Dataflow analysis

Dataflow analysis: what is it?

- A common framework for expressing algorithms that compute information about a program
- Why is such a framework useful?

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- A common framework for expressing algorithms that compute information about a program
- Why is such a framework useful?
- Provides a common language, which makes it easier to:
  - communicate your analysis to others
  - compare analyses
  - adapt techniques from one analysis to another
  - reuse implementations (eg: dataflow analysis frameworks)

Data flow analysis

- IMPORTANT!
  - Data flow analysis should never tell us that a transformation is safe when in fact it is not.
  - When doing data flow analysis we must be
    - Conservative
      - Do not consider information that may not preserve the behavior of the program
    - Aggressive
      - Try to collect information that is as exact as possible, so we can get the greatest benefit from our optimizations.

Global Iterative Data Flow Analysis

- Global:
  - Performed on the control flow graph
  - Goal = to collect information at the beginning and end of each basic block
- Iterative:
  - Construct data flow equations that describe how information flows through each basic block and solve them by iteratively converging on a solution.

Global Iterative Data Flow Analysis

- Components of data flow equations
  - Sets containing collected information
    - **In** (or **entry**) set: information coming into the BB from outside (following flow of dats)
    - **gen** set: information generated/collected within the BB
    - **kill** set: information that, due to action within the BB, will affect what has been collected outside the BB
    - **out** (or **exit**) set: information leaving the BB
  - Functions (operations on these sets)
    - **Transfer functions** describe how information changes as it flows through a basic block
    - **Meet functions** describe how information from multiple paths is combined.

Global Iterative Data Flow Analysis

- Algorithm sketch
  - Typically, a bit vector is used to store the information.
    - For example, in reaching definitions, each bit position corresponds to one definition.
  - We use an iterative fixed-point algorithm.
  - Depending on the nature of the problem we are solving, we may need to traverse each basic block in a forward (top-down) or backward direction.
    - The order in which we "visit" each BB is not important in terms of algorithm correctness, but is important in terms of efficiency.
  - In & Out sets should be initialized in a conservative and aggressive way.

```
Initialize gen and kill sets
Initialize in or out sets (depending on "direction")
while there are no changes in in and out sets {
    for each BB {
        apply meet function
        apply transfer function
    }
}
```

Typical problems

- Reaching definitions
  - For each use of a variable, find all definitions that reach it.
- Upward exposed uses
  - For each definition of a variable, find all uses that it reaches.
- Live variables
  - For a point p and a variable v, determine whether v is live at p.
- Available expressions
  - Find all expressions whose value is available at some point p.
- Very Busy expressions
  - Find all expressions whose value will be used in all the next paths

Reaching definitions

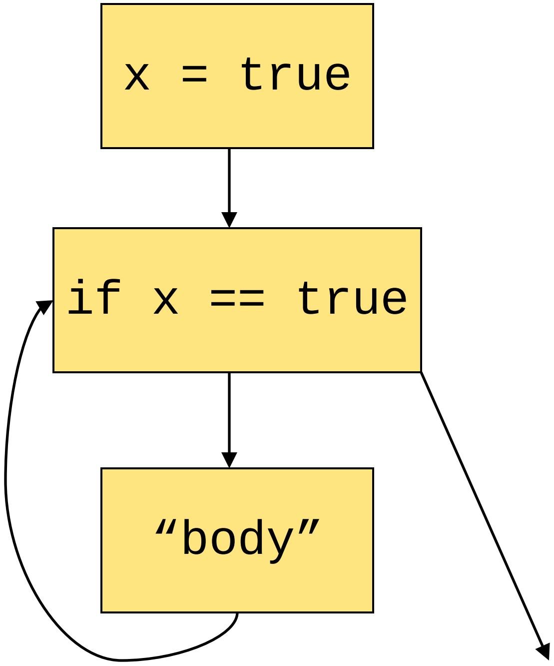
- Determine which definitions of a variable may reach each use of the variable.
  - For each use, list the definitions that reach it. This is also called a ud-chain.
  - In global data flow analysis, we collect such information at the endpoints of a basic block, but we can do additional local analysis within each block.
- Uses of reaching definitions :
  - constant propagation
    - we need to know that all the definitions that reach a variable assign it to the same constant
  - copy propagation
    - we need to know whether a particular copy statement is the only definition that reaches a use.
  - code motion
    - we need to know whether a computation is loop-invariant

Something obvious

```
boolean x = true;
while (x) {
   . . . // no change to x
}
```

- Doesn't terminate.
- Proof: only assignment to x is at top, so x is always true.

