Homework 2
ECE/CS 498 DS Spring 2019
Issued: 02/14/19
Due: 02/22/19

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Problem 1

1. Weather (sunny or rainy) and Location (town or highway) have the potential to cause disengagements of autonomous vehicles. These disengagements could lead to accidents. Given the Bayes Net in Figure 1, answer the following questions:

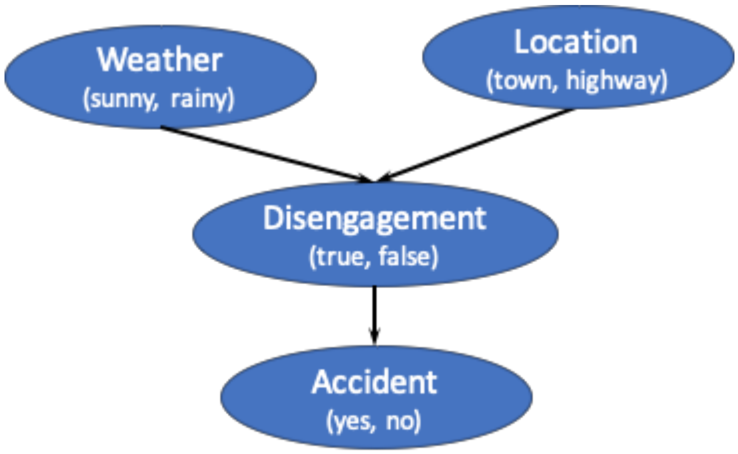

Figure 1

Weather	**Probability**
sunny	0.7
rainy	0.3
Location	**Probability**
town	0.8
highway	0.2

Disengagement Conditional Probability Table (CPT)			
Weather	**Location**	**Disengagement= true**	**Disengagement= false**
sunny	town	0.05	0.95
sunny	highway	0.01	0.99
rainy	town	0.15	0.85
rainy	highway	0.05	0.95

Accident CPT		
Disengagement	**Accident=yes**	**Accident=no**
true	0.4	0.6
false	0.01	0.99

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A. How many parameters are needed to define the conditional probability distribution of the Bayes Net given in Figure 1.

8 parameters. $P(W)\,P(L)\,P(D|W,L)\,P(A|D)$
$1+1+4+2=8$

B. Construct the **joint probability distribution** of Weather, Location, and Disengagement

$P(D|W,L)=\frac{P(D,W,L)}{P(W,L)}$

$\Rightarrow P(D,W,L)=P(D|W,L)\,P(W,L)$

Weather	Location	Disengagement=true	Disengagement=false
sunny	town	$0.05\times0.7\times0.8=0.028$	$0.95\times0.7\times0.8=0.532$
sunny	highway	$0.01\times0.7\times0.2=0.0014$	$0.99\times0.7\times0.2=0.1386$
rainy	town	$0.15\times0.3\times0.8=0.036$	$0.85\times0.3\times0.8=0.204$
rainy	highway	$0.05\times0.3\times0.2=0.003$	$0.95\times0.3\times0.2=0.057$

C. Calculate the probability of the following hypotheses
Let
a. A = Accident
b. D = Disengagement
c. W = Weather
d. L = Location

$P(A|B)=\frac{P(AB)}{P(B)}$

$P(W,L,D,A)=\sum P(W)\,P(L)\,P(D|W,L)\,P(A|D)=\sum P(W,L,D)\,P(A|D)$

$P(A)=\sum P(A|D)\,P(D)=0.4\times(0.028+0.0014+0.036+0.003)+0.01\times(0.532+0.1386+0.204+0.057)=0.036676$

Hypothesis		Probability
H0	$P(W=sunny, L=town \mid A=yes)$	$(0.028\times0.4+0.532\times0.01)/0.036676$ $=0.4504$
H1	$P(W=sunny, L=highway \mid A=yes)$	$(0.0014\times0.4+0.1386\times0.01)/0.036676$ $=0.05306$
H2	$P(W=rainy, L=town \mid A=yes)$	$(0.036\times0.4+0.204\times0.01)/0.036676$ $=0.4482$
H3	$P(W=rainy, L=highway \mid A=yes)$	$(0.003\times0.4+0.057\times0.01)/0.036676$ $=0.04826$

D. Apply the MAP decision rule to the 4 hypotheses above.

$\because$ P(H0) is max

$\therefore$ Sunny & Town is the most possible potential cause of accidents.

