# Package 'Rsurrogate'

July 15, 2019

Type Package

Index

Explained by Surrogate Marker Information				
Version 2.1				
<b>Date</b> 2019-07-10				
Author Layla Parast				
Maintainer Layla Parast <pre><pre></pre></pre>				
<b>Description</b> Provides functions to estimate the proportion of treatment effect on the primary outcome that is explained by the treatment effect on the surrogate marker.				
License GPL				
Imports stats, survival, Matrix				
NeedsCompilation no				
R topics documented:				
Aug.R.s.surv.estimate				
delta.estimate				
delta.s.estimate				
delta.s.surv.estimate				
delta.surv.estimate				
delta.t.surv.estimate				
d_example				
d_example_surv				
fieller.ci				
me.variance.estimate				

R.s.estimate.me22R.s.surv.estimate26R.t.surv.estimate31

**34** 

Aug.R.s.surv.estimate Calculates the augmented estimator of the proportion of treatment effect explained by the surrogate marker information measured at a

specified time and primary outcome information up to that specified time

# **Description**

This function calculates the augmented version of the proportion of treatment effect on the primary outcome explained by the surrogate marker information measured at  $t_0$  and primary outcome information up to  $t_0$ . Variance estimates and 95 % confidence intervals for the augmented estimates are provided automatically; three versions of the confidence interval are provided: a normal approximation based interval, a quantile based interval and Fieller's confidence interval, all using perturbation-resampling. The user can also request an estimate of the incremental value of surrogate marker information.

## Usage

```
Aug.R.s.surv.estimate(xone, xzero, deltaone, deltazero, sone, szero, t,
weight.perturb = NULL, landmark, extrapolate = FALSE, transform = FALSE,
basis.delta.one, basis.delta.zero, basis.delta.s.one = NULL,
basis.delta.s.zero = NULL, incremental.value = FALSE)
```

## **Arguments**

xone	numeric vector, the observed event times in the treatment group, $X = min(T,C)$ where T is the time of the primary outcome and C is the censoring time.
xzero	numeric vector, the observed event times in the control group, $X = min(T,C)$ where T is the time of the primary outcome and C is the censoring time.
deltaone	numeric vector, the event indicators for the treatment group, $D = I(T < C)$ where T is the time of the primary outcome and C is the censoring time.
deltazero	numeric vector, the event indicators for the control group, $D = I(T < C)$ where T is the time of the primary outcome and C is the censoring time.
sone	numeric vector; surrogate marker measurement at $t_0$ for treated observations, assumed to be continuous. If $X_{1i} < t_0$ , then the surrogate marker measurement should be NA.
szero	numeric vector; surrogate marker measurement at $t_0$ for control observations, assumed to be continuous. If $X_{1i} < t_0$ , then the surrogate marker measurement should be NA.
t	the time of interest.
weight.perturb	weights used for perturbation resampling.
landmark	the landmark time $t_0$ or time of surrogate marker measurement.
extrapolate	TRUE or FALSE; indicates whether the user wants to use extrapolation.
transform	TRUE or FALSE; indicates whether the user wants to use a transformation for

basis.delta.one

the surrogate marker.

either a vector of length  $n_1$  or a matrix with  $n_1$  rows; this is the basis transformation used for augmentation of  $\hat{\Delta}(t)$  for treated observations only, all values must be numeric

basis.delta.zero

either a vector of length  $n_0$  or a matrix with  $n_0$  rows; this is the basis transformation used for augmentation of  $\hat{\Delta}(t)$  for control observations only, all values must be numeric

basis.delta.s.one

either a vector of length  $n_1$  or a matrix with  $n_1$  rows; this is the basis transformation used for augmentation of  $\hat{\Delta}_S(t,t_0)$  for treated observations only, all values must be numeric; default is to assume this is the same as basis.delta.one i.e. that the same basis transformation is used for both quantities

basis.delta.s.zero

either a vector of length  $n_0$  or a matrix with  $n_0$  rows; this is the basis transformation used for augmentation of  $\hat{\Delta}_S(t,t_0)$  for control observations only, all values must be numeric; default is to assume this is the same as basis.delta.zero i.e. that the same basis transformation is used for both quantities

incremental.value

TRUE or FALSE; indicates whether the user would like to see the incremental value of the surrogate marker information, default is FALSE.

#### **Details**

Please see R.s. surv. estimate documention for details about the estimates before augmentation is performed. Recent work has shown that augmentation can lead to improvements in efficiency by taking advantage of the association between baseline information, denoted here as Z, and the primary outcome. This function calculates the augmented estimates of the quantities of interest. For example, the augmented version of  $\hat{\Delta}(t)$  is defined as:

$$\hat{\Delta}(t)^{AUG} = \hat{\Delta}(t) + \gamma \{ n_1^{-1} \sum_{i=1}^{n_1} h(Z_{1i}) - n_0^{-1} \sum_{i=1}^{n_0} h(Z_{0i}) \}$$

where  $Z_{gi}, i=1,2,\cdots,n_g$  are i.i.d. random vectors of baseline covariates from treatment group g and  $h(\cdot)$  is a basis transformation given a priori. Due to treatment randomization,  $\{n_1^{-1}\sum_{i=1}^{n_1}h(Z_{1i})-n_0^{-1}\sum_{i=1}^{n_0}h(Z_{0i})\}$  converges to zero in probability as the sample size goes to infinity and thus the augmented estimator converges to the same limit as the original counterparts. The quantity  $\gamma$  is selected such that the variance of  $\hat{\Delta}(t)^{AUG}$  is minimized. That is,  $\gamma=(\Xi_{12})(\Xi_{22})^{-1}$  where

$$\Xi_{12} = \operatorname{cov}\{\hat{\Delta}(t), n_1^{-1} \sum_{i=1}^{n_1} h(Z_{1i}) - n_0^{-1} \sum_{i=1}^{n_0} h(Z_{0i})\},$$

$$\Xi_{22} = \operatorname{var}\{n_1^{-1} \sum_{i=1}^{n_1} h(Z_{1i}) - n_0^{-1} \sum_{i=1}^{n_0} h(Z_{0i})\}$$

and thus we can obtain  $\hat{\Delta}(t)^{AUG}$  by replacing  $\gamma$  with a consistent estimator,  $\hat{\gamma}$  obtained using perturbation-resampling. A similar approach is used to obtain  $\hat{\Delta}_S(t)^{AUG}$  and thus construct

$$\hat{R}_S(t,t_0)^{AUG} = 1 - \frac{\hat{\Delta}_S(t,t_0)^{AUG}}{\hat{\Delta}(t)^{AUG}}.$$

#### Value

A list is returned:

aug.delta the estimate,  $\hat{\Delta}(t)^{AUG}$ . aug.delta.s the estimate,  $\hat{\Delta}_S(t,t_0)^{AUG}$ .

aug.R.s the estimate,  $\hat{R}_S(t,t_0)^{AUG}$ .

aug.delta.var the variance estimate of  $\hat{\Delta}(t)^{AUG}$ .

aug.delta.s.var

the variance estimate of  $\hat{\Delta}_S(t, t_0)^{AUG}$ .

aug.R.s.var the variance estimate of  $\hat{R}_S(t,t_0)^{AUG}$ .

conf.int.normal.aug.delta

a vector of size 2; the 95% confidence interval for  $\hat{\Delta}(t)^{AUG}$  based on a normal approximation.

conf.int.quantile.aug.delta

a vector of size 2; the 95% confidence interval for  $\hat{\Delta}(t)^{AUG}$  based on sample quantiles of the perturbed values.

conf.int.normal.aug.delta.s

a vector of size 2; the 95% confidence interval for  $\hat{\Delta}_S(t,t_0)^{AUG}$  based on a normal approximation.

conf.int.quantile.aug.delta.s

a vector of size 2; the 95% confidence interval for  $\hat{\Delta}_S(t,t_0)^{AUG}$  based on sample quantiles of the perturbed values.

conf.int.normal.R.s

a vector of size 2; the 95% confidence interval for  $\hat{R}_S(t,t_0)^{AUG}$  based on a normal approximation.

conf.int.quantile.aug.R.s

a vector of size 2; the 95% confidence interval for  $\hat{R}_S(t,t_0)^{AUG}$  based on sample quantiles of the perturbed values..

conf.int.fieller.aug.R.s

a vector of size 2; the 95% confidence interval for  $\hat{R}_S(t,t_0)^{AUG}$  based on Fieller's approach.

aug.delta.t the estimate,  $\hat{\Delta}_T(t, t_0)^{AUG}$ ; if incremental.vaue = TRUE.

aug.R.t the estimate,  $\hat{R}_T(t,t_0)^{AUG}$ ; if incremental.vaue = TRUE.

aug.incremental.value

the estimate,  $\hat{IV}_S(t, t_0)^{AUG}$ ; if incremental.vaue = TRUE.

aug.delta.t.var

the variance estimate of  $\hat{\Delta}_T(t, t_0)^{AUG}$ ; if incremental value = TRUE.

aug.R.t.var the variance estimate of  $\hat{R}_T(t,t_0)^{AUG}$ ; if incremental.vaue = TRUE.

aug.incremental.value.var

the variance estimate of  $\hat{IV}_S(t, t_0)^{AUG}$ ; if incremental value = TRUE.

aug.conf.int.normal.delta.t

a vector of size 2; the 95% confidence interval for  $\hat{\Delta}_T(t, t_0)^{AUG}$  based on a normal approximation; if incremental vaue = TRUE.

aug.conf.int.quantile.delta.t

a vector of size 2; the 95% confidence interval for  $\hat{\Delta}_T(t,t_0)^{AUG}$  based on sample quantiles of the perturbed values; if incremental value = TRUE.

aug.conf.int.normal.R.t

a vector of size 2; the 95% confidence interval for  $\hat{R}_T(t,t_0)^{AUG}$  based on a normal approximation; if incremental vaue = TRUE.

aug.conf.int.quantile.R.t

a vector of size 2; the 95% confidence interval for  $\hat{R}_T(t,t_0)^{AUG}$  based on sample quantiles of the perturbed values; if incremental value = TRUE.

Aug.R.s.surv.estimate 5

```
aug.conf.int.fieller.R.t a vector of size 2; the 95% confidence interval for \hat{R}_T(t,t_0)^{AUG} based on Fieller's approach, described above; if incremental.vaue = TRUE.  
aug.conf.int.normal.iv a vector of size 2; the 95% confidence interval for \hat{IV}_S(t,t_0)^{AUG} based on a normal approximation; if incremental.vaue = TRUE.  
aug.conf.int.quantile.iv a vector of size 2; the 95% confidence interval for \hat{IV}_S(t,t_0)^{AUG} based on sample quantiles of the perturbed values; if incremental.vaue = TRUE.
```

#### Note

If the treatment effect is not significant, the user will receive the following message: "Warning: it looks like the treatment effect is not significant; may be difficult to interpret the residual treatment effect in this setting". If the treatment effect is negative, the user will receive the following message: "Warning: it looks like you need to switch the treatment groups" as this package assumes throughout that larger values of the event time are better. If the observed support of the surrogate marker for the control group is outside the observed support of the surrogate marker for the treatment group, the user will receive the following message: "Warning: observed supports do not appear equal, may need to consider a transformation or extrapolation".

