Problem 1

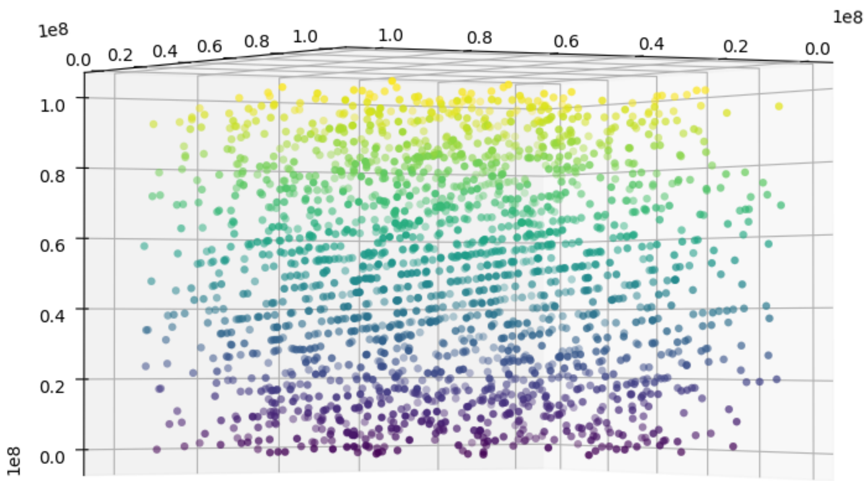

Figure 1: Plot of the randomly generated integers using the C standard library. Notice the evident lines going from left to right, indicating the presence of a set of planes with which the numbers are being generated on. I reduced the number of points to 2000 so that I could pick out these lines more clearly.

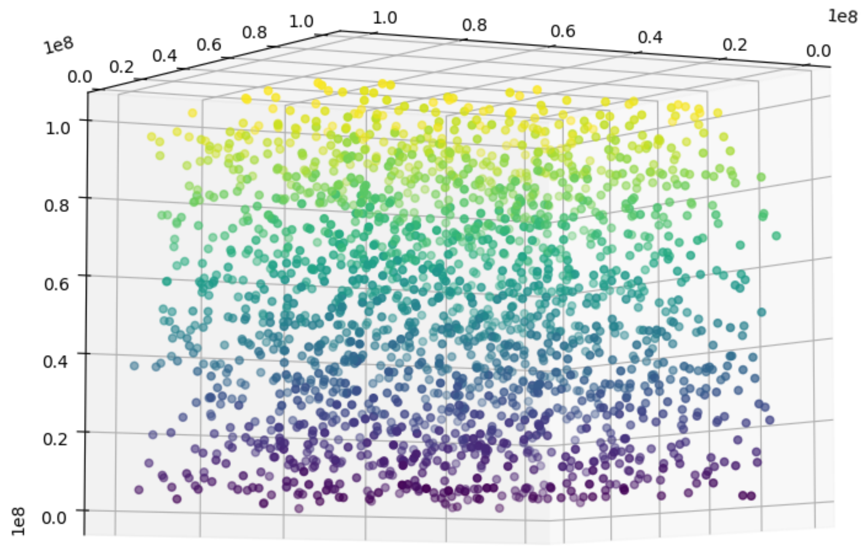

Figure 2: Plot of the randomly generated integers using numpy. At the same viewing angle, there exists no evidence of the planes seen in Figure 1.

After attempting to test the results on my local machine, I was unable to get it to work so I just scraped it and decided not to include it.

Problem 2

Assuming the PDF's are all centered about 0, we have the following PDF and their associated *quantile functions* retrieved from their corresponding wikipedia pages (or derived in class). These quantile functions are obtained by taking the inverse of the CDF's. Note that in all of the below equations, $q \in [0, 1]$ and is uniformily distributed.

