SOLUTION

EE 3350 HW 4: Angle Modulation Due: October 24, 2006

1. Determine the bandwidth in Hz of the angle modulated signal
$\varphi(t) = 10 \cos(2\pi 10^8 t + 200 \cos 2\pi 10^3 t)$.

2. Consider the angle modulated signal
$\varphi(t) = 10 \cos(2\pi 10^8 t + 3 \sin 2\pi 10^3 t)$.
(a) Assume that the signal is PM. Find the bandwidth of the PM signal when the message frequency is (1) doubled, and (2) is halved.
(b) Repeat part (a) assuming that the angle modulated signal is FM.

3. An FM signal is given by
$\varphi(t) = 10 \cos(2\pi 10^6 t + 5 \sin 2\pi 10^3 t)$.
Determine and sketch the magnitude spectrum of the signal $\varphi(t)$. [Note: sketch only those sidebands that are within the "bandwidth" of the FM signal.]

4. An angle modulated signal is given by the following expression:

$$\varphi_{EM}(t) = 5 \cos(\omega_c t + 40 \sin 500\pi t + 20 \sin 1000\pi t + 10 \sin 2000\pi t)$$

a. Determine the frequency deviation Δf, in Hz.
b. Estimate the bandwidth, in Hz, of the angle modulated signal by Carson's rule. If the angle modulated signal is a phase modulated signal with the phase deviation constant, k_p is 5 radians per volt, determine the message signal m (t).
c. If the angle modulated signal is a frequency modulated signal with a frequency deviation constant, k_f is 20,000 π radians/sec per volt, determine the message signal m(t).

5. An FM signal $\varphi_{FM}(t) = 5 \cos(2\pi 10^6 t + \sin 20{,}000\, \pi t)$ is input a square-law nonlinearity (with the characteristic: $y = 2x^2$, where x is the input and y the output). The output of the nonlinearity y(t) is filtered by an ideal band pass filter with center frequency 2.03 MHz and bandwidth 10 kHz to produce the final output z(t). Determine z(t) and sketch its magnitude spectrum.

6. A message signal $m(t) = 4 \cos 2\pi 1000t$ modulates a carrier frequency to produce a frequency modulated signal with a resulting modulation index (i.e. frequency deviation ratio) of 2.

(a) What is the estimate of the bandwidth of the FM signal?
(b) The message signal m(t) is replaced by a new message signal $m(t) = 4 \cos 2\pi 1000t + 4 \cos 2\pi 3000t$. What is the estimate of the bandwidth of this new FM signal?

7. The message signal $m(t) = \{10 (\sin 2\pi\, 200t) / \pi t\}$ frequency modulates an appropriate carrier signal with a modulation index of 6. (a) Write the expression for the FM waveform. (You do not need to integrate m(t) in your answer.) ; (b) What is the maximum frequency deviation of the modulated signal? (c) Find the bandwidth of the modulated signal.

8. The carrier $c(t) = 100 \cos 2\pi\, 10^8\, t$ is frequency modulated by the signal $m(t) = 5 \cos 2\pi\, 10000\, t$. The (peak) frequency deviation is 20 kHz. (a) Determine the amplitude and frequency of all signal components that have a power level of at least 10% of the power of the FM signal. (b) What is the bandwidth of the FM signal?

9. A signal m(t) frequency modulates a 100 kHz carrier to produce the following narrowband FM signal:

   $\varphi_{NB\,FM}(t) = 5 \cos (2\pi.10^5\, t + 0.0050 \sin 2\pi\ 10^4\, t\,)$.

   Generate (block diagram design) the wideband FM signal $\varphi_{WBFM}(t)$ with a carrier frequency of 150 MHz and a (peak) frequency deviation of 100 kHz. Assume that the following are available for the design:

   - Frequency Multipliers of any (integer) value
   - A local oscillator whose frequency can be tuned to any value between 100 MHz to 300 MHz
   - An ideal Band pass filter with tunable center frequency and bandwidth.

   Your block diagram design must clearly specify the carrier frequencies and frequency deviations at all logical points, as well as the center frequency and bandwidth of the Band pass filter.