#### Author(s)

Layla Parast

#### References

Tian L, Cai T, Zhao L, Wei L. On the covariate-adjusted estimation for an overall treatment difference with data from a randomized comparative clinical trial. Biostatistics 2012; 13(2): 256-273.

Garcia TP, Ma Y, Yin G. Efficiency improvement in a class of survival models through model-free covariate incorporation. Lifetime Data Analysis 2011; 17(4): 552-565.

Zhang M, Tsiatis AA, Davidian M. Improving efficiency of inferences in randomized clinical trials using auxiliary covariates. Biometrics 2008; 64(3): 707-715.

Parast L, Cai T and Tian L. Evaluating Surrogate Marker Information using Censored Data. Under Review.

```
#computationally intensive
#Aug.R.s.surv.estimate(xone = d_example_surv$x1, xzero = d_example_surv$x0,
#deltaone = d_example_surv$delta1, deltazero = d_example_surv$delta0,
#sone = d_example_surv$s1, szero = d_example_surv$s0, t=3, landmark = 1,
#basis.delta.one = d_example_surv$z1 , basis.delta.zero = d_example_surv$z0)
```

6 delta.estimate

delta.estimate

Calculates treatment effect

#### **Description**

This function calculates the treatment effect estimate, the difference in the average outcome in the treatment group minus the control group. This function is intended to be used for a fully observed continuous outcome. The user can also request a variance estimate, estimated using perturbating-resampling, and a 95% confidence interval. If a confidence interval is requested two versions are provided: a normal approximation based interval and a quantile based interval, both use perturbation-resampling.

## Usage

```
delta.estimate(yone,yzero, var = FALSE, conf.int = FALSE, weight = NULL,
weight.perturb = NULL)
```

#### **Arguments**

yone numeric vector; primary outcome for treated observations. yzero numeric vector; primary outcome for control observations.

var TRUE or FALSE; indicates whether a variance estimate for delta is requested,

default is FALSE.

conf.int TRUE or FALSE; indicates whether a 95% confidence interval for delta is re-

quested, default is FALSE.

weight a n1+n0 by x matrix of weights where n1 = length of yone and n0 = length

of yzero, default is null; generally not supplied by use but only used by other

functions.

weight.perturb a n1+n0 by x matrix of weights where n1 = length of yone and n0 = length of

yzero, default is null; generally used for confidence interval construction and

may be supplied by user.

#### **Details**

Let  $Y^{(1)}$  and  $Y^{(0)}$  denote the primary outcome under the treatment and primary outcome under the control,respectively. The treatment effect,  $\Delta$ , is the expected difference in  $Y^{(1)}$  compared to  $Y^{(0)}$ ,  $\Delta = E(Y^{(1)} - Y^{(0)})$ . We estimate  $\Delta$  as

$$\hat{\Delta} = n_1^{-1} \sum_{i=1}^{n_1} Y_{1i} - n_0^{-1} \sum_{i=1}^{n_0} Y_{0i}$$

where  $Y_{1i}$  is the observed primary outcome for person i in the treated group,  $Y_{0i}$  is the observed primary outcome for person i in the control group, and  $n_1$  and  $n_0$  are the number of individuals in the treatment and control group, respectively. Randomized treatment assignment is assumed throughout this package.

Variance estimation and confidence interval construction are performed using perturbation-resampling. Specifically, let  $\left\{V^{(b)}=(V_{11}^{(b)},...V_{1n_1}^{(b)},V_{01}^{(b)},...V_{0n_0}^{(b)})^T,b=1,....,D\right\}$  be  $n\times D$  independent copies of a positive random variables V from a known distribution with unit mean and unit variance. Let

$$\hat{\Delta}^{(b)} = \frac{\sum_{i=1}^{n_1} V_{1i}^{(b)} Y_{1i}}{\sum_{i=1}^{n_1} V_{1i}^{(b)}} - \frac{\sum_{i=1}^{n_0} V_{0i}^{(b)} Y_{0i}}{\sum_{i=1}^{n_0} V_{0i}^{(b)}}.$$

delta.s.estimate 7

The variance of  $\hat{\Delta}$  is obtained as the empirical variance of  $\{\hat{\Delta}^{(b)}, b=1,...,D\}$ . In this package, we use weights generated from an Exponential(1) distribution and use D=500. We construct two versions of the 95% confidence interval for  $\hat{\Delta}$ : one based on a normal approximation confidence interval using the estimated variance and another taking the 2.5th and 97.5th empirical percentiles of  $\hat{\Delta}^{(b)}$ .

## Value

A list is returned:

delta the estimate,  $\hat{\Delta}$ , described above.

var the variance estimate of  $\hat{\Delta}$ ; if var = TRUE or conf.int = TRUE.

conf.int.normal

a vector of size 2; the 95% confidence interval for  $\hat{\Delta}$  based on a normal approximation of the second confidence interval for  $\hat{\Delta}$ 

imation; if conf.int = TRUE.

conf.int.quantile

a vector of size 2; the 95% confidence interval for  $\hat{\Delta}$  based on sample quantiles of the perturbed values, described above; if conf.int = TRUE.

#### Author(s)

Layla Parast

## **Examples**

```
data(d_example)
names(d_example)
delta.estimate(yone=d_example$y1, yzero=d_example$y0)
```

delta.s.estimate

Calculates model-based or robust residual treatment effect

# **Description**

This function calculates the model-based or robust estimate of the residual treatment effect i.e. the hypothetical treatment effect if the distribution of the surrogate in the treatment group looks like the distribution of the surrogate in the control group. Ideally, this function is only used as a helper function and is not directly called.

## Usage

```
delta.s.estimate(sone, szero, yone, yzero, weight.perturb = NULL, number="single",
type="robust", warn.te = FALSE, warn.support = FALSE, extrapolate = FALSE,
transform = FALSE)
```

# **Arguments**

sone

numeric vector or matrix; surrogate marker for treated observations, assumed to be continuous. If there are multiple surrogates then this should be a matrix with  $n_1$  (number of treated observations) rows and n.s (number of surrogate markers) columns.

8 delta.s.estimate

szero numeric vector or matrix; surrogate marker for control observations, assumed to

be continuous. If there are multiple surrogates then this should be a matrix with  $n_0$  (number of control observations) rows and n.s (number of surrogate markers)

columns.

yone numeric vector; primary outcome for treated observations.

yzero numeric vector; primary outcome for control observations.

weight.perturb a  $n_1 + n_0$  by x matrix of weights where  $n_1 = \text{length of yone}$  and  $n_0 = \text{length of}$ 

yzero; generally used for variance estimation and confidence interval construc-

tion, default is null.

number specifies the number of surrogate markers; choices are "multiple" or "single",

default is "single".

type specifies the type of estimation; choices are "robust" or "model", default is "ro-

bust".

warn.te value passed from R.s. estimate function to control warnings; user does not need

to specify.

warn.support value passed from R.s. estimate function to control warnings; user does not need

to specify.

extrapolate TRUE or FALSE; indicates whether the user wants to use extrapolation.

transform TRUE or FALSE; indicates whether the user wants to use a transformation for

the surrogate marker.

#### **Details**

Details are included in the documentation for R.s. estimate.

## Value

 $\hat{\Delta}_S$ , the model-based or robust residual treatment effect estimate.

#### Note

If the treatment effect is not significant, the user will receive the following message: "Warning: it looks like the treatment effect is not significant; may be difficult to interpret the residual treatment effect in this setting". If the treatment effect is negative, the user will receive the following message: "Warning: it looks like you need to switch the treatment groups" as this package assumes throughout that higher values are better. In the single marker case with the robust estimation approach, if the observed support of the surrogate marker for the control group is outside the observed support of the surrogate marker for the treatment group, the user will receive the following message: "Warning: observed supports do not appear equal, may need to consider a transformation or extrapolation".

# Author(s)

Layla Parast

# References

Parast, L., McDermott, M., Tian, L. (2015). Robust estimation of the proportion of treatment effect explained by surrogate marker information. Statistics in Medicine, 35(10):1637-1653.

Wang, Y., & Taylor, J. M. (2002). A measure of the proportion of treatment effect explained by a surrogate marker. Biometrics, 58(4), 803-812.

delta.s.surv.estimate 9

#### **Examples**

```
data(d_example)
names(d_example)
delta.s.estimate(yone=d_example$y1, yzero=d_example$y0, sone=d_example$s1.a, szero=
d_example$s0.a, number = "single", type = "robust")
delta.s.estimate(yone=d_example$y1, yzero=d_example$y0, sone=d_example$s1.a, szero=
d_example$s0.a, number = "single", type = "model")
delta.s.estimate(yone=d_example$y1, yzero=d_example$y0, sone=cbind(d_example$s1.a,
d_example$s1.b, d_example$s1.c), szero=cbind(d_example$s0.a, d_example$s0.b, d_example$s0.c),
number = "multiple", type = "robust")
delta.s.estimate(yone=d_example$y1, yzero=d_example$y0, sone=cbind(d_example$s1.a,
d_example$s1.b, d_example$s1.c), szero=cbind(d_example$s0.a, d_example$s0.b, d_example$s0.c),
number = "multiple", type = "model")
```

delta.s.surv.estimate Calculates robust residual treatment effect accounting for surrogate marker information measured at a specified time and primary outcome information up to that specified time

## **Description**

This function calculates the robust estimate of the residual treatment effect accounting for surrogate marker information measured at  $t_0$  and primary outcome information up to  $t_0$  i.e. the hypothetical treatment effect if both the surrogate marker distribution at  $t_0$  and survival up to  $t_0$  in the treatment group look like the surrogate marker distribution and survival up to  $t_0$  in the control group. Ideally this function is only used as a helper function and is not directly called.

