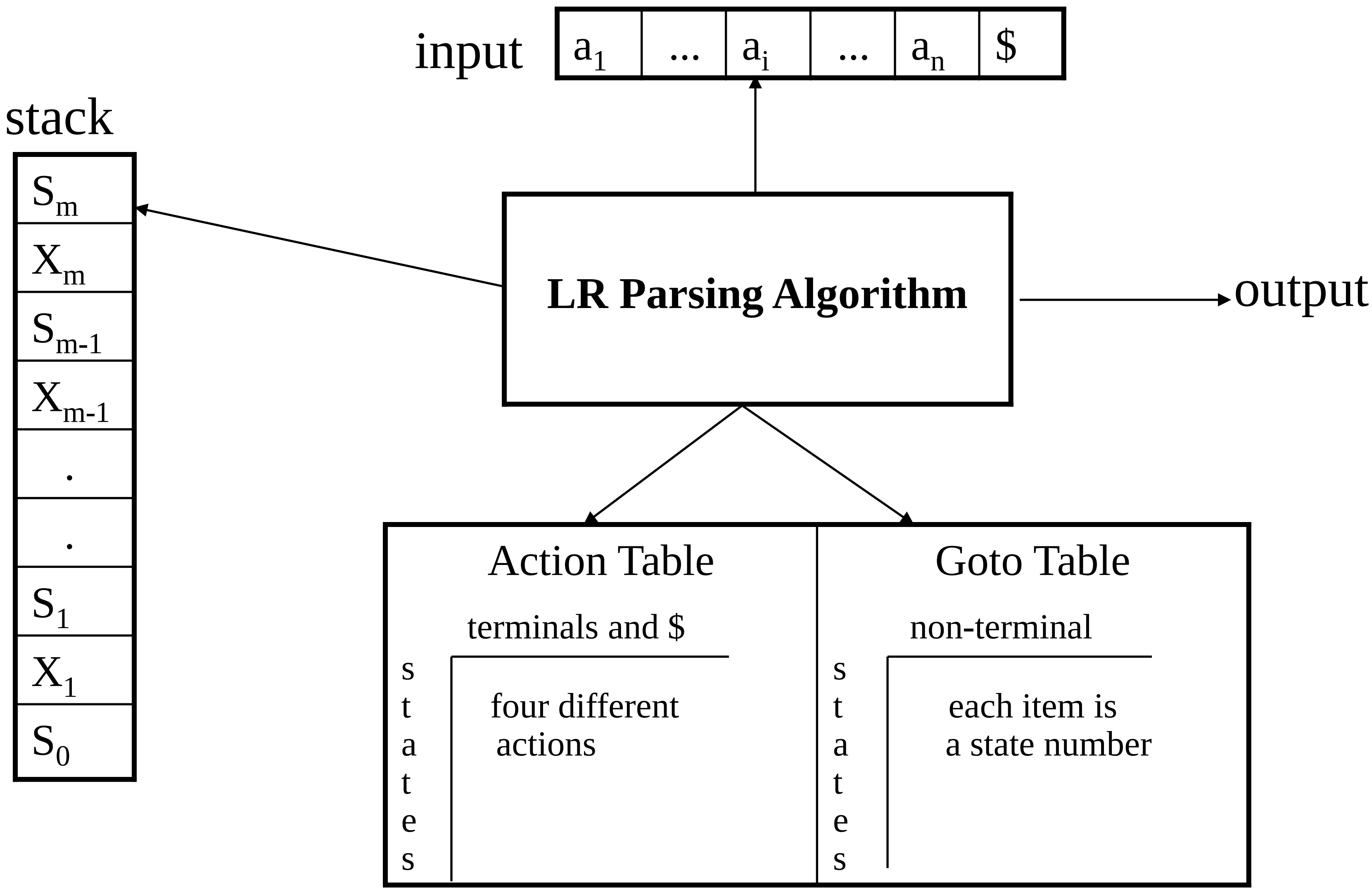
k lookahead

- LR parsing is attractive because:
  - LR parsers can be constructed to recognize virtually all programming language constructs for which CFGs can be written
  - LR parsing is most general non-backtracking shift-reduce parsing, yet it is still efficient
  - Class of grammars that can be parsed using LR methods is a proper superset of the class of grammars that can be parsed with predictive parsers or LL methods

$$\text{LL(1)-Grammars} \subset \text{LR(1)-Grammars}$$

  - LR-parser can detect a syntactic error as soon as it is possible to do so a left-to-right scan of the input

LR Parsing Algorithm



A Configuration of LR Parsing Algorithm

- A configuration of a LR parsing is:

$$(\underbrace{S_o\ X_1\ S_1\ \ldots\ X_m\ S_m}_{\text{Stack}},\ \underbrace{a_i\ a_{i+1}\ \ldots\ a_n\ \$}_{\text{Rest of Input}})$$

- S_m and a_i decides the parser action by consulting the parsing action table (*Initial Stack* contains just S_o)
- S_o : does not represent any grammar symbol

- A configuration of a LR parsing represents the right sentential form:

$$X_1\ \ldots\ X_m\ a_i\ a_{i+1}\ \ldots\ a_n\ \$$$

Actions of A LR-Parser

1. **shift s** -- shifts the next input symbol and the state **s** onto the stack

   $(S_o\ X_1\ S_1 \ldots X_m\ S_m,\ a_i\ a_{i+1} \ldots a_n\ \$\ ) \Rightarrow (S_o\ X_1\ S_1 \ldots X_m\ S_m\ a_i\ s,\ a_{i+1} \ldots a_n\ \$\ )$

2. **reduce A→β** (or **rn** where n is a production number)
   - pop 2|**β**| (=r) items from the stack;
   - then push **A** and **s** where **s=goto[s_{m-r}, A]**

   $(S_o\ X_1\ S_1 \ldots X_m\ S_m,\ a_i\ a_{i+1} \ldots a_n\ \$\ ) \Rightarrow (S_o\ X_1\ S_1 \ldots X_{m-r}\ S_{m-r}\ A\ s,\ a_i \ldots a_n\ \$\ )$

   - Output is the reducing production A→β

3. **Accept** – Parsing successfully completed

4. **Error** -- Parser detected an error (an empty entry in the action table)

