# Measures of Central Tendency

Md. Sabbir Hossain

MNS, BRAC UNIVERSITY

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# Outline



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# Measures of Center

# Measures of Center

- The measure of central tendency / The measure of location / The measure of average is defined as the statistical measure that identifies a single value as representative of an entire distribution.
- It aims to provide an accurate description of the entire data.
- It is the single value that is most typical/ representative of the collected data.
- Simpson and Kafka defined it as A measure of central tendency is a typical value around which other figures congregate.
- In most datasets i.e. population or sample i.e. the values show a distinct tendency to group or cluster around a central point.
- This tendency of clustering the values around the center of the series is usually called central tendency.
- There are different measures of of central tendency.

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# Arithmetic Mean

# The Arithmetic Mean

- Arithmetic mean is a mathematical average and it is the most popular measures of location (central tendency).
- It is frequently referred to as mean it is obtained by dividing sum of the values of all observations in a series by the number of items constituting the series.
- Arithmetic mean (AM) for
  - Sample observation is denoted by $\bar{x}$
  - Population mean is denoted by $\mu$
- For a set of observations $x_1, x_2, \ldots x_n$, the following is the formula for the Arithmetic mean

$$\bar{X} = \frac{1}{n} \Sigma x_i$$

$$= \frac{1}{n}(x_1 + x_2 + \cdots + x_n)$$

# Arithmetic Mean: Example for Ungrouped Data

**Example**

- The marks (out of 25) obtained by 8 students in a class test are 10, 19, 12, 21, 18, 20, 11, and 19. What is the arithmetic mean of the marks obtained by the students?

**Solution**

- To find Arithmetic mean of the marks obtained by the students Using the arithmetic mean formula,

$$\text{Arithmetic mean} = \frac{1}{n} \Sigma x_i = \frac{\text{Sum of observations}}{\text{Number of observations}}$$

$$= \frac{(10 + 19 + 12 + 21 + 18 + 20 + 11 + 19)}{8}$$

$$= 16.25$$

# Exercise

- There are 10 salespeople employed by Midtown Ford. The number of new cars sold last month by the respective salespeople were: 15, 23, 4, 19, 18, 10, 10, 8, 28, 19.
- A mail-order company counted the number of incoming calls per day to the companys toll-free number during the first 7 days in May: 14, 24, 19, 31, 36, 26, 17.
- AAA Heating and Air Conditioning completed 30 jobs last month with a mean revenue of $5,430 per job. The president wants to know the total revenue for the month. Based on the limited information, can you compute the total revenue? What is it?

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# Arithmetic Mean for Group Data

- Recall the notational form of a frequency distribution:

| Class Interval | Class Mark | Frequency |
|---|---|---|
| $L_1 - U_1$ | $x_1$ | $f_1$ |
| $L_2 - U_2$ | $x_2$ | $f_2$ |
| $L_3 - U_3$ | $x_3$ | $f_3$ |
| $\dots$ | $\dots$ | $\dots$ |
| $L_i - U_i$ | $x_i$ | $f_i$ |
| $\dots$ | $\dots$ | $\dots$ |

- For a frequency distribution with $x_i$ and $f_i$ denoting the class frequency and class mark (mid value) of the $i$th class ($i = 1, 2, \dots, k$), the measures of central tendency can be obtained as:

# Arithmetic Mean: Example

Consider Height of 100 University students (in inches)

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 70 | 70 | 70 | 70 | 70 | 70 | 69 | 71 | 71 | 69 |
| 67 | 67 | 67 | 66 | 66 | 67 | 68 | 68 | 67 | 66 |
| 67 | 67 | 66 | 67 | 68 | 67 | 66 | 66 | 67 | 67 |
| 67 | 68 | 66 | 70 | 70 | 66 | 67 | 66 | 67 | 67 |
| 68 | 67 | 65 | 64 | 64 | 64 | 64 | 63 | 63 | 64 |
| 67 | 68 | 67 | 67 | 67 | 66 | 67 | 68 | 67 | 66 |
| 65 | 64 | 64 | 64 | 63 | 64 | 63 | 63 | 63 | 65 |
| 70 | 69 | 71 | 70 | 70 | 67 | 66 | 69 | 70 | 70 |
| 70 | 70 | 70 | 70 | 71 | 70 | 70 | 73 | 72 | 73 |
| 72 | 74 | 74 | 73 | 73 | 60 | 61 | 61 | 61 | 61 |

