EC 502 HW3 R Notebook

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Data Import, Clean, and Set-up

Import libraries

```
suppressPackageStartupMessages({
  library(tidyverse)
  library(ggplot2)
})
```

Load the dataset:

```
country_data = read.csv("cross_country_data.csv")
head(country_data, 10)
```

Country	Year	SavingsRate	RealGDP	LaborForce	HumanCapital	PhysicalCapital
Albania	1980	0.3934749	9981.908	0.9249071	2.395582	42834.52
Albania	1981	0.4208891	10555.427	0.9514943	2.445612	45231.09
Albania	1982	0.4482948	10866.670	0.9771618	2.496687	47943.00
Albania	1983	0.4195175	10986.740	1.0006725	2.548830	50831.43
Albania	1984	0.3768510	10849.229	1.0246358	2.588237	53751.48
Albania	1985	0.3867719	11042.415	1.0539739	2.624506	56010.12
Albania	1986	0.3833166	11660.205	1.0901078	2.648264	58482.48
Albania	1987	0.3503911	11568.308	1.1281416	2.672236	61068.25
Albania	1988	0.3539001	11403.967	1.1635048	2.696425	63407.10
Albania	1989	0.3725055	12526.185	1.1956922	2.720833	66093.03

Calculate (real) GDP per worker in 1981 and 2010:

```
country_data <- country_data %>%
  transform(output_per_worker = RealGDP / LaborForce)

country_data_gdp_per_worker_1981_2010 <- country_data %>%
  filter(Year %in% c(1981, 2010)) %>%
  select(Country, Year, output_per_worker) %>%
  pivot_wider(names_from = Year,
              values_from = output_per_worker,
              names_prefix = "output_per_worker_")

head(country_data_gdp_per_worker_1981_2010, 5)
```

Country	output_per_worker_1981	output_per_worker_2010
Albania	11093.53	24308.51
Argentina	24358.72	37174.97
Australia	52400.84	72687.02
Austria	49189.03	71225.79
Bahrain	74160.12	43394.25

Average savings rate:

```
summary_country_data <- country_data %>%
  group_by(Country) %>%
  summarise_at(vars(SavingsRate), list(savings_mean = mean))

head(summary_country_data, 10)
```

Country	savings_mean
Albania	0.2787880
Argentina	0.1849724
Australia	0.2874728
Austria	0.2803114
Bahrain	0.2586320
Bangladesh	0.1658248
Barbados	0.0913618
Belgium	0.2698151
Belize	0.1453019
Benin	0.1075688

Average growth rate of the labor force and average growth rate of GDP per worker (from 1981 to 2010):

t = 1981

T = 2010 - 1981 = 29

formula: 1/29 * ln(val at 2010 / val at 1981)

(log in R has base e)

```
# GET SAVINGS MEAN
summary_country_data <- country_data %>%
  group_by(Country) %>%
  summarise_at(vars(SavingsRate), list(savings_mean = mean))

# GET AVG GROWTH RATE OF LABOR (n) and AVG GROWTH RATE OF GDP PER WORKER (g)
summary_country_data <- country_data %>%
  filter(Year %in% c(1981, 2010)) %>%
  select(Country, Year, LaborForce, output_per_worker) %>%
  pivot_wider(names_from = Year,
              values_from = c(output_per_worker, LaborForce),
              names_sep = "_",
              names_prefix = "year_") %>%
  transform(avg_growth_labor =
              (1/29) * log(LaborForce_year_2010 / LaborForce_year_1981)) %>%
  transform(avg_growth_gdp_per_worker =
              (1/29) * log(output_per_worker_year_2010 / output_per_worker_year_1981)) %>%
  left_join(summary_country_data, by = "Country")

head(summary_country_data %>%
       select(Country, avg_growth_labor, avg_growth_gdp_per_worker, savings_mean),
     5)
```

Country	avg_growth_labor	avg_growth_gdp_per_worker	savings_mean
Albania	-0.0012733	0.0270505	0.2787880
Argentina	0.0123858	0.0145774	0.1849724
Australia	0.0204163	0.0112841	0.2874728
Austria	0.0093191	0.0127650	0.2803114
Bahrain	0.0556962	-0.0184793	0.2586320

