Tutorial: Kernel Methods

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Abstract

Here comes the Abstract

1 Introduction to Kerenl Method

1.1 What Is Kernel and Kernel Trick

1.2 Why Use Kernel and How Kernel Can Be Used

2 Common Kernel

2.1 Hilber Space and RKHS

Definition 2.1 (Inner Product Space)**.** An inner product space $\mathcal{X}$ is a vector space with an associated inner product $\langle \cdot, \cdot \rangle : \mathcal{X} \to \mathbb{R}$ that satisfies:

- Symmetry: $\langle \mathbf{x}, \mathbf{y} \rangle = \langle \mathbf{y}, \mathbf{x} \rangle$
- Linearity: $\langle a \cdot \mathbf{x}, \mathbf{y} \rangle = a \cdot \langle \mathbf{x}, \mathbf{y} \rangle$ and $\langle \mathbf{w} + \mathbf{x}, \mathbf{y} \rangle = \langle \mathbf{w}, \mathbf{y} \rangle + \langle \mathbf{x}, \mathbf{y} \rangle$
- Positive Semi-Definiteness: $\langle \mathbf{x}, \mathbf{x} \rangle \geq 0$

The inner product space is strict if $\langle \mathbf{x}, \mathbf{x} \rangle = 0$ iff $\mathbf{x} = 0$

Definition 2.2 (Hilbert Space)**.** A strict inner product space $\mathcal{F}$ is a Hilbert space if it is:

- Complete: Every Cauchy sequence $\{h_i \in \mathcal{F}\}_{i=1}^{\infty}$ converges to an element $h \in \mathcal{F}$
- Separable: There is a countable subset $\hat{\mathcal{F}} = \{h_i \in \mathcal{F}\}_{i=1}^{\infty}$ such that for all $h \in \mathcal{F}$ and $\epsilon > 0$, there exists $h_i \in \hat{\mathcal{F}}$ such that $\|h_i - h\| < \epsilon$.

The interval $[0, 1]$, the reals $\mathbb{R}$, the complex numbers $\mathbb{C}$ and Euclidean spaces $\mathbb{R}^D$ are all Hilber spaces.

2.2 Kerenl Function

2.3 Polynomial Kernel

2.4 Gaussian Kernel

3 Kernel Machines

3.1 Kernel PCA

3.2 Kernel SVM

3.3 Kerenl Ridge Regression

3.4 Kernel Logistic Regression

4 Output Kernel

4.1 Kernel in Output Space

4.2 PLST and CPLST

4.3 Other Possible Output Kernel Techniques

5 Conclusion

Here comes the Conclusion