Fundamentals of Wave Simulation: Topics

October 9, 2017

(**P**): Programming Topic: 30 min presentation + Programming + 5 Pages paper

(**L**): Lecutres: 60 min presentation + 5 Pages paper

$\star$: indicate estimated difficulty

1 Introduction to hyperbolic PDEs and Shallow Water Equations(Ioannis Kouroudis). (L)$\star$

Introduce linear and nonlinear hyperbolic PDEs: [2] [1, Chapter 13]

- Conservation Laws.
- Solve the advection Equation
- The Weak Solution.
- Rankine Hugoniot condition.
- The Riemann Problem.
- hyperbolic PDEs
- Characteristic curves and how do they differ for linear and non linear PDEs ?
- Admissibility Conditions (Entropy and Lax).
- Shock, rarefaction waves and contact discontinuities.
- Solve a Riemann problem analytically. (SWE: Dam-Break problem)

2 Finite Volume Method: Acoustic Wave Equation (David Frank)(P)$\star$

Give an Introduction to the Acoustic Wave Equation. Extend the provided code to simulate the Acoustic wave equation in 1D show various examples.[1, Chapter 2 ,p23-32]

- Introduce the continuity equation
- Introduce the momentum equation [1, Chapter 14, p291-292]

- Introduce the equation of state [1, Chapter 14, p292-293]
- What is impedance ?
- Finite Volume Method: Solve the Riemann problem at the interfaces analytically (Why is this unproblematic in this case ?)
- Calculate various analytic solutions (periodic boundary conditions) and perform an Error Analysis [1, Chapter 2 ,p29-32] for your method.

If you have time extend the code to a 2D simulation or simulate a wave propagating through different media (e.g. Air and Water).

3 Finite Volume Method: Elastic Wave Equation (Subhan-Jamal Sohail)(P)⋆

Give an introduction to the Elastic Wave Equation. Extend the provided code to simulate the Elastic wave equation in 1D show various examples. [1, Chapter 2 ,p35-41] (use equation 2.95)

- Finite Volume Method: Solve the Riemann problem at the interfaces analytically (Why is this unproblematic in this case ?)
- Explain P and S-Waves.
- Take a look at the two Dimensional case, what happens to the P-wave ?
- Calculate various analytic solutions (periodic boundary conditions) and perform an Error Analysis for your method.

If you have time extend the code to a 2D simulation or simulate a wave propagating through different media.

4 Finite Volume Method: Shallow Water Equations (Fukushi Sato)(P)⋆

Introduce the Shallow Water Equations. Introduce the Finite Volume Method for Shallow Water Equations. [1, Chapter 4, p64-83], [1, Chapter 13] Lay an eye on important properties as wave speed and the CFL condition. Use the provided code to implement the Finite Volume Method for the simple Advection Equation and the Shallow Water Equations. As flux use the Lax Friedrichs method for fluctuations.

Run both PDEs for a initial Gaussian hump (in water height for SWE with zero velocity) an a discontinuity.

- Explain Godunov's method. [1, Chapter 12, p227-232]
- What do you observe or both sets of PDEs ?
- What is wrong in the discontinuous case for SWE ?

5 Riemann Solvers: Lax Friedrichs and Rusanov Flux (Nathan Brei) (P)⋆

Use the provided code to implement the Finite Volume Method for the Shallow Water Equations. [1, Chapter 4,p 64 -71][1, Chapter 13] Implement the Lax Friedrichs and Rusanov flux (Local Lax Friedrichs Flux) to compute numerical fluctuations. [1, Chapter 12,p 232 - 233], Run the Scenario with a Gaussian initial condition and the dam break problem.

- What do you observe comparing both Solvers ?
- How does the penalty term work, explain numerical viscosity [1, Chapter 12, p227-232] [1, Chapter 12, p323-327]

6 Riemann Solvers: Roe Flux (Dewitte Thiebout) (P)⋆⋆

Use the provided code to implement the Finite Volume Method for the Shallow Water Equations. Implement the Roe flux for SWE to compute numerical fluctuations. [1, Chapter 4 p 84-85][1, Chapter 4 p 84-85] [1, Chapter 15 p 315-322] Run the Scenario with a Gaussian initial condition and the dam break problem.

- Derive the Roe solver for SWE
- What are the advantages and disadvantages of a linearized Riemann solver
- Why does the Roe solver need a Entropy Fix ?

