Capital Market & Investment – FINC

Overview of financial markets

- Financial market: venues for allocating securities to fund and facilitate production and consumption
- Financial securities: standardized contracts (assets), issued by firms or governments, that pay cash to their owners and are usually transferable (tradable)
- Primary markets: Firms/government issue securities to fund productive business activities
- Secondary markets: Investors reallocate securities to fund consumption

Functionality of Financial Market

- Capital Allocation: access to funding, external finance, internally generated fund
- Consumption smoothing: allow people to consume even they are not working
- Risk Sharing: use diversification to reduce investor's riks
- Price discovery: Market price indicate where capital is / is not needed

$$\text{Tobin's Q: } Q = \frac{\text{firm market value}}{bookvalue} \begin{cases} Q > 1, & \textbf{more investment needed} \\ Q < 1, & \textbf{selling assets to increase value} \end{cases}$$

Primary and Secondary Market

- Primary: Governments and firms issue securities to investors
- Secondary: Trading of existing securities on exchanges

Primary Market study: Eventbrite

- IPO
  Before IPO: positive gross profit, fast growth, small labor force

  $$\text{Procedures: } \begin{cases} \text{files a Form S-1 registration statement with the SEC} \\ \text{SEC then has a cooling off period when it conduct investigattion} \\ \text{Underwriter then creates a draft prospectus to take on a ``road show''} \\ \text{underwriter and company determine final IPO price based on roadshow} \\ \text{Syndicate then allocates shares to investors} \\ \text{first day of trading: investing public can first buy stock on an exchange} \end{cases}$$

  $$\text{IPO performance: } \begin{cases} \text{IPO with SEC in Sep 20, 2018} \longrightarrow \text{10M shares for \$230M} \\ \text{Offering share price is \$23, above expected \$19 - \$21} \\ \text{6 banks are underwriter} \end{cases}$$

- Detail Analysis:
  Eventbrite sold 10M shares (13% of 76M outstanding)
  Total capital raised in 12 years of prior funding is $330M
  Buyers of the 10M shares are risk sharing
- Underwriter: help company determine amount of money to be raised
- underwriting agreement: is firm commitment where underwriter agrees to assume risk of entire inventory of stock issued in the IPO and the sale of the stock to the public at the IPO price
- Syndicate: a group of underwriters that share in the risk of the IPO offering

Secondary Market Study: Eventbrite

- None

Motivation for company to go public

- Raise money
- Provide exit for shareholders
- Use publicity to spur growth
- Provide executives with incentives

Types of investors

- Individual Investors
- Corporations
- Institutional investors

Case Study: Investment Decision

- Key formula: **Assets = Liabilities = Debt + Equity**
- Key point: when making investment, we analyse the return based on our original **Equity**
- Return: the **expected** return of an investment
- Risk: = "standard deviation of return"
- Leverage: the ratios of the borrow amount to the equity
  $100 in fund, with $20 net worth
  leverage ratio is 4, portfolio is 5 times as volatile

Valuation and returns in investment

- Payoffs: payment amount received (not return, don't care about gains or losses)
- Required returns: based on market condition
- Expected returns: based on analysis

Security types:

- Bonds: lend money to a firm / government
- stocks: own part of the firm
- derivatives: based on other assets
  Commercial Paper: unsecured (no collateral) short-term debt that is often rolled over

Case study: Lehman Brothers

- Before crisis: leverage: $\begin{cases} 20:1 & market \\ 31:1 & book \end{cases}$,
  the firm decide to give out dividend and increase ratio, to give positive signal
- historical event: Fed bail out Bear Stearns give people the impression that even if bad things happened, the Fed would come out and help
- investment decision criteria: $\begin{cases} \text{Net present value, prices (what you pay) vs value (what you get)} \\ \text{expected return v.s. required return / hurdle rate(from similar assets)} \end{cases}$
- What happened:
  filed Chapter 11 bankruptcy on Sep 15, 2008
  Lehman's key businesses were sold
  CalPERS lost $300M

Fixed Income - Bond Market Study

Bond Market size overview

- Implication on the rates: Long rate predicts economic trend

Fixed Income Market in US

- US Treasure: Federal Debt: T-bills, notes, bonds
- Mortgage Bonds: GNMA, FNMA, FHLMC
- Corporates
- Municipal bons:
- Agencies and GSEs:
- Private Label Backed (ABS):
- Money Markets:
- Repurchase Agreement Market (REPO):

Detail Calculation: prices and yields

- Yield (YTM): is the IRR calculated based on prices and cash flow (can change)
  need to notice the compounding / reinvestment period
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Bond Risk

