pr3_ii_math548_midterm_Lazizbek

March 19, 2024

1 Math 548, Midterm. Problem 3. LS

Problem 3

You would like to use the fixed-point iteration method to find the roots of f (x) = x - x^2 = 0. Consider the following two formulations.

1.

`x = x + 2(x-x^2).`

2.

`x = x - (x - x^2) / (1 - 2x).`

a) For each formulation carry out the iterations first using the starting value 0.8 and then, using the starting value 0.2.

b) Comment and justify your observations.

2 Problem 3. (ii)

x = x - (x - x^2) / (1 - 2x)

[ ]:
```
import pandas as pd

initials = list([0.8, 0.2])
steps = list()
approximations = list()
epsilon = 0.000000001

for i in range(2):
  x0 = initials[i]
  M = 10
  try:
    for k in range(M):
      steps.append(k)
      approximations.append(x0)
      x1 = x0 - (x0 - (x0)**2) / (1 - 2*x0)
      if abs(x0-x1) < epsilon:
```

```
      print(f"\nWhen x0={initials[i]}, |g'({initials[i]})| < 1, so iteration␣
↪converges with tolerance of {epsilon} in {k} steps as follows:")
      break
    x0 = x1


  d = {'step k = ': steps, 'approximation x = ': approximations}
  df = pd.DataFrame(data=d)
  print(df)
  steps = []
  approximations = []

 except:
  print(f"\nWhen x0={initials[i]}, |g'({initials[i]})|>=1, so iteration␣
↪diverges in {k} steps as follows:")
  steps.pop()
  approximations.pop()
  d = {'step k = ': steps, 'approximation x = ': approximations}
  df = pd.DataFrame(data=d)
  print(df)
  steps = []
  approximations = []
```

```
When x0=0.8, |g'(0.8)| < 1, so iteration converges with tolerance of 1e-09 in 4
steps as follows:
   step k =   approximation x =
0          0            0.800000
1          1            1.066667
2          2            1.003922
3          3            1.000015
4          4            1.000000

When x0=0.2, |g'(0.2)| < 1, so iteration converges with tolerance of 1e-09 in 4
steps as follows:
   step k =   approximation x =
0          0        2.000000e-01
1          1       -6.666667e-02
2          2       -3.921569e-03
3          3       -1.525902e-05
4          4       -2.328306e-10
```

[ ]:
```
# !sudo apt-get install texlive-xetex texlive-fonts-recommended␣
↪texlive-plain-generic
```

[ ]:
```
# !jupyter nbconvert --to pdf /content/Math548_hw6_Lazizbek.ipynb
```