

# Capital Market & Investment – FINC

## Overview of financial markets

- Financial market: venues for allocating securities to fund and facilitate production and consumption
- Financial securities: standardized contracts (assets), issued by firms or governments, that pay cash to their owners and are usually transferable (tradable)
- Primary markets: Firms/government issue securities to fund productive business activities
- Secondary markets: Investors reallocate securities to fund consumption

## Functionality of Financial Market

- Capital Allocation: access to funding, external finance, internally generated fund
- Consumption smoothing: allow people to consume even they are not working
- Risk Sharing: use diversification to reduce investor's risks
- Price discovery: Market price indicate where capital is / is not needed

$$\text{Tobin's Q: } Q = \frac{\text{firm market value}}{\text{book value}} \begin{cases} Q > 1, & \text{more investment needed} \\ Q < 1, & \text{selling assets to increase value} \end{cases}$$

## Primary and Secondary Market

- Primary: Governments and firms issue securities to investors
- Secondary: Trading of existing securities on exchanges

## Primary Market study: Eventbrite

- IPO
  - Before IPO: positive gross profit, fast growth, small labor force
  - Procedures:  $\begin{cases} \text{files a Form S-1 registration statement with the SEC} \\ \text{SEC then has a cooling off period when it conduct investigation} \\ \text{Underwriter then creates a draft prospectus to take on a "road show"} \\ \text{underwriter and company determine final IPO price based on roadshow} \\ \text{Syndicate then allocates shares to investors} \\ \text{first day of trading: investing public can first buy stock on an exchange} \end{cases}$
  - IPO performance:  $\begin{cases} \text{IPO with SEC in Sep 20, 2018} \rightarrow 10\text{M shares for } \$230\text{M} \\ \text{Offering share price is } \$23, \text{ above expected } \$19 - \$21 \\ \text{6 banks are underwriter} \end{cases}$
- Detail Analysis:
  - Eventbrite sold 10M shares (13% of 76M outstanding)
  - Total capital raised in 12 years of prior funding is \$330M
  - Buyers of the 10M shares are risk sharing
- Underwriter: help company determine amount of money to be raised
- underwriting agreement: is firm commitment where underwriter agrees to assume risk of entire inventory of stock issued in the IPO and the sale of the stock to the public at the IPO price
- Syndicate: a group of underwriters that share in the risk of the IPO offering

## Secondary Market Study: Eventbrite

- None

## Motivation for company to go public

- Raise money
- Provide exit for shareholders
- Use publicity to spur growth
- Provide executives with incentives

## Types of investors

- Individual Investors
- Corporations
- Institutional investors

## Case Study: Investment Decision

- Key formula: **Assets = Liabilities + Debt + Equity**
- Key point: when making investment, we analyse the return based on our original **Equity**
- Return: the **expected** return of an investment
- Risk: = "standard deviation of return"
- Leverage: the ratios of the borrow amount to the equity  
\$100 in fund, with \$20 net worth  
leverage ratio is 4, portfolio is 5 times as volatile

## Valuation and returns in investment

- Payoffs: payment amount received (not return, don't care about gains or losses)
- Required returns: based on market condition
- Expected returns: based on analysis

## Security types:

- Bonds: lend money to a firm / government
- stocks: own part of the firm
- derivatives: based on other assets  
Commercial Paper: unsecured (no collateral) short-term debt that is often rolled over

## Case study: Lehman Brothers

- Before crisis: leverage:  $\begin{cases} 20 : 1 & \text{market} \\ 31 : 1 & \text{book} \end{cases}$ ,  
the firm decide to give out dividend and increase ratio, to give positive signal
- historical event: Fed bail out Bear Stearns give people the impression that even if bad things happened, the Fed would come out and help
- investment decision criteria:  $\begin{cases} \text{Net present value, prices (what you pay) vs value (what you get)} \\ \text{expected return v.s. required return / hurdle rate (from similar assets)} \end{cases}$
- What happened:  
filed Chapter 11 bankruptcy on Sep 15, 2008  
Lehman's key businesses were sold  
CalPERS lost \$300M

