Guide to the paper "*Best Estimates for Reserves*"

http://www.casact.org/dare/index.cfm?fuseaction=view&abstrID=5144

Overview

- **Introduction**
- **Guide to the Extended Link Ratio Family (ELRF) modelling framework**
  - Mathematical framework
  - Diagnostic tools for evaluating a method's fit
- **Guide to the Probabilistic Trend Family (PTF) modelling framework**
  - Mathematical framework
  - How models are built
  - Diagnostics for assessing a model's fit: does the model work?

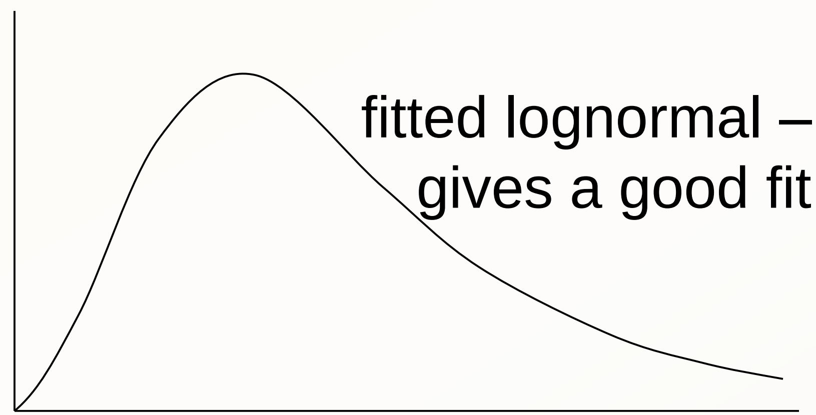
Introduction

What does it mean to say a model gives a good fit?

e.g. lognormal fit to claim size distribution

The fitted distribution is regard as the probabilistic mechanisms for creating the data as a sample. In other words, you can regard x_1 to x_n as a sample from the fitted model.

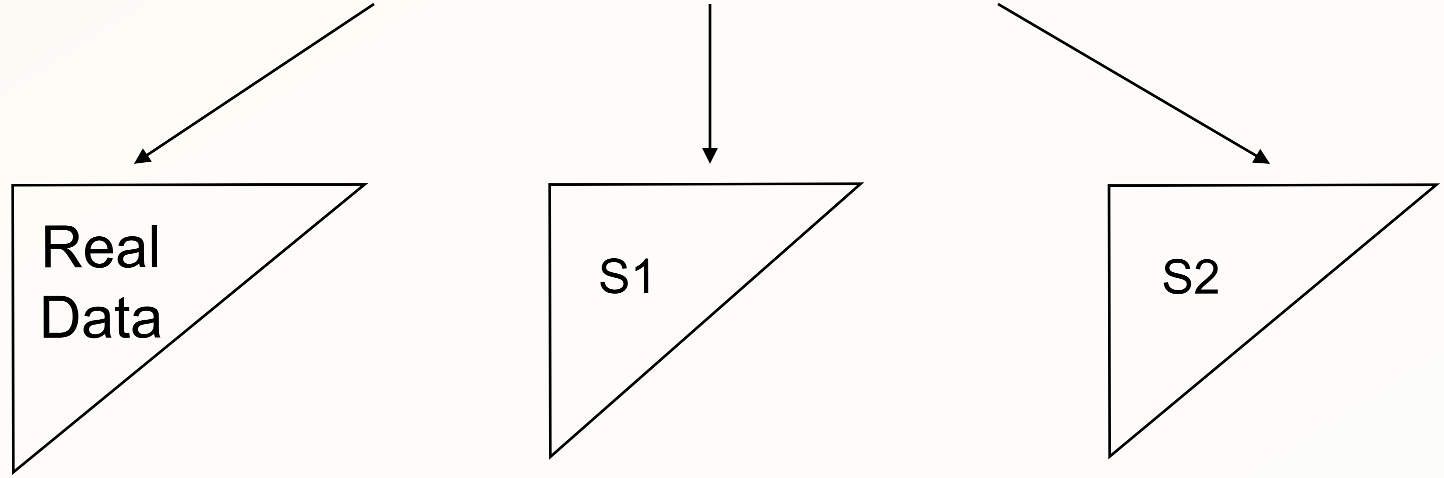
- Real Sample: $x_1,\ldots,x_n$
- Fitted Distribution
- Random Sample from fitted distribution: $y_1,\ldots,y_n$



y's look like x's: — If we simulate y_1 to y_n from the fitted distribution (to the x's) then the y values 'look like' the x's.

Introduction

PTF model fitted to the real data describes the trend structure in the three directions and the quality of the process variability around the trend structure.



The fitted model fits a distribution to every cell. The triangle is regarded as a sample path from the fitted model. If we simulate other triangles (S) from the fitted model they are indistinguishable from the real triangle. They have similar trends, trend changes in same periods, same amount of random variation about trends.

Extended Link Ratio Family (ELRF) Modelling Framework

Link ratio techniques can be formulated as regressions: start with the cumulative loss development array

0	1	2	3	4	5	6	7	8	9
5,012	8,269	10,907	11,805	13,539	16,181	18,009	18,608	18,662	18,834
106	4,285	5,396	10,666	13,782	15,599	15,599	16,272	16,807	
3,410	8,992	13,873	16,141	18,735	22,214	22,863	23,466		
5,655	11,555	15,766	21,266	23,425	26,083	27,067			
1,092	9,565	15,836	22,169	25,955	26,180				
1,513	6,445	11,702	12,935	15,852					
557	4,020	10,946	12,314						
1,351	6,947	13,112							
3,133	5,395								
2,063									

8,269 / 5,012 = 1.65

8,992 / 3,410 = 2.64

- These data are analysed by Mack (1993)

Incurred Losses, Historical Loss Development Study, 1991 Ed. (RAA)

ELRF modelling framework

We can **graph** one development period versus the previous development period:

5,012	8,269	10,907	11,805	13,539	16,181	18,009	18,608	18,662	18,834
106	4,285	5,396	10,666	13,782	15,599	15,599	16,272	16,807	
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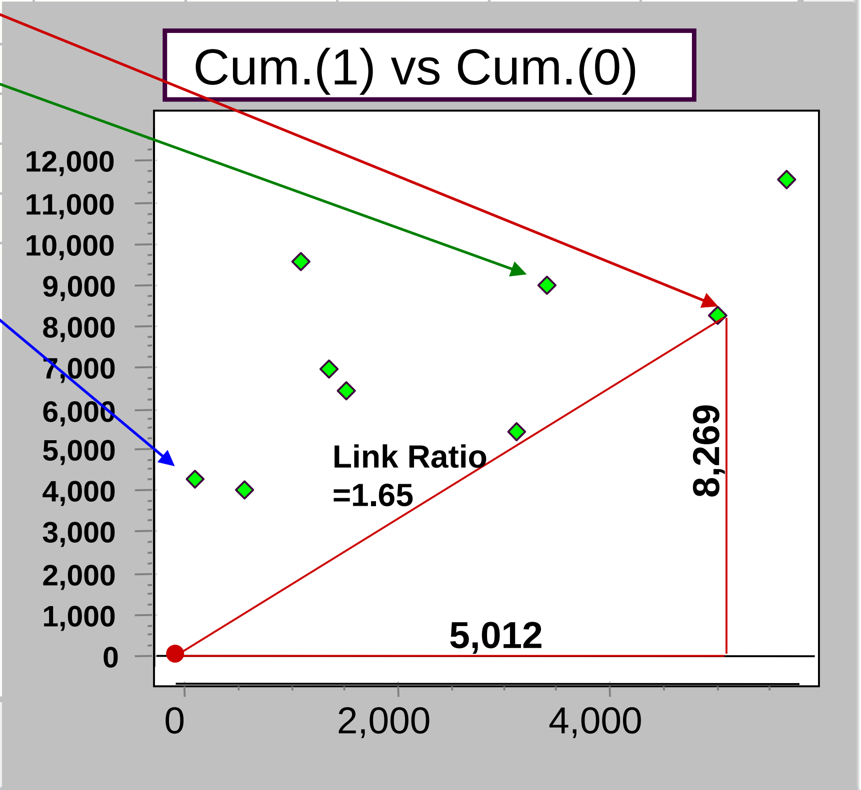
557	4,020	10,946	12,314						
1,351	6,947	13,112							
3,133	5,395								
2,063									

	0~1	1~2
1981	1.64984	1.31902
1982	40.42453	1.25928
1983	2.63695	1.54282
1984	2.04332	1.36443
1985	8.75916	1.65562
1986	4.25975	1.81567
1987	7.21724	2.72289
1988	5.14212	1.88743
1989	1.72199	

Excerpt of link ratio table for Mack method (volume weighted average).



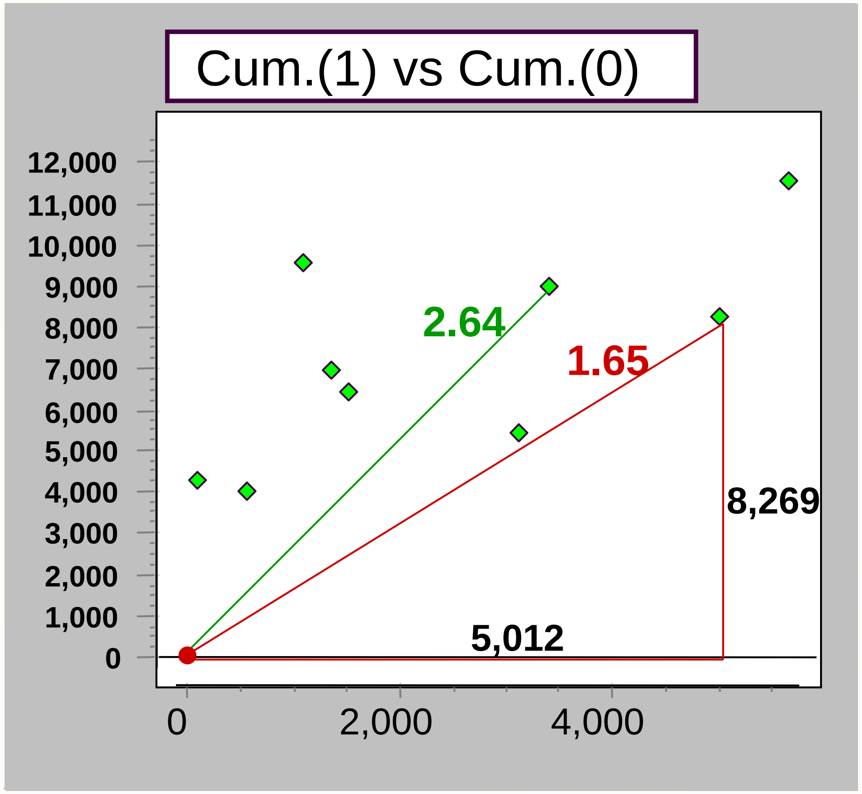
ELRF modelling framework

What is the SLOPE of this line?

Slope = rise/run (*the* ratio)

= 8,269/5,012

= **1.65**



Any link ratio is the slope of a line (equivalently a trend) through the <u>origin</u>.

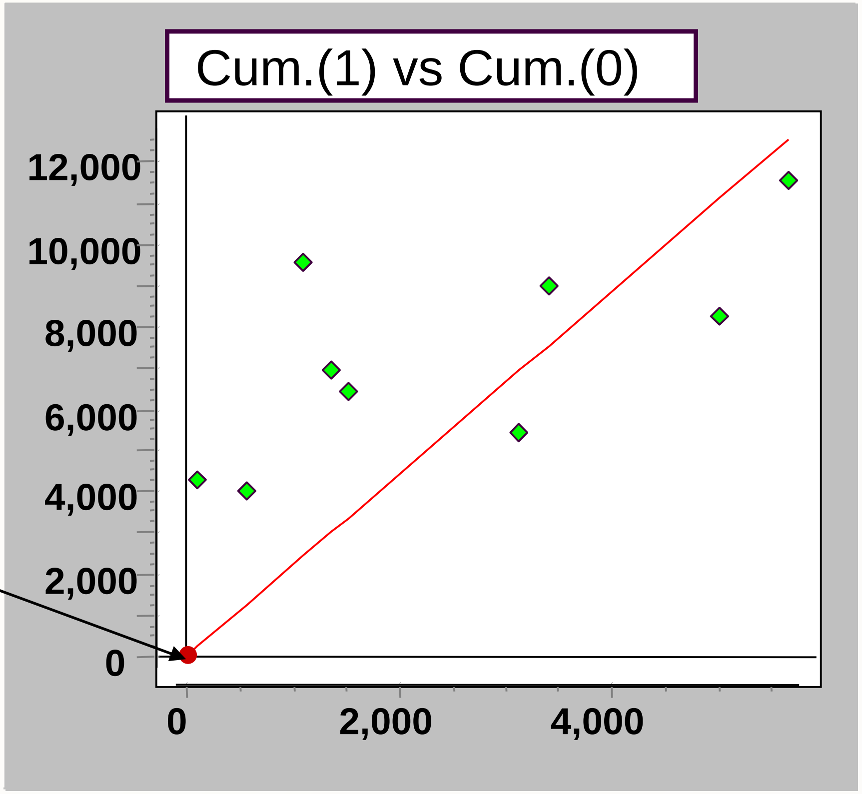
ELRF modelling framework

Select link ratios to be typical ("average") link ratios

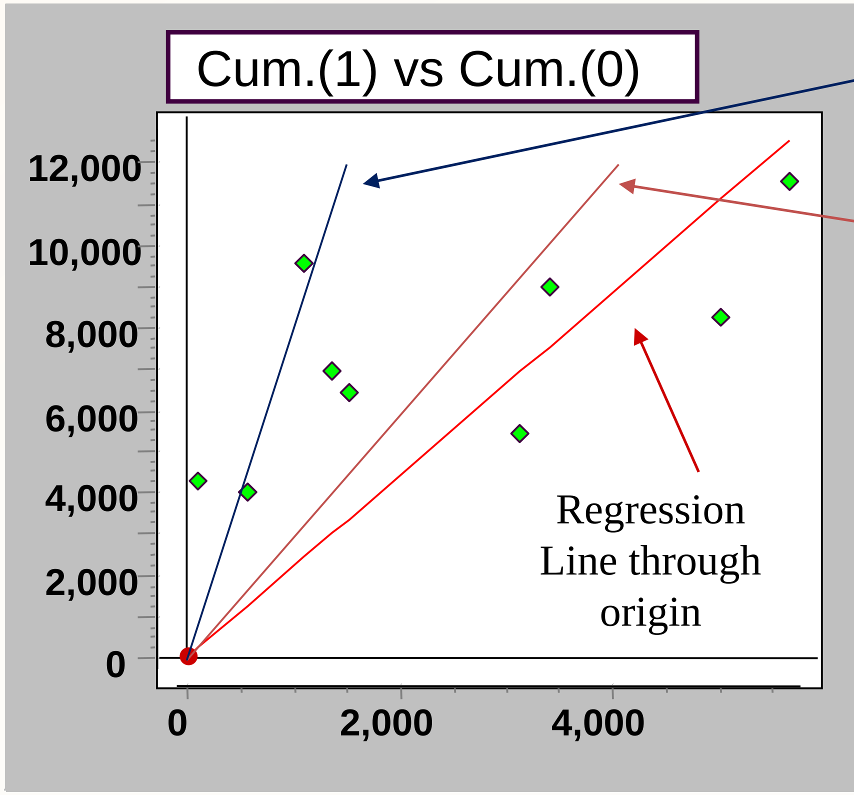
e.g. arithmetic average, Mack method (chain ladder or volume weighted average),

average of last three years, ...

A weighted average ratio is the same as a weighted average trend through the origin.



ELRF modelling framework



Arithmetic
Average ratio
$^{1}/_{n}\ \sum(y/x)$

Mack method ratio =

$\sum {}^{y}/x \cdot x/\sum x = \sum y/\sum x$

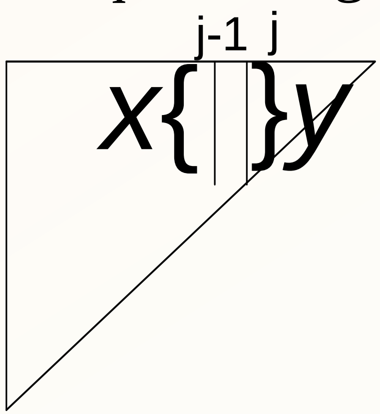
(Volume weighted average, Chain ladder)

BUT a weighted average trend line through the origin is a ***regression*** line through the origin!

Weighted average link ratio techniques are forms of regressions through the origin.

ELRF modelling framework

If link ratios are regressions, we can describe link ratio techniques using formal statistical language.



Regression equation:

$$y = bx + \varepsilon,$$

where $E(\varepsilon) = 0$

and $\mathrm{Var}(\varepsilon) = \sigma^2 x^\delta$

Equivalently

$$E(y \mid x) = bx \quad \text{and} \quad \mathrm{Var}(y \mid x) = \sigma^2 x^\delta$$

$\delta = 0$ corresponds to volume squared weighted average

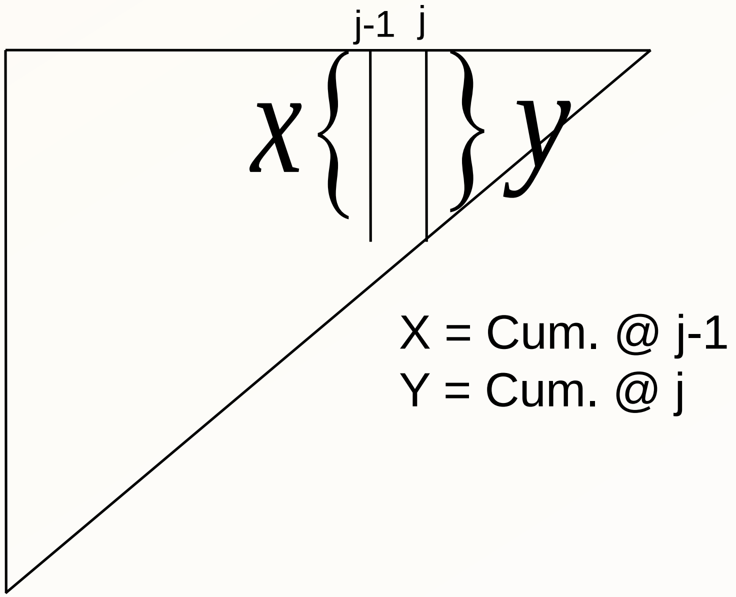
$\delta = 1$ is the Mack method - equivalent to volume weighted average (Chain Ladder - Mack 1993)

$\delta = 2$ is the arithmetic average

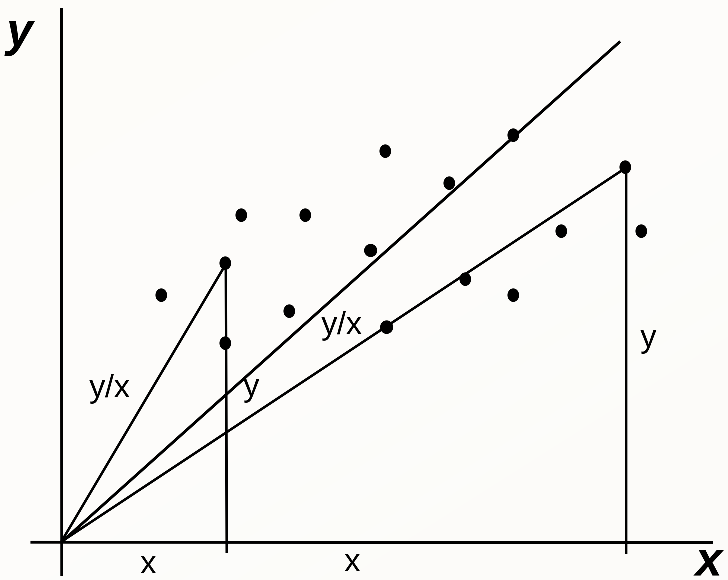
ELRF modelling framework

x is cumulative at dev. j-1 and y is cumulative at dev. j

Link Ratios are a comparison of columns



We can graph the link ratios of Y to X



ELRF modelling framework

To determine an average link ratio (slope) *b* we use **weighted least squares**. Method aims to estimate *b* by minimising

$$\Sigma\, w(y-bx)^2,$$

where weight $w = {}^1/\, x^{\delta} \;\propto\; {}^1/\,\mathrm{Var}(\varepsilon)$

For $\delta = 1$ we obtain

$$\hat{b} = \frac{\Sigma\, x\, y \cdot {}^1/x}{\Sigma\, x^2 \cdot {}^1/x} = \frac{\Sigma\, {}^y/x \cdot x}{\Sigma\, x} = \frac{\Sigma\, y}{\Sigma\, x}$$

Mack (1993)

which is the **Chain Ladder Ratio** (or Volume Weighted Average, or Volume Weighted Ratio); commonly referred to as the ‘Mack method’ in its regression form. There are no differences between the techniques, they are **all** equivalent. The Mack method is just a regression formulation of the chain ladder link ratios.

ELRF modelling framework

$$y = bx + \varepsilon \quad : \quad V(\varepsilon) = \sigma^2 x^\delta$$

$$\text{Minimize} \quad \Sigma w(y - bx)^2$$

$$\text{where} \quad w = 1/x^\delta$$

For other values of δ (0 and 2) we obtain:

$$2. \ \delta = 2, \qquad \hat{b} = \frac{1}{n}\sum \frac{y}{x}$$ Arithmetic Average

$$3. \ \delta = 0, \qquad \hat{b} = \frac{\sum x^2 y/x}{\sum x^2}$$ Weighted average (weighted by volume squared)

What if, for the graph of y versus x, the best line does not go through the origin?
Solution proposed by Murphy (1994) was to add an intercept to the regression equation.

Assessing link ratios in the ELRF modelling framework

What are the major **assumptions** made by the models based on link ratio techniques?

1 $E(\,y \mid x\,) = bx$; and $x \propto \mathrm{Var}(x)$

– i.e. to obtain the mean cumulative at development period *j*, take the cumulative at the previous period and multiply by the link ratio, and as x increases, the variance of x increases.

