Mathias Lazda
260845451

Problem 1:

$$\frac{f(t+dt,x) - f(t-dt,x)}{2dt} = -v\,\frac{f(t,x+dx) - f(t,x-dx)}{2dx}$$

Assume $f(x,t) = \varepsilon^{t} e^{ikx}$. $f_{n,j} = \varepsilon^{j} e^{ikndx}$. The above equation becomes:

$$\frac{\varepsilon^{j+1} e^{ikndx} - \varepsilon^{j-1} e^{ikndx}}{2dt} = -v\,\frac{\left(\varepsilon^{j} e^{ik(n+1)dx} - \varepsilon^{j} e^{ik(n-1)dx}\right)}{2dx}$$

$$\Rightarrow \frac{\varepsilon^{1} - \varepsilon^{-1}}{dt} = -v\,\frac{e^{ikdx} - e^{-ikdx}}{dx}$$

$$\Rightarrow \varepsilon^{2} - 1 = -v\frac{dt}{dx}\left(2i\sin(kdx)\right)\varepsilon$$

$$\frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$\Rightarrow \varepsilon^{2} + 2v\frac{dt}{dx} i \sin(kdx)\,\varepsilon - 1 = 0$$

$$\Rightarrow \varepsilon = \frac{-i\left(\cancel{2}v\frac{dt}{dx}\sin(kdx)\right) \pm \sqrt{4 - \left(2v\frac{dt}{dx}\sin(kdx)\right)^2}}{\cancel{2}}$$

$$\Longrightarrow \quad \varepsilon = -i\left(\frac{v\,dt}{dx}\sin(k\,dx)\right) \pm \sqrt{1-\left(\frac{v\,dt}{dx}\sin(k\,dx)\right)^2}$$

The requirement that energy is conserved requires that $|\varepsilon| \leq 1$. If the CFL condition is satisfied, then $\frac{v\,dt}{dx} \leq 1$ and

$$|\varepsilon|^2 = \cancel{\left(\frac{v\,dt}{dx}\sin(k\,dx)\right)^2} + 1 - \cancel{\left(\frac{v\,dt}{dx}\sin(k\,dx)\right)^2}$$

$$\Longrightarrow \boxed{|\varepsilon| = 1.}$$

$$e = V - \bar{V} \Rightarrow V = e + \bar{V}$$

$$\varepsilon_0 =$$