Multi-Objective optimisation: A practical study based on portfolio management

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by

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Abstract

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The Mandatory Provident Fund (MPF) is an essential pillar of retirement security in Hong Kong, mandating contributions from both employees and employers to ensure a stable financial future for the workforce. This compulsory scheme requires individuals to contribute a fixed percentage of their salary, matched by their employers, culminating in significant monthly contributions to individual MPF accounts. Within this framework, investors are tasked with selecting from a variety of investment portfolios, each designed to cater to different risk tolerance levels and financial objectives. The Mandatory Provident Fund Schemes Authority (MPFA) recommends several investment strategies, ranging from aggressive allocations with a heavy emphasis on equity funds to conservative strategies focusing predominantly on bond investments.

This study delves into the strategies employed by retail investors within the MPF system, particularly those who prefer a 'set and forget' approach to managing

their retirement savings. Despite a general aversion to risk, these investors aim to maximize returns without engaging in frequent portfolio adjustments. Our analysis employs historical data to evaluate the performance of various MPF portfolios over the past decade, assessing their returns, volatility, and risk-adjusted measures like the Sharpe Ratio. This retrospective approach raises critical questions about the reliability of past performance as an indicator of future results and the overall effectiveness of conventional investment strategies in the context of the MPF.

Moreover, this paper explores the applicability of Modern Portfolio Theory (MPT) and convex optimization techniques to enhance MPF portfolio selections, aiming to strike an optimal balance between risk and return. By integrating theoretical models with practical applications, the study seeks to uncover whether traditional investment paradigms, such as those suggested by the MPFA, align with the actual needs and expectations of today's investors.

In conclusion, the paper will offer insights into the potential for redefining MPF investment strategies that not only meet the statutory requirements of the MPF scheme but also adapt to the dynamic financial landscapes and individual risk profiles of Hong Kong's workforce. The ultimate goal is to guide MPF participants towards more informed, data-driven decisions that could lead to improved financial outcomes in their retirement years.

Contents

Chapter 1

Introduction

The Mandatory Provident Fund (MPF) scheme represents a fundamental aspect of retirement planning in Hong Kong, providing a structured and compulsory savings mechanism for the workforce. Established by the government, the scheme mandates contributions from both employees and employers, aiming to ensure a stable financial reservoir for individuals upon retirement. Participants are required to contribute a minimum of 5% of their monthly earnings, which is matched by their employer, thus pooling a significant monthly sum into each individual's MPF account.

Given the diversity of investment options available within the MPF scheme, participants must judiciously select portfolios that not only align with their financial goals but also mirror their risk tolerance levels. The scheme offers a range of investment strategies, from aggressive equity-heavy portfolios to conservative bond-focused approaches. The guidelines provided by the Mandatory Provident Fund Schemes Authority (MPFA) suggest varied allocations, which participants can choose based on their preference for risk exposure and expected returns.

This comprehensive report delves into the optimization of these investment strategies under the MPF framework, employing historical data to scrutinize the performance of different portfolios over the past decade. Through this analysis, the report evaluates the efficacy of these strategies in achieving optimized risk-adjusted returns, assessing their performance through metrics such as the cumulative returns and the Sharpe Ratio.

Structure of the Report:

- **Chapter 2: Problem Formulation and Convex Optimization** This chapter formulates the investment strategy selection as a convex optimization problem, a method well-suited for handling the intricacies of portfolio management under constraints typical of the MPF schemes. It discusses the mathematical foundations and techniques necessary for solving these optimization problems, setting the stage for applying these theoretical models to real-world data.
- **Chapter 3: Implementation Details** Here, the implementation specifics of the optimization models are outlined, detailing the computational approaches and tools used. This chapter provides a walkthrough of the data processing, the use of CVXPY for solving the convex optimization problems, and how the models are tailored to fit the unique characteristics of the MPF investment options.
- **Chapter 4: Results and Conclusion** The outcomes of the optimization processes are presented in this chapter. It evaluates the performance of various MPF portfolios, providing a comparative analysis that highlights the benefits and limitations of each strategy. The chapter concludes with key insights derived from the analysis and recommendations for MPF participants aiming to optimize their retirement investments.
- **Chapter 5: Pareto Optimization and Markowitz Portfolio Theory (MPT)** The final chapter introduces and discusses advanced concepts in portfolio optimization, such as Pareto optimization and Markowitz Portfolio Theory (MPT). This section expands on the theoretical underpinnings of MPT, exploring its relevance and application to the MPF portfolios. We delve into how Pareto optimization can be used to further enhance decision-making in portfolio management by examining the trade-offs between risk and return and identifying the efficient frontier. Additionally, this chapter critically addresses the drawbacks of Modern Portfolio Theory, evaluating its focus on variance as a risk measure and the challenges associated with relying on historical data for future predictions. These discussions serve not only to elucidate the practical

