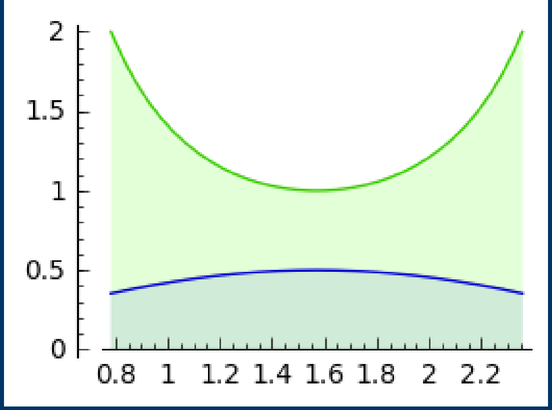
# AULA 5
## Comprimento, arco e volume de um sólido em revolução

Área entre curvas.

Determine the area of the region bounded between the curves

$f(x) = \frac{1}{2}\sin(x)$ $g(x) = \csc^2(x)$ on $[\pi/4, \pi/2]$

```
sage: f(x)= 1/2* sin(x)
sage: f(x)
1/2*sin(x)
sage: g(x)= csc (x)^2
sage: g(x)
csc(x)^2
sage: h= plot ((f(x),g(x)),x,pi /4 ,3* pi /4, figsize
=3, fill
....:  = True )
sage: h
Launched png viewer for Graphics object consisting of
4 graphics primitives
```




Recall that $\csc(x)$ is greater than 1 in this interval. Hence, $\csc^2(x)$ is greater than $\sin(x)$ since $-1 \leqslant \sin(x) \leqslant 1$. Therefore, to calculate the area between $f(x)$ and $g(x)$ on this interval is:

```
sage: integral (g(x)-f(x),x,pi /4 ,3* pi /4)
-1/2*sqrt(2) + 2
```

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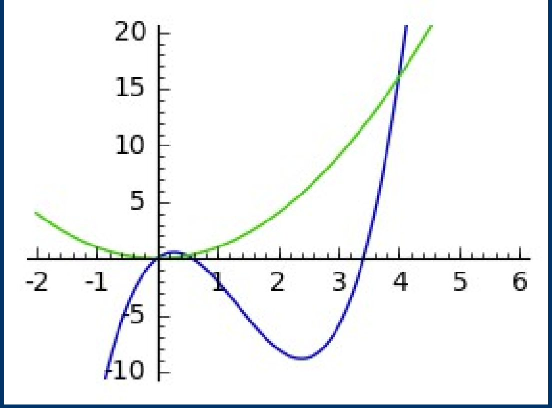
Área entre curvas. Determine the area of the region bounded between the curves

$f(x) = 2x(x^2 - 4x + 2)$ and $g(x) = x^2$

```
sage: f(x) =2* x*(x^2 -4*x +2)
sage: f(x)
2*(x^2 - 4*x + 2)*x
sage: g(x)=x^2
sage: g(x)
x^2
sage: h= plot ((f(x),g(x)),x ,-2,6, figsize =3,
ymin =-10 , ymax =20)
sage: h
Launched png viewer for Graphics object consisting
of 2 graphics primitives
```




solve for the intersection points:

```
sage: solve (f(x)==g(x),x)
[x == 4, x == (1/2), x == 0]
```

the area enclosed between those curves is:

```
sage: integral (f(x)-g(x),x ,0 ,1/2) + integral (g(x)-f(x),x,1/2 ,4)
517/16
```

# AULA 5
## Comprimento, arco e volume de um sólido em revolução

Volume of Solids of Revolution: The Methods of Discs

Let the radius of the cylinder be $R$, the height is $h$, then the volume is:

$$V = \pi R^2 h$$

we can approximate $S$ by discs, that is, the cylinder obtained by revolving each rectangle, constructed by a Riemann sum of $f$ relative to a partition $P = (x_0, x_1, x_2, ..., x_n)$ of $[a, b]$.

it means that the volume of the ith cylinder which corresponding to the ith rectangle is $V_i = \pi[f(x_i^*)]^2 \Delta x$. So, an approximation to the volume of $S$ is given by the Riemann sum:

$$\mathrm{Vol}(S) \approx \sum_{i=1}^{n} V_i = \pi \sum_{i=1}^{n} [f(x_i^*)]^2 \Delta x$$

