Taylor Series :- We define the Taylor series expansion of $f(x)$ about $x=a$ to be the series

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \cdots + \frac{f^{(n)}(a)(x-a)^n}{n!} + \cdots$$

$$= \sum_{n=0}^{\infty} \frac{f^{(n)}(a)(x-a)^n}{n!}$$

When we truncate this series we obtain the Taylor polynomial of $f(x)$ about $x=a$ of degree n denoted by

$$P_{n,a}(x) = f(a) + f'(a)\,(x-a) + \frac{f''\,(a)}{2!}(x-a)^2 + \cdots + \frac{f^{(n)}(a)\,(x-a)^n}{n!}$$

Example :- Find the Taylor series expansion of $f(x) = \cos x$ about $x = \frac{\pi}{2}$.

Solution :- Given that

$$f(x) = \cos x$$

$$f\left(\frac{\pi}{2}\right) = 0, \qquad f'''(x) = \sin x$$

$$f'''(\pi/2) = 1$$

$f'(x) = -\sin x \quad f'(\pi/2) = -1$

$f''(x) = -\cos x \quad f''(\pi/2) = 0$

CamScanner

Therefore,

$$\cos x = f(\pi/2) + f'(\pi/2)(x-\pi/2) + \frac{f''(\pi/2)}{2!}(x-\pi/2)^2$$

$$+ \frac{f'''(\pi/2)}{3!}(x-\pi/2)^3 + \cancel{D}^{IV} \ldots$$

$$= -(x-\pi/2) + \frac{(x-\pi/2)^3}{3!} + \cdots$$

Maclaurin series :- We define the Maclaurin series expansion of $f(x)$ about $x=0$ to be the series

$$\cancel{f(x) \simeq f(0) + f'(0)\frac{2x^2}{2!} + f''(0)\frac{x^3}{3!} + f'''(0)\frac{x^4}{}}$$

$$f(x) \simeq f(0) + f'(0)\frac{x}{1!} + f''(0)\frac{x^2}{2!} + f'''(0)\frac{x^3}{3!} + \cdots$$

$$\simeq \sum_{n=0}^{\infty} \frac{f^{(n)}(0)x^n}{n!}.$$

Example :- Find the Maclaurin series expansion of $\frac{1}{1-x}$.

Solution :- Given that

$$f(x) = \frac{1}{1-x}.$$

Example:- find the Maclaurin series expansion of $f(x) = \frac{1}{1-x}$.

Solution:- Given that

$$f(x) = \frac{1}{1-x}$$

$f(0) = 1$ $\qquad$ $f'(x) = \frac{+1}{(1-x)^2}$ $\qquad$ $f'(0) = +1$.

$f''(x) = \frac{2}{(1-x)^3}$ $\qquad$ $f''(0) = 2$

$f'''(x) = \frac{+6}{(1-x)^4}$ $\qquad$ $f'''(0) = +6$.

$$\therefore \frac{1}{1-x} = f(0) + f'(0)\,x + f''(0)\frac{x^2}{2!} + f'''(0)\frac{x^3}{3!} + \cdots$$

$$= 1 + x + x^2 + x^3 + \cdots$$

Ans.