Software for
Algebra and Geometry Experimentation - SAGE

Um tour no SAGE:

sage: prompt; you do not have to add it.

```
sage: 2+3
5
```

The caret symbol means "raise to a power".

```
sage: 30.2^55
2.51409603225866e81
```

Compute the inverse of a 2 x 2 matrix:

```
sage: matrix([[1,2],[3,4]])^(-1)
[ -2   1]
[ 3/2 -1/2]
```

O cálculo diferencial e seus conceitos principais.

integrate a simple function:

```
sage: x = var('x') # create a symbolic variable
sage: integrate(sqrt(x)*sqrt(1+x), x)
1/4*((x + 1)^(3/2)/x^(3/2) + sqrt(x + 1)/sqrt(x))/((x + 1)^2/x^2 - 2*(x + 1)/x + 1)
- 1/8*log(sqrt(x + 1)/sqrt(x) + 1) + 1/8*log(sqrt(x + 1)/sqrt(x) - 1)
```

Solve a quadratic equation:

```
sage: a=var('a')
sage: S=solve(x^2+x==a,x);S
[x == -1/2*sqrt(4*a + 1) - 1/2, x == 1/2*sqrt(4*a + 1) - 1/2]
```

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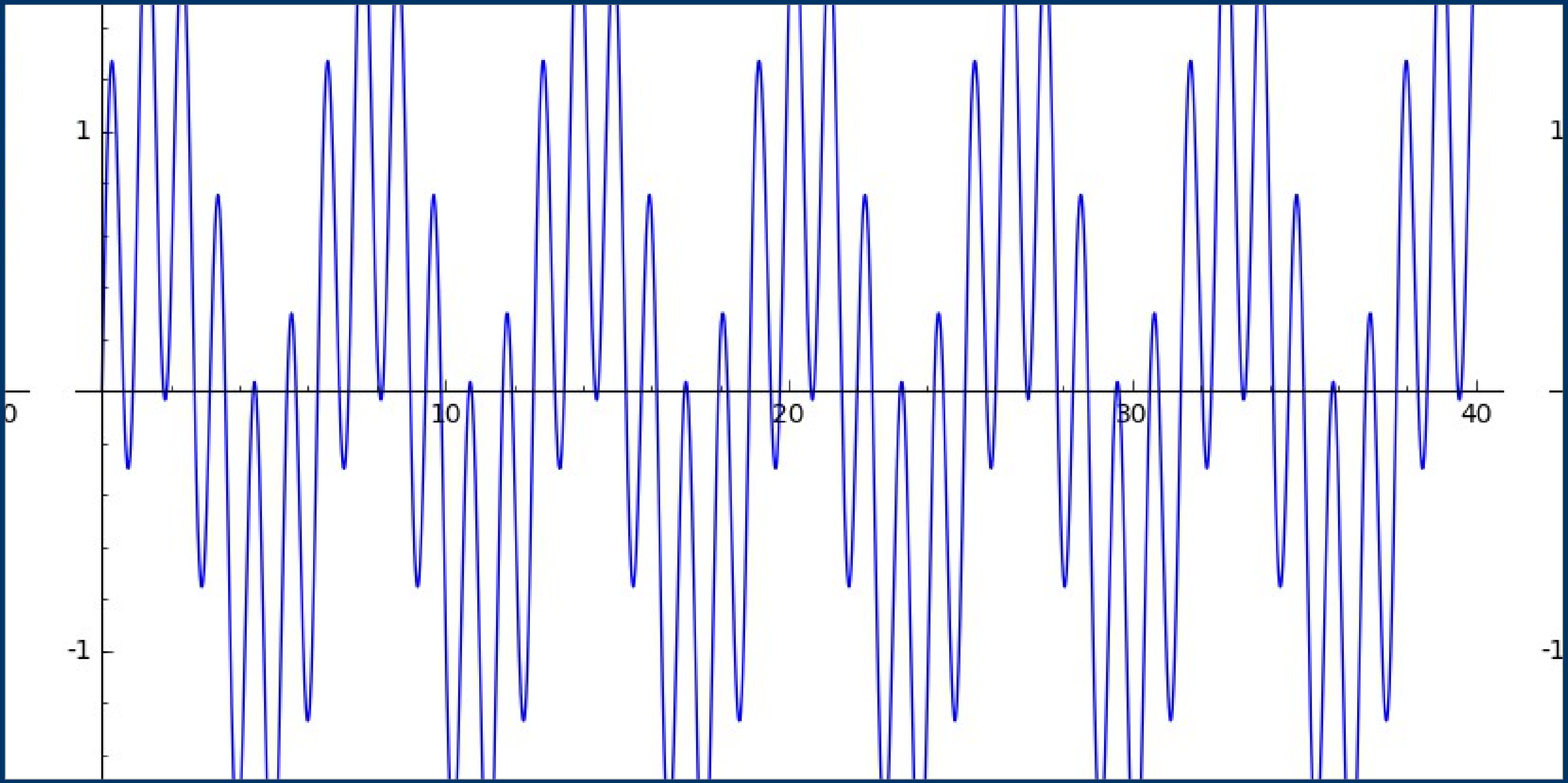
result is a list of equalities:

```
sage: S[0].rhs()
```