As $n \to \infty$, we obtain the exact volume of $S$:

$$\mathrm{Vol}(S) = \pi \lim_{n\to\infty} \sum_{i=1}^{n} [f(x_i^*)]^2 \Delta x = \pi \int_a^b [f(x)]^2 dx$$

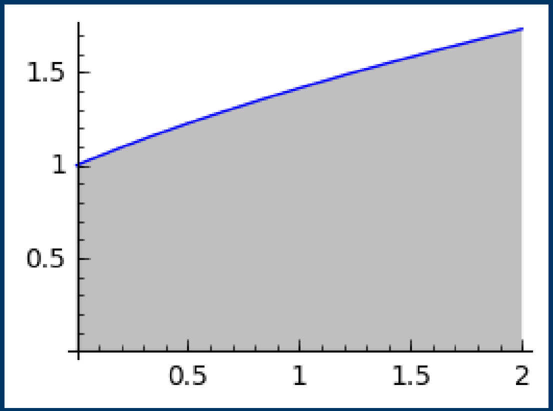
# AULA 5

## Comprimento, arco e volume de um sólido em revolução

Volume of Solids of Revolution:  The Methods of Discs

Find the volume of the solid of revolution obtained by rotating the region bounded by the graph of $f(x) = \sqrt{x+1}$, the x-axis, and the vertical line $x = 2$

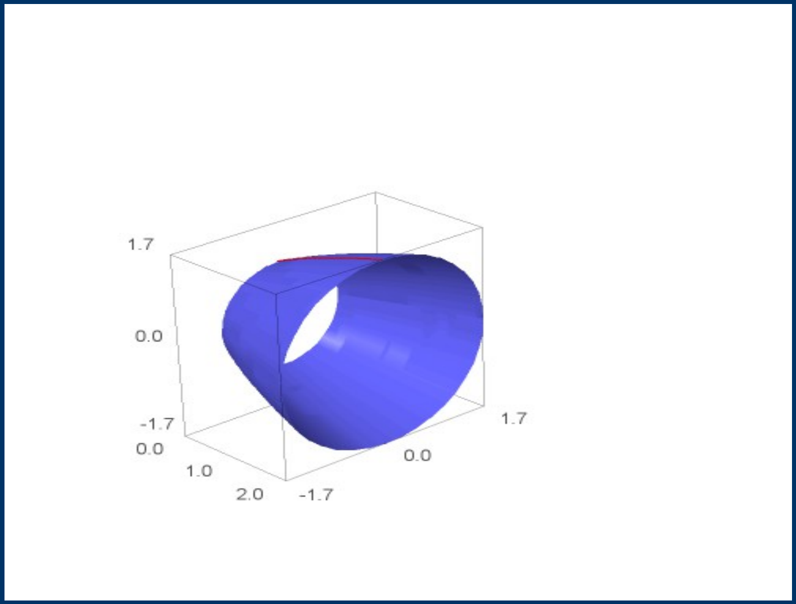
```
sage: var('u')
u
sage: f(u)= sqrt (u +1)
sage: h= plot (f(u),u ,0 ,2, figsize =3, fill = True )
sage: s= revolution_plot3d (f(u) ,(u ,0 ,2) , show_curve
=True , opacity =7, parallel_axis ='x'). show
( aspect_ratio =(1 ,1 ,1))
Launched jmol viewer for Graphics3d Object
sage: s= revolution_plot3d (f(u) ,(u ,0 ,2) , show_curve
=True , opacity =7, parallel_axis ='x'). show
( aspect_ratio =(3 ,1 ,1))
Launched jmol viewer for Graphics3d Object
```

