Problem 1

See `problem_1.txt` or the attached Jupyter Notebook for printed output. By running Jon's original code, we obtain a χ^2 value of 15267.938. After updating the parameters in the original code, the χ^2 value decreased all the way down to 3272.203377.

Given that mean of χ^2 is 2501 with standard deviation 70, our obtained values of both 15267 and 3272 fall extremely far outside of the 1σ region (both fall outside $>10\sigma$), consequently, I would not consider these values an acceptable fit.

Problem 2

See `planck_fit_params.txt` or attached Jupyter Notebook for printed output. Using Levenberg-Marquardt, I was able to estimate the best fit parameters. One major point to highlight in my code is that in order to make sure that τ didn't blow up (or down) to some un-physical value (which it tended to do and `CAMB` became unhappy), I included a line of code that checks if, after updating the parameters, the parameter became un-physical, a new step size was used and it tried to calculate the parameters again.
The result of the simulation yielded a χ^2 of 2577.93, which now falls extremely close to 1σ from the mean of 2501, so I would consider these values a strong fit.

Problem 3

See `planck_chaintxt` for χ^2 and chain. After running the MCMC simulation, I got the following results. Figures 1 and 2 highlight χ^2 as a function of steps as well as the burn in period, which occurs at around step 400. Figures 3 and 4 portray the amplitude of each parameter as a function of time, with and without the burn in period respectively. Plots highlight that the parameters look like noise and appear to have converged. This is confirmed in Figure 5, which suggests that the chain has indeed converged due to the visible 'knee' in each parameter's power spectrum. Lastly, Figure 6 is a corner plot which provides the best fit values and their errors, making sure to only use data after the burn in period is complete. One can also find these printed in the attached Jupyter Notebook. Calculating Ω_Λ, I find that using the best fit parameters,

$$\Omega_\Lambda = 1 - \left(\frac{100}{H_0}\right)^2 (\Omega_b h^2 + \omega_C h^2) = 0.69 \tag{1}$$

with error,

$$\Delta_{\Omega_\Lambda} = \Omega_\Lambda \left[\left(\frac{\Delta\Omega_b h^2}{\Omega_b h^2}\right)^2 + \left(\frac{\Delta\Omega_C h^2}{\Omega_C h^2}\right)^2 + 2\left(\frac{2\Delta H_0}{H_0}\right)^2\right]^{1/2} = 0.03 \tag{2}$$

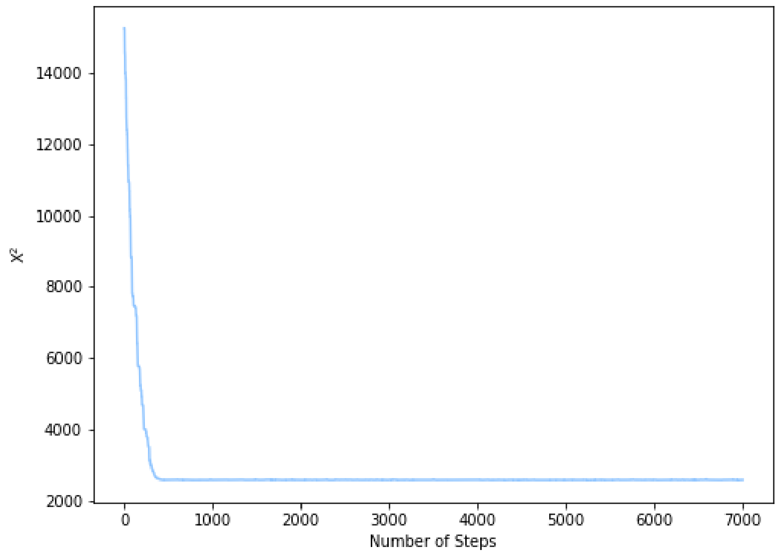

Figure 1: χ^2 as a function of step. Notice that the burn in period occurs from roughly steps $0 - 400$ (see Figure 2).

