Propositional Logic and Truth Tables

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Outline

Truth Tables

- Negation:

p	$\neg p$
T	F
F	T

- Conjunction:

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Truth Tables

- Disjunction:

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

- Implication:

p	q	$p \Rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Truth Tables

- Equivalence:

p	q	$p \Leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Examples using Truth Tables

- Construct the Truth Tables for the following logical formulæ:

1. $\neg p \vee q$
2. $\neg p \vee \neg q$
3. $\neg(p \wedge q)$
4. $(p \vee q) \Rightarrow ((p \wedge q) \vee q)$

Truth Table Example 1

- Construct the Truth Table for $\neg p \vee q$

p	q	$\neg p$	$\neg p \vee q$
T	T		
T	F		
F	T		
F	F		

Truth Table Example 1

- Construct the Truth Table for $\neg p \vee q$

p	q	$\neg p$	$\neg p \vee q$
T	T	F	
T	F	F	
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Truth Table Example 1

- Construct the Truth Table for $\neg p \vee q$

p	q	$\neg p$	$\neg p \vee q$
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	T

Truth Table Example 2

- Construct the Truth Table for $\neg p \vee \neg q$

p	q	$\neg p$	$\neg q$	$\neg p \vee \neg q$
T	T			
T	F			
F	T			
F	F			

Truth Table Example 2

- Construct the Truth Table for $\neg p \vee q$

p	q	$\neg p$	$\neg q$	$\neg p \vee \neg q$
T	T	F		
T	F	F		
F	T	T		
F	F	T		

Truth Table Example 2

- Construct the Truth Table for $\neg p \vee \neg q$

p	q	$\neg p$	$\neg q$	$\neg p \vee \neg q$
T	T	F	F	
T	F	F	T	
F	T	T	F	
F	F	T	T	

Truth Table Example 2

- Construct the Truth Table for $\neg p \vee \neg q$

p	q	$\neg p$	$\neg q$	$\neg p \vee \neg q$
T	T	F	F	F
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

Truth Table Example 3

- Construct the Truth Table for $\neg(p \wedge q)$

p	q	$p \wedge q$	$\neg(p \wedge q)$
T	T		
T	F		
F	T		
F	F		

Truth Table Example 3

- Construct the Truth Table for $\neg(p \wedge q)$

p	q	$p \wedge q$	$\neg(p \wedge q)$
T	T	T	
T	F	F	
F	T	F	
F	F	F	

Truth Table Example 3

- Construct the Truth Table for $\neg(p \wedge q)$

p	q	$p \wedge q$	$\neg(p \wedge q)$
T	T	T	F
T	F	F	T
F	T	F	T
F	F	F	T

Compare

- Compare the Truth Tables for $\neg p \vee \neg q$ and $\neg(p \wedge q)$

p	q	$\neg p \vee \neg q$	$\neg(p \wedge q)$
T	T	F	F
T	F	T	T
F	T	T	T
F	F	T	T

Truth Table Example 4

- Construct the Truth Table for $(p \vee q) \Rightarrow ((p \wedge q) \vee q)$

p	q	$p \vee q$	$p \wedge q$	$(p \wedge q) \vee q)$	$(p \vee q) \Rightarrow ((p \wedge q) \vee q)$
T	T				
T	F				
F	T				
F	F				

Truth Table Example 4

- Construct the Truth Table for $(p \vee q) \Rightarrow ((p \wedge q) \vee q)$

p	q	$p \vee q$	$p \wedge q$	$(p \wedge q) \vee q)$	$(p \vee q) \Rightarrow ((p \wedge q) \vee q)$
T	T	T			
T	F	T			
F	T	T			
F	F	F			

Truth Table Example 4

- Construct the Truth Table for $(p \vee q) \Rightarrow ((p \wedge q) \vee q)$

p	q	$p \vee q$	$p \wedge q$	$(p \wedge q) \vee q)$	$(p \vee q) \Rightarrow ((p \wedge q) \vee q)$
T	T	T	T		
T	F	T	F		
F	T	T	F		
F	F	F	F		

Truth Table Example 4

- Construct the Truth Table for $(p \vee q) \Rightarrow ((p \wedge q) \vee q)$

p	q	$p \vee q$	$p \wedge q$	$(p \wedge q) \vee q)$	$(p \vee q) \Rightarrow ((p \wedge q) \vee q)$
T	T	T	T	T	
T	F	T	F	F	
F	T	T	F	T	
F	F	F	F	F	

Truth Table Example 4

- Construct the Truth Table for $(p \vee q) \Rightarrow ((p \wedge q) \vee q)$

p	q	$p \vee q$	$p \wedge q$	$(p \wedge q) \vee q)$	$(p \vee q) \Rightarrow ((p \wedge q) \vee q)$
T	T	T	T	T	T
T	F	T	F	F	F
F	T	T	F	T	T
F	F	F	F	F	T

Additional Exercises

- Construct the Truth Tables for the following logical formulæ:

1. $p \wedge \neg q \Rightarrow q \vee p$
2. $(p \wedge q) \wedge \neg(p \vee q)$
3. $\neg(p \wedge r) \Leftrightarrow \neg(r \wedge p)$
4. $p \vee q \vee \neg q$
5. $p \Leftrightarrow (\neg p \wedge q)$
6. $q \wedge r \wedge \neg r$

Exclusive OR

- We have seen the use of Truth Tables for constructing the truth values for compound propositions.
- As another example, construct the Truth Table for the "Exclusive OR" (XOR):

$$(p \vee q) \wedge \neg(p \wedge q)$$

Exclusive OR

- Construct the Truth Table for $(p \vee q) \wedge \neg(p \wedge q)$

p	q	$p \vee q$	$p \wedge q$	$\neg(p \wedge q)$	$(p \vee q) \wedge \neg(p \wedge q)$
T	T				
T	F				
F	T				
F	F				

Exclusive OR

- Construct the Truth Table for $(p \vee q) \wedge \neg(p \wedge q)$

p	q	$p \vee q$	$p \wedge q$	$\neg(p \wedge q)$	$(p \vee q) \wedge \neg(p \wedge q)$
T	T	T			
T	F	T			
F	T	T			
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Exclusive OR

- Construct the Truth Table for $(p \vee q) \wedge \neg(p \wedge q)$

p	q	$p \vee q$	$p \wedge q$	$\neg(p \wedge q)$	$(p \vee q) \wedge \neg(p \wedge q)$
T	T	T	T		
T	F	T	F		
F	T	T	F		
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Exclusive OR

- Construct the Truth Table for $(p \vee q) \wedge \neg(p \wedge q)$

p	q	$p \vee q$	$p \wedge q$	$\neg(p \wedge q)$	$(p \vee q) \wedge \neg(p \wedge q)$
T	T	T	T	F	
T	F	T	F	T	
F	T	T	F	T	
F	F	F	F	T	

Exclusive OR

- Construct the Truth Table for $(p \vee q) \wedge \neg(p \wedge q)$

p	q	$p \vee q$	$p \wedge q$	$\neg(p \wedge q)$	$(p \vee q) \wedge \neg(p \wedge q)$
T	T	T	T	F	F
T	F	T	F	T	T
F	T	T	F	T	T
F	F	F	F	T	F

Tautology

- Some compound propositions contain only **T** in the last column of their truth tables.
- A compound proposition which is true under all possible assignments of truth values to its prime propositions is called a tautology or a valid proposition.
- Example: $p \vee \neg p$

p	$\neg p$	$p \vee \neg p$
T		
F		

Tautology

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- Example: $p \vee \neg p$

p	$\neg p$	$p \vee \neg p$
T	F	T
F	T	T

Contradiction

- Similarly, some compound propositions contain only **F** in the last column of their truth tables.
- A compound proposition which is false under all possible assignments of truth values to its prime propositions is called a contradiction or an inconsistent proposition.
- Example: $p \wedge \neg p$

p	$\neg p$	$p \wedge \neg p$
T		
F		

Contradiction

- Similarly, some compound propositions contain only **F** in the last column of their truth tables.
- A compound proposition which is false under all possible assignments of truth values to its prime propositions is called a contradiction or an inconsistent proposition.
- Example: $p \wedge \neg p$

p	$\neg p$	$p \wedge \neg p$
T	F	
F	T	

Contradiction

- Similarly, some compound propositions contain only **F** in the last column of their truth tables.
- A compound proposition which is false under all possible assignments of truth values to its prime propositions is called a contradiction or an inconsistent proposition.
- Example: $p \wedge \neg p$

p	$\neg p$	$p \wedge \neg p$
T	F	F
F	T	F

Contingent Proposition

- A compound proposition which is neither a tautology nor a contradiction is called a contingent proposition.
- Example: $p \Rightarrow \neg p$

p	$\neg p$	$p \Rightarrow \neg p$
T		
F		

Contingent Proposition

- A compound proposition which is neither a tautology nor a contradiction is called a contingent proposition.
- Example: $p \Rightarrow \neg p$

p	$\neg p$	$p \Rightarrow \neg p$
T	F	
F	T	

Contingent Proposition

- A compound proposition which is neither a tautology nor a contradiction is called a contingent proposition.
- Example: $p \Rightarrow \neg p$

p	$\neg p$	$p \Rightarrow \neg p$
T	F	F
F	T	T

Tautology/Contradiction Examples

What can you say about the following compound propositions?:

- $p \wedge (q \vee p)$
- $p \wedge \neg(q \Rightarrow p)$
- $p \Rightarrow q \Leftrightarrow \neg p \vee q$
- $p \wedge q \wedge \neg q$

Tautology/Contradiction Exercise

State whether the following compound propositions are tautologies, contradictions or contingent propositions:

- $p \wedge \neg q \Rightarrow q \vee p$
- $(p \wedge q) \wedge \neg(p \vee q)$
- $\neg(p \wedge r) \Leftrightarrow \neg(r \wedge p)$
- $p \vee q \vee \neg q$
- $p \Leftrightarrow (\neg p \wedge q)$
- $q \wedge r \wedge \neg r$