Reduce Action

- pop 2|**β**| (=r) items from the stack; let us assume that **β** = $Y_1 Y_2 \ldots Y_r$
- then push **A** and **s** where **s=goto[s$_{m-r}$, A]**

$$(S_o\ X_1\ S_1 \ldots X_{m-r}\ S_{m-r}\ Y_1\ S_{m-r-1} \ldots Y_r\ S_m,\ a_i\ a_{i+1} \ldots a_n\ \$\ )$$

$$\rightarrow (S_o\ X_1\ S_1 \ldots X_{m-r}\ S_{m-r}\ A\ s,\ a_i \ldots a_n\ \$\ )$$

- In fact, $Y_1 Y_2 \ldots Y_r$ is a handle

$$X_1 \ldots X_{m-r}\ A\ a_i \ldots a_n\ \$ \Rightarrow X_1 \ldots X_m\ Y_1 \ldots Y_r\ a_i\ a_{i+1} \ldots a_n\ \$$$

(SLR) Parsing Tables for Expression Grammar

1) $E \rightarrow E+T$
2) $E \rightarrow T$
3) $T \rightarrow T*F$
4) $T \rightarrow F$
5) $F \rightarrow (E)$
6) $F \rightarrow id$

	Action Table							Goto Table		
state	**id**	**+**	*****	**(**	**)**	**$**		**E**	**T**	**F**
0	s5			s4				1	2	3
1		s6				acc				
2		r2	s7		r2	r2				
3		r4	r4		r4	r4				
4	s5			s4				8	2	3
5		r6	r6		r6	r6				
6	s5			s4					9	3
7	s5			s4						10
8		s6			s11					
9		r1	s7		r1	r1				
10		r3	r3		r3	r3				
11		r5	r5		r5	r5				

Actions of A (S)LR-Parser -- Example

stack	input	action	output
0	id*id+id$	shift 5	
0id5	*id+id$	reduce by F→id	F→id
0F3	*id+id$	reduce by T→F	T→F
0T2	*id+id$	shift 7	
0T2*7	id+id$	shift 5	
0T2*7id5	+id$	reduce by F→id	F→id
0T2*7F10	+id$	reduce by T→T*F	T→T*F
0T2	+id$	reduce by E→T	E→T
0E1	+id$	shift 6	
0E1+6	id$	shift 5	
0E1+6id5	$	reduce by F→id	F→id
0E1+6F3	$	reduce by T→F	T→F
0E1+6T9	$	reduce by E→E+T	E→E+T
0E1	$	accept	

Constructing SLR Parsing Tables – LR(0) Item

- **LR(0) item:** a production of G a dot at the some position of the right side

  Ex: $A \rightarrow aBb$ Possible LR(0) Items: (four different possibilities)

  $A \rightarrow \bullet aBb$

  $A \rightarrow a \bullet Bb$

  $A \rightarrow aB \bullet b$

  $A \rightarrow aBb \bullet$

- Sets of LR(0) items: states of *action* and *goto* table of the SLR parser
- Collection of sets of LR(0) items (**the canonical LR(0) collection**) is the basis for constructing SLR parsers
- *Augmented Grammar*: G' is

  G with a new production rule S'→S where S' is the new starting symbol

The Closure Operation

I: set of LR(0) items for a grammar G

closure(I): Set of LR(0) items constructed from I by the two rules:

1. Initially, every LR(0) item in I is added to closure(I)

2. If $A \rightarrow \alpha \bullet B\beta$ is in closure(I) and $B \rightarrow \gamma$ is a production rule of G; then $B \rightarrow \bullet \gamma$ will be in the closure (I)

We will apply this rule until no more new LR(0) items can be added to closure(I)

The Closure Operation -- Example

$E' \rightarrow E$
$E \rightarrow E+T$
$E \rightarrow T$
$T \rightarrow T*F$
$T \rightarrow F$
$F \rightarrow (E)$
$F \rightarrow id$

closure($\{E' \rightarrow \bullet E\}$) =

$\{\ E' \rightarrow \bullet E$ kernel items
$E \rightarrow \bullet E+T$
$E \rightarrow \bullet T$
$T \rightarrow \bullet T*F$
$T \rightarrow \bullet F$
$F \rightarrow \bullet (E)$
$F \rightarrow \bullet id\ \}$

Kernel and Non-kernel Items

- Each set of items formed by taking the closure of a set of kernel items
- We are really interested in kernel items
- Non-kernel items can be removed to save storage
- Non-kernel items could be generated by the closure process

Goto Operation

- If I is a set of LR(0) items and X is a grammar symbol (terminal or non-terminal), then goto (I,X) is defined as follows:
  - If $A \rightarrow \alpha \bullet X\beta$ in I then every item in **closure**($\{A \rightarrow \alpha X \bullet \beta\}$) will be in goto (I,X)

Example:

$I = \{ \quad E' \rightarrow \bullet E, \quad E \rightarrow \bullet E+T, \quad E \rightarrow \bullet T,$

$\quad T \rightarrow \bullet T*F, \; T \rightarrow \bullet F,$

$\quad F \rightarrow \bullet (E), \quad F \rightarrow \bullet id \; \}$

$goto(I,E) = \{ E' \rightarrow E \bullet, E \rightarrow E \bullet +T \}$

$goto(I,T) = \{ E \rightarrow T \bullet, T \rightarrow T \bullet *F \}$

$goto(I,F) = \{ T \rightarrow F \bullet \}$

$goto(I,() = \{ F \rightarrow (\bullet E), E \rightarrow \bullet E+T, E \rightarrow \bullet T, T \rightarrow \bullet T*F, T \rightarrow \bullet F,$

$\quad F \rightarrow \bullet (E), F \rightarrow \bullet id \}$

$goto(I,id) = \{ F \rightarrow id \bullet \}$

Construction of The Canonical LR(0) Collection

- To create the SLR parsing tables for a grammar G, we will create the canonical LR(0) collection of the grammar G'

- ***Algorithm***:

  C is { closure({S'→ • S}) }

  repeat the followings until no more set of LR(0) items can be added to ***C***.

      for each I in ***C*** and each grammar symbol X

          if goto(I,X) is not empty and not in ***C***

              add goto(I,X) to ***C***

- goto function is a DFA on the sets in C.