For the Gaussian distribution with standard deviation σ:

$$\text{PDF} = \frac{1}{\sigma\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{x}{\sigma}\right)^2\right]; \tag{1}$$

$$x = \sigma\sqrt{2}\text{erf}^{-1}(2q-1). \tag{2}$$

For the Lorentzian distribution with scaling factor γ:

$$\text{PDF} = \frac{1}{\pi\gamma}\frac{1}{1+\left(\frac{x}{\gamma}\right)^2}; \tag{3}$$

$$x = \gamma\tan(\pi(q-0.5)). \tag{4}$$

For the power law distribution with power α:

$$\text{PDF} = x^{-\alpha}; \tag{5}$$

$$x = q^{\frac{1}{1-\alpha}}. \tag{6}$$

With these, we can generate either gaussian, lorentzian or power law distributed values.

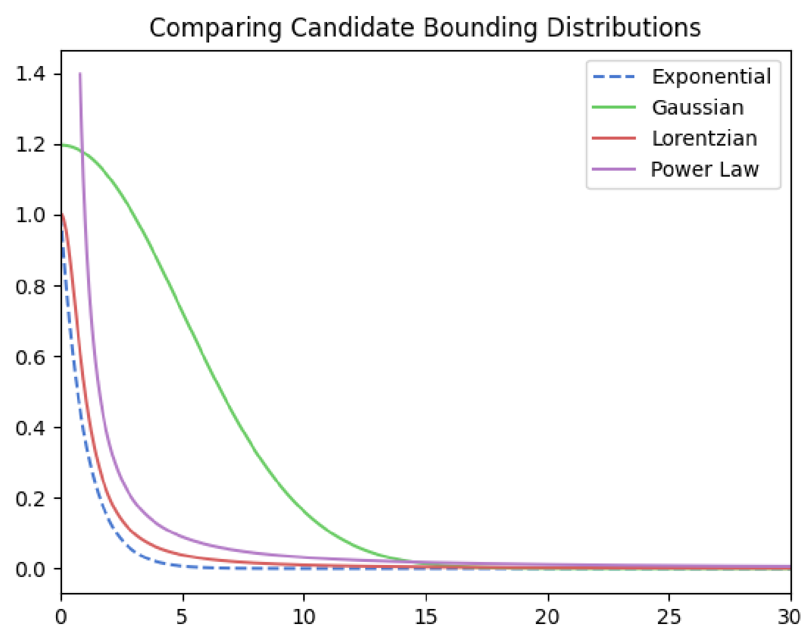

Figure 3: Comparison of the candidate bounding distribution functions. The only possible bounding function that works for all $x \geq 0$ is the Lorentzian distribution. The Gaussian distribution will eventually decay below the power law for large x. The power law diverges at $x = 0$ and so cannot be used to bound the exponential.

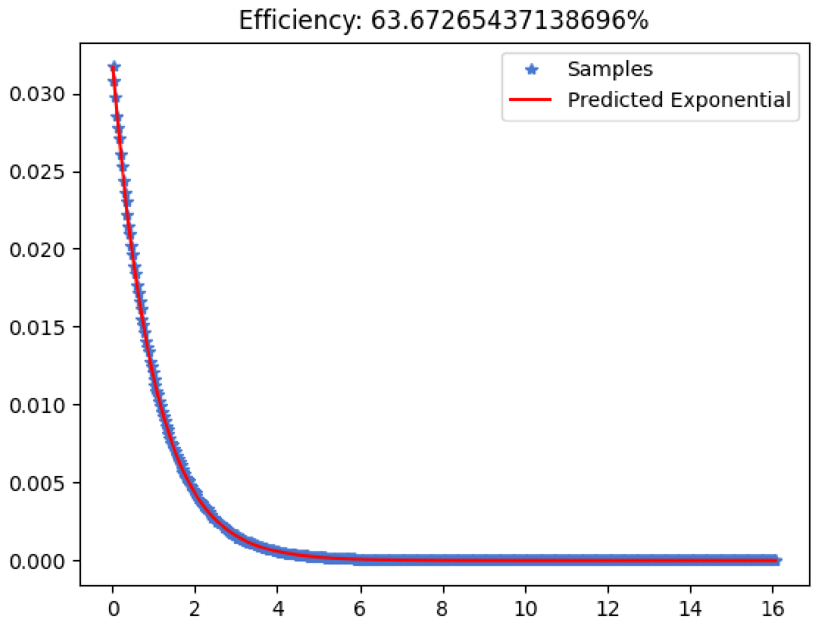

Figure 4: Histogram of exponentially distributed values from a Lorentzian. I was able unable to achieve a higher efficiency than 63% despite playing around with different values of γ. Plot clearly shows that the generated values are exponentially distributed. $\gamma = 1$ and the scaling amplitude is π.