application of these theories but also to offer a balanced view, highlighting both the strengths and limitations of MPT within the broader framework of economic theories in portfolio management

Chapter 2

Portfolio making as a convex optimization problem

In the domain of portfolio optimization, the key objectives are to maximize expected returns and minimize risk, which is quantified as the variance of portfolio returns. This chapter introduces the mathematical formulation of these objectives and sets the foundation for developing a convex optimization model to solve the portfolio optimization problem.

2.1 Statistical Measures of Portfolio Performance

At the heart of portfolio optimization lie two crucial statistical measures: the expected return and the standard deviation of the returns, which serve as proxies for the reward and risk, respectively.

- **Expected Return:** The expected return of a portfolio is a statistical measure that represents the average of all probable returns. It is calculated as the weighted average of the expected returns of individual assets within the portfolio. Mathematically, the expected return R_p of a portfolio can be expressed as follows:

$$R_p = \sum_{i=1}^{n} w_i r_i$$

where w_i represents the weight of the i -th asset in the portfolio, r_i is the expected return of the i -th asset, and n is the number of assets in the portfolio.

- **Standard Deviation and Variance :** The standard deviation of a portfolio quantifies the dispersion of potential returns around the expected return, serving as a primary measure of risk. Variance, the square of the standard deviation, captures the average squared deviations from the expected return, providing a more precise metric of risk spread. In the context of portfolio theory, the variance σ_p^2 of the returns is calculated using the covariance matrix of the returns of the assets comprising the portfolio. The formula for portfolio variance is:

$$\sigma_p^2 = \mathbf{w}^\top \Sigma \mathbf{w}$$

Here, $\mathbf{w}$ represents the vector of portfolio weights, indicating the proportion of the total portfolio value allocated to each asset. Σ, the covariance matrix, is a $n \times n$ matrix where each element Σ_{ij} measures the covariance between the returns of the i-th and j-th assets. Covariance assesses the degree to which two assets move in tandem; a positive covariance indicates that assets typically increase or decrease in value together, whereas a negative covariance signifies that they move in opposite directions. Mathematically, it is defined as:

$$\Sigma_{ij} = \mathrm{Cov}(r_i, r_j)$$

The covariance matrix Σ plays a critical role in portfolio theory, as it helps investors understand the relationships between different assets and thus better manage the risk of their investments.

- **The Sharpe Ratio :** In addition to evaluating portfolios based on their expected returns and risk (variance), it is essential to consider the risk-adjusted performance of the portfolio. One of the most widely recognized measures for this is the Sharpe Ratio, which provides a means to adjust returns for their risk. The Sharpe Ratio is calculated as the difference between the returns of the portfolio and the risk-free rate, divided by the standard deviation of the portfolio's returns. This ratio is a useful metric for comparing the relative risk-adjusted return between portfolios and is defined as follows:

$$\text{Sharpe Ratio} = \frac{r_p - r_f}{\sigma_p}$$

Where:

- r_p is the annualized return of the portfolio.
- r_f is the risk-free rate, which is often considered as the return on a 10-year government bond.
- σ_p is the standard deviation of the portfolio's returns, which serves as a proxy for the total risk.

The Sharpe Ratio is particularly valuable in the context of portfolio optimization as it allows investors to assess how much additional return they are receiving for each unit of increase in risk. Portfolios with higher Sharpe Ratios are generally preferred because they signify greater returns for equivalent or lesser risk.

