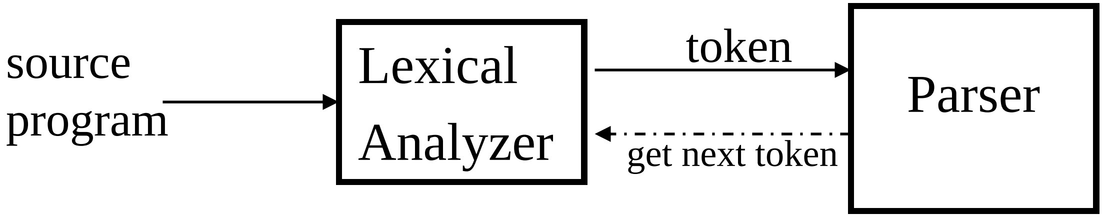
CS 346: Compilers

Lexical Analyzer

Lexical Analyzer

- **Lexical Analyzer**
  - reads the source program character by character to produce tokens
  - doesn’t return a list of tokens at one shot
  - returns a token when the parser asks a token from it



Token

- Token represents a set of strings described by a pattern
  - Identifier: a set of strings which start with a letter continues with letters and digits
  - The actual string is called as *lexeme*
  - Tokens: identifier, number, addop, delimeter, …
- Attribute:
  - additional information specific to a lexeme

- For simplicity, a token may have a single attribute which holds the required information for that token
  - For identifiers, this attribute is a pointer to the symbol table and the symbol table holds the actual attributes for that token

Some attributes:

• <id,attr>	where attr is pointer to the symbol table
• <assgop,_>	no attribute is needed (if there is only one assignment operator)
• <num,val>	where val is the actual value of the number.

- Token type and its attribute uniquely identifies a lexeme
- ***Regular expressions:*** widely used technique to specify patterns

Terminology of Languages

- **Alphabet** : a finite set of symbols (a, b, X etc.)
- **String** :
  - Finite sequence of symbols on an alphabet
  - Sentences and words are also represented in terms of string
  - ε: empty string
  - |s| : length of string s
- **Language**: sets of strings over some fixed alphabet
  - ∅: the empty set is a language
  - {ε} : the set containing empty string is a language
  - The set of well-formed C programs is a language
  - The set of all possible identifiers is a language
- **Operators on Strings**:
  - *Concatenation*: xy represents the concatenation of strings x and y

$s \varepsilon = s \qquad \varepsilon s = s$

$s^n = s s s .. s$ (n times) $\qquad s^0 = \varepsilon$

Operations on Languages

- Concatenation:
  - $L_1L_2 = \{ s_1s_2 \mid s_1 \in L_1 \text{ and } s_2 \in L_2 \}$
- Union
  - $L_1 \cup L_2 = \{ s \mid s \in L_1 \text{ or } s \in L_2 \}$
- Exponentiation:
  - $L^0 = \{\varepsilon\}$ $L^1 = L$ $L^2 = LL$
- Kleene Closure
  - $L^* = \bigcup_{i=0}^{\infty} L^i$
- Positive Closure
  - $L^+ = \bigcup_{i=1}^{\infty} L^i$

Example

$L_1 = \{a,b,c,d\}$ $L_2 = \{1,2\}$

- $L_1L_2 = \{a1,a2,b1,b2,c1,c2,d1,d2\}$
- $L_1 \cup L_2 = \{a,b,c,d,1,2\}$
- L_1^3 = all strings with length three (using a,b,c,d}
- L_1^* = all strings using letters a,b,c,d and empty string
- L_1^+ = doesn’t include the empty string

Regular Expressions

- Convenient and most popular way to describe tokens of a programming language

- A regular expression is built up of simpler regular expressions (using defining rules)

- Each regular expression denotes a language

- **regular set :** Language denoted by a regular expression

Regular Expressions (Rules)

Regular expressions over alphabet Σ

Reg. Expr	Language it denotes
ε	$\{\varepsilon\}$
$a \in \Sigma$	$\{a\}$
$(r_1) \mid (r_2)$	$L(r_1) \cup L(r_2)$
$(r_1)(r_2)$	$L(r_1) L(r_2)$
$(r)^*$	$(L(r))^*$
(r)	$L(r)$

- $(r)^+ = (r)(r)^*$
- $(r)? = (r) \mid \varepsilon$

Regular Expressions (cont.)

- We may remove parentheses by using precedence rules
  - * highest
  - concatenation next
  - | lowest
- $ab^*|c$ means $(a(b)^*)|(c)$

- Ex:
  - $\Sigma = \{0,1\}$
  - $0|1$ => $\{0,1\}$
  - $(0|1)(0|1)$ => $\{00,01,10,11\}$
  - 0^* => $\{\varepsilon,0,00,000,0000,\ldots.\}$
  - $(0|1)^*$ => all strings with 0 and 1, including the empty string

Regular Definitions

- Writing regular expression for some languages may be difficult
- Alternative: *regular definitions*
- Assign names to regular expressions and we can use these names as symbols to define other regular expressions

- A ***regular definition*** is a sequence of the definitions of the form:

$d_1 \rightarrow r_1$

$d_2 \rightarrow r_2$

.

$d_n \rightarrow r_n$

where d_i is a distinct name and

r_i is a regular expression over symbols in

$\Sigma \cup \{d_1, d_2, \ldots, d_{i-1}\}$

basic symbols

previously defined names

Regular Definitions (cont.)

- Ex: Identifiers in Pascal

  letter → A | B | ... | Z | a | b | ... | z
  digit → 0 | 1 | ... | 9
  id → letter (letter | digit) *

  - If we try to write the regular expression representing identifiers without using regular definitions, that regular expression will be complex.