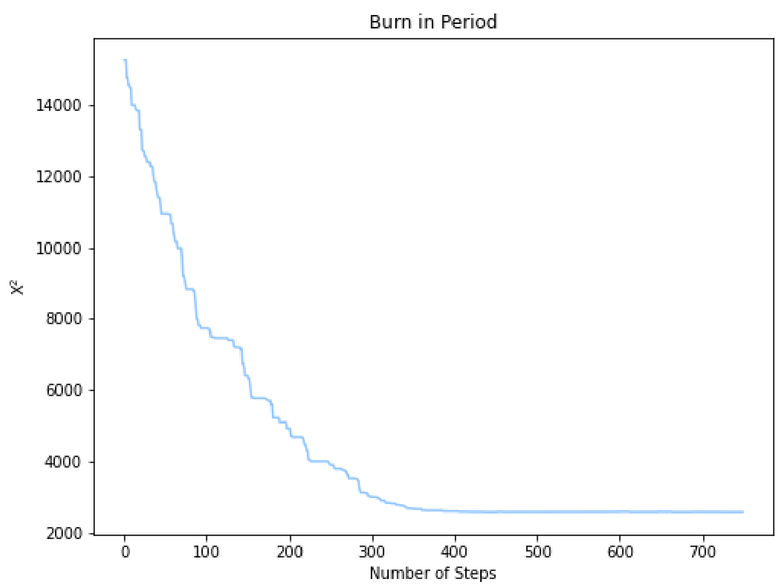

Figure 2: Burn in period of χ^2. Ends at roughly step 400.

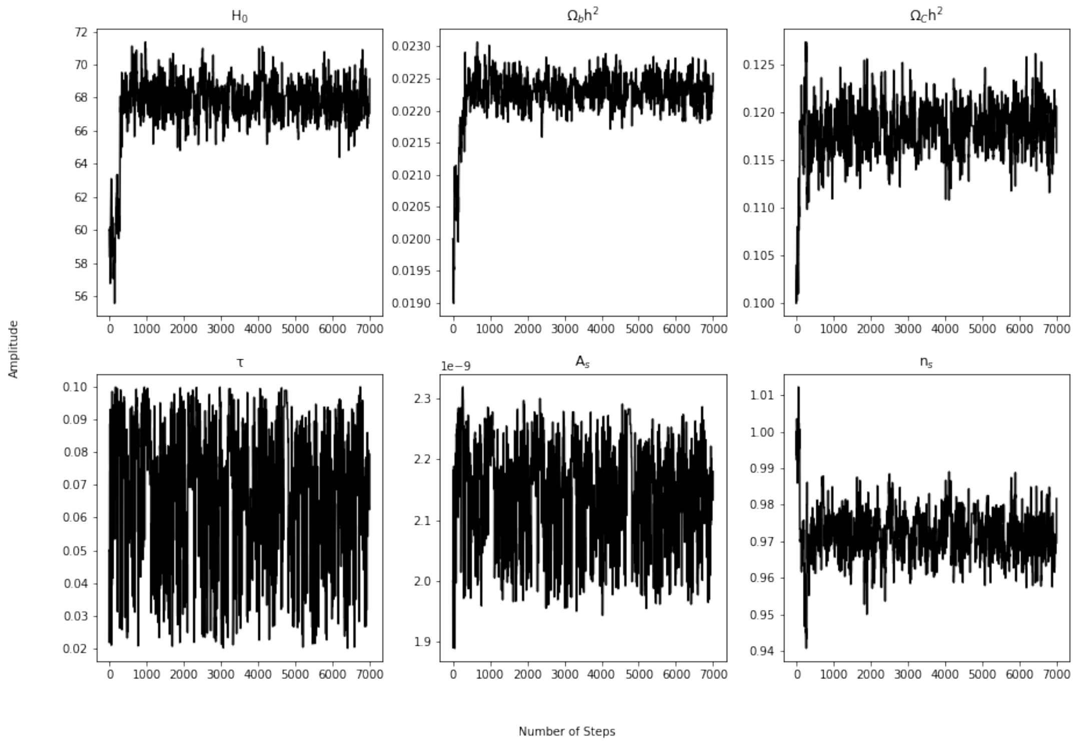

Figure 3: Amplitudes of parameters as a function of time, including burn in period. One can see that after the burn in period, amplitudes fluctuate randomly about some average (see Figure 4).

