MAT 110

Differential Calculus and Coordinate Geometry

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Some basic limits

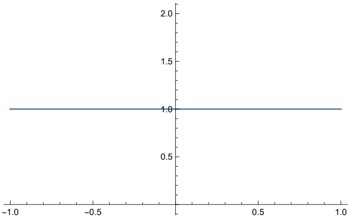
Theorem

Let a and k be real numbers

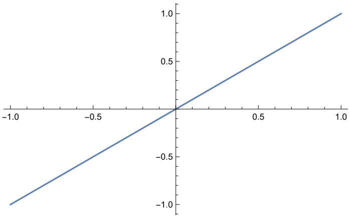
$$\lim_{x\to a} k = k; \quad \lim_{x\to a} x = a; \quad \lim_{x\to 0^-} \frac{1}{x} = -\infty; \quad \lim_{x\to 0^+} \frac{1}{x} = +\infty$$

Cont.

$$\lim_{x \to a} k = k$$



$$\lim_{x \to a} x = a$$



More simple theorems

Theorem

let a be a real number, and suppose that

$$\lim_{x \to a} f(x) = L_1 \ \textit{and} \ \lim_{x \to a} g(x) = L_2$$

Then

$$\lim_{x \to a} (f(x) \pm g(x)) = \lim_{x \to a} f(x) \pm \lim_{x \to a} g(x) = L_1 \pm L_2$$

$$\lim_{x \to a} (f(x) \times g(x)) = \lim_{x \to a} f(x) \times \lim_{x \to a} g(x) = L_1 \times L_2$$

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)} = \frac{L_1}{L_2} \qquad L_2 \neq 0$$

$$\lim_{x \to a} \sqrt[n]{f(x)}) = \sqrt[n]{\lim_{x \to a} f(x)} = \sqrt[n]{L_1}$$

Can be true for one-sided limits too.

in words

- The limit of a sum is the sum of the limits.
- The limit of a difference is the difference of the limits.
- The limit of a product is the product of the limits.
- The limit of a quotient is the quotient of the limits, provided the limit of the denominator is not zero.
- The limit of an nth root is the nth root of the limit.

$$\lim_{x \to a}(k \times g(x)) = \lim_{x \to a} k \times \lim_{x \to a} g(x) = k \times \lim_{x \to a} g(x)$$

• A constant factor can be moved through a limit symbol.

See example 4

Limits of polynomials

Find

$$\lim_{x\to 5}(x^2 - 4x + 3)$$

Theorem

For any polynomial

$$P(x) = c_0 + c_1 x + \cdots + c_n x^n$$

and any real number a,

$$\lim_{x\to a} P(x) = c_0 + c_1 a + \cdots + c_n a^n = P(a)$$

Find

$$\lim_{x\to 1}(x^7 - 2x^5 + 1)^{35}$$

Limits of Rational function

Theorem

Let

$$f(x) = \frac{p(x)}{q(x)}$$

be a rational function, and let a be any real number

- If $q(a) \neq 0$, then $\lim_{x\to a} f(x) = f(a)$
- If $q(a) = 0$ but $p(a) \neq 0$, then $\lim_{x\to a} f(x)$ does not exist

Find limits

$$\lim_{x\to 2} \frac{5x^3+4}{x-3} \qquad \text{ans:} \quad -44$$

$$\lim_{x\to 4^+} \frac{2-x}{(x-4)(x+2)} \qquad \text{ans:} \quad -\infty$$

Find

- $\lim\limits_{x\to 3} \dfrac{x^2-6x+9}{x-3}$ ans: 0
- $\lim\limits_{x\to -4} \dfrac{2x+8}{x^2+x-12}$ ans: $-\dfrac{2}{7}$
- $\lim\limits_{x\to 5} \dfrac{x^2-3x-10}{x^2-10x+25}$ ans: does not exist

Hints for last one:

$$\lim_{x\to 5^-} \frac{x+2}{x-5} \qquad \lim_{x\to 5^+} \frac{x+2}{x-5}$$

Limits involving radicals

Find

$$\lim_{x \to 1} \frac{x-1}{\sqrt{x}-1} \qquad \text{ans: } 2$$

Limits of Piece-wise function

$$f(x) = \begin{cases} \frac{1}{x+2}; & x < -2 \\ x^2 - 5; & -2 < x \leq 3 \\ \sqrt{x+13}; & x > 3 \end{cases}$$

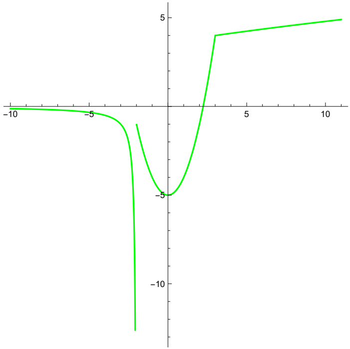
Find

$$\lim_{x \to -2} f(x) \qquad \text{ans: does not exist}$$

$$\lim_{x \to 0} f(x) \qquad \text{ans:} -5$$

$$\lim_{x \to 3} f(x) \qquad \text{ans:}4$$

Limits of piecewise function



HW

Exercise 1.2: 3-32, 37-40

Problem

Let

$$f(x) = \begin{cases} \frac{x^2-9}{x+3}; x \neq -3 \\ k; x = -3 \end{cases}$$

- Find k so that

$$f(-3) = \lim_{x \to -3} f(x) \qquad \text{ans:} k = -6$$

- With k assigned the value $\lim_{x \to -3} f(x)$, show that $f(x)$ can be expressed as a polynomial ans: $p(x) = x - 3$