- Interest rate risk: different maturity is different to the impact of interest rate
- default risk
- Term structure / yield curve:

ytm : yield to maturity

- what is it: The internal rate of return, calculated each time with market condition (usually calculated based on semi-annual compounding)
- When interest rate is 1. also semi-annual compounding and 2. remains the same across time frame, ytm equals to interest rate.
  A situation when ytm $\neq$ interest rate: interest rate are different across time-frame
  interest rate (from different time frame) $\rightarrow$ bond price $\rightarrow$ ytm
- Yield is therefore a somewhat a standardized measure of interest rate / market trend. We can say that ytm is a indicator for interest rate, which is an indicator for market trend
- Therefore: YTM offers us a convenient way to calculate bond price (though the yield is derived from interest rate, the 2 results should match)
- TODO : Input factor: Maturity Output factor: Yield and bond price: higher yield means market investment opportunity is higher, bond price is lower

yield curve

- Shape: x-axis: maturity ; y-axis: yield
- Trend: a growing trend, but grow gradually slowly
- Interpretation:
  for longer maturity, people is asking for higher yield to compensate the risk ;
  as maturity gets longer, yield is changing less, meaning that the risk change is less for the same unit
- Different value of yield curve:
  yield curve moves up: rate for lending and borrowing is high, economic is less active ;
  yield curve moves down, rate is less, economic is more active.

Duration - price sensitivity to yield change

- Shape: x-axis: yield ; y-axis: bond price
- Trend: a decreasing trend, the decrease get slows
- A general characteristic:
  maturity higher, coupon rate lower, yield lower $\rightarrow$ duration is lower
- 2 types of duration: $\begin{cases} \text{MaD: weighted average of maturity} \\ \text{MoD: MaD/(1+ytm/2)} \end{cases}$
- From MoD to predict price change: MoD*change in ytm = price change percentage

convexity - The rate of change of duration

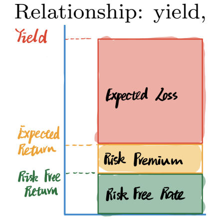
- it is decreasing, meaning that duration is growing in a slower speed

Application of duration:

- The century bond: due to convexity, the duration of a century bond is not that bad
- And because of convexity, the duration is lower in higher yields ()
- A shorter duration bond, will have a large cashflow more nearer than a long duration bond. This cash flow is subject to the discount of yield.
- Scenario1 : looks yield from low to high: a little change in yield at first, will introduce a huge change in longer duration bond's price, because its future great amount of cash flow is dicounted a lot. While shorter duration bond is less impacted. A change in yield at a larger frame, will inpose a same size of effect on short duration bond, but since long duration bond have been discounted a lot, it is less depreciated because the long-maturity part's present value barely change, and the short-maturity part have little cash flow that not gonna impact a lot
- Scenarios 2: looks yield from high to low. The longer duration bond gets a quicker boost in rising the value, for the reason that it provides a reliable, stable cash flow in the future.

Default Risk

- Yield & Expected Return:
  1. yield is the promised rate of return
  2. expected return is the return calculated based on your expectation of the cash flow
- Relationship: yield, expected return, risk free return



- risk & return in asset classes:
  stock $\rightarrow$ junk bond / high yield bond $\rightarrow$ investment grade bond $\rightarrow$ treasury

Portfolio Choice - combine bond with stock

Equity Valuation

- The concept of Equity: residual claim of a firm's cash flow (received after debt payout)
- CAPM model: $k = r_f + \beta(r - r_f)$
- Stock Prices vs. Values

$$\begin{cases} \text{1. PRICES may differ from fundamental VALUES} & \\ \text{2. Eventually, we expect convergence } P_n = V_n & \text{(market becomes efficient in n years)} \\ & \text{(basis for value investing)} \\ \text{3. If investor don't sell the stock} & V_0 = \dfrac{D_0(1+g)}{k-g} \rightarrow P_0 = \dfrac{D_0(1+g)}{E(r)-g} \\ \text{we are taking the price for now} & \\ \text{see what E[r] it gives us} & \\ \rightarrow \text{Expected return = divs yield + growth in divs} & E(r) = \dfrac{D_0(1+g)}{P_0} + g \\ \text{Things change when switch investment horizon} & \end{cases}$$

- Alternative method for long-run (LR) expected return: Profitability and Growth
  This is good for company with stable ROE but not stable dividend

$$\begin{cases} \text{When deriving the E(r) for long run: earnings yield (E/P) instead of dividend yield (D/P)} \\ \text{Sustainable LR payout: growth = (1 – payout)} \times \text{ROE} \end{cases}$$

Dividend Discount Model (DDM)

- Basic Idea : Value (V) equals PV of dividends

$$\begin{cases} \text{Forecast cash flows} \\ \text{Apply discount rate (k) to cash flows} \end{cases}$$