Theorem 2.1. *The covariance matrix* Σ *is positive semidefinite.*

Proof. Covariance matrix Σ is calculated by the formula,

$$\boldsymbol{\Sigma} \triangleq E\{(\mathbf{r} - \bar{\mathbf{r}})(\mathbf{r} - \bar{\mathbf{r}})^T\}.$$

For an arbitrary real vector u, we can write,

$$\begin{aligned}
\mathbf{u}^T\boldsymbol{\Sigma}\mathbf{u} &= \mathbf{u}^T E\{(\mathbf{r} - \bar{\mathbf{r}})(\mathbf{r} - \bar{\mathbf{r}})^T\}\mathbf{u} \\
&= E\{\mathbf{u}^T(\mathbf{r} - \bar{\mathbf{r}})(\mathbf{r} - \bar{\mathbf{r}})^T\mathbf{u}\} \\
&= E\{s^2\} \\
&= \sigma_s^2.
\end{aligned}$$

Where σ_s is the variance of the zero-mean scalar random variable s, that is,

$$s = \mathbf{u}^T(\mathbf{r} - \bar{\mathbf{r}}) = (\mathbf{r} - \bar{\mathbf{r}})^T\mathbf{u}.$$

Thus,

$$\mathbf{u}^T \boldsymbol{\Sigma} \mathbf{u} = \sigma_s^2 \geq 0.$$

which shows that Σ is a positive semidefinite matrix. ■

2.2 Problem Formulation

In our portfolio optimization model, we aim to achieve two primary objectives: maximize the expected return and minimize the risk associated with the returns. This requires an optimal selection of the portfolio weights $\mathbf{w}$, which control the proportion of the total investment allocated to each asset in the portfolio.

The expected return of the portfolio is calculated as the weighted sum of the individual asset returns, where the weights reflect the proportion of the total investment in each asset. On the other hand, risk is quantified by the variance of the portfolio's returns, dependent on these weights and the covariances between the asset returns.

Given these considerations, we aim to select $\mathbf{w}$ such that both the expected return is maximized and the risk is minimized. The selection of weights is also subject to the following constraints: the weights must be non-negative (i.e., no short-selling is allowed), and they must sum to one (ensuring all available capital is fully invested). The optimization problem can be mathematically formulated as follows:

$$\begin{aligned} \max_{\mathbf{w}} \quad & \mathbf{w}^T \mathbf{r} \\ \min_{\mathbf{w}} \quad & \mathbf{w}^T \Sigma \mathbf{w} \\ \text{s.t.} \quad & \mathbf{w} \geq 0 \\ & \mathbf{1}^T \mathbf{w} = 1 \end{aligned} \tag{1}$$

This formulation lays the groundwork for applying convex optimization techniques to find the optimal portfolio weights that balance return and risk, adhering to the practical constraints of the investment environment.

2.3 Convex Approach

In addressing the dual objectives of maximizing returns and minimizing risk in portfolio optimization, we employ convex optimization techniques that leverage the properties of convex functions and constraints to ensure global optimality. The optimization problems are structured as follows:

2.3.1 Maximizing the Expected Return (Quadratic Constrained Quadratic Programming)

To maximize the expected return within a specified level of risk tolerance, we set up the optimization problem as follows:

$$\begin{aligned} \max_{\mathbf{w}} \quad & \mathbf{w}^T \mathbf{r} \\ \text{s.t.} \quad & \mathbf{w} \geq 0 \\ & \mathbf{1}^T \mathbf{w} = 1 \\ & \mathbf{w}^T \Sigma \mathbf{w} \leq \sigma_{\max}^2 \end{aligned} \tag{2}$$

This formulation seeks to maximize the expected return of the portfolio, constrained by the portfolio weights summing to one (ensuring full investment of the capital) and maintaining a total risk (as measured by the portfolio variance) below a pre-defined threshold $\sigma_{\max}^2$. The convexity of this problem is ensured since the objective function, $\mathbf{w}^T \mathbf{r}$, is linear (and hence convex), and the constraint $\mathbf{w}^T \Sigma \mathbf{w} \leq \sigma_{\max}^2$ is a convex constraint due to Σ being a positive semidefinite matrix (as established in Theorem 2.1).