# Fixed Income - Bond Market Study

## Bond Market size overview

- Implication on the rates: Long rate predicts economic trend

## Fixed Income Market in US

- US Treasury: Federal Debt: T-bills, notes, bonds
- Mortgage Bonds: GNMA, FNMA, FHLMC
- Corporates
- Municipal bonds:
- Agencies and GSEs:
- Private Label Backed (ABS):
- Money Markets:
- Repurchase Agreement Market (REPO):

## Detail Calculation: prices and yields

- Yield (YTM): is the IRR calculated based on prices and cash flow (can change)  
need to notice the compounding / reinvestment period
- 

## Bond Risk

- Interest rate risk: different maturity is different to the impact of interest rate
- default risk
- Term structure / yield curve:

## ytm : yield to maturity

- what is it: The internal rate of return, calculated each time with market condition (usually calculated based on semi-annual compounding)
- When interest rate is 1. also semi-annual compounding and 2. remains the same across time frame, ytm equals to interest rate.  
A situation when  $ytm \neq$  interest rate: interest rate are different across time-frame  
interest rate (from different time frame)  $\rightarrow$  bond price  $\rightarrow$  ytm
- Yield is therefore a somewhat a standardized measure of interest rate / market trend. We can say that ytm is a indicator for interest rate, which is an indicator for market trend
- Therefore: YTM offers us a convenient way to calculate bond price (though the yield is derived from interest rate, the 2 results should match)
- TODO : Input factor: Maturity Output factor: Yield and bond price: higher yield means market investment opportunity is higher, bond price is lower

## yield curve

- Shape: x-axis: maturity ; y-axis: yield
- Trend: a growing trend, but grow gradually slowly
- Interpretation:  
for longer maturity, people is asking for higher yield to compensate the risk ;  
as maturity gets longer, yield is changing less, meaning that the risk change is less for the same unit
- Different value of yield curve:  
yield curve moves up: rate for lending and borrowing is high, economic is less active ;  
yield curve moves down, rate is less, economic is more active.

## Duration - price sensitivity to yield change

- Shape: x-axis: yield ; y-axis: bond price
- Trend: a decreasing trend, the decrease get slows
- A general characteristic:  
maturity higher, coupon rate lower, yield lower  $\rightarrow$  duration is lower
- 2 types of duration:  $\begin{cases} \text{MaD: weighted average of maturity} \\ \text{MoD: } \text{MaD} / (1 + ytm / 2) \end{cases}$
- From MoD to predict price change:  $\text{MoD} * \text{change in ytm} = \text{price change percentage}$

## convexity - The rate of change of duration

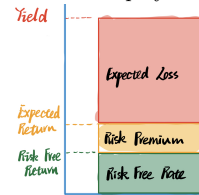
- it is decreasing, meaning that duration is growing in a slower speed

## Application of duration:

- The century bond: due to convexity, the duration of a century bond is not that bad
- And because of convexity, the duration is lower in higher yields ( )
- A shorter duration bond, will have a large cashflow more nearer than a long duration bond. This cash flow is subject to the discount of yield.
- Scenario1 : looks yield from low to high: a little change in yield at first, will introduce a huge change in longer duration bond's price, because its future great amount of cash flow is discounted a lot. While shorter duration bond is less impacted. A change in yield at a larger frame, will impose a same size of effect on short duration bond, but since long duration bond have been discounted a lot, it is less depreciated because the long-maturity part's present value barely change, and the short-maturity part have little cash flow that not gonna impact a lot
- Scenarios 2: looks yield from high to low. The longer duration bond gets a quicker boost in rising the value, for the reason that it provides a reliable, stable cash flow in the future.

## Default Risk

- Yield & Expected Return:
  1. yield is the promised rate of return
  2. expected return is the return calculated based on your expectation of the cash flow
- Relationship: yield, expected return, risk free return



- risk & return in asset classes:  
stock  $\rightarrow$  junk bond / high yield bond  $\rightarrow$  investment grade bond  $\rightarrow$  treasury

# Portfolio Choice - combine bond with stock

## Equity Valuation

- The concept of **Equity**: residual claim of a firm's cash flow (received after debt payout)
- CAPM model:  $k = r_f + \beta(r - r_f)$
- Stock Prices vs. Values
  - 1. PRICES may differ from fundamental VALUES
  - 2. Eventually, we expect convergence  $P_n = V_n$  (market becomes efficient in n years) (basis for value investing)
  - 3. If investor don't sell the stock we are taking the price for now see what E[r] it gives us
$$V_0 = \frac{D_0(1+g)}{k-g} \rightarrow P_0 = \frac{D_0(1+g)}{E(r)-g}$$