  (A|...|Z|a|...|z) ((A|...|Z|a|...|z) | (0|...|9)) *

- Ex: Unsigned numbers in Pascal

  digit → 0 | 1 | ... | 9
  digits → digit $^+$
  opt-fraction → (. digits) ?
  opt-exponent → (E (+|-)? digits) ?
  unsigned-num → digits opt-fraction opt-exponent

Finite Automata

- Recognizer
  - program that takes a string x, and answers “yes” if x is a sentence of that language, and “no” otherwise
- We call the recognizer of the tokens as a *finite automaton*
- Finite automaton
  - Deterministic Finite Automaton (DFA)
  - Non-deterministic Finite Automaton (NFA)
  - DFA and NFA recognize regular sets
- For lexical analyzer, DFA or NFA can be used
- Which one to use?
  - deterministic – faster recognizer, but it may take more space
  - non-deterministic – slower, but it may take less space
  - Deterministic automatons are widely used for lexical analyzers

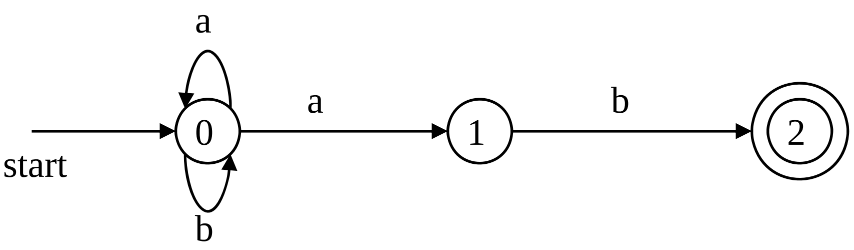
Finite Automata

- For lexical analysis
  - *Algorithm1*: Regular Expression ➔ NFA ➔ DFA (two steps: first to NFA, then to DFA)
  - *Algorithm2*: Regular Expression ➔ DFA (directly convert a regular expression into a DFA)

Non-Deterministic Finite Automaton (NFA)

- A non-deterministic finite automaton (NFA) is a mathematical model that consists of:
  - S - a set of states
  - Σ - a set of input symbols (alphabet)
  - move – a transition function move to map state-symbol pairs to sets of states.
  - s_0 - a start (initial) state
  - F – a set of accepting states (final states)
- ε- transitions are allowed in NFAs
  - we can move from one state to another one without consuming any symbol
- A NFA accepts a string x, if and only if there is a path from the starting state to one of accepting states such that edge labels along this path spell out x

NFA (Example)



Transition graph of the NFA

0 is the start state s_0
{2} is the set of final states F
Σ = {a,b}
S = {0,1,2}
Transition Function:

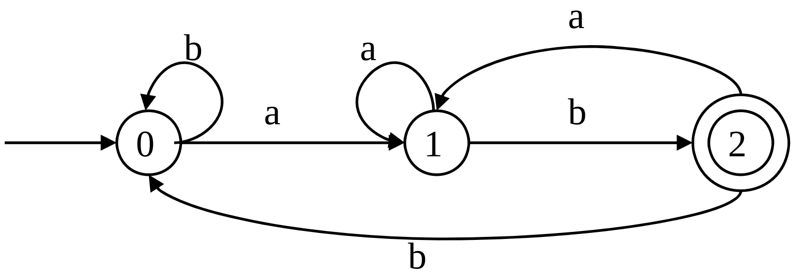
	a	b
0	{0,1}	{0}
1	_	{2}
2	_	_

The language recognized by this NFA is $(a|b)^*$ a b

Deterministic Finite Automaton (DFA)

- Deterministic Finite Automaton (DFA) is a special form of a NFA
  - no state has ε- transition
  - for each symbol *a* and state *s*, there is at most one labeled edge *a* leaving *s*

    i.e. transition function is from pair of state-symbol to state (not set of states)



The language recognized by

this DFA is also $(a|b)^*$ a b

Implementing a DFA

Le us assume that the end of a string is marked with a special symbol (say *eos*). The algorithm for recognition will be as follows:

```
s ← $s_0$                          { start from the initial state }
c ← nextchar                       { get the next character from the input string }
while (c != eos) do{ do until the end of the string }
    begin
            s ← move (s, c)        { transition function }
            c ← nextchar
    end
if (s in F) then                   { if s is an accepting state }
    return "yes"
else
    return "no"
```

Implementing a NFA

```
S ← ε-closure({s_0})          { set all of states can be accessible from s_0 by ε-transitions }
c ← nextchar
while (c != eos)
    begin
        s ← ε-closure(move(S,c))      { set of all states that can be accessible from a state in S
        c ← nextchar                    by a transition on c }
    end
if (S∩F != Φ) then                    { if S contains an accepting state }
    return "yes"
else
    return "no"
```

Converting A Regular Expression into A NFA (Thomson's Construction)

- One way to convert a regular expression into a NFA

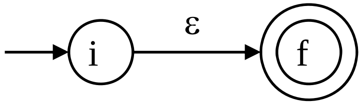
- Many others do exist !

- Thomson's Construction is simple and systematic
  - *It guarantees that the resulting NFA will have exactly one final state, and one start state*

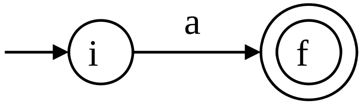
- Construction starts from simplest parts (alphabet symbols)
  - To create a NFA for a complex regular expression, NFAs of its sub-expressions are combined to create its NFA

Thomson's Construction (cont.)

