Final Report

Portfolio Optimization and Surgery

Derivative Securities - MGT-6081-A

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1. Introduction

Portfolio diversification and optimization has always been an important problem in finance. It is a process of deciding distribution of capital among predefined set of assets and then measure its performance. Sharpe ratio is used as a metric to measure the performance of a Portfolio. In this project, we selected stocks from various sectors and create a sound portfolio by weights using optimization. Later, we use options to this portfolio to improve its performance aka Sharpe Ratio. We tried multiple types of options to achieve the best possible portfolio performance.

2. Data

A set of 6 stocks were selected from diverse sectors to see the performance of portfolio clearly. The following are the stocks picked,

Apple Inc (AAPL) - Technology Sector

JP Morgan (JPM) - Financial Sector

Pfizer (PFE) - Pharma Sector

Tesla Inc (TSLA) - Automobile Sector

Chevron (CVX) - Energy Sector

Delta Airlines (DAL) - Airlines Sector

The 10-year historical closing prices data from 2011 – 2021 is collected from Yahoo-Finance for our analysis.

3. Technique

The log-returns are calculated from the closing prices of respective stocks. The correlation matrix of the initial stock portfolio is as follows,

1.0000	0.4070	0.3427	0.3265	0.3728	0.2890
0.4070	1.0000	0.4829	0.2702	0.6436	0.5239
0.3427	0.4829	1.0000	0.1915	0.4177	0.3136
0.3265	0.2702	0.1915	1.0000	0.2743	0.2204
0.3728	0.6436	0.4177	0.2743	1.0000	0.4238
0.2890	0.5239	0.3136	0.2204	0.4238	1.0000

Since we selected stocks from diverse sectors, the stock returns are not correlated. This helps in achieving perfect diversification with increased returns and reduced risk.

Initial Weights:

Continuing, we find the weights of the stocks in our portfolio which maximizes the returns and minimizes the variance (risk). We consider the Sharpe-ratio as the metric to our maximizing function.

$$\text{Sharpe-Ratio} = \frac{Returns}{Variance}$$

The following are the weights of the stocks in our portfolio resulting in a better Sharpe ratio,

AAPL	JPM	PFE	TSLA	CVX	DAL
49.32%	0.13%	17.80%	30.54%	0.00%	2.21%

As expected AAPL and TSLA were given higher weights since the individual annual returns of respective stocks were 24.28% and 49.59%. But higher weights of AAPL compared to TSLA is due to TSLA's high variance. Notably Chevron (CVX) showed annual returns of mere 3.7% over the 10 years and thereby rightly set to 0 weights. We consider these weights as a base for our analysis in the next part.

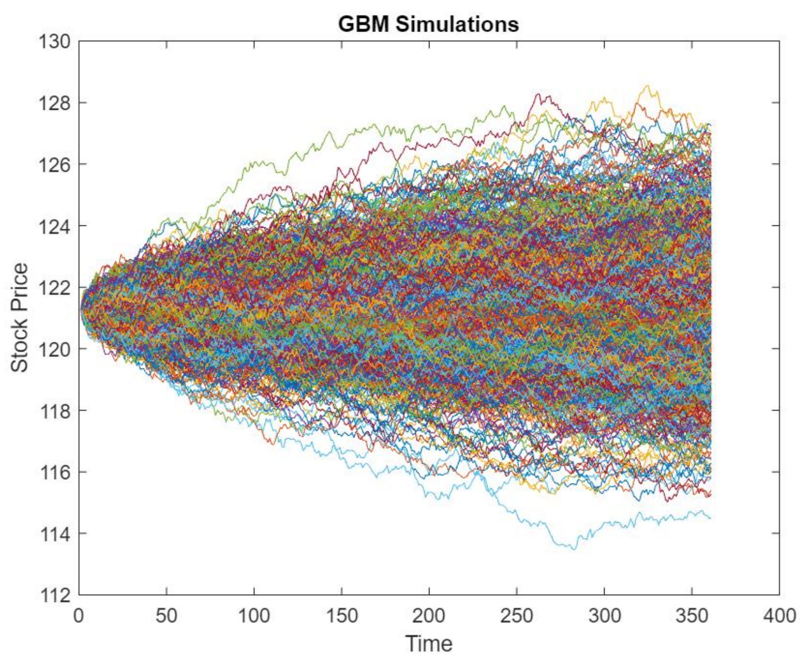
The Sharpe ratio of the above portfolio of stocks based on last 10-yr data is **1.08.**

Simulating Paths:

We now simulate the future 1-yr path of all the stocks using various models.