2 No trend in the calendar period (diagonal) direction

– Link ratio techniques do not measure calendar trends. Changing calendar trends can be seen in the residuals but they cannot be measured.

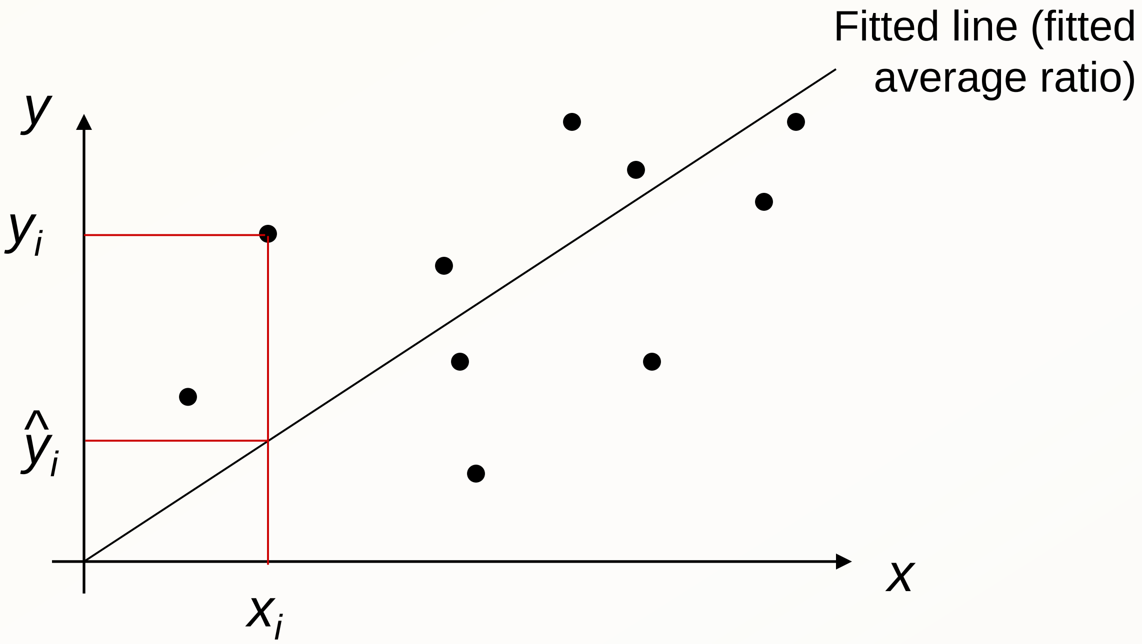
3 Normality

– If error structure is not normal, then the estimates of the ratios are no longer efficient.

Assessing link ratios in the ELRF modelling framework

Fitted values

- value given by the model (the value on the line) called the fitted (or predicted) value, $\hat{y}$

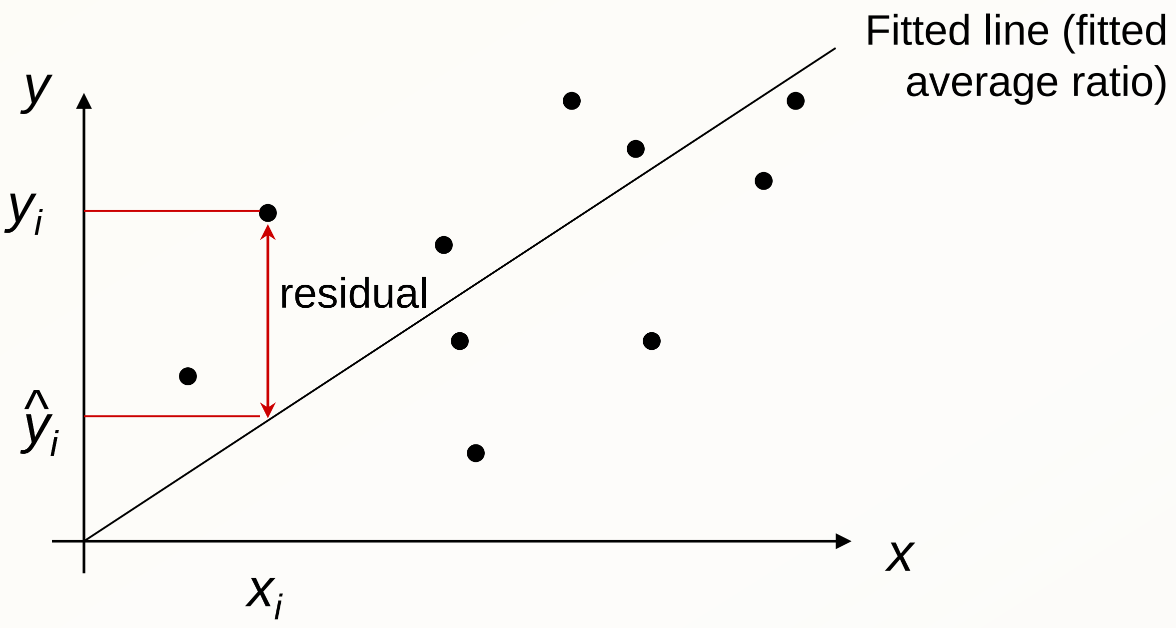


Assessing link ratios in the ELRF modelling framework

Residual = Observed value - Fitted value

That is, Residuals are best interpreted as:

Trend(s) in the data minus Trend(s) estimated by the method



Assessing link ratios in the ELRF modelling framework

Residual Analysis

Residual = Data – Fit

Raw residuals have different standard deviations

- need to adjust to make them comparable

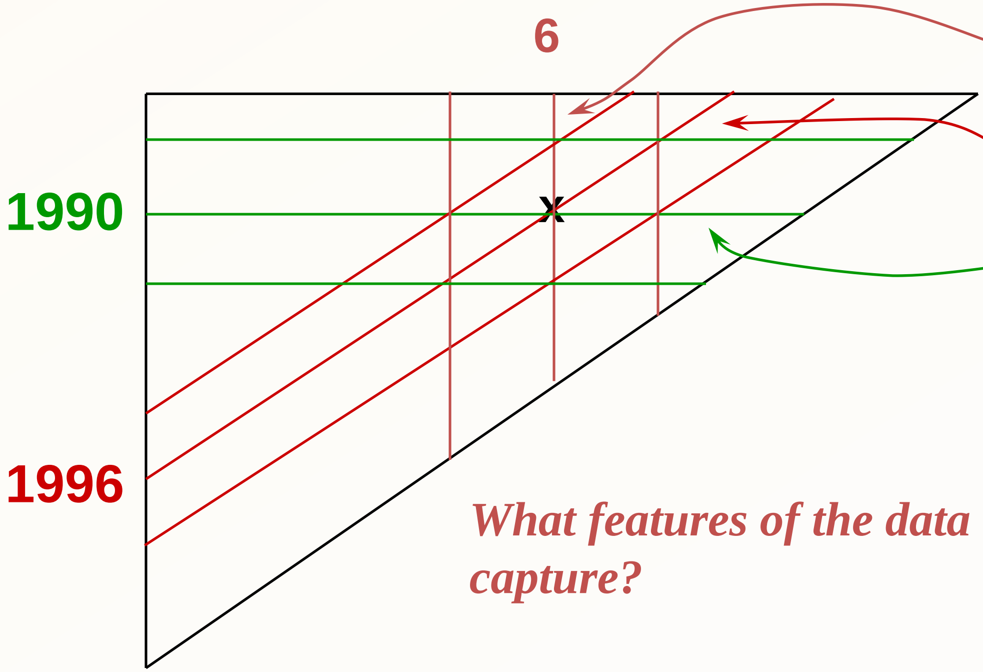
Many model checks use *standardized residuals*

$$\text{Standardized residual} = \frac{\text{Residual}}{\text{Std. dev.(residual)}}$$

Assessing link ratios in the ELRF modelling framework

What can we do with the residuals?

e.g.:



Plot vs dev. yr

Plot vs cal. yr

Plot vs acci. yr

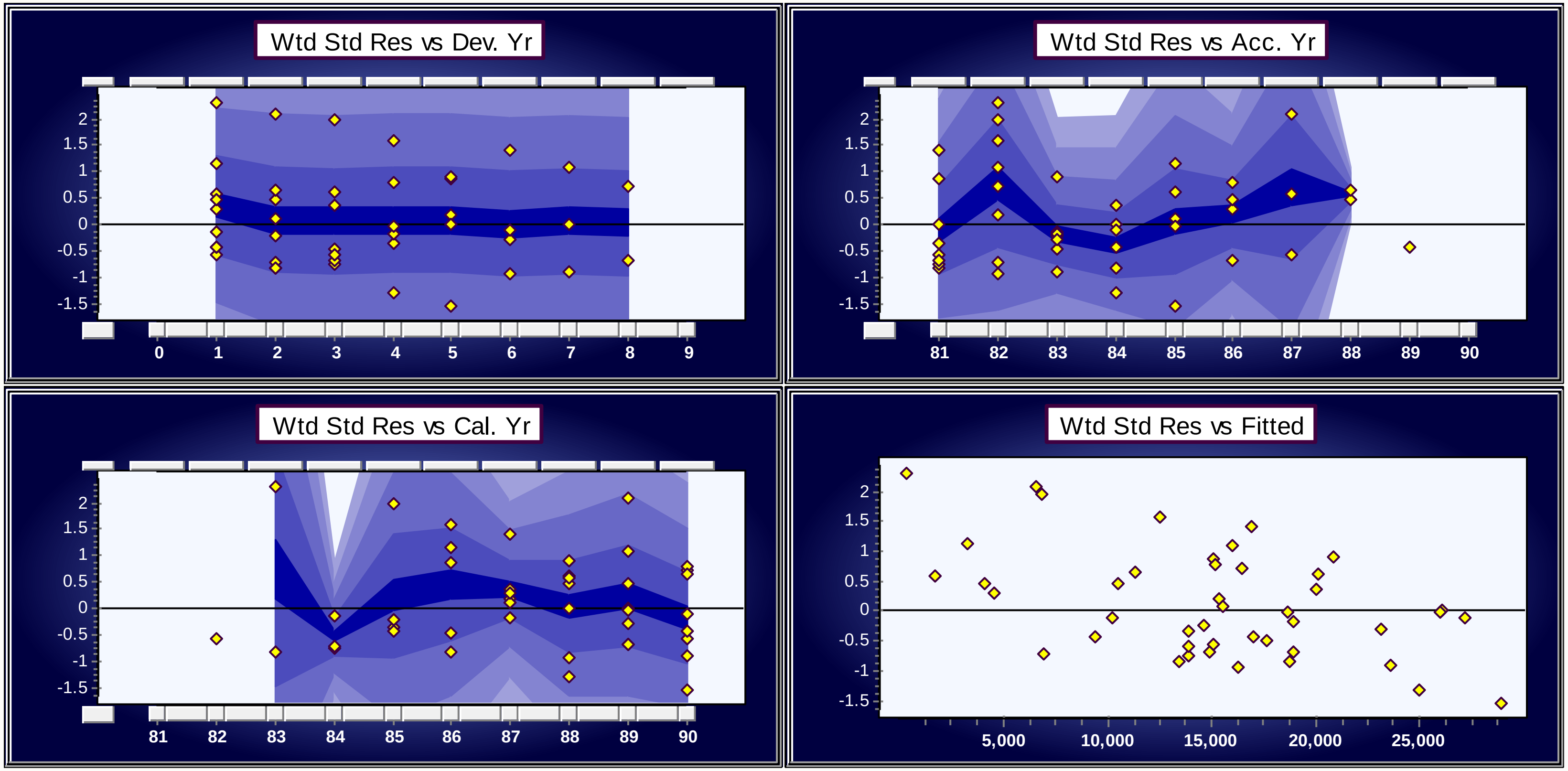
Plot vs fitted value

What features of the data does this model not capture?

Residual plots should appear **random** about 0**,** *without pattern*. The first requirement is necessary for a model to be good, but it is not sufficient.

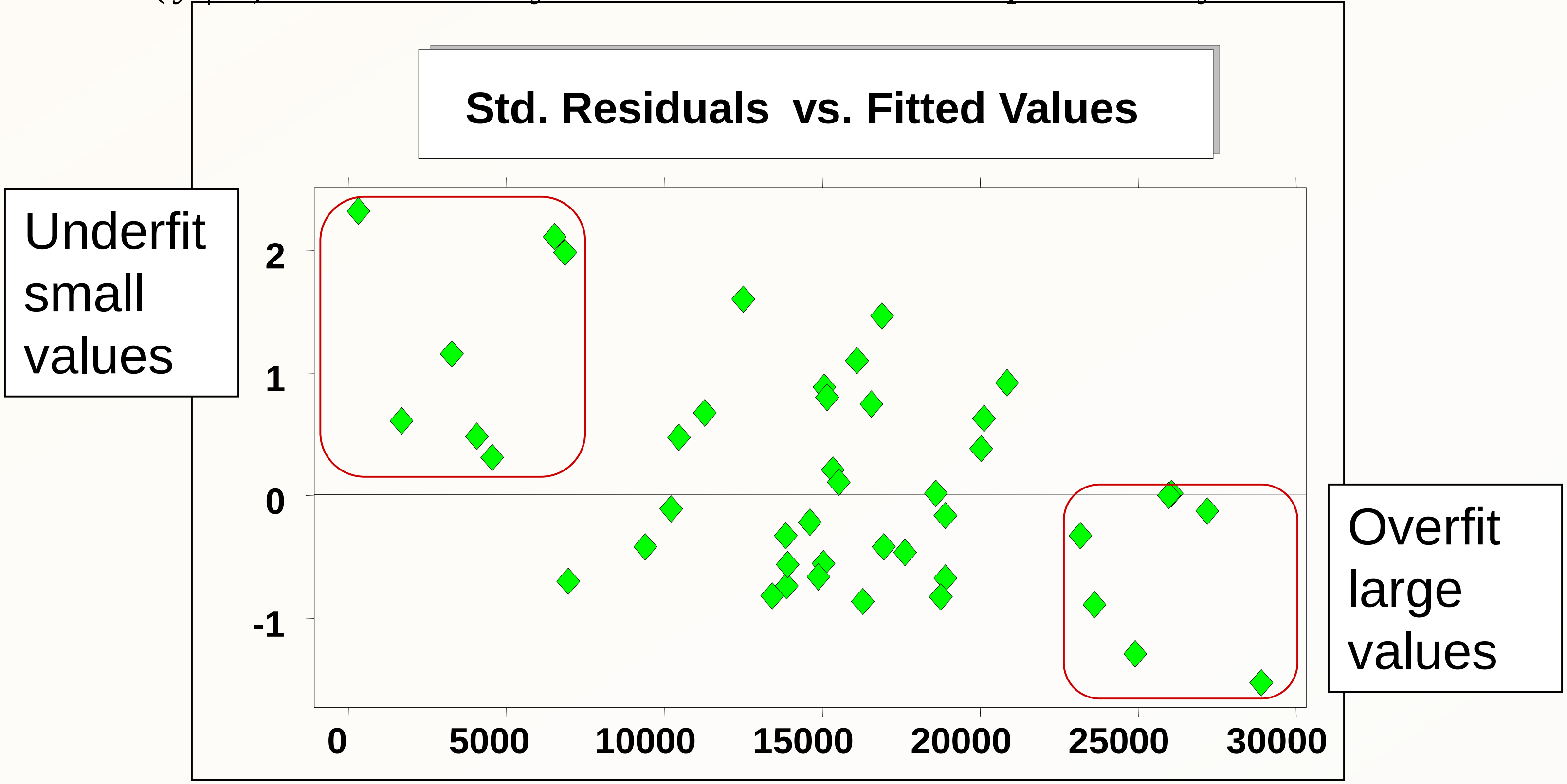
Complete residuals Historical Loss Development Study

- The Mack method is applied to the data as analysed by Mack (1993) and the weighted standardised residuals are plotted against the three directions and against the fitted values. The downward trend in the residuals versus the fitted values indicates the need for an intercept.



Assessing link ratios in the ELRF modelling framework

Is E(*y* | *x*) = *bx* satisfied by the Historical Loss Development Study data?

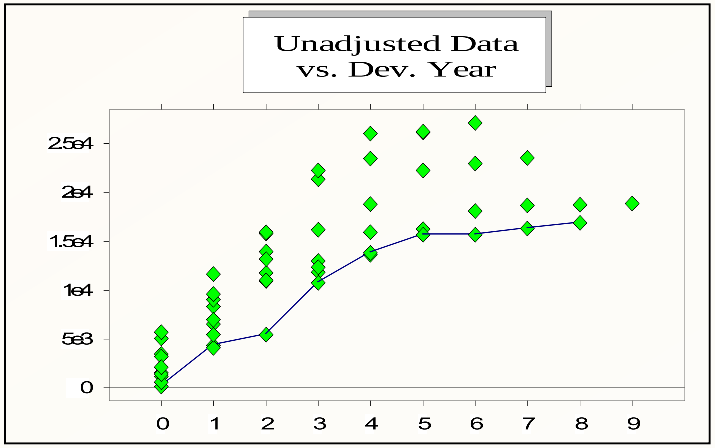


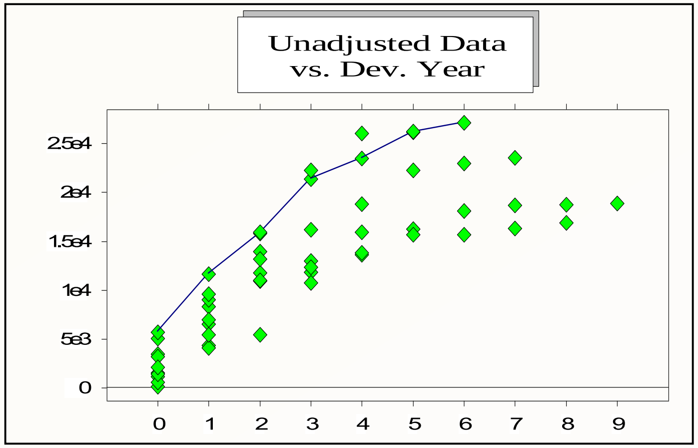
The method seems to overfit the big values and underfit the small values. Does this mean we need an intercept?