Question (1) Descriptive Statistics

a) number of countries

```
country_data %>%  summarise(num_countries = n_distinct(Country))
```

num_countries
118

b) mean (across countries) of output per worker 1981, output per worker 2010, savings rate, growth of labor force, growth rate of GDP per worker

```
summary_country_data %>%
  summarise_at(vars(output_per_worker_year_1981,
                    output_per_worker_year_2010,
                    savings_mean,
                    avg_growth_labor,
                    avg_growth_gdp_per_worker),
               list(mean = mean)) %>%
  mutate(across(everything(), ~ round(., digits = 4))) %>%
  pivot_longer(cols=everything(), names_to = "variable", values_to = "value")
```

variable	value
output_per_worker_year_1981_mean	27042.8747
output_per_worker_year_2010_mean	34551.1683
savings_mean_mean	0.2003
avg_growth_labor_mean	0.0225
avg_growth_gdp_per_worker_mean	0.0113

c) standard deviation (of values above)

```
summary_country_data %>%
  summarise_at(vars(output_per_worker_year_1981,
                    output_per_worker_year_2010,
                    savings_mean,
                    avg_growth_labor,
                    avg_growth_gdp_per_worker), list(sd = sd)) %>%
  mutate(across(everything(), ~ round(., digits = 4))) %>%
  pivot_longer(cols=everything(), names_to = "variable", values_to = "value")
```

variable	value
output_per_worker_year_1981_sd	37952.7301
output_per_worker_year_2010_sd	33102.0465
savings_mean_sd	0.0688
avg_growth_labor_sd	0.0131
avg_growth_gdp_per_worker_sd	0.0181

d) min and max of vals above

Min:

```
summary_country_data %>%
  summarise_at(vars(output_per_worker_year_1981,
                    output_per_worker_year_2010,
                    savings_mean,
                    avg_growth_labor,
                    avg_growth_gdp_per_worker),
               list(min = min)) %>%
  mutate(across(everything(), ~ round(., digits = 4))) %>%
  pivot_longer(cols=everything(), names_to = "variable", values_to = "value")
```

variable	value
output_per_worker_year_1981_min	886.8293
output_per_worker_year_2010_min	967.6390
savings_mean_min	0.0479
avg_growth_labor_min	-0.0141
avg_growth_gdp_per_worker_min	-0.0347

Max:

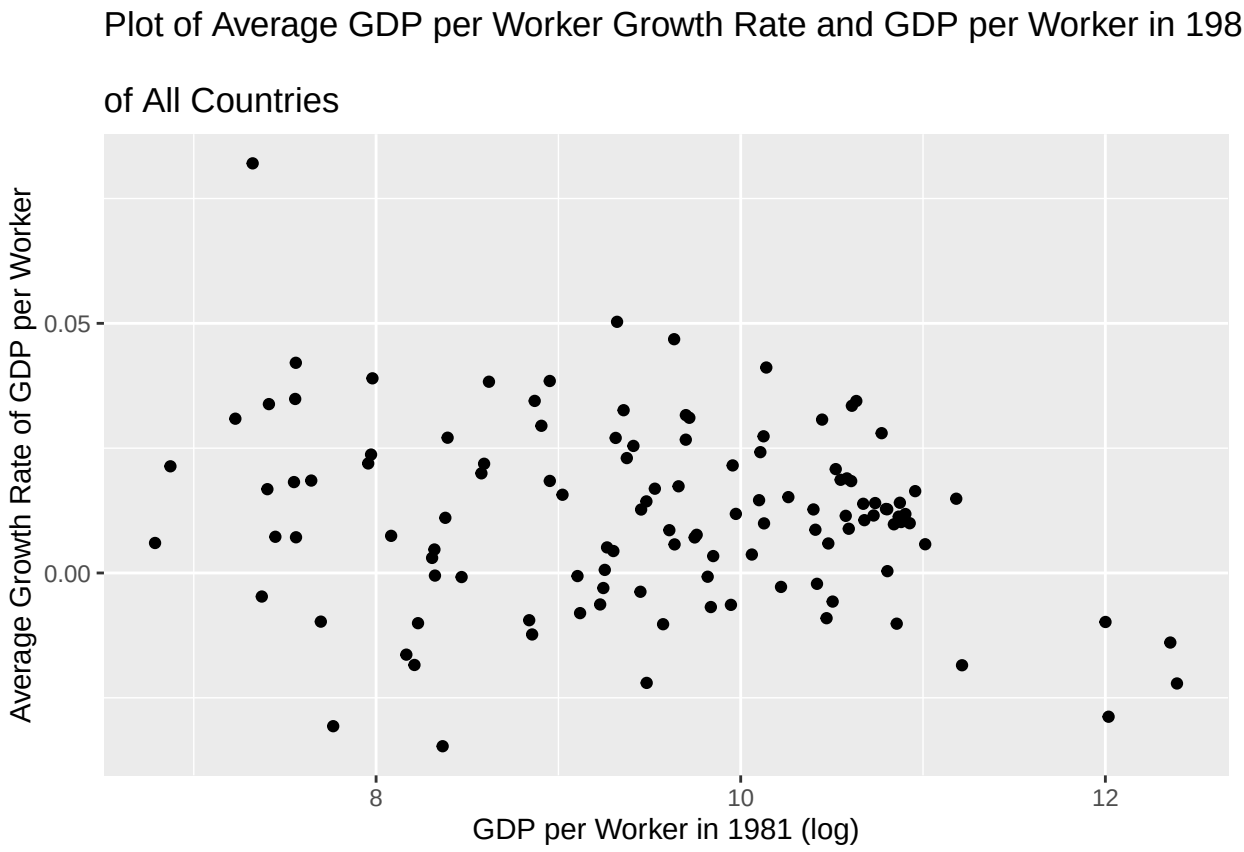
```
summary_country_data %>%
  summarise_at(vars(output_per_worker_year_1981,
                    output_per_worker_year_2010,
                    savings_mean,
                    avg_growth_labor,
                    avg_growth_gdp_per_worker),
               list(max = max)) %>%
  mutate(across(everything(), ~ round(., digits = 4))) %>%
  pivot_longer(cols=everything(), names_to = "variable", values_to = "value")
```

variable	value
output_per_worker_year_1981_max	241085.8648
output_per_worker_year_2010_max	155184.3351
savings_mean_max	0.4455
avg_growth_labor_max	0.0830
avg_growth_gdp_per_worker_max	0.0820

Question (2) Unconditional Convergence

Plot:

```
summary_country_data %>%
  ggplot(aes(x=log(output_per_worker_year_1981), y=avg_growth_gdp_per_worker)) +
  geom_point() +
  labs(title="Plot of Average GDP per Worker Growth Rate and GDP per Worker in 1981
       \nof All Countries ",
        x ="GDP per Worker in 1981 (log)", y = "Average Growth Rate of GDP per Worker")
```



OLS Regression:

```
ols_data <- data.frame(
  # y value:
  avg_growth_gdp_per_worker = summary_country_data[['avg_growth_gdp_per_worker']],
  # x value:
  log_output_per_worker_year_1981 = log(summary_country_data[['output_per_worker_year_1981']])
)
head(ols_data, 3)
```

avg_growth_gdp_per_worker	log_output_per_worker_year_1981
0.0270505	9.314117
0.0145774	10.100645
0.0112841	10.866678

```r
ols_model <- lm(avg_growth_gdp_per_worker~log_output_per_worker_year_1981, data=ols_data)
summary(ols_model)
```

```
##
## Call:
## lm(formula = avg_growth_gdp_per_worker ~ log_output_per_worker_year_1981,
##     data = ols_data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.04915 -0.01204  0.00226  0.01044  0.06475
##
## Coefficients:
##                                 Estimate Std. Error t value Pr(>|t|)
## (Intercept)                     0.037238   0.012788   2.912  0.00431 **
## log_output_per_worker_year_1981 -0.002724   0.001332  -2.045  0.04312 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.0179 on 116 degrees of freedom
## Multiple R-squared:  0.0348, Adjusted R-squared:  0.02648
## F-statistic: 4.182 on 1 and 116 DF,  p-value: 0.04312
```

From the above OLS model, we get the formula where g = 0.037238 + -0.002724 * log(Y/L) + 0.0179

The estimate for b1(hat) is -0.002724 and its standard error is 0.001332

This implies that the living standards in nations that are initially poorer (lower GDP per worker in 1981) fall behind compared to wealthier nations, indicated by the negative b1 coefficient. However, the low R-squared value (3.48%) suggests that while GDP per worker in 1981 does have some predictive power, there are likely other important factors not included in the model.