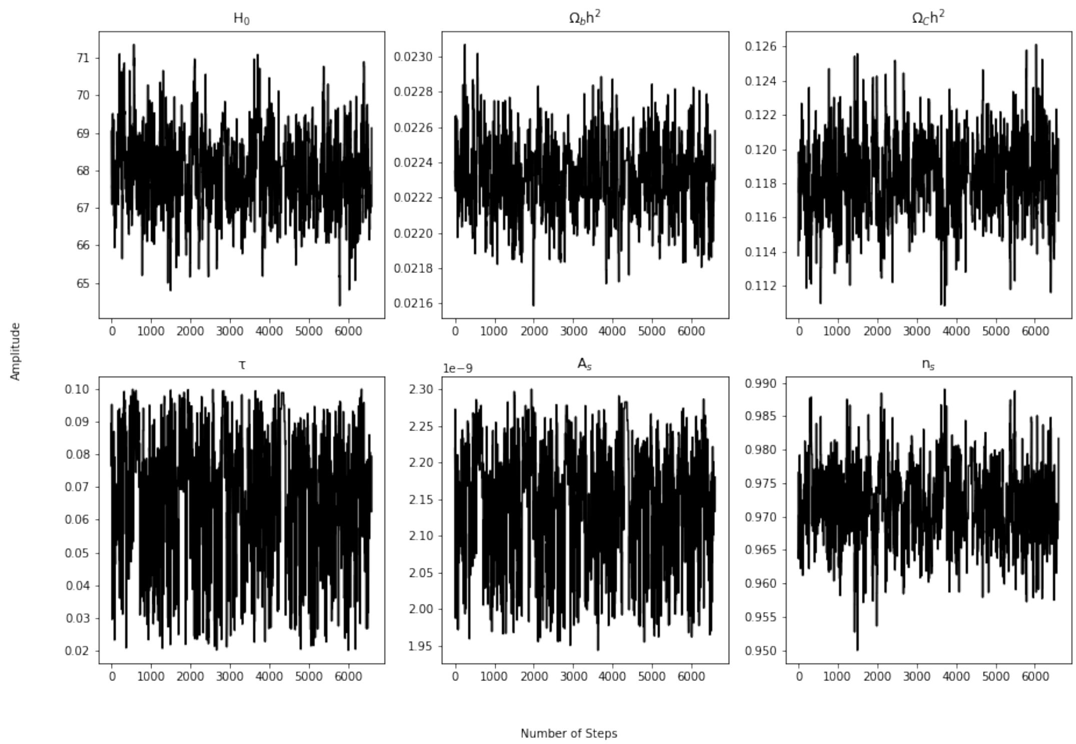

Figure 4: Amplitudes of parameters as a function of time with burn in period removed. Amplitudes tend to fluctuate about some mean and look like noise.

