

CSL302: Compiler Design

Top Down Parsing - LL(1)

Vishwesh Jatala

Assistant Professor

Department of CSE

Indian Institute of Technology Bhilai

vishwesh@iitbhilai.ac.in



Acknowledgement

- Today's slides are modified from that of
 - *Stanford University:*
 - <https://web.stanford.edu/class/archive/cs/cs143/cs143.1128/>

LL(1) Parse Tables

$E \rightarrow \text{int}$

$E \rightarrow (E \text{ Op } E)$

$\text{Op} \rightarrow +$

$\text{Op} \rightarrow *$

	int	()	+	*
E	int	$(E \text{ Op } E)$			
Op				+	*

LL(1) Parse Tables

$E \rightarrow AS$

$A \rightarrow a$

$S \rightarrow s$

$\text{FIRST}(E) = \{a\}$

$\text{FIRST}(A) = \{a\}$

$\text{FIRST}(S) = \{s\}$

	a	s
E	$E \rightarrow AS$	
A	$A \rightarrow a$	
S		$S \rightarrow s$

LL(1) Parse Tables

E → **AS**

A → **a** | **Tb**

T → **t**

S → **s**

FIRST(E) = {a, t}

LL(1) Parse Tables

$E \rightarrow AS$

$A \rightarrow \epsilon$

$S \rightarrow s$

$\text{FIRST}(E) = \{s\}$

FIRST Sets

- Algorithm to compute $\text{FIRST}(X)$:
 1. If X is a terminal, then $\text{FIRST}(X) = \{ X \}$.
 2. If $X \rightarrow \epsilon$ is a production, then add ϵ to $\text{FIRST}(X)$.
 3. If $X \rightarrow Y_1 Y_2 \dots Y_k$ is a production for $k \geq 1$, and
for some $i \leq k$, $Y_1 Y_2 \dots Y_{i-1}$ derives the empty string, and a is
in $\text{FIRST}(Y_i)$, then add a to $\text{FIRST}(X)$.
If $Y_1 Y_2 \dots Y_k$ derives the empty string, then add ϵ to
 $\text{FIRST}(X)$.

LL(1) Parse Tables

$E \rightarrow AS$

$A \rightarrow a \mid Tb$

$T \rightarrow t$

$S \rightarrow s$

$$\begin{aligned} \text{FIRST}(E) &= \text{FIRST}(A) \\ &= \{a\} \cup \text{FIRST}(T) \\ &= \{a, t\} \end{aligned}$$

LL(1) Parse Tables

$E \rightarrow AS$

$A \rightarrow \epsilon$

$S \rightarrow s$

$\text{FIRST}(E) = \text{FIRST}(AS)$

$\text{FIRST}(A) = \{\epsilon\}$

$\text{FIRST}(S) = \{s\}$

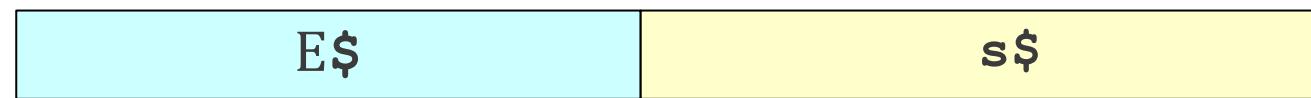
$\text{FIRST}(E) = \{s\}$

LL(1) Parsing

(1) $E \rightarrow AS$

(2) $A \rightarrow \epsilon$

(3) $S \rightarrow s$



LL(1) Parsing

- (1) $E \rightarrow AS$
- (2) $A \rightarrow \epsilon$
- (3) $S \rightarrow s$

E\$	s\$
AS\$	s\$

Apply $A \rightarrow \epsilon$, if s follows A

$$\begin{aligned}\text{FOLLOW}(A) &= \text{FIRST}(S) \\ &= \{s\}\end{aligned}$$

FOLLOW Sets

- Place \$ in FOLLOW (S), where S is the start symbol, and \$ is the input right endmarker.
- If there is a production $A \rightarrow \alpha B \beta$, then everything in FIRST(β) except for ϵ is placed in FOLLOW(B).
- If there is a production $A \rightarrow \alpha B$, or a production $A \rightarrow \alpha B \beta$ where FIRST(β) contains ϵ , then everything in FOLLOW(A) is in FOLLOW(B).