Question (3) MRW Revisited

```
ols_data_mrw <- data.frame(
  # y value:
  avg_growth_gdp_per_worker = summary_country_data[['avg_growth_gdp_per_worker']],
  # x1 value:
  log_output_per_worker_year_1981 = log(summary_country_data[['output_per_worker_year_1981']]),
  # x2 value:
  log_avg_growth_labor_plus_0.05 = log(summary_country_data[['avg_growth_labor']] + 0.05 )
)

head(ols_data_mrw, 5)
```

avg_growth_gdp_per_worker	log_output_per_worker_year_1981	log_avg_growth_labor_plus_0.05
0.0270505	9.314117	-3.021529
0.0145774	10.100645	-2.774417
0.0112841	10.866678	-2.653331
0.0127650	10.803426	-2.824823
-0.0184793	11.213982	-2.247187

```
ols_model_mrw <- lm(avg_growth_gdp_per_worker~log_output_per_worker_year_1981+log_avg_growth_labor_plus_
summary(ols_model_mrw)
```

```
##
## Call:
## lm(formula = avg_growth_gdp_per_worker ~ log_output_per_worker_year_1981 +
##     log_avg_growth_labor_plus_0.05, data = ols_data_mrw)
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -0.047635 -0.009087 -0.001348  0.008121  0.059462
##
## Coefficients:
##                                  Estimate Std. Error t value Pr(>|t|)
## (Intercept)                     -0.062898   0.022800  -2.759  0.00675 **
## log_output_per_worker_year_1981 -0.003519   0.001218  -2.889  0.00462 **
## log_avg_growth_labor_plus_0.05  -0.040776   0.007993  -5.101 1.34e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.01623 on 115 degrees of freedom
## Multiple R-squared:  0.2129, Adjusted R-squared:  0.1992
## F-statistic: 15.55 on 2 and 115 DF,  p-value: 1.051e-06
```

The estimate for b1(hat) is -0.003519 and its standard error is 0.001218, which implies a higher impact of savings on living standards than before, as this value is more negative.

The value of alpha is implied to be 0.003519, which means labor share is around (1 - 0.003519) 0.996481, or the majority of output. This is not consistent with the standard assumption of 2/3.

The R-squared is 0.2129, which indicates that the model does not do a good job of indicating the variance in the model, and there is not much predictive power the coefficients have. The model only explains 21.29% of the variance.

Question (4) HJ Levels Accounting

A = (Y/L) * (K/Y)^(1-a/a) * (L/H)

where a = 1/3:

A = (Y/L) * (K/Y)^(2/9) * (L/H)

Calculate parts of the equation:

```
hj_data <- country_data %>%
  filter(Year == 2010) %>%
  transform(capital_per_output = PhysicalCapital / RealGDP) %>%  # capital per output in 2010
  transform(recip_human_cap = 1 / HumanCapital) %>%  # reciprocal of human capital per person
  select(Country, Year, capital_per_output, recip_human_cap, HumanCapital)

head(hj_data %>% select(-HumanCapital), 5)
```

Country	Year	capital_per_output	recip_human_cap
Albania	2010	4.034723	0.3328644
Argentina	2010	3.398883	0.3547817
Australia	2010	3.613600	0.2951154
Austria	2010	3.385438	0.3521654
Bahrain	2010	3.982973	0.3500738

Estimate A:

```
hj_data <- hj_data %>%
  left_join(summary_country_data %>% select(Country, output_per_worker_year_2010), by="Country") %>%
  transform(A_estimate = output_per_worker_year_2010 * capital_per_output^(2/9) * recip_human_cap) %>%
  transform(log_A_estimate = log(A_estimate))

head(hj_data %>%  select(A_estimate, log_A_estimate), 5)
```

A_estimate	log_A_estimate
11031.92	9.308548
17309.58	9.759015
28538.78	10.259019
32890.93	10.400952
20652.43	9.935588

