1 Algebraic form and conjugation of a complex number

Exercise 1.1

Simplify:

1. $(-2+3i)+(7-8i)$
2. $(4i-3)-(1+10i)$
3. $(\sqrt{2}+i)\cdot(3-\sqrt{3}i)$
4. $\frac{2-3i}{5+4i}$

Exercise 1.2

Find the real numbers x, y satisfying the given equations:

1. $x(2+3i)+y(4-5i)=6-2i$
2. $(x-i)\cdot(2-yi)=11-23i$
3. $\frac{x}{2-3i}+\frac{y}{3+2i}=1$

Exercise 1.3

Solve the given equations in the set of complex numbers:

1. $z^2+3\bar{z}=0$
2. $2z+(1+i)\bar{z}=1-3i$
3. $z^2-z+1=0$
4. $\frac{z+1}{\bar{z}-1}=-1$
5. $(z+\bar{z})+i(z-\bar{z})=2i-6$
6. $(i-3)z=5+i-z$
7. $\frac{1-3i}{3z+2i}=\frac{2i-3}{5-2iz}$

Exercise 1.4

For which values of the real parameters a, b the equation $3\bar{z}-2z=a+bi$ has a solution?

Exercise 1.5

On the complex plane, draw sets of numbers z that satisfy the given conditions:

1. $\operatorname{Im}[(1+2i)z-3i]<0$
2. $\operatorname{Re}(z-i)^2\geq 0$
3. $z^2=2\operatorname{Re}(iz)$
4. $\operatorname{Re}\left(z^3\right)\geq\operatorname{Im}\left(z^3\right)$

Exercise 1.6

Sketch the set of all complex numbers z for which the number $\omega=\frac{z}{z+i}$ is

1. real
2. purely imaginary

Exercise 1.7

Points $z_1=-1+2i$, $z_2=i$, and $z_4=2+4i$ are the vertices of the parallelogram. Find the position of vertex z_3 of this parallelogram.

2 Modulus and argument of a complex number. The trigonometric form of a complex number.

Exercise 2.1

Calculate the modules of the given complex numbers:

1. $4i$
2. $12i-5$
3. $\sqrt{7}+\sqrt{29}i$
4. $(\sqrt{5}-\sqrt{3})+(\sqrt{5}+\sqrt{3})i$
5. $\sin\alpha+i\cos\alpha$, and $\alpha\in\mathbb{R}$

Exercise 2.2

Give a geometric interpretation of the modulus of difference of complex numbers. Using this interpretation, draw sets of complex numbers a satisfying the given conditions:

1. $|z+1-2i|=3$
2. $2\leq|z+i|<4$
3. $|(1+i)z-2|\geq 4$
4. $\left|\frac{z+3}{z-2i}\right|\geq 1$
5. $\operatorname{Re}(z+1)<0$ and $|i-z|\leq 3$
6. $\left|z^2+4\right|\leq|z-2i|$

Exercise 2.3

Write down the given complex numbers in trigonometric form:

1. $-\sqrt{5}$
2. $-6+6i$
3. $-2i$
4. $\sqrt{3}+i$
5. $\sin\alpha - i\cos\alpha$
6. $1-i\cot\alpha$

Exercise 2.4

Draw sets of complex numbers z satisfying the given conditions:

1. $\frac{\pi}{6} < \arg z \leq \frac{2\pi}{3}$
2. $\arg(z+2-i) = \pi$
3. $\pi \leq \arg[(-1+i)z] \leq \frac{3\pi}{2}$
4. $\frac{\pi}{2} < \arg\left(z^3\right) < \pi$
5. $\arg\left(\frac{i}{z}\right) = \frac{3\pi}{4}$
6. $\frac{2\pi}{3} \leq \arg(3i-z) \leq \frac{5\pi}{6}$

3 Exponential form of a complex number. Root of complex numbers.

Exercise 3.1

Using the exponential form of the complex number z solve the given equations:

1. $(\bar{z})^6 = 4\left|z^2\right|$
2. $\frac{|z|^2 z}{(\bar{z})^3} = -1$
3. $z^7 = \bar{z}$
4. $|z|^3 = iz^3$
5. $(\bar{z})^2\left|z^2\right| = \frac{4}{z^2}$
6. $\left|z^8\right| = z^4$

Exercise 3.2

Using the definition, calculate the given roots:

1. $\sqrt{4i-3}$
2. $\sqrt[3]{8}$
3. $\sqrt{5-12i}$
4. $\sqrt{-11+60i}$
5. $\sqrt[3]{i}$
6. $\sqrt[4]{16}$

Exercise 3.3

Calculate and draw the given roots on the complex plane:

1. $\sqrt{-2i}$
2. $\sqrt[4]{-8+8\sqrt{3}i}$

by guessing one of the elements of the given elements, calculate the remaining elements of these elements

Exercise 3.4

By guessing one of the elements of the given roots, calculate the remaining elements of these roots:

1. $\sqrt{(3-5i)^2}$
2. $\sqrt[3]{(1+i)^6}$

Exercise 3.5

One of the vertices of the square is the point $z_1 = 4-i$. Find the other vertices of this square if its center is:

1. origin of the coordinate system
2. point: $u=1$
3. point: $u = 7+\sqrt{2}i$