CS 346: Syntax Analyzer

Resource: Textbook

Alfred V. Aho, Ravi Sethi, and Jeffrey D. Ullman, "*Compilers: Principles, Techniques, and Tools*", Addison-Wesley, 1986.

Syntax Analyzer

- *Syntax Analyzer:* creates the syntactic structure of the given source program
  - Parser
- Syntactic structure: *parse tree*
- Syntax of a programming: described by a *context-free grammar (CFG)*

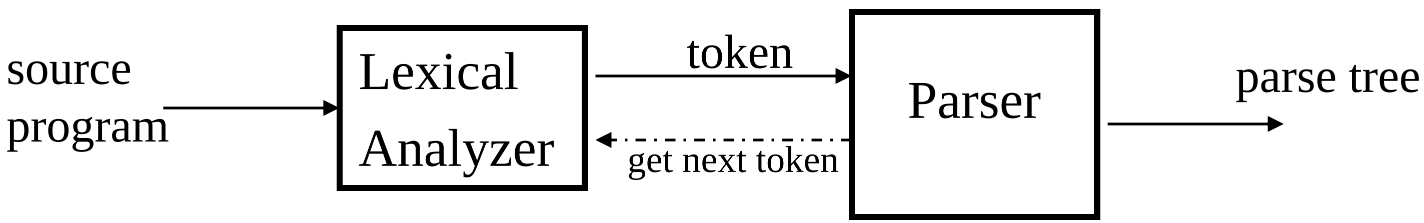
- Steps
  - Parser checks whether a given source program satisfies the rules implied by a CFG or not
  - If it satisfies, the parser creates the parse tree of that program
  - Otherwise the parser gives the error messages

Syntax Analyzer

- CFG
  - gives a precise syntactic specification of a programming language
  - the design of the grammar is an initial phase of the design of a compiler
  - a grammar can be directly converted into a parser by some tools

Parser

- Parser works on a stream of tokens
- Smallest item: token



Parsers (cont.)

- Well-known categories of parsers:

1. **Top-Down Parser**
   - the parse tree created top to bottom, starting from the root
2. **Bottom-Up Parser**
   - the parse created bottom to top; starting from the leaves

- Both top-down and bottom-up parsers scan the input from left to right (*one symbol at a time*)
- Efficient top-down and bottom-up parsers can be implemented only for *sub-classes of CFG*
  - LL for top-down parsing
  - LR for bottom-up parsing

Context-Free Grammars (CFG)

- Inherently recursive structures of a programming language are defined by a CFG
- In a CFG, we have:
  - A finite set of terminals (in our case, this will be the set of tokens)
  - A finite set of non-terminals (syntactic-variables)
  - A finite set of productions rules in the following form

    $A \rightarrow \alpha$ where A is a non-terminal and

    α is a string of terminals and non-terminals (including the empty string); $|A| <= |\alpha|$
  - A start symbol: one of the non-terminal symbols
- Example:

$$E \rightarrow E + E \;|\; E - E \;|\; E * E \;|\; E / E \;|\; - E$$

$$E \rightarrow (E)$$

$$E \rightarrow id$$

Derivations

$E \Rightarrow E+E$

- E+E derives from E
  - we can replace E by E+E

$E \Rightarrow E+E \Rightarrow id + E \Rightarrow id + id$

- A sequence of replacements of non-terminal symbols is called a **derivation** of id + id from E

- In general a derivation step is

  $\alpha A \beta \Rightarrow \alpha \gamma \beta$ if there is a production rule $A \rightarrow \gamma$ in our grammar

  where α and β are arbitrary strings of terminal and non-terminal symbols

$\alpha_1 \Rightarrow \alpha_2 \Rightarrow \ldots \Rightarrow \alpha_n$ (α_n derives from α_1 or α_1 derives α_n)

$\Rightarrow$: derives in one step

$\Rightarrow$: derives in zero or more steps

$\Rightarrow$: derives in one or more steps

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CFG - Terminology

- L(G) is *the language of G* (the language generated by G) which is a set of sentences
- *A sentence of L(G)* is a string of terminal symbols of G
- If S is the start symbol of G then

  ω is a sentence of L(G) iff $S \overset{+}{\Rightarrow} \omega$ where ω is a string of terminals of G

