ECON 21320: Problem Set 4

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1 Linear Utility Model

a. A concrete example that fits this situation could be modelling US college application acceptances and final decisions. For simplicity's' sake, let's assume that the colleges that each student is considering are the same.

   In this case, the individuals i are students applying for college, the products j are the schools that have accepted the students (all of which are the same), and the terms x_j are the characteristics about each of the j schools that are, in a way, "unifiable" across all individuals. Some examples of unifiable characteristics for schools include:

   - safety index value representing the surrounding neighborhood and campus safety in comparison to all other schools in the US
   - tuition costs, percentage of attending students who have post-grad opportunities lined up right after graduating

   The ϵ_{ij} for each individual i and school j represent characteristics about the school that matter to the specific individual in a way that affects their utility. The i.i.d structure of each ϵ_{ij} holds in that a person's relation / affinity with a certain school shouldn't be affected by someone else's relation (described along each example below). Examples of these unique characteristics include:

   - school location relative to student location - does the student want to stay close to home or go to school further away? Whether one student wants to stay closer to home or not and where they live likely doesn't affect other students in the larger spectrum, nor are they likely to be affected by other students.
   - offered specific student programs such as study abroad to India, etc. - there might be a specific experience the individual is looking for special to themselves and a particular school; this is VERY much a specific individual-school specific basis, and therefore just by this definition would abide by the i.i.d structure.

   The utility u_{ij} for each individual i and school j represents how much they want to go to the particular school. y_{ij}, therefore, is 1 if the school j for the individual i is the most desired out of all J schools based on the utilities for that student, and 0 otherwise.

Note in the above described situation we assumed all ϵ_{ij} terms are i.i.d - however, if we more closely analyze the situation and factors we aren't considering, we might see that these terms are actually not i.i.d.

b. Note that the probability of i choosing j was given to us. The likelihood function of observing the choice data for individuals conditional on the product characteristics:

$$\begin{aligned}
L(y=1|x) &= \prod_{i=1}^{N}\prod_{j=1}^{J} P(y=1|x_j)^{y_{ij}} P(y=0|x_j)^{1-y_{ij}} \\
&= \prod_{i=1}^{N}\prod_{j=1}^{J} P(y=1|x_j)^{y_{ij}} \\
&= \prod_{i=1}^{N}\prod_{j=1}^{J} Pr(i \longrightarrow j)^{y_{ij}} \\
&= \prod_{i=1}^{N}\prod_{j=1}^{J} \left(\frac{e^{x_j^{'}\beta}}{\sum_{k=1}^{J} e^{x_k^{'}\beta}}\right)^{y_{ij}} \\
ln(L(y|x)) &= ln\left(\prod_{i=1}^{N}\prod_{j=1}^{J} \left(\frac{e^{x_j^{'}\beta}}{\sum_{k=1}^{J} e^{x_k^{'}\beta}}\right)^{y_{ij}}\right. \\
&= \sum_{i=1}^{N}\sum_{j=1}^{J} y_{ij} ln\left(\frac{e^{x_j^{'}\beta}}{\sum_{k=1}^{J} e^{x_k^{'}\beta}}\right) \\
&= \sum_{i=1}^{N}\sum_{j=1}^{J} y_{ij} ln(e^{x_j^{'}\beta}) - \sum_{i=1}^{N} ln\left(\sum_{k=1}^{J} e^{x_k^{'}\beta}\right) \\
&= \sum_{i=1}^{N}\sum_{j=1}^{J} y_{ij} x_j^{'}\beta - \sum_{i=1}^{N} ln\left(\sum_{k=1}^{J} e^{x_k^{'}\beta}\right)
\end{aligned}$$

c. Deriving the score function by the gradient of the above likelihood function:

$$\frac{dlnL}{d\beta} = \sum_{i=1}^{N}\sum_{j=1}^{J} y_{ij} x_j^{'} - \sum_{i=1}^{N} \frac{\sum_{k=1}^{J} x_k^{'} e^{x_k^{'}\beta}}{\sum_{k=1}^{J} e^{x_k^{'}\beta}}$$

d. i. Code is written in file hw4.py, attached in my canvas submission.
   ii. MLE estimator $\hat{\beta}^{MLE} = [-2.63835286, 3.08439304, 2.0210235]$

e. Code is written in file hw4.py, attached in my canvas submission. Estimator $\hat{\beta} = [-2.2834, 2.8527, 1.8401]$