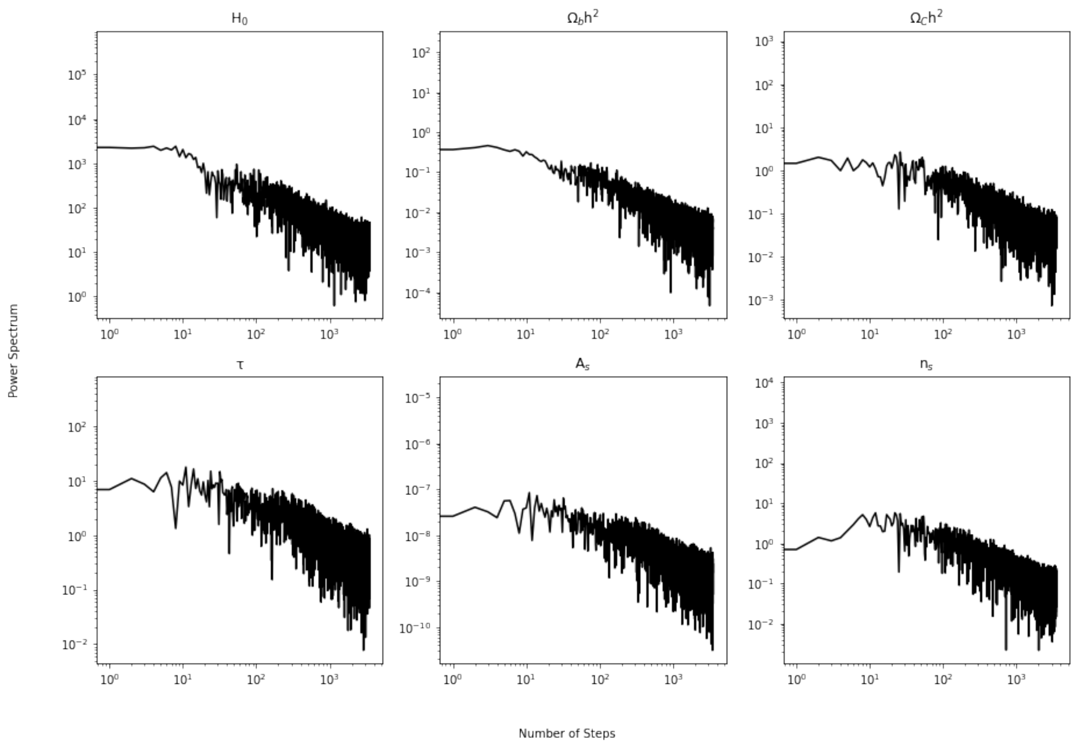

Figure 5: Power spectrum of each parameter. Note that it appears that each chain has converged since a 'knee' is visible for each parameter.

