Atividade Avaliativa de Derivadas

1) a) $f(x) = x^2 + 3x - 1$

$f(x+\Delta x) = (x+\Delta x)^2 + 3(x+\Delta x) - 1$

$f(x+\Delta x) = x^2 + 2x\Delta x + \Delta x^2 + 3x + 3\Delta x - 1$

$f(x+\Delta x) - f(x) = x^2 + 2x\Delta x + \Delta x^2 + 3x + 3\Delta x - 1 - (x^2 + 3x - 1)$

$= \cancel{x^2} + 2x\Delta x + \Delta x^2 \cancel{+3x} + 3\Delta x \cancel{-1} \cancel{-x^2} \cancel{-3x} \cancel{+1}$

$f(x+\Delta x) - f(x) = 2x\Delta x + \Delta x^2 + 3\Delta x$

$f'(x) = \lim_{\Delta x \to 0} \frac{2x\cancel{\Delta x} + \Delta x^{\cancel{2}} + 3\cancel{\Delta x}}{\cancel{\Delta x}} \to f'(x) = \lim_{\Delta x \to 0} 2x + \cancel{\Delta x}^{\,0} + 3 \to \boxed{f'(x) = 2x + 3}$

b) $f(x) = \sqrt{x+3}$

$f(x+\Delta x) = \sqrt{(x+\Delta x)+3}$

$f(x+\Delta x) - f(x) = \sqrt{x+\Delta x+3} - \sqrt{x+3}$

$f'(x) = \frac{\sqrt{x+\Delta x+3} - \sqrt{x+3}}{\Delta x} \cdot \frac{(\sqrt{x+\Delta x+3} + \sqrt{x+3})}{\sqrt{x+\Delta x+3} + \sqrt{x+3}}$

$f'(x) = \lim_{\Delta x \to 0} \frac{\cancel{x} + \Delta x \cancel{+3} \cancel{-x} \cancel{-3}}{\Delta x.(\sqrt{x+\Delta x+3} + \sqrt{x+3})} \to f'(x) = \lim_{\Delta x \to 0} \frac{\cancel{\Delta x}}{\cancel{\Delta x}.(\sqrt{x+\Delta x+3} + \sqrt{x+3})}$

$f'(x) = \lim_{\Delta x \to 0} \frac{1}{\sqrt{x+\Delta x+3} + \sqrt{x+3}} \to \lim_{\Delta x \to 0} \frac{1}{\sqrt{x+0+3} + \sqrt{x+3}} \to \lim_{\Delta x \to 0} \frac{1}{2\sqrt{x+3}} \to f'(x) = \frac{1}{2\sqrt{x+3}}$

c) $f(x) = 1 - 4x^2$

$f(x+\Delta x) = 1 - 4(x+\Delta x)^2$

$= 1 - 4(x^2 + 2x\Delta x + \Delta x^2)$

$= 1 - 4x^2 - 8x\Delta x - 4\Delta x^2$

$f(x+\Delta x) - f(x) = 1 - 4x^2 - 8x\Delta x - 4\Delta x^2 + (1 - 4x^2)$

$= \cancel{1} \cancel{-4x^2} - 8x\Delta x - 4\Delta x^2 \cancel{-1} \cancel{+4x^2}$

$= -8x\Delta x - 4\Delta x^2$

$f'(x) = \lim_{\Delta x \to 0} \frac{-8x\cancel{\Delta x} - 4\Delta x^{\cancel{2}}}{\cancel{\Delta x}} \to f'(x) = \lim_{\Delta x \to 0} -8x - 4\cancel{\Delta x}^{\,0} \to \lim_{\Delta x \to 0} -8x - 4.0 \to f'(x) = -8x$

d) $f(x) = \frac{1}{x+2}$

$f(x+\Delta x) - f(x) = \frac{1}{(x+\Delta x)+2} - \frac{1}{x+2}$

$f(x+\Delta x) = \frac{1}{(x+\Delta x)+2}$

$f'(x) = \lim_{\Delta x \to 0} \frac{\frac{1}{x+\Delta x+2} - \frac{1}{x+2}}{\Delta x} \to f'(x) = \lim_{\Delta x \to 0} \frac{\frac{\cancel{x+2} \cancel{-x} - \Delta x \cancel{-2}}{(x+\Delta x+2).(x+2)}}{\Delta x}$

