AULA 4

Software for Algebra and Geometry Experimentation - SAGE

A integral indefinida: conceitos e propriedades.

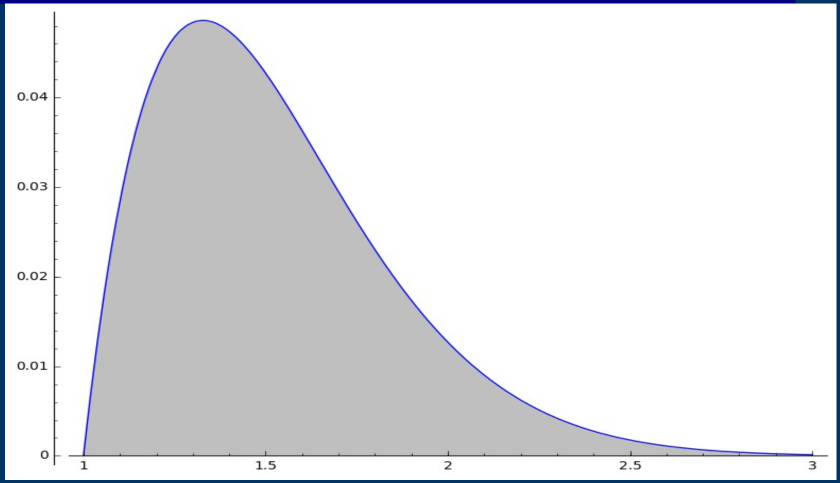
We consider the numerical integration of real functions; for a function $f : I \longrightarrow \mathbb{R}$, where I is an interval of $\mathbb{R}$, we want to approximate:

$$\int_I f(x)\,\mathrm{d}x.$$

For example, let us compute

$$\int_1^3 \exp(-x^2)\log(x)\,\mathrm{d}x.$$

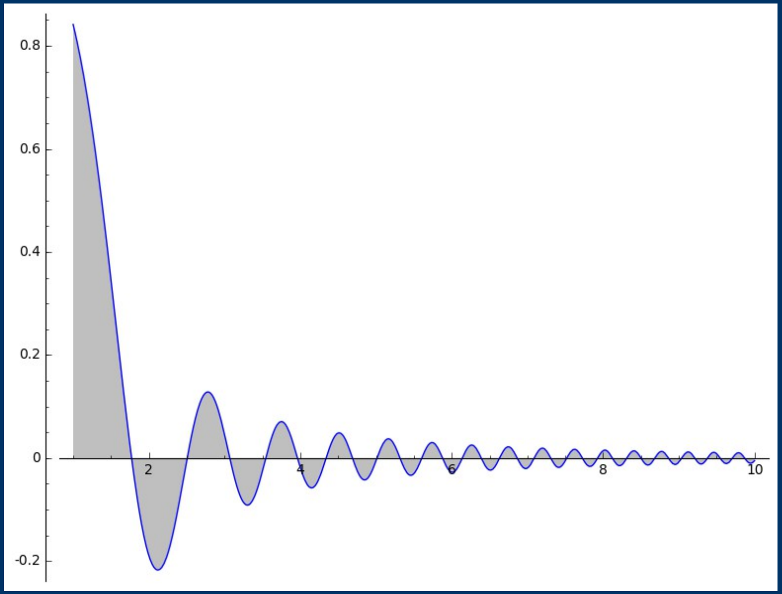
```
sage: x = var('x'); f(x) = exp(-x^2) * log(x)
sage: N(integrate(f, x, 1, 3))
0.035860294991267694
sage: plot(f, 1, 3, fill='axis')
Launched png viewer for Graphics object
consisting of 2 graphics primitives
```



A integral indefinida: conceitos e propriedades.

It is also possible, in principle, to compute integrals on an unbounded interval:

```
sage: N(integrate(sin(x^2)/(x^2), x, 1,
infinity))
0.285736646322853 - 6.93889390390723e-18*I
sage: plot(sin(x^2)/(x^2), x, 1, 10, fill='axis')
Launched png viewer for Graphics object
consisting of 2 graphics primitives
```



Evaluate $\int (x^3 - 3x + 2)dx$

```
sage: integral (x^3 -3*x+2,x)
1/4*x^4 - 3/2*x^2 + 2*x
```

Evaluate $\int x(x^3+2)^2 dx$

```
sage: integral (x*(x ^3+2) ^2,x)
1/8*x^8 + 4/5*x^5 + 2*x^2
```

Evaluate $\int \frac{2x}{\sqrt{x+1}} dx$

```
sage: integral (2*x/( sqrt (x +1)),x)
4/3*(x + 1)^(3/2) - 4*sqrt(x + 1)
sage: integral (2*x/( sqrt (x +1)),x). simplify_full ()
4/3*sqrt(x + 1)*(x - 2)
```

A integral indefinida: conceitos e propriedades.