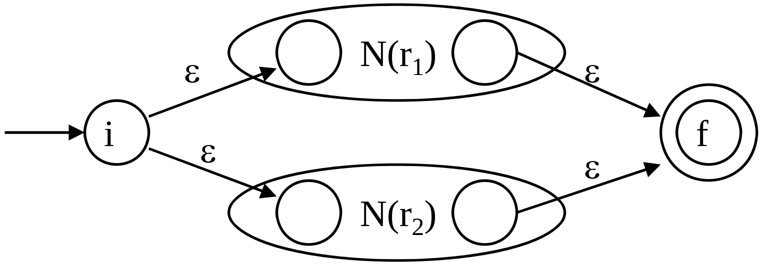
- Recognize an empty string ε



- Recognize a symbol a in the alphabet Σ



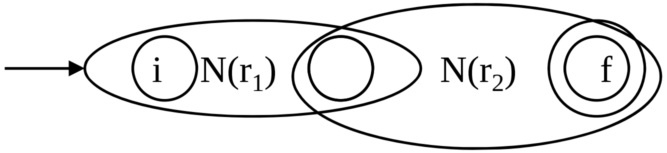
- If N(r_1) and N(r_2) are NFAs for regular expressions r_1 and r_2
  - For regular expression $r_1 | r_2$



NFA for $r_1 | r_2$

Thomson's Construction (cont.)

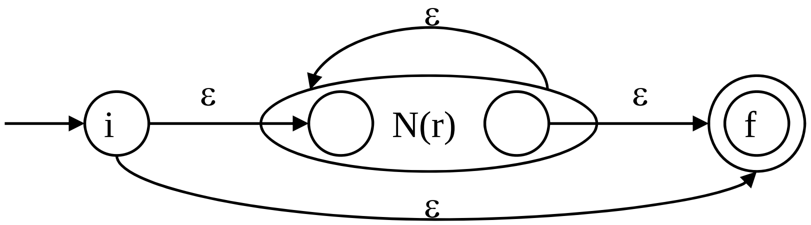
- For regular expression $r_1\ r_2$



Final state of $N(r_2)$ become final state of $N(r_1r_2)$

NFA for $r_1\ r_2$

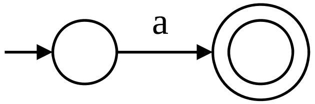
- For regular expression r^*



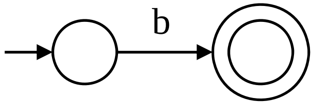
NFA for r^*

Thomson’s Construction (Example - (a|b) * a)

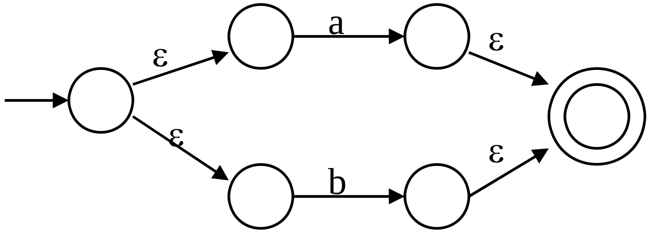
a:



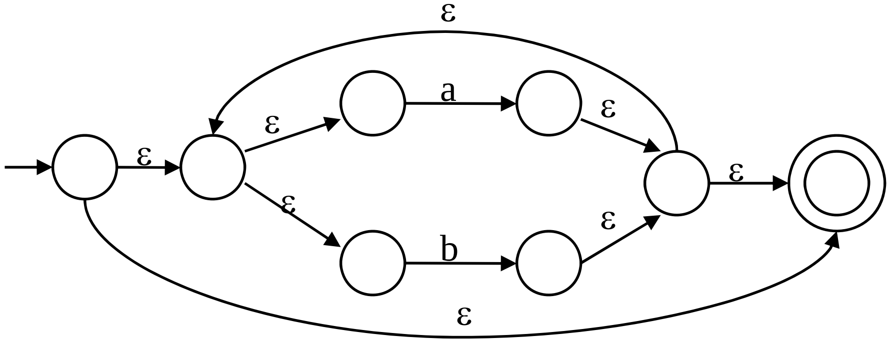
b:



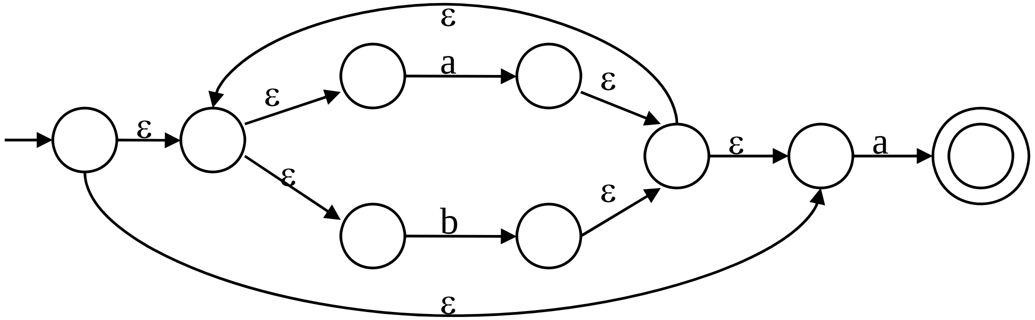
(a | b)



(a|b) *



(a|b) * a



Converting a NFA into a DFA (subset construction)

```
put $\varepsilon$-closure($\{s_0\}$) as an unmarked  state into the set of DFA (DS)
while (there is one unmarked $S_1$ in DS) do
    begin
        mark $S_1$
        for each input symbol a do
            begin
                $S_2 \leftarrow \varepsilon$-closure(move($S_1$,a))
                if ($S_2$ is not in DS) then
                    add $S_2$ into DS as an unmarked state
                transfunc[$S_1$,a] $\leftarrow S_2$
            end
    end
```

ε-closure($\{s_0\}$) is the set of all states, accessible from s_0 by ε-transition.

