

Session 12 Additional Exercise

Problem Statement 1:

A company manufactures LED bulbs with a faulty rate of 30%. If I randomly select 6 chosen LEDs, what is the probability of having 2 faulty LEDs in my sample? Calculate the average value of this process. Also evaluate the standard deviation associated with it.

Solution:

$$p = \text{Success} = 0.7;$$

$$q = \text{Failure} = 0.3$$

1. Calculate Mean

$$\mu = E(x) = n \cdot p$$

Where:

$$n = \text{No. of Trials} = 6$$

$$p = \text{Success ratio} = 0.7$$

$$\begin{aligned}\therefore \text{Mean} &= 6 \cdot 0.7 \\ &= 4.2\end{aligned}$$

2. Standard Deviation

$$\sigma = \sqrt{npq}$$

Where:

$$n = \text{No. of selected Leds} = 6$$

$$p = \text{Success ratio} = 0.7$$

$$q = \text{Failure ratio} = 0.3$$

$$\begin{aligned}\therefore \text{Standard Deviation} &= \sqrt{6 \cdot 0.7 \cdot 0.3} \\ &= 1.12\end{aligned}$$

Problem Statement 2:

Gaurav and Barakha are both preparing for entrance exams. Gaurav attempts to solve 8 questions per day with a correction rate of 75%, while Barakha averages around 12 questions per day with a correction rate of 45%. What is the probability that each of them will solve 5 questions correctly? What happens in cases of 4 and 6 correct solutions? What do you infer from it? What are the two main governing factors affecting their ability to solve questions correctly? Give a pictorial representation of the same to validate your answer.

Solution:

Gaurav:

$$n = 8$$

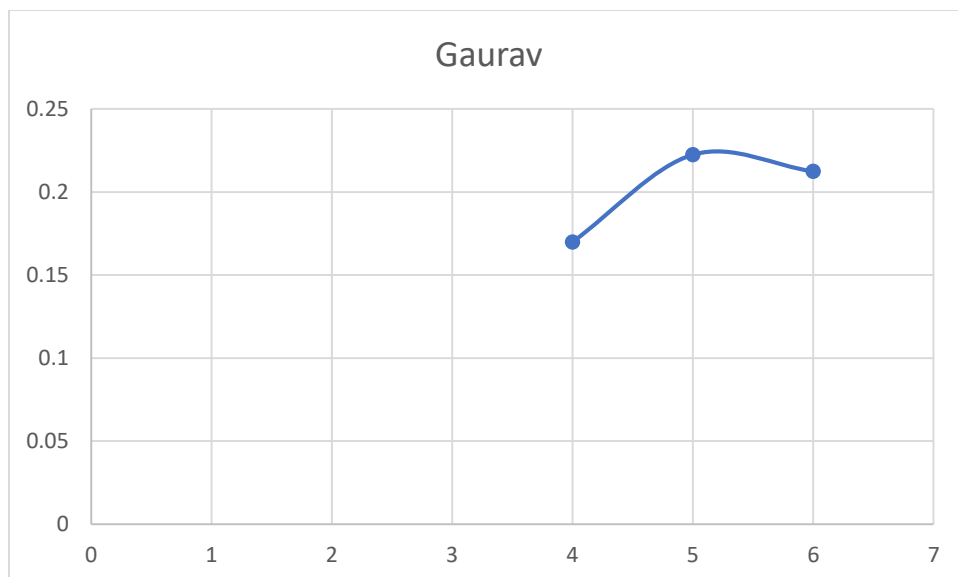
$$p = 0.75$$

$$q = 0.25$$

$$p(x = 5) = {}^8C_5 * (0.75)^5 * (0.25)^3 = 0.155728125$$

$$p(x = 4) = {}^8C_4 * (0.75)^4 * (0.25)^4 = 0.086515625$$

$$p(x = 6) = {}^8C_6 * (0.75)^6 * (0.25)^2 = 0.31146$$



Barakha:

$$n = 12$$

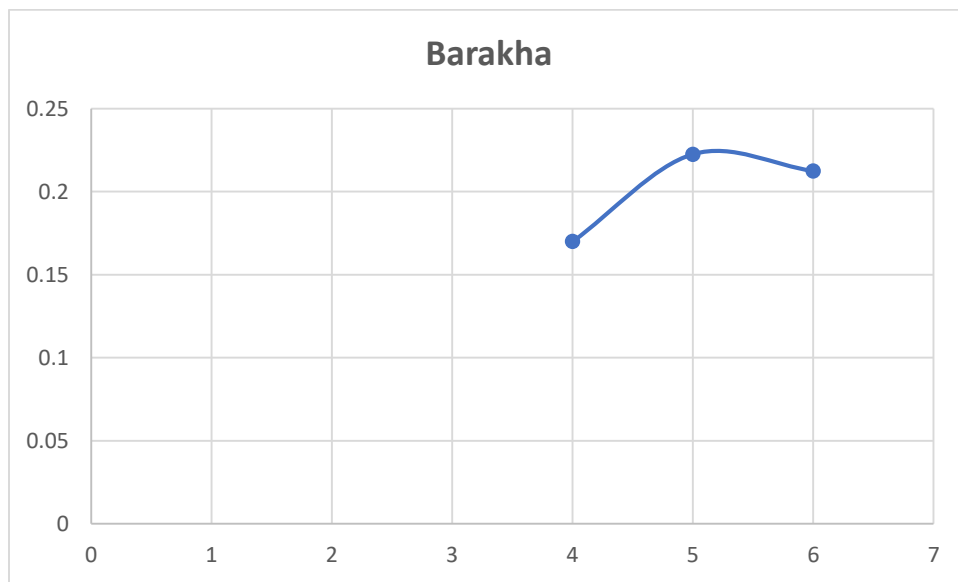
$$p = 0.45$$

$$q = 0.55$$

$$p(x=5) = {}^{12}C_5 * (0.45)^5 * (0.55)^7 = 0.22249$$

$$p(x=4) = {}^{12}C_4 * (0.45)^4 * (0.55)^8 = 0.169962$$

$$p(x=6) = {}^{12}C_6 * (0.45)^6 * (0.55)^6 = 0.212378$$



Problem Statement 3:

Customers arrive at a rate of 72 per hour to my shop. What is the probability of k customers arriving in 4 minutes?

a) 5 customers, b) not more than 3 customers, c) more than 3 customers.

Give a pictorial representation of the same to validate your answer.

Solution:

60 minutes - 72 customers are arriving

4 minutes - ?

$$\frac{4}{60} * 72 = 4.8$$

***2548.09**

a) 5 Customer

$$\frac{e^{(-4.8)} * (4.8)^5}{5!} = \frac{0.008229 * 2548.09}{120} = \mathbf{0.1747}$$

b) Not more than 3

X=0

$$\frac{e^{(-4.8)} * (4.8)^0}{0} = \mathbf{0.008229}$$

X=1

$$\frac{e^{(-4.8)} * (4.8)^1}{1!} = \mathbf{0.008229 * 4.8 = 0.0394992}$$

X = 2

$$\frac{e^{(-4.8)} * (4.8)^2}{2!} = \frac{0.008229 * 23.04}{2} = \mathbf{0.09479}$$

X = 3

$$\frac{e^{(-4.8)} * (4.8)^3}{3!} = \frac{0.008229 * 110.592}{6} = \mathbf{0.151676}$$

$$\mathbf{P(x_i \leq 3) = 0.008229 + 0.0394992 + 0.09479 + 0.151676}$$
$$\mathbf{= 0.2941942}$$

c) More than 3 customers

X = 4

$$\frac{e^{(-4.8)} * (4.8)^4}{4!} = \frac{0.008229 * 530.8}{24} = \mathbf{0.1820}$$

X=5

$$\frac{e^{(-4.8)} * (4.8)^5}{5!} = \frac{0.008229 * 2548.09}{120} = \mathbf{0.1747}$$

$X = 6$

$$\frac{e^{(-4.8)} * (4.8)^6}{6!} = \frac{0.008229 * 12230.59}{720} = 0.1397$$

$$P(x_i > 3) = 0.1820 + 0.1747 + 0.1397 = 0.4964$$



Problem Statement 4:

I work as a data analyst in Aeon Learning Pvt. Ltd. After analyzing data, I make reports, where I have the efficiency of entering 77 words per minute with 6 errors per hour. What is the probability that I will commit 2 errors in a 455-word financial report?

What happens when the no. of words increases/decreases (in case of 1000 words, 255 words)? How is the λ affected? How does it influence the PMF? Give a pictorial representation of the same to validate your answer.

Solution:

6 errors per hour

Per hour words 77*60 =4620

$$1 \text{ error} = \frac{6}{4620} = \frac{1}{770}$$

$$M = \frac{455}{770} = 0.591$$

$$(p_{xi}=2) = \frac{e^{(-0.591)} * (0.591)^2}{2!} = \frac{0.55377 * 0.349281}{2} = 0.09671$$

$$M = \frac{1000}{770} = 1.2987$$

$$(p_{xi}=2) = \frac{e^{(-1.2987)} * (1.2987)^2}{2!} = \frac{0.2731 * 1.6866}{2} = 0.2303$$

$$M = \frac{255}{770} = 0.3311$$

$$(p_{xi}=2) = \frac{e^{(-0.3311)} * (0.3311)^2}{2!} = \frac{0.7181 * 0.1962}{2} = 0.0393621$$

