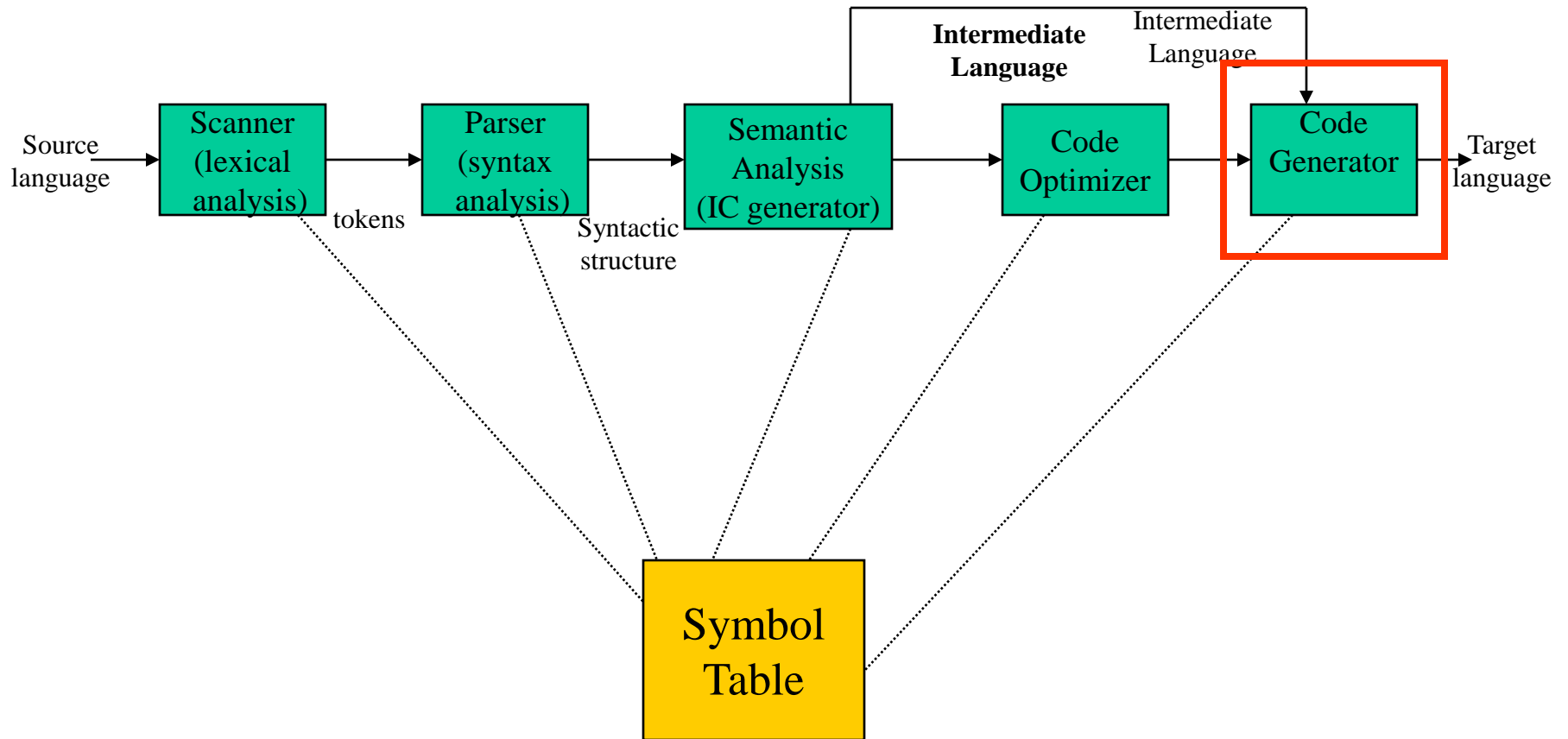


Code Generation-II

CS 346

Compiler Architecture



Register Allocation

How to best use the bounded number of registers?

- Reducing load/store operations
- What are the best values to keep in registers?
- When can we ‘free’ registers?

Complications:

- special purpose registers
- operators requiring multiple registers

Register Allocation Algorithms

- Local (basic block level):
 - **Basic - using liveness information**
 - **Register Allocation using graph coloring**
- Global (CFG)
 - Need to use global liveness information

Basic Code Generation

- Deal with each basic block individually
- Compute liveness information for the block
- Using liveness information, generate code that uses registers as best as possible
- At end, generate code that saves any live values left in registers

.

Concept: Variable Liveness

- For some statement s , variable x is **live** if
 - there is a statement t that uses x
 - there is a path in the CFG from s to t
 - there is no assignment to x on some path from s to t
- **A variable is *live* at a given point in the source code if it could be used before it is defined.**
- Liveness tells us whether we care about the value held by a variable

Example: When is a live?

$a := b + c$

$t1 := a * a$

$b := t1 + a$

$c := t1 * b$

$t2 := c + b$

$a := t2 + t2$

a is live

Assume a, b and c are used
after this basic block

Example: When is b live?



$a := b + c$

$t1 := a * a$

$b := t1 + a$

$c := t1 * b$

$t2 := c + b$

$a := t2 + t2$

Assume a,b and c are used
after this basic block

Computing live status in basic blocks

Input: A basic block.

Output: For each statement, set of live variables

1. Initially all non-temporary variables go into live set (L).
2. for $i = \textit{last}$ statement to *first* statement:
for statement i : $x := y \text{ op } z$
 1. Attach L to statement i .
 2. Remove x from set L.
 3. Add y and z to set L.