<u>Geometric Brownian Motion Model:</u>

We used Multi-Dimensional GBM Model to simulate 1 year path of all the respective stocks. The following is the plot of AAPL generated by GBM.



<u>Merton Model:</u>

For Merton, the parameters for the model are selected based on 10-year historical data. We defined a 'jump' as any return which falls in the top 10% percentile of returns data.

Jump mean --> Calculated mean of all the returns in top 10%.

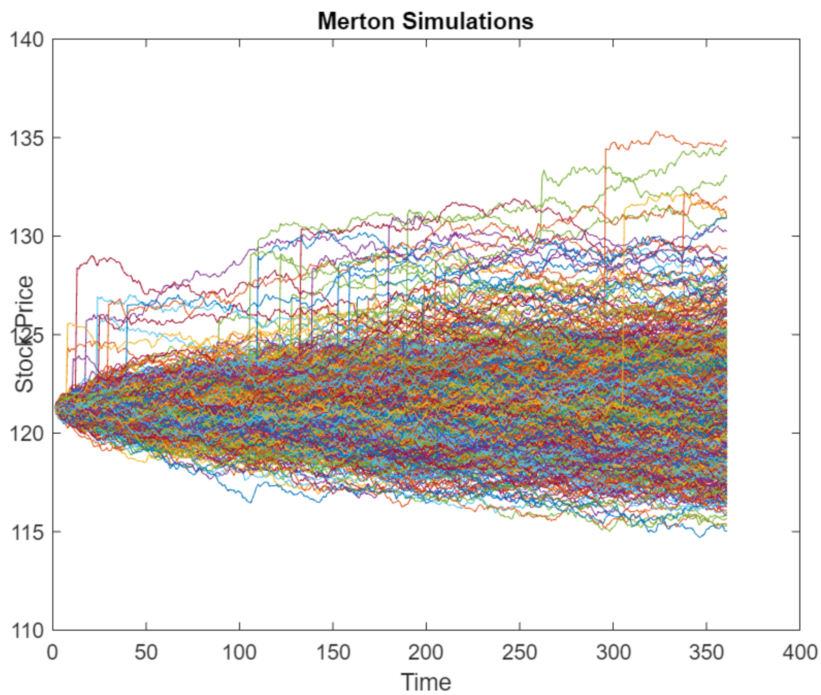
Jump Freq --> Average No of returns in top 10% per year.

Jump Vol --> The standard deviation of the jumps (top 10%).

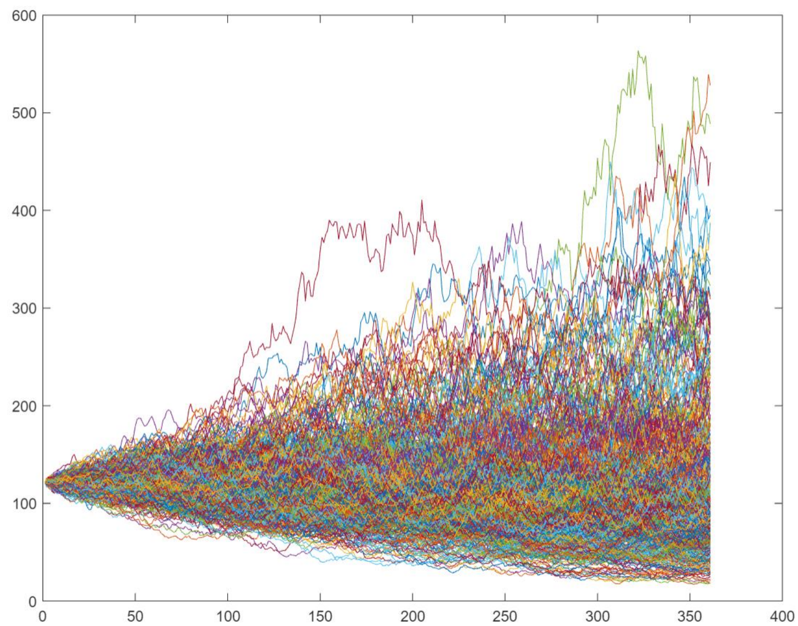
Jump table:

	AAPL	JPM	PFE	TSLA	CVX	DAL
count	252	252	252	252	252	252
mean	0.041842	0.042562	0.030062	0.082845	0.040358	0.059245
std	0.017736	0.02079	0.012295	0.03432	0.025266	0.030986
min	0.027187	0.026883	0.019548	0.050982	0.024814	0.03726
25%	0.030516	0.029936	0.022192	0.058709	0.028831	0.042493
50%	0.03526	0.035669	0.026095	0.070542	0.033725	0.04985
75%	0.046467	0.045706	0.031776	0.093001	0.042194	0.061767
max	0.13771	0.16562	0.085817	0.23652	0.25006	0.301

The following is the plot of AAPL generated by Merton Model.



<u>Heston Model:</u>



<u>CEV Model:</u>

We selected our parameters intuitively and then tuned them appropriately based on the procedure from a research paper (listed in references). The following is the algorithm (written in LATEX) we used.

Process:

$$\alpha S_t = \mu S_t dt + \delta {S_t}^{\theta/2} dB_t \tag{1}$$

- Let ${\Delta_t}^2 = \delta^2 {S_t}^{\theta-2}$
- Let $\hat{\delta}, \hat{\theta}, V_t$ be the estimator for $\delta, \theta, {\Delta_t}^2$
-

$$V_t = \frac{2}{\alpha \Delta t}\left(\frac{{S_{t+\Delta t}}^{1+\alpha} - {S_t}^{1+\alpha}}{(1+\alpha){S_t}^{1+\alpha}} - \frac{S_{t+\Delta t} - S_t}{S_t}\right) \tag{2}$$

- where

$$\alpha = -\frac{13}{11} - \frac{12}{11}\frac{\mu}{{\delta_t}^2} \tag{3}$$

Proof:

$$\alpha({S_t}^{1+\alpha}) = (1+\alpha){S_t}^{\alpha} dS_t + \frac{1}{2}\alpha(1+\alpha){S_t}^{\alpha-1}(\delta^2 {S_t}^{\theta} dt) \tag{4}$$

$$\frac{\alpha({S_t}^{1+\alpha})}{(1+\alpha){S_t}^{1+\alpha}} - \frac{\alpha S_t}{S_t} = \frac{1}{2}\alpha\delta^2 {S_t}^{\theta-2} dt \tag{5}$$

$$\Rightarrow \delta^2 {S_t}^{\theta-2} = \left(\frac{d{S_t}^{1+\alpha}}{(1+\alpha){S_t}^{1+\alpha}} - \frac{dS_t}{S_t}\right)\frac{2}{\alpha dt} \tag{6}$$