Assessing link ratios in the ELRF modelling framework

1982: low incurred development

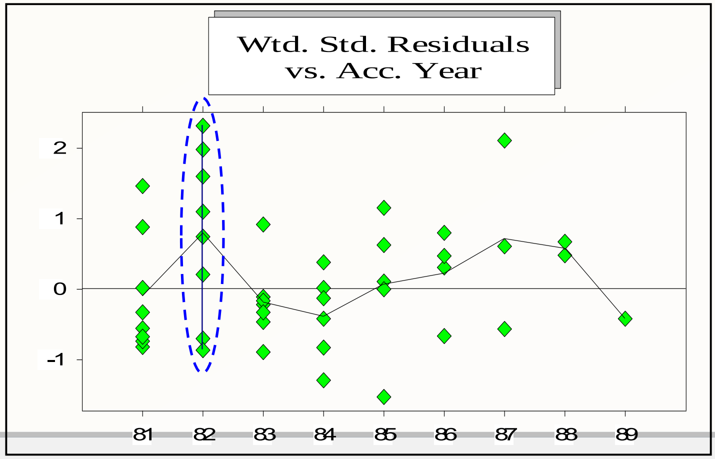
1984: high incurred development

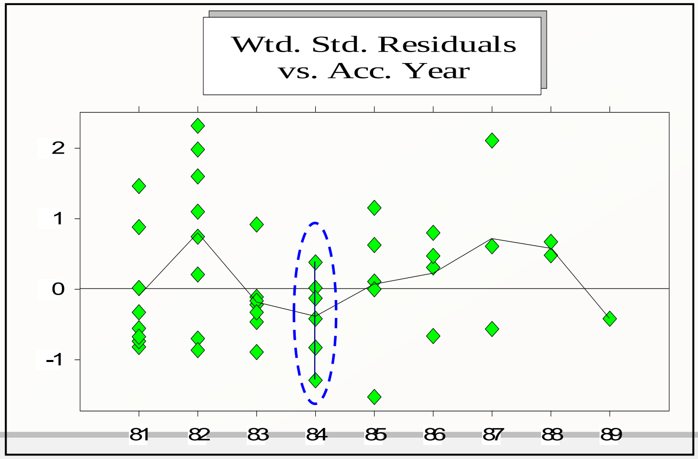




1982 is underfitted

1984 is overfitted

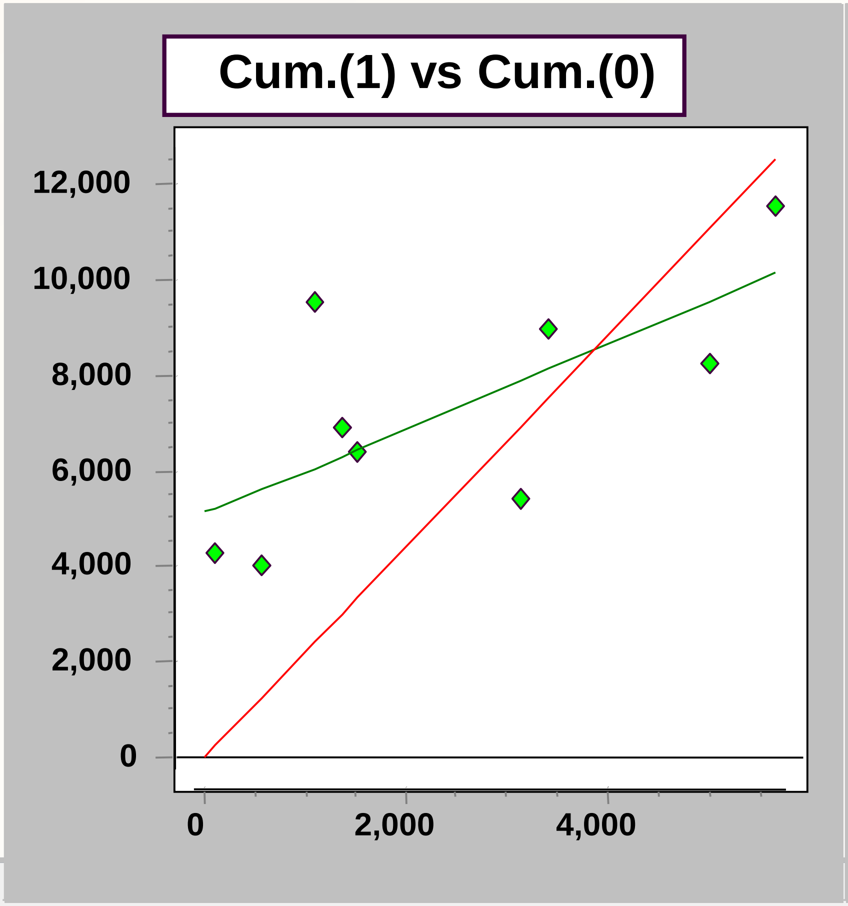


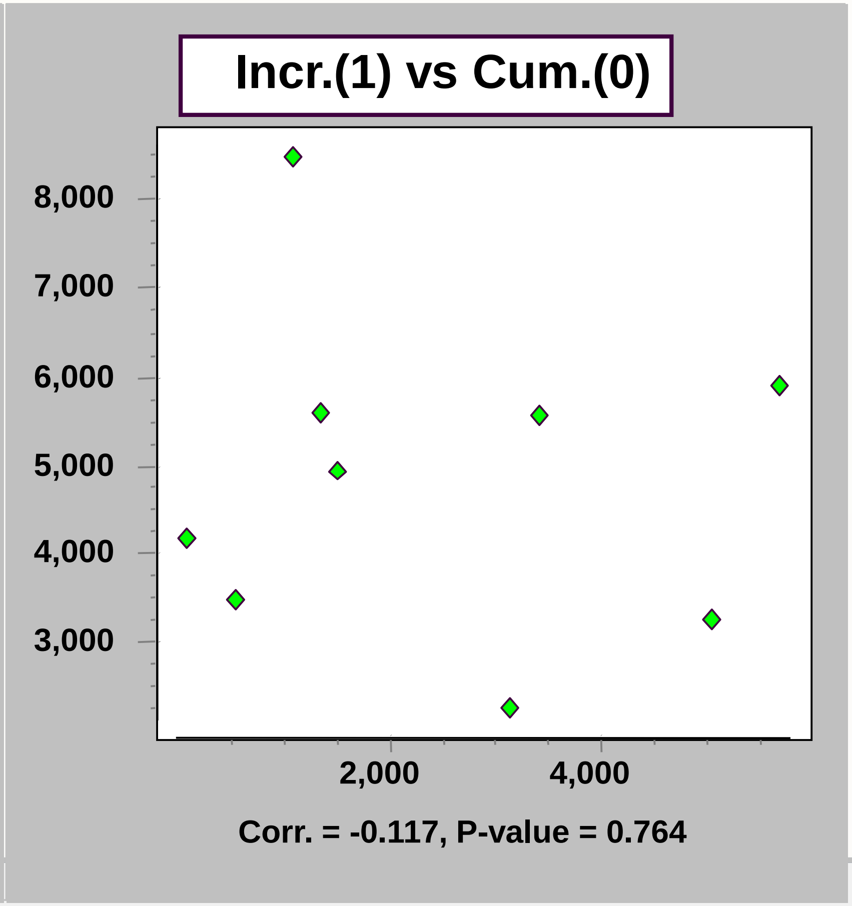


Why is E(y|x) = bx not satisfied by the data?

ELRF modelling framework

- Example where best line to fit the points does not go through the origin.
- The residuals indicated an intercept and you can now see it in the graph of the data Cum(1) vs Cum(0); this is also true for other consecutive periods.





ELRF modelling framework

Intercept (Murphy (1994))

$$y = a + bx + \varepsilon \quad : \quad V(\varepsilon) = \sigma^2 x^\delta$$

Equivalently

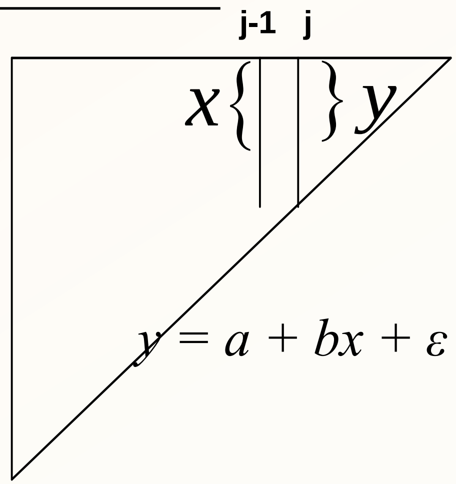
$$p = \underset{\substack{\uparrow\\ \text{Incremental at } j}}{y} - \underset{\substack{\uparrow\\ \text{Cumulative at } j-1}}{x} = a + (b-1)x + \varepsilon \quad : \quad V(\varepsilon) = \sigma^2 x^\delta$$

Is b -1 significant ? Venter (1998)

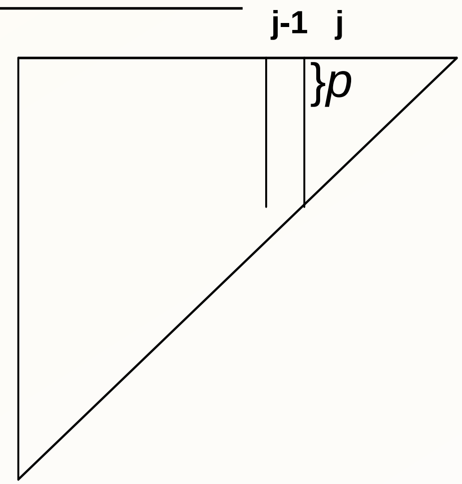
The second formulation of the equation as discussed by Venter, is more useful. We *know* the current cumulative – we are forecasting the incremental conditional on the current cumulative.

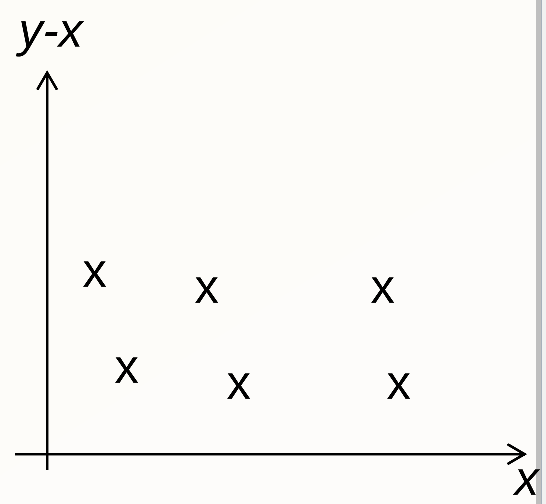
ELRF modelling framework

Cumulative



Incremental





$$p = a + (b-1)x + \varepsilon \quad : \quad V(\varepsilon) = \sigma^2 x^\delta$$

Case (i) $b > 1 \quad a = 0$

Use link ratios for projection

Case (ii) $b = 1 \quad a \neq 0$

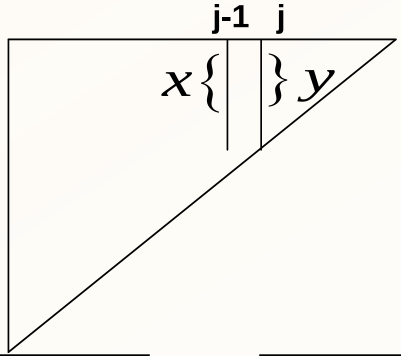
$$\hat{a} = \text{Ave}(\text{Incrementals})$$

Abandon link ratios - No predictive power

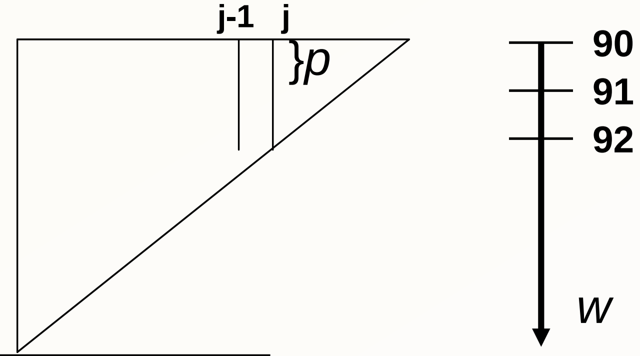
If b = 1 then the incrementals at development year j are not correlated with the cumulatives at development year j – 1. => The link ratio does not have any predictive power.

ELRF modelling framework

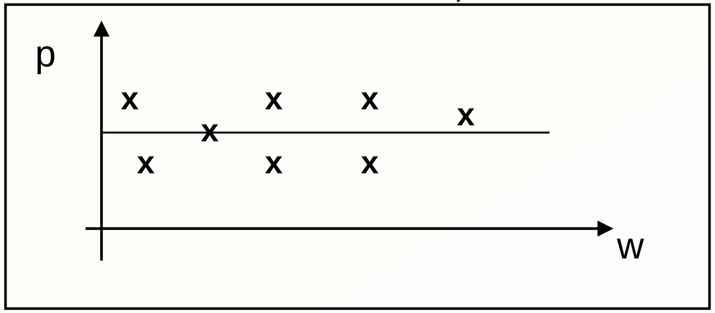
Cumulative



Incremental



Condition 1:



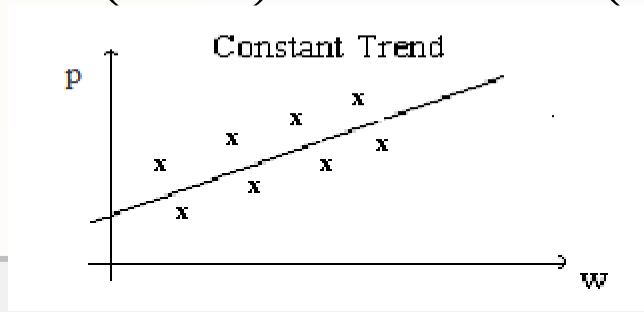
If condition 1 is satisfied, then we assert that

b -1 = 0;

incrementals are not correlated.

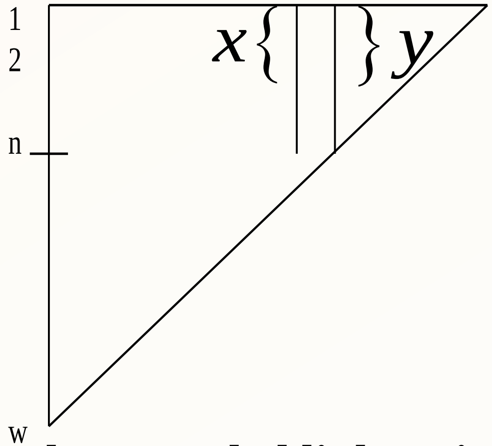
$$p = a + (b-1)x + \varepsilon \quad : \quad V(\varepsilon) = \sigma^2 x^\delta$$

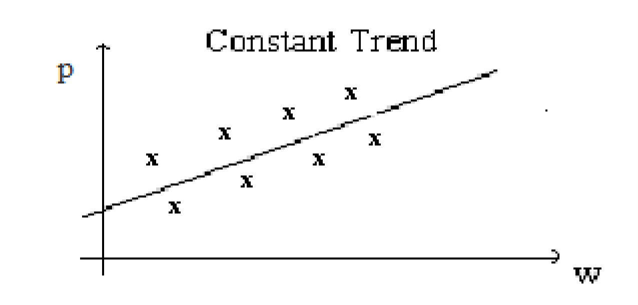
Condition 2:



ELRF modelling framework

- Now introduce trend parameter for incrementals. Most often, the trend has more predictive power than the link ratio and you don't need the link ratio. That is, the link ratio – 1 = 0.





- **This forms the extended link ratio family.** The extended link ratio family includes all models of the following form including where one (or more) of entries are zero.

$$p = a_0 + a_1 w + (b-1)x + \varepsilon$$

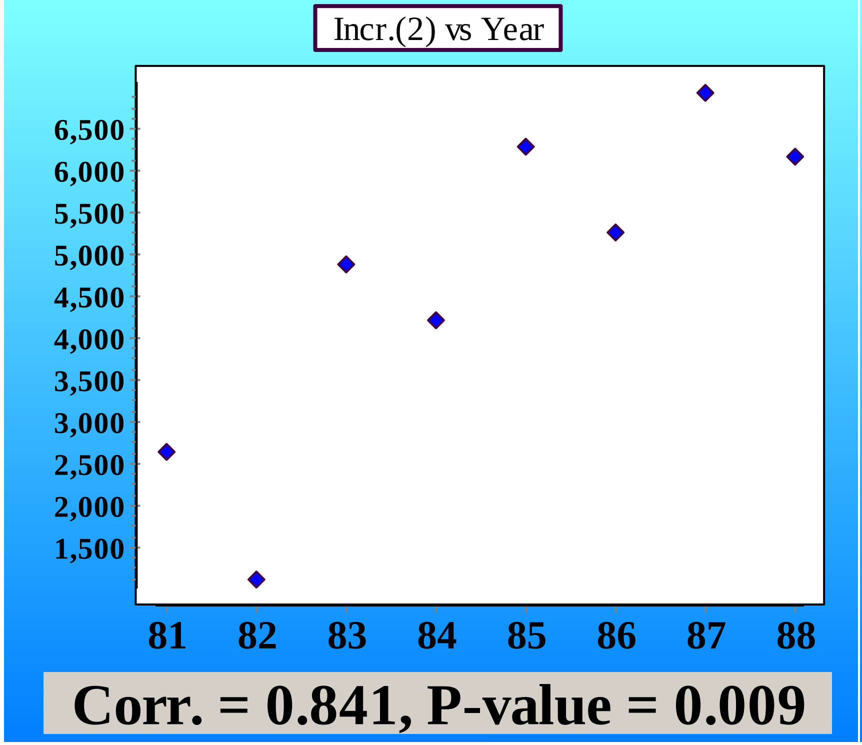
$$a_0 = \text{Intercept}$$

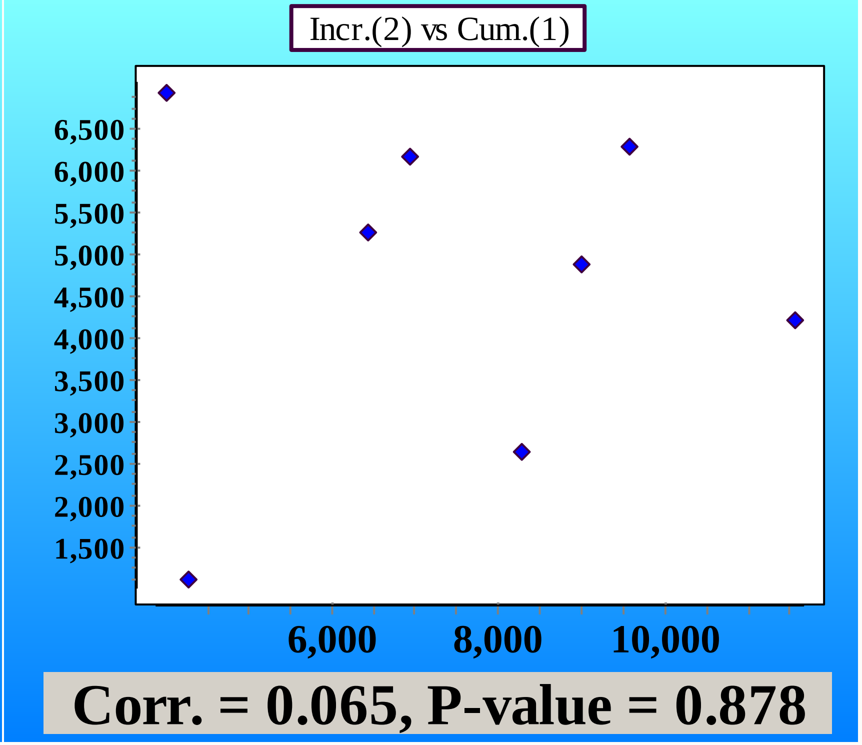
$$a_1 = \text{Trend}$$

$$b = \text{Ratio}$$

ELRF modelling framework

- For the data analyzed by Mack, there is a trend in development year two. That is, condition 2 is satisfied, but in the incremental versus cumulative shows no correlation – in other words b-1 is zero.



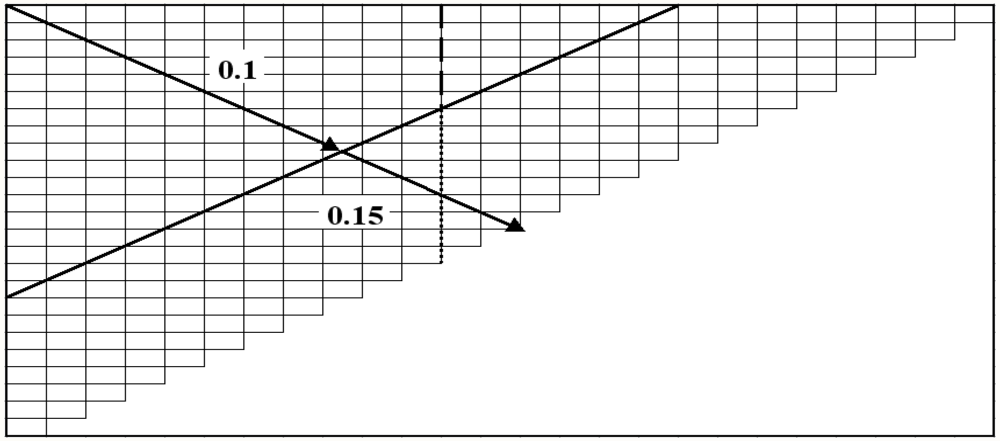


Development Period	Intercept			Trend			Ratio		
	Est.	S.E.	P-Value	Est.	S.E.	P-Value	Est.	Ratio-1	S.E.
0~1	4,462	424	0.00001	****	****	****	1.00000	0.00000	0.00000
1~2	1,796	772	0.05908	772	177	0.00472	1.00000	0.00000	0.00000
2~3	3,374	863	0.00789	****	****	****	1.00000	0.00000	0.00000

ELRF modelling framework

Condition 3: if there is a change in calendar period trend as shown below, then the trend going down the development period is not constant.

Incremental



Review 3 mutually exclusive conditions:

Condition 1: Zero trend

Condition 2: Constant trend, positive or negative

Condition 3: Non-constant trend

Under conditions 2 & 3 link ratios sometimes capture some of the trends; but you have no idea how much! Residuals give some relativities.

ELRF modelling framework

- As a result of conditions 2 & 3 where the Mack method captures some of the calendar trend but without quantifying it, we have no idea of whether the method gives results that are reasonable, too low, or too high. We can hazard a guess by the direction of the residuals, but we still cannot answer the question: by how much?

- An example of both these cases (too low, too high) are given as the final examples in this set of supplementary notes. The triangle ABC gives results from Mack that are too low. For the triangle LR High, as its name suggests, the Mack method produce answers that are far too high – about double what models designed in PTF produce.

- The major difference between the modelling frameworks, link ratio based and probabilistic based, is that the probabilistic method fits a distribution to every cell in the triangle, quantifies calendar trends, and measures the volatility in the data and about the trends. Link ratio methods are information free and do not tell you any information about the triangle. The probabilistic framework provides a natural mechanism of dealing with both these fundamental features of the data. Control over future assumptions in the PTF modelling framework is fully in the hands of the actuary.

Assessing link ratios in the ELRF modelling framework

How does this formal regression methodology in the ELRF framework help us to assess whether link ratio techniques ‘work’?

- We know that when we apply a link ratio technique, we are actually fitting a regression model through the origin.
- Any regression model is based on a set of **assumptions**.
- If the assumptions of a model are not supported by the data then any subsequent calculations made using the model (eg forecasts) are **meaningless** – they are based on the model, not the data.
- Regression methodology allows us to **test** the assumptions made by a model.

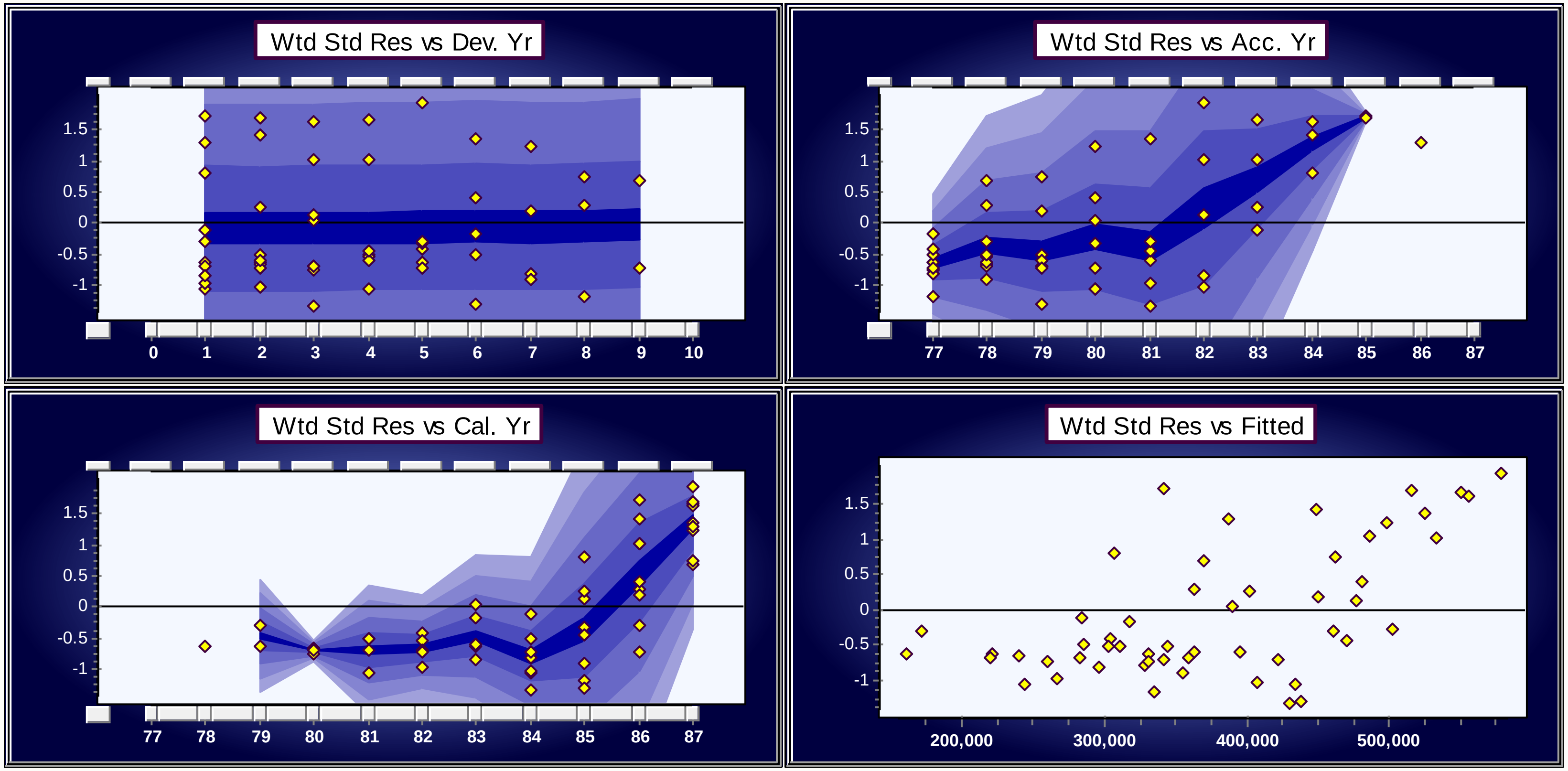
Assessing link ratios in the ELRF modelling framework

Summary of Assumption 1: $E(y \mid x) = bx$

- Very often not satisfied by the data
  - residuals suggest intercept is needed
- Include intercept term:
  - Case 1: ratios abandoned in favour of intercepts
  - Case 2: projection using link ratios
- A relationship between *y* and *x* is often better modelled using a trend parameter
  - Case 3: ratios abandoned in favour of trend terms
- ‘Optimal’ model in the ELRF is not likely to include ratios!