Example

live = {

$a := b + c$

live = {

$t1 := a * a$

live = {

$b := t1 + a$

live = {

$c := t1 * b$

live = {

$t2 := c + b$

live = {

$a := t2 + t2$

live = {a,b,c}

Example Answers

live = {}

a := b + c

live = {}

t1 := a * a

live = {}

b := t1 + a

live = {}

c := t1 * b

live = {}

t2 := c + b

live = {b,c,t2}

a := t2 + t2

live = {a,b,c}

Example Answers

live = {}

a := b + c

live = {}

t1 := a * a

live = {}

b := t1 + a

live = {}

c := t1 * b

live = {b,c}

t2 := c + b

live = {b,c,t2}

a := t2 + t2

live = {a,b,c}

Example Answers

live = {}

a := b + c

live = {}

t1 := a * a

live = {}

b := t1 + a

live = { b, t1 }

c := t1 * b

live = { b, c }

t2 := c + b

live = { b, c, t2 }

a := t2 + t2

live = { a, b, c }

Example Answers

live = {}

a := b + c

live = {}

t1 := a * a

live = {a,t1}

b := t1 + a

live = {b,t1}

c := t1 * b

live = {b,c}

t2 := c + b

live = {b,c,t2}

a := t2 + t2

live = {a,b,c}

Example Answers

live = {}

a := b + c

live = {a}

t1 := a * a

live = {a, t1}

b := t1 + a

live = {b, t1}

c := t1 * b

live = {b, c}

t2 := c + b

live = {b, c, t2}

a := t2 + t2

live = {a, b, c}

Example Answers

live = {b,c}

a := b + c

live = {a}

t1 := a * a

live = {a,t1}

b := t1 + a

live = { b,t1 }

c := t1 * b

live = {b,c}

t2 := c + b

live = {b,c,t2}

a := t2 + t2

live = {a,b,c}

Basic Code Generation

- Deal with each basic block individually
- Compute liveness information for the block
- Using liveness information, generate code that uses registers as much as possible
- At end, generate code that saves any live values left in registers

Basic Code Generation

Idea: Deal with the instructions from beginning to end.

For each instruction,

- Use registers whenever possible
- A non-live value in a register can be discarded, freeing that register

Data Structures:

- **Register descriptor**- register status (empty, full) and contents (one or more "values"): keeps track of what is there in registers
- **Address descriptor**-the location (or locations) where the current value for a variable can be found (*register, stack, memory*)

Instruction type: $x := y \text{ op } z$

1. Choose R_x , the register where the result (x) will be kept.
 1. If y (or z) is in a register t alone and not live, choose $R_x = t$
 2. Else if there is a free register t, choose $R_x = t$
 3. Else must free up a register for R_x
2. Find R_y . If y is not in a register, generate load into a free register (or R_x)
3. Find R_z . If z is not in a register, generate load into a free register (can use R_x if not used by y).
4. Generate: OP R_x, R_y, R_z

Instruction type: $x := y \text{ op } z$

5. Update information about the current best location of x
6. If x is in a register, update that register's information
7. If y and/or z are not live after this instruction, update register and address descriptors accordingly.

Example Code

	$\text{live} = \{b, c\}$
$a := b + c$	
	$\text{live} = \{a\}$
$t1 := a * a$	
	$\text{live} = \{a, t1\}$
$b := t1 + a$	
	$\text{live} = \{b, t1\}$
$c := t1 * b$	
	$\text{live} = \{b, c\}$
$t2 := c + b$	
	$\text{live} = \{b, c, t2\}$
$a := t2 + t2$	
	$\text{live} = \{a, b, c\}$

Returning to live Example

- Initially

Three Registers: (-, -, -) all empty

current values: (a,b,c,t1,t2) = (m,m,m, -, -)

- instruction 1: $a := b + c$, Live = { a }

$R_a = \$t0, R_b = \$t0, R_c = \$t1$

lw $\$t0, b$

lw $\$t1, c$

add $\$t0, \$t0, \$t1$

Registers: (a, -, -)

current values: ($\$t0$,m,m, -, -)

Don't need to keep track
of b or c since aren't live.

- instruction 2: $t1 := a * a$, $\text{Live} = \{a, t1\}$

$R_{t1} = \$t1$ (since a is live)

`mul $t1, $t0, $t0`

Registers: $(a, t1, -)$ current values: $(\$t0, m, m, \$t1, -)$

- instruction 3: $b := t1 + a$, $\text{Live} = \{b, t1\}$

Since a is not live after call, $R_b = \$t0$

`add $t0, $t1, $t0`

Registers: $(b, t1, -)$ current values: $(m, \$t0, m, \$t1, -)$

- instruction 4: $c := t1 * b$, $\text{Live} = \{b, c\}$

Since $t1$ is not live after call $R_c = \$t1$

`mul $t1, $t1, $t0`

Registers: (b, c, -) current values: (m, \$t0, \$t1, -, -)

- instruction 5: $t2 := c + b$, $\text{Live} = \{b, c, t2\}$

$R_{t2} = \$t2$

`add $t2, $t1, $t0`

Registers: (b, c, t2) current values: (m, \$t0, \$t1, -, \$t2)

- instruction 6: $a := t2 + t2$, $\text{Live} = \{a, b, c\}$

`add $t2, $t2, $t2`

Registers: (b, c, a) current values: (\$t2, \$t0, \$t1, -, -)

- Since end of block, move live variables:

`sw $t2, a`

`sw $t0, b`

`sw $t1, c`

all registers available

all live variables moved to memory

Generated code

```
lw $t0, b
lw $t1, c
add $t0, $t0, $t1
mul $t1, $t0, $t0
add $t0, $t1, $t0
mul $t1, $t1, $t0
add $t2, $t1, $t0
add $t2, $t2, $t2
sw $t2, a
sw $t0, b
sw $t1, c
```

$a := b + c$

$t1 := a * a$

$b := t1 + a$

$c := t1 * b$

$t2 := c + b$

$a := t2 + t2$

Cost = 16

How does this compare to
naïve approach?

Register Allocation with Graph Coloring

Local register allocation-*graph coloring problem*

Uses *liveness* information

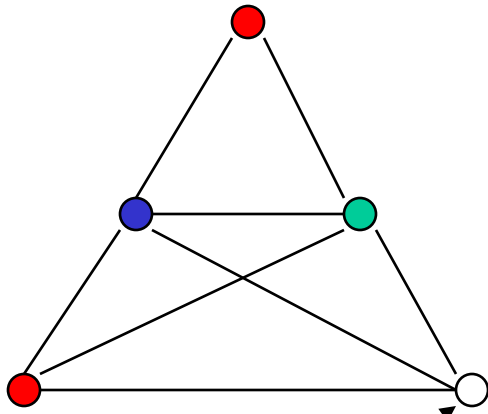
Allocate K registers where each register is associated with one of the K colors

Graph Coloring

- The coloring of a graph $G = (V, E)$ is a mapping $C: V \rightarrow S$, where S is a finite set of colors, such that if edge vw is in E , $C(v) \neq C(w)$
- **NP complete problem** (for more than 2 colors) \Rightarrow no polynomial time solution
- Fortunately there are approximation algorithms!

