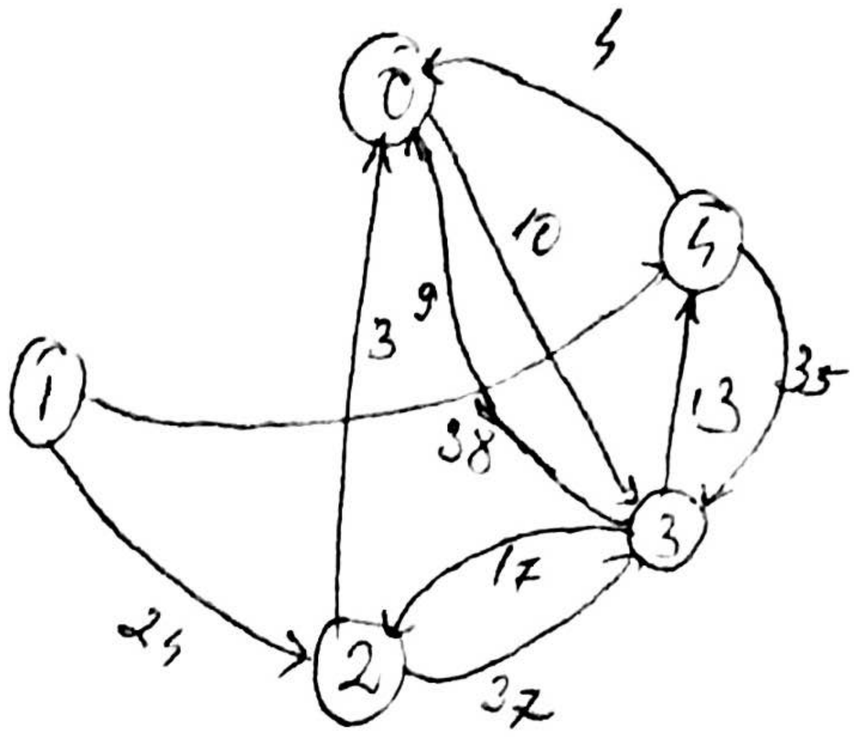
Floyd - Warshall Algorithm



$Cost_i$ = cost matrices

$$Cost_{init} = \begin{pmatrix} inf & inf & inf & 10 & 4 \\ inf & inf & 24 & inf & 38 \\ 3 & inf & inf & 37 & inf \\ 9 & inf & 17 & inf & 13 \\ inf & inf & inf & 35 & inf \end{pmatrix}$$

$next_i$ = successor matrices

$$next_{init} = \begin{pmatrix} -1 & -1 & -1 & 3 & 4 \\ -1 & -1 & 2 & -1 & 4 \\ -1 & -1 & -1 & 3 & -1 \end{pmatrix}$$

$k=0$ - using vertex 0 as an intermediate vertex

$$Cost_0 = \begin{pmatrix} inf & inf & inf & 10 & 4 \\ inf & inf & 24 & inf & 38 \\ 3 & inf & inf & 13 & 7 \\ 9 & inf & 17 & 19 & 13 \\ inf & inf & inf & 35 & inf \end{pmatrix} \quad next_0 = \begin{pmatrix} -1 & -1 & -1 & 3 & 4 \\ -1 & -1 & 2 & -1 & 4 \\ 0 & -1 & -1 & 0 & 0 \\ 0 & -1 & 2 & 0 & 4 \\ -1 & -1 & -1 & 3 & -1 \end{pmatrix}$$

$k=1$ - using vertex 1 as an intermediate vertex

$$Cost_1 = \begin{pmatrix} inf & inf & inf & 10 & 4 \\ inf & inf & 24 & inf & 38 \\ 3 & inf & 13 & 13 & 7 \\ 9 & inf & 17 & 19 & 13 \\ inf & inf & inf & 35 & inf \end{pmatrix} \quad next_1 = \begin{pmatrix} -1 & -1 & -1 & 3 & 4 \\ -1 & -1 & 2 & -1 & 4 \\ 0 & -1 & -1 & 0 & 0 \\ 0 & -1 & 2 & 0 & 4 \\ -1 & -1 & -1 & 3 & -1 \end{pmatrix}$$

$k=2 \Rightarrow$ using vertex 2 as intermediate vertex

$$Cost_2 = \begin{pmatrix} inf & inf & inf & 10 & 4 \\ 27 & inf & 24 & 37 & 31 \\ 3 & inf & inf & 13 & 7 \\ 9 & inf & 17 & 19 & 13 \\ inf & inf & inf & 35 & inf \end{pmatrix} \quad next_2 = \begin{pmatrix} -1 & -1 & -1 & 3 & 4 \\ 2 & -1 & 2 & 2 & 2 \\ 0 & -1 & -1 & 0 & 0 \\ 0 & -1 & 2 & 0 & 4 \\ -1 & -1 & -1 & 3 & -1 \end{pmatrix}$$

$k=3 \Rightarrow$ using vertex 3 as intermediate vertex

$$Cost_3 = \begin{pmatrix} 19 & inf & 27 & 10 & 7 \\ 27 & inf & 24 & 37 & 31 \\ 3 & inf & 30 & 13 & 7 \\ 9 & inf & 17 & 19 & 13 \\ 44 & inf & 52 & 35 & 48 \end{pmatrix} \quad next_3 = \begin{pmatrix} 3 & -1 & 3 & 3 & 4 \\ 2 & -1 & 2 & 2 & 2 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & -1 & 2 & 0 & 4 \\ 3 & -1 & 3 & 3 & 3 \end{pmatrix}$$

$k=4 \Rightarrow$ using vertex 4 as intermediate vertex

$$Cost_4 = \begin{pmatrix} 19 & inf & 27 & 10 & 4 \\ 27 & inf & 24 & 37 & 31 \\ 3 & inf & 30 & 13 & 7 \\ 9 & inf & 17 & 19 & 13 \\ 44 & inf & 52 & 35 & 48 \end{pmatrix} \quad next_4 = \begin{pmatrix} 3 & -1 & 3 & 3 & 4 \\ 2 & -1 & 2 & 2 & 2 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & -1 & 2 & 0 & 4 \\ 3 & -1 & 3 & 3 & 3 \end{pmatrix}$$

The minimum cost walk from startVertex = 0 to endVertex = 2 has the cost $Cost_4(0,2) = 27$ and it is obtained from $next_4$,

~~using line~~ 0: using column 2.

startVertex = 0 ; $next(0,2) = 3$, $next(3,2) = 2$ = endVertex

The minimum cost walk: $0 \xrightarrow{10} 3 \xrightarrow{27} 2$