

# PRACTICE SHEET

Limit :-

① If  $f(x) = \frac{x^2 - 3x + 2}{x - 2}$  show that the limit of  $f(x)$  at  $x = 2$  exists.

② ~~Find~~ Find

a)  $\lim_{x \rightarrow 3} \frac{x^2 - 6x + 9}{x - 3}$

b)  $\lim_{x \rightarrow 3} \frac{x^3 - 9x^2 + x}{x - 3}$

c)  $\lim_{x \rightarrow 3} \frac{x^2 - 2x}{x + 1}$

d)  $\lim_{x \rightarrow \infty} \frac{3^x - 3^{-x}}{3^x + 3^{-x}}$

e)  $\lim_{x \rightarrow +\infty} (\sqrt{x^2 - 3x} - x)$

③ A real function is defined by  $f(x) = \frac{x}{1-x}$ .

a) Find  $\lim_{x \rightarrow 1^+} f(x)$  and  $\lim_{x \rightarrow 1^-} f(x)$ .

b) Find  $\lim_{x \rightarrow \infty} f(x)$  and  $\lim_{x \rightarrow -\infty} f(x)$ .

④ A function is defined as follows:-

$$f(x) = \begin{cases} x^2 + 1 & \text{when } x > 0, \\ 1 & \text{when } x = 0, \\ 1 + x & \text{when } x < 0. \end{cases}$$

Find the value of  $\lim_{x \rightarrow 0} f(x)$ .

⑤ A function  $f(x)$  is defined as

$$f(x) = \begin{cases} 1, & \text{when } x > 0, \\ 0, & \text{when } x = 0 \\ -1, & \text{when } x < 0. \end{cases}$$

Show that  $\lim_{x \rightarrow 0} f(x)$  does not exist.

⑥ A function  $f(x)$  is defined as follows:-

$$f(x) = \begin{cases} e^{\frac{-|x|}{2}} & \text{when } -1 < x < 0, \\ x^2 & \text{when } 0 \leq x < 2. \end{cases}$$

Discuss the existence of  $\lim_{x \rightarrow 0} f(x)$ .

⑦ A function  $f(x)$  is defined as follows:-

$$f(x) = \begin{cases} 2-3x & \text{when } x < 0 \\ 3x-2 & \text{when } x > 0. \end{cases}$$

Find the value of  $\lim_{x \rightarrow 0^+} f(x)$  and  $\lim_{x \rightarrow 0^-} f(x)$ .

Calculus - 10<sup>th</sup> Edition (Howard Anton)

Exercise 3.6  $\rightarrow$  7-45 (Odd Numbers)

⑨ Find a)  $\lim_{x \rightarrow +\infty} (1 + 2x - 3x^5)$ .

b)  $\lim_{x \rightarrow +\infty} \frac{5x^2 - 4x}{2x^2 + 3}$ .

c)  $\lim_{t \rightarrow +\infty} \frac{6 - t^3}{7t^3 + 3}$ .

d)  $\lim_{x \rightarrow -\infty} \frac{\sqrt{5x^2 - 2}}{x + 3}$ .

e)  $\lim_{x \rightarrow +\infty} (\sqrt{x^2 - 3x} - x)$ .

f)  $\lim_{x \rightarrow +\infty} (\sqrt{x^2 + 3} - x)$ .

## Continuity :-

① Determine whether the following functions are continuous at  $x=2$ .

$$a) f(x) = \frac{x^2-4}{x-2}$$

$$b) g(x) = \begin{cases} \frac{x^2-4}{x-2} & , x \neq 2 \\ 3 & x = 2 \end{cases}$$

$$c) h(x) = \begin{cases} \frac{x^2-4}{x-2} & ; x \neq 2 \\ 4 & ; x = 2 \end{cases}$$

$$d) g(x) = \begin{cases} 5-x & , -1 \leq x \leq 2 \\ x^2-1 & , 2 < x < 3 \end{cases}$$

$$e) f(x) = \begin{cases} x+2 & , x < 2 \\ x^2-1 & , x \geq 2 \end{cases}$$



⑤ Find values of  $x$ , if any, at which  $f$  is not continuous ( $x$  is only real value).

a)  $f(x) = 5x^4 - 3x + 7$ .

b)  $f(x) = \sqrt[3]{x-8}$

c)  $f(x) = \frac{x+2}{x^2-4}$

d)  $f(x) = \frac{3}{x} + \frac{x-1}{x^2-1}$ .

④ Find a value of the constant  $k$ , if possible, that will make the function continuous everywhere.

$$a) \quad f(x) = \begin{cases} 7x-2, & x \leq 1. \\ kx^2 & x > 1. \end{cases}$$