- One-stage DDM - required return on equity (k) is constant

$$\begin{cases} \text{stock value from future dividend} & V_0 = \dfrac{D_1}{1+k} + \dfrac{D_2}{(1+k)^2} + \ldots + \dfrac{D_n}{(1+k)^n} \\ \text{the difference of the 2 equations} & \text{one is } D_1 \text{ and one is } D_0 \\ \text{Dividend grows at constant rate} & V_0 = \dfrac{D_0(1+g)}{1+k} + \dfrac{D_0(1+g)^2}{(1+k)^2} + \ldots + \dfrac{D_0(1+g)^n}{(1+k)^n} \\ & V_0 = \dfrac{D_0(1+g)}{k-g} \end{cases}$$

- The Multi-stage DDM: - allows for multiple growth regimes
  1. Economic Stimulus Could Produce Short-run Growth 'Sugar High'
  2. Valuing temporarily higher (lower) growth is similar to valuing an increase (decrease) in the initial dividend (D0)

Valuation multiples

- Advantage of multiples: Avoid directly predicting cash flows and their growth
- Disadvantages of multiples:

$$\begin{cases} \text{Incomparable k, g, and payout of comparable stocks} \\ \text{Inability to identify pricing inefficiencies of comparables} \\ \text{Inconsistencies across different multiples (P/E, P/B, P/S, etc.)} \\ \text{Instability of multiples over time} \end{cases}$$

- Assumption within:

$$\begin{cases} \text{Comparable assets have the same risk, CF growth, and payout as asset being valued} \\ \text{Market pricing of comparables is efficient} \end{cases}$$

- Dividend discount model (DDM) helps explain multiples (i.e., prices)
  g: Stock is relatively expensive because its CFs are growing faster (higher g) than similar stocks
  k: Stock is relatively expensive because its CFs are less risky (lower k, required return) than similar stocks
- Interpret P/E with the DDM

$$\begin{cases} \text{1. P/E is price to earnings, and Dividend = Earning} \times \text{Payout} \\ \text{2. since the DDM gives us } V_0 = \dfrac{D_0(1+g)}{k-g} \text{ we replace dividend and divide each side with } E_0 \\ \dfrac{V_0}{E_0} = \dfrac{Payout(1+g)}{k-g} \text{, similarly, earning yield E/P is } \dfrac{E_0}{V_0} = \dfrac{k-g}{Payout(1+g)} \\ \text{Interpretation 1: Stocks can have high price to earnings (P/E) ratios because g is high} \\ \text{Interpretation 2: Stocks can have high P/E because discount rate (k) is low} \\ \text{Wee need to determine which factor it is that lead to a high P/E !!!} \end{cases}$$

Portfolio Optimization

- We move from long-term return to short-term return

$$\begin{cases} \text{For long-term holding valuation: } E(r) = \dfrac{D_0(1+g)}{P_0} + g \\ \text{For short-term holding valuation: } E(r_1) = \dfrac{V_0(1+k)}{P_0} - 1 \end{cases}$$

- Sharp Ratio: the risk premium per unit of risk: $SR_{us} = \dfrac{E[r_{us}] - f_f}{\sigma_{us}}$
  (As an investor, we always want a high sharp ratio, that's the essense of MVE)
- Mean Variance Efficient MVE: Diversification help us increase the sharp ratio
  (In other words, MVE is the portfolio with the highest sharp ratio)
  (We achieve it by adjusting the weights in different asset, and see the final outcome)
- Mean-Variance Utility: $U_p = E(r_p) - \frac{1}{2}A(\sigma_p)^2$, with "A" being the risk aversion
- Optimal Portfolio Weights (weights in risky asset): $w^*_{MVE} = \frac{E[r_{MVE}] - r_f}{A\sigma^2_{MVE}}$

Applying the CAPM & CAPM Anomalies

- CAPM: build on the idea that everyone holds the unique best portfolio.
  That is the market portfolio: $SR_p \leq SR_m = \frac{E[r_M] - r_f}{\sigma_m}$
- Market portfolio: all risky assets held in proportion to their market capitalization
- For each asset: the weights should equate teh marginal reward-risk ratios
  $E[r_i] - r_f = \beta_{i,MVE} \times (E[r_{MVE}] - r_f)$
  (if one asset is "x" times riskier than the market, we require "x" times excess return)
- Market Risk Premium: $w^*_M = \dfrac{E[r_m] - r_f}{A\sigma^2_m} \longrightarrow E[r_m] - r_f = w^*_m A\sigma^2_m$
  If keep weights in market same, then a rise in A or σ_m ask for higher market risk premium
  If keep the same expectation in market risk premium, a change in A or σ_m changes weights
- SML: Security Market Line $\begin{cases} E[r_i] - r_f = \beta_i (E[r_M] - r_f) \text{ with } \beta_i = \dfrac{\rho_{i,M}\sigma_i}{\sigma_M} = \dfrac{\text{Cov}(r_i, r_M)}{\sigma^2_M} \\ E[r_i] = r_f + \beta_i (E[r_M] - r_f) \\ \text{shows how required returns depend on risk } \beta \\ \text{Required and expected returns are the same in efficient markets} \end{cases}$
- α = expcected return - CAPM required return. A "mispriced" stock has $\alpha \neq 0$