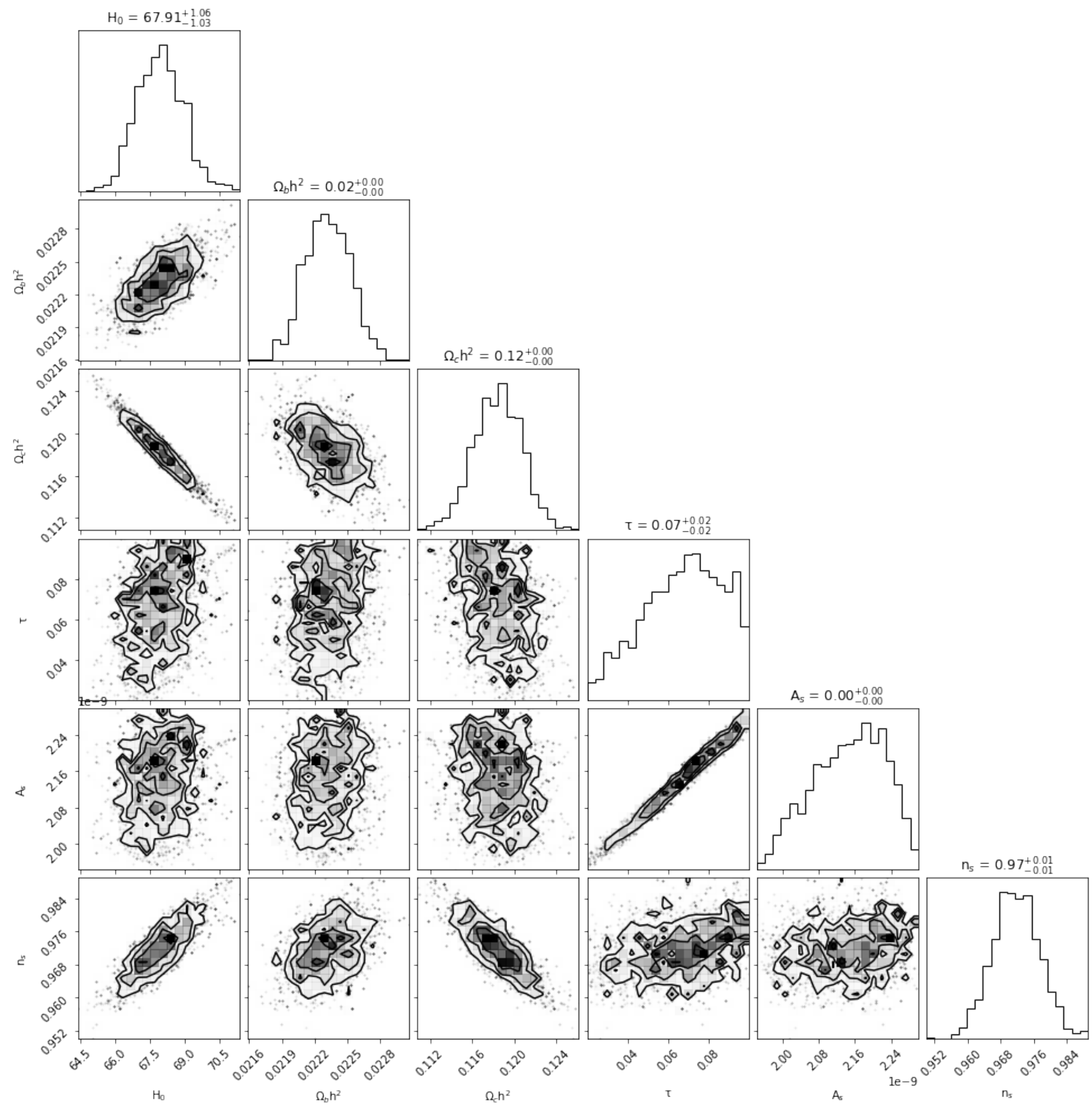

Figure 6: Corner plot of parameters. While all values are quoted, A_s is too small to quote on the figure, but it's value was $2.14 \times 10^{-9} \pm 7.8 \times 10^{-11}$.

Problem 4

See `planck_chain_tauprior.txt` for updated χ^2 and chain. Repeating the same process as Problem 3, but this time taking into account importance sampling, I got the following results. To begin, I re-ran my LM fitter to re-estimate my parameter covariance matrix after including information about the prior on τ. After 200 iterations, it approached a similar χ^2 as in Problem 3 of $\chi^2 = 2579.70$, falling just outside of 1σ from the mean. Note that `curvature_matrix_bestfit_tauprior.txt` contains the updated covariance matrix. After the covariance matrix was re-estimated, I ran my MCMC while taking into account importance sampling. One immediate thing that I noticed was that the acceptance rate was 14.34% compared to 23.85% of the MCMC in Problem 3. A second major thing I noticed was that the χ^2 experienced no burn in phase and immediately started at it's average value and fluctuated about it for the rest of the simulation, as seen in Figure 7. Continuing, the chain seemed to definitely have converged given that Figure 8 highlights that each parameter looks like noise and Figure 9 highlights the presence of visible 'knees' in the power spectrum. Lastly, Figure 10 provides the best fit parameters, highlighting that τ converged to the prior that we gave it and that it's error also converged to 0.0074, so it looks like things have worked as we expected. Applying the same equations as in Problem 3, we find that:

$$\Omega_\Lambda = 0.69 \pm 0.06 \tag{3}$$

Overall, I'd say that even though the error is slightly higher, the plots on the contour plot look more gaussian than in Problem 3, so I would be more content with this answer than the one without a prior on τ, especially given how oddly it acted in the simulations.