Riemann Sums and the Definite Integral

Given a function f on a closed interval $[a,b]$ and a partition $P=\{x_0,x_1,...,x_n\}$ of the interval $[a,b]$, recall that Riemann sum of f over $[a,b]$ relative to P is a sum of the form

$$\sum_{i=1}^{n} f(x_i^*)\Delta x_i,$$

where $\Delta x_i = x_i - x_{i-1}$ and x_i^* is an arbitrary point in the ith subinterval $[x_{i-1}, x_i]$. We assume that $\Delta x_i = \Delta x \ \frac{b-a}{n}$ for all i. A Riemann sum is therefore an approximation to the area of the region between the graph of f and the x-axis along the interval $[a,b]$. The exact area is given by the definite integral of f over $[a,b]$, which is defined to be the limit of its Riemann sums an $n\to\infty$ and is denoted by $\int_a^b f(x)dx$:

$$\int_a^b f(x)dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i^*)\Delta x.$$

A integral indefinida: conceitos e propriedades.

Riemann Sum Using Left Endpoints

```
sage: a,b,nn ,f,x,i, leftrs , xstar = var
('a,b,nn ,f,x,i,leftrs , xstar')
sage: f(x)=x
sage: d=(b-a)/nn
sage: xstar (i)=a+(i -1)*d
sage: leftrs (a,b,nn)=sum(f( xstar (i))*d,i ,1,
nn)
sage: table ([(i,n( leftrs (0 ,2 ,i),digits =4) )
for i in range (10 ,110 ,10) ], header_row
=[ 'n','Riemann_Sum '], frame = T
....: rue )
```

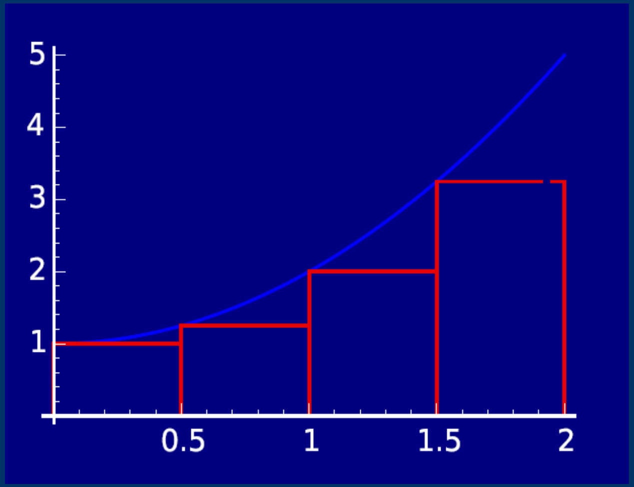
n	Riemann_Sum
10	1.800
20	1.900
30	1.933
40	1.950
50	1.960
60	1.967
70	1.971
80	1.975
90	1.978
100	1.980

```
sage: a,b,nn ,f,x,i, leftrs , xstar = var
('a,b,nn ,f,x,i,leftrs , xstar')
sage: f(x)=x^2+1
sage: d=(b-a)/nn
sage: xstar (i)=a+(i -1)*d
sage: leftrs (a,b,nn)=sum(f( xstar (i))*d,i ,1,
nn)
sage: table ([(i,n( leftrs (0 ,2 ,i),digits =4) )
for i in range (10 ,110 ,10) ], header_row
=[ 'n','Riemann_Sum '], frame = T
....: rue )
```