→ Expected return = divs yield + growth in divs  $E(r) = \frac{D_0(1+g)}{P_0} + g$

Things change when switch investment horizon
- Alternative method for long-run (LR) expected return: Profitability and Growth  
This is good for company with stable ROE but not stable dividend
  - When deriving the E(r) for long run: earnings yield (E/P) instead of dividend yield (D/P)
  - Sustainable LR payout: growth = (1 - payout) × ROE

## Dividend Discount Model (DDM)

- Basic Idea : Value (V) equals PV of dividends
  - Forecast cash flows
  - Apply discount rate (k) to cash flows
- One-stage DDM - required return on equity (k) is constant
  - stock value from future dividend  $V_0 = \frac{D_1}{1+k} + \frac{D_2}{(1+k)^2} + \dots + \frac{D_n}{(1+k)^n}$
  - the difference of the 2 equations one is  $D_1$  and one is  $D_0$**
  - Dividend grows at constant rate  $V_0 = \frac{D_0(1+g)}{1+k} + \frac{D_0(1+g)^2}{(1+k)^2} + \dots + \frac{D_0(1+g)^n}{(1+k)^n}$
  - $V_0 = \frac{D_0(1+g)}{k-g}$
- The Multi-stage DDM: - allows for multiple growth regimes
  - Economic Stimulus Could Produce Short-run Growth 'Sugar High'
  - Valuing temporarily higher (lower) growth is similar to valuing an increase (decrease) in the initial dividend (D0)

## Valuation multiples

- Advantage of multiples: Avoid directly predicting cash flows and their growth
- Disadvantages of multiples:
  - Incomparable k, g, and payout of comparable stocks
  - Inability to identify pricing inefficiencies of comparables
  - Inconsistencies across different multiples (P/E, P/B, P/S, etc.)
  - Instability of multiples over time
- Assumption within:
  - Comparable assets have the same risk, CF growth, and payout as asset being valued
  - Market pricing of comparables is efficient

- Dividend discount model (DDM) helps explain multiples (i.e., prices)
  - g: Stock is relatively expensive because its CFs are growing faster (higher g) than similar stocks
  - k: Stock is relatively expensive because its CFs are less risky (lower k, required return) than similar stocks
- Interpret P/E with the DDM
  - 1. P/E is price to earnings, and Dividend = Earning × Payout
  - 2. since the DDM gives us  $V_0 = \frac{D_0(1+g)}{k-g}$  we replace dividend and divide each side with  $E_0$
  - $\frac{V_0}{E_0} = \frac{Payout(1+g)}{k-g}$ , similarly, earning yield E/P is  $\frac{E_0}{V_0} = \frac{k-g}{Payout(1+g)}$
  - Interpretation 1: Stocks can have high price to earnings (P/E) ratios because g is high
  - Interpretation 2: Stocks can have high P/E because discount rate (k) is low
  - We need to determine which factor it is that lead to a high P/E !!!

## Portfolio Optimization

- We move from long-term return to short-term return
  - For long-term holding valuation:  $E(r) = \frac{D_0(1+g)}{P_0} + g$
  - For short-term holding valuation:  $E(r_1) = \frac{V_0(1+k)}{P_0} - 1$
- Sharp Ratio: the risk premium per unit of risk:  $SR_{us} = \frac{E[r_{us}] - f_f}{\sigma_{us}}$   
(As an investor, we always want a high sharp ratio, that's the essence of MVE)
- Mean Variance Efficient MVE: Diversification help us increase the sharp ratio  
(In other words, MVE is the portfolio with the highest sharp ratio)  
(We achieve it by adjusting the weights in different asset, and see the final outcome)
- Mean-Variance Utility:  $U_p = E(r_p) - \frac{1}{2}A(\sigma_p)^2$ , with "A" being the risk aversion
- Optimal Portfolio Weights (weights in risky asset):  $w_{MVE}^* = \frac{E[r_{MVE}] - r_f}{A\sigma_{MVE}^2}$