# Usage

```
delta.s.surv.estimate(xone, xzero, deltaone, deltazero, sone, szero, t,
weight.perturb = NULL, landmark, extrapolate = FALSE, transform = FALSE)
```

## **Arguments**

xone numeric vector, the observed event times in the treatment group, $X = m$ where T is the time of the primary outcome and C is the censoring time.	in(T,C)
numeric vector, the observed event times in the control group, $X = m$ where T is the time of the primary outcome and C is the censoring time.	in(T,C)
deltaone numeric vector, the event indicators for the treatment group, $D = I(T < C)$ T is the time of the primary outcome and C is the censoring time.	) where
deltazero numeric vector, the event indicators for the control group, $D = I(T < C)$ v is the time of the primary outcome and C is the censoring time.	where T
sone numeric vector; surrogate marker measurement at $t_0$ for treated observations assumed to be continuous. If $X_{1i} < t_0$ , then the surrogate marker measurement should be NA.	
szero numeric vector; surrogate marker measurement at $t_0$ for control observations assumed to be continuous. If $X_{1i} < t_0$ , then the surrogate marker measurement at $t_0$ for control observations.	
should be NA.	ircincin

10 delta.s.surv.estimate

weight.perturb weights used for perturbation resampling.

landmark time  $t_0$  or time of surrogate marker measurement.

extrapolate TRUE or FALSE; indicates whether the user wants to use extrapolation.

transform TRUE or FALSE; indicates whether the user wants to use a transformation for

the surrogate marker.

#### **Details**

Details are included in the documentation for R.s.surv.estimate.

#### Value

 $\hat{\Delta}_S(t,t_0)$ , the robust residual treatment effect estimate accounting for surrogate marker information measured at  $t_0$  and primary outcome information up to  $t_0$ .

#### Note

If the treatment effect is not significant, the user will receive the following message: "Warning: it looks like the treatment effect is not significant; may be difficult to interpret the residual treatment effect in this setting". If the treatment effect is negative, the user will receive the following message: "Warning: it looks like you need to switch the treatment groups" as this package assumes throughout that larger values of the event time are better. If the observed support of the surrogate marker for the control group is outside the observed support of the surrogate marker for the treatment group, the user will receive the following message: "Warning: observed supports do not appear equal, may need to consider a transformation or extrapolation".

## Author(s)

Layla Parast

# References

Parast L, Cai T and Tian L. Evaluating Surrogate Marker Information using Censored Data. Under Review.

```
data(d_example_surv)
names(d_example_surv)
```

delta.surv.estimate 11

delta.surv.estimate Calculates treatment effect in a survival setting

# **Description**

This function calculates the treatment effect in the survival setting i.e. the difference in survival at time t between the treatment group and the control group. The user can also request a variance estimate, estimated using perturbating-resampling, and a 95% confidence interval. If a confidence interval is requested two versions are provided: a normal approximation based interval and a quantile based interval, both use perturbation-resampling.

# Usage

```
delta.surv.estimate(xone, xzero, deltaone, deltazero, t, var = FALSE, conf.int
= FALSE, weight = NULL, weight.perturb = NULL)
```

## **Arguments**

xone	numeric vector, the observed event times in the treatment group, $X = \min(T,C)$ where T is the time of the primary outcome and C is the censoring time.
xzero	numeric vector, the observed event times in the control group, $X = \min(T,C)$ where T is the time of the primary outcome and C is the censoring time.
deltaone	numeric vector, the event indicators for the treatment group, $D = I(T < C)$ where $T$ is the time of the primary outcome and $C$ is the censoring time.
deltazero	numeric vector, the event indicators for the control group, $D = I(T < C)$ where $T$ is the time of the primary outcome and $C$ is the censoring time.
t	the time of interest.
var	TRUE or FALSE; indicates whether a variance estimate for delta is requested, default is FALSE.
conf.int	TRUE or FALSE; indicates whether a $95\%$ confidence interval for delta is requested, default is FALSE.
weight	a $n_1 + n_0$ by $x$ matrix of weights where $n_1 = \text{sample size}$ in treatment group and $n_0 = \text{sample size}$ in the control group, default is null; generally not supplied by use but only used by other functions.
weight.perturb	a $n_1 + n_0$ by $x$ matrix of weights where $n_1$ =sample size in treatment group and $n_0$ = sample size in the control group, default is null; generally used for confidence interval construction and may be supplied by user.

# Details

Let G be the binary treatment indicator with G=1 for treatment and G=0 for control and we assume throughout that subjects are randomly assigned to a treatment group at baseline. Let T denote the time of the primary outcome of interest, death for example. We use potential outcomes notation such that  $T^{(g)}$  denotes the time of the primary outcome under treatment G=g. We define the treatment effect,  $\Delta(t)$ , as the difference in survival rates by time t under treatment versus control,

$$\Delta(t) = E\{I(T^{(1)} > t)\} - E\{I(T^{(0)} > t)\} = P(T^{(1)} > t) - P(T^{(0)} > t)$$

12 delta.surv.estimate

where  $t > t_0$ 

Due to censoring, our data consist of  $n_1$  observations  $\{(X_{1i},\delta_{1i}),i=1,...,n_1\}$  from the treatment group G=1 and  $n_0$  observations  $\{(X_{0i},\delta_{0i}),i=1,...,n_0\}$  from the control group G=0 where  $X_{gi}=\min(T_{gi},C_{gi}),\,\delta_{gi}=I(T_{gi}< C_{gi}),$  and  $C_{gi}$  denotes the censoring time for g=1,0, for individual i. Throughout, we estimate the treatment effect  $\Delta(t)$  as

$$\hat{\Delta}(t) = n_1^{-1} \sum_{i=1}^{n_1} \frac{I(X_{1i} > t)}{\hat{W}_1^C(t)} - n_0^{-1} \sum_{i=1}^{n_0} \frac{I(X_{0i} > t)}{\hat{W}_0^C(t)}$$

where  $\hat{W}_{q}^{C}(\cdot)$  is the Kaplan-Meier estimator of survival for censoring for g=1,0.

Variance estimation and confidence interval construction are performed using perturbation-resampling. Specifically, let  $\left\{V^{(b)}=(V_{11}^{(b)},...V_{1n_1}^{(b)},V_{01}^{(b)},...V_{0n_0}^{(b)})^T,b=1,....,D\right\}$  be  $n\times D$  independent copies of a positive random variables V from a known distribution with unit mean and unit variance. Let

$$\hat{\Delta}^{(b)}(t) = \frac{\sum_{i=1}^{n_1} V_{1i}^{(b)} I(X_{1i} > t)}{\sum_{i=1}^{n_1} V_{1i}^{(b)} \hat{W}_1^{C(b)}(t)} - \frac{\sum_{i=1}^{n_0} V_{0i}^{(b)} I(X_{0i} > t)}{\sum_{i=1}^{n_0} V_{0i}^{(b)} \hat{W}_0^{C(b)}(t)}.$$

In this package, we use weights generated from an Exponential(1) distribution and use D=500. The variance of  $\hat{\Delta}(t)$  is obtained as the empirical variance of  $\{\hat{\Delta}(t)^{(b)}, b=1,...,D\}$ . We construct two versions of the 95% confidence interval for  $\hat{\Delta}(t)$ : one based on a normal approximation confidence interval using the estimated variance and another taking the 2.5th and 97.5th empirical percentiles of  $\hat{\Delta}(t)^{(b)}$ .

## Value

A list is returned:

delta the estimate,  $\hat{\Delta}(t)$ , described above.

the variance estimate of  $\hat{\Delta}(t)$ ; if var = TRUE or conf.int = TRUE.

conf.int.normal

a vector of size 2; the 95% confidence interval for  $\hat{\Delta}(t)$  based on a normal approximation; if conf.int = TRUE.

conf.int.quantile

a vector of size 2; the 95% confidence interval for  $\hat{\Delta}(t)$  based on sample quantiles of the perturbed values, described above; if conf.int = TRUE.

#### Author(s)

Layla Parast

```
data(d_example_surv)
names(d_example_surv)
delta.surv.estimate(xone = d_example_surv$x1, xzero = d_example_surv$x0,
deltaone = d_example_surv$delta1, deltazero = d_example_surv$delta0, t = 3)
```

delta.t.surv.estimate 13

delta.t.surv.estimate Calculates robust residual treatment effect accounting only for primary outcome information up to a specified time

# Description

This function calculates the robust estimate of the residual treatment effect accounting only for primary outcome information up to  $t_0$  i.e. the hypothetical treatment effect if survival up to  $t_0$  in the treatment group looks like survival up to  $t_0$  in the control group. Ideally this function is only used as a helper function and is not directly called.

## Usage

delta.t.surv.estimate(xone, xzero, deltaone, deltazero, t, weight.perturb = NULL,
landmark)

## **Arguments**

xone	numeric vector, the observed event times in the treatment group, $X = \min(T,C)$ where T is the time of the primary outcome and C is the censoring time.
xzero	numeric vector, the observed event times in the control group, $X = min(T,C)$ where T is the time of the primary outcome and C is the censoring time.
deltaone	numeric vector, the event indicators for the treatment group, $D = I(T < C)$ where T is the time of the primary outcome and C is the censoring time.
deltazero	numeric vector, the event indicators for the control group, $D = I(T < C)$ where T is the time of the primary outcome and C is the censoring time.
t	the time of interest.
weight.perturb	weights used for perturbation resampling.
landmark	the landmark time $t_0$ or time of surrogate marker measurement.

# **Details**

Details are included in the documentation for R.t.surv.estimate.

## Value

 $\hat{\Delta}_T(t,t_0)$ , the robust residual treatment effect estimate accounting only for survival up to  $t_0$ .

## Note

If the treatment effect is not significant, the user will receive the following message: "Warning: it looks like the treatment effect is not significant; may be difficult to interpret the residual treatment effect in this setting". If the treatment effect is negative, the user will receive the following message: "Warning: it looks like you need to switch the treatment groups" as this package assumes throughout that larger values of the event time are better.

## Author(s)

Layla Parast

14 d\_example

#### References

Parast L, Cai T and Tian L. Evaluating Surrogate Marker Information using Censored Data. Under Review.

# **Examples**

```
data(d_example_surv)
names(d_example_surv)
```

d\_example

Hypothetical data

# **Description**

Hypothetical data to be used in examples.

# Usage

```
data(d_example)
```

## **Format**

A list with 8 elements representing 500 observations from a control group and 500 observations from a treatment group:

- s1.a First surrogate marker measurement for treated observations.
- s1.b Second surrogate marker measurement for treated observations.
- s1.c Third surrogate marker measurement for treated observations.
- y1 Primary outcome for treated observations.
- s0.a First surrogate marker measurement for control observations.
- s0.b Second surrogate marker measurement for control observations.
- s0.c Third surrogate marker measurement for control observations.
- yo Primary outcome for control observations.

```
data(d_example)
names(d_example)
```

d\_example\_surv 15

d\_example\_surv

Hypothetical survival data

# Description

Hypothetical survival data to be used in examples.