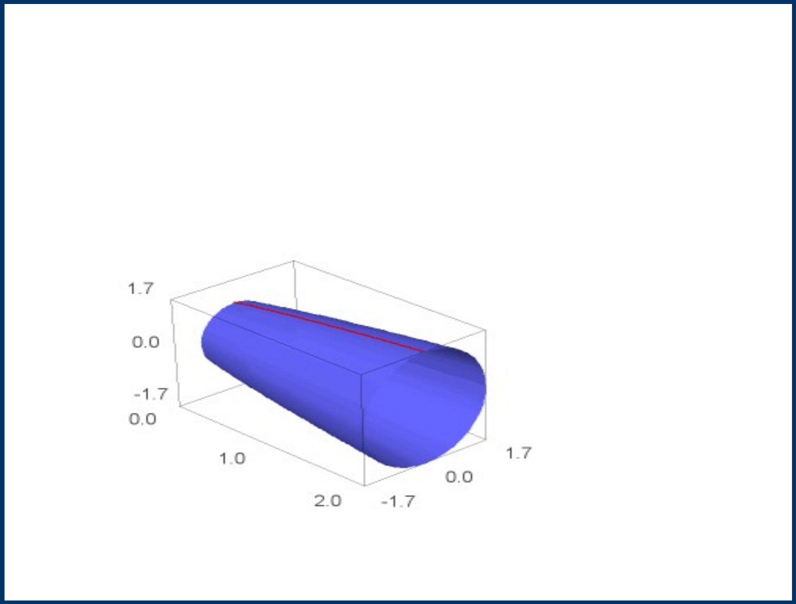


# AULA 5
## Comprimento, arco e volume de um sólido em revolução

Volume of Solids of Revolution:  The Methods of Discs

Find the volume of the solid of revolution obtained by rotating the region bounded by the graph of $f(x) = \sqrt{x+1}$, the x-axis, and the vertical line $x = 2$



# AULA 5
## Comprimento, arco e volume de um sólido em revolução

### The Method of Washers

If a solid of revolution $S$ is generated by revolving a region bounded between two different curves $f(x)$ and $g(x)$ on $[a, b]$ about the $x$-axis, we use washer method. The corresponding volume of $S$ is given by:

$$\mathrm{Vol}(S) = \pi \int_a^b [g(x)]^2 - [f(x)]^2 dx$$

given that $g(x) > f(x)$.

Find the volume of the solid generated by revolving about the $x$-axis the region enclosed by $y = 2x^2 + 1$ and $y = x + 2$.

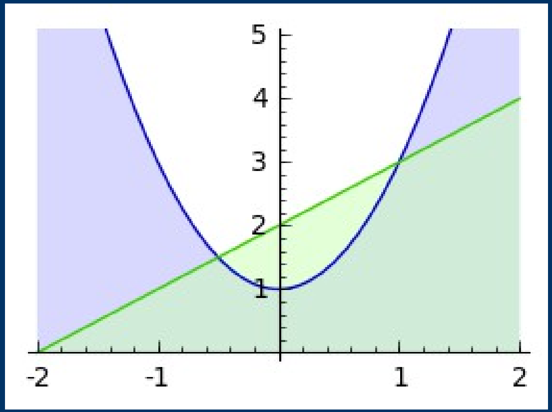
# The Method of Washers

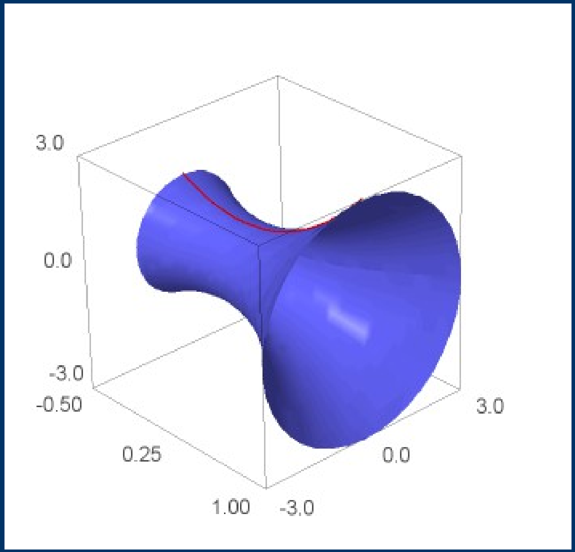
Find the volume of the solid generated by revolving about the x-axis the region enclosed by $y = 2x^2 + 1$ and $y = x + 2$.