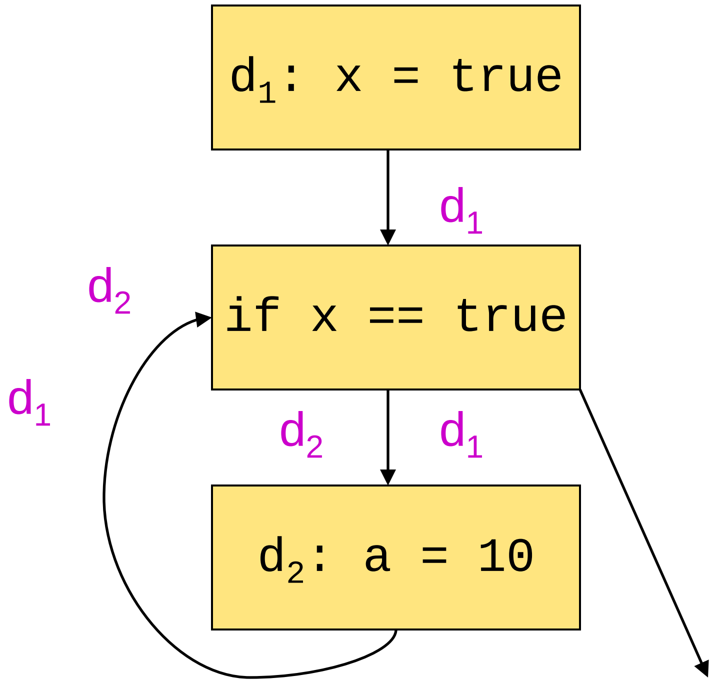
As a Control Flow Graph



Formulation: Reaching Definitions

- Each place some variable x is assigned is a *definition*.
- Ask: for this use of x, where could x last have been defined?
- In our example: only at `x=true`.

