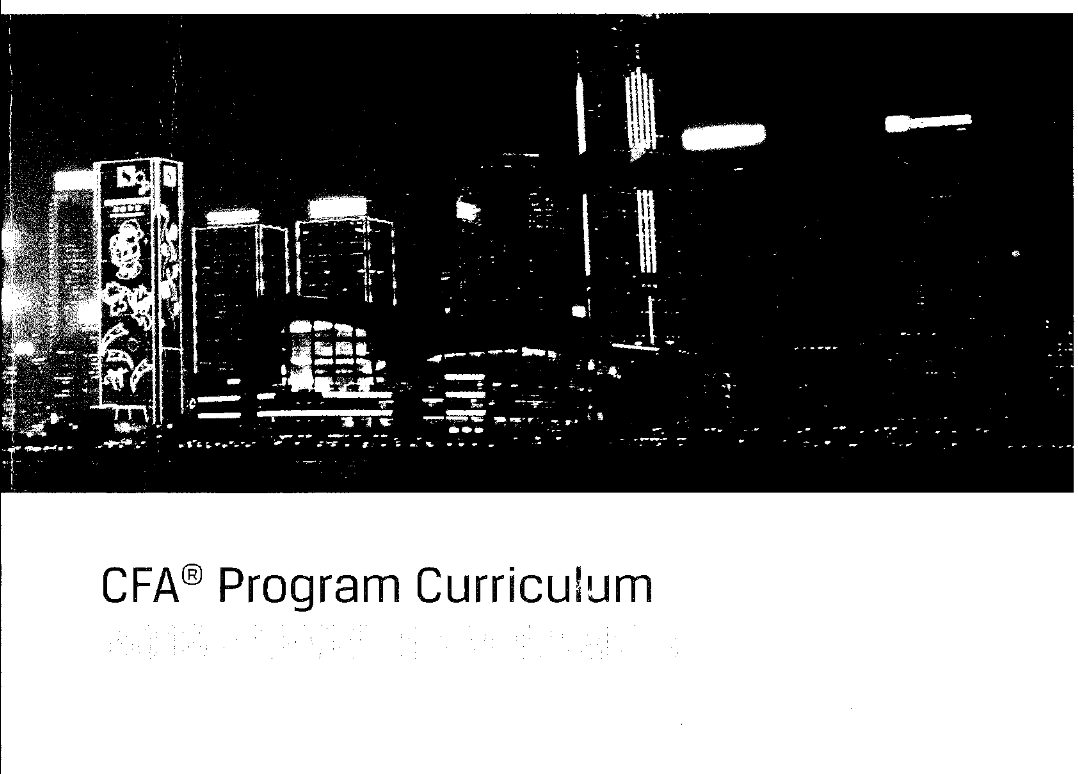

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Table 24 Input Variables and NPV for Scenario Analysis

	Scenario		
Variable	**Pessimistic**	**Most Likely**	**Optimistic**
Unit price	$4.50	$5.00	$5.50
Annual unit sales	35,000	40,000	45,000
Variable cost per unit	$1.60	$1.50	$1.40
Investment in fixed capital	$320,000	$300,000	$280,000
Investment in working capital	$50,000	$50,000	$50,000
Project life	6 years	6 years	6 years
Depreciation (straight-line)	$53,333	$50,000	$46,667
Salvage value	$40,000	$60,000	$80,000
Tax rate	40%	40%	40%
Required rate of return	13%	12%	11%
NPV	−$5,725	$121,157	$269,685
IRR	12.49%	22.60%	34.24%

The most likely scenario is the same as the base case we used above for sensitivity analysis, and the NPV for the most likely scenario is $121,157. To form the pessimistic and optimistic scenarios, managers change several of the assumptions for each scenario. For the pessimistic scenario, several of the input variables are changed to reflect higher costs, lower revenues, and a higher required rate of return. As the table shows, the result is a negative NPV for the pessimistic scenario and an IRR that is less than the pessimistic scenario's 13 percent required rate of return. For the optimistic scenario, the more favorable revenues, costs, and required rate of return result in very good NPV and IRR.

For this example, the scenario analysis reveals the possibility of an unprofitable investment, with a negative NPV and with an IRR less than the cost of capital. The range for the NPV is fairly large compared to the size of the initial investment, which indicates that the investment is fairly risky. This example included three scenarios for which management wants to know the profitability of the investment for each set of assumptions. Other scenarios can be investigated if management chooses to do so.

7.3.3 *Simulation (Monte Carlo) Analysis*

Simulation analysis is a procedure for estimating a probability distribution of outcomes, such as for the NPV or IRR for a capital investment project. Instead of assuming a single value (a point estimate) for the input variables in a capital budgeting spreadsheet, the analyst can assume several variables to be stochastic, following their own probability distributions. By simulating the results hundreds or thousands of times, the analyst can build a good estimate of the distributions for the NPV or IRR. Because of the volume of computations, analysts and corporate managers rely heavily on their personal computers and specialized simulation software such as @RISK.[11] Example 8 presents a simple simulation analysis.

