Linear growth factor of density perturbations

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1. Perturbation equations:

   Time evolution of density contrast:

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G\rho\delta = 0. \tag{1}$$

   Or,

$$\ddot{\delta} + 2H\dot{\delta} - \frac{3}{2}H\Omega_m\delta = 0. \tag{2}$$

   We want to change the time variable to scale factor and then the redshift. The derivate operator:

$$\frac{d}{dt} = Ha\frac{d}{da}. \tag{3}$$

$$\frac{d^2}{dt^2} = (Ha)^2\frac{d^2}{da^2} + Ha(a\frac{dH}{da} + H)\frac{d}{da}. \tag{4}$$

$$\frac{da}{dz} = \frac{-1}{(1+z)^2}. \tag{5}$$

$$\frac{d}{dt} = -H(1+z)\frac{d}{dz}. \tag{6}$$

$$\frac{d^2}{dt^2} = H^2(1+z)^2(\frac{1}{1+z}\frac{d}{dz} + \frac{dH/dz}{H}\frac{d}{dz} + \frac{d^2}{dz^2}) \tag{7}$$

   Finally,Using these equations, we obtain:

$$\frac{d^2\delta}{da^2} = -(\frac{dH/da}{H} + \frac{3}{a})\frac{d\delta}{da} + \frac{3}{2}\frac{\Omega_{m0}}{a^2}\delta. \tag{8}$$

$$E(z) = \frac{H(z)}{H_0} \tag{9}$$

   And,

$$\frac{d^2\delta}{dz^2} = -(\frac{dE/dz}{E(z)} - \frac{1}{1+z})\frac{d\delta}{dz} + \frac{3}{2}\Omega_{m0}\frac{(1+z)}{E^2(z)}\delta \tag{10}$$

   We introduce Growth Rate as f:

$$f = \frac{dln\delta}{dlna} = \frac{1}{\delta}\frac{d\delta}{dlna} \tag{11}$$

Eventually we obtain:

$$\frac{df}{da} = \frac{-f^2}{a} - (\frac{dH/da}{H} + \frac{2}{a})f + \frac{3}{2}\frac{\Omega_{m0}}{a^4 E^2(a)} \tag{12}$$

And,

$$\frac{df}{dz} = \frac{f^2}{(1+z)} - (\frac{dE/dz}{E} - \frac{2}{(1+z)})f - \frac{3}{2}\frac{\Omega_{m0}(1+z)^2}{E^2(z)} \tag{13}$$

2. for calculating $f\sigma_8$:

$$\sigma_8(z) = \sigma_8(0)\frac{\delta(z)}{\delta(0)} \tag{14}$$

$\delta(z=0) = 1$, fixed as the normalization value.

And, $\sigma_8(0) = 0.8$

$$f\sigma_8 = \sigma_8(0)f(z)\delta(z) \tag{15}$$