7 Finite Volume: Multiple Dimensions (Kislaya) (P)⋆⋆

Introduce and implement the Finite Volume Method for multiple dimensions by Dimension splitting. [1, Chapter 19 p438] Introduce the Acoustic Wave Equation for two dimensions. [1, Chapter 18 p425]

- What are the advantages of Dimension splitting.
- Why can one dimensional Riemann solvers be reused ?
- Show various examples

8 Finite Volume: Shallow Water Equations Wetting & Drying by HLLE Solver (Bodhinanda Chandra) (L)(or: (P)) ⋆⋆⋆

Introduce the Shallow Water Equations with bathymetry term.[4] Define the resting lake scenario and the well balanced property. Introduce the HLLE Solver to model wetting and drying cells.

Optional: Implement the HLLE solver (Python script at [5]) for the Shallow Water Equations in the Finite Volume method and show various Simulations.

9 Discontinuous Galerkin method (Ayman Noureldin) (P)⋆⋆

Introduce the second order Runge Kutta Discontinuous Galerkin Method for the Advection equation using the Heun method for time stepping. [3, Chapter 2 p22 -34][3, Chapter 3 p44 - 45, p63-76] Use a Lagrangian Basis on equidistant nodes as test functions and polynomial approximation.
Explain how the Finite Volume Method can be derived from the Discontinuous Galerkin method.
Solve occurring integrals with an exact quadrature formulation (Gauss-Legendre [6]). Compute a solution for the advection Equation [1, Chapter 2 17-19] and do a convergence analysis of your method.

Explain following keywords:

- Weak formulation
- Weak and strong solution
- Mass and stiffness Matrices
- Reference element
- Energy conservation

10 Discontinuous Galerkin: Nodal representation (Dominik Volland) (P)⋆⋆

Implement the second order Runge Kutta Discontinuous Galerkin method for the Advection equation using the Heun method for time stepping. [3, Chapter 2 p22 -34][3, Chapter 3 p44 - 45, p63-76] Solve occurring integrals with an exact quadrature formulation (Gauss-Legendre [6]) Use a Lagrangian Basis on Equidistant and Legendre nodes as test functions and polynomial approximation.

- How does the choice of Nodal set affect the sparsity of all Matrices, the accuracy of the polynomial approximation, the condition of the Mass Matrix.
- Do a numerical analysis: Simulate a Gaussian wave for both bases and compare the errors for different mesh resolutions.

11 Discontinuous Galerkin & Shallow Water Equations: Well balanced scheme (Emily Bourne) (P)⋆⋆⋆

Implement the second order Runge Kutta Discontinuous Galerkin method for the Shallow Water Equations with bathymetry source term using the Heun method for time stepping. Only the PDE: [4] [3, Chapter 2 p22 -34][3, Chapter 3 p44 - 45, p63-76]

Use a Lagrangian Basis on equidistant nodes as test functions and polynomial approximation and the Rusanov Flux to compute fluctuations along the interfaces.
Solve occurring integrals with an exact quadrature formulation (Gauss-Legendre [6].) Use the same approximation for the bathymetry as for the physical quantities.

- Show why the resting lake stays constant for this formulation.
- Show various simulations with bathymetry.

12 DG Limiting: Limiting (Ashwary Pande) (L)⋆⋆⋆

Introduce Limiting methods for Discontinuous Galerkin schemes [3, Chapter 5.6, p136 - 157]

- What is Gibbs Phenomenon ?
- Explain Filtering.
- What are Total Variation Diminishing methods ?
- Introduce the Min-Mod Limiter.

References

[1] LeVeque, R. (2002). Finite Volume Methods for Hyperbolic Problems (Cambridge Texts in Applied Mathematics). Cambridge: Cambridge University Press. doi:10.1017/CBO9780511791253

[2] Hyperbolic Conservation Laws An Illustrated Tutorial, Lectures Notes for Modelling and Optimization of Flows on Networks Cetraro School 2007. `https://www.math.psu.edu/bressan/PSPDF/clawtut09.pdf`

[3] Hesthaven, Jan S., and Tim Warburton. Nodal discontinuous Galerkin methods: algorithms, analysis, and applications. Springer Science & Business Media, 2007.

[4] LeVeque, Randall J., and David L. George. "High-resolution finite volume methods for the shallow water equations with bathymetry and dry states." Advanced numerical models for simulating tsunami waves and runup 10 (2008): 43-73. `http://faculty.washington.edu/rjl/pubs/catalina04/catalina.pdf`

[5] `https://github.com/clawpack/riemann/blob/master/src/shallow_1D_py.py`

[6] `https://en.wikipedia.org/wiki/Gaussian_quadrature`