The Canonical LR(0) Collection -- Example

I_0: E' → .E
E → .E+T
E → .T
T → .T*F
T → .F
F → .(E)
F → .id

I_1: E' → E.
E → E.+T

I_2: E → T.
T → T.*F

I_3: T → F.

I_4: F → (.E)
E → .E+T
E → .T
T → .T*F
T → .F
F → .(E)
F → .id

I_5: F → id.

I_6: E → E+.T
T → .T*F
T → .F
F → .(E)
F → .id

I_7: T → T*.F
F → .(E)
F → .id

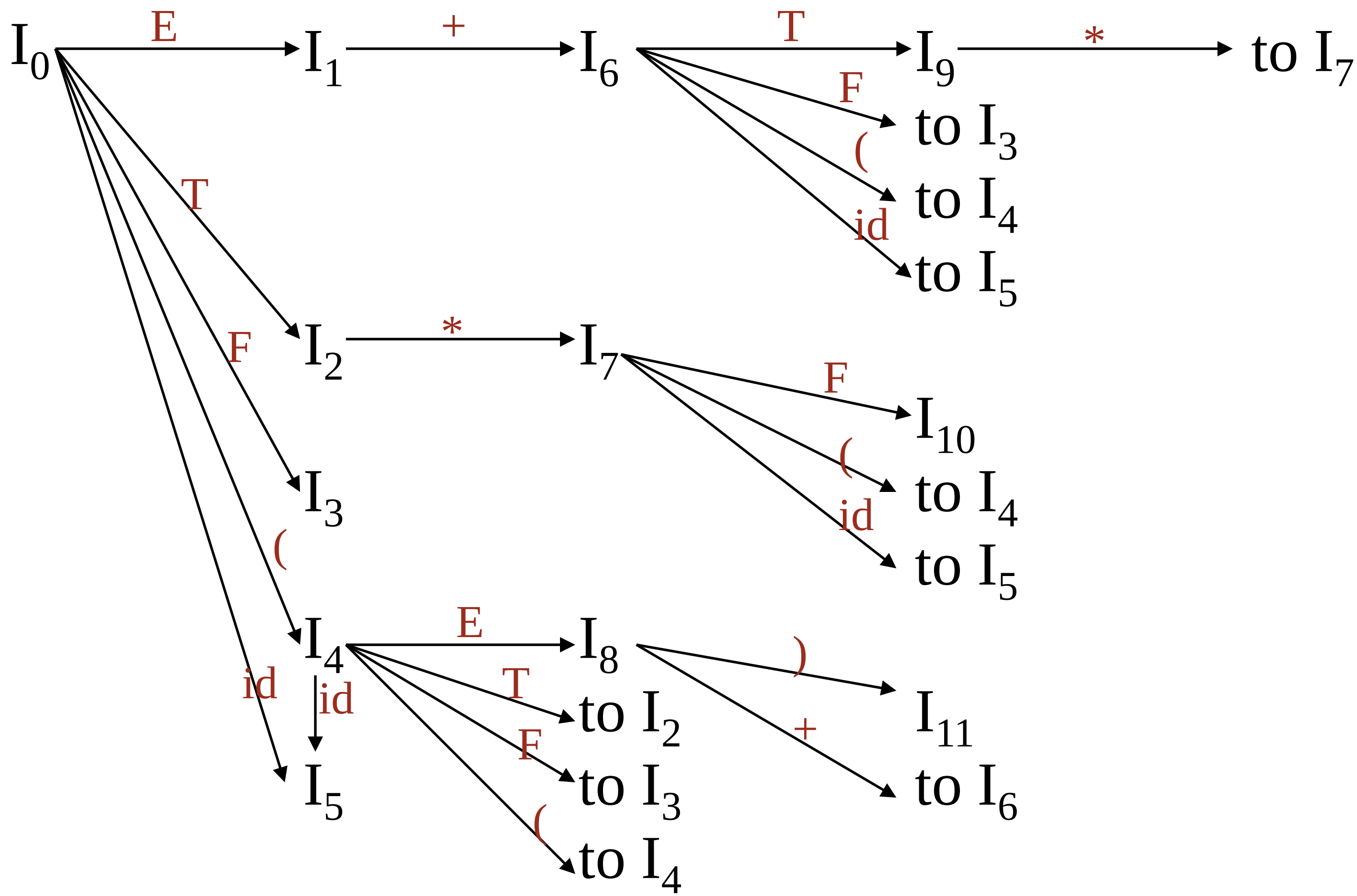
I_8: F → (E.)
E → E.+T

I_9: E → E+T.
T → T.*F

I_{10}: T → T*F.

I_{11}: F → (E).

Transition Diagram (DFA)



Constructing SLR Parsing Table

(of an augmented grammar G')

1. Construct the canonical collection of sets of LR(0) items for G'

   $C \leftarrow \{I_0, \ldots, I_n\}$

2. Create the parsing action table as follows
   - If *a* is a terminal, $A \rightarrow \alpha.a\beta$ in I_i and goto $(I_i, a) = I_j$ then action[i, a] is ***shift j***
   - If $A \rightarrow \alpha.$ is in I_i, then action[i,a] is ***reduce*** $\boldsymbol{A \rightarrow \alpha}$ for all a in FOLLOW(A) where $A \neq S'$
   - If $S' \rightarrow S.$ is in I_i, then action[i,\$] is ***accept***
   - If any conflicting actions generated by these rules, the grammar is not SLR(1)

3. Create the parsing goto table
   - for all non-terminals A, if goto $(I_i, A) = I_j$ then goto [i, A]=j

4. All entries not defined by (2) and (3) are errors

5. Initial state of the parser contains $S' \rightarrow .S$

The Canonical LR(0) Collection -- Example

I_0: $E' \rightarrow .E$
$E \rightarrow .E+T$
$E \rightarrow .T$
$T \rightarrow .T*F$
$T \rightarrow .F$
$F \rightarrow .(E)$
$F \rightarrow .id$