#### Usage

```
data(d_example_surv)
```

#### **Format**

A list with 8 elements representing 500 observations from a control group and 500 observations from a treatment group:

- s1 Surrogate marker measurement for treated observations; this marker is measured at time = 0.5. For observations that experience the primary outcome or are censored before 0.5, this value is NA.
- x1 The observed event or censoring time for treated observations; X = min(T, C) where T is the time of the primary outcome and C is the censoring time.
- delta1 The indicator identifying whether the treated observation was observed to have the event or was censored; D = 1\*(T < C) where T is the time of the primary outcome and C is the censoring time.
- so Surrogate marker measurement for control observations; this marker is measured at time = 0.5. For observations that experience the primary outcome or are censored before 0.5, this value is NA.
- x0 The observed event or censoring time for control observations; X = min(T, C) where T is the time of the primary outcome and C is the censoring time.
- delta0 The indicator identifying whether the control observation was observed to have the event or was censored; D = 1\*(T < C) where T is the time of the primary outcome and C is the censoring time.
- z1 A baseline covariate value for treated observations.
- z0 A baseline covariate value for control observations.

```
data(d_example_surv)
names(d_example_surv)
```

16 fieller.ci

· ·				
fie	וב	IΔr	^	$\sim 1$
1 7 2	ΞΙ.	TCI		-

Constructs Fieller's confidence interval.

# **Description**

Constructs Fieller's confidence interval.

## Usage

```
fieller.ci(perturb.delta.s, perturb.delta, delta.s, delta)
```

## **Arguments**

perturb.delta.s

numeric vector; the perturbed values for  $\hat{\Delta}_S$ , the residual treatment effect estimate, used in variance estimation and confidence interval construction.

perturb. delta numeric vector; the perturbed values for  $\hat{\Delta}$ , the treatment effect estimate, used

in variance estimation and confidence interval construction.

delta.s the residual treatment effect,  $\Delta_S$ , estimate,  $\hat{\Delta}_S$ .

delta the treatment effect,  $\Delta$ , estimate,  $\hat{\Delta}$ .

## **Details**

See documention for R.s.estimate for more detail.

# Value

Returns a vector of length 2, lower bound of the 95% confidence interval and upper bound of the 95% confidence interval.

#### Author(s)

Layla Parast

## References

Fieller, Edgar C. (1954). Some problems in interval estimation. Journal of the Royal Statistical Society. Series B (Methodological), 175-185.

Fieller, E. C. (1940). The biological standardization of insulin. Supplement to the Journal of the Royal Statistical Society, 1-64.

Parast, L., McDermott, M., Tian, L. (2016). Robust estimation of the proportion of treatment effect explained by surrogate marker information. Statistics in Medicine, 35(10):1637-1653.

me.variance.estimate 17

me.variance.estimate Estimates measurement error variance given replicate data.

## **Description**

Estimates measurement error variance given replicate data using a simple components of variance analysis.

# Usage

me.variance.estimate(replicates)

# **Arguments**

replicates

matrix of data where each row indicates a subject and each column is a replicated measurement; columns can have NAs when subjects have different numbers of measurements.

#### **Details**

Estimates measurement error variance given replicate data using a simple components of variance analysis.

## Value

estimate of measurement error variance

## Author(s)

Layla Parast

## References

Carroll, R. J., Ruppert, D., Crainiceanu, C. M., and Stefanski, L. A. (2006). Measurement error in nonlinear models: a modern perspective. Chapman and Hall/CRC.

Parast, L., Garcia, TP, Prentice, RL, Carroll, RJ (2019+). Robust Methods to Correct for Measurement Error when Evaluating a Surrogate Marker. Under Review.

R.s.estimate

Calculates the proportion of treatment effect explained

# **Description**

This function calculates the proportion of treatment effect on the primary outcome explained by the treatment effect on the surrogate marker(s). This function is intended to be used for a fully observed continuous outcome. The user can also request a variance estimate and a 95% confidence interval, both estimated using perturbating-resampling. If a confidence interval is requested three versions are provided: a normal approximation based interval, a quantile based interval, and Fieller's confidence interval.

#### Usage

R.s.estimate(sone, szero, yone, yzero, var = FALSE, conf.int = FALSE,
weight.perturb = NULL, number = "single", type = "robust",extrapolate = FALSE,
transform = FALSE)

# **Arguments**

sone	numeric vector or matrix; surrogate marker for treated observations, assumed to be continuous. If there are multiple surrogates then this should be a matrix with $n_1$ (number of treated observations) rows and n.s (number of surrogate markers) columns.
szero	numeric vector; surrogate marker for control observations, assumed to be continuous. If there are multiple surrogates then this should be a matrix with $n_0$ (number of control observations) rows and n.s (number of surrogate markers) columns.
yone	numeric vector; primary outcome for treated observations, assumed to be continuous.
yzero	numeric vector; primary outcome for control observations, assumed to be continuous.
var	TRUE or FALSE; indicates whether a variance estimate is requested, default is FALSE.
conf.int	TRUE or FALSE; indicates whether a 95% confidence interval is requested, default is FALSE
weight.perturb	a $n_1 + n_0$ by $x$ matrix of weights where $n_1 = \text{length of yone}$ and $n_0 = \text{length of yzero}$ ; used for perturbation-resampling, default is null.
number	specifies the number of surrogate markers; choices are "multiple" or "single", default is "single"
type	specifies the type of estimation; choices are "robust" or "model" or "freedman", default is "robust"
extrapolate	TRUE or FALSE; indicates whether the user wants to use extrapolation.
transform	TRUE or FALSE; indicates whether the user wants to use a transformation for the surrogate marker.

## **Details**

Let  $Y^{(1)}$  and  $Y^{(0)}$  denote the primary outcome under the treatment and primary outcome under the control,respectively. Let  $S^{(1)}$  and  $S^{(0)}$  denote the surrogate marker under the treatment and the surrogate marker under the control,respectively. The residual treatment effect is defined as

$$\Delta_S = \int_{-\infty}^{\infty} E(Y^{(1)}|S^{(1)} = s)dF_0(s) - \int_{-\infty}^{\infty} E(Y^{(0)}|S^{(0)} = s)dF_0(s),$$

where  $\Delta_S(s) = E(Y^{(1)}|S^{(1)} = s) - E(Y^{(0)}|S^{(0)} = s)$  and  $F_0(\cdot)$  is the marginal cumulative distribution function of  $S^{(0)}$ , the surrogate marker measure under the control. The proportion of treatment effect explained by the surrogate marker, which we denote by  $R_S$ , can be expressed using a contrast between  $\Delta_S$  and  $\Delta$ :

$$R_S = {\Delta - \Delta_S}/{\Delta} = 1 - \Delta_S/{\Delta}.$$

The definition and estimation of  $\Delta$  is described in the delta.estimate documentation.

R.s.estimate 19

A flexible model-based approach to estimate  $\Delta_S$  in the single marker setting is to specify:

$$E(S^{(0)}) = \alpha_0 \quad \text{and} \quad E(S^{(1)}) - E(S^{(0)}) = \alpha_1,$$
 
$$E(Y^{(0)}|S^{(0)}) = \beta_0 + \beta_1 S^{(0)} \quad \text{and} \quad E(Y^{(1)}|S^{(1)}) = (\beta_0 + \beta_2) + (\beta_1 + \beta_3) S^{(1)}.$$

It can be shown that when these models hold,  $\Delta_S=\beta_2+\beta_3\alpha_0$ . Thus, reasonable estimates for  $\Delta_S$  and  $R_S$  using this approach would be  $\hat{\Delta}_S=\hat{\beta}_2+\hat{\beta}_3\hat{\alpha}_0$  and  $\hat{R}_S=1-\hat{\Delta}_S/\hat{\Delta}$ .

For robust estimation of  $\Delta_S$  in the single marker setting, we estimate  $\mu_1(s) = E(Y^{(1)}|S^{(1)} = s)$  nonparametrically using kernel smoothing:

$$\hat{\mu}_1(s) = \frac{\sum_{i=1}^{n_1} K_h (S_{1i} - s) Y_{1i}}{\sum_{i=1}^{n_1} K_h (S_{1i} - s)}$$

where  $S_{1i}$  is the observed  $S^{(1)}$  for person  $i, Y_{1i}$  is the observed  $Y^{(1)}$  for person  $i, K(\cdot)$  is a smooth symmetric density function with finite support,  $K_h(\cdot) = K(\cdot/h)/h$  and h is a specified bandwidth. As in most nonparametric functional estimation procedures, the choice of the smoothing parameter h is critical. To eliminate the impact of the bias of the conditional mean function on the resulting estimator, we require the standard undersmoothing assumption of  $h = O(n_1^{-\delta})$  with  $\delta \in (1/4, 1/3)$ . To obtain an appropriate h we first use bw.nrd to obtain  $h_{opt}$ ; and then we let  $h = h_{opt}n_1^{-c_0}$  with  $c_0 = 0.25$ . We then estimate  $\Delta_S$  as

$$\hat{\Delta}_S = \sum_{i=1}^{n_0} \frac{\hat{\mu}_1(S_{0i}) - Y_{0i}}{n_0}$$

where  $S_{0i}$  is the observed  $S^{(0)}$  for person i and  $Y_{0i}$  is the observed  $Y^{(0)}$  for person i. Lastly, we estimate  $R_S$  as  $\hat{R}_S = 1 - \hat{\Delta}_S/\hat{\Delta}$ .

This function also allows for estimation of  $R_S$  using Freedman's approach. Let Y denote the primary outcome, S denote the surrogate marker, and G denote the treatment group (0 for control, 1 for treatment). Freedman's approach to calculating the proportion of treatment effect explained by the surrogate marker is to fit the following two regression models:

$$E(Y|G) = \gamma_0 + \gamma_1 I(G=1)$$
 and  $E(Y|G,S) = \gamma_{0S} + \gamma_{1S} I(G=1) + \gamma_{2S} S$ 

and estimating the proportion of treatment effect explained, denoted by  $R_S$ , as  $1 - \hat{\gamma}_{1S}/\hat{\gamma}_1$ .

This function also estimates  $R_S$  in a multiple marking setting. A flexible model-based approach to estimate  $\Delta_S$  in the multiple marker setting is to specify models for E(Y|G,S) and  $E(S_j|G)$  for each  $S_j$  in  $S=\{S_1,...S_p\}$  (where p is the number of surrogate markers). Without loss of generality, consider the case where there are three surrogate markers,  $S=\{S_1,S_2,S_3\}$  and one specifies the following linear models:

$$E(Y^{(0)}|S^{(0)}) = \beta_0 + \beta_1 S_1^{(0)} + \beta_2 S_2^{(0)} + \beta_3 S_3^{(0)}$$

$$E(Y^{(1)}|S^{(1)}) = (\beta_0 + \beta_4) + (\beta_1 + \beta_5) S_1^{(1)} + (\beta_2 + \beta_6) S_2^{(1)} + (\beta_3 + \beta_7) S_3^{(1)}$$

$$E(S_j^{(0)}) = \alpha_j, \quad j = 1, 2, 3.$$

It can be shown that when these models hold

$$\Delta_S = \beta_4 + \beta_5 \alpha_1 + \beta_6 \alpha_2 + \beta_7 \alpha_3.$$

Thus, reasonable estimates for  $\Delta_S$  and  $R_S$  here would be easily obtained by replacing the unknown regression coefficients in the models above by their consistent estimators.