~~SPRING~~ FALL 06

HW 4.1

HW 4 Solutions

EE 3350

1. $\phi(t) = 10\cos(2\pi.10^8 t + 200\cos 2\pi 10^3 t)$

$= A\cos(\omega_c t + \psi(t))$

where

$\psi(t) = 200\cos 2\pi.10^3 t$

$\Delta\omega = \left|\frac{d\psi(t)}{dt}\right|_{max}$ [Because $\omega_i(t) = \frac{d\psi(t)}{dt} + \omega_c$]

$\frac{d\psi(t)}{dt} = -4\pi.10^5 \sin 2\pi.10^3 t$

$\left|\frac{d\psi(t)}{dt}\right|_{max} = 4\pi\times 10^5$ rad/sec $= \Delta\omega$.

$\Delta f_{FM} = \frac{\Delta\omega}{2\pi} = \frac{4\pi\times 10^5}{2\pi}$ Hz $= \boxed{200\text{ kHz}}$

$B = 1$ kHz.

$BW_{FM} = 2(\Delta f_{FM} + B)$ Hz

$= \boxed{402\text{ kHz}}$

Hw 4.2

2. $\phi(t) = 10\cos(2\pi \cdot 10^8 t + 3\sin 2\pi \cdot 10^3 t)$

$\phi(t) = A\cos(\omega_c t + \psi(t))$

a: PM

$\psi(t) = k_p m(t) = 3\sin 2\pi 10^3 t$

$\Delta\omega = \left|\frac{d\psi(t)}{dt}\right|_{max} = 6\pi \times 10^3$ rad/sec

$\Delta f = 3$ kHz. $BW = 2(\Delta f + B)$ Hz

$= 2(3+1) = 8$ kHz.

(i) Frequency of $m(t)$ is doubled.

i.e.: $\psi(t) = 3 \cdot \sin 2\pi \cdot 2\times10^3 t$

$\frac{d\psi(t)}{dt} = 12\pi \cdot 10^3 \cos 2\pi \cdot 2 \cdot 10^3 t$

$\Delta\omega = \left|\frac{d\psi}{dt}\right|_{max} = 12\pi \cdot 10^3$ rad/sec

or $\Delta f = 6$ kHz

$BW = 2(\Delta f + B) = 2(6+2)$ kHz $= \boxed{16 \text{ kHz}}$

(ii) Frequency is halved.

i.e. $\psi(t) = 3\sin 2\pi \cdot \frac{1}{2} \cdot 10^3 t$

$\frac{d\psi(t)}{dt} = 3\pi \cdot 10^3 \cos 2\pi \cdot \frac{1}{2} \cdot 10^3 t$

$\Delta\omega = \left|\frac{d\psi(t)}{dt}\right|_{max} = 3\pi \cdot 10^3$ rad/sec

or $\Delta f = 1.5$ kHz

$BW = 2(\Delta f + B) = 2(1.5 + 0.5) = \boxed{4 \text{ kHz}}$

2. (continued)

<u>b</u>. FM.

$$\varphi(t) = k_f \int_{-\infty}^{t} m(\lambda)\, d\lambda.$$

$$= k_f \int_{-\infty}^{t} [\alpha \cos \omega_m \lambda]\, d\lambda$$

$$= \frac{k_f \alpha}{\omega_m} \sin \omega_m t. \longrightarrow \beta.$$

Thus $\beta = 3$ for FM. [<u>Note</u>: β depends on $m(t)$, and specifically on B, the BW of $m(t)$.

$B = 1$ kHz $\quad \Delta f = \beta \cdot B = 3 \cdot 1$ kHz $= 3$ kHz.

$BW = 2(\Delta f + B)$ Hz $= 2(3+1) = \boxed{8 \text{ kHz}}$

(i) Frequency is doubled

$\Longrightarrow B = 2$ kHz.

$\beta = \dfrac{k_f \alpha}{\omega_m} \leftarrow$

Note:

When ω_m doubles, β halves.

$\beta = \frac{3}{2}. \Rightarrow \Delta f = \beta \cdot B = \frac{3}{2} \cdot 2 = 3$ kHz

$BW = 2(\Delta f + B) = 2(3+2) = \boxed{10 \text{ kHz}}$

(ii) Frequency is halved:

$B = \frac{1}{2}$ kHz. $\quad \beta = 6$. (why?)