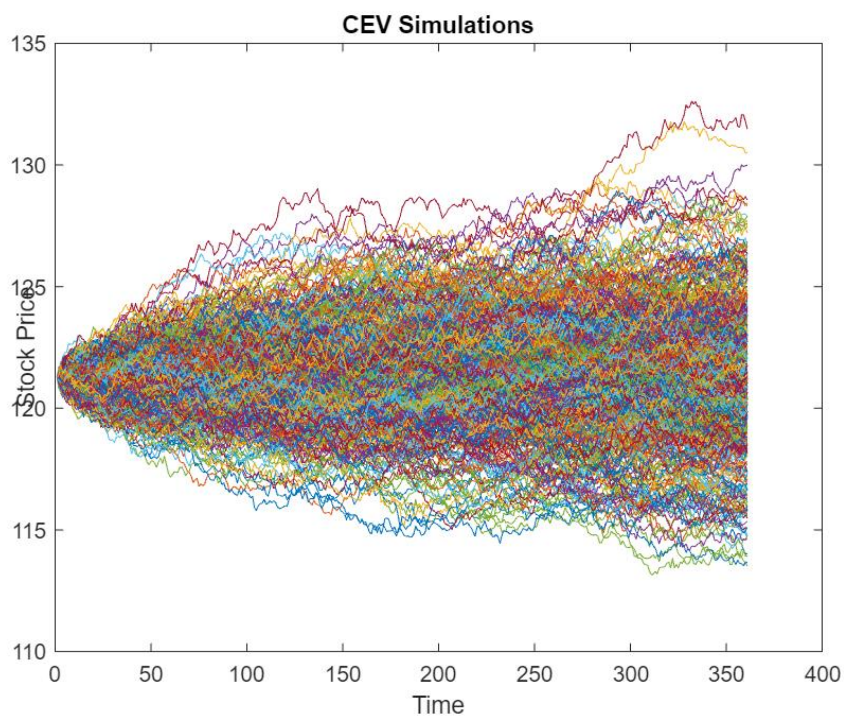
- $\hat{\mu}$ estimate for μ
- Algorithm:
    1. Start with initial guess for α
    2. Calculate V_t using α in equation 2
    3. Calculate new α using V_t and $\hat{\mu}$ in equation 3
    4. Iterate step 2 and 3 until $||\alpha^{new} - \alpha|| < \sum$, where $\sum$ is a set error threshold
    5. $$min \sum_{t=1}^{n-1} (\ln V_t - \ln {\Delta_t}^2)^2$$

    $\Rightarrow$ obtain δ, θ

Stock gammas:

	AAPL	JPM	PFE	TSLA	CVX	DAL
0	1.0729	0.78716	1.0264	1.0535	-0.46365	0.70169

The following is the plot of AAPL generated by CEV Model.



Adding Options:

We add the below options to our data to further improve our returns and thereby Sharpe ratio. The options are priced on different simulated paths (testing simulation) in contrast to the simulated paths used for stocks (pricing simulation). The following are the options we used,

- European Call options with multiple strikes across current trading price on each stock.
- European Put options with multiple strikes across current trading price on each stock.
- Exotic Options include,
    - Payoff of max (max – min, 0)
    - Payoff of max (final – min, 0)
    - Payoff of max (max – final, 0)

A total of 114 options in total were considered to improve the efficiency of our portfolio.

We also assumed that short selling is allowed for our options while performing LPP Optimization (Code attached).

<u>Stocks + European Calls + European Puts + Max-Min options</u>

Model	**Sharpe-Ratio**
Geometric Brownian Motion Model	**3.9670**
Merton Model	**2.6812**
Heston Model	**3.0121**
CEV Model	**3.3630**

Note: The relatively lower Sharpe ratio from Merton model might be due to the increased volatility resulting from 'jumps'.

Using Machine Learning Techniques

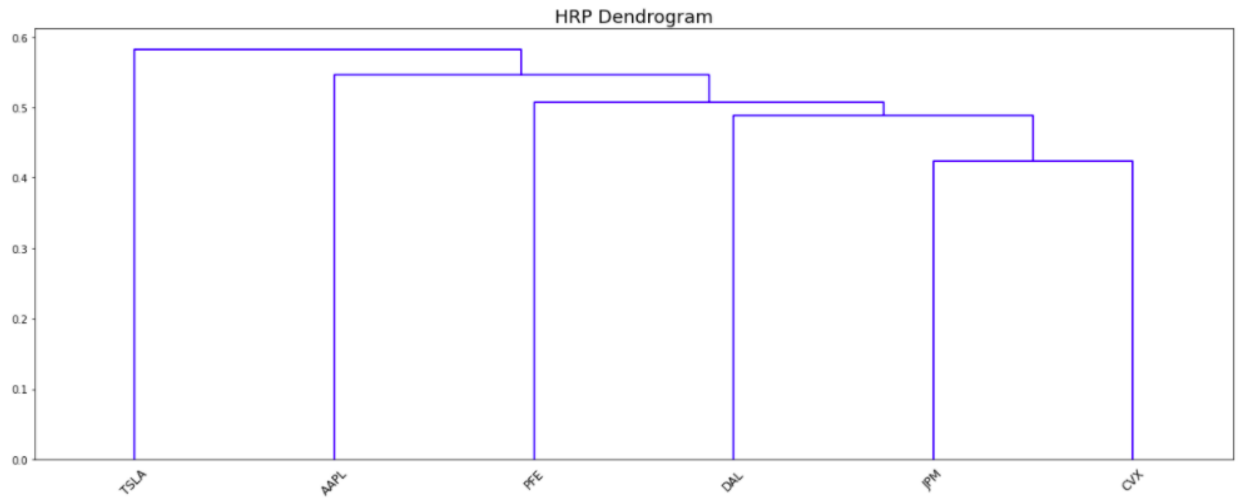
We have tried a method called Hierarchical Risk Parity to decide weights for stocks in our portfolio. In simple words, we cluster same risk assets in hierarchal order. This technique comprises of mainly 3 steps - hierarchal clustering, matrix seriation, and recursive bisection.