Complete residuals ABC

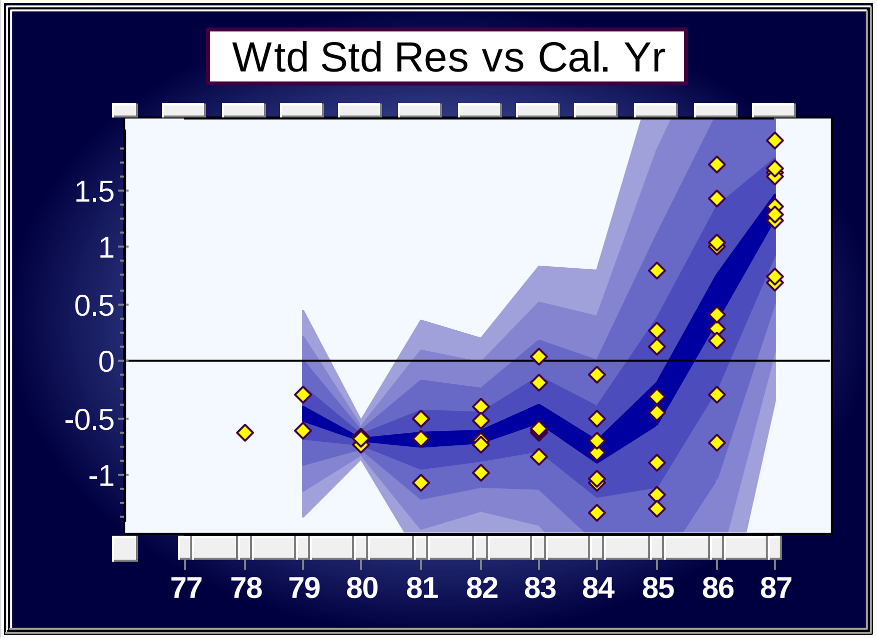
- The Mack method is applied to the data ABC and plotted against the three directions and residuals versus fitted value. The fact that the method can't capture the calendar year trends suggests a modelling framework that has calendar year trends, accident year trends, and development year trends.



Assessing link ratios in the ELRF modelling framework

Assumption 2: Are there trends in the calendar period direction?

Any time there are calendar year shifts, the Mack method (Chain ladder ratios) will not capture it.



- Real data commonly have changing calendar period trends
- Indicated here by Chain Ladder Ratio (Mack method) model residuals
- Assumption 2 (no calendar trends) is not supported by this data

Assessing link ratios in the ELRF modelling framework

- Cumulatives “smear” over the breaks
- With randomness and cumulation means no hope of seeing the trend changes
- Lack control over future calendar year trend assumptions
- Even the ‘best’ model in the ELRF modelling framework can’t capture such trend changes
- A modelling framework which includes calendar trend changes by definition should be used.

Summary of the ELRF modelling framework

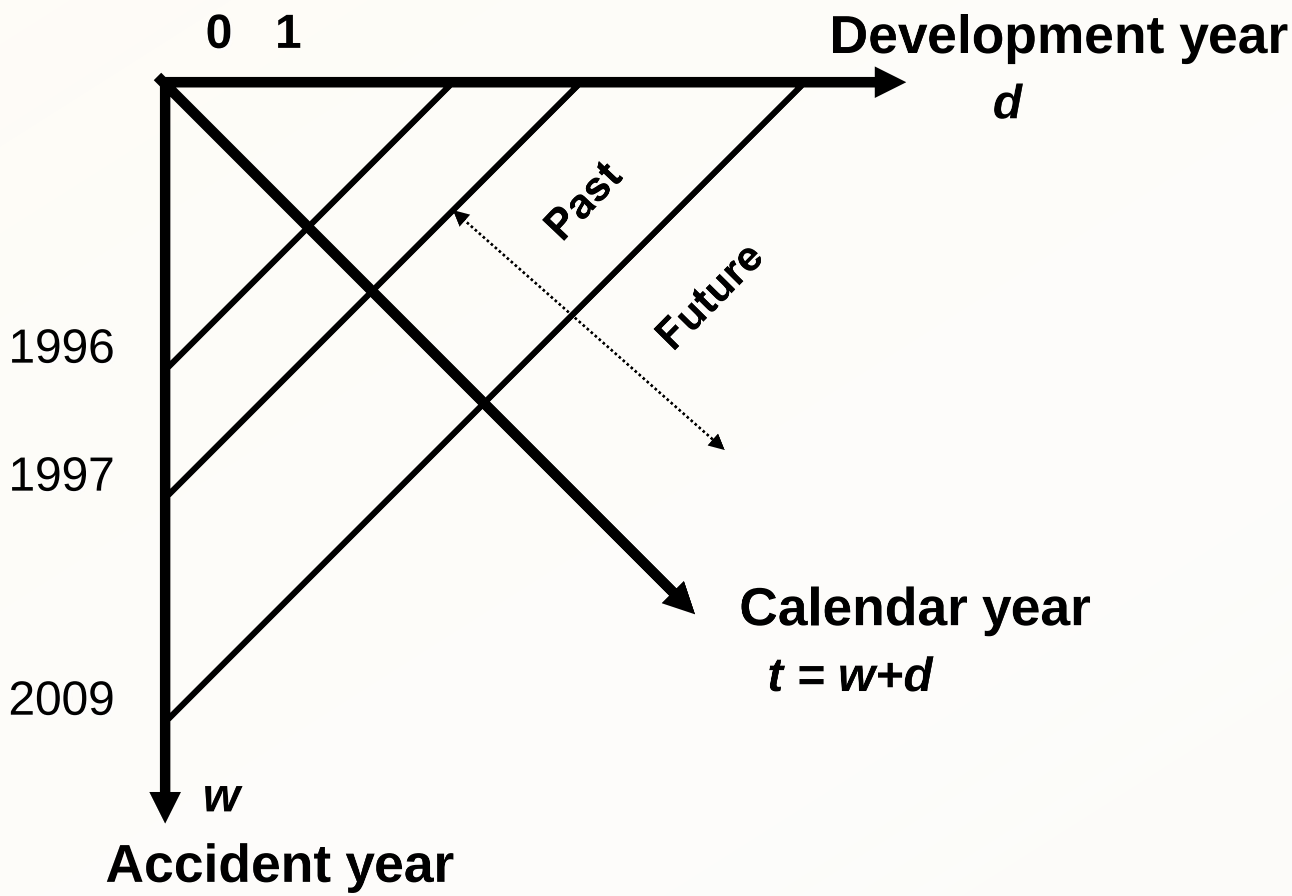
- Average link ratios are regression estimators through the origin.
- Regression methods allow us to *test* the assumptions of a model.
- Often find that major assumptions of link ratios models are not satisfied by the data
  - lack predictive power
  - cannot capture calendar trend changes
  - non-normality.
- Need to work from a different paradigm.

Where to from here?

- very important to check if the model is appropriate!
- Link ratios techniques
  - do not relate the numbers in the triangle in any meaningful way – they do not tell a story about the data
  - do not give control over future assumptions
- changing trends (e.g. against calendar years) suggests modelling trend changes
- non-normality and nonlinear trends suggests transformation - particularly log transform
- Thus we introduce the Probabilistic Trend Family (PTF) modelling framework.

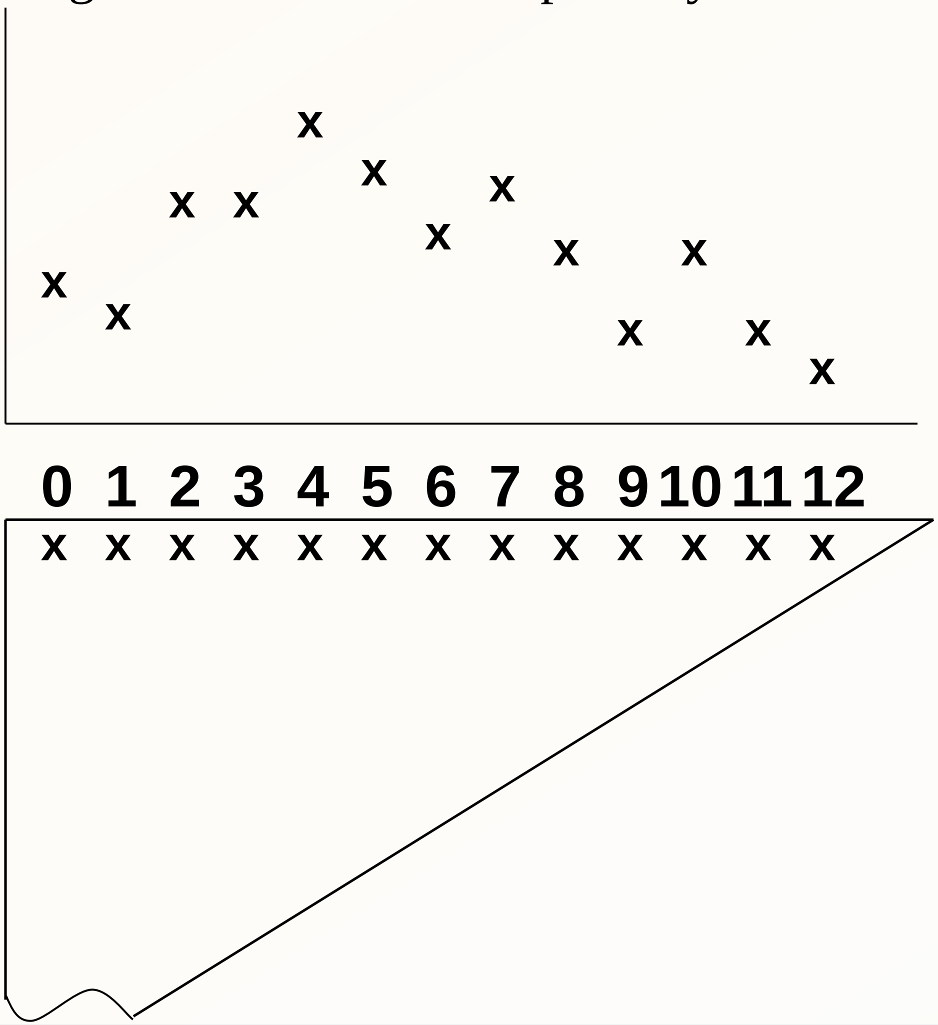
The study of trends

Trends occur in three directions:



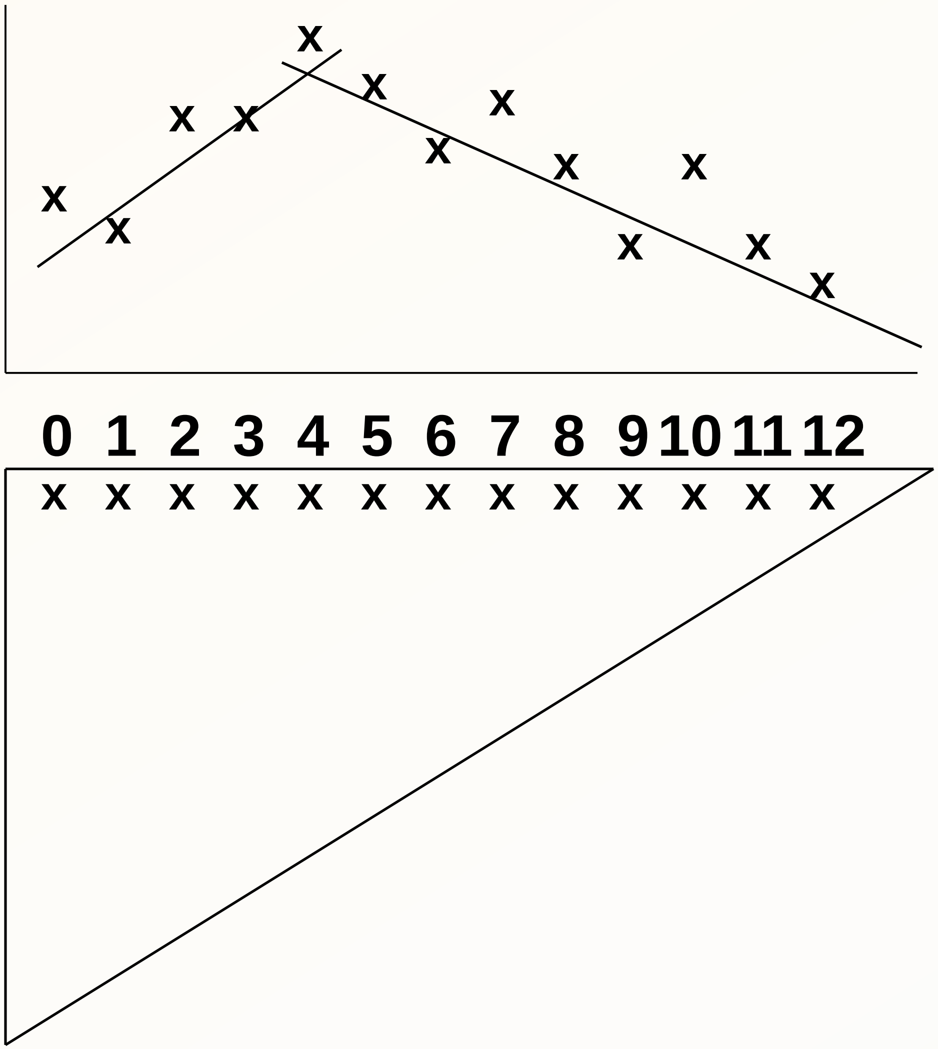
Trend example: development period

- e.g. trends in the development year direction



If we graph the data for an accident year against development year, we can see changing trends.

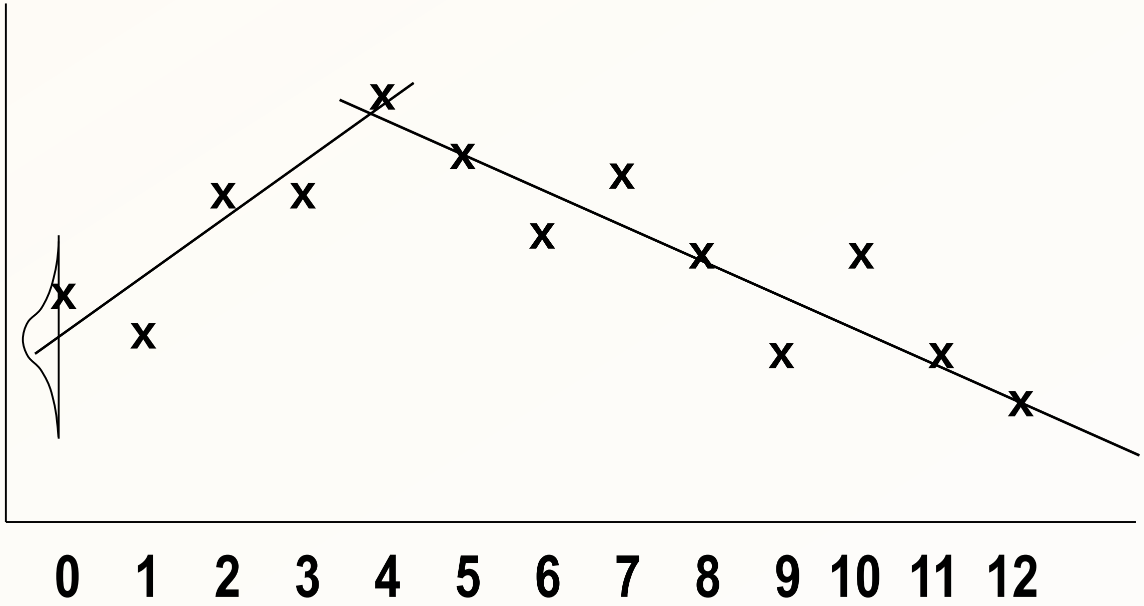
Trend example: development period



Could put a line through the points, using a ruler.

Or could do something formally, using regression.

Trend example: development period



The model is not just the trends in the mean, but the distribution about the mean

(*Data = Trends + Random Fluctuations*)

Introduction to the Probabilistic Trend Family (PTF) modelling framework

The probabilistic trend family (PTF) modelling framework is used to identify (build) the most appropriate model for the data. Such a model, consists of two components:

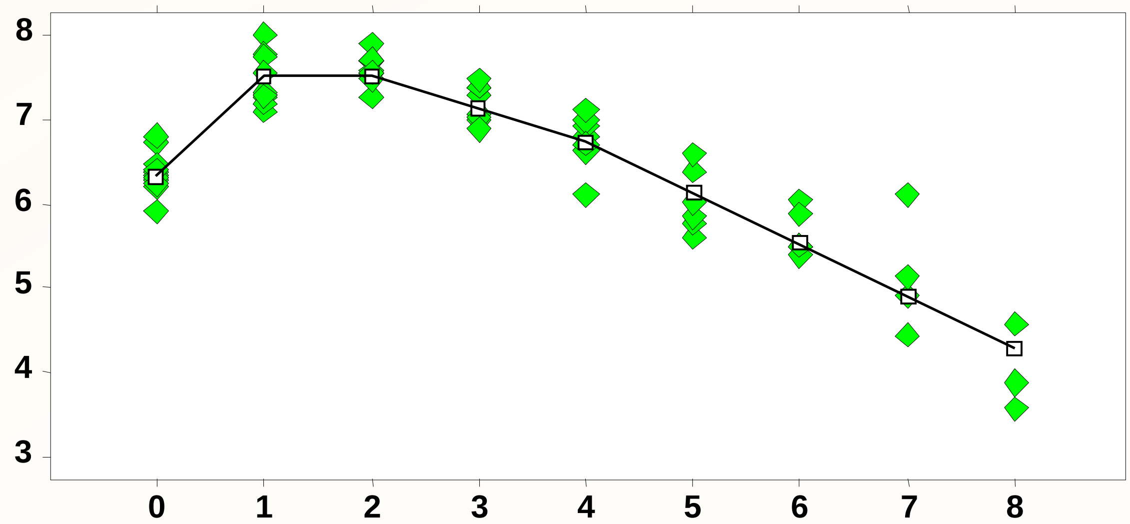
Model the mean in each cell by:

- Trends in the three directions
  - Development period
  - Accident period
  - Calendar period
- Measure the quality of the volatility around the trend structure.

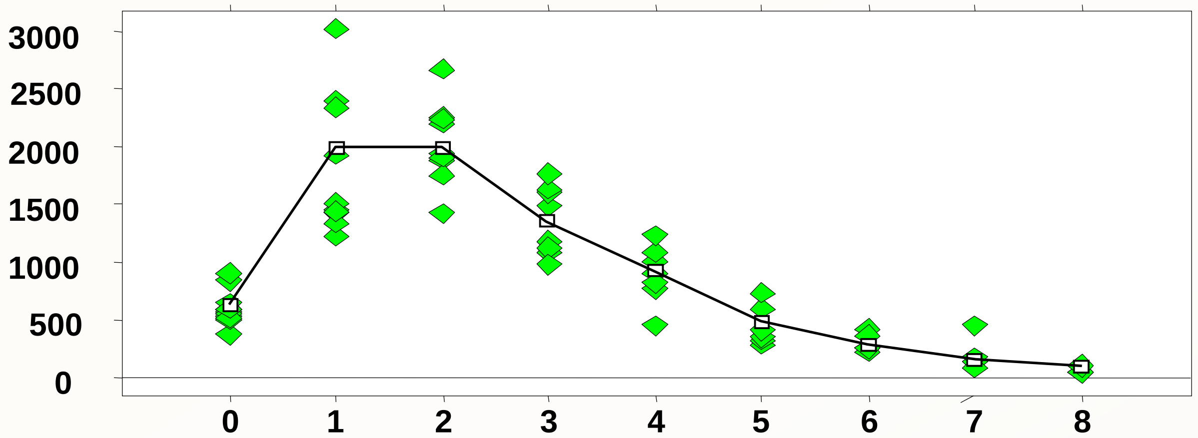
Equivalent to fitting a distribution to every cell on the log scale. The means of the distributions are related through the trend structure.

