Practice Sheet # 7

Rolle's and Mean Value Theorem

a. Verify the hypothesis of Rolle's Theorem for the following functions:

1. $f(x) = x^2 - 6x + 8; \quad [2,4]$
2. $f(x) = \cos x; \quad [\pi/2, 3\pi/2]$
3. $f(x) = \dfrac{x}{2} - \sqrt{x}; \quad [0, 4].$

b. Verify the hypothesis of Mean Value Theorem for the following functions:

1. $f(x) = x^3 + x - 4; \quad [-1, 2]$
2. $f(x) = \sqrt{x+1}; \quad [0, 3]$
3. $f(x) = \sqrt{25 - x^2}; \quad [0, 5].$

Maclaurin and Taylor Series

1. Find the Taylor series for the following functions:

   (i) $\sin x$, at $x_0 = \dfrac{\pi}{2}$. $\qquad$ (ii) $\ln x$, at $x_0 = 2$.

2. Expand $y = \ln x$ in the power of $x - 2$ and $y = e^{ax}$ in the power of $x - 1$.
3. Find the Maclaurin series for the function e^{ax} and $\cos x$.
4. Find the Maclaurin polynomial p_0, p_1, p_2, p_3 for $e^x \cos x$.
5. Expand $y = \ln(x+1)$ and $y = \dfrac{\sin x}{\cos x}$ in the power of x.

Indeterminate Forms

Find the limit using L' Hospital's rule:

1. $\lim\limits_{x\to 1} \dfrac{\ln x}{x-1}$, 2. $\lim\limits_{x\to 3} \dfrac{x-3}{3x^2 - 13x + 12}$, 3. $\lim\limits_{x\to 0}\left(\dfrac{1}{x^2} - \dfrac{\cos 3x}{x^2}\right)$, 4. $\lim\limits_{x\to \pi} \dfrac{\sin x}{x - \pi}$

5. $\lim\limits_{x\to 0} \dfrac{x - \tan^{-1} x}{x^3}$, 6. $\lim\limits_{x\to +\infty} \dfrac{e^{3x}}{x^2}$, 7. $\lim\limits_{x\to 0} \dfrac{a^x - 1 - x\log a}{x^2}$, 8. $\lim\limits_{x\to 0} (e^x + x)^{\frac{1}{x}}$

9. $\lim\limits_{x\to 0}\left(\dfrac{1}{x} - \dfrac{1}{\sin x}\right)$, 10. $\lim\limits_{x\to \pi} (x - \pi)\cot x$, 11. $\lim\limits_{x\to 0} \dfrac{\ln(\sin x)}{\ln(\tan x)}$, 12. $\lim\limits_{x\to \infty} xe^{-x}$

13. $\lim\limits_{x\to 0} \dfrac{\sin 2x}{x}$, 14. $\lim\limits_{x\to 0} \dfrac{\sin x}{x^2}$, 15. $\lim\limits_{x\to \infty} \dfrac{x}{e^x}$, 16. $\lim\limits_{x\to 0}\left(\dfrac{1}{x} - \dfrac{1}{xe^x}\right)$, 17. $\lim\limits_{x\to 0} \dfrac{\sin 2x}{\sin 5x}$.