I_1: $E' \rightarrow E.$
$E \rightarrow E.+T$

I_2: $E \rightarrow T.$
$T \rightarrow T.*F$

I_3: $T \rightarrow F.$

I_4: $F \rightarrow (.E)$
$E \rightarrow .E+T$
$E \rightarrow .T$
$T \rightarrow .T*F$
$T \rightarrow .F$
$F \rightarrow .(E)$
$F \rightarrow .id$

I_5: $F \rightarrow id.$

I_6: $E \rightarrow E+.T$
$T \rightarrow .T*F$
$T \rightarrow .F$
$F \rightarrow .(E)$
$F \rightarrow .id$

I_7: $T \rightarrow T*.F$
$F \rightarrow .(E)$
$F \rightarrow .id$

I_8: $F \rightarrow (E.)$
$E \rightarrow E.+T$

I_9: $E \rightarrow E+T.$
$T \rightarrow T.*F$

I_{10}: $T \rightarrow T*F.$

I_{11}: $F \rightarrow (E).$

Parsing Tables of Expression Grammar

Action Table / Goto Table

state	id	+	*	(	)	$		E	T	F
0	s5			s4				1	2	3
1		s6				acc				
2		r2	s7		r2	r2				
3		r4	r4		r4	r4				
4	s5			s4				8	2	3
5		r6	r6		r6	r6				
6	s5			s4					9	3
7	s5			s4						10
8		s6			s11					
9		r1	s7		r1	r1				
10		r3	r3		r3	r3				
11		r5	r5		r5	r5				

SLR(1) Grammar

- An LR parser using SLR(1) parsing tables for a grammar G is called as the SLR(1) parser for G
- If a grammar G has an SLR(1) parsing table, it is called SLR(1) grammar (or SLR grammar in short)
- *Every SLR grammar is unambiguous, but every unambiguous grammar is not a SLR grammar*

shift/reduce and reduce/reduce conflicts

- **shift/reduce conflict:** choice between a shift operation or reduction for a terminal
- **reduce/reduce conflict:** If a state does not know whether it will make a reduction operation using the production rule `i` or `j` for a terminal
- If the SLR parsing table of a grammar G has a conflict, we say that the grammar is not SLR grammar

Conflict Example

$S \rightarrow L=R$
$S \rightarrow R$
$L \rightarrow *R$
$L \rightarrow id$
$R \rightarrow L$

I_0: $S' \rightarrow .S$
$S \rightarrow .L=R$
$S \rightarrow .R$
$L \rightarrow .*R$
$L \rightarrow .id$
$R \rightarrow .L$

I_1: $S' \rightarrow S.$

I_2: $S \rightarrow L.=R$
$R \rightarrow L.$

I_3: $S \rightarrow R.$

I_4: $L \rightarrow *.R$
$R \rightarrow .L$
$L \rightarrow .*R$
$L \rightarrow .id$

I_5: $L \rightarrow id.$

I_6: $S \rightarrow L=.R$
$R \rightarrow .L$
$L \rightarrow .*R$
$L \rightarrow .id$

I_7: $L \rightarrow *R.$

I_8: $R \rightarrow L.$

I_9: $S \rightarrow L=R.$

Problem

FOLLOW(R)={=,$}

= → shift 6
= → reduce by $R \rightarrow L$

shift/reduce conflict

Conflict Example2

$S \to AaAb$

$S \to BbBa$

$A \to \varepsilon$

$B \to \varepsilon$

I_0: $S' \to .S$

$S \to .AaAb$

$S \to .BbBa$

$A \to .$

$B \to .$

Problem

FOLLOW(A)={a,b}

FOLLOW(B)={a,b}

a → reduce by $A \to \varepsilon$

→ reduce by $B \to \varepsilon$

reduce/reduce conflict

b → reduce by $A \to \varepsilon$

→ reduce by $B \to \varepsilon$

reduce/reduce conflict

Constructing Canonical LR(1) Parsing Tables

- In SLR method, the state i makes a reduction by $A \rightarrow \alpha$ when the current token is a:
  - if $A \rightarrow \alpha\bullet$ in I_i and a is in FOLLOW(A)
- In some situations, βA cannot be followed by the terminal a in a right-sentential form when $\beta\alpha$ and the state i are on the top stack. This means that making reduction in this case is not correct.

$S \rightarrow AaAb$ $\qquad S \Rightarrow AaAb \Rightarrow Aab \Rightarrow ab$ $\qquad S \Rightarrow BbBa \Rightarrow Bba \Rightarrow ba$

$S \rightarrow BbBa$

$A \rightarrow \varepsilon$ $\qquad Aab \Rightarrow \varepsilon\, ab$ $\qquad Bba \Rightarrow \varepsilon\, ba$

$B \rightarrow \varepsilon$ $\qquad AaAb \Rightarrow Aa\, \varepsilon\, b$ $\qquad BbBa \Rightarrow Bb\, \varepsilon\, a$

An Example

$S \rightarrow L{=}R$
$S \rightarrow R$
$L \rightarrow {*}R$
$L \rightarrow id$
$R \rightarrow L$

I_0: $S' \rightarrow .S$
$S \rightarrow .L{=}R$
$S \rightarrow .R$
$L \rightarrow .{*}R$
$L \rightarrow .id$
$R \rightarrow .L$

I_1: $S' \rightarrow S.$

I_2: $S \rightarrow L.{=}R$
$R \rightarrow L.$

I_3: $S \rightarrow R.$

I_4: $L \rightarrow {*}.R$
$R \rightarrow .L$
$L \rightarrow .{*}R$
$L \rightarrow .id$

I_5: $L \rightarrow id.$

I_6: $S \rightarrow L{=}.R$
$R \rightarrow .L$
$L \rightarrow .{*}R$
$L \rightarrow .id$

I_7: $L \rightarrow {*}R.$

I_8: $R \rightarrow L.$

I_9: $S \rightarrow L{=}R.$

Problem

FOLLOW(R)={=,$}

= → shift 6
→ reduce by $R \rightarrow L$

- If L is reduced to R then the contents appear as: R= (no right sentential form can derive it)
- $R \rightarrow L$ is **INVALID** for the input symbol "="

LR(1) Item

- To avoid some of invalid reductions, the states need to carry more information

- Extra information is put into a state by including a terminal symbol as a second component in an item

- A LR(1) item is:

  $A \rightarrow \alpha . \beta, a$ where **a** is the look-ahead of the LR(1) item

  (**a** is a terminal or end-marker)

LR(1) Item (cont.)