Introduction to PTF

Real data show the same features as the model



Log scale

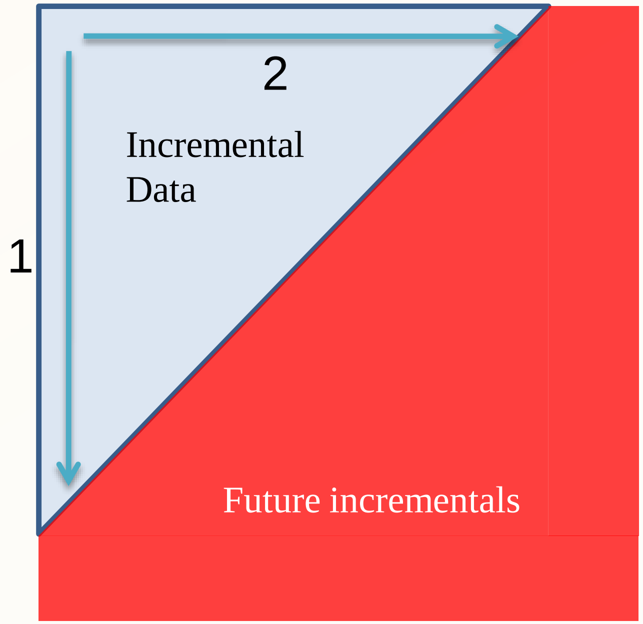


Original scale

- spread related to mean

Introduction to PTF

- Let's step back a little and consider the type of problem we are trying to solve:



Essentially we are wanting to project the future incrementals(*) in the red section of the triangle (which may be extended to the right, below, or both) for each cell in order to calculate the total remaining to be paid.

The total has a distribution of possible outcomes with all the features of a distribution: mean, standard deviation, and other properties. In order for the distribution of the total to be meaningful, the calculation of the total must be directly related to the individual cells, and the projection of the individual cells must be relatable to every other cell in the triangle.

These concepts are illustrated with a series of examples.

First, an example is given where the data are constant, then where the data come from the same distribution in all cells. After this, we consider the simpler problem of projecting a single cell in one direction. Then we examine projecting multiple cells both down and across. The initial examples do not have calendar trends.

(*) This applies to any incremental triangle, including paid, incurred, and any type of count triangle.

Example 1: constant data

- Consider the following ‘data’.

22,026	22,026	22,026	22,026	22,026	22,026	22,026	22,026	22,026	22,026	22,026	22,026	22,026	22,026	22,026
22,026	22,026	22,026	22,026	22,026	22,026	22,026	22,026	22,026	22,026	22,026	22,026	22,026	22,026	
22,026	22,026	22,026	22,026	22,026	22,026	22,026	22,026	22,026	22,026	22,026	22,026	22,026		
22,026	22,026	22,026	22,026	22,026	22,026	22,026	22,026	22,026	22,026	22,026	22,026			
22,026	22,026	22,026	22,026	22,026	22,026	22,026	22,026	22,026	22,026	22,026				
22,026	22,026	22,026	22,026	22,026	22,026	22,026	22,026	22,026	22,026					
22,026	22,026	22,026	22,026	22,026	22,026	22,026	22,026	22,026						
22,026	22,026	22,026	22,026	22,026	22,026	22,026	22,026							
22,026	22,026	22,026	22,026	22,026	22,026	22,026								
22,026	22,026	22,026	22,026	22,026	22,026									
22,026	22,026	22,026	22,026	22,026										
22,026	22,026	22,026	22,026											
22,026	22,026	22,026												
22,026	22,026													
22,026														

The data are constant, and the best prediction is simply to project this constant for every cell. The constant is the ‘best estimate’ given the information to date.

22,026	22,026	22,026	22,026	22,026	22,026	22,026	22,026	22,026	22,026	22,026	22,026	22,026	22,026	22,026
22,026	22,026	22,026	22,026	22,026	22,026	22,026	22,026	22,026	22,026	22,026	22,026	22,026	22,026	22026
22,026	22,026	22,026	22,026	22,026	22,026	22,026	22,026	22,026	22,026	22,026	22,026	22,026	22026	22026
22,026	22,026	22,026	22,026	22,026	22,026	22,026	22,026	22,026	22,026	22,026	22,026	22026	22026	22026
22,026	22,026	22,026	22,026	22,026	22,026	22,026	22,026	22,026	22,026	22,026	22026	22026	22026	22026
22,026	22,026	22,026	22,026	22,026	22,026	22,026	22,026	22,026	22,026	22026	22026	22026	22026	22026
22,026	22,026	22,026	22,026	22,026	22,026	22,026	22,026	22,026	22026	22026	22026	22026	22026	22026
22,026	22,026	22,026	22,026	22,026	22,026	22,026	22,026	22026	22026	22026	22026	22026	22026	22026
22,026	22,026	22,026	22,026	22,026	22,026	22,026	22026	22026	22026	22026	22026	22026	22026	22026
22,026	22,026	22,026	22,026	22,026	22,026	22026	22026	22026	22026	22026	22026	22026	22026	22026
22,026	22,026	22,026	22,026	22,026	22026	22026	22026	22026	22026	22026	22026	22026	22026	22026
22,026	22,026	22,026	22,026	22026	22026	22026	22026	22026	22026	22026	22026	22026	22026	22026
22,026	22,026	22,026	22026	22026	22026	22026	22026	22026	22026	22026	22026	22026	22026	22026
22,026	22,026	22026	22026	22026	22026	22026	22026	22026	22026	22026	22026	22026	22026	22026
22,026	22,026	22026	22026	22026	22026	22026	22026	22026	22026	22026	22026	22026	22026	22026

If the data are variable, then the projection should project **distributions** with a mean and standard deviation; not a single value.

Example 2: known data (one mean, low variance)

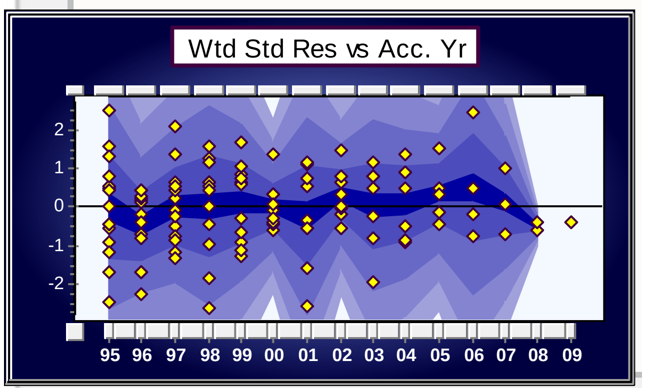
- Consider the following 'data' on a log scale.

9.808	10.130	9.452	10.147	9.694	10.219	9.833	10.755	10.118	10.470	9.620	9.840	9.216	10.390	9.979
10.119	9.767	9.758	9.727	9.452	10.027	10.011	10.057	10.073	9.923	10.107	9.278	9.850	9.461	
9.946	9.822	9.748	10.053	10.186	10.400	9.615	9.909	9.709	10.119	10.625	9.575	10.146		
10.469	9.411	10.368	9.167	9.673	10.170	10.178	10.335	9.979	10.145	10.119	9.835			
10.171	9.582	10.236	9.688	9.885	10.494	10.162	10.217	10.308	9.783	9.629				
9.795	9.948	10.395	10.077	9.848	9.960	9.857	9.891	10.074	9.995					
9.490	9.815	10.325	9.869	10.343	10.144	9.182	10.217	9.816						
9.929	10.083	10.186	9.976	10.231	9.984	10.437	9.813							
10.339	9.378	10.222	9.911	10.123	10.223	9.724								
10.253	10.125	9.701	9.716	10.404	9.820									
10.457	10.132	10.084	9.937	9.846										
10.736	9.928	9.749	10.136											
9.767	10.001	10.292												
9.799	9.854													
9.852														

The data are not constant, but all the cells come from the same distribution and this distribution is known (thus the title) – a normal distribution on a log scale with a mean of 10 and a variance of 0.1. (All the data were independently simulated from this distribution).

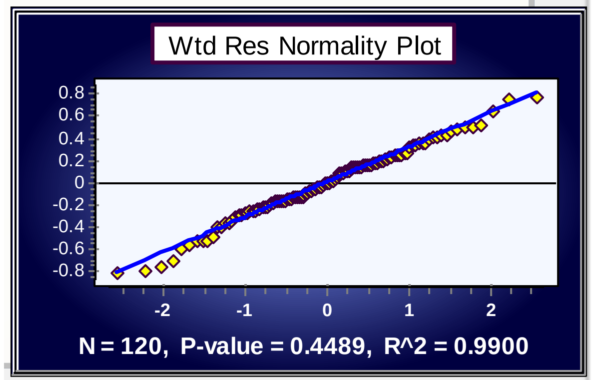
Assuming the true distribution is not known, the best projection is to estimate the distribution from the data, and then project that distribution to all remaining cells.

The mean and variance of the data, on a log scale, are: 9.978 and 0.097 respectively (to three decimal places). The standard deviation is: 0.311 again to three decimal places.



If the data are plotted versus accident period (left) there are no patterns in the data and the dots are scattered randomly around the zero mark. This is the same as a plot of the data after removing the average and standardising. The points plotted are the differences between the data and the fitted model (in this case the model is simply the mean). The differences (residuals) are then standardised. Residuals are simply the trend(s) in the data less the trend(s) in a model.

We can also test the normality of the residuals as shown in the result on the right. The p-value is greater than a 5% significance level so the assumption of normality is unable to be rejected.



Example 2: known data (one mean, low variance)

9.978	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**
9.808	10.13	9.452	10.15	9.694	10.22	9.833	10.76	10.12	10.47	9.62	9.84	9.216	10.39	9.979
9.978	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**
10.119	9.767	9.758	9.727	9.452	10.027	10.011	10.057	10.073	9.923	10.107	9.278	9.850	9.461	0.312
9.978	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**
9.946	9.822	9.748	10.053	10.186	10.400	9.615	9.909	9.709	10.119	10.625	9.575	10.146	0.312	0.312
9.978	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**
10.469	9.411	10.368	9.167	9.673	10.170	10.178	10.335	9.979	10.145	10.119	9.835	0.312	0.312	0.312
9.978	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**
10.171	9.582	10.236	9.688	9.885	10.494	10.162	10.217	10.308	9.783	9.629	0.312	0.312	0.312	0.312
9.978	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**
9.795	9.948	10.395	10.077	9.848	9.960	9.857	9.891	10.074	9.995	0.312	0.312	0.312	0.312	0.312
9.978	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**
9.490	9.815	10.325	9.869	10.343	10.144	9.182	10.217	9.816	0.312	0.312	0.312	0.312	0.312	0.312
9.978	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**
9.929	10.083	10.186	9.976	10.231	9.984	10.437	9.813	0.312	0.312	0.312	0.312	0.312	0.312	0.312
9.978	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**
10.339	9.378	10.222	9.911	10.123	10.223	9.724	0.312	0.312	0.312	0.312	0.312	0.312	0.312	0.312
9.978	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**
10.253	10.125	9.701	9.716	10.404	9.820	0.312	0.312	0.312	0.312	0.312	0.312	0.312	0.312	0.312
9.978	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**
10.457	10.132	10.084	9.937	9.846	0.312	0.312	0.312	0.312	0.312	0.312	0.312	0.312	0.312	0.312
9.978	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**
10.736	9.928	9.749	10.136	0.312	0.312	0.312	0.312	0.312	0.312	0.312	0.312	0.312	0.312	0.312
9.978	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**
9.767	10.001	10.292	0.312	0.312	0.312	0.312	0.312	0.312	0.312	0.312	0.312	0.312	0.312	0.312
9.978	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**
9.799	9.854	0.312	0.312	0.312	0.312	0.312	0.312	0.312	0.312	0.312	0.312	0.312	0.312	0.312
9.978	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**	**9.978**
9.852	0.312	0.312	0.312	0.312	0.312	0.312	0.312	0.312	0.312	0.312	0.312	0.312	0.312	0.312

- The complete projection (log scale) is therefore the values on the lower right of the completed ‘square’. For emphasis, each cell in the triangle has the projected mean (bold type). In the original data cells, the blue values are the original data.
- In the projected cells, the red numbers are the *standard deviation* for the projections for each cell (the measured *variance* is 0.097).
- For **every cell** in the triangle a normal distribution has been fitted – both a mean and a standard deviation.
- The distribution for the total is simply the distribution of the sum of the normal distributions in each projected cell. Since each individual cell comes from an independent normal distribution the sum is also normally distributed with a mean of the sum of the individual means and a variance of the sum of the individual variances.
- The total (log scale) is therefore normally distributed with a mean of:
  - 1,047.69
- And a standard deviation of:
  - 3.20

Example 3: 'known' data with development trend

- For emphasis, data with zero variance is used to illustrate the trend. The data are transformed to a log scale (below right) so the percentage trends can be seen.

22,027	16,318	12,088	8,955	6,634	4,915	3,641	2,697	1,998	1,480	1,097	812	602	446	330
22,026	16,317	12,088	8,955	6,634	4,915	3,641	2,697	1,998	1,480	1,097	812	602	446	
22,026	16,317	12,089	8,955	6,634	4,915	3,641	2,697	1,998	1,480	1,097	812	602		
22,026	16,318	12,088	8,955	6,634	4,915	3,641	2,697	1,998	1,480	1,097	812			
22,026	16,318	12,088	8,955	6,634	4,915	3,641	2,697	1,998	1,480	1,097				
22,026	16,318	12,089	8,955	6,634	4,915	3,641	2,697	1,998	1,480					
22,027	16,317	12,088	8,955	6,634	4,915	3,641	2,697	1,998						
22,027	16,318	12,088	8,955	6,634	4,915	3,641	2,697							
22,026	16,318	12,088	8,955	6,634	4,915	3,641								
22,027	16,318	12,088	8,955	6,634	4,915									
22,027	16,318	12,088	8,955	6,634										
22,026	16,318	12,088	8,955											
22,027	16,318	12,088												
22,026	16,318													
22,026														

Since there is no variation in the data, projection, as with first example, is incredibly simple. Either projection is done down each column, or from left to right, the projected values remain the same. If we go from left to right (log scale), then we decrease the previous value by 0.3 down the column, we just project the last known value.

10.000	9.700	9.400	9.100	8.800	8.500	8.200	7.900	7.600	7.300	7.000	6.700	6.400	6.100	5.799
10.000	9.700	9.400	9.100	8.800	8.500	8.200	7.900	7.600	7.300	7.000	6.700	6.400	6.100	
10.000	9.700	9.400	9.100	8.800	8.500	8.200	7.900	7.600	7.300	7.000	6.700	6.400		
10.000	9.700	9.400	9.100	8.800	8.500	8.200	7.900	7.600	7.300	7.000	6.700			
10.000	9.700	9.400	9.100	8.800	8.500	8.200	7.900	7.600	7.300	7.000				
10.000	9.700	9.400	9.100	8.800	8.500	8.200	7.900	7.600	7.300					
10.000	9.700	9.400	9.100	8.800	8.500	8.200	7.900	7.600						
10.000	9.700	9.400	9.100	8.800	8.500	8.200	7.900							
10.000	9.700	9.400	9.100	8.800	8.500	8.200								
10.000	9.700	9.400	9.100	8.800	8.500									
10.000	9.700	9.400	9.100	8.800										
10.000	9.700	9.400	9.100											
10.000	9.700	9.400												
10.000	9.700													
10.000														

Example 4: ‘known’ data with development trend

- The same trend structure (-30% decay) was used, starting with a mean of 10 (log scale), but where noise was added to the data. Log scale is on the lower right. Can you still see the development trend?

28,775	12,457	21,554	5,607	9,226	4,384	3,351	3,654	2,215	2,553	1,915	644	584	387	562	?
28,531	8,091	15,606	13,910	7,303	3,993	2,681	4,160	2,831	587	1,603	885	581	323		
23,020	21,990	9,652	11,104	6,340	5,308	3,212	1,876	1,334	1,962	831	885	813			
14,225	18,268	18,303	7,729	4,630	2,236	2,754	2,813	2,338	965	2,068	849				
27,091	18,953	14,006	7,505	7,410	4,943	3,283	2,122	3,058	1,160	888					
14,061	15,296	8,864	5,197	6,240	4,955	3,446	2,968	2,455	1,982						
16,365	13,583	9,572	12,868	13,806	4,081	4,742	4,094	3,357							
17,743	12,531	8,520	7,578	4,974	4,198	2,142	1,207								
29,126	14,712	10,175	11,333	7,547	6,283	6,079									
16,338	13,195	15,211	11,468	7,204	4,532										
29,089	20,130	12,909	5,760	6,230											
29,353	13,033	7,900	10,297												
13,106	12,278	10,265													
18,243	15,190														
22,631															

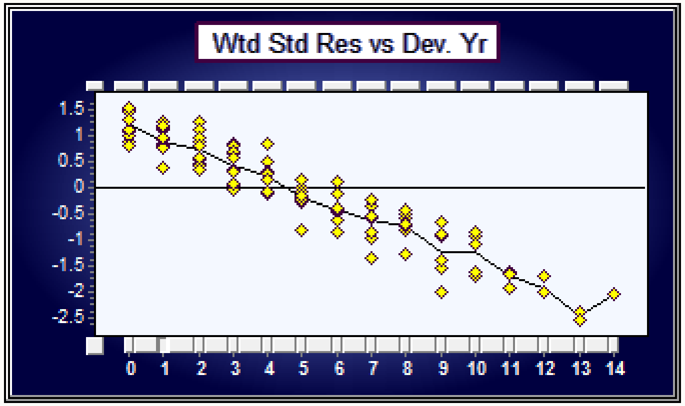
Let’s examine the process of building a complete model relating all the cells in the triangle with distributions starting with the projection of a single cell. Assume for now we **only** have the data highlighted in yellow and we wish to project the ?.

How would you do it?

10.267	9.430	9.978	8.632	9.130	8.386	8.117	8.204	7.703	7.845	7.557	6.468	6.370	5.958	6.332	?
10.259	8.999	9.655	9.540	8.896	8.292	7.894	8.333	7.948	6.375	7.380	6.786	6.365	5.778		
10.044	9.998	9.175	9.315	8.755	8.577	8.075	7.537	7.196	7.582	6.723	6.786	6.701			
9.563	9.813	9.815	8.953	8.440	7.712	7.921	7.942	7.757	6.872	7.634	6.744				
10.207	9.850	9.547	8.923	8.911	8.506	8.097	7.660	8.026	7.056	6.789					
9.551	9.635	9.090	8.556	8.739	8.508	8.145	7.996	7.806	7.592						
9.703	9.517	9.167	9.462	9.533	8.314	8.464	8.317	8.119							
9.784	9.436	9.050	8.933	8.512	8.342	7.669	7.096								
10.279	9.596	9.228	9.335	8.929	8.746	8.713									
9.701	9.488	9.630	9.347	8.882	8.419										
10.278	9.910	9.466	8.659	8.737											
10.287	9.475	8.975	9.240												
9.481	9.416	9.236													
9.812	9.628														
10.027															

Example 4: ‘known’ data with development trend

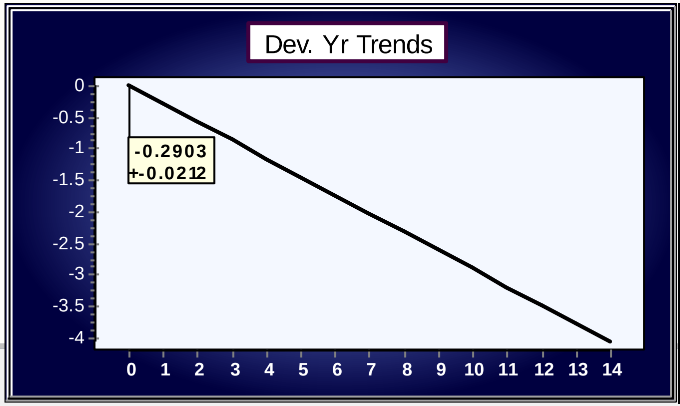
Well, first you would plot the data (right) and observe an obvious downward trend in the points (ignore the scale). You could guess at this trend by eye and place a line on the graph. This slope of the line should correspond roughly to the average decay. Regression is a technique for mathematically estimating the slope of the line from the data to minimise the distance between the data and the line. Residuals are just the differences between the fitted line and the data – usually standardised to a common scale.

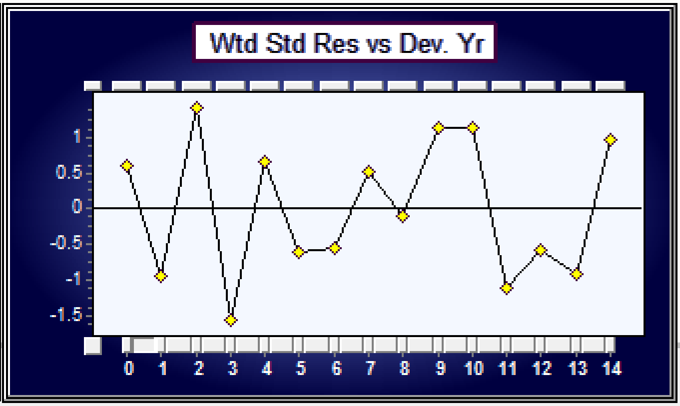


The regression line using least squares is fitted below. For the second accident period, the slope of this line is estimated to be -29.03% with a standard error of the slope of: 2.12%. What does this tell us? This means on average as we move to the right along the data, we expect the payments to reduce by about 29% on average (not the 30% we used since we’re estimating the parameter). We then transform back to the dollar scale with the final projection being as in the rightmost column. The red value is the standard deviation – this time of **log-normal distribution** as the data is normally distributed on a log scale, log-normal when transformed back to the dollar scale.