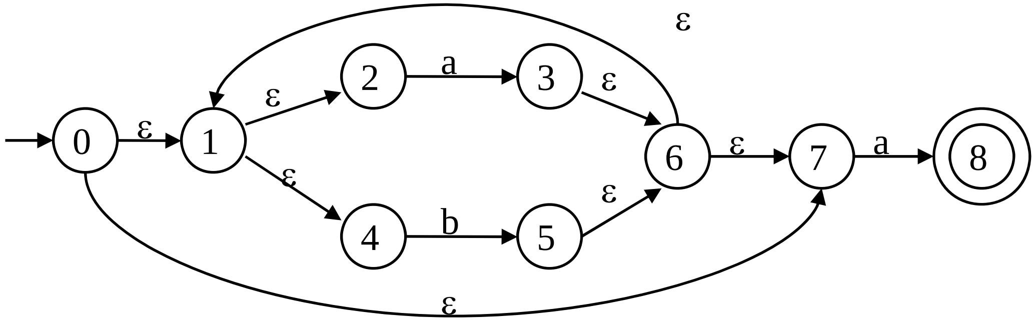
set of states to which there is a transition on a from a state s in S_1

- a state S in DS is an accepting state of DFA if a state in S is an accepting state of NFA
- the start state of DFA is ε-closure($\{s_0\}$)

Computing ε-closure(T)

```
push all states of T onto stack;
initialize ε-closure(T) to T;
While (stack is not empty) {
    Pop t, the top element, off the stack;
    For (each state u with an edge from t to u labeled ε)
        if  (u is not in ε-closure(T) ){
            add u to ε-closure(T)
            push u onto stack;
        }
}
```

Converting a NFA into a DFA (Example)



S_0 = ε-closure({0}) = {0,1,2,4,7} S_0 into DS as an unmarked state

⇓ mark S_0

ε-closure(move(S_0,a)) = ε-closure({3,8}) = {1,2,3,4,6,7,8} = S_1 S_1 into DS

ε-closure(move(S_0,b)) = ε-closure({5}) = {1,2,4,5,6,7} = S_2 S_2 into DS

transfunc[S_0,a] 🡨 S_1 transfunc[S_0,b] 🡨 S_2

⇓ mark S_1

ε-closure(move(S_1,a)) = ε-closure({3,8}) = {1,2,3,4,6,7,8} = S_1

ε-closure(move(S_1,b)) = ε-closure({5}) = {1,2,4,5,6,7} = S_2

transfunc[S_1,a] 🡨 S_1 transfunc[S_1,b] 🡨 S_2

⇓ mark S_2

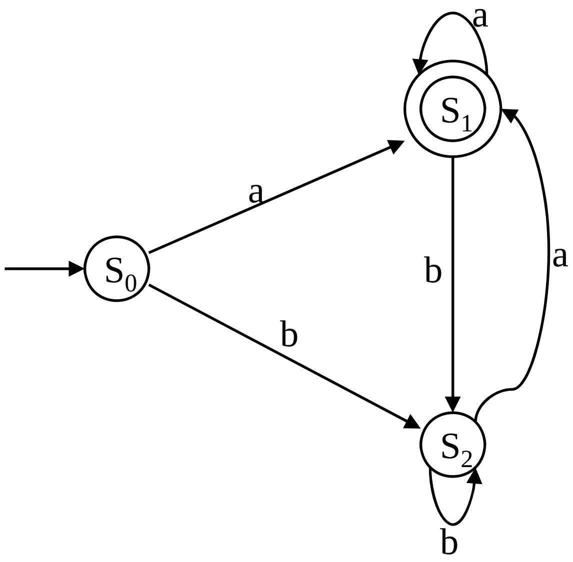
ε-closure(move(S_2,a)) = ε-closure({3,8}) = {1,2,3,4,6,7,8} = S_1

ε-closure(move(S_2,b)) = ε-closure({5}) = {1,2,4,5,6,7} = S_2

transfunc[S_2,a] 🡨 S_1 transfunc[S_2,b] 🡨 S_2

Converting a NFA into a DFA (Example – cont.)

S_0 is the start state of DFA since 0 is a member of S_0={0,1,2,4,7}
S_1 is an accepting state of DFA since 8 is a member of S_1 = {1,2,3,4,6,7,8}



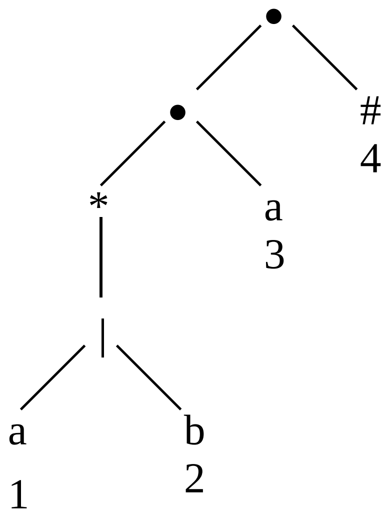
Converting Regular Expressions Directly to DFAs

- Regular expression can be directly converted into a DFA (without creating a NFA first)
- Augment the given regular expression by concatenating it with a special symbol #

  r ➔ (r)# augmented regular expression
- Create a syntax tree for this augmented regular expression
- Syntax tree
  - *Leaves*: alphabet symbols (including # and the empty string) in the augmented regular expression
  - *Intermediate nodes*: operators
- Number each alphabet symbol (including #) depending upon the positions

Regular Expression ➔ DFA (cont.)