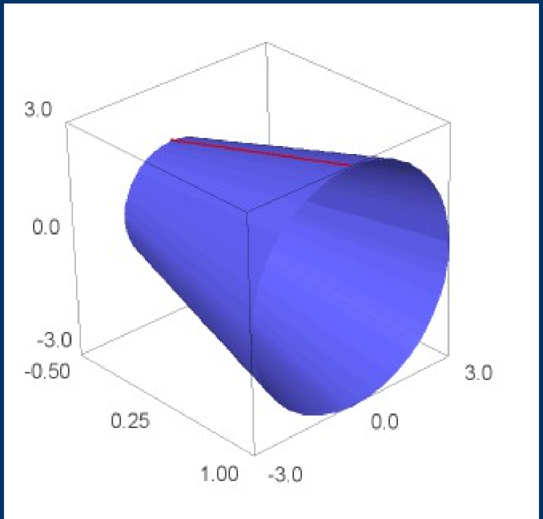
```
sage: var('u')
u
sage: f(u) =2* u ^2+1
sage: f(u)
2*u^2 + 1
sage: g(u)=u+2
sage: g(u)
u + 2
sage: h= plot ((f(u),g(u)),u ,-2,2, figsize =3, ymin =0, ymax =5, fill =True )
sage: h
Launched png viewer for Graphics object consisting of 4 graphics primitives
sage: solve (f(u)==g(u),u)
[u == 1, u == (-1/2)]
sage: var('u,F')
(u, F)
sage: f(u)
2*u^2 + 1
sage: F= revolution_plot3d (f(u) ,(u , -1/2 ,1) , show_curve =True , opacity =7,
parallel_axis ='x' )
sage: F
Launched jmol viewer for Graphics3d Object
sage: var('u,G')
(u, G)
sage: g(u)
u + 2
sage: G= revolution_plot3d (f(u) ,(u , -1/2 ,1) , show_curve =True , opacity =7,
parallel_axis ='x' )
sage: G
Launched jmol viewer for Graphics3d Object
```

# The Method of Washers

Find the volume of the solid generated by revolving about the $x$-axis the region enclosed by $y = 2x^2 + 1$ and $y = x + 2$.