25,007	18,652	13,918	10,390	7,760	5,798	4,334	3,241	2,425	1,815	1,359	1,019	763	572	429	322
28,775	12,457	21,554	5,607	9,226	4,384	3,351	3,654	2,215	2,553	1,915	644	584	387	562	128

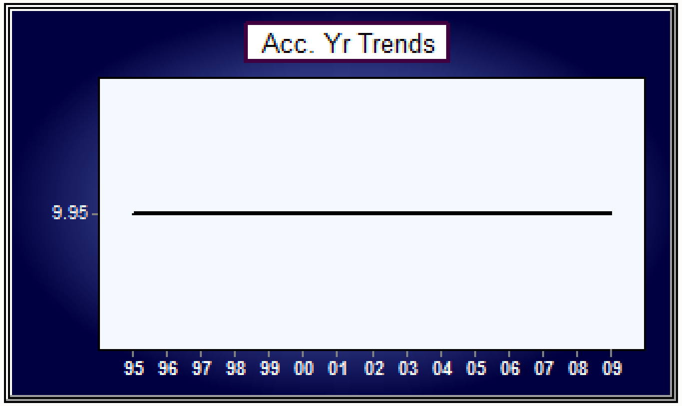
The residuals are shown to the below right after fitting the regression line.





Example 4: ‘known’ data with development trend

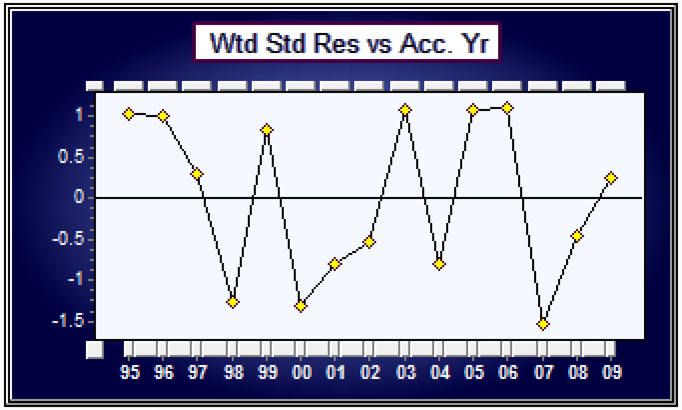
10.267	9.430	9.978	8.632	9.130	8.386	8.117	8.204	7.703	7.845	7.557	6.468	6.370	5.958	6.332
10.259	8.999	9.655	9.540	8.896	8.292	7.894	8.333	7.948	6.375	7.380	6.786	6.365	5.778	
10.044	9.998	9.175	9.315	8.755	8.577	8.075	7.537	7.196	7.582	6.723	6.786	6.701		
9.563	9.813	9.815	8.953	8.440	7.712	7.921	7.942	7.757	6.872	7.634	6.744			
10.207	9.850	9.547	8.923	8.911	8.506	8.097	7.660	8.026	7.056	6.789				
9.551	9.635	9.090	8.556	8.739	8.508	8.145	7.996	7.806	7.592					
9.703	9.517	9.167	9.462	9.533	8.314	8.464	8.317	8.119						
9.784	9.436	9.050	8.933	8.512	8.342	7.669	7.096							
10.279	9.596	9.228	9.335	8.929	8.746	8.713								
9.701	9.488	9.630	9.347	8.882	8.419									
10.278	9.910	9.466	8.659	8.737										
10.287	9.475	8.975	9.240											
9.481	9.416	9.236												
9.812	9.628													
10.027														
?														



We can use the same process of projecting down the column (assuming we only have the green highlighted values) as used for projecting across the row. In this case, however, we can use the average as we ‘know’ no changes have occurred in the accident level. This can be confirmed by the residuals (right) which confirm no change in level. The above has procedure has applications to pricing and writing future underwriting periods since we are extending to future accident periods.

The mean of the column on a log scale is: 9.95 (above right).

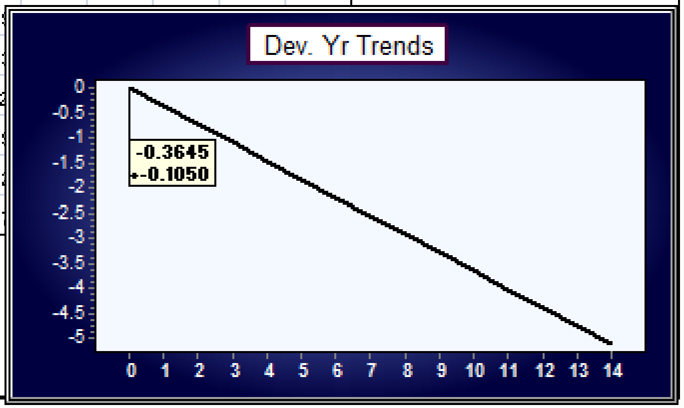
On a dollar scale, this mean of the log-normal corresponds to 21,942; and the standard deviation of the log normal to: 6,860.



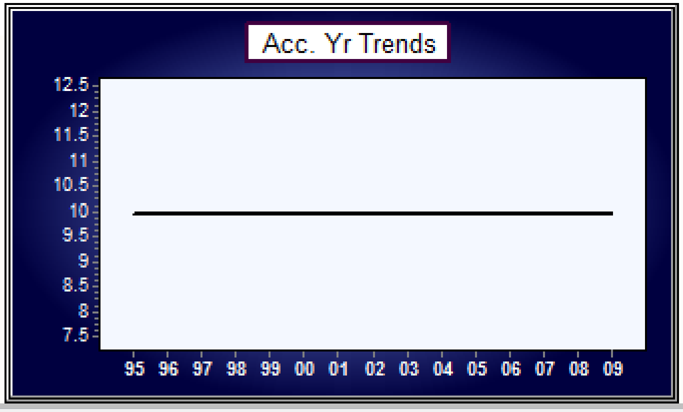
Example 4: ‘known’ data with development trend

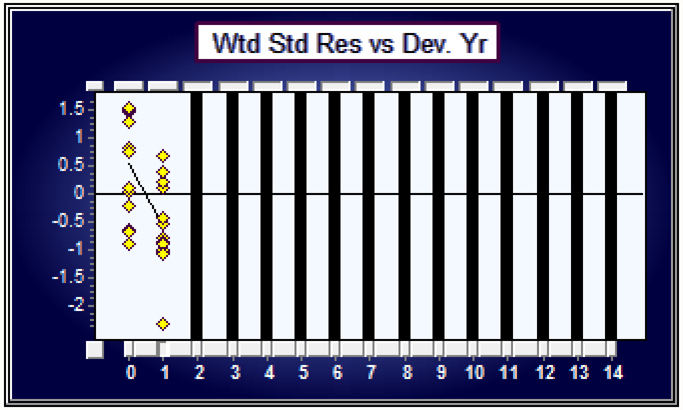
This approach is readily extended to multiple projections with trends, eg: projecting the following three cells.

10.267	9.430	9.978	8.632	9.130	8.386	8.117	8.204	7.703	7.845	7.557	6.468	6.370	5.958	6.332
10.259	8.999	9.655	9.540	8.896	8.292	7.894	8.333	7.948	6.375	7.380	6.786	6.365	5.778	
10.044	9.998	9.175	9.315	8.755	8.577	8.075	7.537	7.196	7.582	6.723	6.786	6.701		
9.563	9.813	9.815	8.953	8.440	7.712	7.921	7.942	7.757	6.872	7.634	6.744			
10.207	9.850	9.547	8.923	8.911	8.506	8.097	7.660	8.026	7.056	6.789				
9.551	9.635	9.090	8.556	8.73[illegible]										
9.703	9.517	9.167	9.462	9.53[illegible]										
9.784	9.436	9.050	8.933	8.51[illegible]										
10.279	9.596	9.228	9.335	8.92[illegible]										
9.701	9.488	9.630	9.347	8.88[illegible]										
10.278	9.910	9.466	8.659	8.73[illegible]										
10.287	9.475	8.975	9.240											
9.481	9.416	9.236												
9.812	9.628													
10.027	?													
?	?													



21,793	15,139
18,243	15,190
21,793	15,139
22,631	4,368
21,793	15,139
6,272	4,368





Since there are only two development periods, the development parameter trend is fairly uncertain (the standard error of the parameter is quite high).

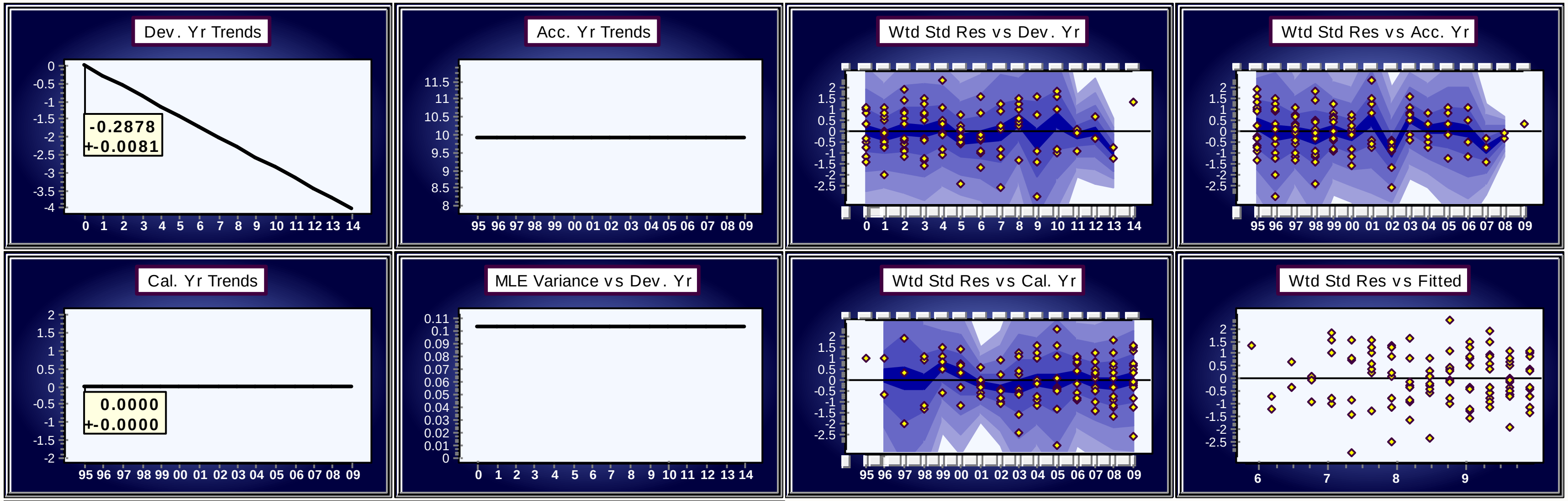
From the residuals above, **without** the trend fitted, development 0 is clearly higher (on average) than development 1.

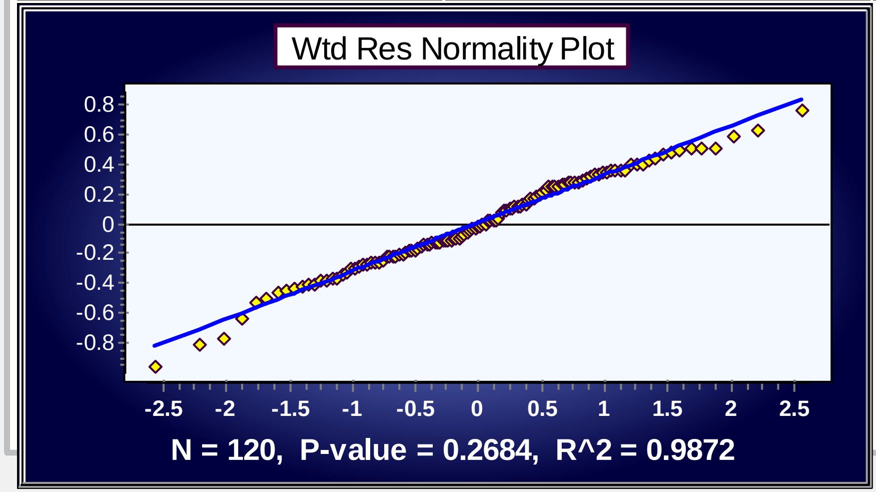
Using this model, we have related all the cells in the triangle available to the model (highlighted yellow). The cell in the first accident period, first development period (10.267) is related, by the model, to every other highlighted cell. Using this model we can project the ‘?’s by extending the model as before.

Projections on the dollar scale are shown on the far left. Again, bold black is the mean of the fitted lognormal distribution, red is the standard deviation, blue is the observed data.

Example 4: ‘known’ data with development trend

As per the previous slides, the regression analysis can be extended by using all the data in the triangle. This leads to the following model and residual displays.





The residuals are scattered randomly over the three directions (although accident year 2002 looks low). The assumption of normality is unable to be rejected.

The trend structure described, -28.78%+_0.81% development trend and the mean accident level of 9.9267+_0.0481 with a process variance of 0.1032 is statistically the same as the structure used to create the data.

The probabilistic modelling framework has enabled the construction of a model which describes the salient features of the data while relating **every** cell in the triangle.

Example 4: 'known' data with development trend

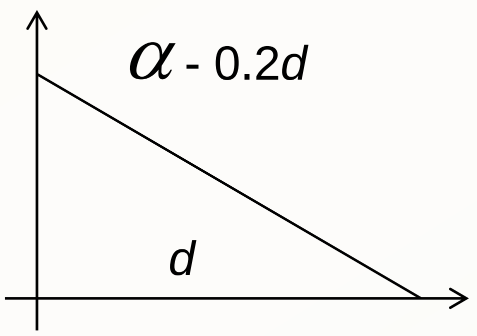
The completed table with a **log-normal distribution** fitted to each cell in the triangle, with the projected distributions and their sums displayed in the lower right section of the triangle. As before, blue numbers are observed, black are fitted means, and red numbers are the standard deviation of the log-normal distribution. The burgundy numbers of the aggregate totals are also standard deviations, but the distribution of the sums are not necessarily log-normally distributed since they are the sum of log-normals.

	Cal. Per. Total	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	Outstanding	Ultimate
1995	21,579	21,579	16,178	12,129	9,094	6,819	5,114	3,835	2,876	2,157	1,618	1,214	911	683	513	385	0	97,869
	28,775	28,775	12,457	21,554	5,607	9,226	4,384	3,351	3,654	2,215	2,553	1,915	644	584	387	562	0	0
1996	37,757	21,579	16,178	12,129	9,094	6,819	5,114	3,835	2,876	2,157	1,618	1,214	911	683	513	385	385	91,471
	40,988	28,531	8,091	15,606	13,910	7,303	3,993	2,681	4,160	2,831	587	1,603	885	581	323	131	131	131
1997	49,886	21,579	16,178	12,129	9,094	6,819	5,114	3,835	2,876	2,157	1,618	1,214	911	683	513	385	897	89,226
	52,665	23,020	21,990	9,652	11,104	6,340	5,308	3,212	1,876	1,334	1,962	831	885	813	174	131	223	223
1998	58,981	21,579	16,178	12,129	9,094	6,819	5,114	3,835	2,876	2,157	1,618	1,214	911	683	513	385	1,580	78,759
	57,428	14,225	18,268	18,303	7,729	4,630	2,236	2,754	2,813	2,338	965	2,068	849	230	174	131	330	330
1999	65,800	21,579	16,178	12,129	9,094	6,819	5,114	3,835	2,876	2,157	1,618	1,214	911	683	513	385	2,491	92,910
	78,148	27,091	18,953	14,006	7,505	7,410	4,943	3,283	2,122	3,058	1,160	888	306	230	174	131	463	463
2000	70,914	21,579	16,178	12,129	9,094	6,819	5,114	3,835	2,876	2,157	1,618	1,214	911	683	513	385	3,705	69,168
	74,108	14,061	15,296	8,864	5,197	6,240	4,955	3,446	2,968	2,455	1,982	406	306	230	174	131	633	633
2001	74,749	21,579	16,178	12,129	9,094	6,819	5,114	3,835	2,876	2,157	1,618	1,214	911	683	513	385	5,323	87,792
	67,080	16,365	13,583	9,572	12,868	13,806	4,081	4,742	4,094	3,357	539	406	306	230	174	131	852	852
2002	77,625	21,579	16,178	12,129	9,094	6,819	5,114	3,835	2,876	2,157	1,618	1,214	911	683	513	385	7,481	66,374
	63,967	17,743	12,531	8,520	7,578	4,974	4,198	2,142	1,207	717	539	406	306	230	174	131	1,136	1,136
2003	79,783	21,579	16,178	12,129	9,094	6,819	5,114	3,835	2,876	2,157	1,618	1,214	911	683	513	385	10,357	95,613
	75,659	29,126	14,712	10,175	11,333	7,547	6,283	6,079	954	717	539	406	306	230	174	131	1,508	1,508
2004	81,401	21,579	16,178	12,129	9,094	6,819	5,114	3,835	2,876	2,157	1,618	1,214	911	683	513	385	14,192	82,139
	73,637	16,338	13,195	15,211	11,468	7,204	4,532	1,271	954	717	539	406	306	230	174	131	1,997	1,997
2005	82,615	21,579	16,178	12,129	9,094	6,819	5,114	3,835	2,876	2,157	1,618	1,214	911	683	513	385	19,306	93,425
	88,730	29,089	20,130	12,909	5,760	6,230	1,694	1,271	954	717	539	406	306	230	174	131	2,645	2,645
2006	83,526	21,579	16,178	12,129	9,094	6,819	5,114	3,835	2,876	2,157	1,618	1,214	911	683	513	385	26,125	86,708
	97,197	29,353	13,033	7,900	10,297	2,259	1,694	1,271	954	717	539	406	306	230	174	131	3,507	3,507
2007	84,209	21,579	16,178	12,129	9,094	6,819	5,114	3,835	2,876	2,157	1,618	1,214	911	683	513	385	35,220	70,868
	76,294	13,106	12,278	10,265	3,015	2,259	1,694	1,271	954	717	539	406	306	230	174	131	4,659	4,659
2008	84,721	21,579	16,178	12,129	9,094	6,819	5,114	3,835	2,876	2,157	1,618	1,214	911	683	513	385	47,349	80,783
	71,441	18,243	15,190	4,026	3,015	2,259	1,694	1,271	954	717	539	406	306	230	174	131	6,207	6,207
2009	85,106	21,579	16,178	12,129	9,094	6,819	5,114	3,835	2,876	2,157	1,618	1,214	911	683	513	385	63,527	86,158
	85,205	22,631	5,382	4,026	3,015	2,259	1,694	1,271	954	717	539	406	306	230	174	131	8,296	8,296
	Total Fitted/Paid		2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020	2021	2022	2023	Total Reserve	Total Ultimate
Cal. Per.	1,038,651		63,527	47,349	35,220	26,125	19,306	14,192	10,357	7,481	5,323	3,705	2,491	1,580	897	385	237,939	1,269,262
Total	1,031,323		8,296	6,207	4,659	3,507	2,645	1,997	1,508	1,136	852	633	463	330	223	131	14,296	14,296

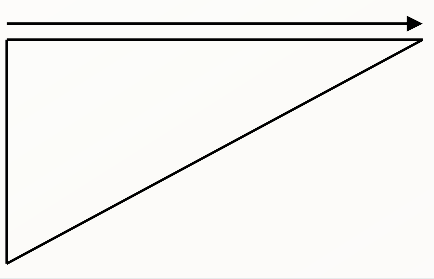
Example 5: M3IR5

Thus far, the examples have included neither changing trends nor calendar trends. The effect of calendar trends on the accident and development levels are now illustrated. We start, as usual, with the following specified model – no process variability – with data shown on a log scale.

0	1	2	3	4	5	6	7	8	9	10	11	12	13
11.513	11.313	11.113	10.913	10.713	10.513	10.313	10.113	9.913	9.713	9.513	9.313	9.113	8.913
11.513	11.313	11.113	10.913	10.713	10.513	10.313	10.113	9.913	9.713	9.513	9.313	9.113	
11.513	11.313	11.113	10.913	10.713	10.513	10.313	10.113	9.913	9.713	9.513	9.313		
11.513	11.313	11.113	10.913	10.713	10.513	10.313	10.113	9.913	9.713	9.513			
11.513	11.313	11.113	10.913	10.713	10.513	10.313	10.113	9.913	9.713				
11.513	11.313	11.113	10.913	10.713	10.513	10.313	10.113	9.913					
11.513	11.313	11.113	10.913	10.713	10.513	10.313	10.113						
11.513	11.313	11.113	10.913	10.713	10.513	10.313							
11.513	11.313	11.113	10.913	10.713	10.513								
11.513	11.313	11.113	10.913	10.713									
11.513	11.313	11.113	10.913										
11.513	11.313	11.113											
11.513	11.313												
11.513													



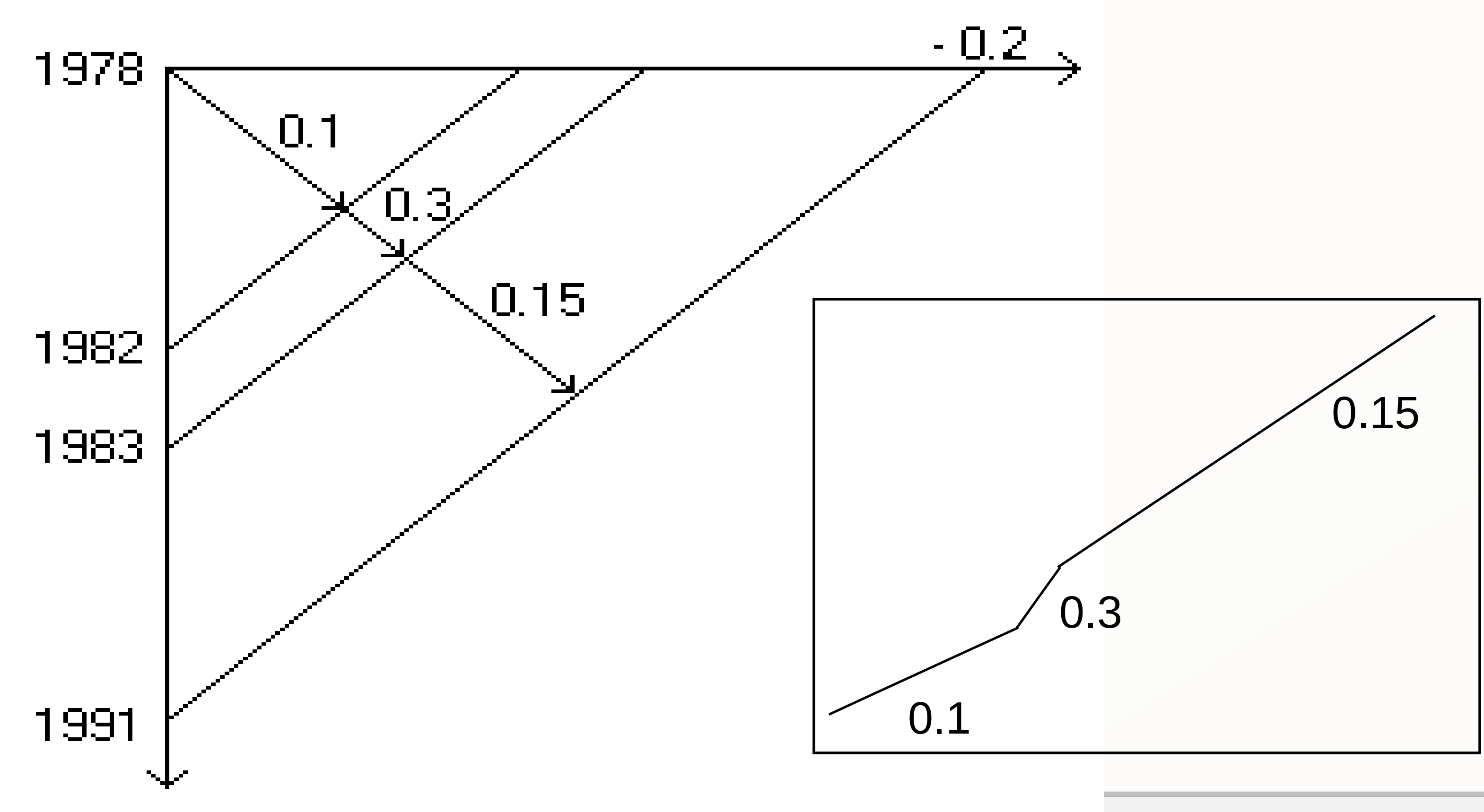
alpha = 11.513



PAID LOSS = EXP(alpha - 0.2d)

Example 5: M3IR5: axioms of trends

Add three new calendar year trends on a log scale to the data. Any calendar year trend projects in the other two directions to produce resultant trends.