## Applying the CAPM

- CAPM: build on the idea that everyone holds the unique best portfolio.  
That is the market portfolio:  $SR_p \leq SR_m = \frac{E[r_M] - r_f}{\sigma_m}$
- Market portfolio: all risky assets held in proportion to their market capitalization
- For each asset: the weights should equate the marginal reward-risk ratios  
 $E[r_i] - r_f = \beta_{i,MVE} \times (E[r_{MVE}] - r_f)$   
(if one asset is "x" times riskier than the market, we require "x" times excess return)
- Market Risk Premium:  $w_M^* = \frac{E[r_M] - r_f}{A\sigma_m^2} \rightarrow E[r_M] - r_f = w_M^* A\sigma_m^2$   
If keep weights in market same, then a rise in A or  $\sigma_m$  ask for higher market risk premium  
If keep the same expectation in market risk premium, a change in A or  $\sigma_m$  changes weights
- SML: Security Market Line
  - $E[r_i] - r_f = \beta_i (E[r_M] - r_f)$  with  $\beta_i = \frac{\rho_{i,M}\sigma_i}{\sigma_M} = \frac{\text{Cov}(r_i, r_M)}{\sigma_M^2}$
  - $E[r_i] = r_f + \beta_i (E[r_M] - r_f)$
  - shows how required returns depend on risk  $\beta$
  - Required and expected returns are the same in efficient markets
- $\alpha$  = expected return - CAPM required return. A "mispriced" stock has  $\alpha \neq 0$

- SCL: Security Characteristic Line:  $r_{it} - r_{ft} = \alpha_i + \beta_i (r_{mt} - r_{ft}) + e_{it}$

$$\underbrace{\sigma_i^2}_{\text{total variance / risk}} = \underbrace{\beta_i^2 \sigma_M^2}_{\text{Systematic risk}} + \underbrace{\sigma_{e_i}^2}_{\text{Idiosyncratic risk}}$$

Systematic risks contribute to the overall risk of any portfolio.  
Idiosyncratic risks can be diversified away in a portfolio

The SCL regression R<sup>2</sup> is (systematic risk)/(total risk)  $R^2 = \frac{\beta_i^2 \sigma_M^2}{\sigma_i^2}$

## CAPM Anomalies

- Anomalies → alpha:  $\left\{ \begin{array}{l} \text{Size (market capitalization): small beats big, but this effect is eaker recently} \\ \text{Value (B/M): high book to market ratio beats the growth} \\ \text{Beta: low beta beats high bta in risk-adjusted return (alpha)} \\ \text{Momentum: Past winner perform bad in the future} \end{array} \right.$
- How to quantify the effect:  $\left\{ \begin{array}{l} \text{SMB: real excess return - theoretical return} \\ \text{HML (value premium): retur of high B/M - return of low B/M} \\ \text{Beta:} \\ \text{UMD Momentum: up stock - down stocks} \end{array} \right.$
- The Fama and French Three-Factor Model:  
 $r_{it} - r_{ft} = \alpha_i^{\text{FF3}} + \beta_i (r_{mt} - r_{ft}) + s_i \text{SMB}_t + h_i \text{HML}_t + e_{it}$   
 $\left\{ \begin{array}{l} \text{Market Factor: } \text{MktRf}_t = r_{mt} - r_{ft} \\ \text{HML (value premium): retur of high B/M - return of low B/M} \\ \text{Beta:} \\ \text{Momentum Factor: } \text{UMD}_t = r_{Ut} - r_{Dt} \end{array} \right.$   
 $\underbrace{r_{it} - r_{ft}}_{\text{asset excess return}} = \alpha_i^{\text{FF3}} + \beta_i (r_{mt} - r_{ft}) + s_i \text{SMB}_t + h_i \text{HML}_t + e_{it}$   
Coefficients  $s_i$  and  $h_i$  are size and value betas (loadings)  
Small (big) firms usually have positive (negative)  $s_i$  values  
Value (growth) firms usually have positive (negative)  $h_i$  values
- Information Ratio:  
Meaning: extra return we can obtain from analysis, compare to,  
Scenario: For an investor holding the market, how much will Sharpe ratio improve if optimally weights BRK?