- When β (in the LR(1) item $A \rightarrow \alpha.\beta,a$) is not empty, the look-ahead does not have any effect

- When β is empty ($A \rightarrow \alpha.,a$), then

  Reduce by $A \rightarrow \alpha$ only if the next input symbol is **a** (not for any terminal in FOLLOW(A))

- A state contains $A \rightarrow \alpha.,a_1$ where $\{a_1,\ldots,a_n\} \subseteq$ FOLLOW(A)

$$\ldots$$

$$A \rightarrow \alpha.,a_n$$

Canonical Collection of Sets of LR(1) Items

- Process: similar to the construction of the canonical collection of the sets of LR(0) items,

  but *closure* and *goto* operations work a little bit different

closure(I) : (where I is a set of LR(1) items)

- every LR(1) item in I is in closure(I)
- if $A \rightarrow \alpha \bullet B\beta, a$ in closure(I) and

  $B \rightarrow \gamma$ is a production rule of G;

then $B \rightarrow .\gamma, b$ belongs to closure(I) for each terminal b in FIRST(βa)

goto operation

- If I is a set of LR(1) items and X is a grammar symbol (terminal or non-terminal),

  then goto (I, X) defined as follows:
  - If $A \rightarrow \alpha.X\beta$, a in I

    then every item in **closure(**$\{\mathbf{A \rightarrow \alpha X.\beta, a}\}$**)** belongs to goto (I,X)

Construction of The Canonical LR(1) Collection

- ***Algorithm***:

  C is { closure({S'→.S,$}) }

  repeat the followings until no more set of LR(1) items can be added to ***C***.

  for each I in ***C*** and each grammar symbol X

  if goto (I, X) is not empty and not in ***C***

  add goto (I, X) to ***C***

- goto function is a DFA on the sets in C

A Short Notation for The Sets of LR(1) Items

- A set of LR(1) items containing the following items

$$A \rightarrow \alpha . \beta, a_1$$

$$\ldots$$

$$A \rightarrow \alpha . \beta, a_n$$

can be written as

$$A \rightarrow \alpha . \beta, a_1/a_2/\ldots/a_n$$

Canonical LR(1) Collection -- Example

$S \rightarrow AaAb$

$S \rightarrow BbBa$

$A \rightarrow \varepsilon$

$B \rightarrow \varepsilon$

I_0: $S' \rightarrow .S\ ,\$$

$S \rightarrow .AaAb\ ,\$$

$S \rightarrow .BbBa\ ,\$$

$A \rightarrow .\ ,a$

$B \rightarrow .\ ,b$

$\xrightarrow{S}$ I_1: $S' \rightarrow S.\ ,\$$

$\xrightarrow{A}$ I_2: $S \rightarrow A.aAb\ ,\$$ $\xrightarrow{a}$ to I_4

$\xrightarrow{B}$ I_3: $S \rightarrow B.bBa\ ,\$$ $\xrightarrow{b}$ to I_5

I_4: $S \rightarrow Aa.Ab\ ,\$$ $\xrightarrow{A}$ I_6: $S \rightarrow AaA.b\ ,\$$ $\xrightarrow{b}$ I_8: $S \rightarrow AaAb.\ ,\$$

$A \rightarrow .\ ,b$

I_5: $S \rightarrow Bb.Ba\ ,\$$ $\xrightarrow{B}$ I_7: $S \rightarrow BbB.a\ ,\$$ $\xrightarrow{a}$ I_9: $S \rightarrow BbBa.\ ,\$$

$B \rightarrow .\ ,a$

Canonical LR(1) Collection – Example 2

$S' \to S$
1) $S \to L=R$
2) $S \to R$
3) $L \to *R$
4) $L \to id$
5) $R \to L$

I_0: $S' \to .S,\$$
$S \to .L=R,\$$
$S \to .R,\$$
$L \to .*R,=$
$L \to .id,=$
$R \to .L,\$$

$I_0 \xrightarrow{S} I_1$: $S' \to S.,\$$

$I_0 \xrightarrow{L} I_2$: $S \to L.=R,\$$ $\longrightarrow$ to I_6
$R \to L.,\$$

$I_0 \xrightarrow{R} I_3$: $S \to R.,\$$

$I_0 \xrightarrow{*} I_4$: $L \to *.R,=$
$R \to .L,=$
$L \to .*R,=$
$L \to .id,=$

$\xrightarrow{R}$ to I_7
$\xrightarrow{L}$ to I_8
$\xrightarrow{*}$ to I_4
$\xrightarrow{id}$ to I_5

$I_0 \xrightarrow{id} I_5$: $L \to id.,=$

I_6: $S \to L=.R,\$$
$R \to .L,\$$
$L \to .*R,\$$
$L \to .id,\$$

$\xrightarrow{R}$ to I_9
$\xrightarrow{L}$ to I_{10}
$\xrightarrow{*}$ to I_{11}
$\xrightarrow{id}$ to I_{12}

I_7: $L \to *R.,=$

I_8: $R \to L.,=$

I_9: $S \to L=R.,\$$

I_{10}: $R \to L.,\$$

I_{11}: $L \to *.R,\$$
$R \to .L,\$$
$L \to .*R,\$$
$L \to .id,\$$

$\xrightarrow{R}$ to I_{13}
$\xrightarrow{L}$ to I_{10}
$\xrightarrow{*}$ to I_{11}
$\xrightarrow{id}$ to I_{12}

I_{12}: $L \to id.,\$$

I_{13}: $L \to *R.,\$$

I_4 and I_{11}

I_5 and I_{12}

I_7 and I_{13}

I_8 and I_{10}

Construction of LR(1) Parsing Tables

1. Construct the canonical collection of sets of LR(1) items for G'.
   $C \leftarrow \{I_0, \ldots, I_n\}$

2. Create the parsing action table as follows
   - If a is a terminal, $A \rightarrow \alpha \bullet a\beta, b$ in I_i and $goto(I_i, a) = I_j$ then action[i,a] is ***shift j.***
   - If $A \rightarrow \alpha \bullet, a$ is in I_i, then action[i,a] is ***reduce*** $\boldsymbol{A \rightarrow \alpha}$ where $A \neq S'$.
   - If $S' \rightarrow S \bullet, \$$ is in I_i, then action[i,$] is ***accept***.
   - If any conflicting actions generated by these rules, the grammar is not LR(1)