2.3.2 Minimizing the Portfolio Variance aka the Risk (Quadratic Programming)

Conversely, when our primary focus is on minimizing risk, the problem is structured to minimize the portfolio variance while ensuring a minimum level of expected return:

$$\begin{aligned} \min_{\mathbf{w}} \quad & \mathbf{w}^T \Sigma \mathbf{w} \\ \text{s.t.} \quad & \mathbf{w} \geq 0 \\ & \mathbf{1}^T \mathbf{w} = 1 \\ & \mathbf{w}^T \mathbf{r} \geq r_{\min} \end{aligned} \tag{3}$$

Here, the objective is to minimize the variance of the portfolio, a convex function as Σ is a positive semidefinite matrix. The constraints are designed to ensure that all investments are non-negative (no short-selling), the total investment sums to one, and the expected return meets or exceeds a predefined minimum $r_{\min}$. These constraints maintain the convexity of the problem, allowing for efficient solutions using convex optimization methods.

The formulation and optimization techniques presented here lay the groundwork for further exploration and implementation, which is detailed in subsequent chapters of this report. Chapter 3 provides a comprehensive implementation of the optimization models discussed, using various programming tools to practically apply the theoretical constructs developed in this chapter. Chapter 4 synthesizes the results derived from these implementations, offering a conclusive analysis of the optimization strategies and their effectiveness in achieving the dual objectives of maximizing returns and minimizing risk.

Furthermore, Chapter 5 delves into the broader implications of these optimization problems, discussing the general problem referenced as (1) within the context of Pareto optimization. This discussion extends to the foundational elements of Markowitz Portfolio Theory (MPT), demonstrating how the concepts addressed here serve as a special case within the larger framework of economic theories in portfolio management. Additionally, this chapter also explores the drawbacks of Modern

Portfolio Theory, critically analyzing its reliance on variance as a sole measure of risk and its dependence on historical data for predicting future returns. These discussions not only highlight the practical relevance of convex optimization approaches but also situate our discussion within the significant economic theories that have shaped modern portfolio management strategies, providing a balanced perspective on both the strengths and limitations of MPT.

Chapter 3

Implementation Details

In this section we provide the details regarding the choice of data, its procurement and its processing. We also take a look at the implementation steps to formulate the convex optimization problem and solve it using relevant software packages and libraries.

3.1 Dataset

The daily price data for funds used in the MPF scheme was not available publicly. Therefore, we utilized the available daily stock price data of 15 assets to simulate the portfolio. The data is sourced from a larger set of data available on Kaggle. The original dataset contained over 25 assets and their historical price data from 2000 to 2021.

The dataset was cleaned by dropping all the insignificant features such as the opening price, volume traded, and highest price of an asset on a particular date. These features are rendered insignificant as we are making our predictions solely on the closing prices. A few assets had missing or NaN values for a few dates. We used a KNN (K-Nearest Neighbors) based strategy to fill in these values. We calculated the mean of the 5 neighboring values and filled in the missing values with this mean.

	Date	DRREDDY	EICHERMOT	GAIL	GRASIM	HDFC
0	2000-01-03	1508.25	48.85	68.70	438.30	293.50
1	2000-01-04	1628.95	51.40	66.35	437.15	304.05
2	2000-01-05	1568.05	55.55	63.20	439.60	292.80
3	2000-01-06	1661.55	60.00	64.95	474.80	296.45
4	2000-01-07	1529.10	64.65	62.65	512.80	286.55

FIGURE 3.1: Part of the dataset used for our formulation

3.2 CVXPY

CVXPY is an open-source Python-embedded modeling language for convex optimization problems. It lets you express your problem in a natural way that follows the math, rather than in the restrictive standard form required by solvers.

```
# our expected min return is 0.09% per year
r_min_annualized = 0.09

# create cvxpy variable to minimize
w = cp.Variable(num_funds)
r_min = r_min_annualized / 252

# construct the objective function and constraints
obj = cp.Minimize(w.T @ cov_annualized @ w)
const = [
    cp.sum(w) == 1, w >= 0,
    w.T @ r_annualized- r_min >= 0
]

# solve it!
prob = cp.Problem(obj, const)
opt_v = prob.solve()
```

