Session 11 Additional Exercise

Problem Statement 1:

The marks awarded for an assignment set for a Year 8 class of 20 students were as follows:

6 7 5 7 7 8 7 6 9 7 4 10 6 8 8 9 5 6 4 8

Calculate the mean, median, mode and standard deviation

Solution:

Mean $\bar{X} = \dfrac{\sum_{i=1}^{n} X_i}{n}$

$$\bar{X} = \frac{6+7+5+7+7+8+7+6+9+7+4+10+6+8+8+9+5+6+4+8}{20} = \frac{137}{20}$$

$$\bar{X} = 6.85$$

Median: rearrange the data in numerical order:

4 ,4,5,5,6,6,6,6,7,7,7,7,7,8,8,8,8,9,9,10

The Median is half way between the two readings i.e. (7+7) / 2 = 7

Mode:

The mode is the most common occurring value is 7

standard deviation:

	Data (X)	Mean($\overline{x}$)	x-$\overline{x}$	$(x - \overline{x})^2$
1	**6**	6.85	0.85	0.7225
2	7	6.85	0.15	0.0225
3	5	6.85	-1.85	3.4225
4	7	6.85	0.15	0.0225
5	7	6.85	0.15	0.0225
6	8	6.85	1.15	1.3225
7	7	6.85	0.15	0.0225
8	6	6.85	0.85	0.7225
9	9	6.85	2.15	4.6225
10	7	6.85	0.15	0.0225
11	**4**	6.85	-2.85	8.1225
12	10	6.85	3.15	9.9225
13	6	6.85	0.85	0.7225
14	8	6.85	1.15	1.3225
15	8	6.85	1.15	1.3225
16	9	6.85	2.15	4.6225
17	5	6.85	-1.85	3.4225
18	6	6.85	0.85	0.7225
`19	4	6.85	-2.85	8.1225
20	8	6.85	1.15	1.3225
Total:	137			50.55
	(mean) 6.85			

$$S = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \overline{x})^2}{n-1}}$$

$$S = \sqrt{\frac{50.5}{19}} = \sqrt{2.65789} = 1.63$$

Problem Statement 2:

The number of calls from motorists per day for roadside service was recorded for a particular month:

28, 122, 217, 130, 120, 86, 80, 90, 140, 120, 70, 40, 145, 113, 90, 68, 174, 194, 170, 100, 75, 104, 97, 75, 123, 100, 75, 104, 97, 75, 123, 100, 89, 120, 109

Calculate the mean, median, mode and standard deviation for the problem statements 1&

Solution:

Mean $\bar{X} = \dfrac{\sum_{i=1}^{n} X_i}{n}$

$\bar{X} = \dfrac{3763}{35}$ **= 107.5142**

Mode is Odd take the middle value 100

Mode:

The mode is the most common occurring value is 75

standard deviation:

	Data (X)	Mean($\bar{x}$)	x-$\bar{x}$	$(x-\bar{x})^2$
1	28	107.5142	-79.5142	6322.508002
2	122	107.5142	14.4858	209.8384016
3	217	107.5142	109.4858	11987.1404
4	130	107.5142	22.4858	505.6112016
5	120	107.5142	12.4858	155.8952016
6	86	107.5142	-21.5142	462.8608016
7	80	107.5142	-27.5142	757.0312016
8	90	107.5142	-17.5142	306.7472016
9	140	107.5142	32.4858	1055.327202
10	120	107.5142	12.4858	155.8952016
11	70	107.5142	-37.5142	1407.315202
12	40	107.5142	-67.5142	4558.167202
13	145	107.5142	37.4858	1405.185202
14	113	107.5142	5.4858	30.09400164
15	90	107.5142	-17.5142	306.7472016
16	68	107.5142	-39.5142	1561.372002
17	174	107.5142	66.4858	4420.361602
18	194	107.5142	86.4858	7479.793602
`19	170	107.5142	62.4858	3904.475202
20	100	107.5142	-7.5142	56.46320164
21	75	107.5142	-32.5142	1057.173202

22	104	107.5142	-3.5142	12.34960164
23	97	107.5142	-10.5142	110.5484016
24	75	107.5142	-32.5142	1057.173202
25	123	107.5142	15.4858	239.8100016
26	100	107.5142	-7.5142	56.46320164
27	75	107.5142	-32.5142	1057.173202
28	104	107.5142	-3.5142	12.34960164
29	97	107.5142	-10.5142	110.5484016
30	75	107.5142	-32.5142	1057.173202
31	123	107.5142	15.4858	239.8100016
32	100	107.5142	-7.5142	56.46320164
33	89	107.5142	-18.5142	342.7756016
34	120	107.5142	12.4858	155.8952016
35	109	107.5142	1.4858	2.20760164
Total:	3763			52616.74286

$$S = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}}$$

$$S = \sqrt{\frac{52616.74286}{34}} = \sqrt{1547.55126} = 39.3389$$

Problem Statement 3:

The number of times I go to the gym in weekdays, are given below along with its associated probability: x = 0, 1, 2, 3, 4, 5

f(x) = 0.09, 0.15, 0.40, 0.25, 0.10, 0.01

Calculate the mean no. of workouts in a week. Also evaluate the variance involved in it.

Solution:

Mean:

X	0	1	2	3	4	5
f(x)	0.09	0.15	0.40	0.25	0.10	0.01

μ = 0*0.09 + 1*0.15 + 2*0.40 + 3*0.25 + 4*0.10+5*0.01

μ = 0 +0.15+0.80+0.75+0.40+0.05 = 2.15

Variance:

$$\boldsymbol{\sigma^2} = (\mathbf{0}-\mathbf{2.15})^2(\mathbf{0.09}) + (\mathbf{1}-\mathbf{2.15})^2(\mathbf{0.15}) + (\mathbf{2}-\mathbf{2.15})^2(\mathbf{0.40})$$

$$+ (\mathbf{3}-\mathbf{2.15})^2(\mathbf{0.25}) + (\mathbf{4}-\mathbf{2.15})^2(\mathbf{0.10}) + (\mathbf{5}-\mathbf{2.15})^2(\mathbf{0.01})$$

$$\boldsymbol{\sigma^2} = 0.416025 + 0.416025 + 0.009 + 0.180625 + 0.34225 + 0.081225 = 1.2275$$

$$\sigma = \sqrt{1.2275} = 1.1079$$

Problem Statement 4:

Let the continuous random variable D denote the diameter of the hole drilled in an aluminum sheet. The target diameter to be achieved is 12.5mm. Random disturbances in the process often result in inaccuracy. Historical data shows that the distribution of D can be modelled by the

PDF f(d) = $20e^{-20(d-12.5)}$, d ≥12.5. If a part with diameter > 12.6 mm needs to be scrapped, what is the proportion of those parts? What is the CDF when the diameter is of 11 mm? What is your conclusion regarding the proportion of scraps?

Solution:

P(D >12.60) = $\int_{12.6}^{\infty} f(d)dd = \int_{12.6}^{\infty} 20e^{-20(d-12.5)}\, dd = -e^{-20(d-12.5)} \int_{12.6}^{\infty}$

= 0.135

What proportion of parts is between 12.5 and 12.6 millimeters?

P(12.5 < D < 12.6) = $\int_{12.5}^{12.6} f(d)dd = -e^{-20(d-12.5)} \int_{12.5}^{12.6}$ = 0.865

the total area under f(d) equals 1, we can also calculate

P (12.5 < D < 12.6) = 1 – P (D > 12.6) = 1-0.135 = 0.865