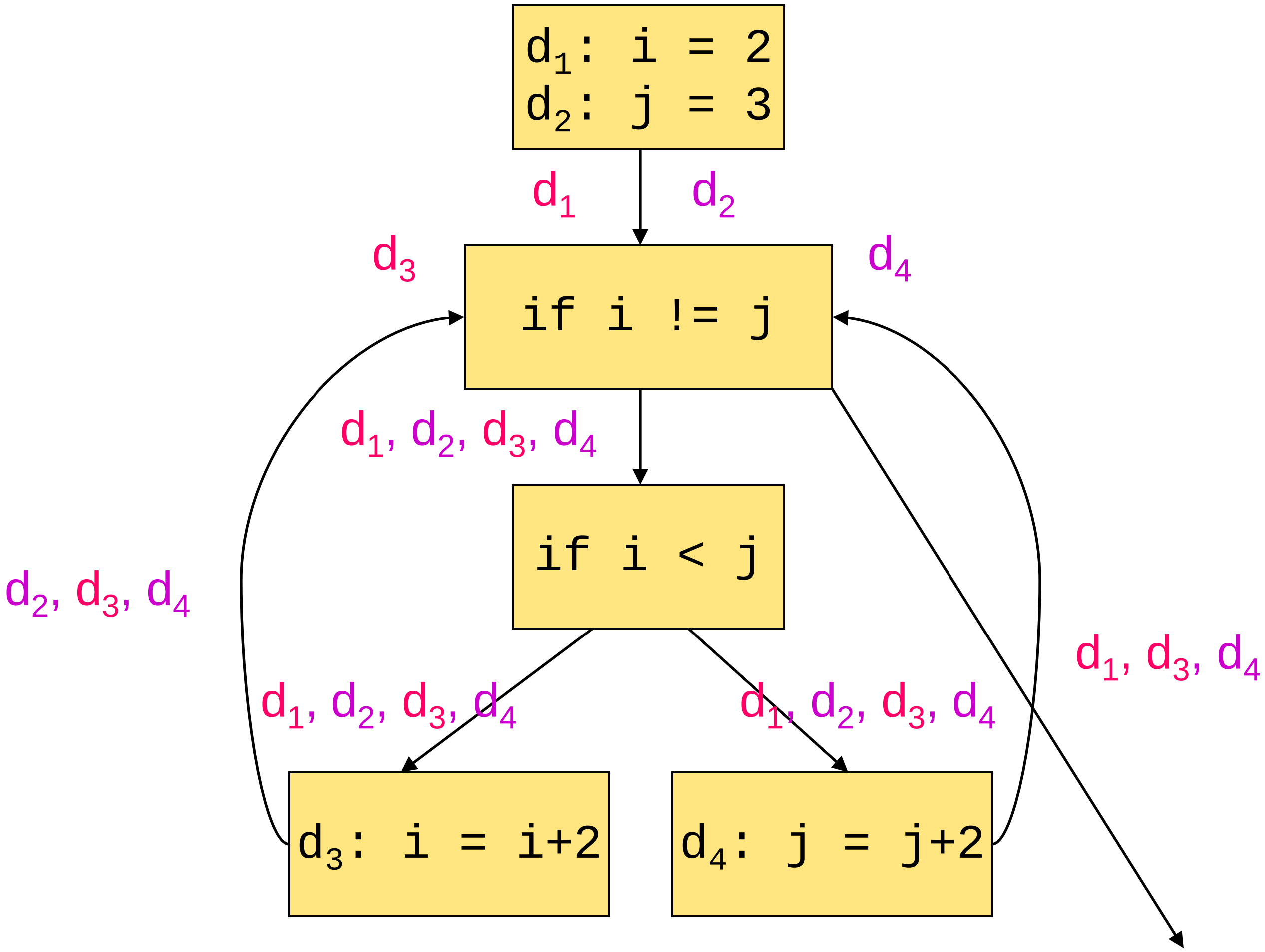
Example: Reaching Definitions



Clincher

- Since at `x == true`, d_1 is the only definition of x that reaches, it must be that x is true at that point.
- The conditional is not really a conditional and can be replaced by a branch.

The Control Flow Graph



DFA is Sufficient Only

- In this example, i can be defined in two places, and j in two places.
- No obvious way to discover that `i!=j` is always true.
- But OK, because reaching definitions is sufficient to catch most opportunities for *constant folding* (replacement of a variable by its only possible value).

Be Conservative!

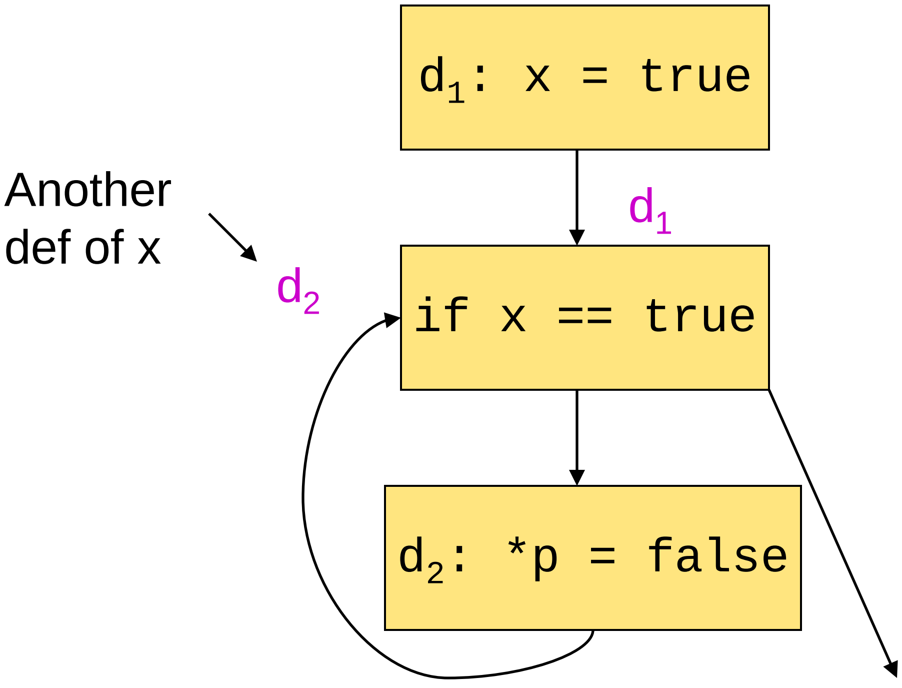
- (Code optimization only)
- It's OK to discover a subset of the opportunities to make some code-improving transformation.
- It's not OK to think you have an opportunity that you don't really have.

Example: Be Conservative

```
boolean x = true;
while (x) {
    . . . *p = false; . . .
}
```

- Is it possible that p points to x?

As a Control Flow Graph



Possible Resolution

- Just as data-flow analysis of "reaching definitions" can tell what definitions of x might reach a point, another DFA can eliminate cases where p definitely does not point to x.
- Example: the only definition of p is `p = &y` and there is no possibility that y is an alias of x.

