Higher Order Derivatives :-

The derivative f' of a function f itself a function and hence may have a derivative of its own. If f' is differentiable then its derivative is denoted by f'' and is called the second derivative of f. As long as we have differentiability we can continue the process of ~~ob~~ differentiating to obtain third, fourth, fifth and even higher derivatives of f. These are called higher derivatives or successive derivatives.

$$f'(x) = 12x^3 - 6x^2 + 2x - 4$$

$$f''(x) = 36x^2 - 12x + 2$$

$$f'''(x) = 72x - 12$$

$$f^{(4)}(x) = 72$$

$$f^{(5)}(x) = 0 \quad (n \geq 5).$$

The Chain Rule :-

If g is differentiable at x and f is differentiable at $g(x)$ then the composition $f \circ g$ is differentiable at x. Therefore,

$$\frac{d}{dx}(f \circ g(x)) = \frac{d}{dx}\left[f(g(x))\right]$$

$$= f'(g(x))\, g'(x)$$

Example :- Evaluate $\frac{dy}{dx}$ of $y = \tan^2 x$.

Solution :- Given that,

$$y = \tan^2 x \qquad\qquad f(x) = x^2$$

$$= \left[\tan x\right]^2 \qquad\qquad g(x) = \tan x$$

Applying chain rule.

$$\frac{d}{dx}\left[f(g(x))\right] = \cancel{f(x)}\ f(g(x))\, g'(x)$$

$$\frac{d}{dx}\left[\tan^2 x\right] = 2\tan x\ \sec^2 x$$

Logarithmic Differentiation :-

$$\frac{d}{dx}\left[\ln x\right] = \frac{1}{x}$$

$$\frac{d}{dx}\left[\ln(x^2+1)\right] = \frac{2x}{x^2+1}$$

Example :- Evaluate the derivative of

$$y = \frac{x^2\sqrt[3]{7x-14}}{(1+x^2)^4}$$

Solution :- Given that

$$y = \frac{x^2\sqrt[3]{7x-14}}{(1+x^2)^4}$$

$$\Rightarrow \ln y = \ln\left[\frac{x^2\sqrt[3]{7x-14}}{(1+x^2)^4}\right]$$

$$\Rightarrow \ln y = \ln\left[x^2\sqrt[3]{7x-14}\right] - \ln(1+x^2)^4$$

$$\Rightarrow \ln y = \ln(x^2) + \ln\left(\sqrt[3]{7x-14}\right) - 4\ln(1+x^2)$$

$$\Rightarrow \ln y = 2\ln x + \frac{1}{3}\ln(7x-14) - 4\ln(1+x^2)$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{2}{x} + \frac{7}{3(7x-14)} - \frac{4\ 2x}{1+x^2}$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{2}{x} + \frac{1}{3x-6} - \frac{8x}{1+x^2}$$

$$\Rightarrow \frac{dy}{dx} = y\left[\frac{2}{x} + \frac{1}{3x-6} - \frac{8x}{1+x^2}\right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{x^2\sqrt[3]{7x-14}}{(1+x^2)^4}\left[\frac{2}{x} + \frac{1}{3x-6} - \frac{8x}{1+x^2}\right]$$

Ans.

Explicit Function:- An equation of the form $y = f(x)$ is said to define explicitly y as a function of x because the variable y appears alone on one side of the equation and does not appear at all on the other side.

$$y = \frac{x-1}{x+1}.$$

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Implicit function :- In general, it is not necessary to solve an equation for y in terms of x in order to differentiate the functions defined implicitly by the equation.

$$5y^2 + \sin y = x^2 .$$

Implicit Differentiation :-

$$5y^2 + \sin y = x^2 .$$

$$\Rightarrow \frac{d}{dx}\left(5y^2 + \sin y\right) = \frac{d}{dx}\left(x^2\right)$$

$$\Rightarrow 10y\frac{dy}{dx} + \cos y\frac{dy}{dx} = 2x$$

$$\Rightarrow \frac{dy}{dx}\left(10y + \cos y\right) = 2x$$

$$\Rightarrow \frac{dy}{dx} = \frac{2x}{10y + \cos y}$$

Homework :-

1. $\cos(xy) = e^{x+y}$
2. $xy + e^y = 0$.

Leibnitz Theorem :- Leibnitz rule is a generalization of the product rule. It states that if the functions $f(x)$ and $g(x)$ are differentiable n times then their product $f(x).g(x)$ is also differentiable n times.

$$\frac{d^n}{dx^n}(fg) = \frac{d^n f}{dx^n} g + {}^nC_1 \frac{d^{n-1}f}{dx^{n-1}} \frac{dg}{dx} + {}^nC_2 \frac{d^{n-2}f}{dx^{n-2}} \frac{d^2g}{dx^2}$$

$$+ \cdots + {}^nC_k \frac{d^{n-k}f}{dx^{n-k}} \frac{d^kg}{dx^k} + \cdots + f \frac{d^ng}{dx^n}$$

$$= \sum_{k=0}^{n} {}^nC_k \frac{d^{n-k}f}{dx^{n-k}} \frac{d^kg}{dx^k}.$$

Example :- Evaluate $\frac{d^n}{dx^n}(x^2 e^{ax})$ using Leibnitz theorem.

Solution :- Let,

$$f(x) = e^{ax}, \quad g(x) = x^2.$$

$$\therefore \frac{d^n}{dx^n}(x^2 e^{ax}) = \frac{d^n}{dx^n}(e^{ax}).\, x^2 + {}^nC_1 \frac{d^{n-1}}{dx^{n-1}}(e^{ax}) \frac{d}{dx}(x^2)$$

$$+ {}^nC_2 \frac{d^{n-2}}{dx^{n-2}}(e^{ax}) \frac{d^2}{dx^2}(x^2) + 0.$$

$$= a^n e^{ax}.\, x^2 + n\, a^{n-1} e^{ax}.\, 2x + \frac{n(n-1)}{2} a^{n-2}.\, 2 e^{ax}$$

$= a^n e^{ax} x^2 + 2xna^{n-1} e^{ax} + n(n-1)a^{n-2} e^{ax}$

Ans.

Homework :- If $y = x^{n-1} e^{1/x}$ prove that

$$y_n = (-1)^n \frac{e^{1/x}}{x^{n+1}}.$$

Practice Sheet

Exercise 2.4 – (5-20), 21, 24

Exercise 2.6 – Example → 5, 6,
Ex – 3, 4, 7-40, 43-54.

Exercise 2.3 → 29, 30, 31, 32, 41-48