- SCL: Security Characteristic Line: $r_{it} - r_{ft} = \alpha_i + \beta_i \left(r_{mt} - r_{ft}\right) + e_{it}$
  $$\underbrace{\sigma_i^2}_{\text{total variance / risk}} = \underbrace{\beta_i^2 \sigma_M^2}_{\text{Systematic risk}} + \underbrace{\sigma_{e_i}^2}_{\text{Idiosyncratic risk}}$$
  Systematic risks contribute to the overall risk of any portfolio.
  Idiosyncratic risks can be diversified away in a portfolio
  The SCL regression R2 is (systematic risk)/(total risk) $R^2 = \dfrac{\beta_i^2 \sigma_M^2}{\sigma_i^2}$

Performance Evaluation - Multi Factor Model

- Anomalies $\rightarrow \alpha$: $\begin{cases} \text{Size (market capitalizaion): small beats big, effect gets weaker recently} \\ \text{Value (B/M): high book to market ratio beats the growth} \\ \text{Beta: low beta beats high bta in risk-adjusted return (alpha)} \\ \text{Momentum: Past winner perform bad in the future} \end{cases}$
- possible explanation: $\begin{cases} \text{Size: less study / info in the small firm, high bid-ask spread} \\ \text{Value: the real / intrinsic value is higher than market value} \rightarrow \text{info} \\ \text{Beta: investors forced to sell more volatile in bad times} \\ \text{Momentum: only good if you do day-trade} \end{cases}$
- In the past, the performance is: Momentum > Value > Size > Market
- quantify the effect: $\begin{cases} \text{SMB (size premium): return of small - return of big} \\ \text{HML (value premium): retur of high B/M - return of low B/M} \\ \text{UMD (Momentum premium): up stock - down stocks} \end{cases}$
- The Fama and French Three-Factor Model: $\begin{cases} \text{Market Factor: } MktRf_t = r_{mt} - r_{f_f} \\ \text{Size Factor: } SMB_t = r_{St} - r_{Bt} \\ \text{Value Factor: } HML_t = r_{Ht} - r_{Lt} \end{cases}$
  $$\underbrace{r_{it} - r_{f_t}}_{\text{excess return}} = \alpha_i^{\text{FF3}} + \beta_i \left(r_{mt} - r_{f_t}\right) + \underbrace{s_i SMB_t}_{\text{Size Beta} \times \text{factor}} + \underbrace{h_i HML_t}_{\text{Value Beta} \times \text{factor}} + e_{it}$$
  Coefficients s_i and h_i are size and value betas (loadings)
  Small (big) firms usually have positive (negative) s_i values
  Value (growth) firms usually'have positive (negative) h_i values
- The Fama-French-Carhart Four-Factor Model: $\begin{cases} \text{Market Factor: } MktRf_t = r_{mt} - r_{f_f} \\ \text{Size Factor: } SMB_t = r_{St} - r_{Bt} \\ \text{Value Factor: } HML_t = r_{Ht} - r_{Lt} \\ \text{Momentum Factor: } UMD_t = r_{Ut} - r_{Dt} \end{cases}$
  $$r_{it} - r_{ft} = \alpha_i^{4F} + \underbrace{\beta_i \left(r_{mt} - r_{ft}\right) + s_i SMB_t + h_i HML_t + u_i UMD_t}_{\text{4 factor model benchmark}} + e_{it}$$
- Information Ratio: $IR = \dfrac{\alpha}{\sigma_e}$
  Meaning: extra return we can obtain from analysis, compare to, passive investment
  Scenario: For an investor holding the market, how much will Sharpe ratio improve if optimally weights BRK?
  $SR_{New} = \sqrt{SR_{Init}^2 + IR^2}$
  $SR_{\text{Portfolio}}^2 = SR_{\text{Market}}^2 + \left[\dfrac{\alpha_A}{\sigma\left(e_A\right)}\right]^2$