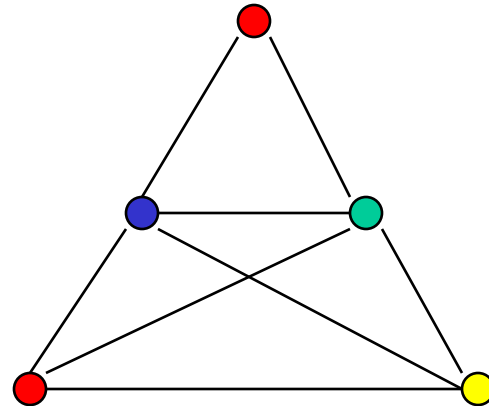
Coloring a graph with K colors

$K = 3$



No color for
this node

$K = 4$



Register Allocation and Graph K-Coloring

K = number of available registers

$G = (V, E)$ where

- **Vertex set:** $V = \{V_s \mid s \text{ is a program variable}\}$
- **Edge:** $V_s V_t \in E$ if s and t can be live at the same time

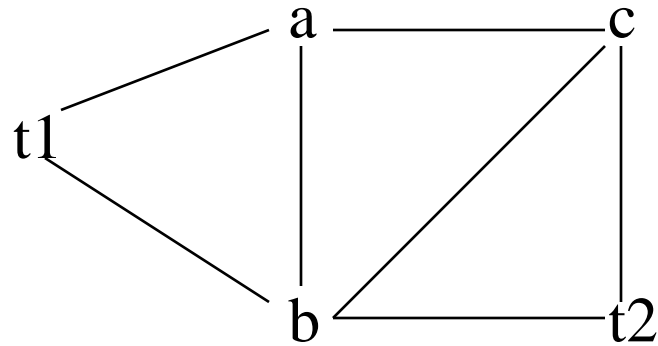
G is an '*interference graph*'

Algorithm: K registers

1. Compute liveness information for the basic block
2. Create interference graph G - one node for each variable, an edge connecting any two variables alive simultaneously

Example: Interference Graph

$a := b + c$	$\{b, c\}$
$t1 := a * a$	$\{a\}$
$b := t1 + a$	$\{t1, a\}$
$c := t1 * b$	$\{b, t1\}$
$t2 := c + b$	$\{b, c\}$
$a := t2 + t2$	$\{b, c, t2\}$
	$\{a, b, c\}$



Algorithm: K registers

3. **Simplify**-For any node m with fewer than K neighbors,
 - a. remove it from the graph
 - b. push it onto a stack
 - c. If $G-m$ can be colored with K colors, so can G
 - d. If we reduce the entire graph, goto step 5

Algorithm: K registers

4. Spill- If we get to the point where we are left with only nodes with degree $\geq K$,

- a. mark some node for potential spilling
- b. remove and push onto stack
- c. back to step 3

Choosing a Spill Node

Potential criteria:

- Random
- Most neighbors
- Longest live range (in code)
 - with or without taking the access pattern into consideration

Algorithm: K registers

5. Assign colors-

- a. start with empty graph
- b. rebuild graph by popping elements off the stack
- c. put them back into the graph
- d. assign them colors different from neighbors

Potential spill nodes may or may not be colorable

Rewriting the code

- Want to be able to remove some edges in the interference graph
 - write variable to memory earlier
 - compute/read in variable later

