

Soal Distribusi Poisson

1. Diket: $n = 192$

probabilitas munculnya satu muka = $\frac{1}{2}$

probabilitas munculnya 6 muka adalah $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{64} = p$

Maka $q = 1 - p = 1 - \frac{1}{64} = \frac{63}{64}$

Dg menggunakan Distribusi Binomial diperoleh

$$P(X=x) = \binom{192}{x} \left(\frac{1}{64}\right)^x \left(\frac{63}{64}\right)^{192-x}, x = 0, 1, 2, \dots, 192$$

Dg menggunakan Distribusi Poisson $\mu = np = 192 \left(\frac{1}{64}\right) = 3$

$$P(X=x) = \frac{e^{-3} (3)^x}{x!}$$

$$P(X=0) = \frac{e^{-3} (3)^0}{0!} = 0,0497$$

$$P(X=1) = \frac{e^{-3} (3)^1}{1!} = \frac{(0,0497) \cdot (3)}{1} = 0,1493$$

$$P(X=2) = \frac{e^{-3} (3)^2}{2!} = \frac{(0,0497) (9)}{2 \times 1} = 0,2240$$

$$P(X=3) = \frac{e^{-3} (3)^3}{3!} = \frac{(0,0497) (27)}{3 \times 2 \times 1} = 0,2240$$

$$P(X=4) = \frac{e^{-3} (3)^4}{4!} = \frac{(0,0497) (81)}{4 \times 3 \times 2 \times 1} = 0,1680$$

$$P(X=5) = \frac{e^{-3} (3)^5}{5!} = \frac{(0,0497) (243)}{5 \times 4 \times 3 \times 2 \times 1} = 0,1006$$

$$P(X=6) = \frac{e^{-3} (3)^6}{6!} = \frac{(0,0497) (729)}{26 \times 5 \times 4 \times 3 \times 2 \times 1} = 0,0503$$

$$2. \text{ Rata-rata } = \mu = n \cdot p = 192 \cdot \left(\frac{1}{64}\right) = 3$$

$$\text{Simpangan baku } \sigma = \sqrt{np} = \sqrt{3} = 1,73$$

$$\text{Variansi } \sigma^2 = \lambda = np = 192 \left(\frac{1}{64}\right) = 3$$

$$\text{Koefisien kemiringan } \gamma_3 = \frac{1}{\sqrt{\mu}} = \frac{1}{\sqrt{3}} = \frac{1}{1,73} = 0,5780$$

$$\text{Koefisien keruncingan } \gamma_4 = 3 + \frac{1}{\mu} = 3 + \frac{1}{3} = \frac{9}{3} + \frac{1}{3} = \frac{10}{3} = 3,33$$

3. Diket: $n = 56$

probabilitas munculnya 1 muka = $\frac{1}{2}$

probabilitas munculnya 3 muka = $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8} = p$

Maka $q = 1 - p = 1 - \frac{1}{8} = \frac{7}{8}$

Dg menggunakan distribusi Poisson $\mu = np = 56 \cdot \left(\frac{1}{8}\right) = 7$

$$P(X=x) = \frac{e^{-\mu} (\mu)^x}{x!}, \text{ dg } e^{-7} = 0,000912$$

$$P(X=0) = \frac{e^{-7} (7)^0}{0!} = 0,0009$$

$$P(X=1) = \frac{e^{-7} (7)^1}{1!} = 0,0063$$

$$P(X=2) = \frac{e^{-7} (7)^2}{2!} = 0,0223$$

$$P(X=3) = \frac{e^{-7} (7)^3}{3!} = 0,0521$$

4. Rata-rata $\mu = n \cdot p = 56 \cdot \left(\frac{1}{8}\right) = 7$

- simpangan baku $\sigma = \sqrt{n \cdot p} = \sqrt{7} = 2,645$

- variansi $\sigma^2 = n \cdot p = 56 \cdot \left(\frac{1}{8}\right) = 7$

- koefisien kemiringan $\sigma_3 = \frac{1}{\sqrt{\mu}} = \frac{1}{\sqrt{7}} = 0,3779$

- koefisien keruncingan $\sigma_4 = 3 + \frac{1}{\mu} = 3 + \frac{1}{7} = \frac{21}{7} + \frac{1}{7} = \frac{22}{7} = 3,14$