LL(1) Parse Tables

$E \rightarrow AS$

$A \rightarrow \epsilon$

$S \rightarrow s$

$\text{FOLLOW}(E) = \{\$\}$

$\text{FIRST}(A) = \text{FIRST}(S) = \{s\}$

$\text{FIRST}(S) = \text{FOLLOW}(E)$
 $= \{\$\}$

LL(1) Parse Tables

$$E \rightarrow AS$$

$$A \rightarrow \epsilon$$

$$S \rightarrow s$$

$$\text{FIRST}(E) = \{s\}$$

$$\text{FIRST}(A) = \{\epsilon\}$$

$$\text{FIRST}(S) = \{s\}$$

$$\text{FOLLOW}(E) = \{\$\}$$

$$\text{FOLLOW}(A) = \{s\}$$

$$\text{FOLLOW}(S) = \{\$\}$$

	s	\$
E	$E \rightarrow AS$	
A	$A \rightarrow \epsilon$	
S	$S \rightarrow s$	

LL(1) Tables with ϵ

Num	\rightarrow Sign Digits
Sign	\rightarrow + - ϵ
Digits	\rightarrow Digit More
More	\rightarrow Digits ϵ
Digit	\rightarrow 0 1 ... 9

LL(1) Tables with ϵ

Num → Sign Digits

Sign → + | - | ϵ

Digits → Digit More

More → Digits | ϵ

Digit → 0 | 1 | ... | 9

	+	-	#	\$
Num				
Sign				
Digits				
More				
Digit				

LL(1) Tables with ϵ

Num → **Sign Digits**
Sign → + | - | ϵ
Digits → **Digit More**
More → **Digits** | ϵ
Digit → 0 | 1 | ... | 9

	Num	Sign	Digit	Digits	More		
	+	-	0	5	0	5	0 5
	0	5	ϵ	1	6	1	6
	1	6		2	7	2	7
	2	7		3	8	3	8
	3	8		4	9	4	9
	4	9					ϵ

	+	-	#	\$
Num				
Sign				
Digits				
More				
Digit				

LL(1) Tables with ϵ

Num	Sign Digits			Num	Sign	Digit	Digits	More
Sign Digits	$\rightarrow +$	$ -$	$ \epsilon$	+	-	0	5	0 5
	\rightarrow Digit More			0	5	1	6	1 6
More	\rightarrow Digits ϵ			1	6	2	7	2 7
Digit	\rightarrow 0 1 ... 9			2	7	3	8	3 8
				3	8	4	9	4 9
				4	9			ϵ

	+	-	#	\$
Num				
Sign				
Digits				
More				
Digit				

LL(1) Tables with ϵ

Num	Sign Digits		Num	Sign	Digit	Digits	More				
Sign Digits	\rightarrow	$+$	$-$	$+$	$-$	0	5	0	5	0	5
	\rightarrow	Digit More		0	5	1	6	1	6	1	6
More	\rightarrow	Digits ϵ		1	6	2	7	2	7	2	7
Digit	\rightarrow	0	$ $	2	7	3	8	3	8	3	8
	$ $	1	$ \dots 9$	3	8	4	9	4	9	4	9
				4	9						ϵ

	+	-	#	\$
Num	Sign Digits	Sign Digits	Sign Digits	
Sign				
Digits				
More				
Digit				

LL(1) Tables with ϵ

Num	\rightarrow	Sign Digits
Sign	\rightarrow	+ - ϵ
Digits	\rightarrow	Digit More
More	\rightarrow	Digits ϵ
Digit	\rightarrow	0 1 ... 9

Num	Sign	Digit	Digits	More
+	-	0	5	0 5
0	5	1	6	1 6
1	6	2	7	2 7
2	7	3	8	3 8
3	8	4	9	4 9
4	9			ϵ

	+	-	#	\$
Num	Sign Digits	Sign Digits	Sign Digits	
Sign				
Digits				
More				
Digit				

LL(1) Tables with ϵ

Num	\rightarrow	Sign Digits
Sign	\rightarrow	+ - ϵ
Digits	\rightarrow	Digit More
More	\rightarrow	Digits ϵ
Digit	\rightarrow	0 1 ... 9

Num	Sign	Digit	Digits	More
+	-	0	5	0 5
0	5	1	6	1 6
1	6	2	7	2 7
2	7	3	8	3 8
3	8	4	9	4 9
4	9			ϵ

	+	-	#	\$
Num	Sign Digits	Sign Digits	Sign Digits	
Sign	+	-		
Digits				
More				
Digit				

LL(1) Tables with ϵ

Num → **Sign Digits**

Sign → + | - | ϵ

Digits → **Digit More**

More Digit → **Digits** | ϵ
→ 0 | 1 | ... | 9

Num	Sign	Digit	Digits	More
+	-	0	5	0 5
0	5	ϵ	1	6 1 6
1	6		2	7 2 7
2	7		3	8 3 8
3	8		4	9 4 9
4	9			ϵ

	+	-	#	\$
Num	Sign Digits	Sign Digits	Sign Digits	
Sign	+	-		
Digits				
More				
Digit				

LL(1) Tables with ϵ

Num → **Sign Digits**
Sign Digits → + | - | ϵ
Sign Digits → **Digit More**
More Digit → **Digits** | ϵ
More Digit → 0 | 1 | ... | 9