Formalization: Reaching definitions Analysis

- A definition D reaches a point *p* if there is a path from D to *p* along which D is not killed.
- A definition D of a variable *x* is killed when there is a redefinition of *x.*
- How can we represent the set of definitions reaching a point?

Reaching definitions

- What is safe?
  - To assume that a definition reaches a point even if it turns out not to.
  - The computed set of definitions reaching a point p will be a superset of the actual set of definitions reaching p
  - It’s a “possible”, not a “definite” property
  - Goal : make the set of reaching definitions as small as possible (i.e. as close to the actual set as possible)

Reaching definitions

- How are the **gen** and **kill** sets defined?
  - **gen**[B] = {definitions that appear in B and reach the end of B}
  - **kill**[B] = {all definitions that never reach the end of B}
- What is the direction of the analysis?
  - forward
  - **out**[B] = **gen**[B] $\cup$ (**in**[B] - **kill**[B])

Reaching definitions

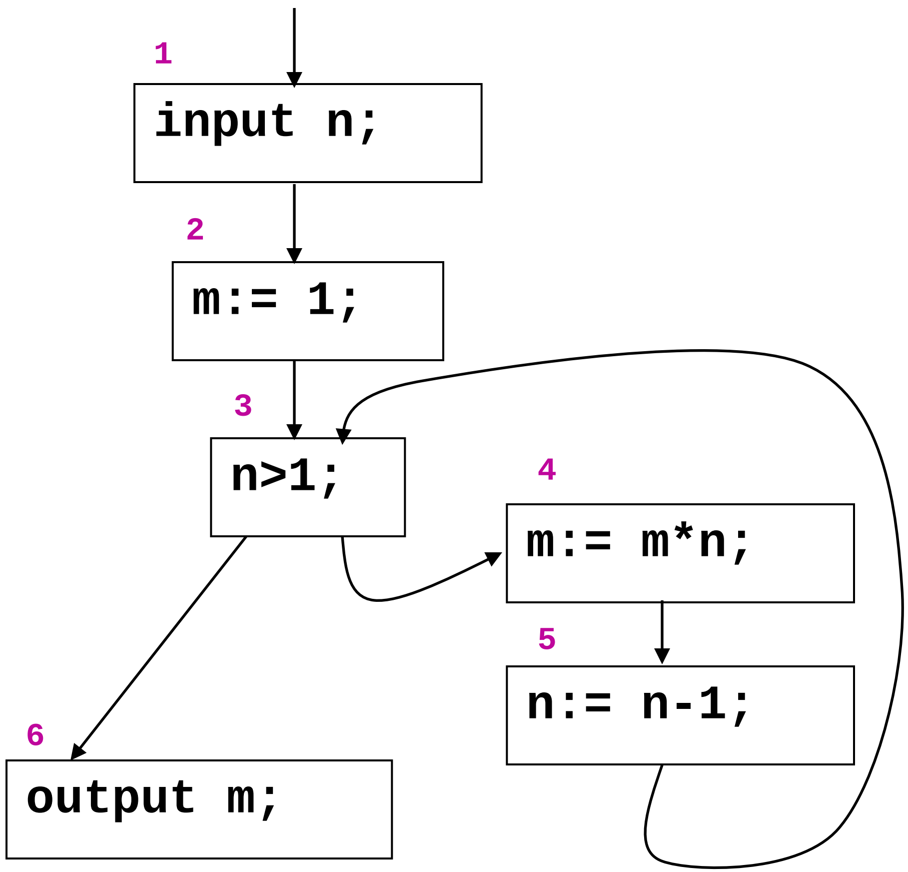
- What is the **confluence** operator?
  - union
  - **in**[P] = $\cup$ **out**[Q], over the predecessors Q of P
- How do we initialize?
  - start small
    - Why? Because we want the resulting set to be as small as possible
  - for each block B initialize **out**[B] = **gen**[B]