FIGURE 3.2: Problem formulation in CVXPY and syntax for solving them

Chapter 4

Results and Conclusion

This section discusses the results obtained through our study. We also discuss the multi-objective optimal solution or the pareto optimal front obtained and its significance

4.1 The Optimal Values

This section describes the optimal results obtained for our problem statement 2. We obtained an optimal risk% of 4.456%. The optimal percentage of principal to be invested in individual assets are mentioned in 4.1.

4.2 Comparative Study

In this section, we evaluate the optimal investment strategy against two other strategies. One of the strategies is to invest equally in all the assets and the other one is to have an aggressive outlook and invest heavily in just one or two assets. As evident from 4.1, the Optimal strategy outperforms the other two strategies and results in greater cumulative wealth over the years. The aggressive strategy(in green) performs better than before towards 2020 as the only two assets we chose to invest

Asset	**Optimal investment%**
DRREDDY	29.24
EICHERMOT	00.00
GAIL	5.15
GRASIM	5.75
HDFC	5.75
HEROHONDA	7.21
HINDALCO	0.97
HINDLEVER	1.35
ICICIBANK	3.74
INDUSINDBK	00.00
INFY	6.72
IOC	18.57
ITC	00.60
KOTAKMAH	00.00
M&M	14.95

TABLE 4.1: Optimal % of total principal to invest in each asset

in made huge amounts of profits. However, the optimal strategy stayed consistent over the years.

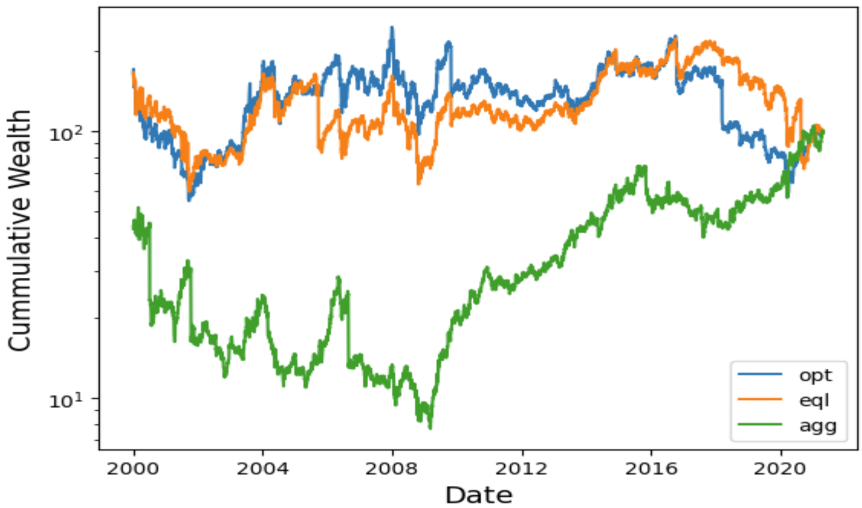

FIGURE 4.1: Comparison of the optimal strategy v/s equal and aggressive strategies

Chapter 5

Pareto Optimization and Markowitz Portfolio Theory (MPT)

5.1 Pareto Optimization

Pareto optimization, also known as Pareto efficiency, is a concept from economics and engineering that helps in decision-making across a multi-objective optimization framework. It is named after Vilfredo Pareto, an Italian economist, who used the concept in the context of economic efficiency and income distribution.

In the realm of optimization, a solution is considered Pareto optimal if no objective can be improved without simultaneously making at least one other objective worse[1]. In mathematical terms, given a set of alternative allocations and a set of individuals, an allocation is defined as Pareto efficient if there is no other allocation where improvements can be made for any individual without worsening the situation for at least one other individual.

The relevance of Pareto optimization in finance, particularly in portfolio management, is profound. It allows for the balancing of multiple conflicting objectives, such as maximizing returns while minimizing risk.