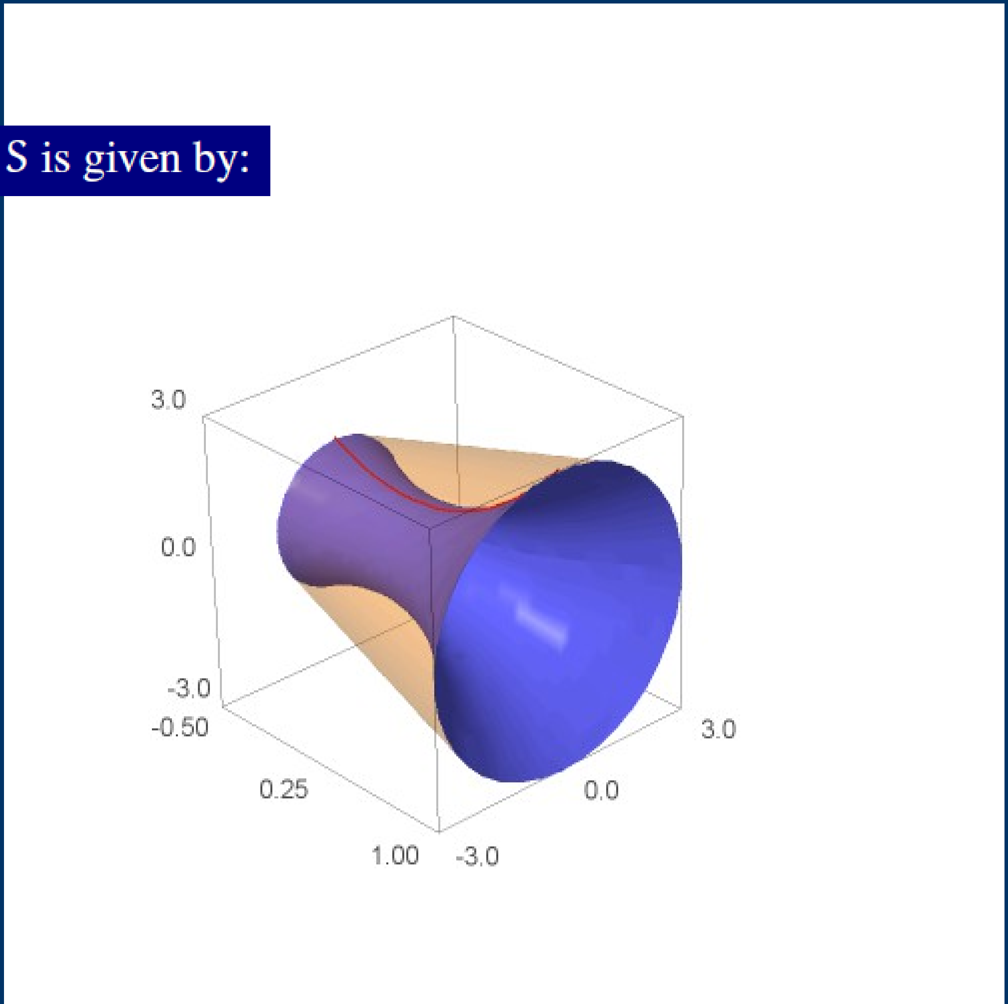



# The Method of Washers

Find the volume of the solid generated by revolving about the x-axis the region enclosed by $y = 2x^2 + 1$ and $y = x + 2$.

```
sage: G= revolution_plot3d (g(u) ,(u , -1/2 ,1) , show_curve =False , opacity =0.2,
rgbcolor=(1,0.5,0),parallel_axis ='x' )
sage: S=G+F
sage: S
Launched jmol viewer for Graphics3d Object
```




Since the curve $f(x)$ is lower than $g(x)$, the volume of $S$ is given by:

```
sage: pi* integral ((g(u)^2-f(u) ^2) ,u ,
-1/2 ,1)

81/20*pi
```

# Polar coordinates

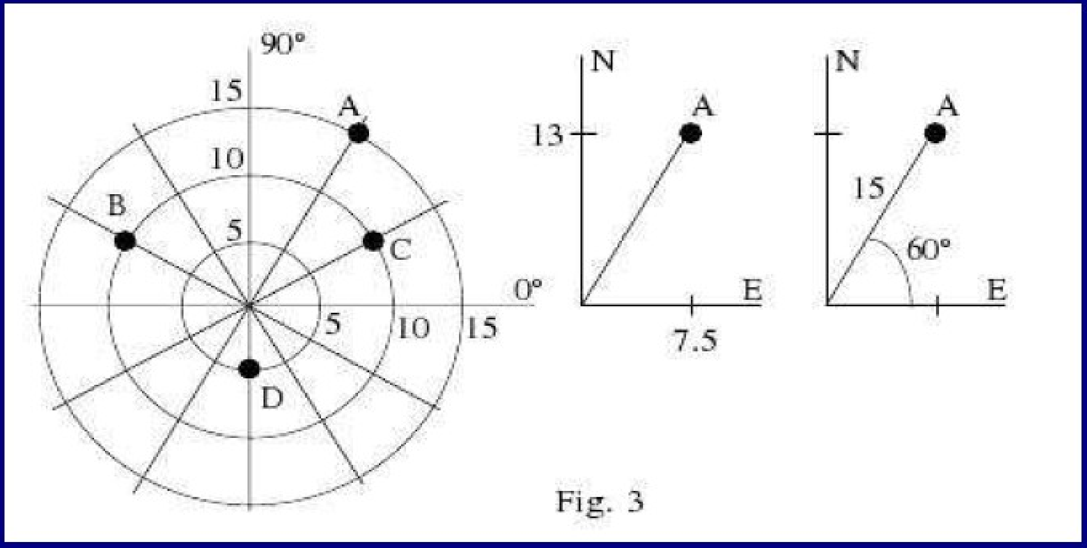


Fig. 3

SOS no ponto A.

Qual a melhor forma de chegarmos até o ponto?

Por coordenadas cartesianas, ou por coordenadas polares?

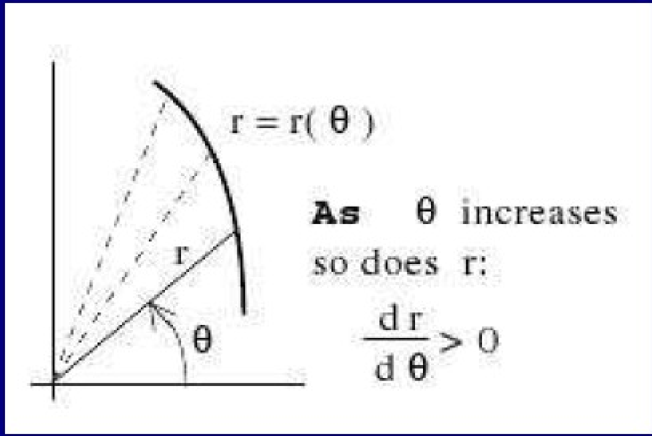
Pontos em Coordenadas Polares:

Origem ou polo => ponto central do sistema de coordenadas.

Eixo principal => uma direção inicial necessária para a determinação angular.

Um par ordenado (r, θ) => r é a distância do polo até o ponto e θ é o ângulo formado entre a reta representada pela distância r e o eixo principal.

