

Differentiability

1. Test the differentiability of the following functions:

(i) $f(x) = \begin{cases} \cos x, & x \geq 0 \\ -\cos x, & x < 0 \end{cases}$ at $x = 0$.

(ii) $f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$ at $x = 0$.

(iii) $f(x) = |x|$ at $x = 0$.

(iv) $f(x) = \begin{cases} x \cos\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$ at $x = 0$.

2. Let $f(x) = \begin{cases} x^2 - 16x, & x < 9 \\ 12\sqrt{x}, & x \geq 9 \end{cases}$. Is $f(x)$ continuous at $x = 9$? Determine whether $f(x)$ is differentiable at $x = 9$.

3. Let $f(x) = \begin{cases} x^2, & x \leq 1 \\ \sqrt{x}, & x > 1 \end{cases}$. Is $f(x)$ continuous at $x = 1$? Determine whether $f(x)$ is differentiable at $x = 1$.

4. Show that $f(x) = \begin{cases} x^2 + 1, & x \leq 1 \\ x, & x > 1 \end{cases}$ is not continuous and differentiable at $x = 1$. Sketch the graph of $f(x)$.