$(a|b)^* a$ ➔ $(a|b)^* a \#$ augmented regular expression



Syntax tree of $(a|b)^* a \#$

- each symbol is numbered (positions)
- each symbol is at a leave
- inner nodes are operators

followpos

Define the function **followpos** for the positions (positions assigned to leaves)

followpos(i) -- set of positions which can follow the position i in the strings generated by the augmented regular expression

For example, $(a \mid b)^{*} a \ \#$

$\quad\ 1 \quad 2 \qquad 3 \ \ 4$

followpos(1) = {1,2,3}
followpos(2) = {1,2,3}
followpos(3) = {4}
followpos(4) = {}

followpos is just defined for leaves, it is not defined for inner nodes.

firstpos, lastpos, nullable

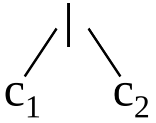
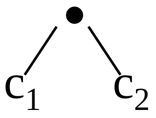
- To evaluate *followpos*, we need three more functions to be defined for the nodes (not just for leaves) of the syntax tree

- **firstpos(n)** -- set of the positions of the **first** symbols of strings generated by the sub-expression rooted by n

- **lastpos(n)** -- set of the positions of the **last** symbols of strings generated by the sub-expression rooted by n

- **nullable(n)** -- *true* if the empty string is a member of strings generated by the sub-expression rooted by n
  false otherwise

How to evaluate firstpos, lastpos, nullable?

n	nullable(n)	firstpos(n)	lastpos(n)
leaf labeled ε	true	Φ	Φ
leaf labeled with position i	false	{i}	{i}
	nullable(c_1) or nullable(c_2)	firstpos(c_1) ∪ firstpos(c_2)	lastpos(c_1) ∪ lastpos(c_2)
	nullable(c_1) and nullable(c_2)	if (nullable(c_1)) firstpos(c_1) ∪ firstpos(c_2) else firstpos(c_1)	if (nullable(c_2)) lastpos(c_1) ∪ lastpos(c_2) else lastpos(c_2)
	true	firstpos(c_1)	lastpos(c_1)

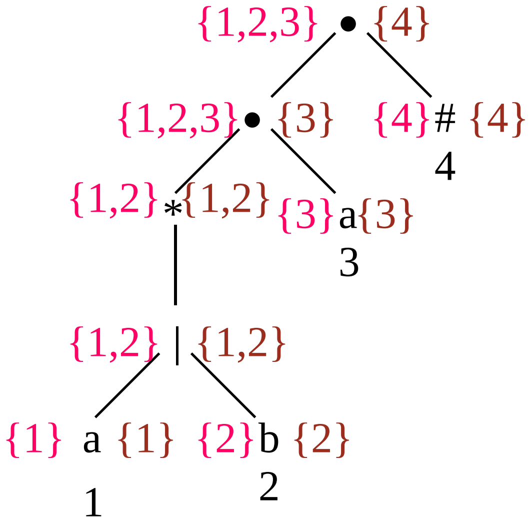
How to evaluate followpos?

Two-rules define the function *followpos*:

1. If **n** is concatenation-node with left child c_1 and right child c_2, and **i** is a position in **lastpos(c_1)**, then all positions in **firstpos (c_2)** are in **followpos (i)**
2. If **n** is a star-node, and **i** is a position in **lastpos(n),** then all positions in **firstpos(n)** are in **followpos(i)**.

- If *firstpos* and *lastpos* have been computed for each node, *followpos* of each position can be computed by making one *depth-first traversal* of the syntax tree

Example -- (a | b) * a



pink – firstpos
blue – lastpos

Then we can calculate followpos

followpos(1) = {1,2,3}
followpos(2) = {1,2,3}
followpos(3) = {4}
followpos(4) = {}

- *After we calculate follow positions, we are ready to create DFA for the regular expression*

Algorithm (RE → DFA)

- Create the syntax tree of (r) #
- Calculate the functions: followpos, firstpos, lastpos, nullable
- Put firstpos (root) into the states of DFA as an unmarked state
- *while* (there is an unmarked state S in the states of DFA) *do*
    - mark **S**
    - *for each* input symbol **a** *do*
        - let $s_1, \ldots, s_n$ are positions in **S** and symbols in those positions are **a**
        - **S'** ← followpos(s_1) $\cup \ldots \cup$ followpos(s_n)
        - **move(S, a) ← S'**
        - if (**S'** is not empty and not in the states of DFA)
            - put **S'** into the states of DFA as an unmarked state

- *the start state of DFA is firstpos (root)*
- *the accepting states of DFA are all states containing the position of #*

Example -- (a | b) * a
1 2 3 4

followpos(1)={1,2,3} followpos(2)={1,2,3} followpos(3)={4} followpos(4)={}

S_1=firstpos(root)={1,2,3}

⇓ mark S_1

a: followpos (1) ∪ followpos(3)={1,2,3,4}=S_2 move(S_1,a)=S_2

b: followpos (2)={1,2,3}=S_1 move(S_1,b)=S_1

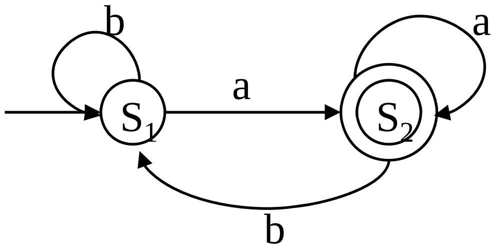
⇓ mark S_2

a: followpos(1) ∪ followpos(3)={1,2,3,4}=S_2 move(S_2,a)=S_2

b: followpos(2)={1,2,3}=S_1 move(S_2,b)=S_1

start state: S_1

accepting states: {S_2}



Example -- (a | ε) b c*

$\qquad\qquad 1 \qquad\qquad 2 \quad 3 \quad 4$

followpos(1)={2}　followpos(2)={3,4}　followpos(3)={3,4}　followpos(4)={}

S_1=firstpos(root)={1,2}

⇓ mark S_1

a: followpos(1)={2}=S_2　　move(S_1,a)=S_2

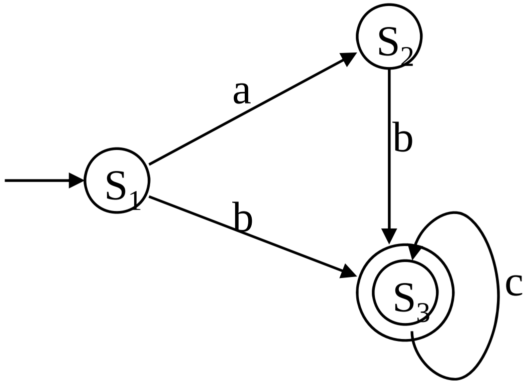
b: followpos(2)={3,4}=S_3　　move(S_1,b)=S_3