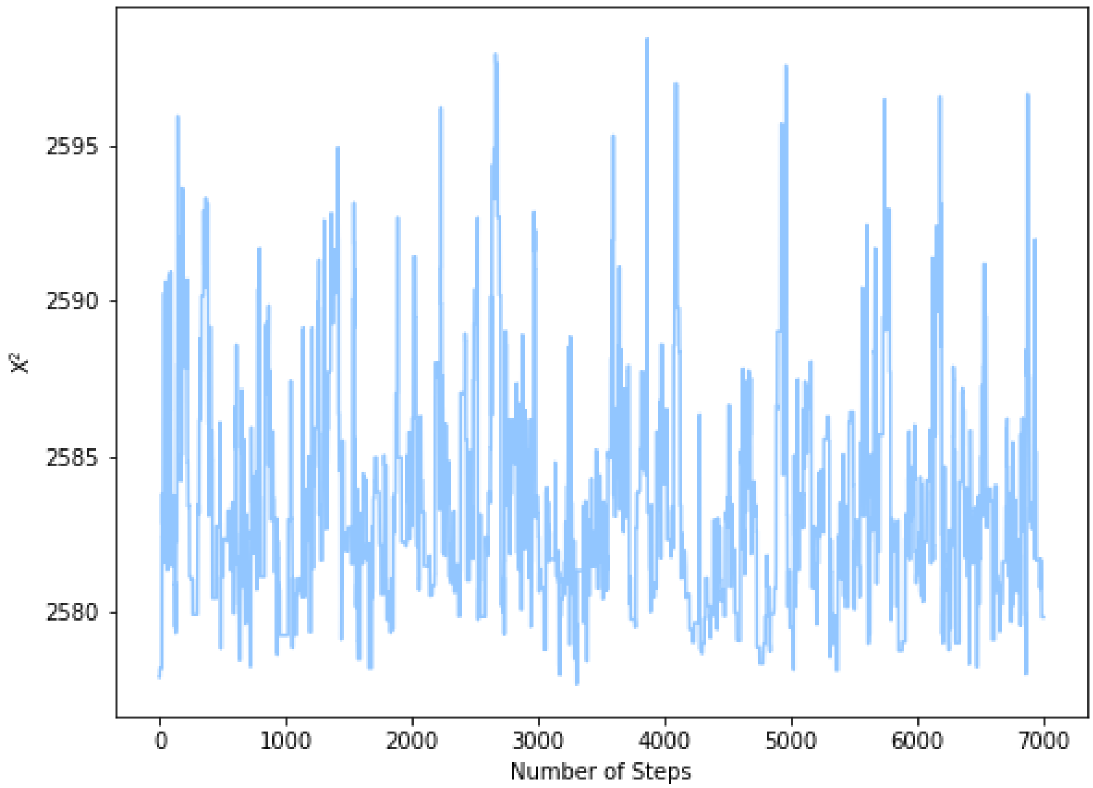

Figure 7: χ^2 as a function of time with a prior on τ. Notice that there is no obvious burn in phase.