O ângulo é positivo no sentido anti-horário e negativo no sentido horário.



# Polar coordinates

We can convert back and forth between cartesian and polar coordinates using that

$$x = r\cos(\theta) \tag{3.1}$$
$$y = r\sin(\theta), \tag{3.2}$$

and in the other direction

$$r^2 = x^2 + y^2 \tag{3.3}$$
$$\tan(\theta) = \frac{y}{x} \tag{3.4}$$

(Thus $r = \pm\sqrt{x^2 + y^2}$ and $\theta = \tan^{-1}(y/x)$.)

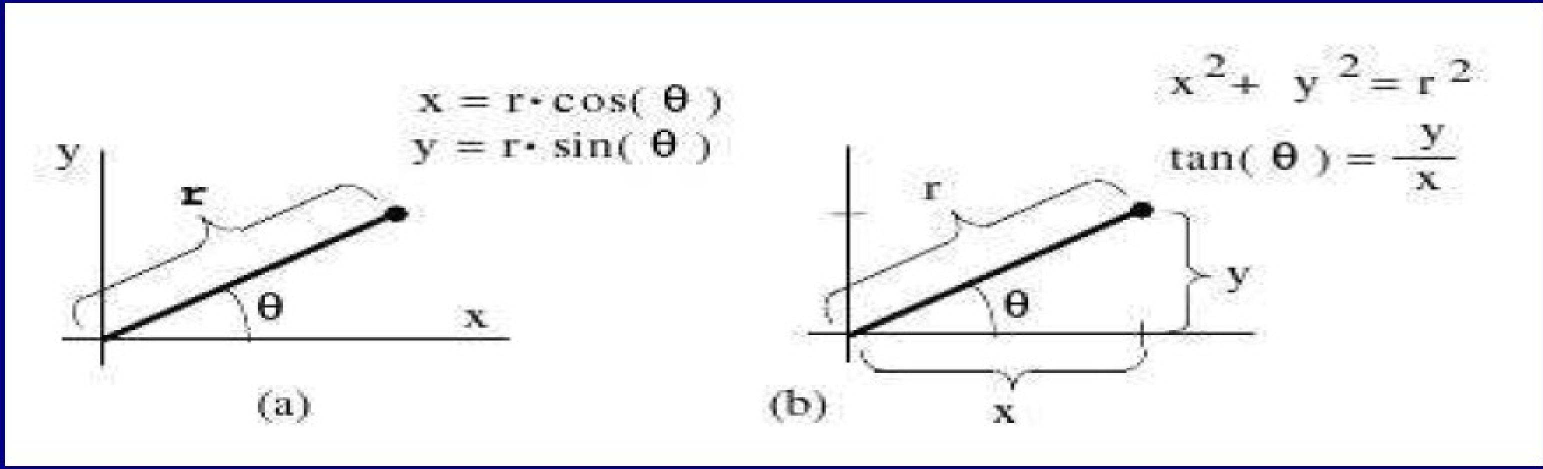
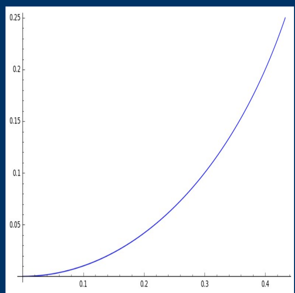


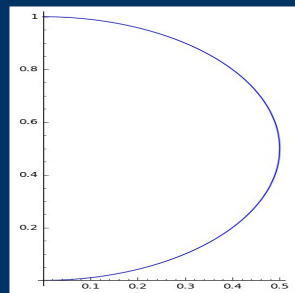

Figure 3.5: Rectangular to polar coordinate conversion.