5.1.1 Application in Portfolio Optimization

When applying Pareto optimization to portfolio management, the objective is to find asset combinations that provide the best possible trade-off between expected returns and risk. This concept directly relates to the creation of an "efficient frontier" in the context of Markowitz Portfolio Theory (MPT)[2]. The efficient frontier represents a set of portfolios that offers the highest expected return for a given level of risk or the lowest risk for a given level of expected return.

Referring to the problem defined in (1), we can consider it a special case of Pareto optimization where the dual objectives of maximizing return and minimizing risk are considered. By plotting various combinations of assets and their respective risks and returns, we can visualize the set of Pareto optimal portfolios as demonstrated by the efficient frontier.

5.1.2 Visualization of the Efficient Frontier

Figure 5.1 illustrates the efficient frontier derived from a set of portfolio combinations, showcasing the optimal trade-offs between risk (standard deviation) and expected returns (mean).

This efficient frontier is foundational in the application of Markowitz Portfolio Theory, which guides investors on how to choose optimally diversified portfolios. MPT, by leveraging the principles of Pareto efficiency, not only aids in maximizing returns but also in effectively managing the risks, thus fulfilling the critical objectives of investment strategies.

5.2 Markowitz Portfolio Theory (MPT)

Markowitz Portfolio Theory (MPT), developed by Harry Markowitz in 1952, revolutionized the field of investment portfolio management through its rigorous mathematical framework. MPT introduces the concept of diversification of assets to optimize the balance between risk and return in a portfolio. This theory posits that

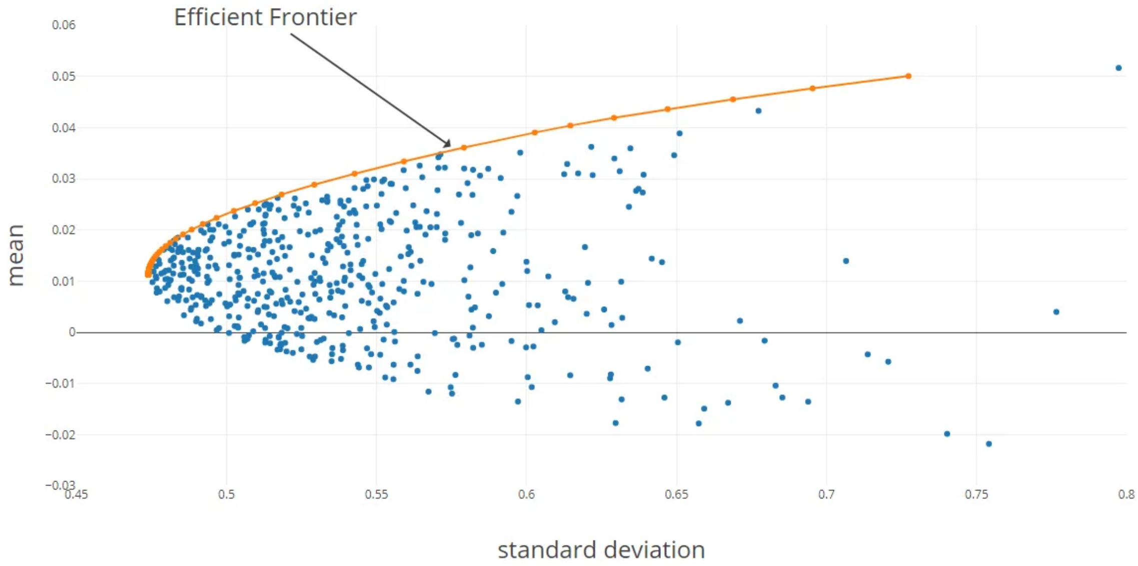

FIGURE 5.1: The Efficient Frontier: Each point on this curve represents a portfolio whose configuration offers the best possible expected return for a given level of risk. Image Source: `https://medium.com/the-modern-scientist/how-to-select-your-mpf-portfolio-wisely-portfolio-optimization-53c9b86621b2` [3]

an investor can construct a portfolio of multiple assets that will maximize returns for a given level of risk, or equivalently minimize risk for a given level of expected return.