11 @RISK is a popular and powerful risk analysis tool sold by Palisade Corporation. @RISK is an add-in for Microsoft Excel that allows simulation techniques to be incorporated into spreadsheet models.

EXAMPLE 8

Capital Budgeting Simulation

Gouhua Zhang has made the following assumptions for a capital budgeting project:

- Fixed capital investment is 20,000; no investment in net working capital is required.
- The project has an expected five-year life.
- The fixed capital is depreciated straight-line to zero over a five-year life. The salvage value is normally distributed with an expected value of 2,000 and a standard deviation of 500.
- Unit sales in Year 1 are normally distributed with a mean of 2,000 and a standard deviation of 200.
- Unit sales growth after Year 1 is normally distributed with a mean of 6 percent and standard deviation of 4 percent. Assume the same sales growth rate for Years 2–5.
- The sales price is 5.00 per unit, normally distributed with a standard deviation of 0.25 per unit. The same price holds for all five years.
- Cash operating expenses as a percentage of total revenue are normally distributed with a mean and standard deviation of 30 percent and 3 percent, respectively.
- The discount rate is 12 percent and the tax rate is 40 percent.

1 What are the NPV and IRR using the expected values of all input variables?

2 Perform a simulation analysis and provide probability distributions for the NPV and IRR.

Solution to 1:

Table 25 Expected Cash Flows for Simulation Example

Time	0	1	2	3	4	5
Fixed capital	–20,000					
After-tax salvage value						1,200
Price		5.00	5.00	5.00	5.00	5.00
Output		2,000	2,120	2,247	2,382	2,525
Revenue		10,000	10,600	11,236	11,910	12,625
Cash operating expenses		3,000	3,180	3,371	3,573	3,787
Depreciation		4,000	4,000	4,000	4,000	4,000
Operating income before taxes		3,000	3,420	3,865	4,337	4,837
Taxes on operating income		1,200	1,368	1,546	1,735	1,935
Operating income after taxes		1,800	2,052	2,319	2,602	2,902
Depreciation		4,000	4,000	4,000	4,000	4,000
Total after-tax cash flow	–20,000	5,800	6,052	6,319	6,602	8,102

Table 25 (Continued)

Time	0	1	2	3	4	5
NPV (at r = 12 percent)	3,294					
IRR	18.11%					

Based on the point estimates for each variable (the mean values for each), which are shown in Table 25 above, Zhang should find the NPV to be 3,294 and the IRR to be 18.11 percent.

Solution to 2:

Zhang performs a simulation using @RISK with 10,000 iterations. For each iteration, values for the five stochastic variables (price, output, output growth rate, cash expense percentage, and salvage value) are selected from their assumed distributions and the NPV and IRR are calculated. After the 10,000 iterations, the resulting information about the probability distributions for the NPV and IRR is shown in Figure 6 and Table 26.