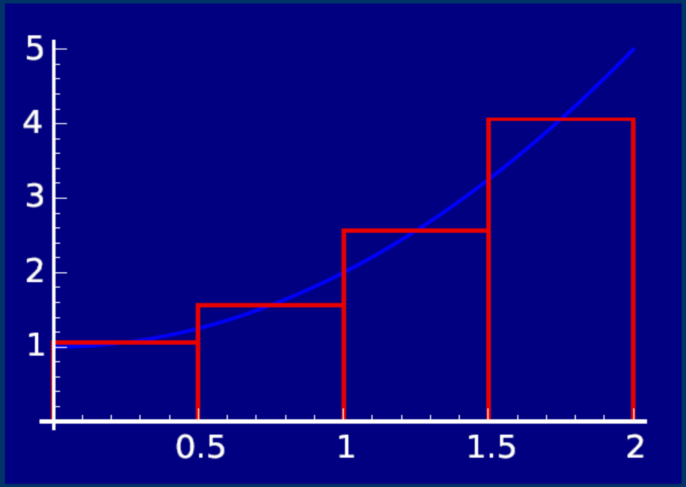
n	Riemann_Sum
10	4.280
20	4.470
30	4.535
40	4.568
50	4.587
60	4.600
70	4.610
80	4.617
90	4.622
100	4.627

A integral indefinida: conceitos e propriedades.

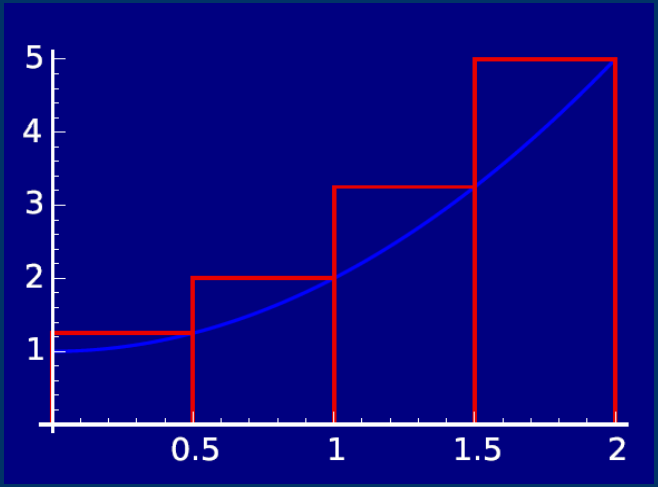
leftrs



midrs



rightrs




```
sage: a,b,nn ,f,x,i, rightrs , xstar =
var('a,b,nn ,f,x,i, rightrs , xstar ')
sage: f(x)=x
sage: d=(b-a)/nn
sage: xstar (i)=a+i*d
sage: rightrs (a,b,nn)=sum (f( xstar
(i))*d,i ,1, nn)
sage: table ([(i,n( rightrs (0 ,2
,i),digits =4) ) for i in range (10 ,
110 ,10) ], header_row
=[ 'n','Riemann_Sum'], frame = True )
sage: limit ( rightrs (0 ,2, nn),nn=
infinity )
2
```

```
sage: a,b,nn ,f,x,i= var('a,b,nn ,f,x,i')
sage: f(x)=x ^2+1
sage: d=(b-a)/nn
sage: xstar (i)=a+(i -1/2) *d
sage: midrs (a,b,nn)=sum(f( xstar
(i))*d,i ,1, nn)
sage: table ([(i,n( midrs (0 ,2
,i),digits =4) ) for i in range (10 ,
110 ,10) ], header_row
=[ 'n','Riemann_Sum '], frame = True )
sage: limit ( midrs (0 ,2 , nn),nn=
infinity )
14/3
```

AULA 5
Utilizando o SAGE para a solução de integrais.

The Fundamental Theorem of Calculus

Part I: Let f(x) is continuous on $[a, b]$, we have:

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

where $F(x)$ is any antiderivative of $f(x)$.

Part II:

$$\text{If } F(x) = \int_a^x f(t)\,dt \text{ , then } F'(x) = f(x)$$

Utilizando o SAGE para a solução de integrais.

INTEGRATION TECHNIQUES: BY PARTS

$$\int x\cos(5x)dx$$

$$\int re^{r/2}dr$$

$$\int (x^2+2x)\cos(x)dx$$

$$\int cos^{-1}(x)dx$$

$$\int t^4\ln(t)dt$$

$$\int t.cossec^2t.dt$$

$$\int \ln\left(\sqrt[3]{x}\right)dx$$

```
sage: integral(x*cos(5*x))
1/5*x*sin(5*x) + 1/25*cos(5*x)

sage: var ('r')
r
sage: integral(r*exp(r/2))
2*(r - 2)*e^(1/2*r)

sage: integral((x^2+2*x)*cos(x))
2*x*cos(x) + (x^2 - 2)*sin(x) + 2*x*sin(x) + 2*cos(x)
```

Utilizando o SAGE para a solução de integrais.