Performance evaluation: skill vs luck

- The CAPM and multifactor models allow us to analyze average returns, risk, and investment strategies:
  $\begin{cases} \text{Average returns: How much is alpha and how much is beta?} \\ \text{Risk: How much is systematic and how much is idiosyncratic?} \\ \text{Strategies: Are we using value/growth or small/big, etc.?} \end{cases}$
- actively managed funds: managers pick the portfolio weights
  Example: ClearBridge Large Cap Growth Fund (Legg Mason) $\begin{cases} \text{people manage the fund} \\ \text{Sold by advisers and brokers} \\ \text{large-cap growth stocks} \\ \text{Annual turnover (trading) 20\%} \end{cases}$
- passively managed index funds: computers pick index weights
  Example: Vanguard 500 Index Fund (Vanguard) $\begin{cases} \text{humans program computer to manage} \\ \text{track performance of S\&P500 Index} \\ \text{Directly sold to investors} \\ \text{Annual turnover (trading) 4\%} \end{cases}$
- Open-ended mutual funds $\begin{cases} \text{don't trade on the open market} \\ \text{executed at net asset value (NAV) at end of day market close} \\ \text{can't watch price fluctuate minute by minute like a stock} \\ \text{repriced based on number of shares bought and sold} \end{cases}$
- Exchange-traded Funds $\begin{cases} \text{traded in the exchange like a stock} \\ \text{fees lower than open-ended funds} \\ \text{Supply increases only when fund holdings are deposited} \end{cases}$
- Closed End Funds $\begin{cases} \text{Similar to ETFs but actively managed} \\ \text{CEFs launch through an IPO, only set amt of shares enter market} \\ \text{supply and demand determining price} \\ \text{CEFs higher fees} \end{cases}$
- Cost adjusted return: Usually, index fund with the lowest expense ratio have higher return

Ticker	Expense Ratio	2000 – 2015 Avg. Return
AAFPX	0.60%	4.71%
PREIX	0.34%	5.01%
VFINX	0.17%	5.16%
SPY	0.09%	5.19%

- whether to include a fund in your investment: Buy the fund if alpha is positive relative to your portfolio
  CAPM Example $\begin{cases} \text{compares a fund to a ``matching'' portfolio} \\ \text{``matching'' portfolio: market and risk-free asset, has same beta as the fund} \\ r_{it} - \underbrace{\left[r_{ft} + \beta_i \left(r_{mt} - r_{ft}\right)\right]}_{\text{net long CAPM benchmark}} = \alpha_i^C + e_{it} \\ \text{The fund } \alpha = \text{its average return - the average return of the matching portfolio} \\ \alpha_i^C = E\left(r_i\right) - \left[\beta_i E\left(r_m\right) + \left(1 - \beta_i\right) r_f\right] \end{cases}$
  4-factor model: $r_{it} - \underbrace{\left[r_{ft} + \beta_i \left(r_{mt} - r_{ft}\right) + s_i SMB_t + h_i HML_t + u_i UMD_t\right]}_{\text{net long 4-factor model benchmark}} = \alpha_i^{4F} + e_{it}$
- Can past alpha predict future alpha?
  $\begin{cases} \text{Individual stocks' alphas do not usually persist} \\ \text{SMB, HML, and UMD portfolio alphas seem persistent} \\ \text{Some positive CAPM alphas persist} \\ \text{no fund group has positive 4-factor } \alpha \text{ (Possibly: Winning funds hold high momentum stocks)} \\ \text{The average fund alpha is negative} \end{cases}$

Behavioral Finance - Mispricing Study

Limits to Arbitrage

- Implementation costs or bans:
- Fundamental risk:
- Noise trader risk: buy/sell reluctenly
- Delegated Arbitrage:

Mispricing of Stocks

- hard to estimate a stock's intrinsic value $\begin{cases} \text{cash flow, hard to be exact} \\ \text{discount rate, hard to determine} \end{cases}$
- estimate value with hindsight (e.g. business fraud)
- Relative equity values not always easy to compute
- Case: Insys Therapeutics: the lending fee, short interest rate

Fundamental Risk

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Options

Options and Leverage - Aribitrage & Risk Neutral Pricing

- A call option is a cheap way of betting that the underlying risky asset will increase in price $\longrightarrow$ A natural alternative is to buy the risky asset with leverage
- Call option on a stock = long stock + short risk-free asset
- Put option on a stock = short stock + long risk-free asset
- No Arbitrage: How to do option Replication:
  1. get option return $\longrightarrow$ payoff that we want to replicate. (Here we have our assumption on the future stock price movement)
  2. use stock price to match option payoff $\longrightarrow$ volatility match
  $\mathbf{\Delta = (C_u - C_d) / (S_u - S_d)}$
  3. match bond with the difference of value between stock and option $\longrightarrow$ principle match
  $\mathbf{B_Q = PV\left[(C_d u - C_H d) / (u - d)\right]}$ the diff between payoff