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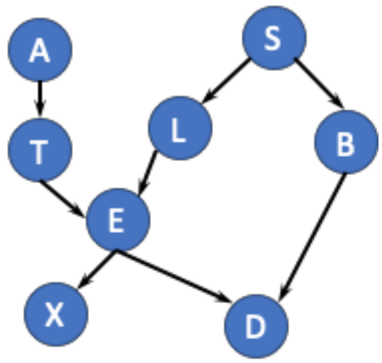
Problem 2



The chest clinic network above concerns the diagnosis of lung disease (tuberculosis, lung cancer, or both, or neither). In this model, a visit to Asia is assumed to increase the probability of tuberculosis. We have the following binary variables:

Variable	
X	positive X-ray
D	dyspnea (shortness of breath)
E	either tuberculosis or lung cancer
T	tuberculosis
L	lung cancer
B	bronchitis
A	a visit to Asia
S	smoker

A. Write down the factorization of the distribution implied by the network.

$P(X,D,E,T,L,B,A,S) = P(A)P(T|A)P(S)P(L|S)P(B|S)P(E|T,L)P(X|E)P(D|E,B)$

B. This video introduces a general way to determine independence relationships in Bayes Net: https://www.coursera.org/lecture/probabilistic-graphical-models/flow-of-probabilistic-influence-1eCp1.

T S D

Example: Is it true that tuberculosis ⫫ smoking | shortness of breath (given shortness of breath, tuberculosis and smoking are independent)?

Solution: There are two trails from T to S: (T, E, L, S) and (T, E, D, B, S). The trail (T, E, L, S) features a collider node E that is opened by the conditioning variable D. The trail is thus active and we do not need to check the second trail because for independence all trails needed to be blocked. The independence relationship does thus generally not hold.

Are the following conditional independence relationships true or false? Explain why.

* A trail $X_1 \cdots X_m$ is active given Z if

- for any v-structure $X_{i-1} \to X_i \leftarrow X_{i+1}$ we have that X_i or one of its descendants $\in Z$

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- no other X_i is in Z.

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L B S

1. lung cancer ⫫ bronchitis | smoking

① L ← S → B : since smoking is known condition, so L and B trail is not active.
② L → E → D ← B : since D is unknown, so this trail is also not active.
⇒ L ⫫ B | S independent (true)

2. a visit to Asia ⫫ smoking | lung cancer

① A → T → E ← L ← S : L is known condition is trail, so this trail is not active
② A → T → E → D ← B ← S : D is a v-structure while it's not ∈ L, so this trail is also not active
⇒ A ⫫ S | L independent (true)

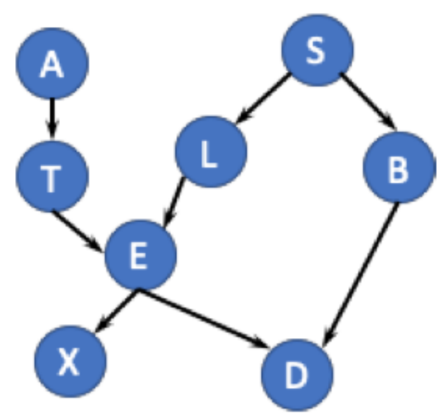
3. a visit to Asia ⫫ smoking | lung cancer, shortness of breath

Z = known condition L + D
for trail A → T → E → D ← B ← S : D is a v-structure
and D ∈ Z ⇒ this trail is active ⇒ A ⫫ S | L, D shows they are not independent (false)

C. Calculate the values for **P(D)**. The conditional probabilities are:

P(A=1) = 0.01	P(S=1) = 0.5
P(T=1|A=1) = 0.05	P(T=1|A=0) = 0.01
P(L=1|S=1) = 0.1	P(L=1|S=0) = 0.01
P(B=1|S=1) = 0.6	P(B=1|S=0)=0.3
P(X=1|E=1) = 0.98	P(X=1|E=0) = 0.05
P(D=1|E=1,B=1) = 0.9	P(D=1|E=1,B=0) = 0.7
P(D=1|E=0,B=1) = 0.8	P(D=1|E=0,B=0) = 0.1
P(E=1|T=1,L=1) = 1	P(E=1|T=1,L=0) = 1
P(E=1|T=0,L=1) = 1	P(E=1|T=0,L=0) = 0

$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$



(Hint: try to come up with a good sequence of calculations.)

2. $P(T=1) = 0.05 \times 0.01 + 0.01 \times 0.99 = 0.0104$

$P(T=0) = 1 - P(T=1) = 0.9896$

$P(B=1) = 0.6 \times 0.5 + 0.3 \times 0.5 = 0.45$

$P(B=0) = 1 - P(B=1) = 0.55$

$P(L=0) = 0.1 \times 0.5 + 0.01 \times 0.5 = 0.055$

$P(L=1) = 1 - P(L=0) = 0.945$

$P(E=1) = 0.0104 \times 0.945 + 0.0104 \times 0.055 + 0.9896 \times 0.945 + 0 = 0.0648$

$P(E=0) = 1 - P(E=1) = 0.9351$

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$$P(D) = \sum_{E,B} P(D \mid E, B)\, P(E)\, P(B)$$

$$= 0.9 \times 0.0648 \times 0.45 + 0.7 \times 0.0648 \times 0.55 + 0.8 \times 0.9351 \times 0.45 + 0.1 \times 0.9351 \times 0.55$$

$$P(D=1) = 0.43925$$

$$P(D=0) = 0.5607$$