3. Create the parsing goto table
   - for all non-terminals A, if $goto(I_i, A) = I_j$ then goto[i, A]=j

4. All entries not defined by (2) and (3) are errors

5. Initial state of the parser contains $S' \rightarrow .S, \$$

Canonical LR(1) Collection – Example 2

$S' \rightarrow S$
1) $S \rightarrow L=R$
2) $S \rightarrow R$
3) $L \rightarrow *R$
4) $L \rightarrow id$
5) $R \rightarrow L$

I_0: $S' \rightarrow .S,\$$
$S \rightarrow .L=R,\$$
$S \rightarrow .R,\$$
$L \rightarrow .*R,=$
$L \rightarrow .id,=$
$R \rightarrow .L,\$$

$\xrightarrow{S}$ I_1: $S' \rightarrow S.,\$$

$\xrightarrow{L}$ I_2: $S \rightarrow L.=R,\$$ $\longrightarrow$ to I_6
$R \rightarrow L.,\$$

$\xrightarrow{R}$ I_3: $S \rightarrow R.,\$$

$\xrightarrow{*}$ I_4: $L \rightarrow *.R,=$
$R \rightarrow .L, =$
$L \rightarrow .*R, =$
$L \rightarrow .id, =$

$\xrightarrow{R}$ to I_7
$\xrightarrow{L}$ to I_8
$\xrightarrow{*}$ to I_4
$\xrightarrow{id}$ to I_5

$\xrightarrow{id}$ I_5: $L \rightarrow id., =$

I_6: $S \rightarrow L=.R,\$$
$R \rightarrow .L,\$$
$L \rightarrow .*R,\$$
$L \rightarrow .id,\$$

$\xrightarrow{R}$ to I_9
$\xrightarrow{L}$ to I_{10}
$\xrightarrow{*}$ to I_{11}
$\xrightarrow{id}$ to I_{12}

I_7: $L \rightarrow *R.,=$

I_8: $R \rightarrow L.,=$

I_9: $S \rightarrow L=R.,\$$

I_{10}: $R \rightarrow L.,\$$

I_{11}: $L \rightarrow *.R,\$$
$R \rightarrow .L,\$$
$L \rightarrow .*R,\$$
$L \rightarrow .id,\$$

$\xrightarrow{R}$ to I_{13}
$\xrightarrow{L}$ to I_{10}
$\xrightarrow{*}$ to I_{11}
$\xrightarrow{id}$ to I_{12}

I_{12}: $L \rightarrow id.,\$$

I_{13}: $L \rightarrow *R.,\$$

I_4 and I_{11}

I_5 and I_{12}

I_7 and I_{13}

I_8 and I_{10}

LR(1) Parsing Tables – (for Example2)

	id	*	=	$	S	L	R
0	s5	s4			1	2	3
1				acc			
2			s6	r5			
3				r2			
4	s5	s4				8	7
5			r4				
6	s12	s11				10	9
7			r3				
8			r5				
9				r1			
10				r5			
11	s12	s11				10	13
12				r4			
13				r3			

no shift/reduce or
no reduce/reduce conflict

$\Downarrow$

so, it is a LR(1) grammar

LALR Parsing Tables

- **LALR** stands for **LookAhead LR**
- LALR parsers are often used in practice because LALR parsing tables are smaller than LR(1) parsing tables
- Number of states in SLR and LALR parsing tables for a grammar G are equal
- But, LALR parsers recognize more grammars than SLR parsers
- *yacc* creates a LALR parser for the given grammar
- A state of LALR parser will be again a set of LR(1) items

Creating LALR Parsing Tables

Canonical LR(1) Parser ➔ LALR Parser

shrink # of states

- Shrink process may introduce a **reduce/reduce** conflict in the resulting LALR parser (so the grammar is NOT LALR)
- But, this shrink process does not produce a **shift/reduce** conflict

The Core of A Set of LR(1) Items

- The core of a set of LR(1) items is the set of its first component

Ex: $S \rightarrow L \bullet =R,\$$ ➔ $S \rightarrow L \bullet =R$ Core

$R \rightarrow L \bullet ,\$$ $R \rightarrow L \bullet$

- Find the states (sets of LR(1) items) in a canonical LR(1) parser with the same cores. Merge them as a single state

$I_1:L \rightarrow id \bullet ,=$ A new state: $I_{12}: L \rightarrow id \bullet ,=$

➔ $L \rightarrow id \bullet ,\$$

$I_2:L \rightarrow id \bullet ,\$$ have same core, merge them

- Do this for all states of a canonical LR(1) parser to get the states of the LALR parser

number of the states of the LALR parser = number of states of the SLR parser for any grammar

Creation of LALR Parsing Tables

- Create the canonical LR(1) collection of the sets of LR(1) items for the given grammar
- Find each core; find all sets having that same core; replace those sets having same cores with a single set which is their union.

  $C=\{I_0,\ldots,I_n\} \Rightarrow C'=\{J_1,\ldots,J_m\}$ where $m \leq n$
- Create the parsing tables (action and goto tables) same as the construction of the parsing tables of LR(1) parser
  - Note that: If $J=I_1 \cup \ldots \cup I_k$ since $I_1,\ldots,I_k$ have same cores

    $\Rightarrow$ cores of goto $(I_1,X),\ldots,$ goto (I_2,X) must be same
  - So, goto $(J,X)=K$, where K is the union of all sets of items having same cores as goto (I_1,X)
- Grammar is LALR(1) if no conflict is introduced
  - possible to introduce reduce/reduce conflicts
  - cannot introduce a shift/reduce conflict

Shift/Reduce Conflict

- Assume that we can introduce a shift/reduce conflict. In this case, a state of LALR parser must have:

$$A \rightarrow \alpha \bullet ,a \qquad \text{and} \qquad B \rightarrow \beta \bullet a\gamma, b$$