$f'(x) = \lim_{\Delta x \to 0} \frac{-\cancel{\Delta x}}{(x+\Delta x+2).(x+2)} \cdot \frac{1}{\cancel{\Delta x}} \to f'(x) = \lim_{\Delta x \to 0} -\frac{1}{(x+\cancel{\Delta x}+2).(x+2)} = \lim_{\Delta x \to 0} -\frac{1}{(x+0+2).(x+2)}$

$f'(x) = \frac{1}{(x+2).(x+2)} \to f'(x) = \frac{1}{(x+2)^2}$

2) a) $f(x) = 5x^2$, $x = 5$

$f(x+\Delta x) = \underline{5(x+\Delta x)^2}$ $\quad f(x+\Delta x) - f(x) = \underline{5(x+\Delta x)^2 - 5x^2}$

$f'(x) = \lim_{\Delta x \to 0} \frac{5(x+\Delta x)^2 - 5x^2}{\Delta x} \rightarrow f'(x) = \lim_{\Delta x \to 0} \frac{5(x^2 + 2x\Delta x + \Delta x^2) - 5x^2}{\Delta x}$

$f'(x) = \lim_{\Delta x \to 0} \frac{\cancel{5x^2} + 10x\Delta x + \cancel{\Delta x^2} - \cancel{5x^2}}{\cancel{\Delta x}} \rightarrow f'(x) = \lim_{\Delta x \to 0} 10x + \cancel{\Delta x}^{\,0} \rightarrow f'(x) = 10x \rightarrow f'(5) = 10.5 \rightarrow \boxed{f'(5) = 50}$

b) $f(x) = -3x + 2$, $x = 2$

$f(x+\Delta x) = -3(x+\Delta x) + 2$ $\quad f(x+\Delta x) - f(x) = -3(x+\Delta x) + 2 + 3x - 2$

$f'(x) = \lim_{\Delta x \to 0} \frac{-3(x+\Delta x) + 2 + 3x - 2}{\Delta x} \rightarrow f'(x) = \lim_{\Delta x \to 0} \frac{\cancel{-3x} - 3\cancel{\Delta x} \cancel{+2} + \cancel{3x} \cancel{-2}}{\cancel{\Delta x}} \rightarrow f'(x) = 3 \rightarrow \boxed{f'(2) = 3}$

c) $f(x) = x^2 - 6x + 2$, $x = 3$

$f(x+\Delta x) = (x+\Delta x)^2 - 6(x+\Delta x) + 2$
$= \underline{x^2 + 2x\Delta x + \Delta x^2 - 6x - 6\Delta x + 2}$

$f(x+\Delta x) - f(x) = x^2 + 2x\Delta x + \Delta x^2 - 6x - \Delta x + 2 - (x^2 - 6x + 2)$
$= \cancel{x^2} + \underline{2x\Delta x} + \underline{\Delta x^2} - \cancel{6x} - \underline{6\Delta x} + \cancel{2} - \cancel{x^2} + \cancel{6x} - \cancel{2}$

$f'(x) = \lim_{\Delta x \to 0} \frac{2x\cancel{\Delta x} + \Delta x^2 - 6\cancel{\Delta x}}{\cancel{\Delta x}} \rightarrow f'(x) = \lim_{\Delta x \to 0} 2x + \cancel{\Delta x}^{\,0} - 6 \rightarrow \lim_{\Delta x \to 0} 2x + 0 - 6 = 2x - 6$

$f'(x) = 2x - 6 \rightarrow f'(3) = 2.3 - 6 \rightarrow f'(3) = 6 - 6 \rightarrow \boxed{f'(3) = 0}$