$\Delta f = 6 \cdot 1/2 = 3$ kHz

$BW = 2(\Delta f + B) = 2(3 + 1/2) = \boxed{7 \text{ kHz}}$

Note on #2:

This problem illustrates the effect of changing the bandwidth of $m(t)$ on the bandwidths BW_{PM} and BW_{FM}. It illustrates that BW_{PM} is more sensitive to the change in B (BW of $m(t)$) than BW_{FM}.

PM:

$$BW_{PM} = 2(\Delta f_{PM} + B)$$

$$\Delta f_{PM} = \frac{k_p}{2\pi} \left|\frac{dm(t)}{dt}\right|_{max}$$

→ depends on B.

FM:

$$BW_{FM} = 2(\Delta f_{FM} + B)$$

$$\Delta f_{FM} = \frac{k_f m_p}{2\pi}$$, which does not depend on the bandwidth of $m(t)$.

However, BW_{FM} does depend on the BW of $m(t)$ through the expression

$$BW_{FM} = 2(\Delta f_{FM} + B).$$

Hw 4.5

3. $\phi_{FM}(t) = 10\cos(2\pi \cdot 10^6 t + 5\sin 2\pi 10^3 t)$

The above signal is an FM signal.

$\Longrightarrow$ $A = 10$, $f_c = 1\,MHz$, $\omega_c = 2\pi \times 10^6$ rad/sec

$\beta = 5$, $f_m = 1\,kHz$, $\omega_m = 2\pi \times 10^3$ rad/sec

For a tone modulated FM,

$$\phi_{FM}(t) = A \sum_{n=-\infty}^{\infty} J_n(\beta)\cos((\omega_c + n\omega_m)t).$$

We have to determine the spectrum within the bandwidth of the FM signal.

For $\beta = 5$, we have only $\beta + 1 = 6$ sidebands on each side of ω_c. Thus, within the signal bandwidth, the FM signal can be written as:

$$\tilde{\phi}_{FM}(t) = 10\sum_{n=-6}^{6} J_n(5)\cos((\omega_c + n\omega_m)t)$$

Spectrum:

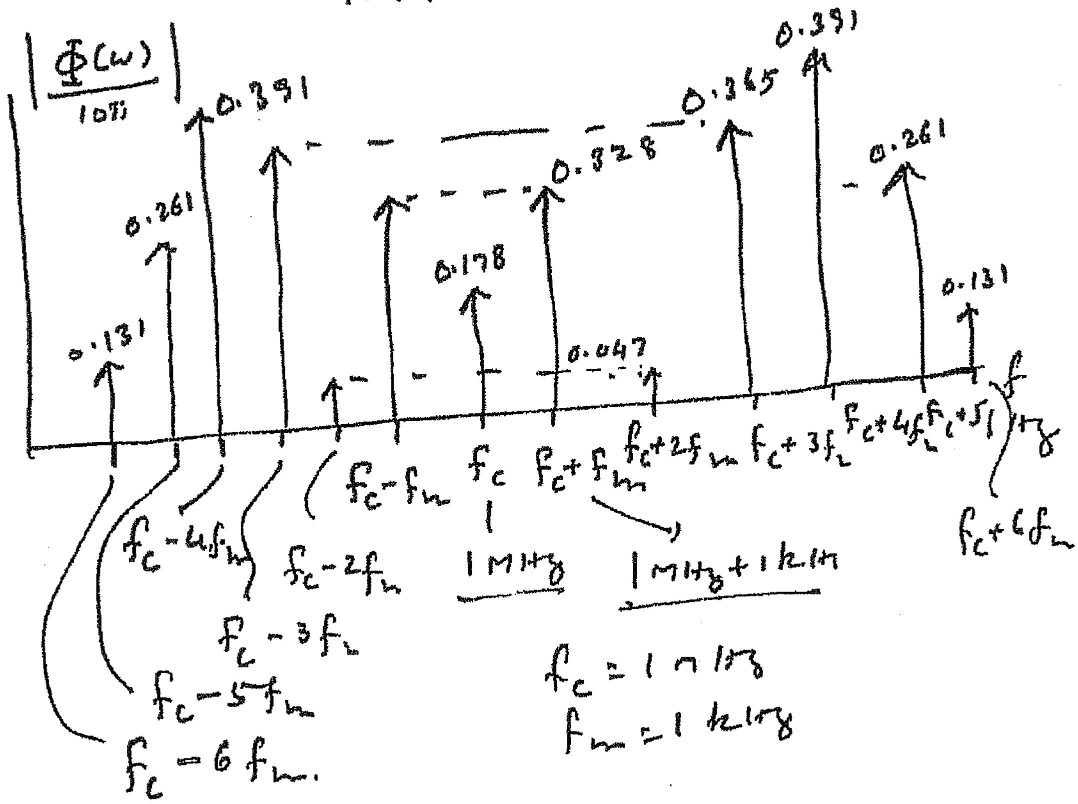
$$\tilde{\Phi}(\omega) = 10\pi \sum_{n=-6}^{6} J_n(5)\left[\delta(\omega - (\omega_c + n\omega_m)) + \delta(\omega + (\omega_c + n\omega_m))\right].$$