# Arithmetic Mean for Group data

| Height (inch) | Frequency ($f_i$) | Mid values ($x_i$) | $f_i x_i$ |
|---|---|---|---|
| 60-63 | 5 | 61.5 | 307.5 |
| 63-66 | 18 | 64.5 | 1161 |
| 66-69 | 42 | 67.5 | 2835 |
| 69-72 | 27 | 70.5 | 1903.5 |
| 72-75 | 8 | 73.5 | 588 |

$$\overline{x} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i}$$

$$= \frac{6795}{100}$$

$$= 67.95$$

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# Properties of Arithmetic Mean

- Every set of interval and ratio level data has a mean.
- A set of data has only one mean. The mean is unique.
- All the values are included in computing the mean.
- The mean is a useful measure for comparing two or more populations.
- The arithmetic mean is the only measure of central tendency where the sum of the deviations of each value from the mean will always be zero. Expressed symbolically:

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# Advantages and Disadvantages of Arithmetic mean

## Advantages

- It is easy to understand simple to calculate.
- It is easy to understand even if some of the details of the data are lacking.
- It is based on all the values.
- It is rigidly defined .
- It is not based on the position in the series.

## Disadvantages

- It is affected by extreme values.
- It cannot be calculated for open end classes.
- Cannot be used in case of qualitative data.
- It can not be located graphically.

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# Median

# Median

- Median is a central value of the distribution, or the value which divides the distribution in two equal parts, each part containing equal number of items.
- Thus it is the central value of the variable, when the values are arranged in order of magnitude.

**Formula**

- For a set of observations $x_1, x_2, \ldots x_n$, the following is the formula for the Median
  - for odd $n$, $\frac{(n+1)}{2}^{\text{th}}$ ordered observation
  - for even $n$, $\frac{1}{2}\{(\frac{n}{2})^{\text{th}}$ordered observation $+ (\frac{n}{2} + 1)^{\text{th}}\}$th ordered observation

# Example: n is Odd

- The ages of a family of seven members are given as 12, 7, 2, 34, 17, 21 and 19. Find the median age.

**Formula**

- Step 1: Count the total number of elements, $n =?$ Here $n = 7$, an odd number.
- Step 2: Arrange the values in ascending order : 2, 7, 12, 17, 19, 21, 34
- Step 3: Median: $M_e$ = Value of $\frac{(n+1)}{2}^{th}$ observation = Value of $\frac{(n+1)}{2}^{th}$ observation = Value of $4^{\text{th}}$ observation = 17.
- Step 4: Median Age of the family is 17 years.

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# Example: n is Even

- The ages of a family of eight members are given as 12, 7, 2, 34, 17, 21, 19 and 40. Find the median age.

**Formula**

- Step 1: Count the total number of elements, $n=$? Here $n = 8$, an odd number.
- Step 2: Arrange the values in ascending order : 2, 7, 12, 17, 19, 21, 34, 40
- Step 3: Median: $M_e = \frac{1}{2}\{$Value of $(\frac{n}{2})^{\text{th}}$observation + Value of $(\frac{n}{2}+1)^{\text{th}}$observation$\}$ = $\frac{1}{2}\{$Value of $4^{\text{th}}$observation+Value of $5^{\text{th}}$observation$\} = 18$.
- Step 4: Median Age of the family is 18 years

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## Determination of Median: Grouped Data

**Median**

$$\text{Median, Me} = L_0 + \frac{\frac{n}{2} - F_{-Me}}{f_m} \times C$$

Where,

- $L_0 =$ Lower class boundary of the median class,
- $n =$ Number of items in the data,
- $F_{-Me} =$ Sum of frequencies of all classes lower than the median class (Cumulative frequency of previous class),
- $f_m =$ Frequency of the median class,
- $C =$ Size of class interval.