For robust estimation of S  $\Delta_S$  in the multiple marker setting, we use a two-stage procedure combining the model-based approach and the nonparametric estimation procedure from the single marker setting. Specifically, we use a working semiparametric model:

$$E(Y^{(1)}|S^{(1)} = S) = \beta_0 + \beta_1 S_1^{(1)} + \beta_2 S_2^{(1)} + \beta_3 S_3^{(1)}$$

and define  $Q^{(1)} = \hat{\beta}_0 + \hat{\beta}_1 S_1^{(1)} + \hat{\beta}_2 S_2^{(1)} + \hat{\beta}_3 S_3^{(1)}$  and  $Q^{(0)} = \hat{\beta}_0 + \hat{\beta}_1 S_1^{(0)} + \hat{\beta}_2 S_2^{(0)} + \hat{\beta}_3 S_3^{(0)}$  to reduce the dimension of S in the first stage and in the second stage, we apply the robust approach used in the single marker setting to estimate its surrogacy.

To use Freedman's approach in the presence of multiple markers, the markers are simply additively entered into the second regression model.

Variance estimation and confidence interval construction are performed using perturbation-resampling. Specifically, let  $\left\{V^{(b)}=(V_{11}^{(b)},...V_{1n_1}^{(b)},V_{01}^{(b)},...V_{0n_0}^{(b)})^T,b=1,....,D\right\}$  be  $n\times D$  independent copies of a positive random variables V from a known distribution with unit mean and unit variance. Let

$$\hat{\Delta}^{(b)} = \frac{\sum_{i=1}^{n_1} V_{1i}^{(b)} Y_{1i}}{\sum_{i=1}^{n_1} V_{1i}^{(b)}} - \frac{\sum_{i=1}^{n_0} V_{0i}^{(b)} Y_{0i}}{\sum_{i=1}^{n_0} V_{0i}^{(b)}}.$$

The variance of  $\hat{\Delta}$  is obtained as the empirical variance of  $\{\hat{\Delta}^{(b)}, b=1,...,D\}$ . In this package, we use weights generated from an Exponential(1) distribution and use D=500. Variance estimates for  $\hat{\Delta}_S$  and  $\hat{R}_S$  are calculated similarly. We construct two versions of the 95% confidence interval for each estimate: one based on a normal approximation confidence interval using the estimated variance and another taking the 2.5th and 97.5th empirical percentile of the perturbed quantities. In addition, we use Fieller's method to obtain a third confidence interval for  $R_S$  as

$$\left\{1 - r : \frac{(\hat{\Delta}_S - r\hat{\Delta})^2}{\hat{\sigma}_{11} - 2r\hat{\sigma}_{12} + r^2\hat{\sigma}_{22}} \le c_\alpha\right\},\,$$

where  $\hat{\Sigma} = (\hat{\sigma}_{ij})_{1 \leq i,j \leq 2}$  and  $c_{\alpha}$  is the  $(1 - \alpha)$ th percentile of

$$\left\{ \frac{\{\hat{\Delta}_S^{(b)} - (1 - \hat{R}_S)\hat{\Delta}^{(b)}\}^2}{\hat{\sigma}_{11} - 2(1 - \hat{R}_S)\hat{\sigma}_{12} + (1 - \hat{R}_S)^2\hat{\sigma}_{22}}, b = 1, \dots, C \right\}$$

where  $\alpha = 0.05$ .

Note that if the observed supports for S are not the same, then  $\hat{\mu}_1(s)$  for  $S_{0i}=s$  outside the support of  $S_{1i}$  may return NA (depending on the bandwidth). If extrapolation = TRUE, then the  $\hat{\mu}_1(s)$  values for these surrogate values are set to the closest non-NA value. If transform = TRUE, then  $S_{1i}$  and  $S_{0i}$  are transformed such that the new transformed values,  $S_{1i}^{tr}$  and  $S_{0i}^{tr}$  are defined as:  $S_{gi}^{tr} = F([S_{gi} - \mu]/\sigma)$  for g = 0, 1 where  $F(\cdot)$  is the cumulative distribution function for a standard normal random variable, and  $\mu$  and  $\sigma$  are the sample mean and standard deviation, respectively, of  $(S_{1i}, S_{0i})^T$ .

## Value

A list is returned:

R. s the estimate,  $\hat{R}_S$ , described above.

R.s.var the variance estimate of  $\hat{R}_S$ ; if var = TRUE or conf.int = TRUE.

conf.int.normal.R.s

a vector of size 2; the 95% confidence interval for  $\hat{R}_S$  based on a normal approximation; if conf.int = TRUE.

R.s.estimate 21

conf.int.quantile.R.s

a vector of size 2; the 95% confidence interval for  $\hat{R}_S$  based on sample quantiles of the perturbed values, described above; if conf.int = TRUE.

conf.int.fieller.R.s

a vector of size 2; the 95% confidence interval for  $\hat{R}_S$  based on Fieller's approach, described above; if conf.int = TRUE.

For all options other then "freedman", the following are also returned:

delta the estimate,  $\hat{\Delta}$ , described in delta.estimate documentation.

delta.s the estimate,  $\hat{\Delta}_S$ , described above.

delta.var the variance estimate of  $\hat{\Delta}$ ; if var = TRUE or conf.int = TRUE.

delta.s.var the variance estimate of  $\hat{\Delta}_S$ ; if var = TRUE or conf.int = TRUE.

conf.int.normal.delta

a vector of size 2; the 95% confidence interval for  $\hat{\Delta}$  based on a normal approximation; if conf.int = TRUE.

conf.int.quantile.delta

a vector of size 2; the 95% confidence interval for  $\hat{\Delta}$  based on sample quantiles of the perturbed values, described above; if conf.int = TRUE.

conf.int.normal.delta.s

a vector of size 2; the 95% confidence interval for  $\hat{\Delta}_S$  based on a normal approximation; if conf.int = TRUE.

conf.int.quantile.delta.s

a vector of size 2; the 95% confidence interval for  $\hat{\Delta}_S$  based on sample quantiles of the perturbed values, described above; if conf.int = TRUE.

## Note

If the treatment effect is not significant, the user will receive the following message: "Warning: it looks like the treatment effect is not significant; may be difficult to interpret the proportion of treatment effect explained in this setting". If the treatment effect is negative, the user will receive the following message: "Warning: it looks like you need to switch the treatment groups" as this package assumes throughout that higher values are better. In the single marker case with the robust estimation approach, if the observed support of the surrogate marker for the control group is outside the observed support of the surrogate marker for the treatment group, the user will receive the following message: "Warning: observed supports do not appear equal, may need to consider a transformation or extrapolation"

# Author(s)

Layla Parast

#### References

Freedman, L. S., Graubard, B. I., & Schatzkin, A. (1992). Statistical validation of intermediate endpoints for chronic diseases. Statistics in medicine, 11(2), 167-178.

Parast, L., McDermott, M., Tian, L. (2016). Robust estimation of the proportion of treatment effect explained by surrogate marker information. Statistics in Medicine, 35(10):1637-1653.

Wang, Y., & Taylor, J. M. (2002). A measure of the proportion of treatment effect explained by a surrogate marker. Biometrics, 58(4), 803-812.

Fieller, Edgar C. (1954). Some problems in interval estimation. Journal of the Royal Statistical Society. Series B (Methodological), 175-185.

Fieller, E. C. (1940). The biological standardization of insulin. Supplement to the Journal of the Royal Statistical Society, 1-64.

## **Examples**

```
data(d_example)
names(d_example)
R.s.estimate(yone=d_example$y1, yzero=d_example$y0, sone=d_example$s1.a, szero=d_example$s0.a,
number = "single", type = "robust")
R.s.estimate(yone=d_example$y1, yzero=d_example$y0, sone=cbind(d_example$s1.a,d_example$s1.b,
d_example$s1.c), szero=cbind(d_example$s0.a, d_example$s0.b, d_example$s0.c),
number = "multiple", type = "model")
```

R.s.estimate.me

Calculates the proportion of treatment effect explained correcting for measurement error in the surrogate marker

# **Description**

This function calculates the proportion of treatment effect on the primary outcome explained by the treatment effect on a surrogate marker, correcting for measurement error in the surrogate marker. This function is intended to be used for a fully observed continuous outcome. The user must specify what type of estimation they would like (parametric or nonparametric estimation of the proportion explained, denoted by R) and what estimator they would like (see below for details).

# Usage

```
R.s.estimate.me(sone, szero, yone, yzero, parametric = FALSE, estimator = "n", me.variance, extrapolate = TRUE, transform = FALSE, naive = FALSE, Ronly = TRUE)
```

# **Arguments**

sone	numeric vector or matrix; surrogate marker for treated observations, assumed to be continuous. If there are multiple surrogates then this should be a matrix with $n_1$ (number of treated observations) rows and n.s (number of surrogate markers) columns.
szero	numeric vector; surrogate marker for control observations, assumed to be continuous. If there are multiple surrogates then this should be a matrix with $n_0$ (number of control observations) rows and n.s (number of surrogate markers) columns.
yone	numeric vector; primary outcome for treated observations, assumed to be continuous.
yzero	numeric vector; primary outcome for control observations, assumed to be continuous.
parametric	TRUE or FALSE; indicates whether the user wants the parametric approach to be used (TRUE) or nonparametric (FALSE).
estimator	options are "d","q","n" for parametric and "q","n" for nonparametric; "d" stands for the disattenuated estimator, "q" stands for the SIMEX estimator with quadratic extrapolation, "n" stands for the SIMEX estimator with a nonlinear extrapolation.

me.variance the variance of the measurement error; must be provided.

extrapolate TRUE or FALSE; indicates whether the user wants to use extrapolation.

transform TRUE or FALSE; indicates whether the user wants to use a transformation for

the surrogate marker.

naive TRUE or FALSE; indicates whether the user wants the naive estimate (not cor-

recting for measurement error) to also be calculated

Ronly TRUE or FALSE; indicates whether the user wants only R (and corresponding

variance and confidence intervals) to be returned.

#### **Details**

Details can be found in Parast, L., Garcia, TP, Prentice, RL, Carroll, RJ (2019+). Robust Methods to Correct for Measurement Error when Evaluating a Surrogate Marker. Under Review.

Please email parast@rand.org if you would like a copy of this article.