Mag. Spectrum:

$$|\tilde{\Phi}(\omega)| = (10\pi) \sum_{n=-6}^{6} |J_n(5)| \left[\delta(\omega - (\omega_c + n\omega_m)) + \delta(\omega + (\omega_c + n\omega_m))\right]$$

$J_0(5) = -0.178$
$J_1(5) = -0.328$
$J_2(5) = 0.047$ (from the tables)
$J_3(5) = 0.365$
$J_4(5) = 0.391$
$J_5(5) = 0.261$
$J_6(5) = 0.131$

Mag. spectrum: [only +ve freq. shown; it is scaled by 10π. Freq. axis is shown in Hz]



Hw. 4.7

4.

An angle modulated signal is given by the following expression:

$\varphi(t)$

$$\varphi_{EM}(t) = 5\cos(\omega_c t + \underbrace{40\sin 500\pi t + 20\sin 1000\pi t + 10\sin 2000\pi t}_{\varphi(t)}) \quad \text{— (1)}$$

a. Determine the (peak) frequency deviation Δf, in Hz.
b. Estimate the bandwidth, in Hz, of the angle modulated signal by Carson's rule.
c. If the angle modulated signal is a phase modulated signal with the phase deviation constant, k_p is 5 radians per volt, determine the message signal $m(t)$
d. If the angle modulated signal is a frequency modulated signal with a frequency deviation constant, k_f is 20,000 π radians/sec per volt, determine the message signal $m(t)$.

a.

$$\omega_{i_{max}} - \omega_c = \left| \frac{d}{dt}\left[40\sin 500\pi t + 20\sin 1000t + 10\sin 2000t\right]\right|_{max}$$

$$= \left| 20{,}000\pi\cos 500\pi t + 20{,}000\pi\cos 1000t + 20{,}000\pi\cos 2000t\right|$$

$$= 60{,}000\pi \text{ rad/sec}$$

$$\Rightarrow \boxed{\Delta f_{EM} = 30 \text{ kHz}}$$

b.

$$BW_{EM} = 2(\Delta f_{EM} + BW) = 2(30+1)\text{kHz} = \boxed{62 \text{ kHz}}$$

c.

$k_p = 5$ rad/volt

$$\phi_{PM}(t) = A\cos(\omega_c t + k_p \cdot m(t)) \quad \text{— (2)}$$

Making correspondences between (1) & (2), we have

$$\boxed{m(t) = 8\sin 500\pi t + 4\sin 1000t + 2\sin 2000t}$$

d

$k_f = 20{,}000\pi$ rad/sec/volt

$$\omega_i(t) = k_f \frac{d\varphi(t)}{dt} \quad \text{— (3)}$$

(3.4)

$$= k_f\, m(t)$$

$$\boxed{m(t) = \cos 500\pi t + \cos 1000\pi t + \cos 2000\pi t}$$

Hw 4.8

5.