Problem 3

Begin with the bounds of u:

$$0 \leq u \leq \sqrt{\text{PDF}(v/u)} = \sqrt{\lambda e^{-\lambda \frac{v}{u}}} \tag{7}$$

Solving for v,

$$u^2 = \lambda e^{-\lambda \frac{v}{u}} \tag{8}$$

$$\frac{u^2}{\lambda} = e^{-\lambda \frac{v}{u}} \tag{9}$$

$$-\ln\left(\frac{u^2}{\lambda}\right) = \lambda \frac{v}{u} \tag{10}$$

$$v = -\frac{u}{\lambda} \ln\left(\frac{u^2}{\lambda}\right) \tag{11}$$

For $0 \leq u \leq 1$,

$$0 \leq v \leq -\frac{u}{\lambda} \ln\left(\frac{u^2}{\lambda}\right) \tag{12}$$

Taking the derivative of the right hand side with respect to u and setting it to 0 (used mathematica), we find that this occurs for

$$u = \frac{\pm\sqrt{\lambda}}{e}. \tag{13}$$

Neglecting the negative solution (since we want our upper bound to be positive, and after working out the algebra, the positive solution ensures a positive bound) and plugging this into the upper bound of the inequality, we find that the inequality becomes:

$$\boxed{0 \leq v \leq \frac{2}{e\sqrt{\lambda}}} \tag{14}$$

This result, combined with

$$\boxed{0 \leq u \leq 1} \tag{15}$$

allows us to use the ratio of uniforms. See code for the application of these results and a plot is given below.

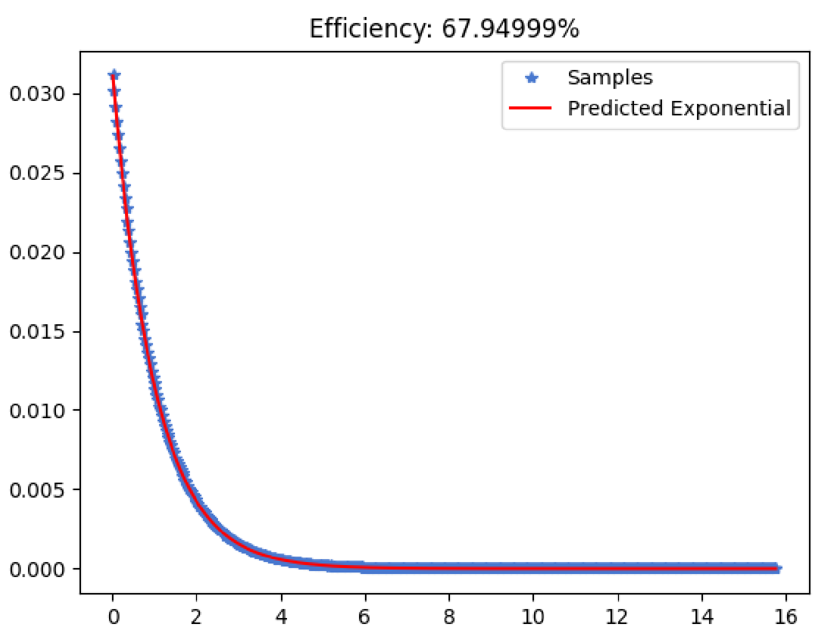

Figure 5: Exponentially distributed values obtained using the ratio of uniform numbers. With an efficiency of 67%, it performed a little bit better than the rejection method though still in the same ballpark.