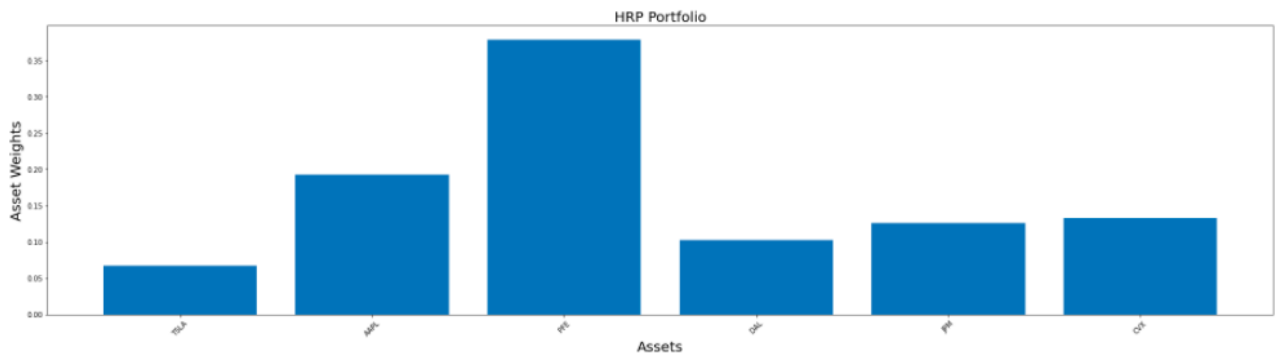
Step 1 – Hierarchical clustering divides assets using correlation matrix into various clusters forming a binary tree.

Step 2 - Matrix seriation is used to rearrange the data to show the inherent clusters clearly.

Step 3 – It is main step in which weights are assigned. We start from top of tree and keep dividing weights further based on covariance matrix.

We used python library called *portfoliolab* to apply this method. Our Hierarchal Tree and weights of assets in portfolio using this method were:





4. Conclusion

An important learning aspect of this project is to generate future stock prices and returns using sophisticated models. This project gave an insight into how to select various parameters for our models.

The Sharpe Ratio given by basic portfolio optimization techniques can be further improved by adding options to it. By doing so, the returns of the portfolio are improved, keeping the volatility in check. We also noticed that the Merton resulted in a relatively lower Sharpe ratio compared to other models as it included jump, which increases volatility of returns.

5. References

- https://www.researchgate.net/publication/254417409_Estimation_in_the_Constant_Elasticity_of_Variance_Model
- https://hudsonthames.org/an-introduction-to-the-hierarchical-risk-parity-algorithm/
- Lohre, Harald and Rother, Carsten and Schäfer, Kilian Axel, Hierarchical Risk Parity: Accounting for Tail Dependencies in Multi-Asset Multi-Factor Allocations (January 23, 2020). https://ssrn.com/abstract=3513399

6. Appendix

Repository for this project is at https://github.com/layoups/derivative-securities