-1/2*sqrt(4*a + 1) – ½

Plot various useful functions:

```
sage: show(plot(sin(x) + sin(6*x), 0, 40))
Launched png viewer for Graphics object
consisting of 1 graphics primitive
```



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Create a 500 x 500 matrix of random numbers:

```
sage: m = random_matrix(RDF,500)
sage: m
500 x 500 dense matrix over Real Double
Field (use the '.str()' method to see the
entries)
sage: str(m)
```

```
 0.6361575561318267      0.43228051944679047      -0.4425473436266165
4788703780377      0.1568287058287503      0.7995937575945371
3236489      0.6245361978564246      0.763410021558371      -0.43965
5      -0.5929912389203189      -0.6668546230171153      -0.7963808871
).03241401855744308      -0.07777175248589674      -0.8921120762714068
2502705111168      0.6720884197775883      0.18000354127385787      -0
7521728      0.38702082978886376      0.5213416413055516      -0.33986
)      0.7130039582422922      -0.19007809922061591      -0.8973834577
).04511752494257326      -0.09951692029946813      -0.3296778642088387
)692278068804      0.43385508558199715      0.24159454962418425
7162014      -0.6796092580624595      0.635333234663285      0.3723
7      0.26595619534115933      -0.25076996176335387      0.6086259840
-0.0874690420886306      0.27390205884563557      0.6984811944544886
1887281277638      0.003460316939720931      0.2665639041256176      -0
3037958      -0.14977575587937553      -0.48717805896763156      0.07034
)      -0.492178917278705      0.9495506846570929      0.575354160
 0.7118822801742108      0.751257006368643      0.5103819266501086
```

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The eigenvalues of the matrix and plot them:

```
sage: e = m.eigenvalues()
sage: e#about 2 seconds
sage: w = [(i, abs(e[i])) for i in range(len(e))]
sage: show(points(w))
```

Eigenvalues and eigenvectors of matrices

Eigenvalues and eigenvectors are often introduced to students in the context of linear algebra courses focused on matrices.[23][24] Furthermore, linear transformations can be represented using matrices,[1][2] which is especially common in numerical and computational applications.[25]

Consider n-dimensional vectors that are formed as a list of n scalars, such as the three-dimensional vectors

$$x = \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix} \quad \text{and} \quad y = \begin{bmatrix} -20 \\ -60 \\ -80 \end{bmatrix}.$$

These vectors are said to be scalar multiples of each other, or parallel or collinear, if there is a scalar λ such that

$$x = \lambda y.$$

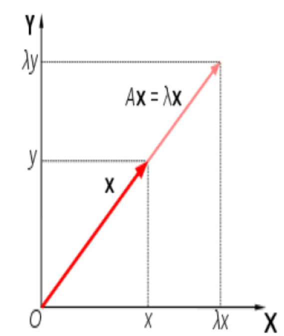
In this case $\lambda = -1/20$.