Formal specification

- The reaching Definition Analysis is specified by the folllowing equations:
- For each program point,

$$RD_{in}(p) = \begin{cases} \iota & \text{if p is the initial point in the control graph} \\ \bigcup \{ RD_{out}(q) \mid \text{there is an arrow from q to p} \} \end{cases}$$

$$RD_{out}(p) = gen_{RD}(p) \cup (RD_{in}(p) \setminus kill_{RD}(p))$$



- $RD_{in}(1) = \{(n,?),(m,?)\}$
  $RD_{out}(1) = \{(n,?),(m,?)\}$
- $RD_{in}(2) = \{(n,?),(m,?)\}$
  $RD_{out}(2) = \{(n,?),(m,2)\}$
- $RD_{in}(3) = RD_{out}(2) \cup RD_{out}(5)$
  $= \{(n,?),(n,5),(m,2),(m,4)\}$
  $RD_{out}(3) = \{(n,?),(n,5),(m,2),(m,4)\}$
- $RD_{in}(4) = \{(n,?),(n,5),(m,2),(m,4)\}$
  $RD_{out}(4) = \{(n,?),(n,5),(m,4)\}$
- $RD_{in}(5) = \{(n,?),(n,5),(m,4)\}$
  $RD_{out}(5) = \{(n,5),(m,4)\}$
- $RD_{in}(6) = \{(n,?),(n,5),(m,2),(m,4)\}$
  $RD_{out}(6) = \{(n,?),(n,5),(m,2),(m,4)\}$

Algorithm

- Input: Control Graph Diagram
- Output : RD

- Steps:
  - step 1 (inizialization):
    - $RD_{in}(p)$ is the emptyset for each p
    - $RD_{in}(1) = \iota = \{(x,?) \mid x \text{ is a program variable}\}$

- Step 2 (iteration)
  - Flag =TRUE;
    while Flag
    Flag = FALSE;
    for each program point p
    new = U{f(RD,q) | (q,p) is an edge of the graph}
    if $RD_{in}(p)$!= new
    Flag = TRUE;
    $RD_{in}(p)$ = new;

where $f(RD,q) = gen_{RD}(q) \cup (RD_{in}(q) \setminus kill_{RD}(q))$

Example

```
[ input n; ]1
[ m:= 1; ]2
  [ while n>1 do ]3
     [ m:= m * n; ]4
     [ n:= n - 1; ]5
[ output m; ]6
```

- $RD_{in}(1) = \{(n,?), (m,?)\}$
- $RD_{in}(2) = \{(n,?), (m,?)\}$
- $RD_{in}(3) = \{(n,?), (n,5), (m,2), (m,4)\}$
- $RD_{in}(4) = \{(n,?), (n,5), (m,4)\}$
- $RD_{in}(5) = \{(n,5), (m,4)\}$
- $RD_{in}(6) = \{(n,?), (n,5), (m,2), (m,4)\}$

Using Reaching Definition analysis for Global Constant Folding

Constant Folding

- By using the Reaching Definitions Analysis, we can now formally define the rules for global constant folding optimizations.
- If P is a program, we denote by RD the minimal solution of the Reaching Definition Analysis for P.
- A statement S in P can be tranformed in a more optimized statement, by applying one of the rules below, and we'll use the notation:

$$RD \vdash S \rhd S'$$

Rule 1

1. $RD \vdash [x := a]^{\nu} \triangleright [x := a[y \rightarrow n]]^{\nu}$

   if $y \in FV(a) \quad \wedge \quad (y,?) \notin RD_{entry}(\nu) \quad \wedge$
   for every $(z,\mu) \in RD_{entry}(\nu)$: (z=y ⇨ $[. . .]^{\mu}$ è $[y:=n]^{\mu}$)

   The rule says that a variable can be substituted by a contant value if the Reaching Definition Analysis ensures that this is the only value that the variable can hold.

   $a[y \rightarrow n]$ means that in the expression a, variable y is substituted by value n

   FV(a) denotes the set of free variables in the expression a.

Rule 2

2. RD |– $[x := a]^{\nu} \triangleright [x := n]^{\nu}$

   if $FV(a)=\emptyset \wedge a \notin Num \wedge$ the value of a is n

- The rule says that an expression can be evaluated at compile time if it contains no free variables.

Composition rules

3. $RD \vdash S_1 \triangleright S'_1 \Rightarrow$

$$RD \vdash S_1 ; S_2 \triangleright S'_1 ; S_2$$

4. $RD \vdash S_2 \triangleright S'_2 \Rightarrow$

$$RD \vdash S_1 ; S_2 \triangleright S_1 ; S'_2$$

- These rules say that the transformation of a sub-statement (here a sequential statement) can be extended to the whole statement.