⇓ mark S_2

b: followpos(2)={3,4}=S_3　　move(S_2,b)=S_3

⇓ mark S_3

c: followpos(3)={3,4}=S_3　　move(S_3,c)=S_3



start state: S_1

accepting states: {S_3}

Minimizing Number of States of a DFA

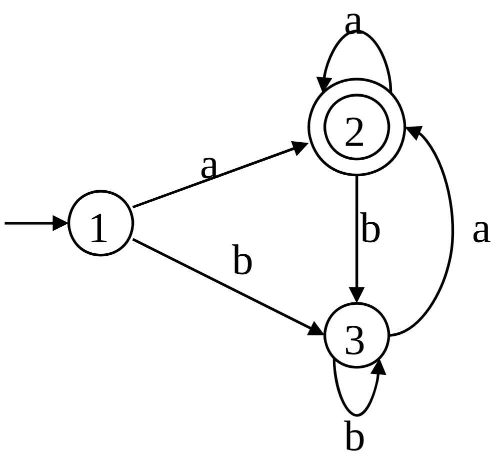
- partition the set of states into two groups:
    - G_1 : set of accepting states
    - G_2 : set of non-accepting states

- For each new group G
    - partition G into subgroups such that states s_1 and s_2 are in the same group iff for all input symbols a, states s_1 and s_2 have transitions to states in the same group

- Start state: the group containing the start state of the original DFA

- Accepting states: the groups containing the accepting states of the original DFA

Minimizing DFA - Example

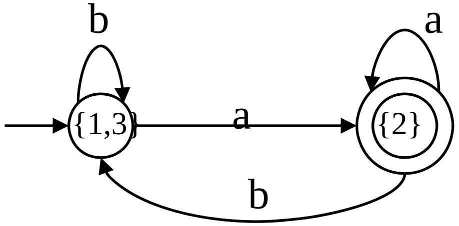


$G_1 = \{2\}$
$G_2 = \{1,3\}$

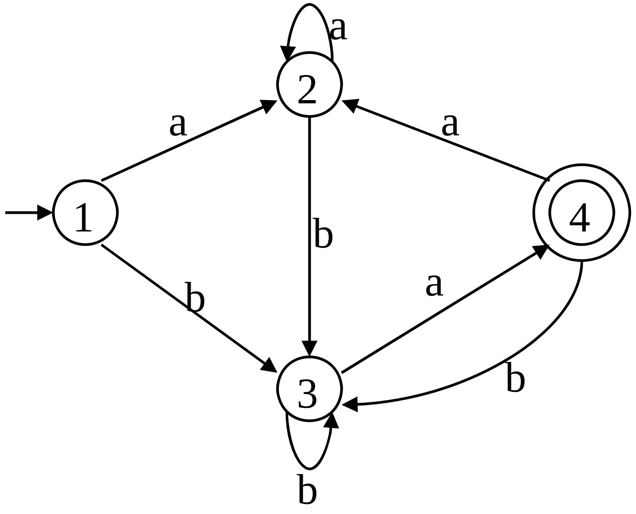
G_2 cannot be partitioned because

move(1,a)=2	move(1,b)=3
move(3,a)=2	move(3,b)=3

So, the minimized DFA (with minimum states)



Minimizing DFA – Another Example



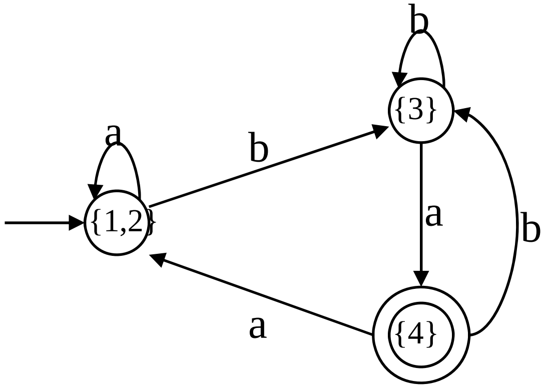
Groups: {1,2,3} {4}

{1,2}
no more partitioning

{3}

a	b
1->2	1->3
2->2	2->3
3->4	3->3

So, the minimized DFA



An Example

Grammar Fragment (Pascal)

stmt → **if** *expr* **then** *stmt*
| **if** *expr* **then** *stmt* **else** *stmt*
| ε
expr → *term* **relop** *term*
| *term*
term → **id** | **num**