- If G is a context-free grammar, L(G) is a *context-free language*
- Two grammars are *equivalent* if they produce the same language

$$S \overset{*}{\Rightarrow} \alpha$$

- If α contains non-terminals, it is called as a *sentential* form of G

- If α does not contain non-terminals, it is called as a *sentence* of G

Derivation: Example

$$E \Rightarrow -E \Rightarrow -(E) \Rightarrow -(E+E) \Rightarrow -(id+E) \Rightarrow -(id+id)$$

OR

$$E \Rightarrow -E \Rightarrow -(E) \Rightarrow -(E+E) \Rightarrow -(E+id) \Rightarrow -(id+id)$$

- At each derivation step, we can choose any of the non-terminals in the sentential form of G for the replacement
- **left-most derivation:** always chooses the left-most non-terminal in each derivation step
- **right-most derivation:** always chooses the right-most non-terminal in each derivation step

Left-Most and Right-Most Derivations

Left-Most Derivation

$$E \underset{lm}{\Rightarrow} -E \underset{lm}{\Rightarrow} -(E) \underset{lm}{\Rightarrow} -(E+E) \underset{lm}{\Rightarrow} -(id+E) \underset{lm}{\Rightarrow} -(id+id)$$

Right-Most Derivation

$$E \underset{rm}{\Rightarrow} -E \underset{rm}{\Rightarrow} -(E) \underset{rm}{\Rightarrow} -(E+E) \underset{rm}{\Rightarrow} -(E+id) \underset{rm}{\Rightarrow} -(id+id)$$

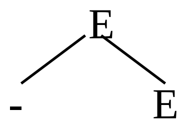
- top-down parsers: finds the left-most derivation of the given source program

- bottom-up parsers: finds the right-most derivation of the given source program in the reverse order

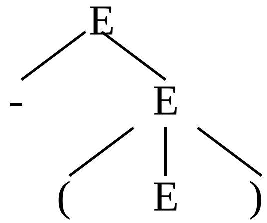
Parse Tree

- Intermediate nodes: Inner nodes of a parse tree
- Leaves: Terminal symbols
- A parse tree can be seen as a graphical representation of a derivation

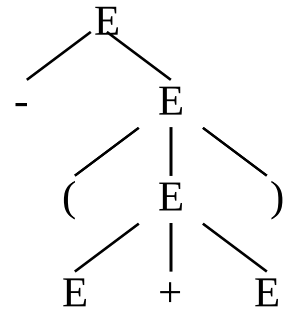
$E \Rightarrow -E$



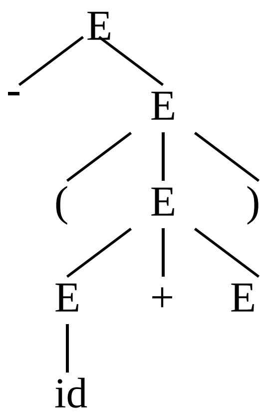
$\Rightarrow -(E)$



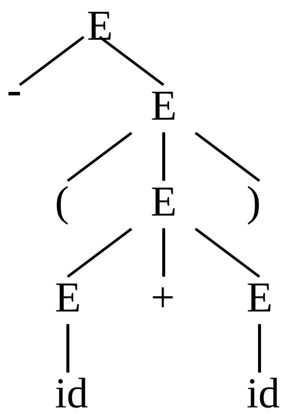
$\Rightarrow -(E+E)$



$\Rightarrow -(id+E)$



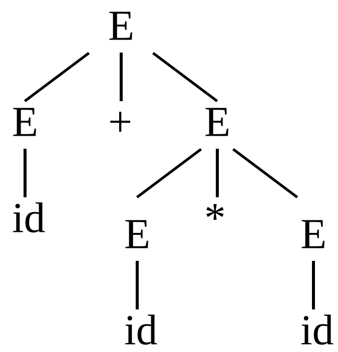
$\Rightarrow -(id+id)$



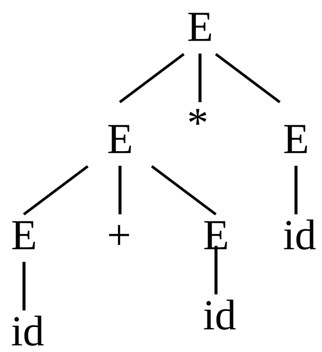
Ambiguity

- A grammar that produces more than one parse tree for a sentence is called as an ***ambiguous*** grammar

$$E \Rightarrow E+E \Rightarrow id+E \Rightarrow id+E*E \Rightarrow id+id*E \Rightarrow id+id*id$$



$$E \Rightarrow E*E \Rightarrow E+E*E \Rightarrow id+E*E \Rightarrow id+id*E \Rightarrow id+id*id$$



