

Assignment - 3

Let consider a sample dataset have one $inp (X_i^2)$ & one $out (Y_i^2)$, & number of samples 4. Develop a simple linear regression model using Stochastic Gradient Descent Optimizer.

Sample (i)	X_i^2	Y_i^2
1	0.2	3.4
2	0.4	3.8
3	0.6	4.2
4	0.8	4.6

* \rightarrow i) Do manual calculations for 2 iterations with 1st two samples.

Stochastic Gradient Descent Algorithm:

Step 1: $m=1, c=1, \eta=0.1, \text{epochs}=2, N_s=2$

Step 2: $iter=1$

Step 3: $\text{sample}=1 (i=1)$

Step 4: $E = \frac{1}{2} (y_i - m x_i - c)^2$

$$\begin{aligned}
 \frac{\partial E}{\partial m} &= -(y_i - m x_i - c) x_i = -(y_i - m x_i - c) x_i \\
 &= -(3.4 - (1)(0.2) - 1) 0.2 \\
 &= -0.44//
 \end{aligned}$$

$$\begin{aligned}
 \frac{\partial E}{\partial c} &= -(y_i - m x_i - c) = -(y_i - m x_i - c) \\
 &= -2.2.
 \end{aligned}$$

$$\begin{aligned}
 \text{Step 5: } \Delta m &= -\eta \cdot \frac{\partial E}{\partial m} = -0.1 (-0.44) \\
 &= 0.044//
 \end{aligned}$$

$$\Delta c = -\eta \cdot \frac{\partial \epsilon}{\partial c} = -(-0.1)(-2.2) = -0.22$$

Step 6: $m = m + \Delta m = 1 - 0.044 = 0.956 //$

$$c = c + \Delta c = 1 - 0.22 = 0.78 //$$

Step 7: Sample = $1+1=2$ ($i=2$)

Steps: if (sample > 2) no,
else goto step 4.

Step 4: $\frac{\partial \epsilon}{\partial m} = -(y_i - mx_i - c)x_i$

$$= -(y_2 - mx_2 - c)x_2$$

$$= -(3.8 - (0.956 \times 0.4) - 0.78)(0.4)$$

$$= -1.05504 //$$

$$\frac{\partial \epsilon}{\partial c} = -(y_i - mx_i - c)$$

$$= -(3.8 - (0.956 \times 0.4) - 0.78)$$

$$= -2.6376 //$$

Step 5: $\Delta m = -\eta \cdot \frac{\partial \epsilon}{\partial m} = -(-0.1)(-1.05504) = -0.105504 //$

$$\Delta c = -\eta \cdot \frac{\partial \epsilon}{\partial c} = -(-0.1)(-2.6376) = -0.26376 //$$

Step 6: $m = m + \Delta m, = 0.956 - 0.105504 = 0.8504 //$

$$c = c + \Delta c = 0.78 - 0.26376 = 0.51624 //$$

Step 7: Sample = $2+1=3$ ($i=3$).

Step 8: $3 > 2$
if (sample $> n_s$) yes \Rightarrow goto step 9.

Step 9: if iter > 2 epochs) no
else goto step 3.

Step 3: Sample = 1 $i = 1$

Step 4: $\frac{\partial \mathcal{L}}{\partial m} = -(y_i - m x_i - c) x_i$
 $= -(3.4 - (0.8504)(0.2) - 0.51624)(0.2)$
 $= -0.5427321 //$

$$\frac{\partial \mathcal{L}}{\partial c} = -(y_i - m x_i - c)$$
$$= -(3.4 - (0.8504)(0.2) - 0.51624)$$
$$= -2.7136 //$$

Step 5: $\Delta m = -\eta \frac{\partial \mathcal{L}}{\partial m} = -(-0.1)(-0.5427) = -0.05427321 //$

$$\Delta c = -\eta \frac{\partial \mathcal{L}}{\partial c} = -(-0.1)(-2.7136) = -0.27136 //$$

Step 6: $m = m + \Delta m = 0.8504 - 0.05427 = 0.79622 //$

$$c = c + \Delta c = \textcircled{0.96376} - 0.27136 = 0.24488 //$$

0.51624

Step 7: Sample = 1 + 1 = 2. $i = 2$

Step 8: $2 > 2$
if (sample $> n_s$) no
else goto step 4.

Step 4: $\frac{\partial \mathcal{L}}{\partial m} = -(y_i - m x_i - c) x_i$
 $= -(3.8 - (0.79622 \times 0.4) - 0.24488)(0.4)$
 $= -1.29468 //$

$$\frac{\partial \mathcal{L}}{\partial c} = -(y_i - mx_i - c) = -(3.8 - (0.79622)(0.4) - 0.24488) = -3.236 //$$

Step 5: $\Delta m = -\eta \cdot \frac{\partial \mathcal{L}}{\partial m} = -(-0.1)(-1.99) = -0.129 //$

$$\Delta c = -\eta \cdot \frac{\partial \mathcal{L}}{\partial c} = -(-0.1)(-3.236) = -0.323 //$$

Step 6: $m = m + \Delta m = 0.79622 + (-0.129) = 0.66 //$
 $c = c + \Delta c = 0.24488 - 0.323 = -0.079 //$

Step 7: $\text{Sample} = 2 + 1 = 3$

Step 8: $\overset{3 > 2}{\text{if (Sample} > N)} \text{ yes goto step 9.}$

Step 9: $\text{iter} = 2 + 1 = 3$

Step 10: $\overset{3 > 2}{\text{if (iter} > \text{epochs})} \text{ yes} \Rightarrow \text{goto step 11.}$

Step 11: $\text{print}(m, c) \quad \boxed{m = 0.66}, \boxed{c = -0.079}$