Title: On the evolution of the set of limiting probabilities of first order properties for sparse random graphs.

Abstract: This is joint work with Marc Noy and Tobias Müller. It is known that for any first order property of graphs P, the limit probability that the random graph $G(n, c/n)$ satisfies P as n tends to infinity exists and varies in a smooth way with c. An immediate consequence of this is that first order properties cannot individually "capture" the phase transition that occurs in $G(n, c/n)$ when c=1.

We consider the set of limiting probabilities

$$L_c := \{ \lim_{n->\infty} \Pr(G(n, c/n) \text{ satisfies } P \,|\, P \text{ first order property } \}.$$

We ask the question of whether we can "detect" the phase transition in $G(n, c/n)$ through the changes in L_c. We arrive at a negative answer and show that there is a constant $c_0 \simeq 0.93...$ such that the closure $\overline{L_c}$ of L_c is the whole interval $[0, 1]$ when $c \geq c_0$ and $\overline{L_c}$ is a finite union of disjoint intervals when $c < c_0$. Moreover, the number of intervals forming $\overline{L_c}$ tends to infinity as c tends to zero. The same question can be asked in the setting of random uniform hypergraphs and similar results are obtained.