Ambiguity (cont.)

- For the most parsers, the grammar must be unambiguous

- Unambiguous grammar

  ➔ unique selection of the parse tree for a sentence

- Disambiguation

  --Necessary to eliminate the ambiguity in the grammar during the design phase of the compiler

  - Design unambiguous grammar
  - Choose one of the parse trees of a sentence to restrict to this choice

Ambiguity (cont.)

stmt → **if** expr **then** stmt |
if expr **then** stmt **else** stmt | otherstmts

if E_1 **then** **if** E_2 **then** S_1 **else** S_2

Interpretation-1: S2 being executed when E_1 is false (thus attaching the else to the first if)

if E_1 **then** (**if** E_2 **then** S_1) **else** S_2

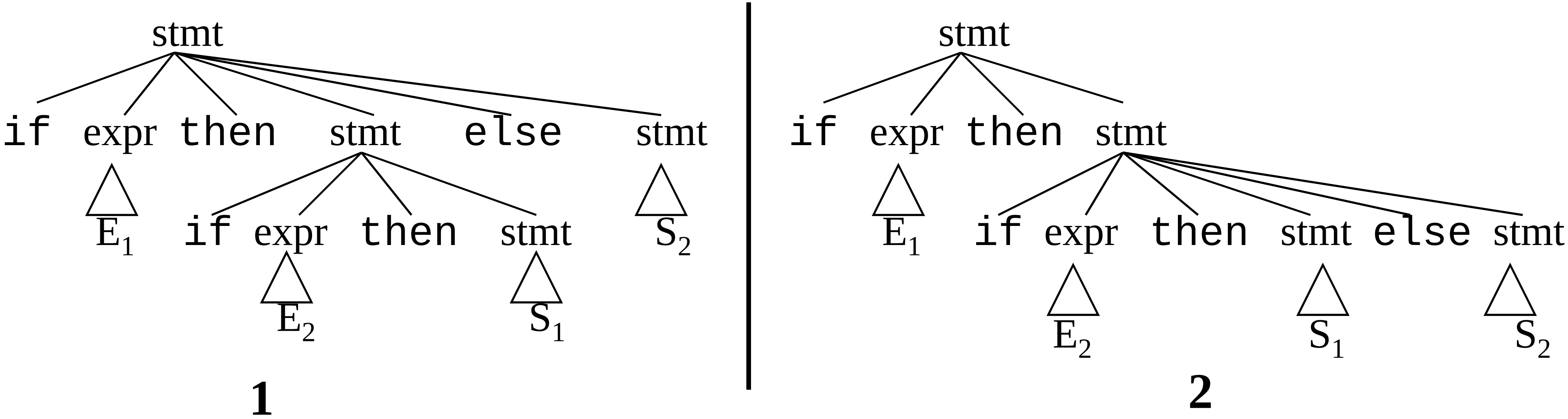
Interpretation-I1: E_1 is true and E_2 is false (thus attaching the else to the second if)

if E_1 **then** (**if** E_2 **then** S_1 **else** S_2)

Ambiguity (cont.)

stmt → `if` expr `then` stmt |
`if` expr `then` stmt `else` stmt | otherstmts

`if` E_1 `then` `if` E_2 `then` S_1 `else` S_2



Ambiguity (cont.)

