pr3_i_math548_midterm_Lazizbek

March 19, 2024

1 Math 548, Midterm. Problem 3. LS

Problem 3

You would like to use the fixed-point iteration method to find the roots of $f(x) = x - x^2 = 0$. Consider the following two formulations.

```
1.

x = x + 2(x-x^2).

2.

x = x - (x - x^2) / (1 - 2x).
```

- a) For each formulation carry out the iterations first using the starting value 0.8 and then, using the starting value 0.2.
- b) Comment and justify your observations.

```
#Problem 3. (i)

**x = x + 2*(x-x^2).**
```

```
[1]: import pandas as pd
     initials = list([0.8, 0.2])
     steps = list()
     approximations = list()
     epsilon = 0.000001
     for i in range(2):
       x0 = initials[i]
       M = 10
       try:
         for k in range(M):
           steps.append(k)
           approximations.append(x0)
           x1 = x0 + 2*(x0-(x0)**2)
           if abs(x0-x1) < epsilon:
             print(f"\nWhen x0={initials[i]}, |g'({initials[i]})| < 1, so iteration_
      ⇒converges with tolerance of {epsilon} in {k} steps as follows:")
```

```
break
    x0 = x1
  d = {'step k = ': steps, 'approximation x = ': approximations}
  df = pd.DataFrame(data=d)
  print(df)
  steps = []
  approximations = []
except:
  print(f"\nWhen x0=\{initials[i]\}, |g'(\{initials[i]\})|>=1, so iteration_{\sqcup}\}

→diverges in {k} steps as follows:")
  steps.pop()
  approximations.pop()
  d = {'step k = ': steps, 'approximation x = ': approximations}
  df = pd.DataFrame(data=d)
  print(df)
  steps = []
  approximations = []
```

```
step k =
              approximation x =
0
           0
                         0.800000
1
           1
                         1.120000
2
           2
                         0.851200
           3
3
                         1.104517
4
           4
                         0.873635
5
           5
                         1.094429
6
           6
                         0.887738
7
           7
                         1.087057
8
           8
                         0.897786
9
                         1.081319
              approximation x =
   step k =
0
           0
                         0.200000
1
           1
                         0.520000
           2
2
                         1.019200
3
           3
                         0.980063
           4
4
                         1.019142
5
           5
                         0.980125
6
           6
                         1.019085
7
           7
                         0.980186
8
           8
                         1.019028
9
                         0.980247
```

```
[]: # !sudo apt-get install texlive-xetex texlive-fonts-recommended → texlive-plain-generic
```

 $\begin{tabular}{ll} [\]: \ & \textit{!jupyter nbconvert ---to pdf /content/Math548_hw6_Lazizbek.ipynb.} \end{tabular}$