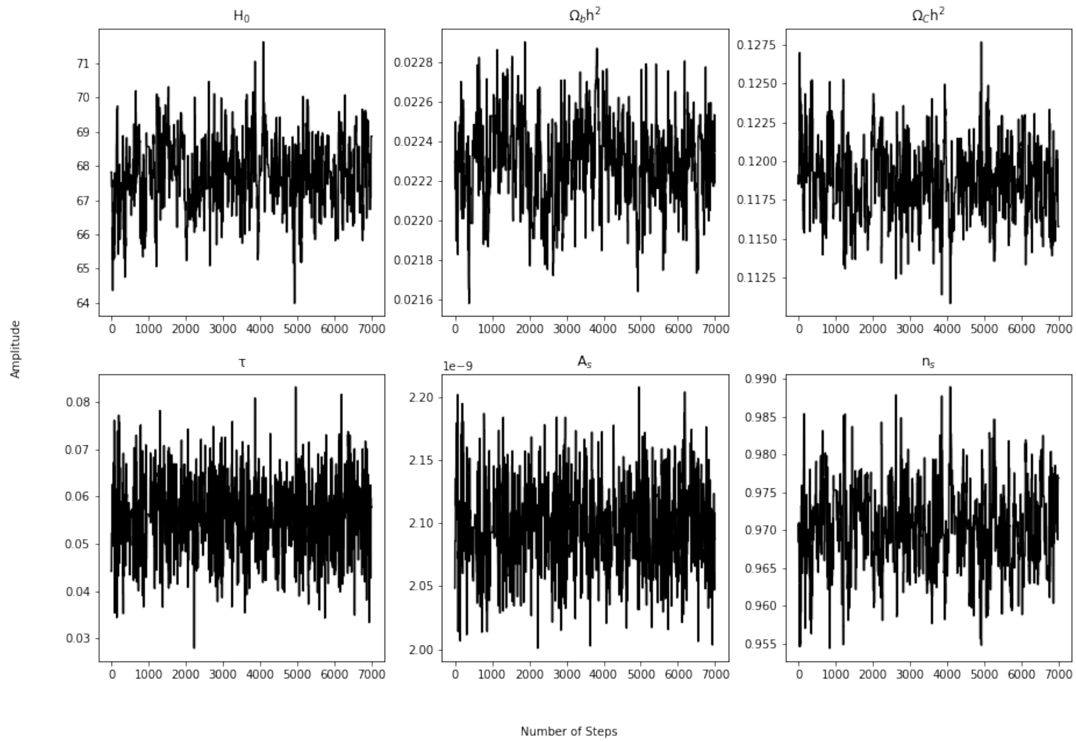

Figure 8: Amplitudes of parameters as a function of time. Notice that each parameter appears to look like noise.

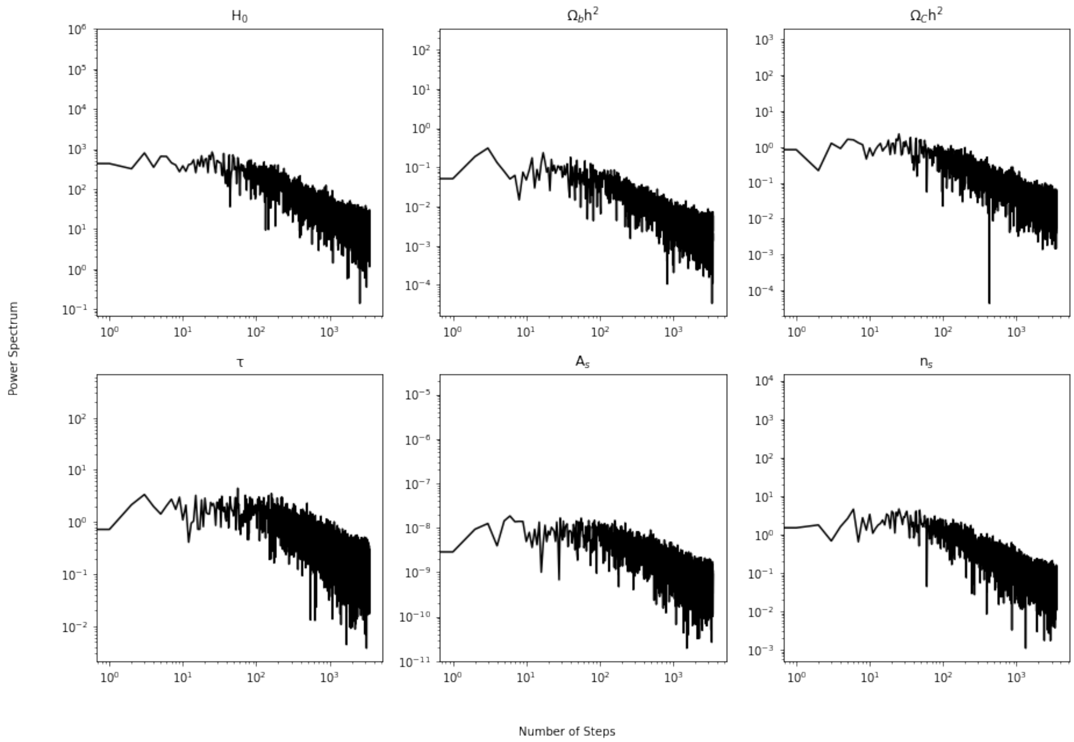

Figure 9: Power spectrum of each parameter. Note that it appears that each chain has converged since a 'knee' is visible for each parameter.