- We prefer the second parse tree (else matches with closest if)

So, we have to disambiguate our grammar to reflect this choice

- Unambiguous grammar:

stmt → matchedstmt | unmatchedstmt

matchedstmt → if expr then matchedstmt else matchedstmt |
otherstmts

unmatchedstmt → if expr then stmt |
if expr then matchedstmt else unmatchedstmt

Ambiguity – Operator Precedence

- Ambiguous grammars (because of ambiguous operators) can be disambiguated according to the precedence and associativity rules

$$E \rightarrow E+E \mid E*E \mid E\hat{}E \mid id \mid (E)$$

$\Downarrow$ disambiguate the grammar

precedence:

^ (right to left)

* (left to right)

\+ (left to right)

$$E \rightarrow E+T \mid T$$
$$T \rightarrow T*F \mid F$$
$$F \rightarrow G\hat{}F \mid G$$
$$G \rightarrow id \mid (E)$$

Left Recursion

- A grammar is ***left recursive*** if it has a non-terminal A such that there is a derivation

  $$A \stackrel{+}{\Rightarrow} A\alpha \qquad \text{for some string } \alpha$$

- Top-down parsing techniques **cannot** handle left-recursive grammars
  - Conversion of left-recursive grammar into an equivalent non-recursive grammar is essential
- Possible ways of left-recursion
  - may appear in a single step of the derivation (*immediate left-recursion*) or
  - may appear in more than one step of the derivation

Immediate Left-Recursion

$A \rightarrow A\,\alpha \mid \beta$ where β does not start with A

$\Downarrow$ eliminate immediate left recursion

$A \rightarrow \beta\, A'$

$A' \rightarrow \alpha\, A' \mid \varepsilon$ an equivalent grammar

In general,

$A \rightarrow A\,\alpha_1 \mid \ldots \mid A\,\alpha_m \mid \beta_1 \mid \ldots \mid \beta_n$ where $\beta_1 \ldots \beta_n$ do not start with A

$\Downarrow$ eliminate immediate left recursion

$A \rightarrow \beta_1\, A' \mid \ldots \mid \beta_n\, A'$

$A' \rightarrow \alpha_1\, A' \mid \ldots \mid \alpha_m\, A' \mid \varepsilon$ an equivalent grammar

Immediate Left-Recursion -- Example

$E \rightarrow E+T \mid T$

$T \rightarrow T*F \mid F$

$F \rightarrow id \mid (E)$

$\Downarrow$ eliminate immediate left recursion

$E \rightarrow T E'$

$E' \rightarrow +T E' \mid \varepsilon$

$T \rightarrow F T'$

$T' \rightarrow *F T' \mid \varepsilon$

$F \rightarrow id \mid (E)$

Left-Recursion -- Problem

- A grammar cannot be *immediately left-recursive*, but it still can be *left-recursive*
- Just elimination of the immediate left-recursion does not guarantee a grammar which is not left-recursive

$S \rightarrow Aa \mid b$

$A \rightarrow Sc \mid d$ This grammar is not immediately left-recursive, but it is still left-recursive

$\underline{S} \Rightarrow Aa \Rightarrow \underline{S}ca$ or

$\underline{A} \Rightarrow Sc \Rightarrow \underline{A}ac$ causes to a left-recursion

- Solution: *eliminate all left-recursions from the grammar*

Eliminate Left-Recursion -- Algorithm

- Arrange non-terminals in some order: $A_1 \dots A_n$

- for i **from** 1 **to** n **do** {

  - **for** j **from** 1 **to** i-1 **do** {

    replace each production

    $$A_i \rightarrow A_j \gamma$$

    by

    $$A_i \rightarrow \alpha_1 \gamma \mid \dots \mid \alpha_k \gamma$$

    where $A_j \rightarrow \alpha_1 \mid \dots \mid \alpha_k$

  }

  - eliminate immediate left-recursions among A_i productions

}

Eliminate Left-Recursion -- Example

$S \rightarrow Aa \mid b$
$A \rightarrow Ac \mid Sd \mid f$

- Order of non-terminals: S, A

for S:

- we do not enter the inner loop.
- there is no immediate left recursion in S.

for A:

- Replace $A \rightarrow Sd$ with $A \rightarrow Aad \mid bd$
  So, we will have $A \rightarrow Ac \mid Aad \mid bd \mid f$
- Eliminate the immediate left-recursion in A

$$A \rightarrow bdA' \mid fA'$$
$$A' \rightarrow cA' \mid adA' \mid \varepsilon$$