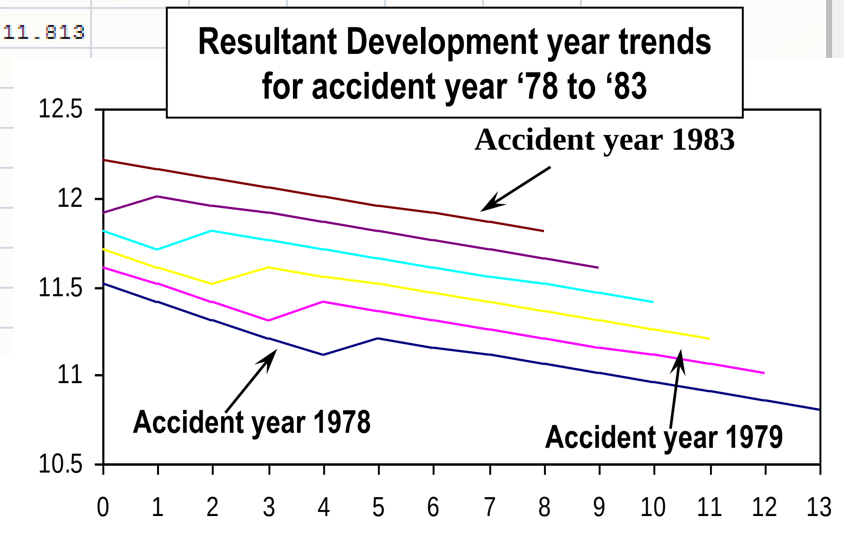


Example 5: M3IR5

- Log of data (alpha - 0.2d) + calendar trends
- Arrows apply to first row and column respectively and show the direction of the net trends.

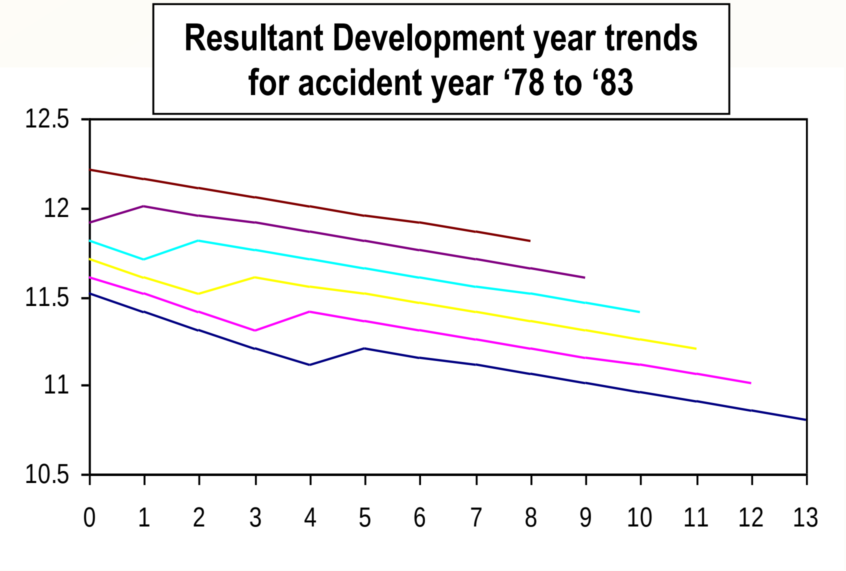
1978	11.513	11.413	11.313	11.213	11.113	11.213	11.163	11.113	11.063	11.013	10.963	10.913	10.863	10.813
1979	11.613	11.513	11.413	11.313	11.413	11.363	11.313	11.263	11.213	11.163	11.113	11.063	11.013	
1980	11.713	11.613	11.513	11.613	11.563	11.513	11.463	11.413	11.363	11.313	11.263	11.213		
1981	11.813	11.713	11.813	11.763	11.713	11.663	11.613	11.563	11.513	11.463	11.413			
1982	11.913	12.013	11.963	11.913	11.863	11.813	11.763	11.713	11.663	11.613				
1983	12.213	12.163	12.113	12.063	12.013	11.963	11.913	11.863	11.813					
1984	12.363	12.313	12.263	12.213	12.163	12.113	12.063	12.013						
1985	12.513	12.463	12.413	12.363	12.313	12.263	12.213							
1986	12.663	12.613	12.563	12.513	12.463	12.413								
1987	12.813	12.763	12.713	12.663	12.613									
1988	12.963	12.913	12.863	12.813										
1989	13.113	13.063	13.013											
1990	13.263	13.213												
1991	13.413													

Note kick up is in different development periods for each accident year. Both accident and development trends are affected by the change in calendar year trends.



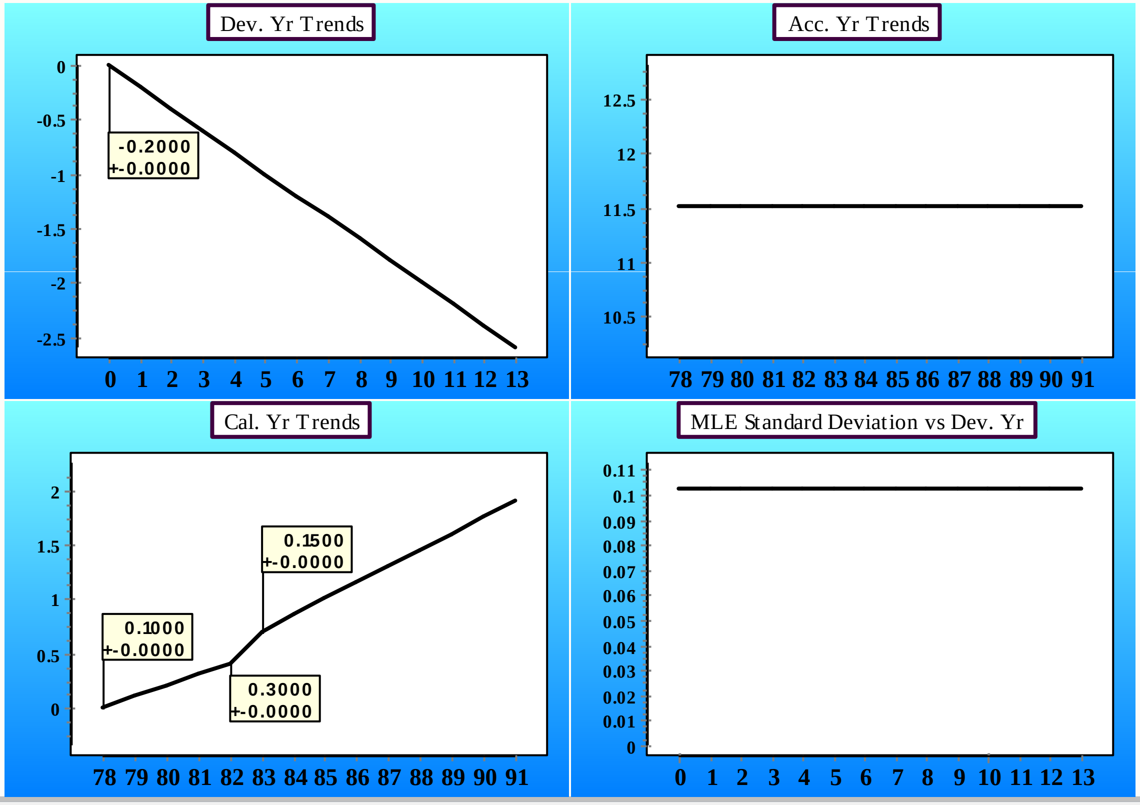
Example 5: M3IR5

- Consider accident year 1978. Along development years 0-4 the resultant trend is -0.2+0.1 (=-0.1). From development year 4-5 the resultant trend is: -0.2+0.3 (=0.1), and from development year 5 to 13 the resultant trend is: -0.2+0.15 (=-0.5).
- Accident year 1979 is 0.1 higher than 1978 but the kick up of the 0.1 is one development period earlier.
- For development year 4, as you step down the accident years, the first trend is 0.3, and the subsequent trends are each 0.15. For development year 7, the difference is 0.05 between successive accident years.
- For development year 2, it is 0.1 for two accident years going down, then 0.3, then 0.15 thereafter.



Example 5: M3IR5

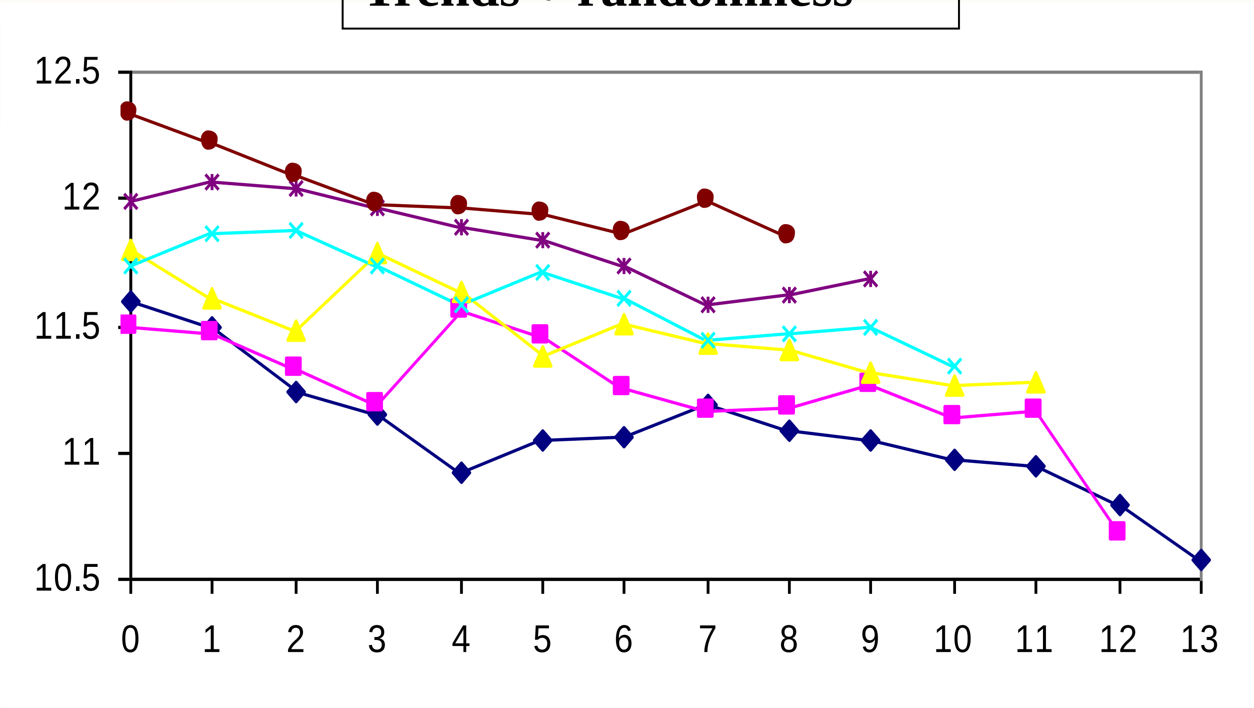
- The model display below shows the trend structure in each direction and the process variability in the lower right.



Example 5: M3IR5

Now add some process variation: in this case random numbers from a normal distribution with mean 0 and standard deviation 0.1.

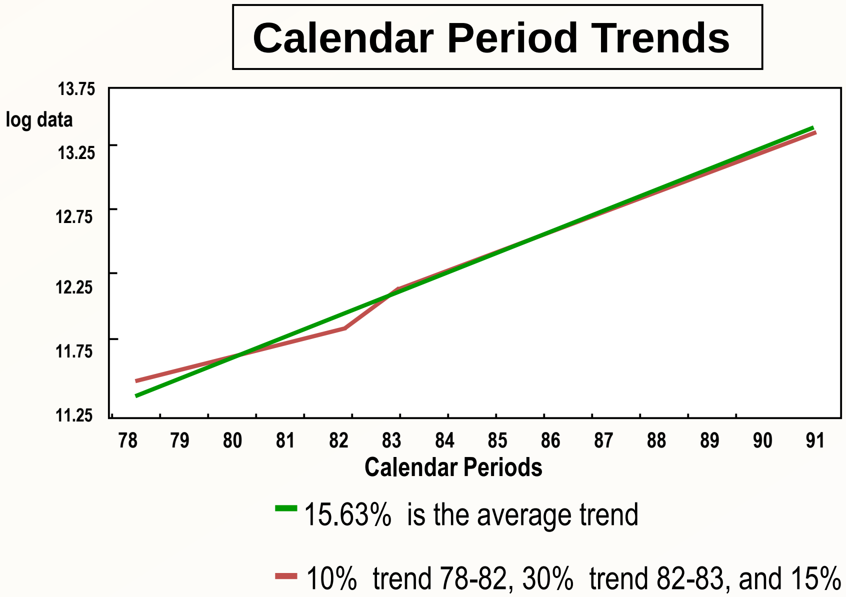
Trends + randomness

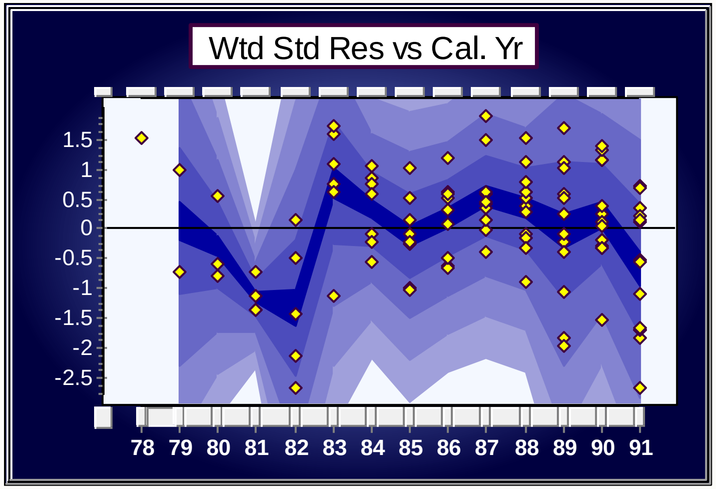


Changing trends hard to pick without removing main development and payment year trend.

Example 5: M3IR5

Fitting the average calendar year trend to three calendar year trends; after fitting the average development year trend and a zero trend along the accident years.





The residuals represent trend in data minus trend estimated by the method. From 78-82 the difference is 10%-15.63%, for 82-83, the differences is the 30%-15.63% and from 83 to 91, the difference is 15% - 15.63%.

Summary of trend concepts

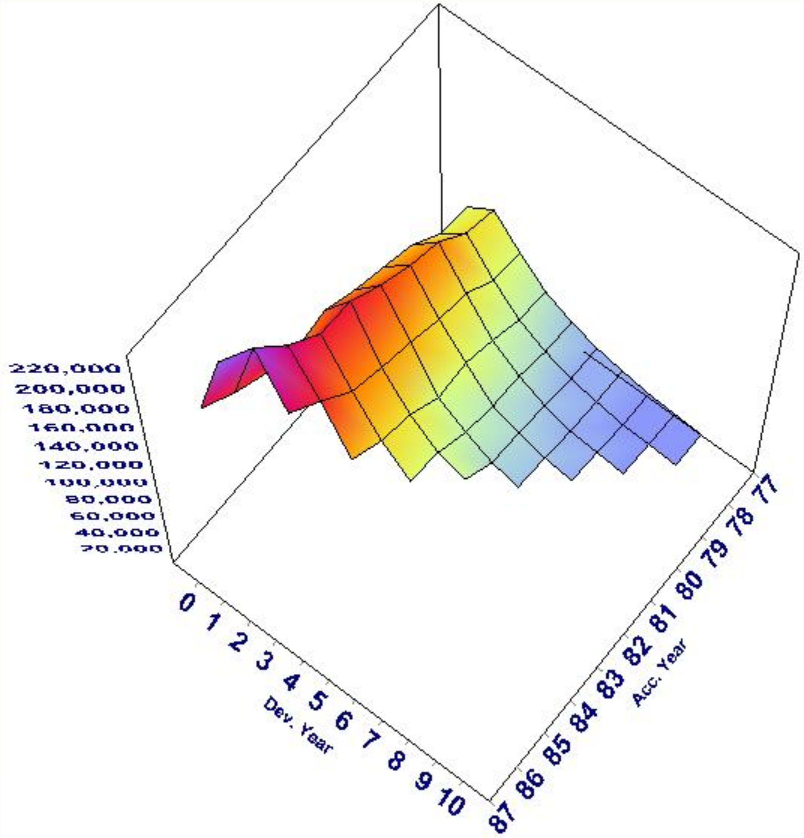
- Trends in calendar year direction project onto the other two directions and vice versa
- You cannot determine calendar period trend changes without including parameters in that direction
- Changing trends can be hard to pick up in the presence of noise, unless main trends are removed first (regression as a form of adjustment)
- Modelling a changing trend as a single trend will result in pattern in the residual plots

Summary of trend concepts

- If the modelling framework "works", it should be hard to differentiate between real data and data simulated from an identified model for that data
  - trends are the 'same' and change in the same periods
  - amount of randomness is the same
- If you create (simulate) data, you should be able to identify the (known) changing trends in the data; mean forecasts should usually be within about 2 standard errors of the true mean

Example 6: ABC

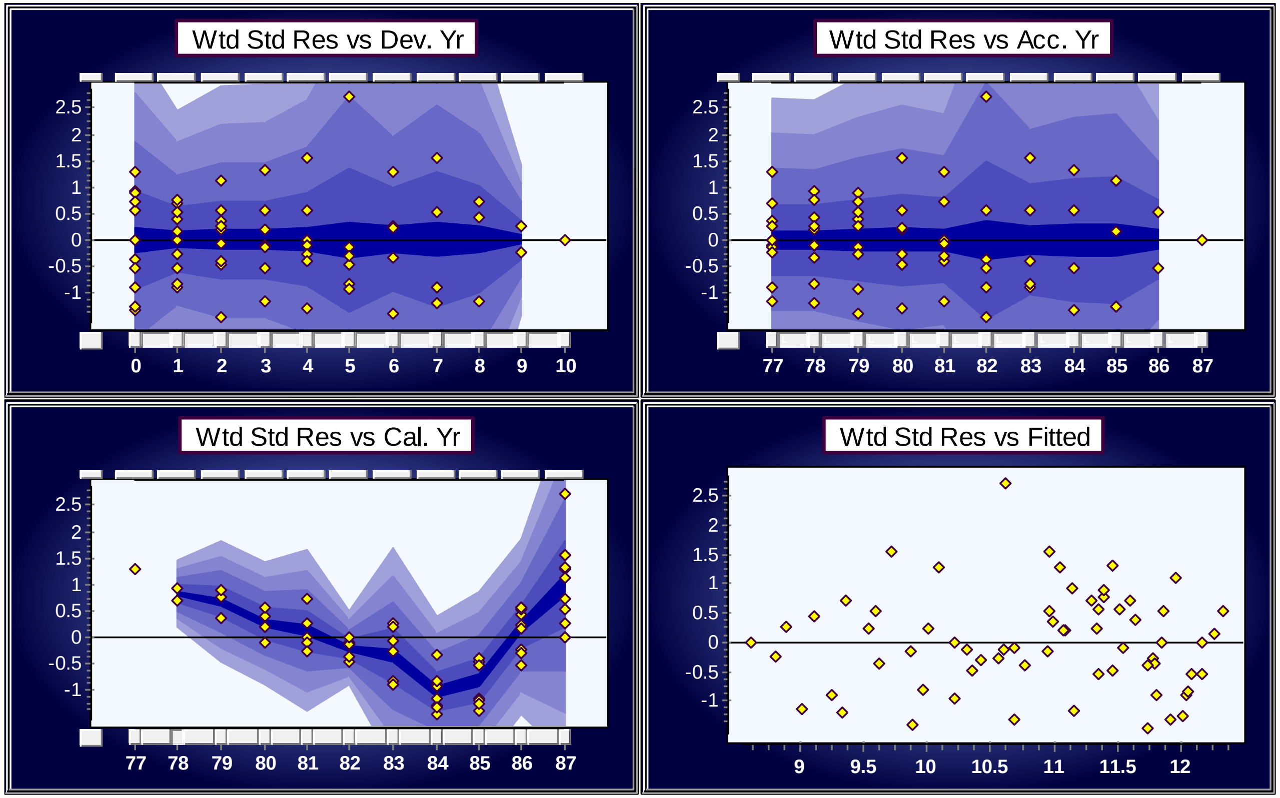
- Smooth data, that is, data with very little process variability, can conceal calendar year trend changes.



