**Course preview.**

**Game Features:**

Prisoner’s Dilemma (PD) Game囚徒困境

Either **confess**, or **Not confess**.

**Repetition or not …**

**One-shot**: only one round for players.

**Repeated Games**: Same game, multiple rounds.

**Dynamic Games**: …

Distinction between game type: Knowledge information: Cost of other player?

**Solution concepts,**

\* Mathematical rules decide how to play.

\* Players are rational, plays optimally. (Maximize expected utility, Minimize expected cost.)

1. **MinMax Solution:**

* Minimize a player’s Maximum (worst) expected cost – Secruity strategy.

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Prisoner’s Dilemma Game example,

-- matrix are used to evaluate said cost ~~/ utility~~.

1. **Best-response (BR),**

Play the strategy that gives lowest cost given your opponent’s strategy.

For player I, suppose his opponents’ (all expect I ) plays U\_i

*J\_i (u\_i^\*,u\_(-i) )≤J\_i (U\_i,u\_(-i) )for All u\_i taken from Ω\_i*

1. **Nash Equilibrium (NE). solution:**

All players play B. Best response to strageties.

(Within PD Game, Matrix readily shows result of Nash Equilibria. )

*Formulation As per Wiki.*

**Example: Battle of Sexes Game**

A couple (2 player) decide to attend opera / football (2 options)

Both have cost matrix respectively,

Two pure strategy Nash equilibria, There is also a Nash Equilibrium in mixed strategies.(randomized) – players go to their preferred event more often than the other.

Nash theorem: Every finite game has a mixed strategy Nash equilibrium.

Classical equilibrium analysis is based on NE, or NE’s further refinements.

Why NE arises? …

**Learning in Games**

1. **Adaptive Learning**

“Myopic”, simple and rule-of-thumb rules.

1. **Evolutionary (Population) Dynamics:**

Selection of strageties according to performance against the aggregate and random mutations.

1. **Bayesian Learning (not covered in course**
2. **CA Games:**

BR play./ dynamics

1. **Population games:**

Imitation Dynamics (ID) & Replicator Dynamics

1. **Dynamic games.**

…

**Evaluations:**

**Problem sets 30%**

**Final Exam 30%**

Last lecture: Nov. 28th ?

**Project / Paper 40%**

Two (2) weeks after Final Exam to finalize.

**Lecture 1 20250905:**

**2 Player Zero-Sum Finite Games (Matrix)** ***“2PZS”***

* Definition, Security Strategy
* MinMax Solution
* No regret property
* Saddle-point equilibrium solution

Action set = Ω (Finite in this case)

Index = I (Finite in this case)

Cost = J\_i

Action of player = U\_i

P\_i = Player “I”.

By definition, J1 = Ω\_1 x

By definition, g is a zero-sum game, if (Cost sums to 0.)

For

For

Thereby:

Thus, it is to their best intention that:

P1 minimized with respect to

P2 minimized with respect to , or MAXIMIZES w.r.t.

On the empheses of Ω\_i being a finite action set:

m\_1 = amount of actions. “J” is the index.

Action sets: , Let Wherein the jth action of P1, can be notated as:

Now, for P1 selecting jth action = u\_1 = e\_1j,

For P2 selecting kth action = u\_2 = e\_2k,

There outcomes: , a real value that belongs to a matrix:

Is the cost matrix of featured game, with dimention .

Where Player 1 selects jth action, corresbonding to jth **row** “--”of

Where Player 2 selects kth action, corresbonding to jth **column** “ | ” of

**Example: Matching pennies:**

Two selectes a pennie’s tail or head, rewarded if of they do not coincide.

P1 selects row, P2 selects column.

As an example, if P1 selects Head, P2 selects Head, they outcomes -1. Or as formulation:

Infact, in terms of matrix multiplication:

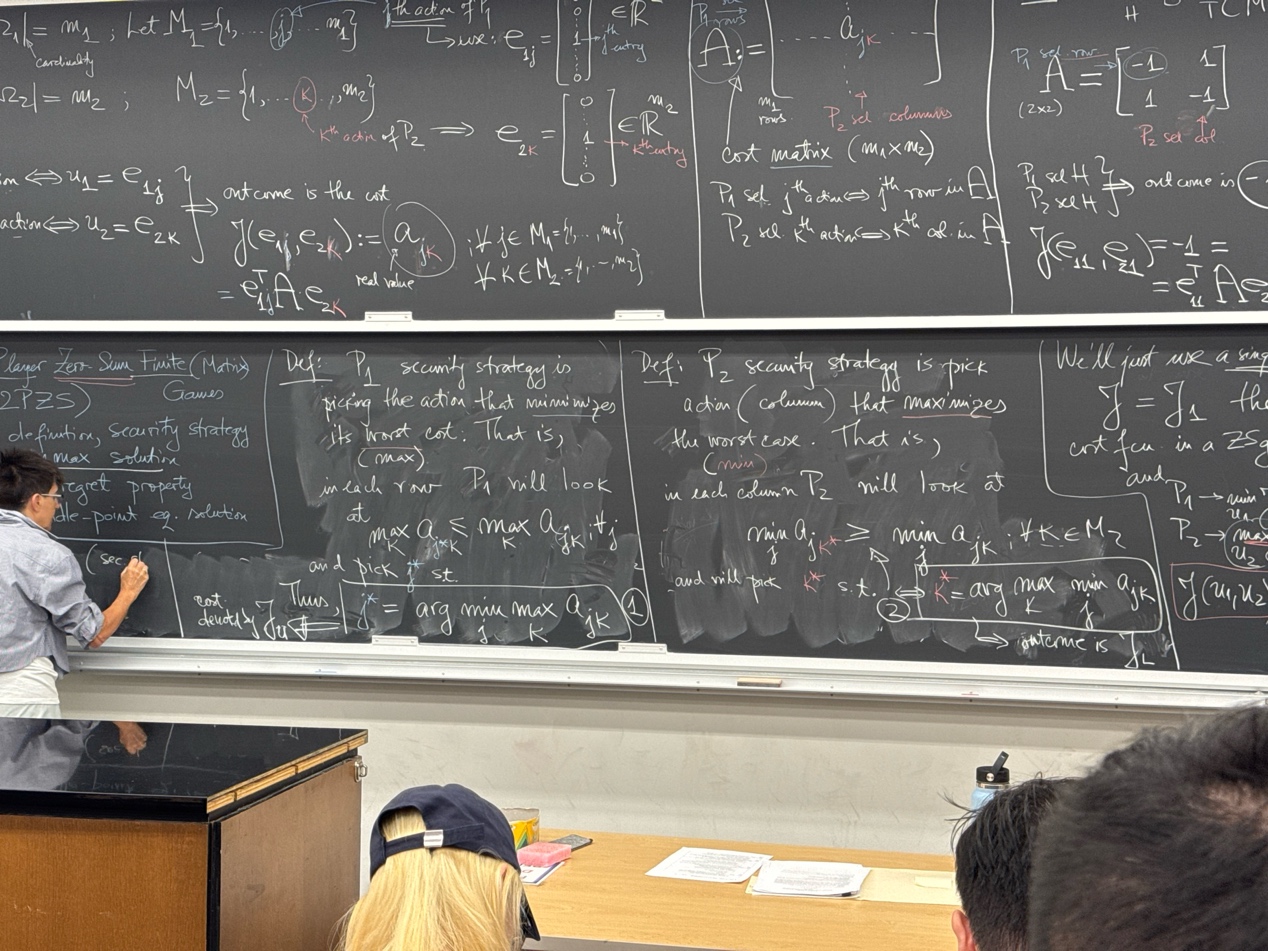
This brings us back to the general formulation:

*“There outcomes: , a real value that belongs to a matrix…”*

**Definition: Security Strategy**

The Strategy where P1 will pick the action that minimizes its worst(its maximum) cost, that is:

In each row P1 will look at, the maximum outcom of all selected j:



**Definition: MinMax solution**

MinMax Solution is there the set (j, k) are within j = j\* and k = k\*, as defined within security

And the outcome is :

Such outcome, in general, will be between the resolution between result of Security Strategy.

(“Not equal, in general”)

**Example for Security strategy:**

Outcome matrix:

P1 minimizing the overall outcome, while selecting rows.

P2 maximizing the overall outcome, while selecting columns.

For player 1, they looks **j\*** to result for the minimum of the maximum, across rows.

For P1, j\* = argmin{5,2,4}, which is index **j\* = 2**

For player 2, they looks k\* to result for the maximum of the minimum, across columns.

For P2, k\* = argmax{1 ,2 ,-3 } which is index **k\* = 2**.

Therefore, Minmax set is (j,k) = (2,2) and

Example 2:

Working it out: J\* = 3, J\_u = 2

K\* = 1, J\_L = 0. -> J\_phi = 1

All of the above follows **NO REGRET rule.**

It is worty to discuss **if, both players would regret their decision** given that they could foresee their opponents’ choices?

Does (j\*, k\*) corresbonds to neigther P1 and P2, regreting their choices?

For example 1:

P1: no regret

P2: no regret

For example 2:

P1: **WILL** regret. If they knew P2 will select the 1st column,

P2: **WILL** regret. If they knew P1 will select the 3rd row,

If both player would regret / not regret, that constitutes the Saddle -point equilibrium?.