  The reasoning is: now that future value is the same, the current value must be the same
- The practical side of arbitrage - restrictions:
  1. bid-ask spread
  2. lending fee for short stock
  3. lending fee for rf asset

- Binomial Model for Risk Neutral Pricing
  1. $C_0 = \Delta S_0 + B = \frac{C_u - C_d}{u-d} + \frac{1}{(1+r_f/2)^{2T}}\left(\frac{C_d u - C_u d}{u-d}\right)$
  2. $C_0 = \frac{1}{(1+r_f/2)^{2T}}[\underbrace{\left[\frac{(1+r_f/2)^{2T} - d}{u-d}\right]}_{q} C_u + \underbrace{\left[\frac{u - (1+r_f/2)^{2T}}{u-d}\right]}_{1-q} C_d$
  3. $q = \frac{(1+r_f/2)^{2T} - d}{u-d}$
- Challenge - How to determine volatility forecast

Relationship between parameters and option prices

- $K \uparrow \Rightarrow C_0 \downarrow$
- $S_0 \uparrow \Rightarrow C_0 \uparrow$
- $\sigma \uparrow \Rightarrow (u-d) \uparrow \Rightarrow C_0 \uparrow$
- $T \uparrow \Rightarrow (u-d) \uparrow \Rightarrow C_0 \uparrow$
- $r_f \uparrow \Rightarrow C_0 \uparrow$
- For Call Option
  Early exercise is almost always worse than selling the call
  Exception: Deep in-the-money call on a stock that will pay a big dividend before expires
- For Deep in the money Put Option
  If the risk-free rate is positive, you should exercise early

Challenge - How to model the parameters in pricing option

- $u = e^{\sigma\sqrt{\triangle t}}$
- $d = e^{-\sigma\sqrt{\triangle t}}$
- σ - historical
  1. get daily return r_{cc}
  2. Compute the volatility of returns
  3. annualized daily return volatility: multiply $\sqrt{252}$
- σ - future - need to use relevant history to predict

- notice we need to use continuous compounding

From Binomial to Black Scholes

- Why Binomial isn't enough:
  1. Price Coverage
  2. Exercise before maturity (for the synthetic one)
- This two problem can be solved by:
  1. Use smaller time frame, until infinity (Black Scholes' case). By dividing the time interval to smaller fraction, we can theoretically cover all possible prices at expiration

- No Arbitrage Revisit:
  1. Using the Blackshole's formula, we can get a call option price. Then we need to create a synthetic call. S - 266.63 ; K - 270 ; c - 12.04
  2. TODO: Option 3 Page 14: how to create leveraged synthetic call option
  3. The leverage:
  For stock part: $\mathbf{w = (\Lambda_c \times S_0) / C_0}$;
  For bond part: -986% = (-118.65/12.04)

Put-Call Parity

- $P_0 + S_0 = C_0 + PV(K)$
  (In other words, bonds that par at K)
- Graph representation of the replication
  Page 17
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$P_0 + S_0 = C_0 + PV(K)$	Protective Put
$P_0 = C_0 + PV(K) - S_0$	Synthetic Put
$C_0 = P_0 + S_0 - PV(K)$	Synthetic Cal
$S_0 = C_0 - P_0 + PV(K)$	Synthetic Stock
$PV(K) = S_0 + P_0 - C_0$	Synthetic Bond

- Further discussion:
  1. if a stock paydividend : substract PV(divident) from stock value

Option Strategies

- Speculate events or hedge risk
- synthesize option payoff
- Arbitrage: option and synthetic option prices are close

It is all about Volatility

- Implied Volatility: volatility value that makes option value equals actual market price
- Average of 30-day and 1-year historical non-earnings volatility

Black-Schole's Formulation

- Assume log-normal underlying asset price
- Assume constant volatility
- all options based on the underlying asset, the implied volatility should be the same

Implied Volatility Property

- Term structure of volatility (Maturity)
- Stochastic volatility
- Differ by strike price

1. Skew
2. Smile: extreme strike price have high implied volatility 3. Smirk

dynamic option strategies

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Quantitative Finance – Derivatives

Time value of money

- money has time value, because we can reinvest them
- always picture a timeline in thinking about this problem

Interest

- The easiest way to make money is by earning interest from the bank
- simple interest rate (return calculation): $a_t = a_0 * (1 + it)$
- compound interest rate (keep reinvest): $a_t = a_0(1 + i)^t$
- effective interest rate during a period: $i = \frac{a_n - a_{n-1}}{a_{n-1}}$
- nominal rate of interest: $i^{(m)}$, meaning that this interest i is paid m times a year
  an amount of $\frac{i^{(m)}}{m}$ will be paid at each time interval, therefore at the end $= (1 + \frac{i^{(m)}}{m})^m$
  let m goes to infinity, we have the continuous form $(1 + \frac{r}{m})^{mt} \longrightarrow e^{rt}$