Num	Sign	Digit	Digits	More
+	-	0	5	0 5
0	5	ϵ	1	6 1 6
1	6		2	7 2 7
2	7		3	8 3 8
3	8		4	9 4 9
4	9			ϵ

	+	-	#	\$
Num	Sign Digits	Sign Digits	Sign Digits	
Sign	+	-		
Digits			Digits More	
More				
Digit				

LL(1) Tables with ϵ

Num	\rightarrow	Sign Digits
Sign	\rightarrow	+ - ϵ
Digits	\rightarrow	Digit More
More	\rightarrow	Digits ϵ
Digit	\rightarrow	0 1 ... 9

Num	Sign	Digit	Digits	More
+	-	0 5	0 5	0 5
0 5	ϵ	1 6	1 6	1 6
1 6		2 7	2 7	2 7
2 7		3 8	3 8	3 8
3 8		4 9	4 9	4 9
4 9				ϵ

	+	-	#	\$
Num	Sign Digits	Sign Digits	Sign Digits	
Sign	+	-		
Digits			Digits More	
More				
Digit				

LL(1) Tables with ϵ

Num → **Sign Digits**
Sign Digits → **+ | - | ϵ**
Digits → **Digit More**
More → **Digits | ϵ**
Digit → **0 | 1 | ... | 9**

Num	Sign	Digit	Digits	More
+	-	0	5	0 5
0	5	ϵ	1	6 1 6
1	6		2	7 2 7
2	7		3	8 3 8
3	8		4	9 4 9
4	9			ϵ

	+	-	#	\$
Num	Sign Digits	Sign Digits	Sign Digits	
Sign	+	-		
Digits			Digits More	
More			Digits	
Digit				

LL(1) Tables with ϵ

Num → **Sign Digits**
Sign → + | - | ϵ
Digits → **Digit More**
More → **Digits** | ϵ
Digit → 0 | 1 | ... | 9

	Num	Sign	Digit	Digits	More		
	+	-	0	5	0	5	0 5
	0	5	ϵ	1	6	1	6
	1	6		2	7	2	7
	2	7		3	8	3	8
	3	8		4	9	4	9
	4	9					ϵ

	+	-	#	\$
Num	Sign Digits	Sign Digits	Sign Digits	
Sign	+	-		
Digits			Digits More	
More			Digits	
Digit				

LL(1) Tables with ϵ

Num → **Sign Digits**
Sign → + | - | ϵ
Digits → **Digit More**
More → **Digits** | ϵ
Digit → 0 | 1 | ... | 9

	Num	Sign	Digit	Digits	More		
	+	-	0	5	0	5	0 5
	0	5	ϵ	1	6	1	6
	1	6		2	7	2	7
	2	7		3	8	3	8
	3	8		4	9	4	9
	4	9					ϵ

	+	-	#	\$
Num	Sign Digits	Sign Digits	Sign Digits	
Sign	+	-		
Digits			Digits More	
More			Digits	
Digit			#	

LL(1) Tables with ϵ

Num → **Sign Digits**
Sign → + | - | ϵ
Digits → **Digit More**
More → **Digits** | ϵ
Digit → 0 | 1 | ... | 9

	Num	Sign	Digit	Digits	More		
	+	-	0	5	0	5	0 5
	0	5	ϵ	1	6	1	6
	1	6		2	7	2	7
	2	7		3	8	3	8
	3	8		4	9	4	9
	4	9					ϵ

	+	-	#	\$
Num	Sign Digits	Sign Digits	Sign Digits	
Sign	+	-		
Digits			Digits More	
More			Digits	
Digit			#	

LL(1) Tables with ϵ

Num	Sign Digits		Num	Sign	Digit	Digits	More				
Sign Digits	\rightarrow	$+$	$-$	$+$	$-$	0	5	0	5	0	5
	\rightarrow	Digit More		0	5	1	6	1	6	1	6
More More	\rightarrow	Digits ϵ		1	6	2	7	2	7	2	7
Digit	\rightarrow	0	$ $	2	7	3	8	3	8	3	8
	$ $	1	$ $	3	8	4	9	4	9	4	9
	$ $	\dots	$ $	4	9						ϵ

	+	-	#	\$
Num	Sign Digits	Sign Digits	Sign Digits	
Sign	+	-		
Digits			Digits More	
More			Digits	
Digit			#	

LL(1) Tables with ϵ

Num → **Sign Digits**

Sign → + | - | ϵ

Digits → **Digit More**

More → **Digits** | ϵ

Digit → 0 | 1 | ... | 9

Num	Sign	Digit	Digits	More
+	-	0	5	0 5
0	5	1	6	1 6
1	6	2	7	2 7
2	7	3	8	3 8
3	8	4	9	4 9
4	9			ϵ