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# Median: Example for Group Data

| Height (inch) | Frequency $(f_i)$ | Cumulative Frequency |
|---|---|---|
| 60-63 | 5 | 5 |
| 63-66 | 18 | 23 |
| 66-69 | 42 | 65 |
| 69-72 | 27 | 92 |
| 72-75 | 8 | 100 |

- $\frac{n}{2} = \frac{100}{2} = 50^{\text{th}}$ordered observation, i.e. the class of 66-69.
- $L_0 =$ Lower class boundary of the median class $= 66$,

  $n =$ Number of items in the data $= 100$,

  $F_{-Me} =$ Cumulative frequency of the previous class $= 23$,

  $f_m =$ Frequency of the median class $= 42$,

  $C =$ Size of class interval $= 3$.

$$\text{Median, Me} = L_0 + \frac{\frac{n}{2} - F_{-Me}}{f_m} \times C = 66 + \frac{\frac{100}{2} - 23}{42} \times 3 = 67.93$$

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# Properties of Median

- Every set of ordinal, interval and ratio level data has a median.
- The median is unique; that is, like the mean, there is only one median for a set of data.
- It is not affected by extremely large or small values and is therefore a valuable measure of central tendency when such values do occur.
- It can be computed for a frequency distribution with an open-ended class if the median does not lie in an open ended class.
- It can be computed for ratio-level, interval-level and ordinal-level data.

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# Advantages and Disadvantages of Median

**Advantages**

- Median can be calculated in all distributions.
- Median can be ascertained even with the extreme items.
- It can be located graphically
- It is most useful in dealing with qualitative data.

**Disadvantages**

- It is not based on all the values.
- It is not capable of further mathematical treatment.
- It is affected by fluctuation of sampling.
- In case of even number of values it may not a value from the data.

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# Usage of Median

- Whenever a data set has extreme values, the median is the preferred measure of central location.
- The median is the measure of location most often reported for annual income and property value data.
- A few extremely large incomes or property values can inflate the mean.

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# Mode

# Mode

- Mode is the most frequent value or score in the distribution.
- Generally speaking, mode can be used to describe qualitative data.
- Mode is particularly useful average for discrete data.
- For ungrouped data / categorical variable: mode is the value of the variable for which the frequency is highest
- For a set of observations $x_1, x_2, \ldots, x_n$
  - if $n(x_i)$ denote the number of times the observation $x_i$ occurs,
  - and if $n(x_k) = \max\{(n(x_i)\}$,then Mode = $x_k$ .

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# Mode: Ungrouped Data

**Example 1**

- Find the Mode of the data set 3, 7, 5, 13, 20, 23, 39, 23, 40, 23, 14, 12, 56, 23, 29.
- Count how many times each of the number occurs (can be counted easily by writing them in order): 3, 5, 7, 12, 13, 14, 20, 23, 23, 23, 23, 29, 39, 40, 56
- In this case the mode is 23.

**Example 2**

- 1, 3, 3, 3, 4, 4, 6, 6, 6, 9 .
- 3 appears three times, as does 6. So there are two modes: 3 and 6.

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# Mode: Ungrouped Data

**Example 3**

- 4, 7, 11, 16, 20, 22, 25, 26, 33.
- There exists no Mode.

**Example 4**

- strongly disagree, disagree, somewhat disagree, neutral, somewhat agree, agree, strongly agree, strongly agree, strongly agree.
- "strongly agree" appears 3 times, so the mode is "strongly agree".

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# Mode: Ungrouped Data

**Example 5**

- Ten customers answered the following to the question "Which of the following items do you want for your pizza toppings?"
- Data: Spinach, Pepperoni, Sardines, Olives, Sardines, Sausage, Extra cheese, Sardines, Onions and Tomatoes.
- Count how many times each of the value occurs Spinach, Pepperoni, Sardines, Olives, Sardines, Sausage, Extra cheese, Sardines, Onions and Tomatoes.
- Sardines appears 3 times, so the Mode is "Sardines".

