# Tut02 A1

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Show that Newton's method for optimization solves quadratic problems in one step.

$$f(\underline{x}) = \frac{1}{2}\underline{x}^T \underline{A}\underline{x} + \underline{b}^T\underline{x} + c \qquad \nabla_x f = \underline{A}\underline{x} + \underline{b} \qquad \nabla_{xx} f = \underline{A} = \underline{H}_f(x)$$

Optimality condition:

$$\underline{A}\underline{x}^* + b = \nabla_x f = 0 \implies \underline{x}^* = -\underline{A}^{-1}\underline{b}$$

Newton's method:

$$\underline{x}_{k+1} = \underline{x}_k + \alpha_k \underline{H}_f^{-1}(\underline{x}_k)\, \nabla_x f$$

$$= \underline{x}_k - \alpha_k \underline{H}_f^{-1}(\underline{x}_k) \cdot (\underline{A}\underline{x}_k + \underline{b}) \qquad (\underline{H}_f^{-1} = \underline{A}^{-1})$$

$$= \underline{x}_k - \alpha_k (\underline{A}^{-1}\underline{A}\underline{x}_k + \underline{A}^{-1}\underline{b}) \qquad (\underline{A}^{-1}\underline{A} = \underline{I})$$

$$\underline{x}_{k+1} = \underline{x}_k (\underline{I} - \alpha_k \underline{I}) - \underline{A}^{-1}\underline{b} \overset{!}{=} \underline{x}^* = -\underline{A}^{-1}\underline{b}$$

$$\implies \alpha_k = 1 \qquad \square$$