An FM signal

$$\varphi_{FM}(t) = 5\cos(2\pi.10^6 t + \quad \sin 20{,}000\pi t)$$

is input to a square-law nonlinearity (with the characteristic: $y = 2x^2$, where x is the input and y is the output), and filtered by a bandpass filter. The center frequency of the bandpass filter is 2.03 MHz and the bandwidth is 10 kHz. Determine the output z(t), and sketch its magnitude spectrum.

$$\phi_{FM}(t) \longrightarrow \boxed{2(\ )^2} \xrightarrow{y(t)} \boxed{BPF} \longrightarrow z(t)$$

$$y(t) = 2\phi_{FM}^2(t) = 2A^2\cos^2(\omega_c t + \psi(t))$$

$$= A^2 + \underbrace{A^2\cos(2\omega_c t + 2\psi(t))}_{y_1(t)}$$

Rejected by BPF

For our case,

$$y_1(t) = 25\cos(2\pi \cdot 2 \cdot 10^6 t + 2\sin 20{,}000\pi t)$$

is an FM signal with $f_c = 2\,MHz$, $\beta = 2$ and $f_m = 10\,kHz$

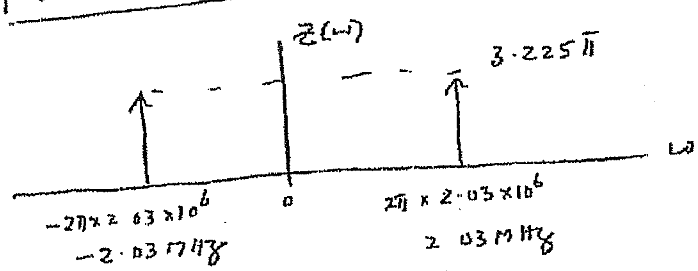
$$y_1(t) = 25\sum_{n=-\infty}^{\infty} J_n(2)\cos\left(2\pi(2\,10^6 + n \cdot 10 \cdot 10^3)t\right)$$

The only tone that will pass through the BPF with CF = 2.03 MHz and BW = 10 kHz will be the tone at 2.03 MHz

$$z(t) = 25 \cdot J_3(2)\cos\left(2\pi(2\,10^6 + 30\,10^3)t\right)$$

$$J_3(2) = 0.129$$

$$\boxed{z(t) = 3.225\cos\left(2\pi(2\,10^6 + 30 \cdot 10^3)t\right)}$$



6. FM:

$m(t) = 4 \cdot \cos 2\pi \cdot 10^3 t \Longrightarrow B = 1\ kHz$

$m_p = 4.$

Given: $\beta = 2 = \dfrac{k_f \cdot m_p}{\omega_m} = \dfrac{k_f \cdot 4}{2\pi \cdot 10^3}$

$\Longrightarrow \boxed{k_f = \frac{\pi}{2} \cdot 10^3 \text{ rad/sec/volt}}$

a. $BW_{FM} = 2(\Delta f_{FM} + B)$. or $2B(\beta+1)$

$= 2 \cdot 1 \cdot (2+1) = \boxed{6\ kHz}$

b. $m(t) = 4 \cos 2\pi \cdot 10^3 t + 4\cos 2\pi \cdot 3 \cdot 10^3 t$

$\Longrightarrow m_p = 8, \quad B = 3\ kHz.$

$\Delta f_{FM} = \dfrac{k_f \cdot m_p}{2\pi} = \dfrac{\frac{\pi}{2} \times 10^3 \times 8}{2\pi} = 4\ kHz$

(or new $\beta = 4$).

$BW_{FM} = 2(\Delta f_{FM} + B) = 2(4+3) = \boxed{14\ kHz}$

7. FM:

$$\phi_{FM}(t) = A\left[\cos w_c t + k_f \int_{-\infty}^{t} m(\lambda)\, d\lambda\right].$$

Given: $\begin{cases} m(t) = 10 \cdot \dfrac{\sin 2\pi\, 200 t}{\pi t} \Rightarrow B = 200\,Hz,\ m_p = 10 \\ \beta = 6. \end{cases}$

$$\Delta f = \beta \cdot B = \frac{k_f \cdot m_p}{2\pi}$$

$$\Rightarrow 6 \times 200 = \frac{k_f \cdot 10}{2\pi} \Rightarrow \boxed{k_f = 240\pi \text{ rad/sec/volt}}$$

a Thus, the FM waveform can be written as

$$\phi_{FM}(t) = A\cos\left[w_c t + 240\pi \int_{-\infty}^{t} \left[10 \cdot \frac{\sin 2\pi \cdot 200\lambda}{\pi\lambda}\right] d\lambda\right]$$

$$= \boxed{A\cos\left[w_c t + 2400\pi \int_{-\infty}^{t} \left[\frac{\sin 2\pi \cdot 200\lambda}{\pi\lambda}\right] d\lambda\right]}$$

b Freq. Dev.