$$SR_{New} = \sqrt{SR_{Init}^2 + IR^2}$$

$$SR_{\text{Portfolio}}^2 = SR_{\text{Market}}^2 + \left[ \frac{\alpha_A}{\sigma(e_A)} \right]^2$$

## Performance evaluation: skill vs luck

- The CAPM and multifactor models allow us to analyze average returns, risk, and investment strategies:

$\left\{ \begin{array}{l} \text{Average returns: How much is alpha and how much is beta?} \\ \text{Risk: How much is systematic and how much is idiosyncratic?} \\ \text{Strategies: Are we using value/growth or small/big, etc.?} \end{array} \right.$

- actively managed funds: managers pick the portfolio weights

$\left\{ \begin{array}{l} \text{people manage the fund} \\ \text{Sold by advisers and brokers} \\ \text{large-cap growth stocks} \\ \text{Annual turnover (trading) 20\%} \end{array} \right.$

- passively managed index funds: computers pick index weights

Example: Vanguard 500 Index Fund (Vanguard)  $\left\{ \begin{array}{l} \text{umans program computer to manage} \\ \text{track performance of S\&P500 Index} \\ \text{Directly sold to investors} \\ \text{Annual turnover (trading) 4\%} \end{array} \right.$

Usually, index fund with the lowest expense ratio have higher return

Ticker	Expense Ratio	2000 – 2015 Avg. Return
AAFPX	0.60%	4.71%
PREIX	0.34%	5.01%
VFINX	0.17%	5.16%
SPY	0.09%	5.19%

- Open-ended mutual funds  $\left\{ \begin{array}{l} \text{don't trade on the open market} \\ \text{executed at net asset value (NAV) at end of day market close} \\ \text{can't watch price fluctuate minute by minute like a stock} \\ \text{repriced based on number of shares bought and sold} \end{array} \right.$

- Exchange-traded Funds  $\left\{ \right.$

- Closed End Funds  $\left\{ \begin{array}{l} \text{Similar to ETFs but actively managed} \\ \text{CEFs launch through an IPO, only set amt of shares enter market} \\ \text{supply and demand determining price} \\ \text{CEFs higher fees} \end{array} \right.$

# Quantitative Finance – Derivatives

## Time value of money

- money has time value, because we can reinvest them
- always picture a timeline in thinking about this problem

## Interest

- The easiest way to make money is by earning interest from the bank
- simple interest rate (return calculation):  $a_t = a_0 * (1 + it)$
- compound interest rate (keep reinvest):  $a_t = a_0(1 + i)^t$
- effective interest rate during a period:  $i = \frac{a_n - a_{n-1}}{a_{n-1}}$
- nominal rate of interest:  $i^{(m)}$ , meaning that this interest  $i$  is paid  $m$  times a year  
an amount of  $\frac{i^{(m)}}{m}$  will be paid at each time interval, therefore at the end  $= (1 + \frac{i^{(m)}}{m})^m$   
let  $m$  goes to infinity, we have the continuous form  $(1 + \frac{r}{m})^{mt} \rightarrow e^{rt}$

## Annuities and Perpetuities

- annuity: fixed payment at each time point  
annuity-immediate: payment at the end of interval  
annuity-due: payment at the beginning of interval
- perpetuities: fixed payment at each time point, until infinity  
perpetuity-immediate: payment at the end of interval  
perpetuity-due: payment at the beginning of interval

## Bonds: a fixed income instrument

- market graph of bonds: what kinds of bonds are out there
- zero coupon bond: no coupon payment, only pay face value at maturity (treat as discount)  
zero rate: the annualized rate of return (annual compound)  $\rightarrow B(t, T)(1 + R(t, T))^{T-t} = 1$   
continuous zero rate:  $\rightarrow B(t, T)e^{(T-t)R(t, T)} = 1$  [treat  $R(t, T)$  as an interest rate]  
k-times per year:  $\rightarrow B(t, T)(1 + \frac{R(t, T)}{m})^m(T-t) = 1$
- Bond with coupon: pay at specific time

Yield-to-Maturity: the interest rate that makes present value of cash flow equals bond price

or we can say that the YTM is the IRR at this point: internal rate of return

it tells us the interest rate that this investment is representing, so that we can compare

