Practice Sheet # 6

Successive Differentiation

a. Find the *n* th derivative of the following functions:

1. $y = x^n$
2. $y = (ax+b)^n$
3. $y = \ln(ax+b)$
4. $y = \dfrac{1}{x+a}$
5. $y = e^{ax}$
6. $y = \sin(ax+b)$
7. $y = \cos(ax+b)$

b. If $y = e^{ax}\sin bx,$ then show that $y_2 - 2ay_1 + (a^2+b^2)y = 0.$

c. If $y = e^{x}\sin x,$ then show that $y_4 + 4y = 0.$

Leibnitz's Theorem

1. If $y = \tan^{-1} x$, then show that $(1+x^2)y_{n+2} + 2(n+1)xy_{n+1} + n(n+1)y_n = 0.$
2. If $y = \cot^{-1} x,$ then show that $(1+x^2)y_{n+2} + 2(n+1)xy_{n+1} + n(n+1)y_n = 0.$
3. If $y\sqrt{1-x^2} = \sin^{-1} x$, then show that $(1-x^2)y_{n+1} - (2n+1)xy_n - n^2 y_{n-1} = 0.$
4. If $y = e^{\tan^{-1} x}$, then show that $(1+x^2)y_{n+2} + (2nx+2x-1)y_{n+1} + n(n+1)y_n = 0.$
5. If $y = e^{m\sin^{-1} x}$, then show that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+m^2)y_n = 0.$
6. If $y = (\sin^{-1} x)^2$, then show that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2 y_n = 0.$
7. If $\log_e y = a\sin^{-1} x$ then show that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+a^2)y_n = 0.$
8. If $y = e^{m\cos^{-1} x}$ then show that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+m^2)y_n = 0.$
9. If $\log_e y = \tan^{-1} x$ then show that $(1+x^2)y_{n+2} + (2nx+2x-1)y_{n+1} + n(n+1)y_n = 0.$
10. If $y = \left(\cos^{-1} x\right)^2$, then show that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2 y_n = 0.$
11. If $\ln y = m\cos^{-1} x,$ then show that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+m^2)y_n = 0.$
12. If $x = \tan(\ln y),$ then show that $(1+x^2)y_{n+2} + (2nx+2x-1)y_{n+1} + n(n+1)y_n = 0.$