Composition rules

5. $RD \vdash S_1 \triangleright S'_1 \Rightarrow$

   $RD \vdash \text{if } [b]^\nu \text{ then } S_1 \text{ else } S_2 \triangleright$

   $\text{if } [b]^\nu \text{ then } S'_1 \text{ else } S_2$

6. $RD \vdash S_2 \triangleright S'_2 \Rightarrow$

   $RD \vdash \text{if } [b]^\nu \text{ then } S_1 \text{ else } S_2 \triangleright$

   $\text{if } [b]^\nu \text{ then } S_1 \text{ else } S'_2$

7. $RD \vdash S \triangleright S' \Rightarrow$

   $RD \vdash \text{while } [b]^\nu \text{ do } S \triangleright$

   $\text{while } [b]^\nu \text{ do } S'$

Example

- Consider the program:
  $[x{:=}10]^1;\ [y{:=}x+10]^2;\ [z{:=}y+10]^3;$

- The minimal solution of the Reaching Definition Analysis is:

- $RD_{\mathtt{in}}(1) = \{(x,?),(y,?),(z,?)\}$
  $RD_{\mathtt{in}}(2) = \{(x,1),(y,?),(z,?)\}$
  $RD_{\mathtt{in}}(3) = \{(x,1),(y,2),(z,?)\}$

- Using RD, we may start applying the rules above:

- RD $\vdash$ $[x{:=}10]^1$; $[y{:=}x{+}10]^2$; $[z{:=}y{+}10]^3$
  $\triangleright [x{:=}10]^1$; $[y{:=}10{+}10]^2$; $[z{:=}y{+}10]^3$

- Here we apply Rule 1, with a=(x+10)
  $RD_{in}(2) = \{(x,1),(y,?),(z,?)\}$

$RD \vdash [y := a]^2 \triangleright [y := a[x \rightarrow 10]]^2$

if $x \in FV(a) \wedge (x,?) \notin RD_{in}(2) \wedge$
for every $(z,\mu) \in RD_{in}2)$: ($z{=}x \Rightarrow [. . .]^\mu$ is $[x{:=}10]^\mu$)

- RD $\vdash$ $[x:=10]^1; [y:=x+10]^2; [z:=y+10]^3$
  $\triangleright [x:=10]^1; [y:=10+10]^2; [z:=y+10]^3$
  $\triangleright [x:=10]^1; [y:=20]^2; [z:=y+10]^3$

- Here we apply Rule 2, whith expression a=(10+10)

  RD $\vdash$ $[y := a]^2 \;\triangleright\; [y := n]^2$

  if $FV(a)=\emptyset \wedge a \notin Num \wedge$ the value of expression a is n

- $RD \vdash [x:=10]^1; [y:=x+10]^2; [z:=y+10]^3$
  $\triangleright [x:=10]^1; [y:=10+10]^2; [z:=y+10]^3$
  $\triangleright [x:=10]^1; [y:=20]^2; [z:=y+10]^3$
  $\triangleright [x:=10]^1; [y:=20]^2; [z:=20+10]^3$

- Here we apply again Rule 1, with $a=(y+10)$

$RD \vdash [z := a]^3 \triangleright [z := a[y \to 20]]^3$

if $y \in FV(a) \wedge (y,?) \notin RD_{in}(3) \wedge$
for every $(w,\mu) \in RD_{in}(3)$: ($w=y \Rightarrow [. . .]^\mu$ is $[y:=20]^\mu$)

- RD |– $[x:=10]^1; [y:=x+10]^2; [z:=y+10]^3$
  $\triangleright [x:=10]^1; [y:=10+10]^2; [z:=y+10]^3$
  $\triangleright [x:=10]^1; [y:=20]^2; [z:=y+10]^3$
  $\triangleright [x:=10]^1; [y:=20]^2; [z:=20+10]^3$
  $\triangleright [x:=10]^1; [y:=20]^2; [z:=30]^3$

- Here we apply again Rule 2 with a=(20+10)

  RD |– $[z := a]^3 \ \triangleright \ [z := n]^3$

  if $FV(a)=\emptyset \wedge a \notin Num \wedge$ the value of expression a is n

- The example above show how to get a sequence of transformations

$$RD \vdash S_1 \triangleright S_2 \triangleright S_3 \triangleright \ . \ . \ . \triangleright S_k$$

- Theoretically, once computed S_2 we shoud re-execute a reaching Definition Analysis to the new program.
- However, if RD is a solution of the Reaching Def. Analysis for S_i and $RD \vdash S_i \triangleright S_{i+1}$, then it is easy to see that RD is also a solution of the Reaching Def. Analysis for S_{i+1}.
  In fact the transformation applies to elements that do not affect at all the Reaching Def. Analysis.