MATH 464. HW #8. Lazizbek.

Ex. 3.17.

$$(1)\quad \begin{cases} \min\ z = 2x_1 + 3x_2 + 3x_3 + x_4 - 2x_5 \\ \text{s.t.}\quad x_1 + 3x_2 + 0x_3 + 4x_4 + x_5 = 2 \\ \qquad x_1 + 2x_2 + 0\cdot x_3 - 3x_4 + x_5 = 2 \\ \qquad -x_1 - 4x_2 + 3x_3 + 0\cdot x_4 + 0\cdot x_5 = 1. \\ \qquad x \geq 0, \quad x \in \mathbb{R}^5 \end{cases}$$

Auxilary problem for the given s.f. problem (1) with $\left(\frac{1}{3}\right)\cdot$ equality constrain 3 is as follows:

$$(2)\quad \begin{cases} \min\ w = 0\cdot x_1 + 0\cdot x_2 + 0\cdot x_3 + 0\cdot x_4 + 0\cdot x_5 + x_6 + x_7 \\ \text{s.t.}\quad x_1 + 3x_2 + 0\cdot x_3 + 4x_4 + x_5 + 0x_6 + 0x_7 = 2 \\ \qquad x_1 + 2x_2 + 0\cdot x_3 - 3x_4 + x_5 + 0\cdot x_6 + 0\cdot x_7 = 2 \\ \qquad \left(\frac{-1}{3}\right)x_1 - \frac{4}{3}x_2 + 1\cdot x_3 + 0\cdot x_4 + 0\cdot x_5 + 0\cdot x_6 + 0\cdot x_7 = \frac{1}{3} \\ \qquad x \geq 0, \quad x \in \mathbb{R}^7 \end{cases}$$

Matrix s.f. of the auxilary problem (2):

$$(3)\quad \begin{cases} \min\ w = c_w^T \cdot x \\ \text{s.t.}\quad Ax = b \\ \qquad x \geq 0 \\ \qquad x \in \mathbb{R}^7 \end{cases} \qquad c_w^T = [0\ 0\ 0\ 0\ 0\ 1\ 1]^T$$

$$A = \begin{bmatrix} 1 & 3 & 0 & 4 & 1 & 1 & 0 \\ 1 & 2 & 0 & -3 & 1 & 0 & 1 \\ -\frac{1}{3} & -\frac{4}{3} & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$b = \left[2,\ 2,\ \frac{1}{3}\right]^T$$

(4) choose basis $\beta_1 = \{6, 7, 3\} \Rightarrow$

(5) $$\begin{cases} x_B = (x_6, x_7, x_3) = \left(2, 2, \frac{1}{3}\right) \geq 0 \\ x_N = (x_1, x_2, x_4\ x_5) = (0, 0, 0, 0) \geq 0 \end{cases} \Rightarrow$$

(6) $$B = [A_6\ A_7\ A_3] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \text{so} \quad x_B = B^{-1} b = b = \begin{bmatrix} 2 \\ 2 \\ \frac{1}{3} \end{bmatrix}$$

(7) $$-z = -c_B^T x_B = -[0\ 0\ 3] \cdot \begin{bmatrix} 2 \\ 2 \\ 1/3 \end{bmatrix} = -1,$$ recall
$$c_z = \begin{array}{c} \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{matrix} \\ [2, 3, 3, 1, -2, 0, 0]^T \end{array}$$

(8) $$-w = -c_{wB}^T x_B = -[1\ 1\ 0] \cdot \begin{bmatrix} 2 \\ 2 \\ 1/3 \end{bmatrix} = -4$$

(9) reduced costs for z:
$$\bar{c}_z = c_z - c_{zB}^T \overbrace{B^{-1}}^{=I} A =$$
$$= [2, 3, 3, 1, -2, 0, 0]^T - [0\ 0\ 3] \cdot I\, A =$$
$$= [2, 3, 3, 1, -2, 0, 0]^T - [0, 0, 3] \cdot \begin{bmatrix} 1 & 3 & 0 & 4 & 1 & 1 & 0 \\ 1 & 2 & 0 & -3 & 1 & 0 & 1 \\ -\frac{1}{3} & -\frac{4}{3} & 1 & 0 & 0 & 0 & 0 \end{bmatrix} =$$
$$= [2, 3, 3, 1, -2, 0, 0]^T - [-1\ -4\ 3\ 0\ 0\ 0\ 0] = [3, 7, 0, 1, -2, 0, 0]$$