Now consider the linear transformation of n-dimensional vectors defined by an n by n matrix A,

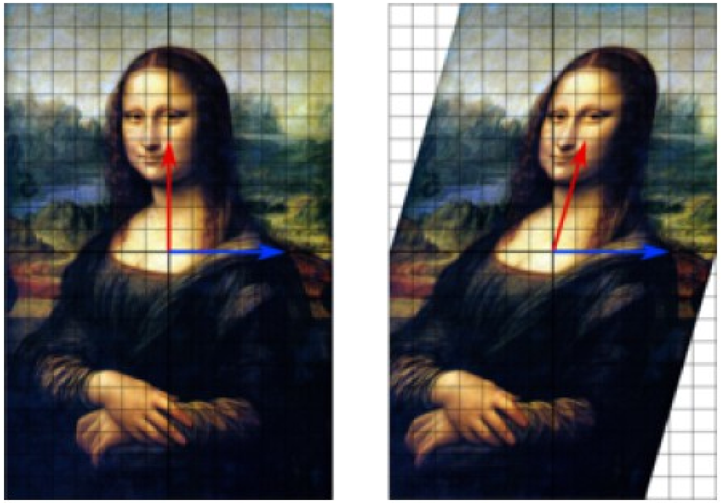
$$Av = w,$$

or

$$\begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1n} \\ A_{21} & A_{22} & \cdots & A_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ A_{n1} & A_{n2} & \cdots & A_{nn} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix} = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix}$$

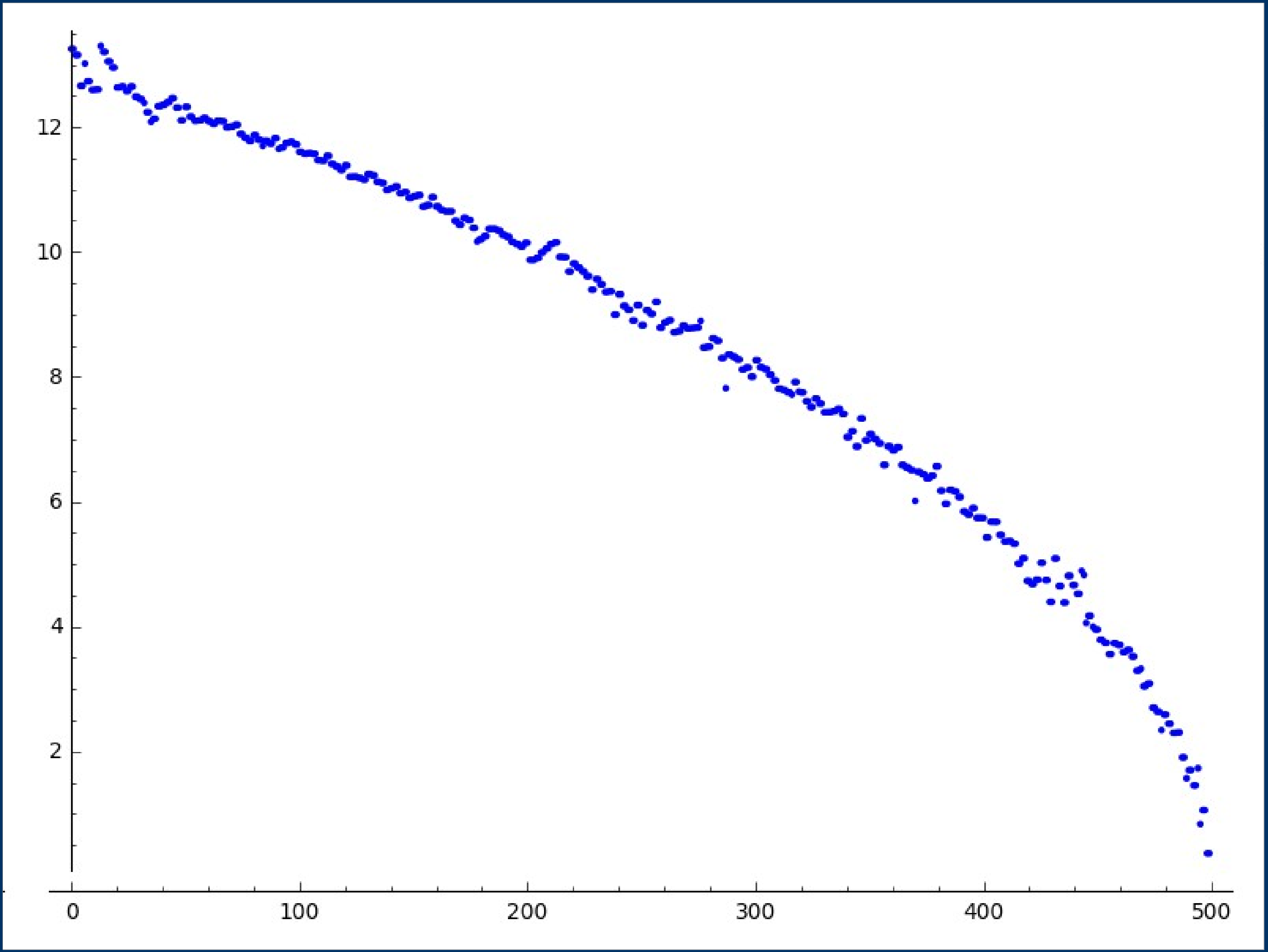


Matrix A acts by stretching the vector x, not changing its direction, so x is an eigenvector of A.