#### Value

A list is returned:

R. naive the naive estimate of the proportion of treatment effect explained by the surro-

gate marker; only if naive = TRUE

R.naive.var the estimated variance of the naive estimate of the proportion of treatment effect

explained by the surrogate marker; only if naive = TRUE

R.naive.CI.normal

the 95% confidence interval using the normal approximation for the naive estimate of the proportion of treatment effect explained by the surrogate marker;

only if naive = TRUE

R.naive.CI.fieller

the 95% confidence interval using Fieller's approach for the naive estimate of the proportion of treatment effect explained by the surrogate marker; only if

naive = TRUE

B1star.naive the naive estimate of the adjusted regression coefficient for treatment; only if

naive = TRUE and Ronly = FALSE and parametric = TRUE

B1star.naive.var

the estimated variance of the naive estimate of the adjusted regression coefficient for treatment; only if naive = TRUE and Ronly = FALSE and parametric =

TRUE

B1star.naive.CI.normal

the 95% confidence interval using the normal approximation for the naive estimate of the adjusted regression coefficient for treatment; only if naive = TRUE

and Ronly = FALSE and parametric = TRUE

deltas.naive the naive estimate of the residual treatment effect; only if naive = TRUE and

Ronly = FALSE and parametric = FALSE

deltas.naive.var

the estimated variance of the naive estimate of the residual treatment effect; only if naive = TRUE and Ronly = FALSE and parametric = FALSE

deltas.naive.CI.normal

the 95% confidence interval using the normal approximation for the naive estimate of the residual treatment effect; only if naive = TRUE and Ronly = FALSE and parametric = FALSE

#### R.corrected.dis

the corrected disattenuated estimate of the proportion of treatment effect explained by the surrogate marker; only if parametric = TRUE and estimator = "d"

#### R.corrected.var.dis

the estimated variance of the corrected disattenuated estimate of the proportion of treatment effect explained by the surrogate marker; only if naive = TRUE

# R.corrected.CI.normal.dis

the 95% confidence interval using the normal approximation for the corrected disattenuated estimate of the proportion of treatment effect explained by the surrogate marker; only if parametric = TRUE and estimator ="d"

#### R.corrected.CI.fieller.dis

the 95% confidence interval using Fieller's approach for the corrected disattenuated estimate of the proportion of treatment effect explained by the surrogate marker; only if parametric = TRUE and estimator ="d"

## B1star.corrected.dis

the corrected disattenuated estimate of the adjusted regression coefficient for treatment; only if parametric = TRUE and estimator = "d" and Ronly = FALSE

# B1star.corrected.var.dis

the estimated variance of the corrected disattenuated estimate of the adjusted regression coefficient for treatment; only if parametric = TRUE and estimator = "d" and Ronly = FALSE

## B1star.corrected.CI.normal.dis

the 95% confidence interval using the normal approximation for the corrected disattenuated estimate of the adjusted regression coefficient for treatment; only if parametric = TRUE and estimator = "d" and Ronly = FALSE

# R.corrected.q the corrected SIMEX (quadratic) estimate of the proportion of treatment effect explained by the surrogate marker; only if estimator = "q"

# R.corrected.var.q

the estimated variance of the corrected SIMEX (quadratic) estimate of the proportion of treatment effect explained by the surrogate marker; only if estimator = "q"

# R.corrected.CI.normal.q

the 95% confidence interval using the normal approximation for the corrected SIMEX (quadratic) estimate of the proportion of treatment effect explained by the surrogate marker; only if estimator = "q"

# R.corrected.CI.fieller.q

the 95% confidence interval using Fieller's approach for the corrected SIMEX (quadratic) estimate of the proportion of treatment effect explained by the surrogate marker; only if estimator = "q"

#### B1star.corrected.q

the corrected SIMEX (quadratic) estimate of the adjusted regression coefficient for treatment; only if estimator = "q" and Ronly = FALSE and parametric = TRUE

# B1star.corrected.var.q

the estimated variance of the corrected SIMEX (quadratic) estimate of the adjusted regression coefficient for treatment; only if estimator = "q" and Ronly = FALSE and parametric = TRUE

# B1star.corrected.CI.normal.q

the 95% confidence interval using the normal approximation for the corrected SIMEX (quadratic) estimate of the adjusted regression coefficient for treatment; only if estimator = "q" and Ronly = FALSE and parametric = TRUE

deltas.corrected.q

the corrected SIMEX (quadratic) estimate of the residual treatment effect; only if estimator = "q" and Ronly = FALSE and parametric = FALSE

deltas.corrected.var.q

the estimated variance of the corrected SIMEX (quadratic) estimate of the residual treatment effect; only if estimator = "q" and Ronly = FALSE and parametric = FALSE

deltas.corrected.CI.normal.q

the 95% confidence interval using the normal approximation for the corrected SIMEX (quadratic) estimate of the residual treatment effect; only if estimator = "q" and Ronly = FALSE and parametric = FALSE

R.corrected.nl the corrected SIMEX (nonlinear) estimate of the proportion of treatment effect explained by the surrogate marker; only if estimator = "q"

R.corrected.var.nl

the estimated variance of the corrected SIMEX (nonlinear) estimate of the proportion of treatment effect explained by the surrogate marker; only if estimator = "q"

R.corrected.CI.normal.nl

the 95% confidence interval using the normal approximation for the corrected SIMEX (nonlinear) estimate of the proportion of treatment effect explained by the surrogate marker; only if estimator = "q"

R.corrected.CI.fieller.nl

the 95% confidence interval using Fieller's approach for the corrected SIMEX (nonlinear) estimate of the proportion of treatment effect explained by the surrogate marker; only if estimator = "q"

B1star.corrected.nl

the corrected SIMEX (nonlinear) estimate of the adjusted regression coefficient for treatment; only if estimator = "q" and Ronly = FALSE and parametric = TRUE

B1star.corrected.var.nl

the estimated variance of the corrected SIMEX (nonlinear) estimate of the adjusted regression coefficient for treatment; only if estimator = "q" and Ronly = FALSE and parametric = TRUE

B1star.corrected.CI.normal.nl

the 95% confidence interval using the normal approximation for the corrected SIMEX (nonlinear) estimate of the adjusted regression coefficient for treatment; only if estimator = "q" and Ronly = FALSE and parametric = TRUE

deltas.corrected.nl

the corrected SIMEX (nonlinear) estimate of the residual treatment effect; only if estimator = "q" and Ronly = FALSE and parametric = FALSE

deltas.corrected.var.nl

the estimated variance of the corrected SIMEX (nonlinear) estimate of the residual treatment effect; only if estimator = "q" and Ronly = FALSE and parametric = FALSE

deltas.corrected.CI.normal.nl

the 95% confidence interval using the normal approximation for the corrected SIMEX (nonlinear) estimate of the residual treatment effect; only if estimator = "q" and Ronly = FALSE and parametric = FALSE

## Author(s)

Layla Parast

26 R.s.surv.estimate

#### References

Parast, L., Garcia, TP, Prentice, RL, Carroll, RJ (2019+). Robust Methods to Correct for Measurement Error when Evaluating a Surrogate Marker. Under Review.

## **Examples**

```
data(d_example)
names(d_example)
R.s.estimate.me(yone=d_example$y1, yzero=d_example$y0, sone=d_example$s1.a, szero=d_example$s0.a,
parametric = TRUE, estimator = "d", me.variance = 0.5, naive= TRUE, Ronly = FALSE)
R.s.estimate.me(yone=d_example$y1, yzero=d_example$y0, sone=d_example$s1.a, szero=d_example$s0.a,
parametric = TRUE, estimator = "q", me.variance = 0.5, naive= FALSE, Ronly = TRUE)
R.s.estimate.me(yone=d_example$y1, yzero=d_example$y0, sone=d_example$s1.a, szero=d_example$s0.a,
parametric = FALSE, estimator = "q", me.variance = 0.5, naive= FALSE, Ronly = TRUE)
```

R.s.surv.estimate

Calculates the proportion of treatment effect explained by the surrogate marker information measured at a specified time and primary outcome information up to that specified time

## **Description**

This function calculates the proportion of treatment effect on the primary outcome explained by the surrogate marker information measured at  $t_0$  and primary outcome information up to  $t_0$ . The user can also request a variance estimate, estimated using perturbating-resampling, and a 95% confidence interval. If a confidence interval is requested three versions are provided: a normal approximation based interval, a quantile based interval and Fieller's confidence interval, all using perturbation-resampling. The user can also request an estimate of the incremental value of surrogate marker information.

## Usage

```
R.s.surv.estimate(xone, xzero, deltaone, deltazero, sone, szero, t,
weight.perturb = NULL, landmark, extrapolate = FALSE, transform = FALSE,
conf.int = FALSE, var = FALSE, incremental.value = FALSE)
```

# Arguments

xone	numeric vector, the observed event times in the treatment group, $X = min(T,C)$ where T is the time of the primary outcome and C is the censoring time.
xzero	numeric vector, the observed event times in the control group, $X = min(T,C)$ where T is the time of the primary outcome and C is the censoring time.
deltaone	numeric vector, the event indicators for the treatment group, $D = I(T < C)$ where T is the time of the primary outcome and C is the censoring time.
deltazero	numeric vector, the event indicators for the control group, $D = I(T < C)$ where T is the time of the primary outcome and C is the censoring time.
sone	numeric vector; surrogate marker measurement at $t_0$ for treated observations, assumed to be continuous. If $X_{1i} < t_0$ , then the surrogate marker measurement should be NA.

R.s.surv.estimate 27

szero numeric vector; surrogate marker measurement at  $t_0$  for control observations, assumed to be continuous. If  $X_{1i} < t_0$ , then the surrogate marker measurement

should be NA.

t the time of interest.

weight.perturb weights used for perturbation resampling.

landmark time  $t_0$  or time of surrogate marker measurement.

extrapolate TRUE or FALSE; indicates whether the user wants to use extrapolation.

transform TRUE or FALSE; indicates whether the user wants to use a transformation for

the surrogate marker.

conf.int TRUE or FALSE; indicates whether a 95% confidence interval for delta is re-

quested, default is FALSE.

var TRUE or FALSE; indicates whether a variance estimate for delta is requested,

default is FALSE.

incremental.value

TRUE or FALSE; indicates whether the user would like to see the incremental value of the surrogate marker information, default is FALSE.

## **Details**

Let G be the binary treatment indicator with G=1 for treatment and G=0 for control and we assume throughout that subjects are randomly assigned to a treatment group at baseline. Let  $T^{(1)}$  and  $T^{(0)}$  denote the time of the primary outcome of interest, death for example, under the treatment and under the control, respectively. Let  $S^{(1)}$  and  $S^{(0)}$  denote the surrogate marker measured at time  $t_0$  under the treatment and the control, respectively.