- This means that a state of the canonical LR(1) parser must have:

$$A \rightarrow \alpha \bullet ,a \qquad \text{and} \qquad B \rightarrow \beta \bullet a\gamma, c$$

But, this state has also a shift/reduce conflict. i.e. the original canonical LR(1) parser has a conflict

(Reason for this, the shift operation does not depend on lookaheads)

Reduce/Reduce Conflict

- For reduce/reduce conflict:

$$I_1 : A \rightarrow \alpha \bullet ,a \qquad\qquad I_2 : A \rightarrow \alpha \bullet ,b$$
$$B \rightarrow \beta \bullet ,b \qquad\qquad B \rightarrow \beta \bullet ,c$$

$$\Downarrow$$

$$I_{12} : A \rightarrow \alpha \bullet ,a/b$$
$$B \rightarrow \beta \bullet ,b/c$$

➔ reduce/reduce conflict

Canonical LR(1) Collection – Example 2

$S' \to S$
1) $S \to L=R$
2) $S \to R$
3) $L \to *R$
4) $L \to id$
5) $R \to L$

I_0: $S' \to .S,\$$
$S \to .L=R,\$$
$S \to .R,\$$
$L \to .*R,=$
$L \to .id,=$
$R \to .L,\$$

S
I_1: $S' \to S.,\$$

*

L
I_2: $S \to L.=R,\$$ → to I_6
$R \to L.,\$$

R
I_3: $S \to R.,\$$

I_4: $L \to *.R,=$
$R \to .L, =$
$L \to .*R, =$
$L \to .id, =$

R → to I_7
L → to I_8
* → to I_4
id → to I_5

id
I_5: $L \to id., =$

I_6: $S \to L=.R,\$$
$R \to .L,\$$
$L \to .*R,\$$
$L \to .id,\$$

R → to I_9
L → to I_{10}
* → to I_{11}
id → to I_{12}

I_7: $L \to *R.,=$

I_8: $R \to L.,=$

I_9: $S \to L=R.,\$$

I_{10}: $R \to L.,\$$

I_{11}: $L \to *.R,\$$
$R \to .L,\$$
$L \to .*R,\$$
$L \to .id,\$$

R → to I_{13}
L → to I_{10}
* → to I_{11}
id → to I_{12}

I_{12}: $L \to id.,\$$

I_{13}: $L \to *R.,\$$

I_4 and I_{11}

I_5 and I_{12}

I_7 and I_{13}

I_8 and I_{10}

Canonical LALR(1) Collection – Example2

S' → S
1) S → L=R
2) S → R
3) L → *R
4) L → id
5) R → L

I_0: S' → •S,$
S → •L=R,$
S → •R,$
L → •*R,=
L → •id,=
R → •L,$

$I_0 \xrightarrow{S} I_1$: S' → S•,$

$I_0 \xrightarrow{L} I_2$: S → L•=R,$ → to I_6
R → L•,$

$I_0 \xrightarrow{R} I_3$: S → R•,$

$I_0 \xrightarrow{*} I_{411}$: L → *•R, =/$
R → •L, =/$
L → •*R, =/$
L → •id, =/$

I_{411}: R → to I_{713}; L → to I_{810}; * → to I_{411}; id → to I_{512}

$I_0 \xrightarrow{id} I_{512}$: L → id•, =/$

I_6: S → L=•R,$
R → •L,$
L → •*R,$
L → •id,$

I_6: R → to I_9; L → to I_{810}; * → to I_{411}; id → to I_{512}

I_9: S → L=R•,$

I_{713}: L → *R•,=/$

I_{810}: R → L•, =/$

Same Cores
I_4 and I_{11}

I_5 and I_{12}

I_7 and I_{13}

I_8 and I_{10}

LALR(1) Parsing Tables – (for Example2)

	id	*****	**=**	**$**	**S**	**L**	**R**
0	s5	s4			1	2	3
1				acc			
2			s6	r5			
3				r2			
4	s5	s4				8	7
5			r4	r4			
6	s12	s11				10	9
7			r3				
8			r5				
9				r1			

no shift/reduce or
no reduce/reduce conflict

$\Downarrow$

so, it is a LALR(1) grammar

LR(1) Parsing Tables – (for Example2)

	id	*	=	$	S	L	R
0	s5	s4			1	2	3
1				acc			
2			s6	r5			
3				r2			
4	s5	s4				8	7
5			r4				
6	s12	s11				10	9
7			r3				
8			r5	r5			
9				r1			
10				r5			
11	s12	s11				10	13
12				r4			
13				r3			

no shift/reduce or
no reduce/reduce conflict

$\Downarrow$

so, it is a LR(1) grammar

Using Ambiguous Grammars

- All grammars used in the construction of LR-parsing tables must be un-ambiguous
- *Can we create LR-parsing tables for ambiguous grammars ?*
  - Yes, but they will have conflicts
  - We can resolve these conflicts in favor of one of them to disambiguate the grammar
  - At the end, we will have again an unambiguous grammar

