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Chapter 1

preliminaries

- General hypergraphs.
- First order language for graphs and hypergraphs.
- The $\mathcal{G}(n, p)$ model and generalizations to hypergraphs.
- E.F. Games.

Escribir todo esto

1.1 Generic hypergraphs

When considering the various definitions of graphs G with no multiple edges, one can notice that there are two main properties which set them apart. First, one can set the edges of G to be directed - i.e., the edge (u, v) is different from the edge (v, u)- or undirected, and second, one can either allow loops - edges of the form (u, u) - or not. The first property is a *commutativity* one, that relates to the *isomorphy* group of our edges: since our edges are of cardinality two, their isomorphy group can either be the trivial one - and they will be directed-, or the whole S_2 - and they will be undirected -. The second property in turn relates to *anti-reflexivity* constraints. While this point of view may seem artificial in the case of graphs, in the framework of hypergraphs it becomes more natural.

Given a natural number, k, we will call $[k]$ to the set $\{1, \ldots, k\}$, and Δ_k to the diagonal $\{\,(i, i) \,|\, 1 \leq i \leq k\,\} \subset [k]^2$.

Definition 1.1.1. Let k be a natural number. Then a **k-hyperedge blueprint**, (Φ, A) consists of

1. a subgroup Φ of the symmetric group S_k, and
2. a subset $A \subset [k]^2 \setminus \Delta_k$ that is Φ-invariant. This means that if $(i, j) \in A$ and $\sigma \in \Phi$ then $(\sigma(i), \sigma(j)) \in A$.

Given a set V, the symmetric group S_k acts in a natural way on V^k in the following way: if $\sigma \in S_k$ and $(v_1, \ldots, v_k) \in V^k$ then we define $\sigma(v_1, \ldots v_k)$ to be the k-tuple $(w_1, \ldots, w_k)$ such that $w_{\sigma(i)} = v_i$ for all $1 \leq i \leq k$. In other words, S_k acts on V^k by permuting coordinates of its elements.

Let $S \subseteq V^k$ and $\Phi \leq S_k$. Then ΦS will denote the set $\{\, \sigma v \mid \sigma \in \Phi,\, v \in S \,\}$. We will say that S is Φ-closed if $\Phi S = S$. If S is Φ-closed, we will call S/φ to the orbit space of S by Φ.

Chapter 2

Almost sure winning conditions for Duplicator

- Classes of j-morphic trees.
- Classes of j-morphic Graphs.
- Cores and j-agreeability.
- Simplicity and richness.
- Almost sure winning strategy.