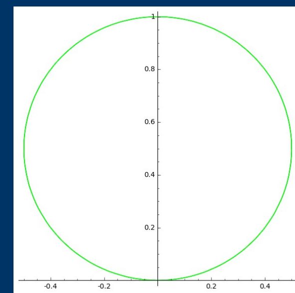
# Polar coordinates

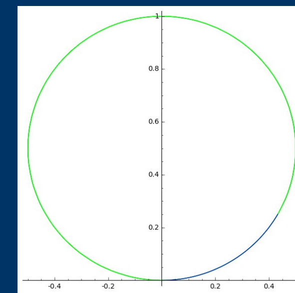
```
sage: P1 = polar_plot(lambda x:sin(x), 0, pi/6, rgbcolor=(0,0,1))
sage: P1
Launched png viewer for Graphics object consisting of 1 graphics primitive
sage: P1 = polar_plot(lambda x:sin(x), 0, pi/6, rgbcolor=(0,0,1))
sage: P1 = polar_plot(lambda x:sin(x), 0, pi/2, rgbcolor=(0,0,1))
sage: P1
Launched png viewer for Graphics object consisting of 1 graphics primitive
sage: P1 = polar_plot(lambda x:sin(x), 0, 2*pi, rgbcolor=(0,1,0))
sage: P1
Launched png viewer for Graphics object consisting of 1 graphics primitive
sage: P2 = polar_plot(lambda x:sin(x), 0, pi/6, rgbcolor=(0,0,1))
sage: P2
Launched png viewer for Graphics object consisting of 1 graphics primitive
sage: show(P1+P2)
Launched png viewer for Graphics object consisting of 2 graphics primitives
sage: help(polar_plot)

sage: polar_plot(sin(5*x)^2, (x, 0, 2*pi),
....: color='blue')
Launched png viewer for Graphics object consisting of 1 graphics primitive
sage: polar_plot(sin(x), (x, 0, 2*pi),color='blue')
Launched png viewer for Graphics object consisting of 1 graphics primitive
sage: polar_plot(abs(sqrt(1 - sin(x)^2)), (x, 0,
....: 2*pi), color='red')
Launched png viewer for Graphics object consisting of 1 graphics primitive
sage: polar_plot([2*sin(x), 2*cos(x)], (x, 0,
....: 2*pi))
Launched png viewer for Graphics object consisting of 2 graphics primitives
sage: polar_plot(sqrt, 0, 2 * pi, fill=True)
Launched png viewer for Graphics object consisting of 2 graphics primitives
sage: polar_plot(cos(4*x) + 1.5, 0, 2*pi, fill=0.5
....: * cos(4*x) + 2.5, fillcolor='orange')
Launched png viewer for Graphics object consisting of 2 graphics primitives
sage: polar_plot(sin(x) + 1, 0, 2*pi, fillcolor='orange')
Launched png viewer for Graphics object consisting of 1 graphics primitive
```

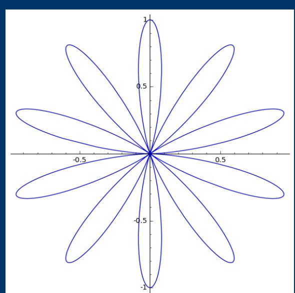
# Polar coordinates

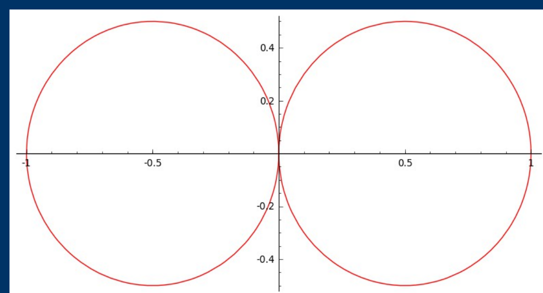


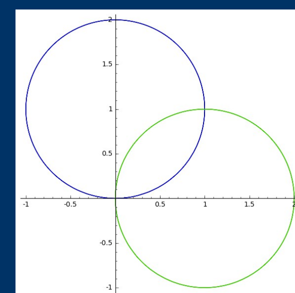


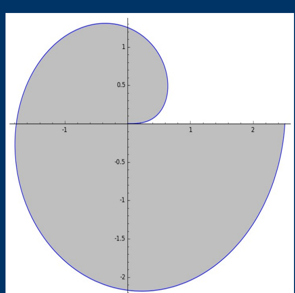


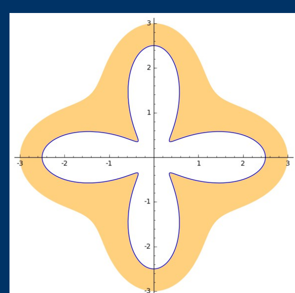


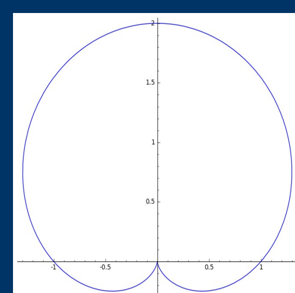












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