MAT 110

Differential Calculus and Coordinate Geometry

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Tangent line problem

Given a function f and point $P(x_0, y_0)$ on its graph, find an equation of the line that is tangent to the graph at P.

The area problem

Given a function f, find the area between the graph of f and an interval $[a, b]$ on the x-axis

Differential calculus arises from tangent line problem
Integral calculus arises from area problem

Misconceptions

Straight line meets a curve at one point $\implies$ line is tangent to the curve
It's a wrong idea
It might be true and also might not be true
Draw graphs of-

- line intersect circle once
- line intersect parabola once trough the axis
- line intersect a graph at multiple points

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Tangent lines and limits

secant line

intersects a graph at least twice
it is called a secant between those two points

Tangent line

limiting position where one point of the secant line(say P) goes very near to the other point(say Q)
So close that we can consider them the same points
mathematically we will say $P \to Q$ or maybe $P - Q \to 0$

Example 1

Find an equation for the tangent line to the parabola $y = x^2$ at the point $P(1,1)$
Answer:

$$y = 2x - 1$$

Hints: Point-slope formula

$$y - y_1 = m(x - x_1)$$

Informally speaking

Limit

If the values of $f(x)$ can be made as close as we like to L by taking value of x sufficiently close to a (but not equal to a) then we write

$$\lim_{x \to a} f(x) = L$$

Or

$$f(x) \to L \text{ as } x \to a$$

Example 2

Numerical evidence to conjecture about the value of

$$\lim_{x \to 1} \frac{x - 1}{\sqrt{x} - 1}$$

x	$f(x)$
0.9999	1.99995
0.99995	1.99997
1.	Indeterminate
1.00005	2.00002
1.0001	2.00005

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Another numerical

For the value of

$$\lim_{x \to 0} \frac{\sin(x)}{x}$$

x	$f(x)$
0.1	0.998334
0.08	0.998934
0.06	0.9994
0.04	0.999733
0.02	0.999933
0.	Indeterminate
-0.02	0.999933
-0.04	0.999733
-0.06	0.9994
-0.08	0.998934
-0.1	0.998334

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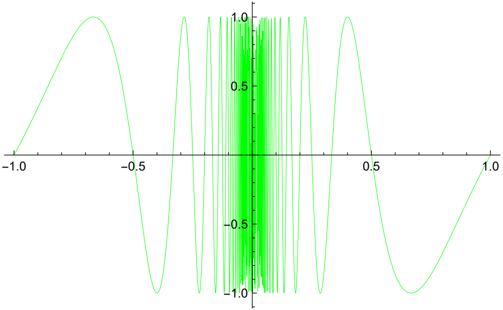
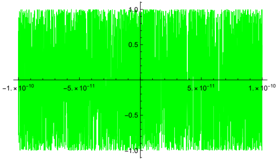
Numerical evidence can't be trusted always

x	$f(x)$	$\sin(\frac{\pi}{x})$
-1	$-\pi$	0
-0.1	-10π	0
-0.01	-100π	0
-0.001	-1000π	0
-0.0001	-10000π	0
0.0001	10000π	0
0.001	1000π	0
0.01	100π	0
0.1	10π	0
1	π	0

what is happening

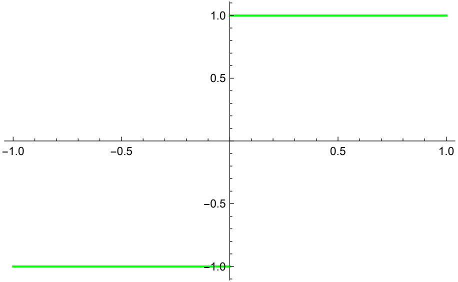




Numerical guesses are not always feasible

One-sided limits

$$f(x) = \frac{|x|}{x} = \begin{cases} 1; & x > 0 \\ -1; & x < 0 \end{cases}$$



$$\lim_{x \to 0^-} \frac{|x|}{x} = -1 \qquad \lim_{x \to 0^+} \frac{|x|}{x} = 1$$

One sided limit

If the values of $f(x)$ can be made as close as we like to L by taking values of x sufficiently close to a (but greater than a), then we write

$$\lim_{x \to a^+} f(x) = L$$

and in case of x sufficiently close to a (but smaller than a) then we write

$$\lim_{x \to a^-} f(x) = L$$

Relationship with two sided limits

The two sided limit of a function $f(x)$ exists at $x = a$ if and only if both of the one-sided limits exists at $x = a$ and have the same value that is

$$\lim_{x \to a} f(x) = L, \text{ iff, } \lim_{x \to a^-} f(x) = L = \lim_{x \to a^+} f(x)$$

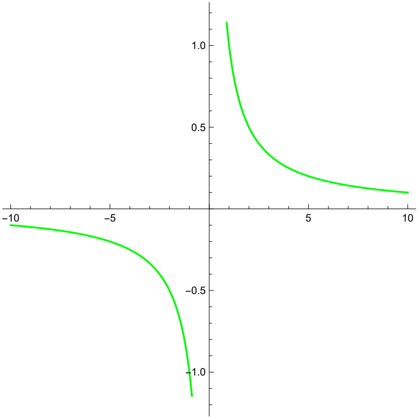
See example 4,5 and 6 in the book

Infinite Limits

Sometimes one-sided or two-sided limits fail to exist because the values of the function increase without bound.

$$\boxed{\lim_{x \to 0^-} \frac{1}{x} = -\infty}$$

$$\boxed{\lim_{x \to 0^+} \frac{1}{x} = +\infty}$$



Informally speaking

One-sided limits

$$\lim_{x \to a^-} f(x) = +\infty \qquad \lim_{x \to a^+} f(x) = +\infty$$

If both of them are true

$$\lim_{x \to a} f(x) = +\infty$$

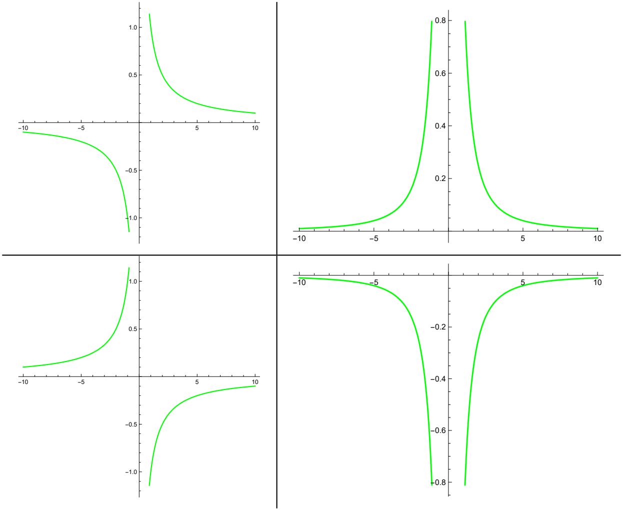
One-sided limits

$$\lim_{x \to a^-} f(x) = -\infty \qquad \lim_{x \to a^+} f(x) = -\infty$$

If both of them are true

$$\lim_{x \to a} f(x) = -\infty$$

Infinite limits and vertical Asymptotes



HW

Exercise 1.1
1-10, 21-30

30. Find equation of tangent line for $y = x^4$ at $(-1, 1)$
Answer:

$$y = -4x - 3$$