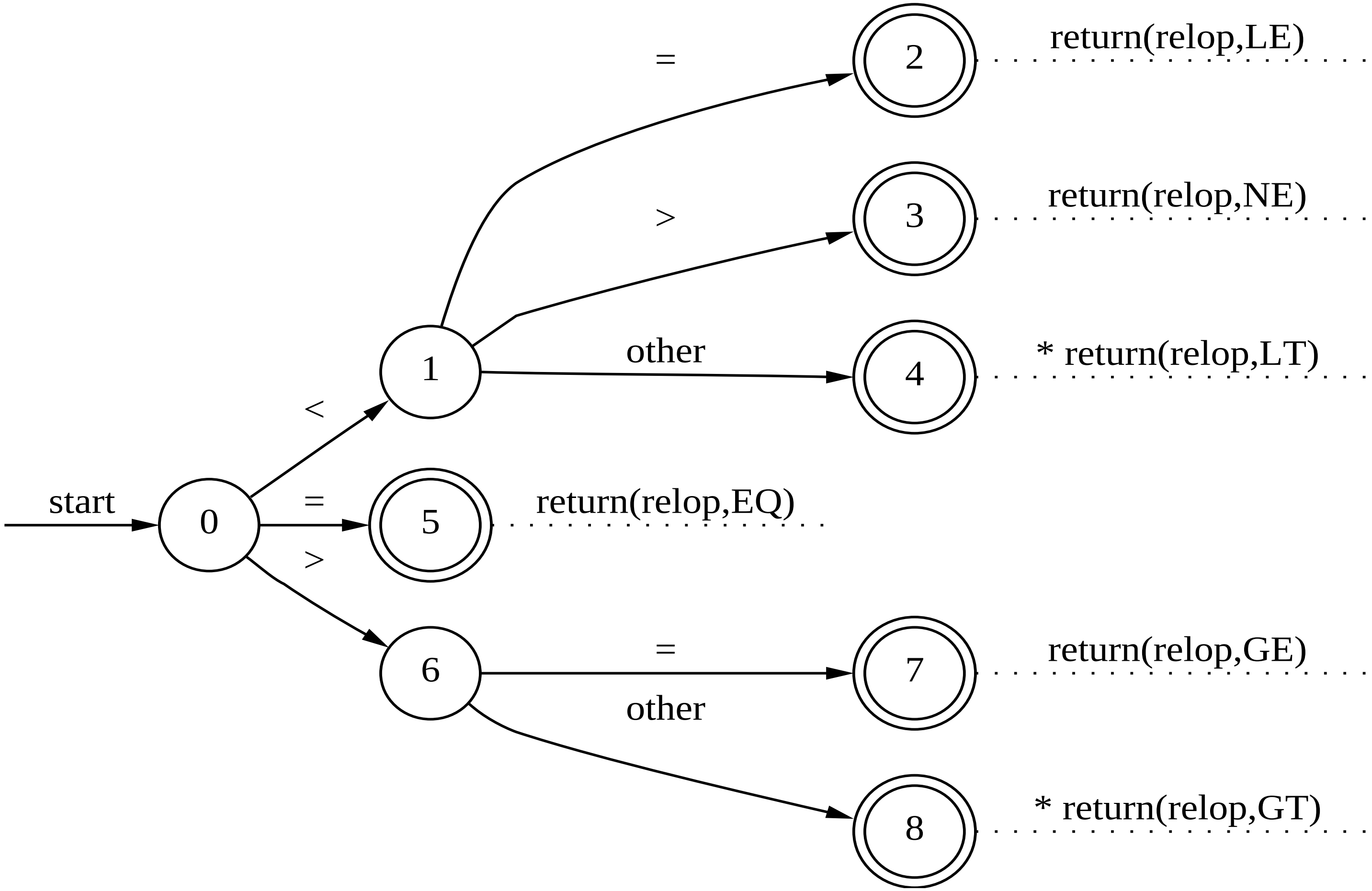
Related Regular Definitions

if → if
then → then
else → else
relop → < | <= | = | <> | > | >=
id → **letter** (**letter** | **digit**)*
num → **digit**$^+$ (. **digit**$^+$)? (E(+|-)? **digit**$^+$)?
delim → **blank** | **tab** | **newline**
ws → **delim**$^+$

Tokens and Attributes

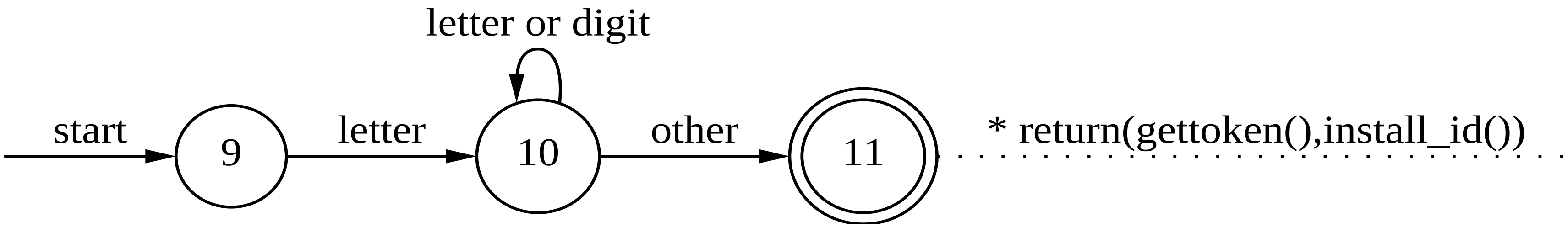
Regular Expression	Token	Attribute Value
ws	-	-
if	if	-
then	then	-
else	else	-
id	id	pointer to entry
num	num	pointer to entry
<	relop	LT
<=	relop	LE
=	relop	EQ
<>	relop	NE
>	relop	GT
=>	relop	GE

Transition Diagram for “relop”