Annuities and Perpetuities

- annuity: fixed payment at each time point
  annuity-immediate: payment at the end of interval
  annuity-due: payment at the beginning of interval
- perpetuities: fixed payment at each time point, until infinity
  perpetuity-immediate: payment at the end of interval
  perpetuity-due: payment at the beginning of interval

Bonds: a fixed income instrument

- market graph of bonds: what kinds of bonds are out there
- zero coupon bond: no coupon payment, only pay face value at maturity (treat as discount)
  zero rate: the annualized rate of return (annual compound) $\longrightarrow B(t,T)(1+R(t,T))^{T-t} = 1$
  continuous zero rate: $\longrightarrow B(t,T)e^{(T-t)R(t,T)} = 1$ [treat R(t,T) as an interest rate]
  k-times per year: $\longrightarrow B_{(t,T)}(1 + \frac{R_{(r,T)}}{m})^m(T - t) = 1$
- Bond with coupon: pay at specific time

  Yield-to-Maturity: the interest rate that makes present value of cash flow equals bond price
  or we can say that the YTM is the IRR at this point: internal rate of return
  it tells us the interest rate that this investment is representing, so that we can compare

  Annual Coupon Bond: $P = \sum \frac{CF_t}{(1+y)^t}$
  Semi annual coupon bond: $P = \sum \frac{CF_t}{(1+\frac{y}{2})^{2t}}$

  So what happened in the market $\longrightarrow$ we calculate the bond price, by the risk-free rate
  Therefore it is easy to imagine:
  if risk-free rate goes up, the current bond price should drop
  if risk-free rate goes down, the current bond price should rise
  In short, the bond price is negatively correlated to the YTM

  It depends on how we structure the dependent variable and independent variables:
  1. Calculate the bond price: the DV is bond price, IDV is the interest rate, negative correlated.
  2, Calculate YTM: the DV is YTM, IDV is bond price, if bond price is high, meaning the YTM must be low

- Relationship between zero coupon bond and coupon bond:
  When a coupon bond pays no coupon, it is the same as zero coupon bond $\longrightarrow$ same price and YTM
  With coupon bond introduce coupon, we have 2 dimension to interpret the results:
  1. how the price will change: with introduction of future cash flow, interest rate does not change, price will rise; with price rise, the YTM will drop
  2. how the YTM will change: not a good direction, stick to PRICE $\longrightarrow$ YTM path
- Since the zero coupon bond has a higher YTM than coupon bond, it is some how confusing because we may think the coupon bond gives us more return.
- We introduce the "Par Yield" to do standardization on the returns:
- Par Bond: the price = face value $\longrightarrow$ issue at par $\longrightarrow$ YTM = annual coupon rate
  Par Yield: when a bond is issue at par, the coupon rate (annualized return) is equal to YTM (calculated annualized return)
- Dollar Duration: the $\frac{dP}{dy}$ the derivatives of price w.r.t. YTM
  Modified Duration: $-\frac{1}{P}\frac{dp}{dy}$, Dur normalized by price
  Dollar Value of 1 basis point (DV01): $= Dur \cdot (-1bp)$

  At a specific price level, the Dollar Duration Dur, decrease as:
  1. maturity increase (Dur is a price sensitivity measure to yield, with T goes up, it is more sensitive)
  2. coupon increases (1. more money to discount, more sensitive. 2. more coupon means price up, then more sensitive)
  3. YTM decrease (look at the graph, when YTM is small, it is steeper)

Forward Rates

- Diagram and calculation: separate the time into different intervals, make investment using respective rates
-

Futures

Options

* Memoryless:

Bonds

* Memoryless:

Financial Assets: Derivatives

Forwards / Futures

- Futures: Traded in the exchange, so basically standardized contract
- Forwards: customized
- Keywords: strike price, maturity day, long, short

  Agencies
- Clearing house: ensure trader fulfill obligations
- Margin: Trader deposit margin to fulfill obligations:
  Initial margin ; Maintenance margin ; Variation margin ;
-

Options

- Vanilla Option / European Option: fixed maturity date, strike price
- American Option: can exercise before maturity date
- Bermudan Option: can exercise on pre-determined date
- Barrier Option: can exercise once cross the barrier level:
  up and out, down and out, up and in, down and in
  1) Callable Bull: if always bull, then it is valid (never drop down barrier) [down and out]
  2) Callable Bear: if always bear, then it is valid (never go up barrier) [up and out]
- Asian Option: the payoff determined by average price of pre-set period of time

  Option strategies: Notice difference between prices and payoff
- Straddle: long 1 call and 1 put with same K and T
- Strangle: long call and put with same K and T
- Bull Spread: make money when price up: Buy call with K_1, sell call with K_2
- Bear Spread: make money when price down: Buy call with K_1, sell call with K_2
- Butterfly: long 2 calls, short call * 2

Warrents

- equity warrants: issue by company. represent the right to subscribe to equity securities
- derivative warrants: issued by 3rd party.