$$\Delta f = \frac{k_f \cdot m_p}{2\pi}\,Hz = \frac{240\pi \times 10}{2\pi} = \boxed{1200\,Hz}$$

c. BW_{FM}:

$$BW_{FM} = 2(\Delta f_{FM} + B) = 2(1200 + 200)\,Hz$$

$$= \boxed{2800\,Hz}$$

HW 4.11

8. **FM:**

$c(t) = 100 \cos 2\pi . 10^8 t$

$m(t) = 5 . \cos(2\pi . 10^4 t)$. , $\Delta f = 20 kHz$
$B = 10 kHz$

$\phi_{FM}(t) = 100 [\cos 2\pi . 10^8 + \beta . \sin(2\pi . 10^4) t]$ — (1)

[Tone Modulation]

Where we need to determine β from Δf and B.

$\beta = \frac{\Delta f}{B} = \frac{20 \times 10^3}{10 \times 10^3} = \boxed{2}$.

(1) can be written as

$\phi_{FM}(t) = 100 \sum_{n=-\infty}^{\infty} J_n(2) \cos[2\pi(10^8 + n 10^4) t]$ — (2)

Let's answer part b first

b. $BW = 2 . B(\beta + 1) = 2 \times 10^4 (2+1) = \boxed{60 kHz}$

a. The power level of FM is: $\frac{100^2}{2}$ watts: $\boxed{5000}$ watts

The power in the k-th sideband, at frequency $f_c + k f_m$ is $|J_k(\beta)|^2 / 2$. watts.

We have to include in our list those sidebands for which $100^2 . |J_k(\beta)|^2 / 2 \geqslant 500.$ $\Rightarrow |J_k(\beta)|^2 \geqslant \frac{1000}{10^4} = 0.1.$

Index	$J_k(2)$	Freq. Hz	Amp. $100 \cdot J_k(2)$	Power $100^2 \cdot J_k^2(\beta)/2$
−4	0.034	$10^8 - 4.10^4$	3.40	5.7785
−3	−0.1289	$10^8 - 3.10^4$	−12.89	83.3
−2	0.3528	$10^8 + 2.10^4$	35 28	622.46
−1	−0.5767	$10^8 - 10^4$	−57.67	1663.1
0	0.2239	10^8	22.39	250.63
1	0.5767	$10^8 + 10^4$	57.67	1663.1
2	0.3528	$10^8 + 2.10^4$	35.28	622.46
3	0.1289	$10^8 + 2.10^4$	12.89	83.13
4	0.034	$10^8 + 2.10^4$	3.40	5.7785

The highlighted (entries in the boxes) components are the only components that have at least 10% of the FM signal power. Beyond 3 components ($\beta+1$), the sideband powers are dropping off rapidly

Hw 4.13

9. $\phi_{NBFM}(t) = 5\cos(2\pi\cdot 10^5 t + 0.0050 \sin 2\pi 10^4 t)$ — (1)

Target: $f_c = 150$ MHz $\quad \Delta f = 100$ kHz

From (1):

$\beta = 0.005, \quad f_m = 10$ kHz $\Rightarrow \Delta f_0 = 50$ Hz

$f_{c_0} = 100$ kHz

Freq. Multiplier

$$N = \frac{\Delta f}{\Delta f_0} = \frac{100\times 10^3}{50} = \boxed{2000} \Rightarrow \Delta f = 100\text{ kHz}$$

This guarantees that

Local oscillator freq.

$|f_{LO} - N\cdot f_{c_0}| = f_c \quad$ or $\quad f_{LO} + N f_{c_0} = f_c$.

$N f_{c_0} = 200$ MHz $\Rightarrow \boxed{f_{LO} = 50\text{ MHz or } 350\text{ MHz}}$

[Note: The problem given has a wrong range for the oscillator].

BPF:

$CF = f_c = 150$ MHz

$BW = 2(\Delta f + f_m) = \boxed{220\text{ kHz}}$