5. Diket: $n = 10$

probabilitas produk yg cacat = $10\% = 0,1 = p = \frac{1}{10}$

Untuk memperoleh 2 alat yg cacat berarti $x = 2$

Dg menggunakan distribusi Poisson $\mu = np = 10 \cdot \left(\frac{1}{10}\right) = 1$

$$P(X=x) = \frac{e^{-\mu} (\mu)^x}{x!}, \text{ dg } e^{-1} = 0,367879$$

$$P(X=2) = \frac{e^{-1} (1)^2}{2!} = 0,1839$$

6. Diket: $n = 3000$

$$p = \frac{2}{2000} = 0,001$$

Dg menggunakan distribusi Poisson $\mu = n \cdot p = (3000)(0,001) = 3$

97. Pak Tegar mendapat paling banyak 2 pasang sepatu, berarti $P(X \leq 2)$

$$P(X \leq 2) = P(X=0) + P(X=1) + P(X=2)$$

$$P(X=0) = \frac{e^{-3}(3)^0}{0!} = 0,0497$$

$$P(X=1) = \frac{e^{-3}(3)^1}{1!} = 0,1493$$

$$P(X=2) = \frac{e^{-3}(3)^2}{2!} = 0,2240$$

$$\begin{aligned} \text{Maka } P(X \leq 2) &= P(X=0) + P(X=1) + P(X=2) \\ &= 0,0497 + 0,1493 + 0,2240 \\ &= 0,423 \end{aligned}$$

b) Pak Toger melempar lebih dari 3 pasang sepatu yg tak memenuhi standar mutu, maka $P(X > 3)$

$$P(X > 3) = 1 - P(X \leq 3)$$

$$\begin{aligned} P(X \leq 3) &= P(X=0) + P(X=1) + P(X=2) + P(X=3) \\ &= 0,0497 + 0,1493 + 0,2240 + 0,2240 \\ &= 0,647 \end{aligned}$$

$$\begin{aligned} P(X > 3) &= 1 - P(X \leq 3) \\ &= 1 - 0,647 \\ &= 0,353 \approx 0,35 \end{aligned}$$

c) - Rata-rata $= \mu = n \cdot p = 3000 \cdot 0,001 = 3$

- Simpangan baku $\sigma = \sqrt{\mu} = \sqrt{3} = 1,73$

7. Diket: $n = 1.000$

$$p = \frac{50}{10.000} = 0,005$$

Dg menggunakan distribusi Poisson $\mu = n \cdot p = 1000 \cdot 0,005 = 5$

a) Peluang toko mendapat komplain dari 7 pelanggan maka $P(X=7)$

$$P(X=x) = \frac{e^{-5}(5)^x}{x!}, \text{ Dg } e^{-5} = 0,006738$$

$$P(X=7) = \frac{e^{-5}(5)^7}{7!} = \frac{526,406}{5040} = 0,1044$$

b) Mendapat komplain dari 5 pelanggan maka $P(X=5)$

$$P(X=5) = \frac{e^{-5}(5)^5}{5!} = \frac{21,0562}{120} = 0,1755$$

c) Mendapat komplain dari 2 pelanggan maka $P(X=2)$

$$P(X=2) = \frac{e^{-5}(5)^2}{2!} = \frac{0,1684}{2} = 0,0842$$

d) Tidak ada komplain maka $P(X=0)$

$$P(X=0) = \frac{e^{-5}(5)^0}{0!} = \frac{0,0067}{1} = 0,0067$$

e) Lebih dari 2 pelanggan maka $P(X > 2) = 1 - P(X \leq 2)$

$$P(x > 2) = 1 - P(x \leq 2)$$

$$= 1 - [P(x=0) + P(x=1) + P(x=2)]$$

$$= 1 - [0,0067 + 0,0337 + 0,0842]$$

$$= 1 - 0,1246$$

$$= 0,8754$$

8. Diket : $\mu = 4$

peluang PD hari esok yg akan terjadi

a) Lampu LED sebanyak 5 maka $P(x=5)$

b) Lampu LED sebanyak 3 maka $P(x=3)$

$$P(x=5) = \frac{e^{-4}(4)^5}{5!}$$

$$= \frac{10,7552}{120}$$

$$= 0,1563$$

$$P(x=3) = \frac{e^{-4}(4)^3}{3!}$$

$$= 1,1722$$

$$= \frac{6}{6}$$

$$= 0,1954$$

9. Diket : $n = 20$

$$p = 0,02$$

probabilitas dari 3 mesin yg mengalami gangguan maka $P(x=3)$

Dg menggunakan distribusi Poisson $\mu = n \cdot p = 20 \cdot 0,02 = 0,4$

$$P(x=3) = \frac{e^{-0,4}(0,4)^3}{3!}, \text{ Dg } e^{-0,4} = 0,67032$$

$$= \frac{0,04290048}{6} = 0,00715 = 0,0072$$

10. Diket : $n = 4000$

$$p = 0,0005$$

Dg menggunakan distribusi Poisson $\mu = n \cdot p = 4000 \cdot 0,0005 = 2$

menghitung 3 orang akan check maka $P(x=3)$

$$P(x=x) = \frac{e^{-2}(2)^x}{x!}, \text{ Dg } e^{-2} = 0,135335$$

$$P(x=3) = \frac{e^{-2}(2)^3}{3!}$$

$$= \frac{1,08268}{6}$$

$$= 0,18044$$