Back to example

$a := b + c$ $\{b, c\}$

$t1 := a * a$ $\{a\}$

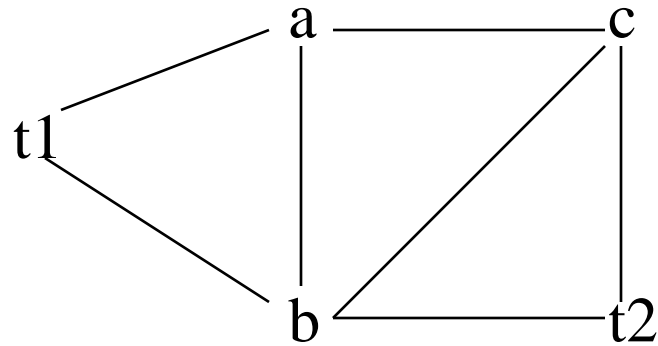
$b := t1 + a$ $\{t1, a\}$

$c := t1 * b$ $\{b, t1\}$

$t2 := c + b$ $\{b, c\}$

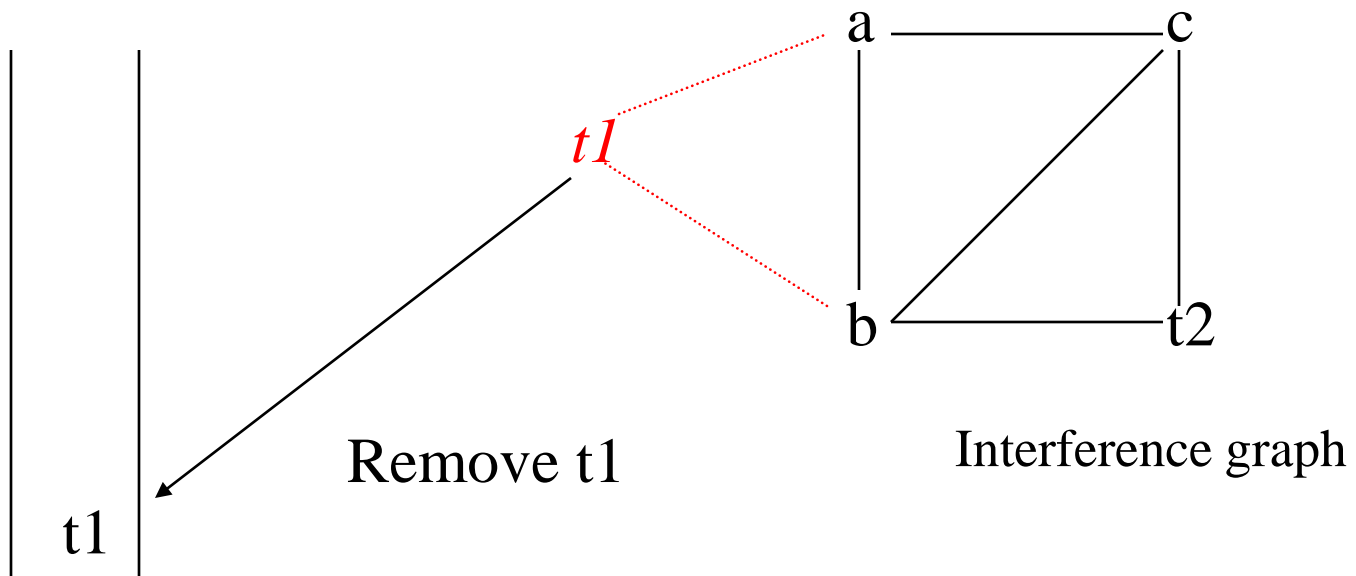
$a := t2 + t2$ $\{b, c, t2\}$

$\{a, b, c\}$



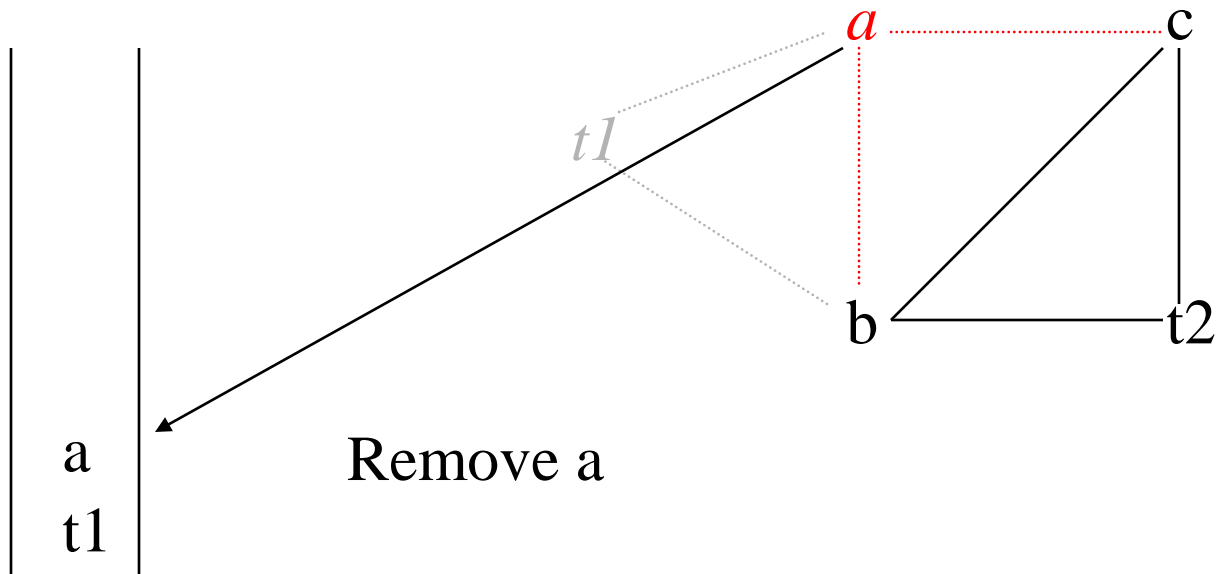
Example, $k = 3$

Assume $k = 3$



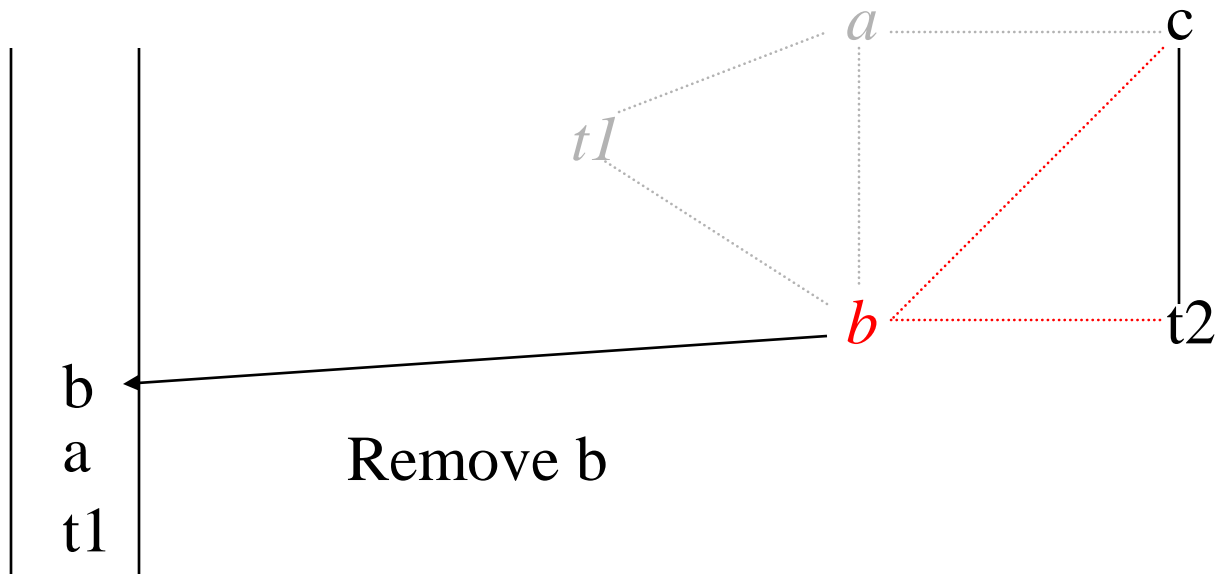
Example

Assume $k = 3$



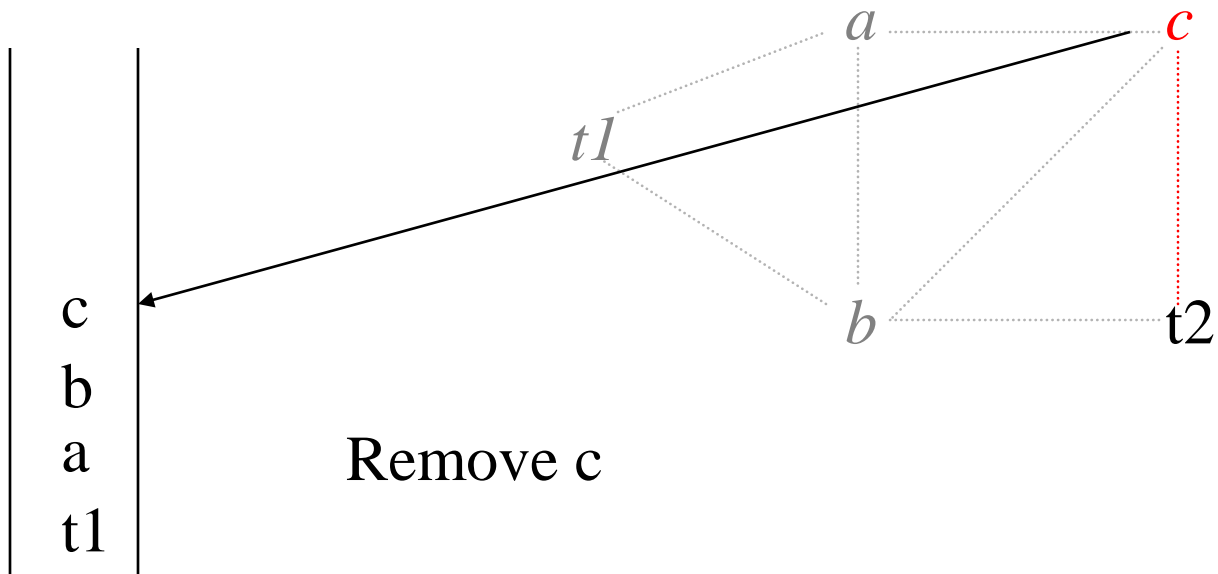
Example

Assume $k = 3$



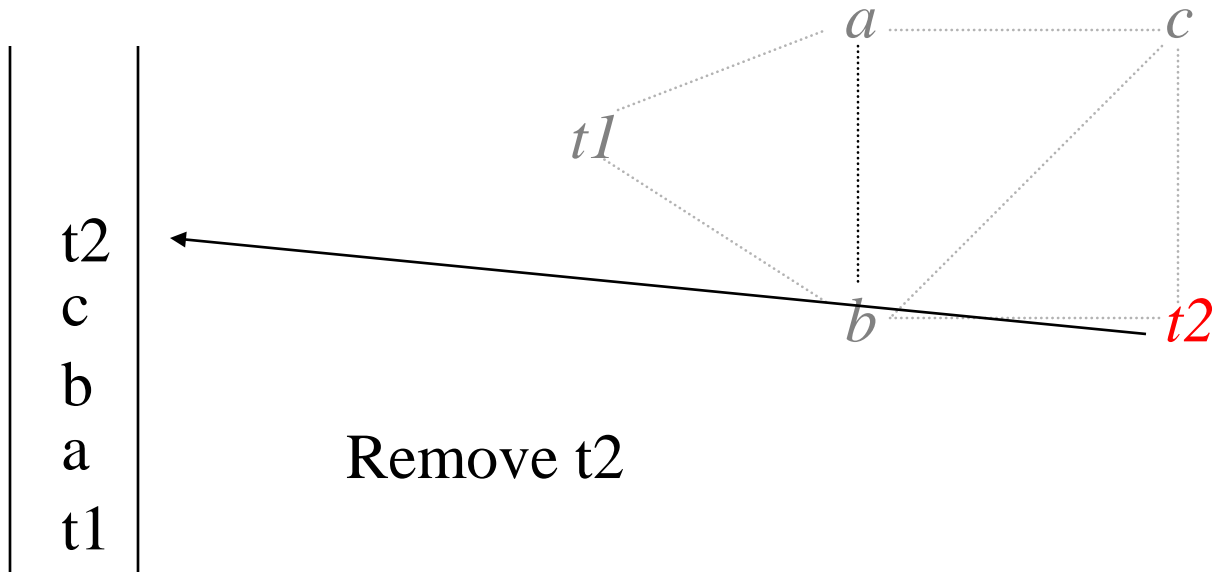
Example

Assume $k = 3$



Example

Assume $k = 3$



Rebuild the graph