Very smooth data (ABC)

Example 6: ABC

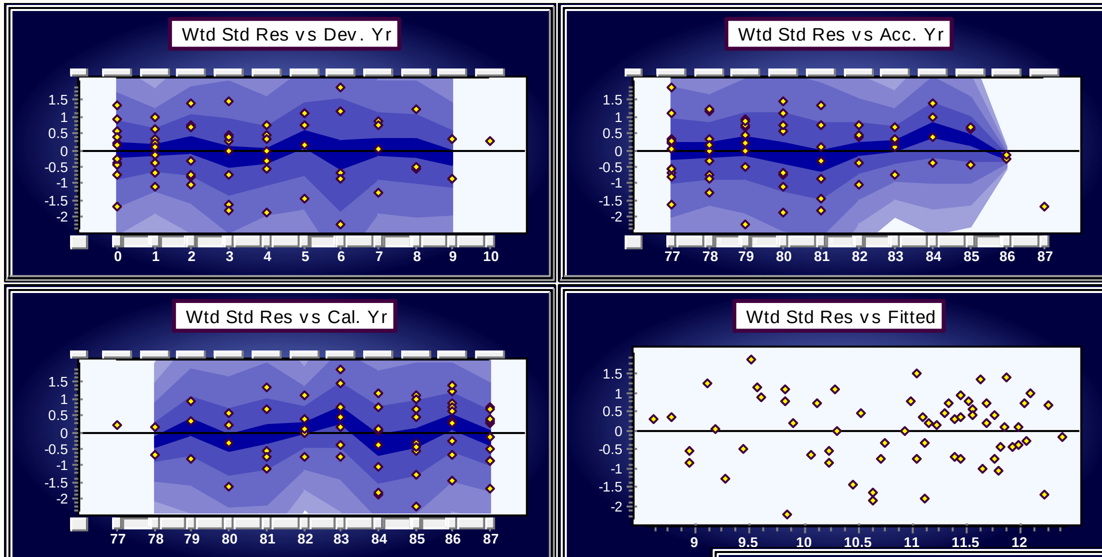
- Residuals below are after removing **all** accident and development year trends. This is a diagnostic model to determine whether there are any left over calendar trends after accounting for accident and development trend changes. This model is not be used for forecasting.



Residuals versus calendar year show clear structure. A good model, reflective of the data, will both adjust for and quantify these trends.

Example ABC: PTF modelling framework

- Identifying a PTF model for this data allows the full description of the changing trends including the volatility around them. Furthermore, we have control over **exactly** what trends are assumed going forward.



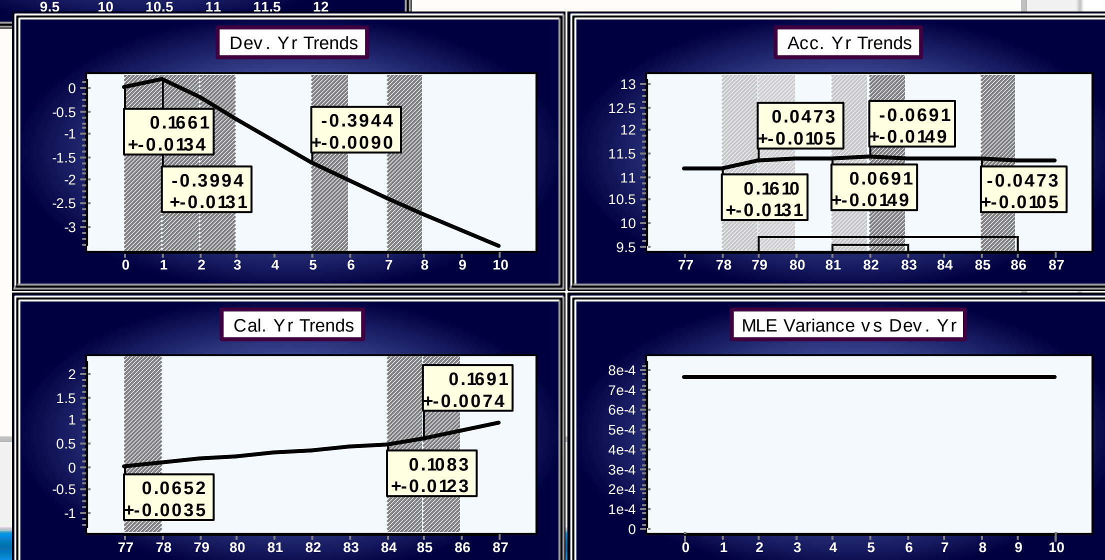
It is only in the PTF modelling framework that we can quantify, and adjust for, the change in calendar trend.

The initial calendar trend is measured at 6.52% +_ 0.35%. This trend changes in 1984-1985 to: 10.83% +_ 1.23%, and again in 1986-1987 to 16.91% +_ 0.74%.

The reserve forecast, assuming the most recent calendar trend continues is:

Mean Reserve	SD Reserve
7,120,007	234,706

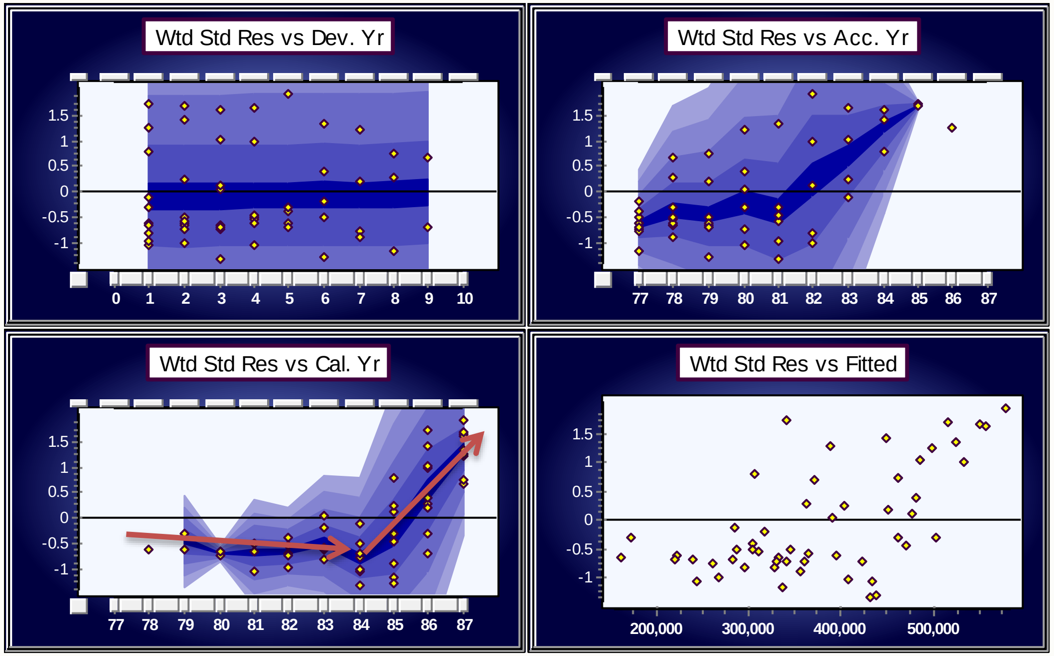
In the PTF modelling framework we have control over the calendar year trends. If we know why the 16.91% +_ 0.74% trend occurred, we can make decisions about whether this trend will continue into the future or whether it would revert to one of the older trends.



Example ABC: ELRF modelling framework

- When you apply the Mack method to this data, we can see it does not capture the calendar year trends (nor the accident year trends).

The residuals versus calendar year show a distinct change in trend from 1984 onwards. (Note this change in trend is not necessarily constant; the superimposed arrows are illustrative only).



Example: LR High

- In this example, the Mack method produces estimates which are far too high. The data triangle is shown below (and then again on a log scale).

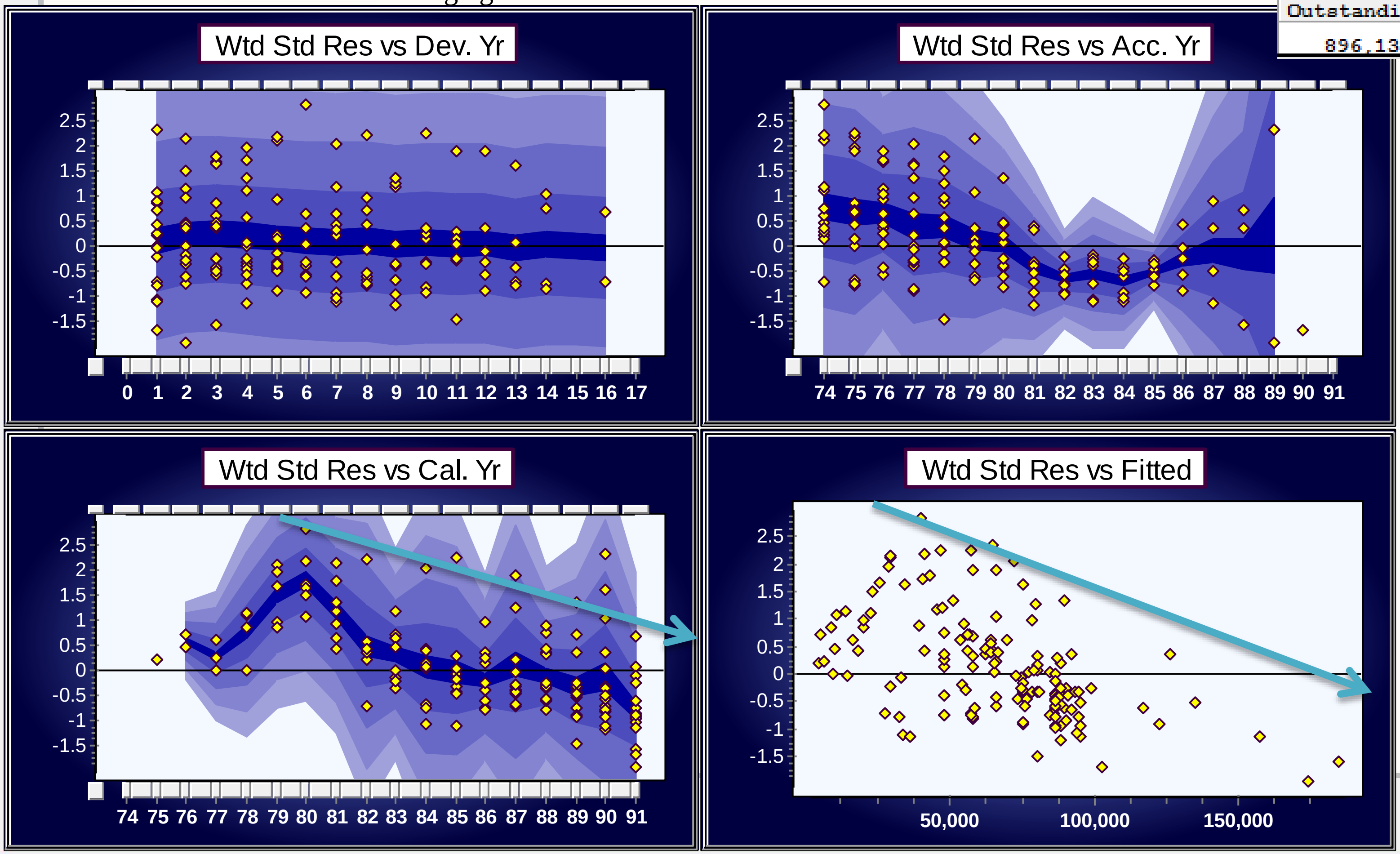
668	4,938	11,468	18,438	26,653	38,080	44,720	46,353	47,343	47,825	47,973	48,023	48,085	48,085	48,115	48,115	48,115	48,115
775	7,023	14,215	23,465	37,210	51,635	55,043	56,323	56,930	57,053	57,605	57,650	57,818	57,818	57,818	57,818	57,898	
925	6,860	18,203	33,218	49,433	59,720	62,935	64,548	65,123	65,450	65,670	65,910	65,928	65,948	65,993	65,993		
1,443	9,693	24,030	42,405	59,410	66,780	70,983	74,140	74,598	74,860	74,993	74,998	74,998	75,123	75,123			
1,273	11,295	30,168	53,045	67,985	74,043	77,135	78,100	78,995	79,723	80,005	79,868	79,913	79,965				
1,575	14,408	40,930	60,263	72,728	81,328	84,695	85,463	85,725	86,050	86,198	86,245	86,310					
2,695	20,165	43,795	65,228	79,518	84,708	87,050	88,705	89,018	89,843	89,893	89,905						
4,115	23,445	45,085	66,630	78,133	83,440	85,718	87,463	87,805	87,903	87,930							
4,385	28,140	51,560	69,643	78,400	83,328	85,153	85,568	85,793	85,945								
4,993	26,570	52,538	72,535	84,470	90,823	93,615	93,928	94,203									
5,410	28,845	53,873	72,918	86,100	91,320	92,375	92,748										
4,805	27,348	53,393	71,220	82,455	88,325	90,148											
4,905	32,633	69,673	96,400	111,153	114,970												
5,823	45,215	95,248	129,883	145,073													
8,358	62,015	130,135	165,508														
9,618	85,428	148,080															
15,225	83,480																
13,628																	

13.41	15.41	16.26	16.73	17.10	17.46	17.62	17.65	17.67	17.68	17.69	17.69	17.69	17.69	17.69	17.69	17.69	17.69
13.56	15.76	16.47	16.97	17.43	17.76	17.82	17.85	17.86	17.86	17.87	17.87	17.87	17.87	17.87	17.87	17.87	
13.74	15.74	16.72	17.32	17.72	17.91	17.96	17.98	17.99	18.00	18.00	18.00	18.00	18.00	18.01	18.01		
14.18	16.09	16.99	17.56	17.90	18.02	18.08	18.12	18.13	18.13	18.13	18.13	18.13	18.13	18.13			
14.06	16.24	17.22	17.79	18.03	18.12	18.16	18.17	18.18	18.19	18.20	18.20	18.20	18.20				
14.27	16.48	17.53	17.91	18.10	18.21	18.25	18.26	18.27	18.27	18.27	18.27	18.27					
14.81	16.82	17.60	17.99	18.19	18.25	18.28	18.30	18.30	18.31	18.31	18.31						
15.23	16.97	17.62	18.01	18.17	18.24	18.27	18.29	18.29	18.29	18.29							
15.29	17.15	17.76	18.06	18.18	18.24	18.26	18.26	18.27	18.27								
15.42	17.10	17.78	18.10	18.25	18.32	18.35	18.36	18.36									
15.50	17.18	17.80	18.10	18.27	18.33	18.34	18.35										
15.39	17.12	17.79	18.08	18.23	18.30	18.32											
15.41	17.30	18.06	18.38	18.53	18.56												
15.58	17.63	18.37	18.68	18.79													
15.94	17.94	18.68	18.92														
16.08	18.26	18.81															
16.54	18.24																
16.43																	

Example: LR High

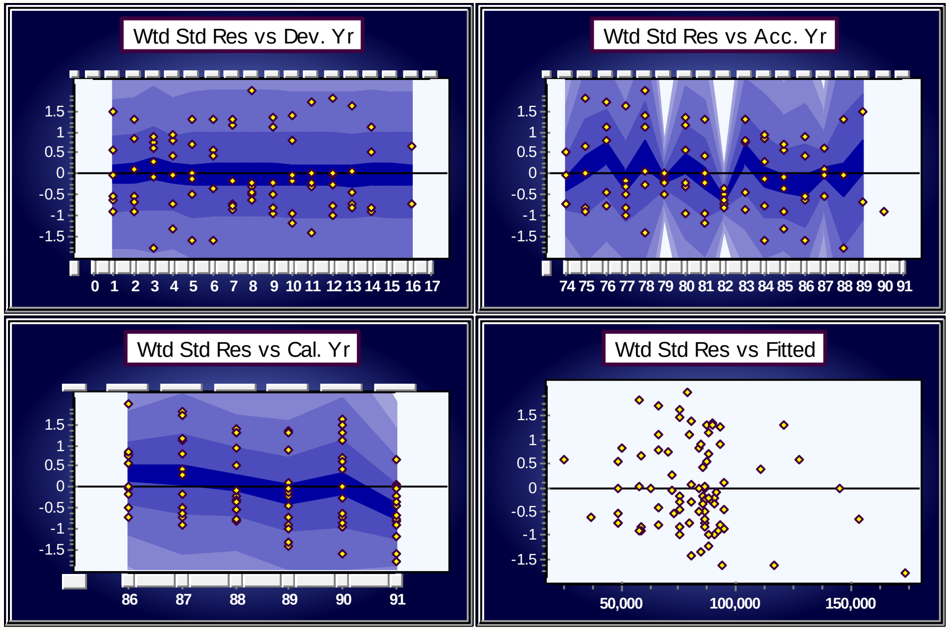
- There are obvious patterns in the residuals, remembering that residuals represent the trend(s) in the data minus the trend(s) in the method. In this example, the trend(s) in the method are much higher than the trends in the data. As a result, the forecast from the Mack method is **far too high**. We do not have control over capturing of calendar trends in the Mack method and, as a result, we must move to a framework which quantifies calendar trend in order to formulate a more correct estimate. The mean estimate of the total outstanding using the Mack method is 896M, with a standard deviation of the distribution of 104M. The arithmetic average gives around 1.1 Billion.

Outstanding	SD
896,133T	104,117T



Example: LR High

- What if we adjust for the calendar year trends for the last 6 calendar years?



Outstanding	SD
549,957T	34,430T

- The residuals versus calendar year look more around zero. This means that the trend estimated by the method is pretty close to the trend in the data. Accordingly the answer is much lower.

- However, although the method now seems to capture the most calendar trend, we don't know what it is, nor can we control the assumption of the future calendar trend. It is all hidden in the method's black box.

Example LR High

- Optimal parameter estimates: best combination of intercepts, trends, and ratios

ELRF Parameter Estimates

Development Period	Intercept Est.	Intercept S.E.	Intercept P-Value	Trend Est.	Trend S.E.	Trend P-Value	Ratio Est.	Ratio Ratio-1	Ratio S.E.	Ratio P-Value
0~1	-105,741	20,672	0.00691	11,392	1,592	0.00202	1.00000	0.00000	0.00000	0.00000
1~2	-73,558	18,019	0.01507	9,450	1,502	0.00326	1.00000	0.00000	0.00000	0.00000
2~3	****	****	****	****	****	****	1.33770	0.33770	0.01975	0.00001
3~4	943	3,743	0.81360	1,098	362	0.03872	1.00000	0.00000	0.00000	0.00000
4~5	5,306	332	0.00002	****	****	****	1.00000	0.00000	0.00000	0.00000
5~6	2,024	239	0.00038	****	****	****	1.00000	0.00000	0.00000	0.00000
6~7	890	268	0.02110	****	****	****	1.00000	0.00000	0.00000	0.00000
7~8	396	107	0.01418	****	****	****	1.00000	0.00000	0.00000	0.00000
8~9	398	123	0.02312	****	****	****	1.00000	0.00000	0.00000	0.00000
9~10	149	39.3	0.01256	****	****	****	1.00000	0.00000	0.00000	0.00000
10~11	42.8	50.3	0.43389	****	****	****	1.00000	0.00000	0.00000	0.00000
11~12	63.8	24.7	0.04957	****	****	****	1.00000	0.00000	0.00000	0.00000
12~13	****	****	****	****	****	****	1.00060	0.00060	0.00031	0.12725
13~14	19.6	10.9	0.17004	****	****	****	1.00000	0.00000	0.00000	0.00000
14~15	0	0	0.00000	****	****	****	1.00000	0.00000	****	****
15~16	****	****	****	****	****	****	1.00076	0.00076	0.00069	0.47080
16~17	****	****	****	****	****	****	1.00000	0.00000	****	****
To Ultimate							1.00000	0.00000	0.00000	****

Delta = 1, AIC = 1,256.5
If the test is to be conducted at an overall 5% level, a parameter would be regarded as insignificant if the corresponding P-Value is greater than 0.002440

- In this example, when an intercept and/or trend is used in the model, the ratio is not used (the ratio is set to 1.

Combining answers

Combining answers from several techniques

- It is a misguided exercise to combine information from several models without assessing their appropriateness (fidelity to data, assumptions, validation)
  - why mix good with bad?
- Answers should not be selected merely on the basis of their similarity to *each other* — not borrowing strength
- Projections from the best models unlikely to come from the centre of the range of answers.

Combining answers

- Misguided to try to fit a continuous probability distribution to the results from a collection of methods.
- Different methods do **not** simply yield random values from some underlying process centered on the correct answer - many methods will share similar biases. e.g. may miss the same features.
- More methods do **not** mean more information about the process.
- The range of answers from a variety of methods does **not** reflect the uncertainty in the process generating the losses.

Summary

1) Ratio techniques and extensions

- Ratios are regressions
- Regressions have assumptions
  - *Know what you assume when using ratios*
- Assumptions need to be checked
  - *When do ratios work?*
- Assumptions often don't hold
  - *What does this suggest?*

Summary

2) Probabilistic Trend Family modelling framework

- Model the incrementals on a logarithmic scale and use the relationship between the multivariate normal and the multivariate log-normal to transform to a dollar scale. (Include parameter uncertainty)
- Identify a parsimonious set of parameters that capture the trends in the three directions recognising that any calendar period trend projects in the other two directions.
- The identified model fits a probability **distribution** to every cell (mean + variance) and relates them through the trend structure.
- Test that all assumptions are met – they usually are.
- The methodology applies to any incremental array: paid losses, incremental incurred losses, case reserve estimates, claim counts.