	+	-	#	\$
Num	Sign Digits	Sign Digits	Sign Digits	
Sign	+	-		
Digits			Digits More	
More			Digits	
Digit			#	

LL(1) Tables with ϵ

Num → **Sign Digits**

Sign → + | - | ϵ

Digits → **Digit More**

More → **Digits** | ϵ

Digit → 0 | 1 | ... | 9

Num	Sign	Digit	Digits	More
+	-	0	5	0 5
0	5	1	6	1 6
1	6	2	7	2 7
2	7	3	8	3 8
3	8	4	9	4 9
4	9			ϵ

	+	-	#	\$
Num	Sign Digits	Sign Digits	Sign Digits	
Sign	+	-	ϵ	
Digits			Digits More	
More			Digits	
Digit			#	

LL(1) Tables with ϵ

Num → **Sign Digits**

Sign → + | - | ϵ

Digits → **Digit More**

More Digit → **Digits** | ϵ
→ 0 | 1 | ... | 9

Num	Sign	Digit	Digits	More
+	-	0	5	0 5
0	5	ϵ	1	6 1 6
1	6		2	7 2 7
2	7		3	8 3 8
3	8		4	9 4 9
4	9			ϵ

	+	-	#	\$
Num	Sign Digits	Sign Digits	Sign Digits	
Sign	+	-	ϵ	
Digits			Digits More	
More			Digits	
Digit			#	

LL(1) Tables with ϵ

Num → **Sign Digits**
Sign Digits → **+ | - | ϵ**
Sign Digits → **Digit More**
More → **Digits | ϵ**
Digit → **0 | 1 | ... | 9**

Num	Sign	Digit	Digits	More
+	-	0	5	0 5
0	5	1	6	1 6
1	6	2	7	2 7
2	7	3	8	3 8
3	8	4	9	4 9
4	9			ϵ

	+	-	#	\$
Num	Sign Digits	Sign Digits	Sign Digits	
Sign	+	-	ϵ	
Digits			Digits More	
More			Digits	
Digit			#	

LL(1) Tables with ϵ

Num → **Sign Digits**
Sign Digits → **+ | - | ϵ**
Sign Digits → **Digit More**
More → **Digits | ϵ**
Digit → **0 | 1 | ... | 9**

Num	Sign	Digit	Digits	More
+	-	0	5	0 5
0	5	1	6	1 6
1	6	2	7	2 7
2	7	3	8	3 8
3	8	4	9	4 9
4	9			ϵ

	+	-	#	\$
Num	Sign Digits	Sign Digits	Sign Digits	
Sign	+	-	ϵ	
Digits			Digits More	
More			Digits	ϵ
Digit			#	

LL(1) Tables with ϵ

Num
Sign
Digits
More

→ **Sign Digits**

→ + | - | ϵ

→ **Digit More**

→ **Digits** | ϵ

Digit → 0 | 1 | ... | 9

Num	Sign	Digit	Digits	More
+	-	0 5	0 5	0 5
0 5	ϵ	1 6	1 6	1 6
1 6		2 7	2 7	2 7
2 7		3 8	3 8	3 8
3 8		4 9	4 9	4 9
4 9				ϵ

	+	-	#	\$
Num	Sign Digits	Sign Digits	Sign Digits	
Sign	+	-	ϵ	
Digits			Digits More	
More			Digits	ϵ
Digit			#	

LL(1) Tables with ϵ

Num → **Sign Digits**
Sign Digits → **+ | - | ϵ**
Digits More → **Digit More**
More Digit → **Digits | ϵ**
Digits → **0 | 1 | ... | 9**

	Num	Sign	Digit	Digits	More
	+	-	0	5	0 5
	0	5	ϵ	1 6	1 6
	1	6		2 7	2 7
	2	7		3 8	3 8
	3	8		4 9	4 9
	4	9			ϵ

	+	-	#	\$
Num	Sign Digits	Sign Digits	Sign Digits	
Sign	+	-	ϵ	
Digits			Digits More	
More			Digits	ϵ
Digit			#	

The Final LL(1) Table Algorithm

- Compute FIRST(A) and FOLLOW(A) for all nonterminals A .
- For each rule $A \rightarrow \omega$, for each terminal $t \in \text{FIRST}^*(\omega)$, set $T[A, t] = \omega$.
 - Note that ϵ is not a terminal.
- For each rule $A \rightarrow \omega$, if $\epsilon \in \text{FIRST}^*(\omega)$, set $T[A, t] = \omega$ for each $t \in \text{FOLLOW}(A)$.

A Formal Characterization of LL(1)

- A grammar is LL(1) if there are no conflicts in the table.
 - Every entry in the LL(1) table is unique

Questions?