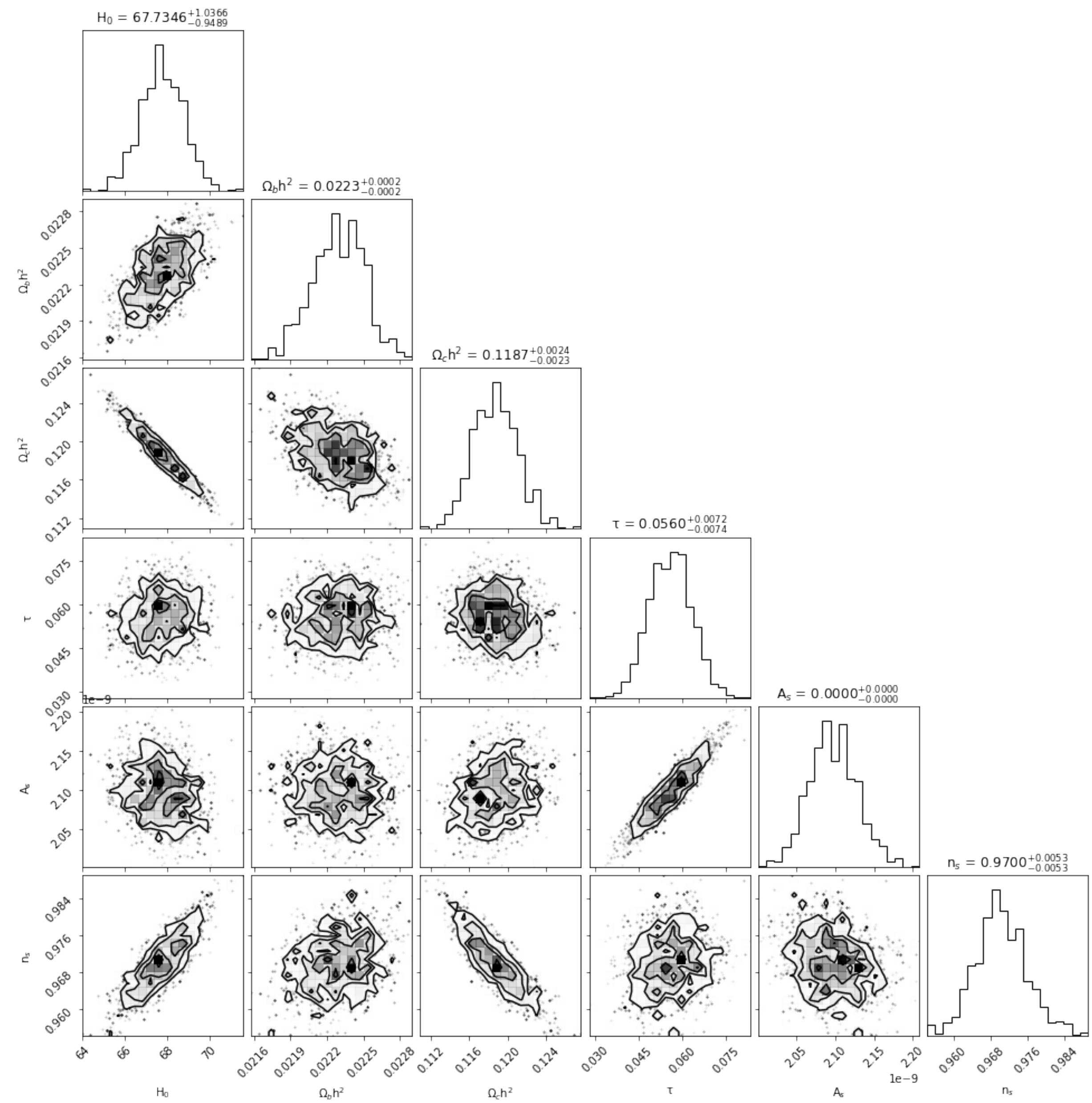

Figure 10: Corner plot highlighting the best fit parameters and their errors. Notice that τ has the same amplitude and error as it's prior, and that since A_s is too small, it's actual value was $2.097 \times 10^{-9} \pm 3 \times 10^{-11}$.