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# Determination of Mode: Grouped Data

**Mode**

$$M_0 = L_0 + \frac{(f_0 - f_{-1})}{(f_0 - f_{-1}) + (f_0 - f_1)} \times C$$

Where,

- $L_0$ = Lower class boundary of the modal class,
- $f_0$ = Frequency of the modal class,
- $f_{-1}$ = Frequency of the pre modal class,
- $f_1$ = Frequency of the post modal class,
- $C$ = width of the modal class

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# Mode Using Grouped data

| Height (inch) | Frequency ($f_i$) |
|---|---|
| 60-63 | 5 |
| 63-66 | 18 |
| 66-69 | 42 |
| 69-72 | 27 |
| 72-75 | 8 |

- $L_0$ = Lower class boundary of the modal class = 66 ,
- $f_0$ = Frequency of the modal class = 42,
- $f_{-1}$ = Frequency of the pre modal class = 18,
- $f_1$ = Frequency of the post modal class = 27,
- $C$ = width of the modal class= 3

$$M_0 = L_0 + \frac{(f_0 - f_{-1})}{(f_0 - f_{-1}) + (f_0 - f_1)} \times C = 66 + \frac{24}{24 + 15} \times 3 = 67.84.$$

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# Properties of Mode

- Unlike the mean and median, Mode is not unique; that is, there may be more than one mode for a set of data.
  - If the data have exactly two modes, the data are bimodal.
  - If the data have more than two modes, the data are multimodal.
- Mode may be calculated for ratio level, interval level, ordinal level and nominal level data.
- Mode of data set may not exist.

# Advantages and Disadvantages of Mode

### Advantages

- Mode is readily comprehensible and easily calculated
- It is the best representative of data
- It is not at all affected by extreme value.
- The value of mode can also be determined graphically.

### Disadvantages

- It is not based on all observations.
- It is not capable of further mathematical manipulation.
- Mode is affected to a great extent by sampling fluctuations.
- Choice of grouping has great influence on the value of mode.

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# Weighted Mean

# Weighted Mean

- The weighted mean is a special case of the arithmetic mean. It occurs when there are several observations of the same value.
- Instead of each data point contributing equally to the final mean, some data points contribute more weight than others. Note: If all the weights are equal, then the weighted mean equals the arithmetic mean.
- It is important to note that all the probabilities or weights must be mutually exclusive (i.e., no two events can occur at the same time) and that the total weights and probabilities must add up to 100%.

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# Weighted Mean: Example

- Weighted means are useful in a wide variety of scenarios.
- For example, a student may use a weighted mean in order to calculate his/her percentage grade in a course. In such an example, the student would multiply the weighing of all assessment items in the course (e.g., assignments, exams, projects, etc.) by the respective grade that was obtained in each of the categories.
- Consider a student with the following grades:

| Item | Weight | Grade |
|---|---|---|
| Assignment | 10% | 70% |
| Attendence | 10% | 65% |
| Midterm Exam | 30% | 70% |
| Final Exam | 50% | 85% |

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# Weighted Mean: Example

- In the example above, we can arrive at the weighted mean by multiplying the weights associated with each assessment item by the grade that the student obtained on each of the items.
- Then, we can sum the products and arrive at the students final grade.
- Here, we see that the student is actually able to get a better than expected grade by doing well in the most heavily weighted component of the course: the final.
- Given the knowledge of the weighing of each assessment element in the course, students can allocate their study time more effectively.

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# Exercise

- The Carter Construction Company pays its hourly employees \$16.50, \$19.00, or \$25.00 per hour. There are 26 hourly employees, 14 of whom are paid at the \$16.50 rate, 10 at the \$19.00 rate, and 2 at the \$25.00 rate. What is the mean hourly rate paid the 26 employees?
- Andrews and Associates specialize in corporate law. They charge \$100 an hour for researching a case, \$75 an hour for consultations, and \$200 an hour for writing a brief. Last week one of the associates spent 10 hours consulting with her client, 10 hours researching the case, and 20 hours writing the brief. What was the weighted mean hourly charge for her legal services?
- The Bookstall Inc. is a specialty bookstore concentrating on used books sold via the Internet. Paperbacks are \$1.00 each, and hardcover books are \$3.50. Of the 50 books sold last Tuesday morning, 40 were paperback and the rest were hard- cover. What was the weighted mean price of a book?