Annual Coupon Bond:  $P = \sum \frac{CF_t}{(1+y)^t}$

Semi annual coupon bond:  $P = \sum \frac{CF_t}{(1+\frac{y}{2})^{2t}}$

So what happened in the market  $\rightarrow$  we calculate the bond price, by the risk-free rate  
Therefore it is easy to imagine:

if risk-free rate goes up, the current bond price should drop

if risk-free rate goes down, the current bond price should rise

In short, the bond price is negatively correlated to the YTM

It depends on how we structure the dependent variable and independent variables:

1. Calculate the bond price: the DV is bond price, IDV is the interest rate, negative correlated.
2. Calculate YTM: the DV is YTM, IDV is bond price, if bond price is high, meaning the YTM must be low

- Relationship between zero coupon bond and coupon bond:

When a coupon bond pays no coupon, it is the same as zero coupon bond  $\rightarrow$  same price and YTM

With coupon bond introduce coupon, we have 2 dimension to interpret the results:

1. how the price will change: with introduction of future cash flow, interest rate does not change, price will rise; with price rise, the YTM will drop
2. how the YTM will change: not a good direction, stick to PRICE  $\rightarrow$  YTM path

- Since the zero coupon bond has a higher YTM than coupon bond, it is some how confusing because we may think the coupon bond gives us more return.

- We introduce the "Par Yield" to do standardization on the returns:

- Par Bond: the price = face value  $\rightarrow$  issue at par  $\rightarrow$  YTM = annual coupon rate  
Par Yield: when a bond is issue at par, the coupon rate (annualized return) is equal to YTM (calculated annualized return)

- Dollar Duration: the  $\frac{dP}{dy}$  the derivatives of price w.r.t. YTM

Modified Duration:  $-\frac{1}{P} \frac{dP}{dy}$ , Dur normalized by price

Dollar Value of 1 basis point (DV01):  $= Dur \cdot (-1bp)$

At a specific price level, the Dollar Duration Dur, decrease as:

1. maturity increase (Dur is a price sensitivity measure to yield, with  $T$  goes up, it is more sensitive)
2. coupon increases (1. more money to discount, more sensitive. 2. more coupon means price up, then more sensitive)
3. YTM decrease (look at the graph, when YTM is small, it is steeper)

## Forward Rates

- Diagram and calculation: separate the time into different intervals, make investment using respective rates

•

## Futures

## Options

\* Memoryless:

## Bonds

\* Memoryless:

# Financial Assets: Derivatives

## Forwards / Futures

- Futures: Traded in the exchange, so basically standardized contract
- Forwards: customized
- Keywords: strike price, maturity day, long, short

Agencies

- Clearing house: ensure trader fulfill obligations
- Margin: Trader deposit margin to fulfill obligations:  
Initial margin ; Maintenance margin ; Variation margin ;
- 

## Options

- Vanilla Option / European Option: fixed maturity date, strike price
- American Option: can exercise before maturity date
- Bermudan Option: can exercise on pre-determined date
- Barrier Option: can exercise once cross the barrier level:  
up and out, down and out, up and in, down and in  
1) Callable Bull: if always bull, then it is valid (never drop down barrier) [down and out]  
2) Callable Bear: if always bear, then it is valid (never go up barrier) [up and out]
- Asian Option: the payoff determined by average price of pre-set period of time

Option strategies: Notice difference between prices and payoff

- Straddle: long 1 call and 1 put with same K and T
- Strangle: long call and put with same K and T
- Bull Spread: make money when price up: Buy call with  $K_1$ , sell call with  $K_2$
- Bear Spread: make money when price down: Buy call with  $K_1$ , sell call with  $K_2$
- Butterfly: long 2 calls, short call \* 2

## Warrents

- equity warrants: issue by company. represent the right to subscribe to equity securities
- derivative warrants: issued by 3rd party.

