Lista 13

a) $\lim\limits_{x\to 0} \dfrac{\operatorname{sen} 3x}{2x} = \dfrac{1}{2} \lim\limits_{x\to 0} \dfrac{\operatorname{sen} 3x}{x} \cdot \dfrac{3}{3} = \dfrac{1}{2} \lim\limits_{x\to 0} \dfrac{3 \operatorname{sen} 3x}{3x} = \dfrac{3}{2} \lim\limits_{x\to 0} \dfrac{\operatorname{sen} 3x}{3x} = \dfrac{3}{2}$

b) $\lim\limits_{x\to 0} \dfrac{\operatorname{sen} x}{4x} = \dfrac{1}{4} \lim\limits_{x\to 0} \dfrac{\operatorname{sen} x}{x} = \dfrac{1}{4}$

c) $\lim\limits_{x\to 0} \dfrac{\operatorname{tg} 2x}{3x} = \lim\limits_{x\to 0} \dfrac{\frac{\operatorname{sen} 2x}{\cos 2x}}{3x} = \lim\limits_{x\to 0} \dfrac{\operatorname{sen} 2x}{3x \cos 2x} = \dfrac{2}{2} \lim\limits_{x\to 0} \dfrac{\operatorname{sen} 2x}{3x \cos 2x} =$

$\dfrac{2}{3} \lim\limits_{x\to 0} \dfrac{\operatorname{sen} 2x}{2x} \cdot \dfrac{1}{\cos 2x} = \dfrac{2}{3} \lim\limits_{x\to 0} \dfrac{\operatorname{sen} 2x}{2x} \cdot \lim \dfrac{1}{\cos 2x} =$

$\dfrac{2}{3} \lim\limits_{x\to 0} \dfrac{1}{\cos 2.0} = \dfrac{2}{3}$

d) $\lim\limits_{x\to 0} \dfrac{\operatorname{sen} 4x}{\operatorname{sen} 3x} = \lim\limits_{x\to 0} \dfrac{\operatorname{sen} 4x}{\operatorname{sen} 4x} \cdot \dfrac{4}{3} = \dfrac{4}{3} \lim\limits_{x\to 0} \dfrac{\operatorname{sen} 4x}{4x} = \dfrac{4}{3} \cdot 1 = \dfrac{4}{3}$

e) $\lim\limits_{x\to 0} \dfrac{\operatorname{tg} 3x}{\operatorname{tg} 5x} = \lim\limits_{x\to 0} \dfrac{\frac{\operatorname{sen} 3x}{\cos 5x}}{\frac{\cos 3x}{\operatorname{sen} 5x}} = 3 \lim\limits_{5x\to 0} \dfrac{\operatorname{sen} 3x \cdot \cos 5x}{\operatorname{sen} 5x \cdot \cos 3x} \cdot \dfrac{3x}{5x} =$

$\dfrac{3}{5} \lim\limits_{x\to 0} \dfrac{\frac{\operatorname{sen} 3x}{3x} \cdot \frac{\cos 3x}{3x}}{\frac{\operatorname{sen} 5x}{5x} \cdot \frac{\cos 5x}{5x}} = \cancel{\dfrac{3}{5}} \quad \dfrac{3}{5}$

f) $\lim\limits_{x\to 0} \dfrac{1 - \cos x}{x} = \lim\limits_{x\to 0} \dfrac{(1-\cos x)}{x} \cdot \dfrac{(1+\cos x)}{(1+\cos x)} = \lim\limits_{x\to 0} \dfrac{(1-\cos^2 x)}{x(1+\cos x)} = \lim\limits_{x\to 0} \dfrac{\operatorname{sen}^2 x}{x(1+\cos x)} =$

$\lim\limits_{x\to 0} \dfrac{\operatorname{sen} x}{x} \cdot \dfrac{\operatorname{sen} x}{1+\cos x} = \lim\limits_{x\to 0} \left(\dfrac{\operatorname{sen} x}{x}\right) \cdot \lim\limits_{x\to 0} \left(\dfrac{\operatorname{sen} x}{1+\cos x}\right) = 1 \cdot \dfrac{\operatorname{sen} 0}{1+\cos 0} = 1 \cdot \dfrac{0}{1+1} = 1 \cdot 0 = 0$

g) $\lim\limits_{x\to 0} \dfrac{1 - \cos x}{x \cdot \operatorname{sen} x} = \lim\limits_{x\to 0} \dfrac{\operatorname{sen} x}{\operatorname{sen} x + x \cos x} = \lim\limits_{x\to 0} \dfrac{\cos(x)}{2 \cdot \cos x - x \cdot \operatorname{sen} x} = \dfrac{\cos 0}{2\cos 0 - 0 \operatorname{sen} 0} = \dfrac{1}{2}$

h) $\lim\limits_{x\to 0} \dfrac{1 - \sec x}{x^2} = \lim\limits_{x\to 0} \dfrac{(1-\sec x)(1+\sec x)}{x^2(1+\sec x)} = \lim\limits_{x\to 0} \dfrac{(1-\sec^2 x)}{x^2(1+\sec x)} = \lim\limits_{x\to 0} \dfrac{1-(1+\operatorname{tg}^2 x)}{x^2(1+\sec x)} =$

$\lim\limits_{x\to 0} \left(\dfrac{-\operatorname{tg}^2 x}{x^2}\right) \cdot \lim\limits_{x\to 0} \dfrac{1}{1+\sec x} = \lim\limits_{x\to 0} \left(\dfrac{\operatorname{tg}}{x}\right) \cdot \lim\limits_{x\to 0} \left(\dfrac{\operatorname{tg}}{x}\right) \cdot \lim\limits_{x\to 0} \dfrac{1}{1+\sec x} = -1 \cdot 1 \cdot \left(\dfrac{1}{1+\sec 0}\right) =$

$-1 \cdot 1 \left(\dfrac{1}{2}\right) = -\dfrac{1}{2}$

i) $\lim_{x \to 0} \frac{tg\, x + sen\, x}{x} = \lim_{x \to 0} \frac{\frac{sen\, x}{cos\, x} + sen\, x}{x} = \lim_{x \to 0} = \frac{(sen\, x).(cos x) + sen\, x}{x}$

$\lim_{x \to 0} = \frac{sen\, \cancel{x}}{\cancel{x}}^{1} \cdot \frac{cos\, x + sen\, x}{x} = \lim_{x \to 0} \frac{cos\, x}{x}^{1} + \frac{sen\, \cancel{x}}{\cancel{x}}^{1} = 1 + 1 = 2$

j) $\lim_{x \to 0} \frac{tg\, x - sen\, x}{sen^2 x} = \lim_{x \to 0} \frac{\frac{sen\, x}{cos\, x} - sen\, x}{sen^2 x} = \lim_{x \to 0} \frac{1 - cos\, x}{sen\, x \, . \, cos\, x} =$

$\lim_{x \to 0} \frac{(1 - cos\, x)(1 + cos\, x)}{sen\, x \, . \, cos\, x \, . (1 + cos\, x)} = \lim_{x \to 0} \frac{1 - cos^2 x}{sen\, x \, . \, cos\, x \, . \, 1 + cos\, x} = \lim_{x \to 0} \frac{sen^2 x}{sen\, x \, . \, cos\, x \, . (1 + cos\, x)} =$

$\lim_{x \to 0} \frac{sen\, x}{cos\, x \, . (1 + cos\, x} = \frac{0}{1 \, . \, 2} = 0$