In this shear mapping the red arrow changes direction but the blue arrow does not. The blue arrow is an eigenvector of this shear mapping because it doesn't change direction, and since its length is unchanged, its eigenvalue is 1.

O cálculo diferencial e seus conceitos principais.



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Large numbers:

```
sage: factorial(100)
93326215443944152681699238856266700490715968264381621468
59296389521759999322991560894146397615651828625369792082
722375825118521
sage: n = factorial(1000000) #about 2.5 seconds
sage: N(pi, digits=100)
3.141592653589793238462643383279502884197169399375105820
9749592307816406286208998628034825342117068
```

Factor a polynomial in two variables:

```
sage: R.<x,y> = QQ[]
sage: R
Multivariate Polynomial Ring in x, y over Rational
Field
sage: F = factor(x^10 + y^10)
sage: F
(x^2 + y^2) * (x^8 - x^6*y^2 + x^4*y^4 - x^2*y^6 + y^8)
sage: F.expand()
```

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Notation of functions:

The symbol f(x) is used to denote a function of x, and is read "f of x". In order to distinguish between different functions, the prefixed letter is changed, as F(x), φ(x), f′(x), etc.

$$f(x) = x^2 - 9x + 14$$
$$f(b + 1) = (b + 1)^2 - 9(b + 1) + 14 = b^2 - 7b + 6$$
$$f(7) = 7^2 - 9 \cdot 7 + 14 = 0$$

$$\varphi(x, y) = \sin(x + y)$$
$$\varphi(a, b) = \sin(a + b)$$

$$F(x, y, z) = 2x + 3y - 12z$$
$$F(m, -m, m) = 2m - 3m - 12m = -13m$$
$$F(3, 2, 1) = 2 \cdot 3 + 3 \cdot 2 - 12 \cdot 1 = 0$$

O cálculo diferencial e seus conceitos principais.

SAGE`s functions:

```
sage: x,y = var("x,y")
sage: f = log(sqrt(x))
sage: f
log(sqrt(x))
sage: f(4).simplify_log()
/opt/sagemath-8.1/local/lib/python2.7/site-packages/IPython/core/interactiveshell.py:2881: DeprecationWarning: Substitution using function-call syntax and unnamed arguments is deprecated and will be removed from a future release of Sage; you can use named arguments instead, like EXPR(x=..., y=...)
See http://trac.sagemath.org/5930 for details.
  exec(code_obj, self.user_global_ns, self.user_ns)
log(2)
sage: f = lambda x: (x^2+1)/2
sage: f(x)
1/2*x^2 + 1/2
sage: f(1)
1
sage: f = lambda x,y: x^2+y^2
sage: f(x,y)
x^2 + y^2
sage: f(3,4)
25
```

O cálculo diferencial e seus conceitos principais.

SAGE`s functions:

```
sage: R.<x> = PolynomialRing(CC,"x")
sage: R
Univariate Polynomial Ring in x over Complex Field with 53 bits of precision
sage: CC
Complex Field with 53 bits of precision
sage: f = x^2+2
sage: f
x^2 + 2.00000000000000
sage: f.roots()
[(-1.41421356237310*I, 1), (1.41421356237310*I, 1)]
sage: f = x^3+x^2+2
sage: f
x^3 + x^2 + 2.00000000000000
sage: f.roots()
[(-1.69562076955986, 1),
 (0.347810384779931 - 1.02885225413669*I, 1),
 (0.347810384779931 + 1.02885225413669*I, 1)]
sage: f = x^4+x^3+x^2+2
sage: f
x^4 + x^3 + x^2 + 2.00000000000000
sage: f.roots()
[(-1.00000000000000 - 1.00000000000000*I, 1), (-1.00000000000000 + 1.00000000000000*I, 1),
 (0.500000000000000 - 0.866025403784439*I, 1), (0.500000000000000 + 0.866025403784439*I, 1)]
sage: I
I
```

THE END AULA 1