The residual treatment effect is defined as

$$\Delta_S(t, t_0) = P(T^{(0)} > t_0) \left\{ \int \psi_1(t|s, t_0) dF_0(s|t_0) - P(T^{(0)} > t|T^{(0)} > t_0) \right\}$$

where  $F_0(\cdot|t_0)$  is the cumulative distribution function of  $S^{(0)}$  conditional on  $T^{(0)} > t_0$  and  $\psi_1(t|s,t_0) = P(T^{(1)} > t|S^{(1)} = s, T^{(1)} > t_0)$ . The proportion of treatment effect explained by the surrogate marker information measured at  $t_0$  and primary outcome information up to  $t_0$ , which we denote by  $R_S(t,t_0)$ , can be expressed using a contrast between  $\Delta_S(t,t_0)$  and  $\Delta(t)$ :

$$R_S(t, t_0) = {\Delta(t) - \Delta_S(t, t_0)}/{\Delta(t)} = 1 - {\Delta_S(t, t_0)}/{\Delta(t)}.$$

The definition and estimation of  $\Delta(t)$  is described in the delta.surv.estimate documentation.

Due to censoring, our data consist of  $n_1$  observations  $\{(X_{1i}, \delta_{1i}, S_{1i}), i=1,...,n_1\}$  from the treatment group G=1 and  $n_0$  observations  $\{(X_{0i}, \delta_{0i}, S_{0i}), i=1,...,n_0\}$  from the control group G=0 where  $X_{gi}=\min(T_{gi},C_{gi}), \delta_{gi}=I(T_{gi}< C_{gi}), C_{gi}$  denotes the censoring time, and  $S_{gi}$  denotes the surrogate marker information measured at time  $t_0$ , for g=1,0, for individual i. Note that if  $X_{gi}< t_0$ , then  $S_{gi}$  should be NA (not available).

To estimate  $\Delta_S(t,t_0)$ , we use a nonparametric kernel Nelson-Aalen estimator to estimate  $\psi_1(t|s,t_0)$  as  $\hat{\psi}_1(t|s,t_0) = \exp\{-\hat{\Lambda}_1(t|s,t_0)\}$ , where

$$\hat{\Lambda}_1(t|s,t_0) = \int_{t_0}^t \frac{\sum_{i=1}^{n_1} I(X_{1i} > t_0) K_h\{\gamma(S_{1i}) - \gamma(s)\} dN_{1i}(z)}{\sum_{i=1}^{n_1} I(X_{1i} > t_0) K_h\{\gamma(S_{1i}) - \gamma(s)\} Y_{1i}(z)},$$

is a consistent estimate of  $\Lambda_1(t|s,t_0) = -\log[\psi_1(t|s,t_0)], Y_{1i}(t) = I(X_{1i} \geq t), N_{1i}(t) = I(X_{1i} \leq t)\delta_i, K(\cdot)$  is a smooth symmetric density function,  $K_h(x) = K(x/h)/h, \gamma(\cdot)$  is a given

monotone transformation function, and h is a specified bandwidth. To obtain an appropriate h we first use bw.nrd to obtain  $h_{opt}$ ; and then we let  $h = h_{opt} n_1^{-c_0}$  with  $c_0 = 0.11$ .

Since  $F_0(s|t_0) = P(S_{0i} \le s|X_{0i} > t_0)$ , we empirically estimate  $F_0(s|t_0)$  using all subjects with  $X_{0i} > t_0$  as

$$\hat{F}_0(s|t_0) = \frac{\sum_{i=1}^{n_0} I(S_{0i} \le s, X_{0i} > t_0)}{\sum_{i=1}^{n_0} I(X_{0i} > t_0)}.$$

Subsequently, we construct an estimator for  $\Delta_S(t, t_0)$  as

$$\hat{\Delta}_S(t, t_0) = n_0^{-1} \sum_{i=1}^{n_0} \left[ \hat{\psi}_1(t|S_{0i}, t_0) \frac{I(X_{0i} > t_0)}{\hat{W}_0^C(t_0)} - \frac{I(X_{0i} > t)}{\hat{W}_0^C(t)} \right]$$

where  $\hat{W}_g^C(\cdot)$  is the Kaplan-Meier estimator of survival for censoring for g=1,0. Finally, we estimate  $R_S(t,t_0)$  as  $\hat{R}_S(t,t_0)=1-\hat{\Delta}_S(t,t_0)/\hat{\Delta}(t)$ .

Variance estimation and confidence interval construction are performed using perturbation-resampling. Specifically, let  $\left\{V^{(b)}=(V_{11}^{(b)},...V_{1n_1}^{(b)},V_{01}^{(b)},...V_{0n_0}^{(b)})^T,b=1,....,D\right\}$  be  $n\times D$  independent copies of a positive random variables V from a known distribution with unit mean and unit variance. Let

$$\hat{\Delta}^{(b)}(t) = \frac{\sum_{i=1}^{n_1} V_{1i}^{(b)} I(X_{1i} > t)}{\sum_{i=1}^{n_1} V_{1i}^{(b)} \hat{W}_1^{C(b)}(t)} - \frac{\sum_{i=1}^{n_0} V_{0i}^{(b)} I(X_{0i} > t)}{\sum_{i=1}^{n_0} V_{0i}^{(b)} \hat{W}_0^{C(b)}(t)}.$$

In this package, we use weights generated from an Exponential(1) distribution and use D=500. The variance of  $\hat{\Delta}(t)$  is obtained as the empirical variance of  $\{\hat{\Delta}(t)^{(b)}, b=1,...,D\}$ . Variance estimates for  $\hat{\Delta}_S(t,t_0)$  and  $\hat{R}_S(t,t_0)$  are calculated similarly. We construct two versions of the 95% confidence interval for each estimate: one based on a normal approximation confidence interval using the estimated variance and another taking the 2.5th and 97.5th empirical percentile of the perturbed quantities. In addition, we use Fieller's method to obtain a third confidence interval for  $R_S(t,t_0)$  as

$$\left\{ 1 - r : \frac{(\hat{\Delta}_S(t, t_0) - r\hat{\Delta}(t))^2}{\hat{\sigma}_{11} - 2r\hat{\sigma}_{12} + r^2\hat{\sigma}_{22}} \le c_{\alpha} \right\},\,$$

where  $\hat{\Sigma} = (\hat{\sigma}_{ij})_{1 \leq i,j \leq 2}$  and  $c_{\alpha}$  is the  $(1 - \alpha)$ th percentile of

$$\left\{ \frac{\{\hat{\Delta}_S^{(b)}(t) - (1 - \hat{R}_S(t, t_0))\hat{\Delta}(t)^{(b)}\}^2}{\hat{\sigma}_{11} - 2(1 - \hat{R}_S(t, t_0))\hat{\sigma}_{12} + (1 - \hat{R}_S(t, t_0))^2\hat{\sigma}_{22}}, b = 1, \dots, C \right\}$$

where  $\alpha = 0.05$ .

Since the definition of  $R_S(t,t_0)$  considers the surrogate information as a combination of both S information and T information up to  $t_0$ , a logical inquiry would be how to assess the incremental value of the S information in terms of the proportion of treatment effect explained, when added to T information up to  $t_0$ . The proportion of treatment effect explained by T information up to  $t_0$  only is denoted as  $R_T(t,t_0)$  and is described in the documentation for R.t.surv.estimate. The incremental value of S information is defined as:

$$IV_S(t, t_0) = R_S(t, t_0) - R_T(t, t_0) = \frac{\Delta_T(t, t_0) - \Delta_S(t, t_0)}{\Delta(t)}.$$

For estimation of  $R_T(t_0)$ , see documentation for R.t.surv.estimate. The quantity  $IV_S(t,t_0)$  is then estimated by  $\hat{IV}_S(t,t_0) = \hat{R}_S(t,t_0) - \hat{R}_T(t,t_0)$ . Perturbation-resampling is used for variance estimation and confidence interval construction for this quantity, similar to the other quantities in this package.

R.s.surv.estimate 29

Note that if the observed supports for S are not the same, then  $\hat{\Lambda}_1(t|s,t_0)$  for  $S_{0i}=s$  outside the support of  $S_{1i}$  may return NA (depending on the bandwidth). If extrapolation = TRUE, then the  $\hat{\Lambda}_1(t|s,t_0)$  values for these surrogate values are set to the closest non-NA value. If transform = TRUE, then  $S_{1i}$  and  $S_{0i}$  are transformed such that the new transformed values,  $S_{1i}^{tr}$  and  $S_{0i}^{tr}$  are defined as:  $S_{gi}^{tr} = F([S_{gi} - \mu]/\sigma)$  for g=0,1 where  $F(\cdot)$  is the cumulative distribution function for a standard normal random variable, and  $\mu$  and  $\sigma$  are the sample mean and standard deviation, respectively, of  $\{(S_{1i}, S_{0i})^T, i \ s.t. X_{gi} > t_0\}$ .

#### Value

A list is returned:

delta the estimate,  $\hat{\Delta}(t)$ , described in delta.estimate documentation.

delta.s the estimate,  $\hat{\Delta}_S(t, t_0)$ , described above.

R. s the estimate,  $\hat{R}_S(t, t_0)$ , described above.

delta.var the variance estimate of  $\hat{\Delta}(t)$ ; if var = TRUE or conf.int = TRUE.

delta.s.var the variance estimate of  $\hat{\Delta}_S(t, t_0)$ ; if var = TRUE or conf.int = TRUE.

R.s.var the variance estimate of  $\hat{R}_S(t, t_0)$ ; if var = TRUE or conf.int = TRUE.

conf.int.normal.delta

a vector of size 2; the 95% confidence interval for  $\hat{\Delta}(t)$  based on a normal approximation; if conf.int = TRUE.

conf.int.quantile.delta

a vector of size 2; the 95% confidence interval for  $\hat{\Delta}(t)$  based on sample quantiles of the perturbed values, described above; if conf.int = TRUE.

conf.int.normal.delta.s

a vector of size 2; the 95% confidence interval for  $\hat{\Delta}_S(t,t_0)$  based on a normal approximation; if conf.int = TRUE.

conf.int.quantile.delta.s

a vector of size 2; the 95% confidence interval for  $\hat{\Delta}_S(t,t_0)$  based on sample quantiles of the perturbed values, described above; if conf.int = TRUE.

conf.int.normal.R.s

a vector of size 2; the 95% confidence interval for  $\hat{R}_S(t,t_0)$  based on a normal approximation; if conf.int = TRUE.

conf.int.quantile.R.s

a vector of size 2; the 95% confidence interval for  $\hat{R}_S(t,t_0)$  based on sample quantiles of the perturbed values, described above; if conf.int = TRUE.

conf.int.fieller.R.s

a vector of size 2; the 95% confidence interval for  $\hat{R}_S(t,t_0)$  based on Fieller's approach, described above; if conf.int = TRUE.

delta.t the estimate,  $\hat{\Delta}_T(t, t_0)$ , described above; if incremental vaue = TRUE.