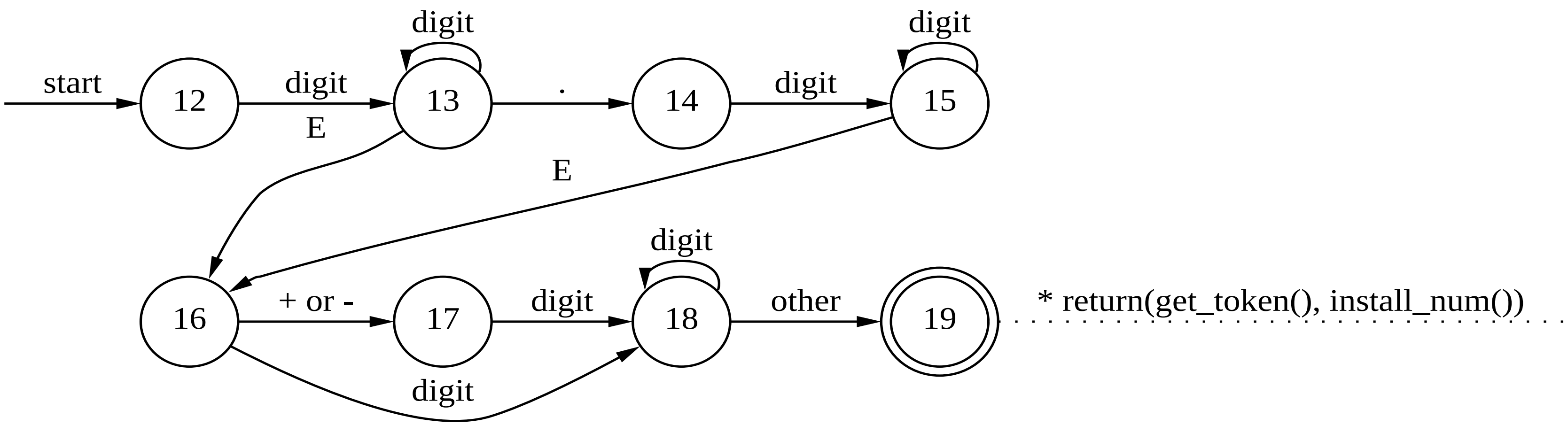


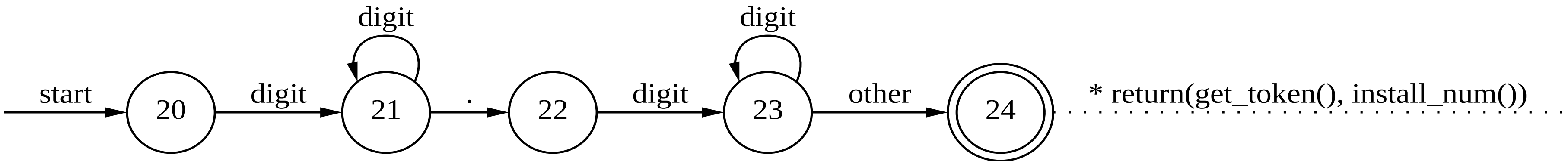
Identifiers and Keywords

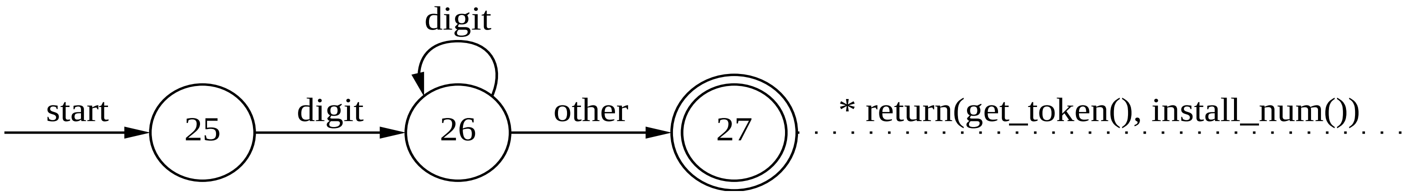
- Share a transition diagram
  - After reaching accepting state, code determines if lexeme is keyword or identifier
  - Easier than encoding exceptions in diagram
- Simple technique is to appropriately initialize symbol table with keywords



Numbers


start
12
digit
13
digit
E
.
14
digit
15
digit
E
16
+ or -
17
digit
18
digit
digit
other
19
* return(get_token(), install_num())


start
20
digit
21
digit
.
22
digit
23
digit
other
24
* return(get_token(), install_num())


start
25
digit
26
digit
other
27
* return(get_token(), install_num())

Order of Transition Diagrams

- Transition diagrams tested in order
- Diagrams with low numbered start states tried before diagrams with high numbered start states
- Order influences efficiency of lexical analyzer

Trying Transition Diagrams

```
int next_td(void) {
  switch (start) {
    case 0: start = 9; break;
    case 9: start = 12; break;
    case 12: start = 20; break;
    case 20: start = 25; break;
    case 25: recover(); break;
    default: error("invalid start state");
  }

  /* Possibly additional actions here */

  return start;
}
```

Finding the Next Token

```
token nexttoken(void) {
  while (1) {
    switch (state) {
      case 0:
        c = nextchar();
        if (c == ' ' || c=='\t' || c == '\n') {
          state = 0;
          lexeme_beginning++;
        }
        else if (c == '<') state = 1;
        else if (c == '=') state = 5;
        else if (c == '>') state = 6;
        else state = next_td();

        break;

      … /* 27 other cases here */
```

The End of a Token

```
token nexttoken(void) {
  while (1) {
    switch (state) {
      … /* First 19 cases */

      case 19:
        retract();
        install_num();
        return(NUM);

        break;

      … /* Final 8 cases */
```

Some Other Issues in Lexical Analyzer

- Lexical analyzer has to recognize the longest possible string
  - Ex: identifier `newval` -- `n` `ne` `new` `newv` `newva` `newval`

- What is the end of a token? Is there any character which marks the end of a token?
  - normally not defined
  - Not an issue if the number of characters in a token is fixed
  - But < ➔ < or <> (in Pascal) (*not fixed*)

  End of an identifier : the characters cannot be in an identifier that can mark the end of token
  - We may need a lookahead
    - In Prolog: p :- X is 1. p :- X is 1.5.

    The dot followed by a white space character can mark the end of a number.

    But if that is not the case, the dot must be treated as a part of the number.

Some Other Issues in Lexical Analyzer (cont.)

- Skipping comments
  - Normally we don't return a comment as a token
  - We skip a comment, and return the next token (which is not a comment) to the parser
  - So, the comments are only processed by the lexical analyzer, and don't complicate the syntax of the language

- Symbol table interface
  - symbol table holds information about tokens (at least lexeme of identifiers)
  - how to implement the symbol table, and what kind of operations?
    - hash table – open addressing, chaining
    - putting into the hash table, finding the position of a token from its lexeme

- Positions of the tokens in the file (for the error handling)