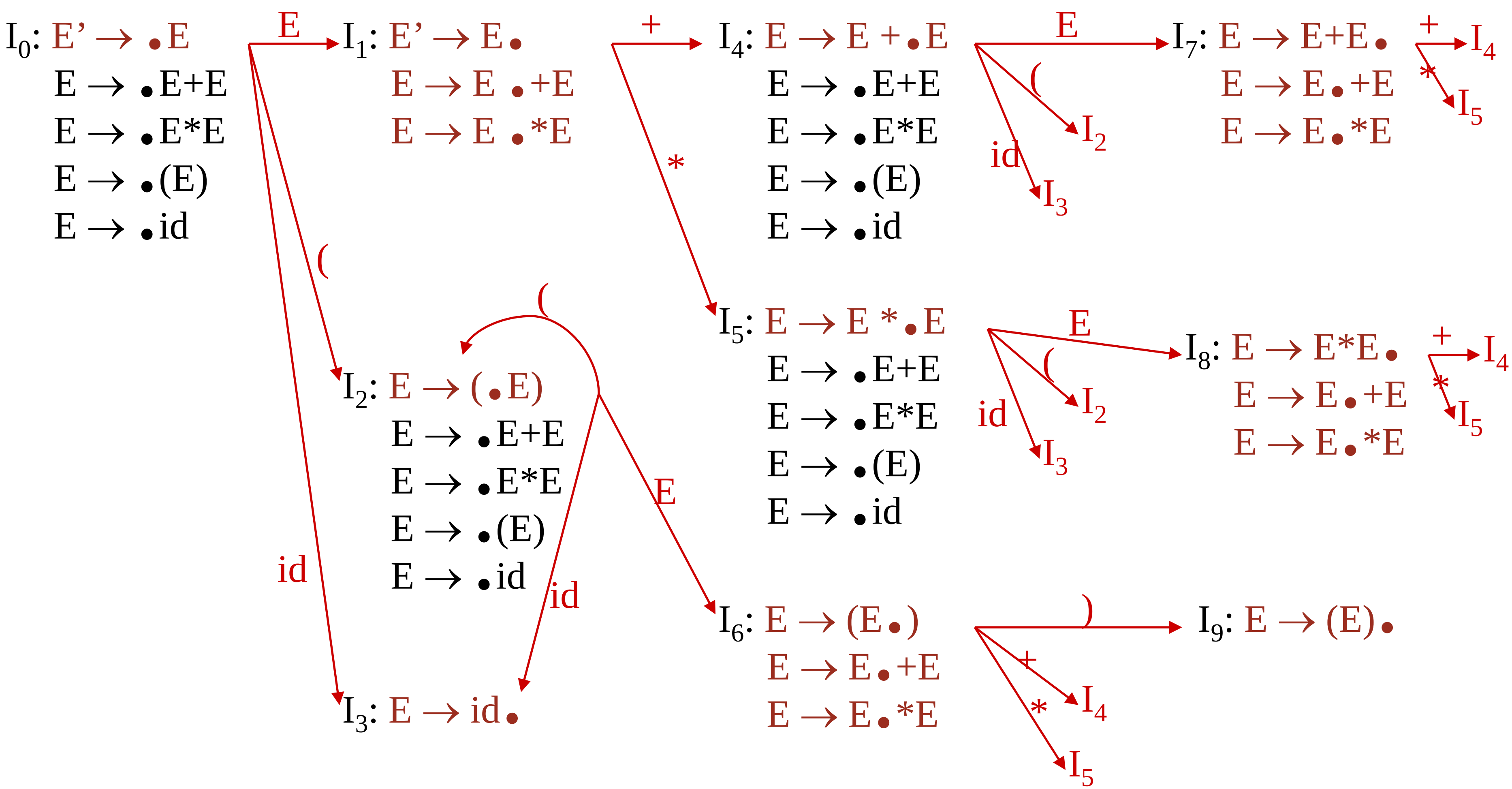
- *Why we want to use an ambiguous grammar?*
  - Some of the ambiguous grammars are **much natural**, and a corresponding unambiguous grammar can be **very complex**

  - Usage of an ambiguous grammar may **eliminate unnecessary reductions**

Ex.

$$E \rightarrow E+E \mid E*E \mid (E) \mid id \quad \Rightarrow \quad \begin{array}{l} E \rightarrow E+T \mid T \\ T \rightarrow T*F \mid F \\ F \rightarrow (E) \mid id \end{array}$$

Sets of LR(0) Items for Ambiguous Grammar



SLR-Parsing Tables for Ambiguous Grammar

FOLLOW(E) = { $, +, *,) }

State I_7 has shift/reduce conflicts for symbols + and *

$$I_0 \xrightarrow{E} I_1 \xrightarrow{+} I_4 \xrightarrow{E} I_7$$

when current token is +

shift ➔ + is right-associative

reduce ➔ + is left-associative

when current token is *

shift ➔ * has higher precedence than +

reduce ➔ + has higher precedence than *

SLR-Parsing Tables for Ambiguous Grammar

FOLLOW(E) = { $, +, *,) }

State I_8 has shift/reduce conflicts for symbols + and *.

$$I_0 \xrightarrow{E} I_1 \xrightarrow{*} I_5 \xrightarrow{E} I_8$$

when current token is *

shift ➔ * is right-associative

reduce ➔ * is left-associative

when current token is +

shift ➔ + has higher precedence than *

reduce ➔ * has higher precedence than +

SLR-Parsing Tables for Ambiguous Grammar

	Action							Goto
	id	**+**	*****	**(**	**)**	**$**		**E**
0	s3			s2				1
1		s4	s5			acc		
2	s3			s2				6
3		r4	r4		r4	r4		
4	s3			s2				7
5	s3			s2				8
6		s4	s5		s9			
7		r1	s5		r1	r1		
8		r2	r2		r2	r2		
9		r3	r3		r3	r3		

Error Recovery in LR Parsing

- LR parser detects an error when it consults the parsing action table and finds an error entry
- *Empty entries in the action table are error entries*
- Errors are never detected by consulting the *goto* table
- Some tricks
  - LR parser will **announce error** as soon as there is no valid continuation for the scanned portion of the input
  - Canonical LR parser (LR(1) parser) will **never make even a single reduction** before announcing an error
  - SLR and LALR parsers may make **several reductions** before announcing an error
  - LR parsers (LR(1), LALR and SLR parsers) will never shift an **erroneous input symbol onto the stack**

Panic Mode Error Recovery in LR Parsing

- Scan down the stack until a state **s** with a goto on a particular non-terminal **A** is found (Get rid of everything from the stack before this state s)
- Discard zero or more input symbols until a symbol **a** is found that can legitimately follow A
  - Symbol a is simply in FOLLOW(A), but this may not work for all situations
- Parser stacks the non-terminal **A** and the state **goto [s, A]**, and it resumes the normal parsing
- Non-terminal A is normally a basic programming block (there can be more than one choice for A)
  - stmt, expr, block, …

Phrase-Level Error Recovery in LR Parsing

- Each empty entry in the action table marked with a specific error routine
- An error routine reflects the error that the user most likely will make in that case
- An error routine
  - inserts the symbols into the stack or the input
  - or, deletes the symbols from the stack and the input
  - or, can do both insertion and deletion
- Error routine may denote
  - missing operand
  - unbalanced right parenthesis