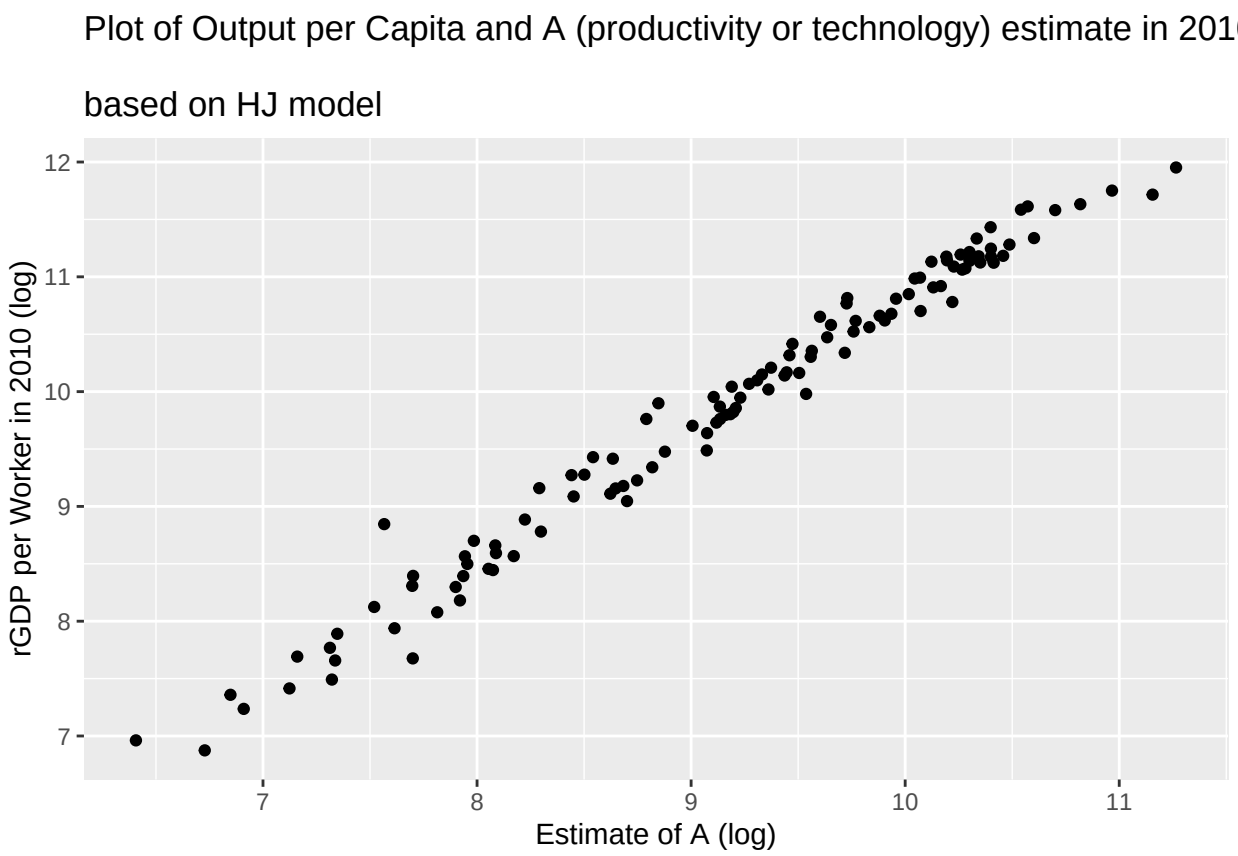
Summary statistics:

```
hj_data %>%
  summarise_at(vars(log_A_estimate), list(log_A_mean = mean, log_A_sd = sd))
```

log_A_mean	log_A_sd
9.155138	1.105463

Plots:

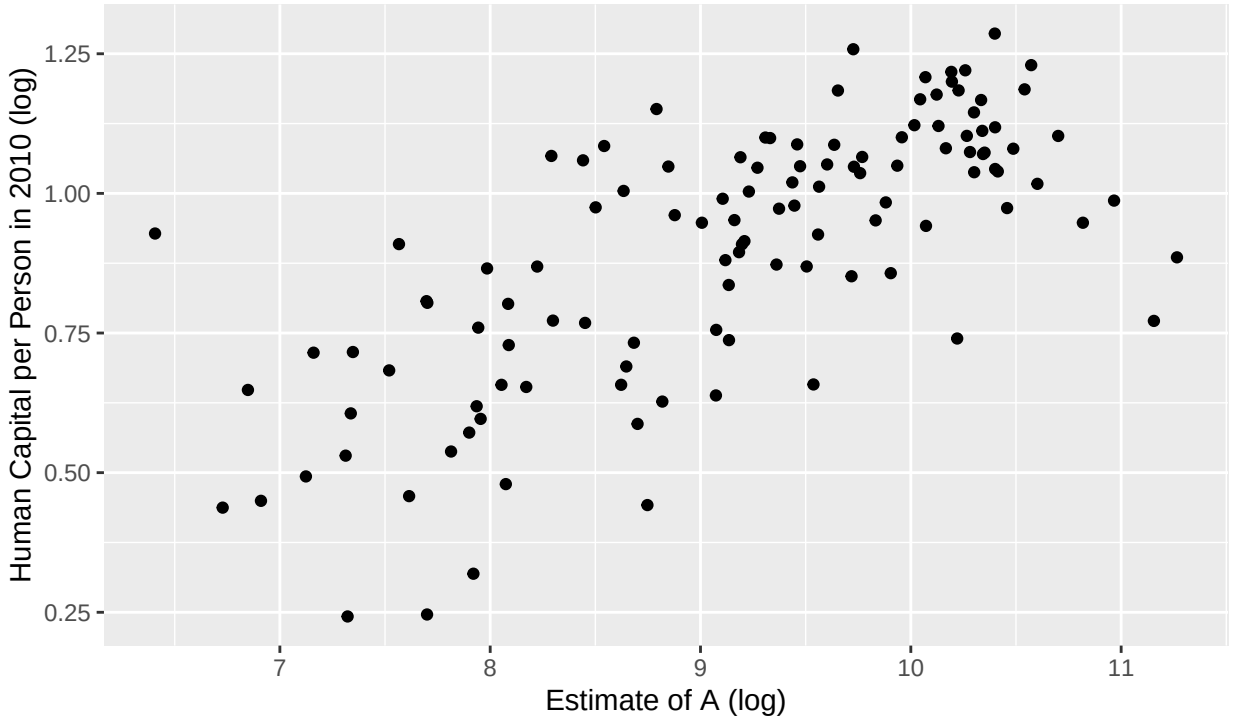
```
hj_data %>%
  ggplot(aes(x=log_A_estimate, y=log(output_per_worker_year_2010))) +
  geom_point() +
  labs(title="Plot of Output per Capita and A (productivity or technology) estimate in 2010
       \nbased on HJ model",
        x ="Estimate of A (log)", y = "rGDP per Worker in 2010 (log)")
```




```
hj_data %>%
  ggplot(aes(x=log_A_estimate, y=log(HumanCapital))) +
  geom_point() +
  labs(title="Plot of Human Capital per Person and A (productivity or technology) estimate
       \nin 2010 based on HJ model",
        x ="Estimate of A (log)", y = "Human Capital per Person in 2010 (log)")
```

Plot of Human Capital per Person and A (productivity or technology) estima

in 2010 based on HJ model




```
hj_ols_model <- lm(log(output_per_worker_year_2010)~log_A_estimate, data=hj_data)
summary(hj_ols_model)
```

```
##
## Call:
## lm(formula = log(output_per_worker_year_2010) ~ log_A_estimate,
##     data = hj_data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.53717 -0.11691 -0.00643  0.11088  0.78249
##
## Coefficients:
##                Estimate Std. Error t value Pr(>|t|)
## (Intercept)    -0.44376    0.13704  -3.238  0.00157 **
## log_A_estimate  1.12432    0.01486  75.654  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.1777 on 116 degrees of freedom
## Multiple R-squared:  0.9801, Adjusted R-squared:   0.98
## F-statistic:  5724 on 1 and 116 DF,  p-value: < 2.2e-16
```

The estimated b1(hat), or the coefficient for A, is 1.12432, and since this is a positive coefficient, the model suggests that living standards (Y/L) is higher in nations with higher productivity (A). The R-squared is 0.9801, which means the model is a good fit and explains 98.01% of the variance in the data.