Swap: 2 parties exchange financial instruments

- Interest rate swap
  A: expect Libor drops ; B: expect Libor ups
  ????
- Benefit: 1) hedge against interest rate exposure ; 2) leverage comparative advantage
  A: AAA rated company ; B: BBB rated company.
  Company A will definitely have less lending interest rate in the market, in all loans
  But that doesn't mean company A can't do better. We can calculate the spread and see what is company A's comparative advantage, and borrow only that kind of loan, other company can use their comparative advantage P38
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Financial Risk: Value at Risk

Return

- Arithmetic Return: $R_t = \frac{S_t - S_{t-1}}{S_{t-1}}$
- Log Return: $log(1 + R_t) = log(\frac{S_t}{S_{t-1}}) = log(S_t) - log(S_{t-1})$

Volatility Measure

- $\sigma_s = \sqrt{\frac{1}{n-1}\sum(\mu_i - \bar{\mu})^2}$
- Then the volatility over N periods: $\sqrt{N}\sigma_s$

VaR: Value-at-Risk

- if there is a distribution graph, starting from left to some point, this region is α
- the rest region to the right is $1 - \alpha$, stands for the confidence: 95%, 99% etc.
- We called α as the "tail probability", since it is the worst scenario
- In a graphical sense, VaR is the $p - th$ quantile of the PnL distribution
- "Relative VaR": $E[PnL] - VaR$

Modeling the PnL distribution

- The fundamental of finding VaR is to model the portfolio PnL distribution
- Parametric Approach:
  Keypoint: construct the mean μ and volatility σ
  Approach: convert to approach "standard normal distribution"
  Then we can calculate VaR ; or extend this to a multi-time period scenario
- Non-Parametric Approach: Historical Simulation
  1. we want to know the price at tomorrow
  2. we convert the a problem: how much will the portfolio value change
  3. based on past period-wise change data, we treat it as distribution
  4. select the change we need

Financial Risk: Hedging, Optimal Hedging

Idea

- Hedge against a instrument that is negatively correlated to your portfolio

Quantify

- we want to minimize variance of new portfolio : $S + NF$, with S original portfolio
- we first construct the variance: $\sigma_s + N^2\sigma_f + 2N\sigma_{s,f}$
- Then N can be solved by using the solution for a square equation

Coursera - Columbia - No Arbitrage

Example of using No-Arbitrage pricing:

- 1. A bond that pays A in 1 year:
  Pricing 1: portfolio that: buy 1 bond, borrow $\frac{A}{1+r}$
  Pricing 2: portfolio that: sell 1 bond, lend $\frac{A}{1+r}$
- 2. Use compounding interest rate for pricing:
- 3. Floating Rate Bond:
  The price of a floating rate bond, is equal to, the face value.
-

Cousera - Columbia - No Arbitrage in various assets

Swaps

- What is it: a trade between fixed and float rates
- Why is it: leverage relative strength, capture trends in other markets
- How does it works: do your best, borrow your weakness
  1. A has relative strength in fixed rate
  2. B has relative strength in floating rate
  3. A will borrow fixed rate, B will borrow floating rate
  4. A borrow floating from B, pays fixed to B
- How to price it: What should be the exchange rate pay to each other
  1. Swap should be of 0 value to each party
  2. Write the Cash Flow PV for one side
  3. Set it to 0 and we calculate the payment rate

Futures

- What is it: An traded an exchange standardized contract
- Why need it: standardized, counter party risk, solved multiple price for same maturity
- How does it works: Margin call (go to initial margin), clearing house
- How to price it:
- Minimum variance hedging (since perfect hedge is impossible):

Options

- In the money, at the money, out of the money
- Put-Call Parity: call + cash = put + stock
  try to buy a stock, and use money to help me
  try to sell a stock, and hold a stock to help me
  $C + Ke^{-rt} = P + S$
  $C + Ke^{-rt} = P + S - D$ (with dividend)
- Bounds of option prices

Binomial Tree Pricing

- 1-period Binomial Tree: price goes up and down with probabilities, allow short sell
- Multi-period Binomial Tree: extend with same prob and up/down scale
- Replications: (buying shares, investing in cash) $\leftrightarrow$ (option)
  make the payoff equals
- European and American Pricing: use risk-neutral prob to price:
  $q = \frac{R-d}{u-d}$. $C_0 = \frac{1}{R}[qC_u + (1-q)C_d]$