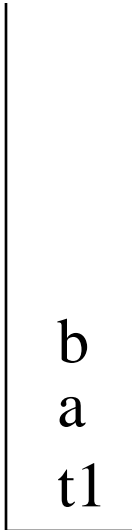
Assume $k = 3$

c
b
a
t1

t2

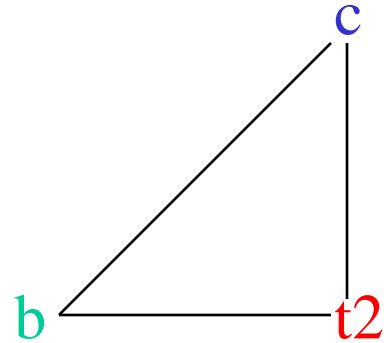
Example

Assume $k = 3$



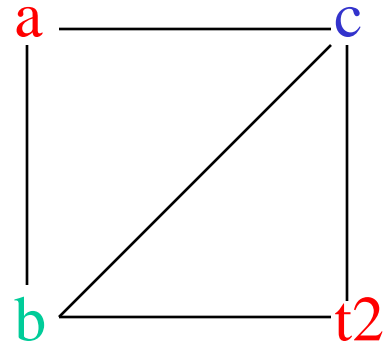
Example

Assume $k = 3$



Example

Assume $k = 3$

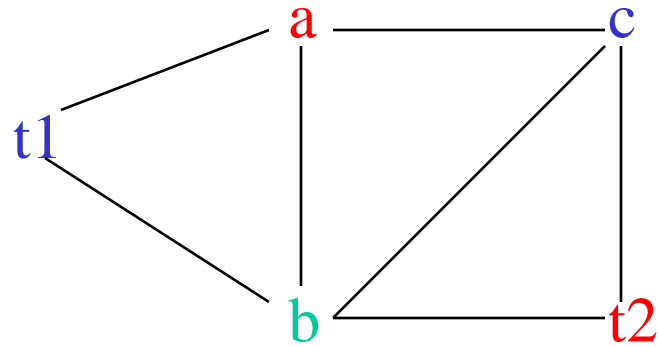


Example

Assume $k = 3$



a	t0
b	t1
c	t2
t1	t2
t2	t0



Back to example

a := b + c
t1 := a * a
b := t1 + a
c := t1 * b
t2 := c + b
a := t2 + t2

a	t0
b	t1
c	t2
t1	t2
t2	t0

```
lw $t1,b
lw $t2,c
add $t0,$t1,$t2
mul $t2,$t0,$t0
add $t1,$t2,$t0
mul $t2,$t2,$t1
add $t0,$t2,$t1
add $t0,$t0,$t0
sw $t0,a
sw $t1,b
sw $t2,c
```

