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Optimization – Exercise 2 – WS 21/22

Constrained and Unconstrained optimization

Exercise 2.1 – For preparation: One-step solvability of unconstrained quadratic functions

Show that the unconstrained optimization problem

$$\min_x \frac{1}{2}x^T Hx + f^T x + c, \qquad H \in \mathbb{R}^{n\times n}, H \succ 0, x, f, c \in \mathbb{R}^n,$$

can be solved with the Newton method for an arbitrary $x_0 \in \mathbb{R}^n$ in one step. How do you have to choose the step length α_0?

Exercise 2.2 – For preparation: Eat yourself happy

You are at a buffet which offers three dishes. For each meal you know from experience how much joy it brings you. You can sum up the nutrition information (per 100g) of the meals, the *joy values* in the shown table. Assume it is dinner time and you have already consumed $\frac{2}{3}$

	kcal	Carbs	Protein	Fat	Iron	Vitamin C	Joy
m_1: Salad with dressing	42	2.9	1.4	3.2	0.9	9.2	17
m_2: Spaghetti Bolognese	235	21.5	10	11.2	2.1	2.7	60
m_3: Ice-cream sundea	104	11	1.7	5.9	0.4	1.5	90
n: Personal nutrition recomm.	2200	200	70	60	12	110	

of your personal daily nutrition recommendation. Your goal is to follow the recommendation, this means that you want to eat, for example, <u>at least</u> 70g of protein or 110mg of Vitamin C. Additionally, you want to maximize the joy you feel during your meal. Unfortunately, you are starting to feel guilty as soon as you are consuming more than 2200kcal.
This leads to the following optimization problem: By choosing the amount of 100g servings for each meal ($x_1 \cdot$ Salad, $x_2 \cdot$ Spaghetti, $x_3 \cdot$ Sundea) you want to maximize

$$\text{happiness} = \text{joy} - \text{guilty conscience}$$

under the constraint that you follow your nutrition recommendation.
The guilty conscience is modelled by

$$\text{guilty conscience} = s\left(\text{kcal}_{\text{eaten}} - \text{kcal}_{\text{max}}\right) \cdot 10^{-3} \left(\text{kcal}_{\text{eaten}} - \text{kcal}_{\text{max}}\right)^2$$
$$s(x) = \frac{1}{\pi}\left(\arctan\left(1000 \cdot x\right) + \frac{\pi}{2}\right)$$

The function $s(x)$ can be interpreted as a differentiable switch that is shown in Figure 1.

a) Formulate the constrained optimization problem, i.e. derive all necessary quantities.

b) Implement the optimization problem in MATLAB. Make the gradient of the cost function available to the solver.

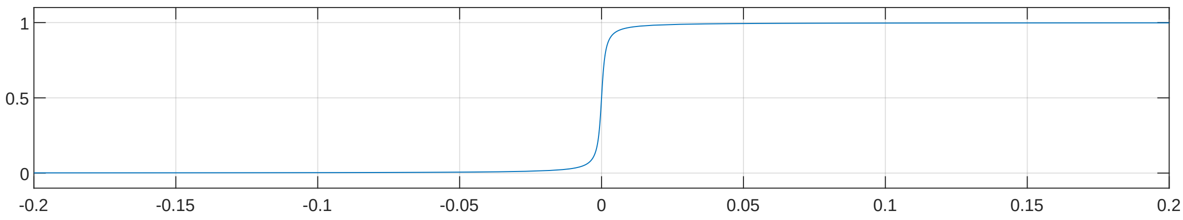

Figure 1: Differentiable switch based on the arctan function.

c) Now consider fulfilling all nutrition values and add the constraint that you want to eat a maximum of 130g fat. How do the nutrition values and your happiness change?

d) The maximum amount of fat shall now be 120g. What changes? Read the output of the solver and interpret the results.

Exercise 2.3 – In class: Nelder-Mead method

The Nelder-Mead method is a gradient-free minimization method, using simplices in the decision space and the function values of their vertices in order to descent onto a possible minimum. The Matlab function `fminsearch` uses this algorithm. We provide an object-oriented implementation, which includes a visualization of the simplices. Please download the files `NelderMeadSimplex.m` and `example_Nelder_Mead.m`. In this implementation each simplex is an object of the `NelderMeadSimplex` class and with its `do_step` method one step of the algorithm is applied and a new simplex is generated. In this exercise we want to discuss how and why the algorithm works:

a) The five different operators (reflect, expand, contract outside, contract inside, shrink) that can be applied to a simplex were presented in the lecture. When would you apply which operator? Discuss this first in small groups, reading the article "fminsearch algorithm" might prove helpful.

b) Visualize the path the simplices take over the Rosenbrock function's topology. Can you interpret the overall behaviour of the algorithm?

c) Vary the parameters `a_in`, `a_out`, and `a_shrink` and judge how the behavior changes.