(10) $$\bar{c}_w = c_w - c_{wB}^T \cdot \overbrace{B^{-1}}^{=I} A = \underbrace{\begin{array}{c} \begin{matrix} & & 3 & & & 6 & 7 \end{matrix} \\ [0\ 0\ 0\ 0\ 0\ 1\ 1]^T \end{array}}_{c_w} - \begin{array}{c} \begin{matrix} 6 & 7 & 3 \end{matrix} \\ [1, 1, 0] \end{array} \cdot I \cdot A$$
$$= [0, 0, 0, 0, 0, 1, 1] - [1, 1, 0] \cdot \begin{bmatrix} 1 & 3 & 0 & 4 & 1 & 1 & 0 \\ 1 & 2 & 0 & -3 & 1 & 0 & 1 \\ -\frac{1}{3} & -\frac{4}{3} & 1 & 0 & 0 & 0 & 0 \end{bmatrix} = [-2\ -5\ 0\ -1\ -2\ 0\ 0]^T$$

reduced cost for w ↑

(11) $\mathcal{U} = B^{-1}A = A$

So, the initial tableau by (4) - ... - (10), (11):

(12)

			x_1	x_2	x_3	x_4	x_5	x_6	x_7	
row ①	$-z =$	-1	3	7	0	1	-2	0	0	$\bar{c}_z$
②	$-w =$	-4	-2	-5	0	-1	-2	0	0	$\bar{c}_w$
③	$x_6 =$	2	1	3	0	4	1	$\boxed{1}$	0	
④	$x_7 =$	2	1	2	0	-3	1	0	$\boxed{1}$	
⑤	$x_3 =$	$\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{4}{3}$	$\boxed{1}$	0	0	0	0	

Status:

- $\beta_1 = \{6, 7, 3\}$
- $x = (0, 0, \frac{1}{3}, 0, 0, 2, 2)$
- $z = 1$
- $w = 4$
- Nonoptimal
- Direction: [ ]
- Valid pivot: ⊙

			x_1	x_2	x_3	x_4	x_5	x_6	x_7
$R1 = r1 + 2 \cdot r_4$		3	5	11	0	-5	0	0	2
$R2 = r2 + 2 \cdot r_4$		0	0	-1	0	-7	0	0	2
$R3 = r3 - r4$	$x_6 =$	0	0	1	0	7	0	$\boxed{1}$	-1
$R4 = r4$	$x_5 =$	2	1	2	0	-3	$\boxed{1}$	0	1
$R5 = r5$	$x_3 =$	$\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{4}{3}$	$\boxed{1}$	0	0	0	0

Status:

- $\beta_2 = \{6, 5, 3\}$
- $x = (0, 0, \frac{1}{3}, 0, 2, 0, 0) \geq 0$
- $z = -3$, $w = 0$
- Nonoptimal, $\bar{c}_{w4} = -7 < 0$
- Degenerate $\Rightarrow$ Bland's Rule!

- I'm going to use Bland's Rule ~~by~~ changing basis to go out of degeneracy:

			x_1	x_2	x_3	x_4	x_5	x_6	x_7
$R1 = r1 + \frac{5}{7} r_3$	$-z =$	3	5	$82/7$	0	0	0	$5/7$	$9/7$
$R2 = r2 + r3$	$-w =$	0	0	0	0	0	0	1	1
$R3 = \frac{1}{7} \cdot r3$	$x_4 =$	0	0	$1/7$	0	$\boxed{1}$	0	$1/7$	$-1/7$
$R4 = r4 + \frac{3}{7} r3$	$x_5 =$	2	1	$17/7$	0	0	$\boxed{1}$	$3/7$	$4/7$
$R5 = r5$	$x_3 =$	$\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{4}{3}$	$\boxed{1}$	0	0	0	0

Status:

- $\beta_3 = \{4, 5, 3\}$ = Initial b.f. Basis
- $x = (0, 0, 1/3, 0, 2, 0, 0) \geq 0$
- $z = -3$, $w = 0 \Rightarrow$ can stop
- Optimal $\bar{c}_z \geq 0$, $\bar{c}_w \geq 0$.
- still degenerate

Yes, we're still at a degenerate point $\boxed{x^* = (0, 0, \frac{1}{3}, 0, 2)}$, possibly there's a way to go out of degenerate point by Bland's Rule with zero step size at △ I pointed out, but I'm leaving here as my answer is matching with the software, too!

math_464_actual_hw8

March 6, 2024

#**Math 464. HW8. Lazizbek**

EXercise 3.17

```
min z = 2 x1 + 3 x2 + 3 x3 +   x4 - 2 x5

s.t.      x1 + 3 x2 + 0 x3 + 4 x4 +   x5 = 2

          x1 + 2 x2 + 0 x3 - 3 x4 +   x5 = 2

          (-1/3) x1 -(4/3)x2 + x3 + 0 x4 + 0 x5 = (1/3)

          x >= 0

          x1, x2,x3,x4, x5 in R
```

1 *Linear Program*

Now let's see the case where **all** decison variables are **reals**

[9]:
```
"""
Script to demonstrate how to solve small mixed integer programs
using the python optimize module in the scipy package
"""
"""
All decision variables are from reals:
"""

import pandas as pd
import numpy as np
import scipy.optimize as opt

# The problem we will solve is:
#
# min z = 2 x1 + 3 x2 + 3 x3 +   x4 - 2 x5
# s.t.      x1 + 3 x2 + 0 x3 + 4 x4 +   x5 = 2
#           x1 + 2 x2 + 0 x3 - 3 x4 +   x5 = 2
#     (-1/3)x1 -(4/3)x2 + x3 + 0 x4 + 0 x5 = (1/3)
#           x >= 0
```

```
#           x1, x2,x3,x4, x5 in R

function_array = list()
solution_array = list()

# First build the objective vector.
c=np.array([2, 3, 3, 1, -2])

# Next, create the coefficient array for the inequality constraints.
# Note that the inequalities must be Ax <= b, so some sign
# changes result when converting >= into <=.
# A = np.array([[   1,    3,    0,     4,     1],\
#                [   1,    2,    0,    -3,     1],\
#                [(-1/3), (-4/3), 1, 0,     0]])
A = None

# Next the right-hand-side vector for the inequalities
# Sign changes can occur here too.
# b = np.array([345 , 50000 , 60000, 0, 0, 0, 0, 0, 0 ])
b = None

#The coefficient matrix for the equality constraints and
# the right hand side vector.
Ae = np.array([[   1,    3,    0,     4,     1],\
              [   1,    2,    0,    -3,     1],\
              [(-1/3), (-4/3), 1, 0,     0]])     # Ae = [[1,1,1,1]]

be = np.array([ 2, 2, (1/3)])

# Next, we provide any lower and upper bound vectors, one
# value for each decision variable.  In this example all
# lower bound are zero and there are no upper bounds.
bounds=((0,np.inf),(0,np.inf),(0,np.inf),(0,np.inf),(0,np.inf))

# Lastly, we can specify which variables are required to be integer.
# If no variables are integer then isint=[];  In our example, only x2
# is integer.
# isint=[]

# The call to the mixed integer solver looks like the following.
# Notice that we pass usual "c" when we have a minimization
# problem, we send "-c" when we have max problem.
# This is because the solver is expecting a minimization.

res=opt.linprog(c,A,b,Ae,be,bounds)

# The result is stored in the dictionary variable "res".
```

```
# In particular, to show the optimal objective value and the
# optimal decision variable values:

print("min z = ", res['fun'])
print("at optimal solution x = ", res['x'])

# print(res['x'])
# print(np.dot(c, res['x']))
# print(res)

# To download:
# !sudo apt-get install texlive-xetex texlive-fonts-recommended␣
 ↪texlive-plain-generic
# !jupyter nbconvert --to pdf /content/math_464_actual_hw2.ipynb
```

```
min z =  -3.0
at optimal solution x =  [0.         0.         0.33333333 0.         2.
]
```