CV. Ex01.

1.1 Pen & Paper.

Homogeneous Coordinates

a). $\tilde{x} = \begin{pmatrix} \tilde{x}_1 \\ \tilde{y}_1 \\ 1 \end{pmatrix}$. $\tilde{l}_1 = \begin{pmatrix} a_1 \\ b_1 \\ c_1 \end{pmatrix}$. $\tilde{l}_2 = \begin{pmatrix} a_2 \\ b_2 \\ c_2 \end{pmatrix}$.

$\because$ $\tilde{x}$ is the intersection point.

$\therefore \begin{cases} a_1x_1 + b_1y_1 + c_1 = 0. \\ a_2x_1 + b_2y_1 + c_2 = 0. \end{cases}$ ans: $\begin{cases} x_1 = \dfrac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1} \\ y_1 = \dfrac{a_2c_1 - a_1c_2}{a_1b_2 - a_2b_1} \end{cases}$

$\because \tilde{l}_1 \times \tilde{l}_2 = \begin{bmatrix} 0 & -c_1 & b_1 \\ c_1 & 0 & -a_1 \\ -b_1 & a_1 & 0 \end{bmatrix} \begin{pmatrix} a_2 \\ b_2 \\ c_2 \end{pmatrix} = \begin{pmatrix} b_1c_2 - b_2c_1 \\ a_2c_1 - a_1c_2 \\ a_1b_2 - a_2b_1 \end{pmatrix} = \begin{pmatrix} \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1} \\ \frac{a_2c_1 - a_1c_2}{a_1b_2 - a_2b_1} \\ 1 \end{pmatrix}$

$\therefore$ $\tilde{x} = \tilde{l}_1 \times \tilde{l}_2$. proved.

b).

$\tilde{l} = (a_1, b_1, c_1)^T$. $\tilde{x}_1 = \begin{pmatrix} x_1 \\ y_1 \\ 1 \end{pmatrix}$. $\tilde{x}_2 = \begin{pmatrix} x_2 \\ y_2 \\ 1 \end{pmatrix}$.

$\because$ $\tilde{l}$ is joined by $\tilde{x}_1$, $\tilde{x}_2$.

$\therefore \begin{cases} a_1x_1 + b_1y_1 + c_1 = 0. \\ a_1x_2 + b_1y_2 + c_2 = 0. \end{cases}$ $(x_1 - x_2)a_1 = (y_2 - y_1)b_1$. one of the ans: ans: $\begin{cases} a_1 = y_2 - y_1 \\ b_1 = x_1 - x_2. \\ c_1 = x_2y_1 - x_1y_2. \end{cases}$

$\because \tilde{x}_1 \times \tilde{x}_2 = \begin{bmatrix} 0 & -1 & y_1 \\ 1 & 0 & -x_1 \\ -y_1 & x_1 & 0 \end{bmatrix} \cdot \begin{pmatrix} x_2 \\ y_2 \\ 1 \end{pmatrix} = \begin{pmatrix} \overset{y_1 - y_2}{\cancel{b_1c_2 - b_2c_1}} \\ x_2 - x_1 \\ x_1y_2 - x_2y_1 \end{pmatrix} = -1 \cdot \begin{pmatrix} y_2 - y_1 \\ x_1 - x_2 \\ x_2y_1 - x_1y_2 \end{pmatrix}$.

$\therefore$ $\tilde{l} = \tilde{x}_1 \times \tilde{x}_2$.

c).

$\begin{cases} x + y + 3 = 0 \\ -x - 2y + 7 = 0 \end{cases}$ $\begin{cases} x = -13 \\ y = 10 \end{cases}$. $x_0 = (-13, 10)$.

$\tilde{l}_1 \times \tilde{l}_2 = \begin{bmatrix} 0 & -3 & 1 \\ 3 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix} \cdot \begin{pmatrix} -1 \\ -2 \\ 7 \end{pmatrix} = \begin{pmatrix} 13 \\ -10 \\ -1 \end{pmatrix} = \begin{pmatrix} -13 \\ 10 \\ 1 \end{pmatrix}$. same.

d). $4x - 3y \pm 20 = 0$. $\tilde{l}$: $\begin{pmatrix} 4 \\ -3 \\ \underset{15}{20} \end{pmatrix}$. or $\begin{pmatrix} 4 \\ -3 \\ \underset{15}{-20} \end{pmatrix}$.

e). $\vec{n} = (\frac{5}{\sqrt{29}}, \frac{2}{\sqrt{29}})^T$ $d = \dfrac{\frac{\sqrt{29}}{5}}{\sqrt{29}} = \frac{1}{5}$.