Figure 6A Distribution for NPV

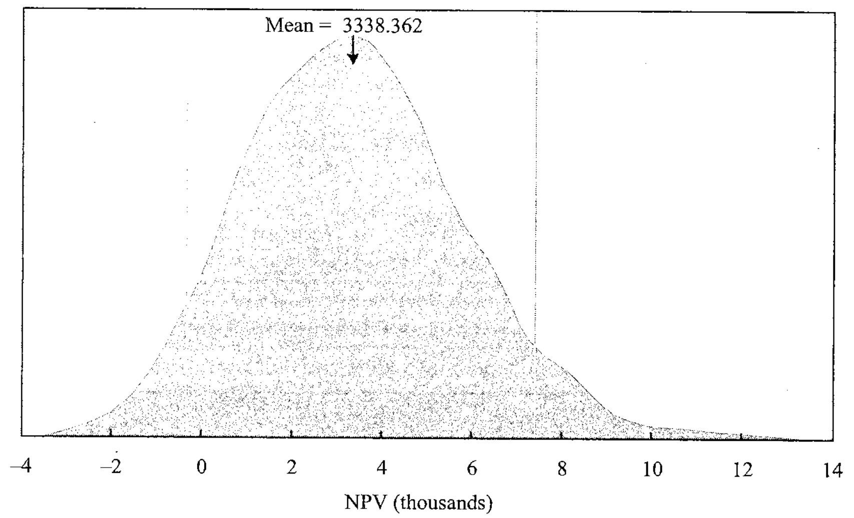

Figure 6B Distribution for IRR

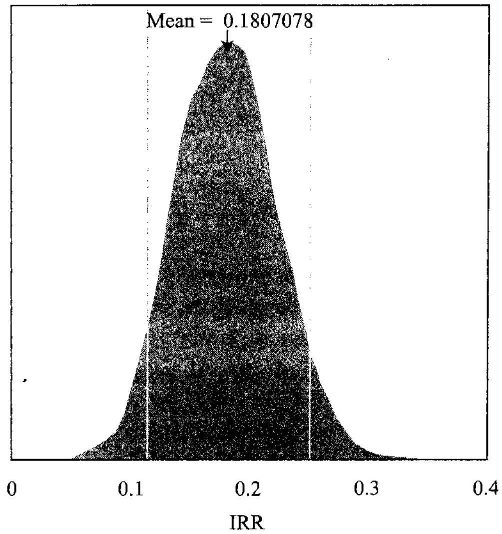

Table 26 Summary Statistics for NPV and IRR

Statistic	NPV	IRR
Mean	3,338	18.07%
Standard deviation	2,364	4.18%
Skewness	0.2909	0.1130
Kurtosis	3.146	2.996
Median	3,236	18.01%
90% confidence interval	−379 to 7,413	11.38% to 25.13%

Correlations between Input Variables and NPV and IRR		
Input Variable	**NPV**	**IRR**
Output	0.71	0.72
Output growth rate	0.49	0.47
Price	0.34	0.34
Cash expense proportion	−0.28	−0.29
Salvage value	0.06	0.05

As the figure shows, the distributions for the NPV and IRR are somewhat normal looking. The means and standard deviations for each are given in Table 26. Both distributions have a slight positive skewness, which means the distributions are skewed to the right. The two kurtosis values are fairly close to 3.0, which means that the distributions are not peaked or fat-tailed compared to the standard normal distribution. The median is the value at which 50 percent of the 10,000 outcomes fall on either side. The 90 percent confidence intervals show that 90 percent of the observations fall between −379 and 7,413 for the

NPV and between 11.38 percent and 25.13 percent for the IRR. Although not shown in the table, 7.04 percent of the observations had a negative NPV and an IRR less than the 12 percent discount rate.

The means of the NPV and IRR from the simulation (in Table 26) are fairly close to their values calculated using point estimates for all of the input variables (in Table 25). This is not always the case, but it is here. The additional information from a simulation is the dispersions of the NPV and IRR. Given his assumptions and model, the simulation results show Zhang the distributions of NPV and IRR outcomes that should be expected. Managers and analysts often prefer to know these total distributions rather than just their mean values.

The correlations in Table 26 can be interpreted as sensitivity measures. Changes in the "output" variable have the highest correlation with NPV and IRR outcomes. The salvage value has the lowest (absolute value) correlation.

This capital budgeting simulation example was not very complex, with only five stochastic variables. The example's five input variables were assumed to be normally distributed—in reality, many other distributions can be employed. Finally, the randomly chosen values for each variable were assumed to be independent. They can be selected jointly instead of independently. Simulation techniques have proved to be a boon for addressing capital budgeting problems.

Sensitivity analysis, scenario analysis, and simulation analysis are well-developed stand-alone risk analysis methods. These risk measures depend on the variation of the project's cash flows. Market risk measures, presented in the next section, depend not only on the variation of a project's cash flows, but also on how those cash flows covary with (or correlate with) market returns.

7.4 Risk Analysis of Capital Investments—Market Risk Methods

When using market risk methods, the discount rate to be used in evaluating a capital project is the rate of return required on the project by a diversified investor. The discount rate should thus be a risk-adjusted discount rate, which includes a premium to compensate investors for risk.[12] This risk premium should reflect factors that are priced or valued in the marketplace. The two equilibrium models for estimating this risk premium are the capital asset pricing model (CAPM) and arbitrage pricing theory (APT). We will discuss the CAPM as a way of finding risk-adjusted discount rates, although you should be aware that other methods can be used.

In the CAPM, total risk can be broken into two components: systematic risk and unsystematic risk. Systematic risk is the portion of risk that is related to the market and that cannot be diversified away. Unsystematic risk is non-market risk, risk that is idiosyncratic and that can be diversified away. Diversified investors can demand a risk premium for taking systematic risk, but not unsystematic risk.[13] Hence, the

12 Our approach to capital budgeting is to discount expected cash flows at a risk-adjusted cost of capital. An alternative approach, which is also conceptually sound, is the "certainty-equivalent method." In this method, certainty-equivalent cash flows (expected cash flows that are reduced to certainty equivalents) are valued by discounting them at a risk-free discount rate. The use of risk-adjusted discount rates is more intuitive and much more popular.

13 The capital asset pricing model uses this intuition to show how risky assets should be priced relative to the market. While the CAPM assigns a single market risk premium for each security, the APT develops a set of risk premia. The CAPM and APT are developed in detail elsewhere in the CFA curriculum.