Generated code: Basic

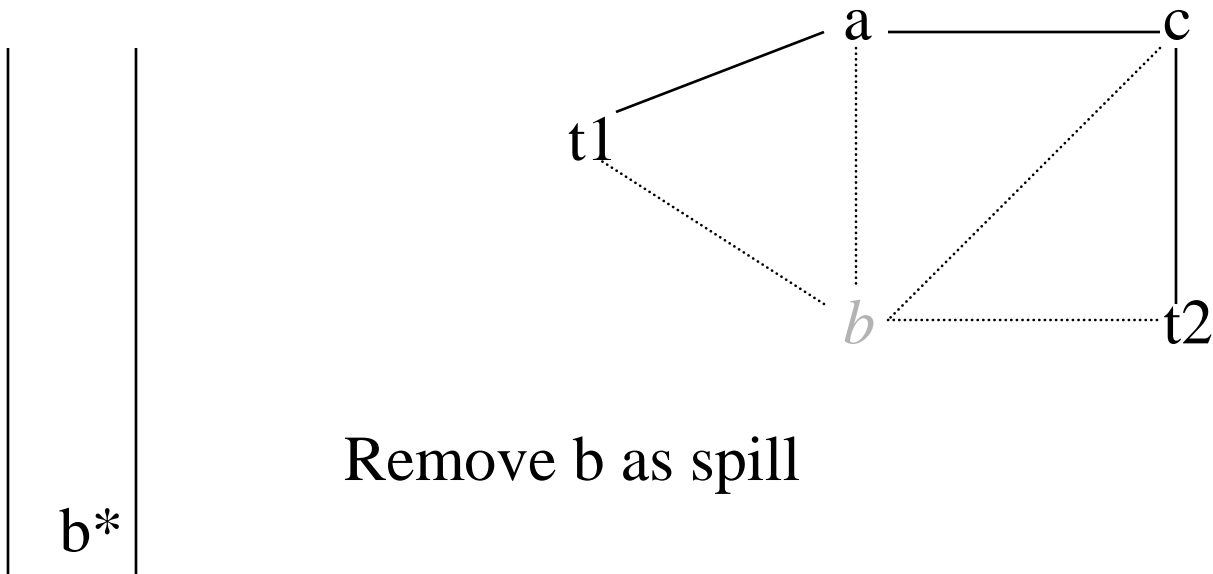
```
lw $t0,b
lw $t1,c
add $t0,$t0,$t1
mul $t1,$t0,$t0
add $t0,$t1,$t0
mul $t1,$t1,$t0
add $t2,$t1,$t0
add $t2,$t2,$t2
sw $t2, a
sw $t0,b
sw $t1,c
```

Generated Code: Coloring

```
lw $t1,b
lw $t2,c
add $t0,$t1,$t2
mul $t2,$t0,$t0
add $t1,$t2,$t0
mul $t2,$t2,$t1
add $t0,$t2,$t1
add $t0,$t0,$t0
sw $t0,a
sw $t1,b
sw $t2,c
```

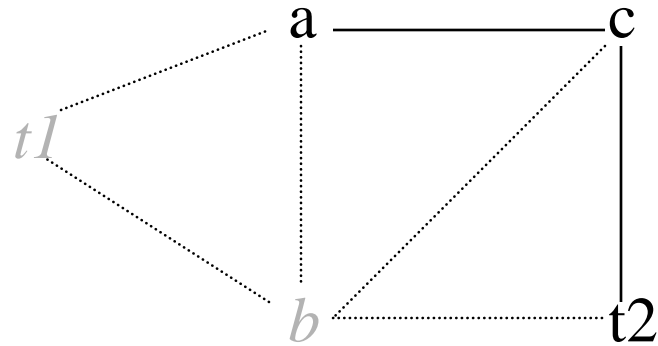
Example, $k = 2$

Assume $k = 2$



Example

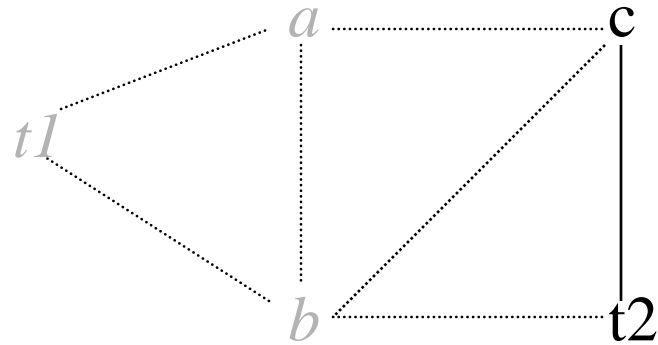
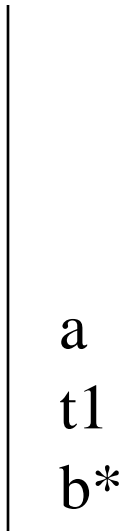
Assume $k = 2$



Remove $t1$

Example

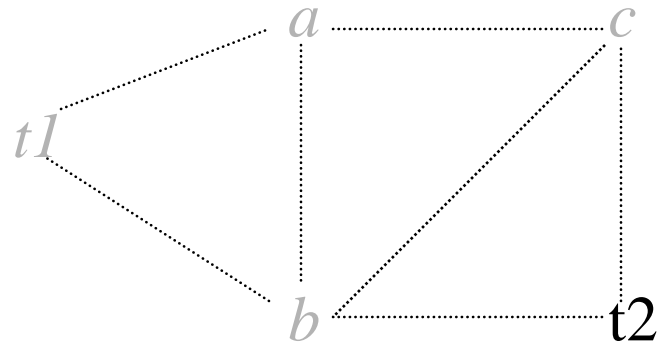
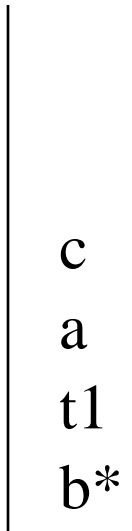
Assume $k = 2$