R. t the estimate,  $\hat{R}_T(t, t_0)$ , described above; if incremental vaue = TRUE.

incremental.value

the estimate,  $\hat{IV}_S(t, t_0)$ , described above; if incremental vaue = TRUE.

delta.t.var the variance estimate of  $\hat{\Delta}_T(t,t_0)$ ; if var = TRUE or conf.int = TRUE and incremental.vaue = TRUE.

R.t.var the variance estimate of  $\hat{R}_T(t,t_0)$ ; if var = TRUE or conf.int = TRUE and incremental.vaue = TRUE.

30 R.s.surv.estimate

incremental.value.var

the variance estimate of  $\hat{IV}_S(t,t_0)$ ; if var = TRUE or conf.int = TRUE and incremental.vaue = TRUE.

conf.int.normal.delta.t

a vector of size 2; the 95% confidence interval for  $\hat{\Delta}_T(t,t_0)$  based on a normal approximation; if conf.int = TRUE and incremental.vaue = TRUE.

conf.int.quantile.delta.t

a vector of size 2; the 95% confidence interval for  $\hat{\Delta}_T(t,t_0)$  based on sample quantiles of the perturbed values, described above; if conf.int = TRUE and incremental value = TRUE.

conf.int.normal.R.t

a vector of size 2; the 95% confidence interval for  $\hat{R}_T(t,t_0)$  based on a normal approximation; if conf.int = TRUE and incremental.vaue = TRUE.

conf.int.quantile.R.t

a vector of size 2; the 95% confidence interval for  $\hat{R}_T(t,t_0)$  based on sample quantiles of the perturbed values, described above; if conf.int = TRUE and incremental value = TRUE.

conf.int.fieller.R.t

a vector of size 2; the 95% confidence interval for  $\hat{R}_T(t,t_0)$  based on Fieller's approach, described above; if conf.int = TRUE and incremental.vaue = TRUE.

conf.int.normal.iv

a vector of size 2; the 95% confidence interval for  $\hat{IV}_S(t,t_0)$  based on a normal approximation; if conf.int = TRUE and incremental vaue = TRUE.

conf.int.quantile.iv

a vector of size 2; the 95% confidence interval for  $\hat{IV}_S(t,t_0)$  based on sample quantiles of the perturbed values, described above; if conf.int = TRUE and incremental.vaue = TRUE.

## Note

If the treatment effect is not significant, the user will receive the following message: "Warning: it looks like the treatment effect is not significant; may be difficult to interpret the residual treatment effect in this setting". If the treatment effect is negative, the user will receive the following message: "Warning: it looks like you need to switch the treatment groups" as this package assumes throughout that larger values of the event time are better. If the observed support of the surrogate marker for the control group is outside the observed support of the surrogate marker for the treatment group, the user will receive the following message: "Warning: observed supports do not appear equal, may need to consider a transformation or extrapolation".

# Author(s)

Layla Parast

#### References

Parast L, Cai T and Tian L. Evaluating Surrogate Marker Information using Censored Data. Under Review.

```
data(d_example_surv)
names(d_example_surv)
```

R.t.surv.estimate 31

R.t.surv.estimate	Calculates the proportion of treatment effect explained by the primary
	outcome information up to a specified time

# Description

This function calculates the proportion of treatment effect on the primary outcome explained by the treatment effect on the primary outcome up to  $t_0$ . The user can also request a variance estimate, estimated using perturbating-resampling, and a 95% confidence interval. If a confidence interval is requested three versions are provided: a normal approximation based interval, a quantile based interval and Fieller's confidence interval, all using perturbation-resampling.

# Usage

```
R.t.surv.estimate(xone, xzero, deltaone, deltazero, t, weight.perturb = NULL,
landmark, var = FALSE, conf.int = FALSE)
```

# Arguments

xone	numeric vector, the observed event times in the treatment group, $X = min(T,C)$ where T is the time of the primary outcome and C is the censoring time.
xzero	numeric vector, the observed event times in the control group, $X = \min(T,C)$ where T is the time of the primary outcome and C is the censoring time.
deltaone	numeric vector, the event indicators for the treatment group, $D = I(T < C)$ where T is the time of the primary outcome and C is the censoring time.
deltazero	numeric vector, the event indicators for the control group, $D = I(T < C)$ where $T$ is the time of the primary outcome and $C$ is the censoring time.
t	the time of interest.
weight.perturb	weights used for perturbation resampling.
landmark	the landmark time $t_0$ or time of surrogate marker measurement.
var	TRUE or FALSE; indicates whether a variance estimate for delta is requested, default is FALSE.
conf.int	TRUE or FALSE; indicates whether a $95\%$ confidence interval for delta is requested, default is FALSE.

# **Details**

Let G be the binary treatment indicator with G=1 for treatment and G=0 for control and we assume throughout that subjects are randomly assigned to a treatment group at baseline. Let T denote the time of the primary outcome of interest, death for example. We use potential outcomes notation

such that  $T^{(g)}$  denotes the time of the primary outcome under treatment G=g. The proportion of treatment effect explained by T observed up to  $t_0$  only is  $R_T(t,t_0)=1-\Delta_T(t,t_0)/\Delta(t)$  where

$$\Delta_T(t, t_0) = P(T^{(0)} > t_0)P(T^{(1)} > t \mid T^{(1)} > t_0) - P(T^{(0)} > t).$$

To estimate  $R_T(t,t_0)$ , we use the estimator  $\hat{R}_T(t,t_0) = 1 - \hat{\Delta}_T(t,t_0)/\hat{\Delta}(t)$  where  $\hat{\Delta}_T(t,t_0) = \hat{\phi}_0(t_0)\hat{\phi}_1(t)/\hat{\phi}_1(t_0) - \hat{\phi}_0(t)$  and  $\hat{\phi}_g(u) = n_g^{-1}\sum_{i=1}^{n_g}\frac{I(X_{gi}>u)}{\hat{W}_g^C(u)}$  for g=1,0 where  $\hat{W}_g^C(\cdot)$  is the Kaplan-Meier estimator of survival for censoring for g=1,0.

## Value

A list is returned:

delta the estimate,  $\hat{\Delta}(t)$ , described in delta.estimate documentation.

delta.t the estimate,  $\hat{\Delta}_T(t, t_0)$ , described above. R.t the estimate,  $\hat{R}_T(t, t_0)$ , described above.

delta.var the variance estimate of  $\hat{\Delta}(t)$ ; if var = TRUE or conf.int = TRUE.

delta.t.var the variance estimate of  $\hat{\Delta}_T(t,t_0)$ ; if var = TRUE or conf.int = TRUE. R.t.var the variance estimate of  $\hat{R}_T(t,t_0)$ ; if var = TRUE or conf.int = TRUE.

conf.int.normal.delta

a vector of size 2; the 95% confidence interval for  $\hat{\Delta}(t)$  based on a normal approximation; if conf.int = TRUE.

conf.int.quantile.delta

a vector of size 2; the 95% confidence interval for  $\hat{\Delta}(t)$  based on sample quantiles of the perturbed values, described above; if conf.int = TRUE.

conf.int.normal.delta.t

a vector of size 2; the 95% confidence interval for  $\hat{\Delta}_T(t,t_0)$  based on a normal approximation; if conf.int = TRUE.

conf.int.quantile.delta.t

a vector of size 2; the 95% confidence interval for  $\hat{\Delta}_T(t,t_0)$  based on sample quantiles of the perturbed values, described above; if conf.int = TRUE.

conf.int.normal.R.t

a vector of size 2; the 95% confidence interval for  $\hat{R}_T(t, t_0)$  based on a normal approximation; if conf.int = TRUE.

conf.int.quantile.R.t

a vector of size 2; the 95% confidence interval for  $\hat{R}_T(t, t_0)$  based on sample quantiles of the perturbed values, described above; if conf.int = TRUE.

conf.int.fieller.R.t

a vector of size 2; the 95% confidence interval for  $\hat{R}_T(t, t_0)$  based on Fieller's approach, described above; if conf.int = TRUE.

## Note

If the treatment effect is not significant, the user will receive the following message: "Warning: it looks like the treatment effect is not significant; may be difficult to interpret the residual treatment effect in this setting". If the treatment effect is negative, the user will receive the following message: "Warning: it looks like you need to switch the treatment groups" as this package assumes throughout that larger values of the event time are better.

## Author(s)

Layla Parast

R.t.surv.estimate 33

# References

Parast L, Cai T and Tian L. Evaluating Surrogate Marker Information using Censored Data. Under Review.

```
data(d_example_surv)
names(d_example_surv)
```

# Index

*Topic augment	Aug.R.s.surv.estimate, 2
Aug.R.s.surv.estimate, 2	delta.s.surv.estimate, 9
*Topic datasets	delta.surv.estimate, 11
d_example, 14	delta.t.surv.estimate, 13
d_example_surv, 15	R.s.surv.estimate, 26
*Topic <b>htest</b>	R.t.surv.estimate, 31
fieller.ci, 16	*Topic <b>univar</b>
me.variance.estimate, 17	delta.estimate, 6
*Topic models	delta.surv.estimate, 11
delta.s.estimate, 7	me.variance.estimate, 17
R.s.estimate, 17	
R.s.estimate.me, 22	Aug.R.s.surv.estimate, 2
*Topic <b>nonparametric</b>	d_example, 14
Aug.R.s.surv.estimate, 2	d_example, 14 d_example_surv, 15
delta.s.estimate,7	delta.estimate, 6
delta.s.surv.estimate,9	delta.s.estimate, 7
delta.surv.estimate, 11	delta.s.surv.estimate, 9
delta.t.surv.estimate, 13	delta.surv.estimate, 11
R.s.estimate, 17	delta.t.surv.estimate, 13
R.s.estimate.me, 22	40104.0.04. 1.0001400, 10
R.s.surv.estimate, 26	fieller.ci,16
R.t.surv.estimate, 31	
*Topic <b>regression</b>	me.variance.estimate, 17
delta.s.estimate, 7	D
R.s.estimate, 17	R.s.estimate, 17
R.s.estimate.me, 22	R.s.estimate.me, 22
*Topic <b>robust</b>	R.s.surv.estimate, 26
Aug.R.s.surv.estimate, 2	R.t.surv.estimate, 31
delta.s.estimate,7	
delta.s.surv.estimate,9	
delta.t.surv.estimate, 13	
R.s.estimate, 17	
R.s.estimate.me, 22	
R.s.surv.estimate, 26	
R.t.surv_estimate, 31	
*Topic <b>smooth</b>	
Aug.R.s.surv.estimate, 2	
delta.s.estimate,7	
delta.s.surv.estimate, 9	
R.s.estimate, 17	
R.s.estimate.me, 22	
R.s.surv.estimate, 26	
*Topic <b>survival</b>	