## Swap: 2 parties exchange financial instruments

- Interest rate swap  
A: expect Libor drops ; B: expect Libor ups  
????
- Benefit: 1) hedge against interest rate exposure ; 2) leverage comparative advantage  
A: AAA rated company ; B: BBB rated company.  
Company A will definitely have less lending interest rate in the market, in all loans  
But that doesn't mean company A can't do better. We can calculate the spread and see  
what is company A's comparative advantage, and borrow only that kind of loan, other  
company can use their comparative advantage P38
-

# Financial Risk: Value at Risk

## Return

- Arithmetic Return:  $R_t = \frac{S_t - S_{t-1}}{S_{t-1}}$
- Log Return:  $\log(1 + R_t) = \log\left(\frac{S_t}{S_{t-1}}\right) = \log(S_t) - \log(S_{t-1})$

## Volatility Measure

- $\sigma_s = \sqrt{\frac{1}{n-1} \sum (\mu_i - \bar{\mu})^2}$
- Then the volatility over N periods:  $\sqrt{N}\sigma_s$

## VaR: Value-at-Risk

- if there is a distribution graph, starting from left to some point, this region is  $\alpha$
- the rest region to the right is  $1 - \alpha$ , stands for the confidence: 95%, 99% etc.
- We called  $\alpha$  as the "tail probability", since it is the worst scenario
- In a graphical sense, VaR is the  $p - th$  quantile of the PnL distribution

- "Relative VaR":  $E[PnL] - VaR$

## Modeling the PnL distribution

- The fundamental of finding VaR is to model the portfolio PnL distribution
- Parametric Approach:  
Keypoint: construct the mean  $\mu$  and volatility  $\sigma$   
Approach: convert to approach "standard normal distribution"  
Then we can calculate VaR ; or extend this to a multi-time period scenario
- Non-Parametric Approach: Historical Simulation
  1. we want to know the price at tomorrow
  2. we convert the a problem: how much will the portfolio value change
  3. based on past period-wise change data, we treat it as distribution
  4. select the change we need

# Financial Risk: Hedging, Optimal Hedging

## Idea

- Hedge against a instrument that is negatively correlated to your portfolio

## Quantify

- we want to minimize variance of new portfolio :  $S + NF$ , with  $S$  original portfolio
- we first construct the variance:  $\sigma_s + N^2\sigma_f + 2N\sigma_{s,f}$
- Then N can be solved by using the solution for a square equation

# Coursera - Columbia - No Arbitrage

## Example of using No-Arbitrage pricing:

- 1. A bond that pays A in 1 year:

Pricing 1: portfolio that: buy 1 bond, borrow  $\frac{A}{1+r}$

Pricing 2: portfolio that: sell 1 bond, lend  $\frac{A}{1+r}$

- 2. Use compounding interest rate for pricing:
- 3. Floating Rate Bond:  
The price of a floating rate bond, is equal to, the face value.
- 

# Cousera - Columbia - No Arbitrage in various assets

## Swaps

- What is it: a trade between fixed and float rates
- Why is it: leverage relative strength, capture trends in other markets
- How does it work: do your best, borrow your weakness
  - A has relative strength in fixed rate
  - B has relative strength in floating rate
  - A will borrow fixed rate, B will borrow floating rate
  - A borrow floating from B, pays fixed to B
- How to price it: What should be the exchange rate pay to each other
  - Swap should be of 0 value to each party
  - Write the Cash Flow PV for one side
  - Set it to 0 and we calculate the payment rate

## Futures

- What is it: A traded on an exchange standardized contract
- Why need it: standardized, counter party risk, solved multiple price for same maturity
- How does it work: Margin call (go to initial margin), clearing house
- How to price it:

- Minimum variance hedging (since perfect hedge is impossible):

## Options

- In the money, at the money, out of the money
- Put-Call Parity: call + cash = put + stock  
try to buy a stock, and use money to help me  
try to sell a stock, and hold a stock to help me  
 $C + Ke^{-rt} = P + S$   
 $C + Ke^{-rt} = P + S - D$  (with dividend)
- Bounds of option prices

## Binomial Tree Pricing

- 1-period Binomial Tree: price goes up and down with probabilities, allow short sell
- Multi-period Binomial Tree: extend with same prob and up/down scale
- Replications: (buying shares, investing in cash)  $\leftrightarrow$  (option)  
make the payoff equals
- European and American Pricing: use risk-neutral prob to price:  
 $q = \frac{R-d}{u-d}$ .  $C_0 = \frac{1}{R} [qC_u + (1-q)C_d]$