5.2.1 Foundations of MPT

At the heart of Markowitz's theory is the idea that the risk and return of a portfolio are not merely the weighted averages of the individual asset risks and returns, but also depend on the covariance between the returns of the assets. By considering both the variance and the covariance of asset returns, MPT provides a more comprehensive measure of portfolio risk than approaches considering only individual asset risks.

MPT utilizes the efficient frontier, a concept we discussed in detail in the previous section. The efficient frontier represents a set of portfolios that are Pareto optimal, meaning no other portfolio exists that has a higher expected return with the same or lower level of risk. This curve is crucial because it embodies the maximum expected return that one can achieve for a given level of risk, according to the theory.

5.2.2 Role of the Efficient Frontier in MPT

The efficient frontier is central to Markowitz Portfolio Theory because it provides the foundational basis for the construction of optimized portfolios. By selecting portfolios that lie on this curve, investors are adhering to the MPT principle that for every level of risk, there is a portfolio that maximizes returns. Each point on the efficient frontier represents a portfolio that has the highest expected return for a given standard deviation of return, or conversely, the lowest risk for a given level of expected return.

From the practical standpoint, the implementation of MPT involves calculating the expected returns, variances, and covariances for all assets considered, and then using these calculations to generate the set of portfolios that comprise the efficient frontier. Investors can then choose a portfolio from this set based on their risk tolerance and investment goals, making the efficient frontier a critical tool in strategic asset allocation.

In conclusion, the discussion of the efficient frontier in the previous section sets the stage for applying MPT in real-world portfolio management. By understanding and utilizing the relationships between risk, return, and diversification as encapsulated by the efficient frontier, investors can make more informed decisions that align with their financial objectives and risk tolerance.

5.3 Drawbacks of Modern Portfolio Theory

Despite its widespread adoption and foundational role in investment strategy, Modern Portfolio Theory (MPT) is not without its criticisms. This section explores the significant drawbacks associated with MPT, highlighting areas where it may fall short in real-world applications.

5.3.1 Focus on Variance Rather Than Downside Risk

One of the primary criticisms of MPT is its reliance on variance as the sole measure of risk. MPT equates higher variance with greater risk, treating all deviations from the average—both positive and negative—as equally undesirable. This approach can significantly underestimate the impact of negative downturns, which are often of more concern to investors than equal magnitude gains. Such an approach to risk assessment may leave portfolios more vulnerable to extreme market conditions, where the potential for severe losses is more critical than underperformance relative to an average.

> "Modern Portfolio Theory equates higher variance with greater risk, treating all deviations from the average equally. This approach can underestimate the impact of negative downturns, making portfolios vulnerable to extreme market conditions where the potential for severe losses is greater."

5.3.2 Past Performance is Not Indicative of Future Results

Another notable limitation of MPT is its dependence on historical data to guide portfolio optimization. MPT assumes that past market behaviors will predict future outcomes, an assumption that often does not hold in the face of unexpected economic or geopolitical events. This reliance on historical data can misalign strategies when unprecedented changes occur, making past performance a poor predictor of future returns and potentially leading to inadequate investment decisions.

> "MPT relies on historical data to guide portfolio optimization, assuming past market behaviors will predict future outcomes. This can misalign strategies when unexpected economic or geopolitical events occur, making past performance a poor predictor of future returns and potentially leading to inadequate investment decisions."

These drawbacks underscore the need for a more nuanced approach to portfolio management that considers more than just variance as a measure of risk and incorporates more forward-looking factors than just historical data. As financial markets continue to evolve, so too must the theories and models we use to understand and navigate them.

Appendix A

Requirements and Source

A.1 List of Tools, Libraries and Frameworks

The implementation of the project uses the following:

- Pandas for working with tabular data
- numpy for matrix operations
- CVXPY for solving the convex optimization problem
- matplotlib for plotting the results and visualization

A.2 Project Source Link

In spirit of open source, all code is made publicly available at the kaggle URL: `https://www.kaggle.com/code/azaki02/convex-optimization`

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