Remove a

Example

Assume $k = 2$

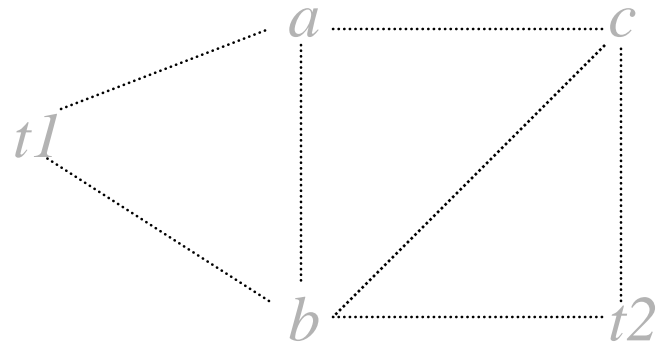


Remove c

Example

Assume $k = 2$

t2
c
a
t1
b*



Remove $t2$

Example

Assume $k = 2$

Can flush b out to memory, creating a smaller window

$a := b + c$ $\{b, c\}$

$t1 := a * a$ $\{a\}$

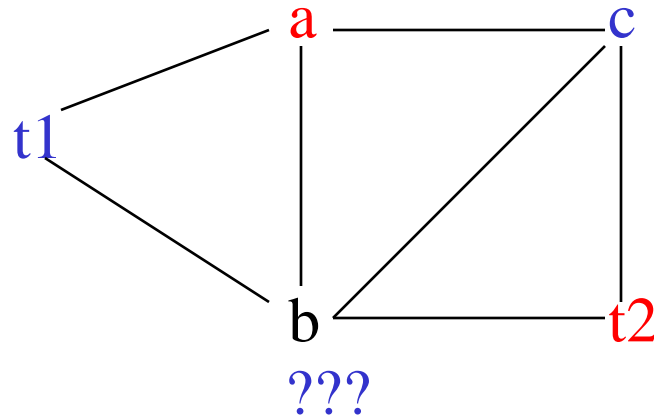
$b := t1 + a$ $\{t1, a\}$

$c := t1 * b$ $\{b, t1\}$

$t2 := c + b$ $\{b, c\}$

$a := t2 + t2$ $\{b, c, t2\}$

$\{a, b, c\}$



After spilling b:

$a := b + c$ $\{b, c\}$

$t1 := a * a$ $\{a\}$

$b := t1 + a$ $\{t1, a\}$

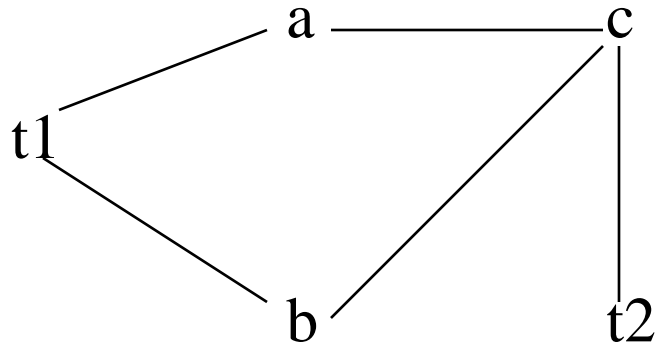
$c := t1 * b$ $\{b, t1\}$

b to memory

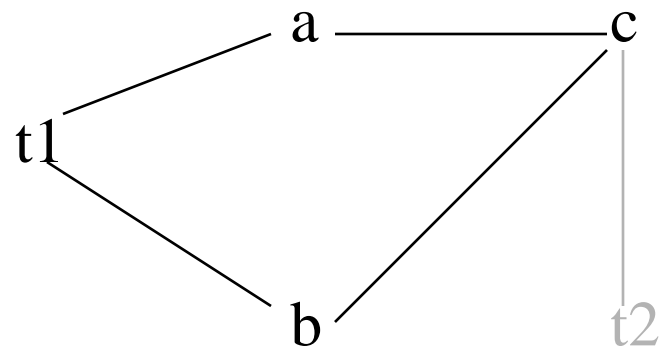
$t2 := c + b$ $\{b, c\}$

$a := t2 + t2$ $\{c, t2\}$

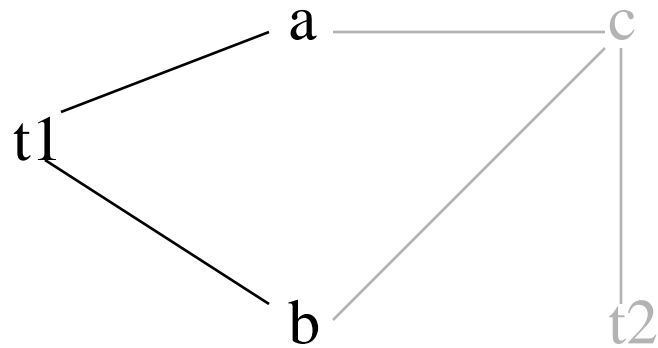
$\{a, c\}$



After spilling b:

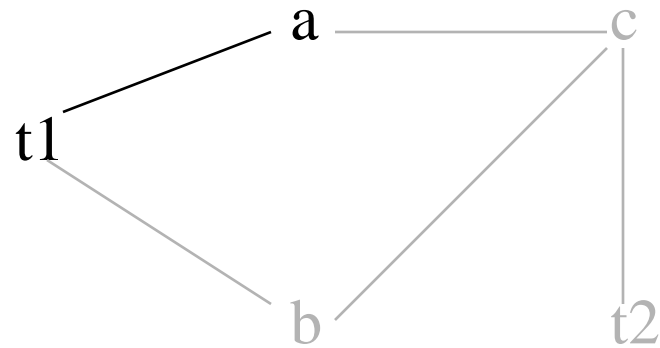
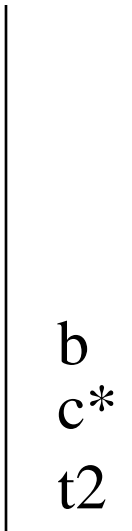


After spilling b:

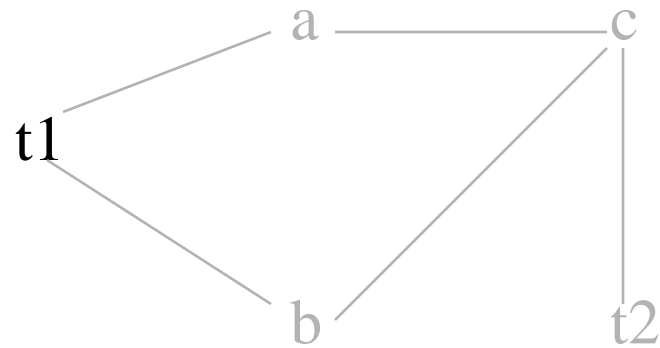
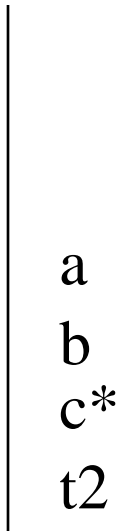


Have to choose c as a potential spill node.

