Let $\overline{w} \in (\mathbb{N})_*$ be a finite set of fixed vertices. Given a vertex $v \in V$ we define the set $E([n],\overline{w};v)$ as the set of loop-free edges $e \in E([n])$ that satisfy $e \cap \overline{w} = \{v\}$. Consider a pattern ε. We define the set $Copies(\varepsilon,[n],V;v)$ as the set of Σ-edges $(e,\chi) \in Copies(\varepsilon,[n])$ satisfying that $e \in E([n],V;v)$ and that $\chi(v) = \tau$.

Theorem 0.1. *Let $\overline{w} \in (\mathbb{N})_*$ be a finite list of vertices and let $\phi(\overline{x})$ be a consistent edge sentence with $len(\overline{w}) = len(\overline{x})$. Let $\overline{v} \subset \overline{w}$. For each $v \in \overline{v}$ let $r(v) \in \mathbb{N}$ and $T_{n,v} = Tr(G_n(\overline{w}),v;r(v))$. For each $v \in \overline{v}$ let $E_v \subset E(\mathbb{N},\overline{w},v)$ be a finite set. Suppose that each $u \in \mathbb{N} \setminus \overline{w}$ belongs at most to one edge $e \in \bigcup_{v\in\overline{v}} E_v$. Then*

$$\lim_{n\to\infty} \Pr\Big(\bigwedge_{\substack{v\in\overline{v}\\ e\in E_v}} e \in E(T_{n,v}) \,\Big|\, \phi(\overline{w}) \bigwedge_{\substack{v\in\overline{v}\\ e\in E_v}} e \in E(G_n)\Big) = 1.$$

Proof. Let $r = \max_{v\in\overline{v}}(r(v))$. Suppose that for some v, some $e \in E_v$ satisfies that $e \in E(G_n)$ but $e \notin E(T_{n,v})$. Let $\overline{u} \in (\mathbb{N})_*$ be a list that contains precisely the vertices in $\overline{w}$ and the ones belonging to any edge $e' \in \bigcup_{v\in\overline{v}}$. Then in $G_n \setminus E(\overline{u})$ there must be path whose length is bounded by $2r$ joining a vertex in e with either another vertex $w \in \overline{u}$ or with a cycle $C \subset G_n \setminus E(\overline{u})$ whose diameter is bounded by r. That is, $SIMPLE_n(\overline{u},r)$ is not satisfied. Notice that the event $\phi(\overline{w}) \bigwedge_{\substack{v\in\overline{v}\\ e\in E_v}} e \in E(G_n)$ can be described by an open sentence whose variables are interpreted as $\overline{u}$. In consequence, using REF we obtain

$$\lim_{n\to\infty} \Pr\big(SIMPLE_n(\overline{u},r) \,\big|\, \phi(\overline{w}) \bigwedge_{\substack{v\in\overline{v}\\ e\in E_v}} e \in E(G_n)\big) = 1,$$

and the result follows. $\square$

Given