INTEGRATION TECHNIQUES: trigonometric integrals(x

$$\int sen^3x.cos^2x.dx$$

$$\int_0^{\pi/2} sen^7\theta.cos^5\theta.d\theta$$

$$\int sen^5(2t).cos^2(2t).dt$$

$$\int_0^{\pi/2} cos^2(\theta)d\theta$$

$$\int_0^{\pi} cos^4(2t)dt$$

$$\int_0^{\pi/2} sen^2x.cos^2x.dt$$

$$\int \sqrt{cos(\theta)}.sen^3(\theta).d\theta$$

```
sage: integral(((sin(x))^3)*(cos(x)^2))
1/5*cos(x)^5 - 1/3*cos(

sage: var('teta')
teta
sage: n(integral(((sin(teta))^7)*((cos(teta))^5), teta, 0, pi()/2))
0.00833333333333333

sage: var('t')
t
sage: integral(((sin(2*t))^5)*(cos(2*t)^2))
-1/14*cos(2*t)^7 + 1/5*cos(2*t)^5 - 1/6*cos(2*t)^3
```

Utilizando o SAGE para a solução de integrais.

INTEGRATION TECHNIQUES: trigonometric substitutions

$$\int \frac{\sqrt{x^2-1}}{x^4}\,dx$$

$$\int_0^a \frac{dx}{(a^2+x^2)^{3/2}}, \ldots a > 0$$

$$\int_2^3 \frac{dx}{(x^2-1)^{3/2}}$$

$$\int_0^{1/2} x\sqrt{1-4x^2}\,dx$$

$$\int \frac{\sqrt{x^2-9}}{x^3}\,dx$$

$$\int_0^a x^2\sqrt{a^2-x^2}\,dx$$

$$\int \frac{x}{\sqrt{x^2-7}}\,dx$$

```
sage: integral((1)/((a^2+x^2)^(3/2)),x)
x/(sqrt(a^2 + x^2)*a^2)
sage: f(x)=integral((1)/((a^2+x^2)^(3/2)),x)
sage: f(x)
x/(sqrt(a^2 + x^2)*a^2)
sage: f(0)
0
sage: f(a)
1/2*sqrt(2)/(sqrt(a^2)*a)
sage: f(a)-f(0)
1/2*sqrt(2)/(sqrt(a^2)*a)
```

```
sage: f(x)=integral((x^2)*sqrt(a^2+x^2),x)
sage: f(x)
-1/8*a^4*arcsinh(x/sqrt(a^2)) - 1/8*sqrt(a^2 + x^2)*a^2*x
+ 1/4*(a^2 + x^2)^(3/2)*x
sage: f(a)-f(0)
-1/8*a^4*arcsinh(a/sqrt(a^2)) - 1/8*sqrt(2)*sqrt(a^2)*a^3
+ 1/2*sqrt(2)*(a^2)^(3/2)*a
```

Utilizando o SAGE para a solução de integrais.

INTEGRATION TECHNIQUES: racional functions by partial fractions

$$\int \frac{x}{x-6}\,dx$$

$$\int \frac{x-9}{(x+5)(x-2)}\,dx$$

$$\int_0^1 \frac{2}{(2x^2+3x+1)}\,dx$$

$$\int_3^4 \frac{x^3-2x^2-4}{x^3-2x^2}\,dx$$

$$\int_{-1}^0 \frac{x^3-4x+1}{x^2-3x+2}\,dx$$

$$\int_1^2 \frac{4y^2-7y-12}{y(y+2)(y-3)}\,dy$$

$$\int_0^1 \frac{x^2+x+1}{(x+1)^2(x+2)}\,dx$$

```
sage: integral((x)/(x-6),x)
x + 6*log(x - 6)
```

```
sage: integral((x-9)/((x+5)*(x-2)),x)
2*log(x + 5) - log(x - 2)
```

```
sage: f(x)=integral((2)/
(2*x^2+3*x+1),x)
sage: f(1)-f(0)
2*log(3) - 2*log(2)
```

References:

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