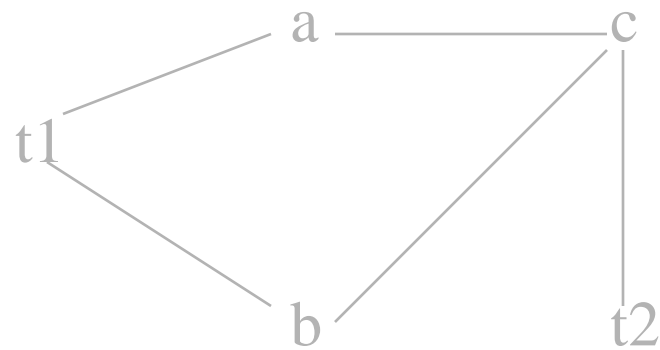
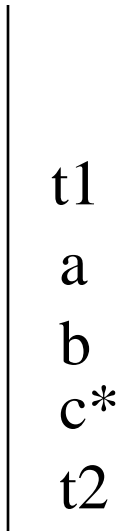
After spilling b:



After spilling b:

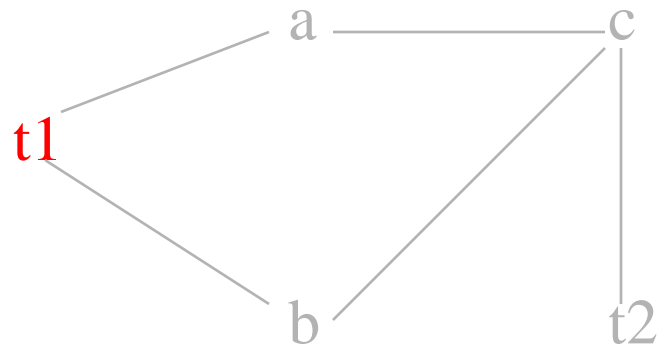


After spilling b:



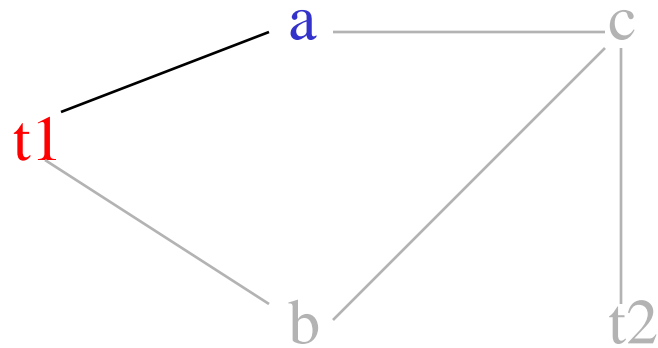
Now rebuild:

a
b
c*
t2

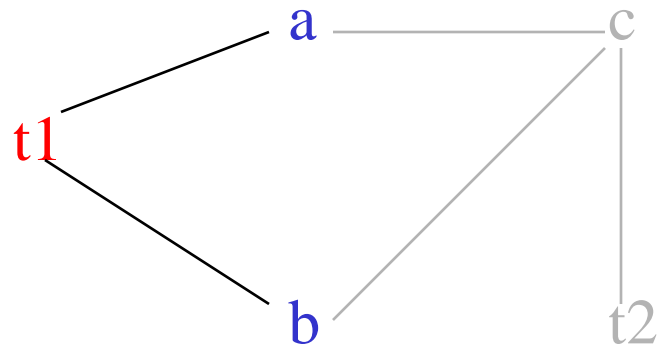
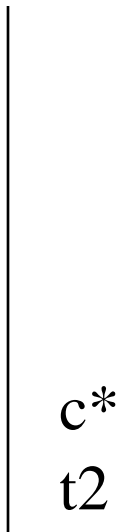


Now rebuild:

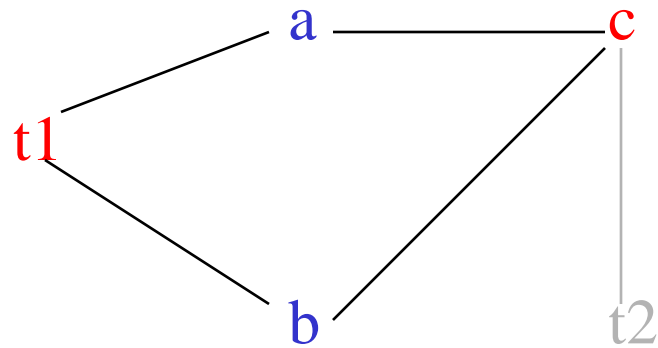
b
c*
t2



Now rebuild:



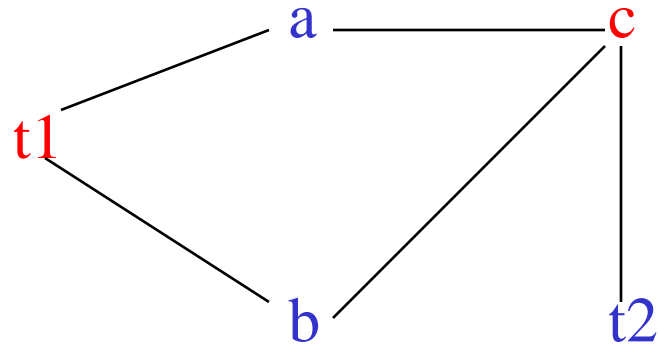
Now rebuild:



Fortunately, there is a color for c

Now rebuild:

a	t0
b	t0
c	t1
t1	t1
t2	t0



The graph is 2-colorable now

The code

a := b + c
t1 := a * a
b := t1 + a
c := t1 * b
b to memory
t2 := c + b
a := t2 + t2

a	t0
b	t0
c	t1
t1	t1
t2	t0

```
lw $t0,b
lw $t1,c
add $t0,$t0,$t1
mul $t1,$t0,$t0
add $t0,$t1,$t0
mul $t1,$t1,$t0
sw $t0,b
add $t0,$t1,$t0
add $t0,$t0,$t0
sw $t0,a
sw $t1,c
```

Spilling (more!)

- **Spilling:** once all nodes have K or more neighbors, pick a node for **spilling**
 - Storage on the stack
- There are many heuristics that can be used to pick a node
 - not in an inner loop

Spilling code

- We need to generate extra instructions to load variables from stack and store them
- These instructions use registers themselves

What to do?

- **Stupid approach:** always keep extra registers handy for shuffling data in and out: **what a waste!**
- **Better approach:** ?

Spilling code

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What to do?
 - **Stupid approach:** always keep extra registers handy for shuffling data in and out: **what a waste!**
 - **Better approach:** rewrite code introducing a new temporary; rerun liveness analysis and register allocation

Rewriting code

- Consider: `add t1 t2`
 - Suppose `t2` is selected for spilling and assigned to stack location `[ebp-24]`
 - Invent new temporary `t35` for just this instruction and rewrite:
 - `mov t35, [ebp - 24]; add t1, t35`
 - Advantage: `t35` has a very short live range and is much less likely to interfere.
 - Rerun the algorithm; fewer variables will spill