ON THE USE OF CONTINUOUS FINITE-ELEMENTS FOR TWO-PHASE FLOW SIMULATIONS IN HORIZONTAL AND NEARLY HORIZONTAL PIPES

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RESUMO – The one-dimensional two-fluid model, employed for simulating two-phase flows in long pipes, is conditionally well-posed and discretized with lower-order space and time schemes. This work investigates the capability of continuous finite-element methods of describing wave growth and potential flow pattern transitions, with stratified smooth flow as an initial condition, using the FEniCS computing platform. The effects of initial conditions, mesh configuration, order of basis functions, and pipe inclination on the stability were investigated through eigenspectra and flow pattern maps of the differential and discrete equations. The numerical model proposed describes potential wave growth, which serves as a baseline for simulating flow pattern transitions in oil and gas systems.

1. INTRODUCTION

Two-phase flows in long pipes are frequently simulated with the one-dimensional two-fluid model (hereafter two-fluid model), proposed by Ishii e Hibiki (2011). In large systems, the pipe lengths are of the order of kilometers and gas bubble diameters of millimeters van Zwieten et al. (2017).

Issa e Kempf (2003) proposed the slug capturing approach by solving the two-fluid model under well-posed initial conditions, simulating the natural growth of instabilities starting from non-equilibrium flows. Issa e Kempf (2003); Liao et al. (2008) used first-order Finite Volume central schemes to avoid excessive numerical diffusion associated with upwind schemes.

This paper proposes a methodology for predicting potential wave growth, solving the two-fluid model. We investigate the spatial discretization with Continuous Galerkin and Taylor-Hood finite elements, often employed for solving the Navier-Stokes equations and other convection-dominated problems Logg et al. (2012).

2. METHODS

The starting point of this work is the slug capturing approach of Issa e Kempf (2003) and the equations of Montini (2010), with stratified smooth flow (see Fig. **??**) as the initial condition, as in the studies Sanderse et al. (2017); van Zwieten et al. (2017).

2.1. Governing equations

The two-fluid model Ishii e Hibiki (2011) consists of a set of transport equations for the phases, including mass, momentum, and energy equations. For isothermal flow, the energy equations are dismissed Issa e Kempf (2003). The governing equations for one-dimensional stratified and slug flow read

$$\partial_t\left(\alpha_k\rho_k\right)+\partial_s\left(\alpha_k\rho_k u_k\right)=0, \tag{1}$$

and

$$\partial_t\left(\alpha_k\rho_k u_k\right)+\partial_s\left(\alpha_k\rho_k u_k^2\right)+\alpha_k\partial_s p_i+\alpha_k\rho_k g\partial_s h_i\cos\beta=-\alpha_k\rho_k g\sin\beta-\sum_{\theta\in\{l,g,w\}}\frac{\tau_{k\theta}P_{k\theta}}{A}, \tag{2}$$

where $\theta \neq k$, w is the pipe wall. The liquid and gas volume fractions sum to one ($\alpha_l+\alpha_g=1$). The liquid, gas, and interface are represented by the subscripts l, g and i, respectively.

2.2. Spatial discretization

The convective terms of equations (1)-(2) should be carefully discretized to prevent issues related to numerical oscillations De Bertodano et al. (2016); Liao et al. (2008); Sanderse et al. (2017).

Equation (**??**) is simultaneously approximated by the variational form of Eq. (**??**):

$$\sum_{\Omega\in\Omega_h}\int_\Omega \partial_t\mathbf{W}_h\ \mathbf{V}\ \mathrm{d}x+\sum_{\Omega\in\Omega_h}\int_\Omega a_A\left(\mathbf{B}_{\mathbf{0},h};\mathbf{W}_h,\mathbf{V}\right)\mathrm{d}x=\sum_{\Omega\in\Omega_h}\int_\Omega \mathbf{C}_{\mathbf{0},h}\ \mathrm{d}x, \tag{3}$$

where $\mathbf{W_h}$ and $\mathbf{V}$ are the vectors of trial and test functions, respectively. The convective term is denoted by a_A. $\Omega\subset\mathbb{R}^n$ is the spatial domain with boundaries $\partial\Omega=\Gamma_D^{in}\cup\Gamma_D^{out}$. At the inlet $\{\alpha_{l,0},u_{l,0},u_{g,0}\}\in\Gamma_D^{in}$, and at the outlet $\{p_{i,0}\}\in\Gamma_D^{out}$.

