Techniques of Differentiation

1. Find the differential coefficients of the following functions with respect to x (i.e. $\dfrac{dy}{dx}$).

$(i)\, y = \sin x \sin 2x \sin 3x,\quad (ii)\, y = \cos ec^3 x,\quad (iii)\, y = \cos 2x \cos 3x,\quad (iv)\, y = \sin^{-1}(x^2),$

$(v)\, y = \tan(\sin^{-1} x),\quad (vi)\, \cot^{-1}\left(\dfrac{1+x}{1-x}\right),\quad (vii)\, \cos^{-1}\left(\dfrac{1-x^2}{1+x^2}\right),\quad (viii)\, \sin^{-1}\left(\dfrac{2x}{1+x^2}\right),$

$(ix)\, \tan^{-1}\left(\dfrac{2x}{1-x^2}\right),\quad (x)\, \tan^{-1}\left(\dfrac{x}{\sqrt{1-x^2}}\right).$

2. Find the differential coefficients of:

$(i)\,(\sin x)^{\ln x},\qquad (ii)\,(\sin x)^{\cos x} + (\cos x)^{\sin x}.$

3. Find the differential coefficients of $\sqrt{\left(\dfrac{1+x}{1-x}\right)}$.

4. Find $\dfrac{dy}{dx}$ in the following cases:

$(i)\, 3x^4 - x^2 y + 2y^3 = 0,\quad (ii)\, x^3 + y^3 + 4x^2 y - 25 = 0,\quad (iii)\, x^y = y^x.$

5. Find $\dfrac{dy}{dx}$ when

$(i)\, x = a\cos^3\theta,\ y = a\sin^3\theta,\quad (ii)\, x = \sin^2\theta,\ y = \tan\theta,\quad (iii)\, x = a\sec^2\theta,\ y = a\tan^2\theta.$

6. Differentiate the left-side functions with respect to the right-side ones:

$(i)\, \cos^{-1}\left(\dfrac{1-x^2}{1+x^2}\right)$ with respect to $\tan^{-1}\left(\dfrac{2x}{1-x^2}\right)$ $(ii)\, x^{\sin^{-1}(x)}$ with respect to $\sin^{-1} x.$

7. Find the differential coefficients of:

$(i)\, \ln\sqrt{\dfrac{1+\sin x}{1-\sin x}},\quad (ii)\, \ln\sqrt{\dfrac{1-\cos x}{1+\cos x}},\quad (iii)\, \tan^{-1}\sqrt{\dfrac{1-\cos x}{1+\cos x}},\quad (iv)\, \tan^{-1}\left(\dfrac{\cos x - \sin x}{\cos x + \sin x}\right)$

$(v)\, \sin\left(2\tan^{-1}\sqrt{\dfrac{1-x}{1+x}}\right).$