So, the resulting equivalent grammar which is not left-recursive is:

$S \rightarrow Aa \mid b$
$A \rightarrow bdA' \mid fA'$
$A' \rightarrow cA' \mid adA' \mid \varepsilon$

Eliminate Left-Recursion – Example2

$S \rightarrow Aa \mid b$
$A \rightarrow Ac \mid Sd \mid f$

- Order of non-terminals: A, S

for A:

- we do not enter the inner loop
- Eliminate the immediate left-recursion in A

$$A \rightarrow SdA' \mid fA'$$
$$A' \rightarrow cA' \mid \varepsilon$$

for S:

- Replace $S \rightarrow Aa$ with $S \rightarrow SdA'a \mid fA'a$
  So, we will have $S \rightarrow SdA'a \mid fA'a \mid b$
- Eliminate the immediate left-recursion in S

$$S \rightarrow fA'aS' \mid bS'$$
$$S' \rightarrow dA'aS' \mid \varepsilon$$

So, the resulting equivalent grammar which is not left-recursive is:

$S \rightarrow fA'aS' \mid bS'$
$S' \rightarrow dA'aS' \mid \varepsilon$
$A \rightarrow SdA' \mid fA'$
$A' \rightarrow cA' \mid \varepsilon$

Left-Factoring

- Top-down *parser without backtracking* (predictive parser) insists that the grammar must be *left-factored*

grammar ➔ a new equivalent grammar suitable for predictive parsing

stmt → `if` expr `then` stmt `else` stmt |
`if` expr `then` stmt

After seeing `if`, we cannot decide which production rule to choose to re-write *stmt* in the derivation

Left-Factoring (cont.)

- In general,

  $A \rightarrow \alpha\beta_1 \mid \alpha\beta_2$ where α is non-empty and the first symbols of β_1 and β_2 (if they have one) are different

- Choice involved when processing α

  A to $\alpha\beta_1$ or

  A to $\alpha\beta_2$

- Re-write the grammar as follows:

  $A \rightarrow \alpha A'$

  $A' \rightarrow \beta_1 \mid \beta_2$ so, we can immediately expand A to $\alpha A'$

Left-Factoring -- Algorithm

- For each non-terminal A with two or more alternatives (production rules) with a common non-empty prefix, let say

$$A \rightarrow \alpha\beta_1 \mid \ldots \mid \alpha\beta_n \mid \gamma_1 \mid \ldots \mid \gamma_m$$

convert it into

$$A \rightarrow \alpha A' \mid \gamma_1 \mid \ldots \mid \gamma_m$$
$$A' \rightarrow \beta_1 \mid \ldots \mid \beta_n$$

Left-Factoring – Example1

$$A \rightarrow \underline{a}bB \mid \underline{a}B \mid cdg \mid cdeB \mid cdfB$$

$$\Downarrow$$

$$A \rightarrow aA' \mid \underline{cd}g \mid \underline{cd}eB \mid \underline{cd}fB$$

$$A' \rightarrow bB \mid B$$

$$\Downarrow$$

$$A \rightarrow aA' \mid cdA''$$

$$A' \rightarrow bB \mid B$$

$$A'' \rightarrow g \mid eB \mid fB$$

Left-Factoring – Example2

$A \rightarrow ad \mid a \mid ab \mid abc \mid b$

$\Downarrow$

$A \rightarrow aA' \mid b$

$A' \rightarrow d \mid \varepsilon \mid b \mid bc$

$\Downarrow$

$A \rightarrow aA' \mid b$

$A' \rightarrow d \mid \varepsilon \mid bA''$

$A'' \rightarrow \varepsilon \mid c$

Non-Context Free Language Constructs

- Some language constructions in the programming languages are not context-free

Example-1: $L1 = \{ \omega c \omega \mid \omega \text{ is in } (a|b)^* \}$

➔ declaring an identifier and checking whether it is declared or not later. We cannot do this with a context-free language. We need semantic analyzer (which is not context-free)

Example-2: $L2 = \{a^n b^m c^n d^m \mid n \geq 1 \text{ and } m \geq 1 \}$

➔ declaring two functions (one with n parameters, the other one with m parameters), and then calling them with actual parameters