3. RESULTS AND DISCUSSION

The results are reported through eigenspectra and flow pattern maps, validated against the literature. To ease validation of the results, the input parameters are the same as Issa e Kempf (2003); Montini (2010); Sanderse et al. (2017). c_g is taken such that for $p_i=p_{i,0}$, the gas density $\rho_g=1.1614\ kg\ m^{-3}$ Sanderse et al. (2017).

3.1. Stiffness analysis of the semi-discrete equations

Using Taylor-Hood elements, the existence of convective modes suggest that certain solutions might potentially grow in time, since they have a positive real part. Differently, using continuous elements, the eigenspectrum is composed of acoustic modes. The imaginary parts of the eigenvalues associated with acoustic and convective waves have different orders of magnitude since acoustic modes have higher frequencies than convective modes. Taylor-Hood function spaces preserve the eigenstructure for the parameters studied, suggesting the capability of such function spaces for describing wave growth.

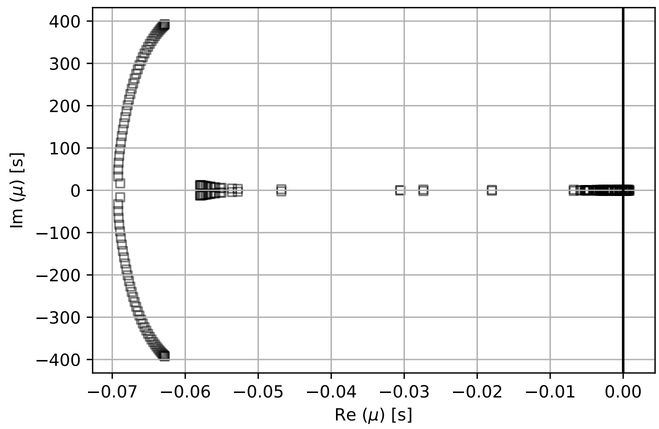

Figura 1 – Taylor-Hood P_2.

3.2. Discrete flow pattern maps

The VKH limits for the fully discrete equations are built with a sequence of superficial velocities, for the time step $\Delta t = 1/40$ as the first approach (See Fig. 2).

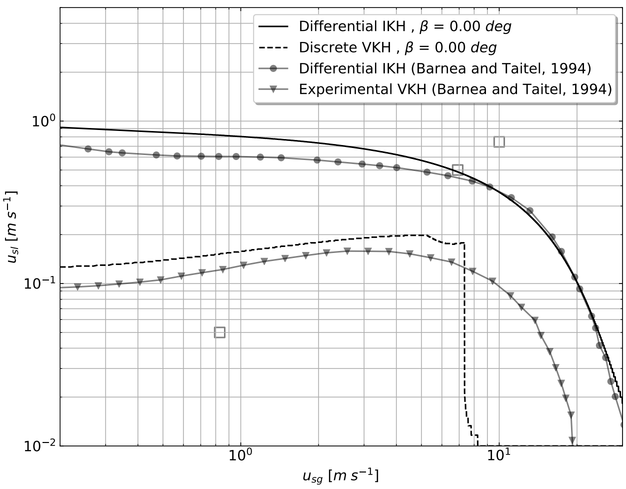

Figura 2 – Discrete flow pattern maps ($D = 0.051\ m$). Taylor-Hood P_2 combined with BDF2.

4. CONCLUSIONS

The stiffness analysis of the semi-discretized equations reveals the influence of pipe inclination on the presence of eigenvalues related to acoustic and convective waves. Convective waves are not attenuated by numerical diffusion present in the numerical schemes. The eigenvalues associated with acoustic waves do not affect the wave growth for smooth initial conditions. Continuous finite element function spaces represent alternatives to solve the two-fluid model equations under smooth initial conditions. Additionally, the discretization with Taylor-Hood mixed finite elements, combined with implicit time schemes, describes potential wave growth.

Discrete flow pattern maps for different pipe inclinations validate the numerical methods proposed during the first time steps in transient simulations. Differential and discrete flow pattern maps based on Kelvin-Helmholtz instabilities present good agreement with experimental data available in the literature. Meanwhile, it is essential to simultaneously validate the actual flow patterns and transition curves through differential flow pattern maps, discrete flow pattern maps, and nonlinear simulations.

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