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# Geometric mean

# The Geometric mean

- Geometric mean is often used for a set of numbers whose values are meant to be multiplied together or are exponential in nature, such as data on the growth of the human population or interest rates of a financial investment.
- We conduct arithmetic mean when observation are independent to each other and perform geometric mean when observations are dependent to each other. a set of observations $x_1, x_2, \ldots, x_n; (x_i > 0, \forall i = 1, 2, \ldots, n)$, the following is the formula for the Geometric mean

$$G = \left(\prod_{i=a}^{n} x_i\right)^{\frac{1}{n}} = Antilog\left\{\frac{1}{n}\sum_{i=1}^{n} \log x_i\right\}$$

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# The Geometric mean: Example

- Lets say you own a piece of art that increases in value by 50% the first year after you buy it, 20% the second year, and 90% the third year.
- What these numbers tell you is that at the end of the first year the value was multiplied by 150% or 1.5, the second year the value at the end of year 1 was multiplied by 120% or 1.2 and at the end of the third year the value at the end of year 2 was multiplied by 190% or 1.9.
- As these are multiplied, what you are looking for is the geometric mean which can be calculated in the following way:

$$(1.5 \times 1.2 \times 1.9)^{(1/3)} \approx 1.51$$

- What the answer of 1.51 is telling you is that if you multiplied your initial investment by 1.51 each year, you would get the same amount as if you had multiplied it by 1.5, 1.2 and 1.9.

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# The Geometric mean: Example

- Art work value year 0: $\$90,000$.
- Art work value year 1: $\$90,000 \times 1.5 = \$135,000$
- Art work value year 2: $\$135,000 \times 1.2 = \$162,000$.
- Art work value year 3: $\$162,000 \times 1.9 = \$307,000$.
- or, using the geometric mean: $\$90,000 * 1.51^3 = \$307,000$.

- The arithmetic mean is the sum of the data items divided by the number of items in the set:
  $(1.5 + 1.2 + 1.9)/3 = 1.53$
- As you can probably tell, adding 1.53 to your initial price wont get you anywhere, and multiplying it will give you the wrong result.
  $\$90,000 \times 1.53 \times 1.53 \times 1.53 = \$322,343.91$

# Exercise

- The return on investment earned by Atkins Construction Company for four success-sive years was 30%, 20%, 40%, and 200%. What is the geometric mean rate of return on investment?
- Sakib bought a share of 100000 tk. for a particular company in 2018. The growth of the share will increase with 10% during 2019. But due to Covid-19 pandemic it decreases with a 5% rate in 2020. In 2021 the growth rate of share increases again with 15%. In 2022, Sakib wants to withdraw the share, how much will he get?

# Harmonic Mean

# The Harmonic Mean

- The Harmonic Mean (HM) is defined as the reciprocal of the average of the reciprocals of the data values.
- Harmonic mean gives less weightage to the large values and large weightage to the small values to balance the values correctly.
- In general, the harmonic mean is used when there is a necessity to give greater weight to the smaller items.
- It is applied in the case of times and average rates.

For a set of observations $x_1, x_2, \ldots, x_n$; $(x_i > 0, \forall i = 1, 2, \ldots, n)$, the following is the formula for the Harmonic mean,

$$H = \left\{\frac{1}{n}\sum_{i=1}^{n}\frac{1}{x_i}\right\}^{-1} = \frac{1}{\left\{\frac{1}{n}\sum_{i=1}^{n}\frac{1}{x_i}\right\}}$$

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# The Harmonic Mean

**Example**

- A boat travels upstream from B to A at a speed of 15 km/hour and down stream from A to B at a speed of 25 km/hour. What is the average speed of the boat?
-

$$H = \left\{\frac{1}{n}\sum_{i=1}^{n}\frac{1}{x_i}\right\}^{-1}$$

$$= \left\{\frac{1}{2}\left(\frac{1}{15}+\frac{1}{25}\right)\right\}^{-1}$$

$$= 18.75 \text{ km/hour} (\text{Not } 20)$$

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# References and Acknowledgement

- Reference Books:
  1. Introductory STATISTICS by Neil A. Weiss, Ph.D.
  2. INTRODUCTORY STATISTICS by PREM S. MANN
- Acknowledgement:
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THANK YOU!