Transformation.

a).
$$\begin{pmatrix}0\\3\\1\end{pmatrix} = \begin{pmatrix}1 & 0 & t_x\\0 & 1 & t_y\\0 & 0 & 1\end{pmatrix}\cdot\begin{pmatrix}1\\2\\1\end{pmatrix}. \quad t_x = -1. \quad t_y = -\cancel{2}1 \qquad M_T: \begin{pmatrix}1 & 0 & -1\\0 & 1 & 1\end{pmatrix}.$$

b). translation Matrix:
$$\begin{pmatrix}1 & 0 & t_1\\0 & 1 & t_2\\0 & 0 & 1\end{pmatrix} = T.$$

$$E(T) = \sum_{i=1}^{N} \| T\tilde{x}_i - \tilde{y}_i \|_2^2. = \sum_{i=1}^{N} \| (x_1^i + t_1 - y_1^i,\ x_2^i + t_2 - y_2^i,\ 1)^T \|_2^2$$

in order to find $T^* = \underset{T}{\operatorname{argmin}}\ E(T)$: $\quad J_E = \left[\frac{\partial(E)}{t_1},\ \frac{\partial(E)}{t_2}\right] = 0^T$

$$\therefore \begin{cases} \sum_{i=1}^{N} \partial\left[(x_1^i + t_1 - y_1^i)^2 + (x_2^i + t_2 - y_2^i)^2 + 1\right] / t_1 = 0. \\ \sum_{i=1}^{N} \partial\left[(x_1^i + t_1 - y_1^i)^2 + (x_2^i + t_2 - y_2^i)^2 + 1\right] / t_2 = 0. \end{cases}$$

$$\therefore \begin{cases} \sum_{i=1}^{N} (2t_1 + 2x_1^i - 2y_1^i) = 0. \\ \sum_{i=1}^{N} (2t_2 + 2x_2^i - 2y_2^i) = 0. \end{cases} \qquad \therefore \begin{cases} t_1 = \frac{1}{N} \sum_{i=1}^{N} (y_1^i - x_1^i). \\ t_2 = \frac{1}{N} \sum_{i=1}^{N} (y_2^i - x_2^i). \end{cases}$$

c).
$$t_1 = \frac{1}{3}(3-0+7-5+5-4) = 2.$$

$$t_2 = \frac{1}{3}(-5-1+6-7\cancel{0}-4-1) = -4.$$

$$T^* = \begin{pmatrix}1 & 0 & 2\\0 & 1 & -4\\0 & 0 & 1\end{pmatrix}.$$

Camera Projections

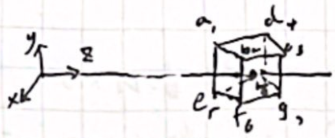
a).
$$R = \begin{bmatrix}1 & 0 & 0\\0 & 0 & -1\\0 & 1 & 0\end{bmatrix}. \quad t = \begin{bmatrix}1\\0\\2\end{bmatrix}. \quad K = \begin{bmatrix}100 & 0 & 25\\0 & 100 & 25\\0 & 0 & 1\end{bmatrix}.$$

$$\tilde{P} = \begin{bmatrix}100 & 0 & 25 & 0\\0 & 100 & 25 & 0\\0 & 0 & 1 & 0\\0 & 0 & 0 & 1\end{bmatrix} \times \begin{bmatrix}1 & 0 & 0 & 1\\0 & 0 & -1 & 0\\0 & 1 & 0 & 2\\0 & 0 & 0 & 1\end{bmatrix} = \begin{bmatrix}100 & 25 & 0 & 150\\0 & 25 & -100 & 50\\0 & 1 & 0 & 2\\0 & 0 & 0 & 1\end{bmatrix}$$

b). $\because \tilde{x}_c = P\cdot\tilde{x}_w \quad \therefore \tilde{x}_w = P^{-1}\cdot\tilde{x}_c.$ inhomogeneous: $(-1, 2, -1)^T$.

$$\therefore P^{-1} = \begin{bmatrix}\frac{1}{100} & 0 & -\frac{1}{4} & -1\\0 & 0 & 1 & -2\\0 & -\frac{1}{100} & \frac{1}{4} & 0\\0 & 0 & 0 & 1\end{bmatrix}. \qquad \tilde{x}_w = \begin{pmatrix}-0.25\\0.5\\-0.25\\0.25\end{pmatrix}$$

c). i). $K = \begin{bmatrix} 5 & 0 & 10 \\ 0 & 5 & 10 \\ 0 & 0 & 1 \end{bmatrix}$.



$\cancel{\{a^T, b^T, c^T, d^T, e^T, f^T, g^T, h^T\}_w}$

$\{a_i^T\}^T_w = \begin{pmatrix} (-10, 10, 5) & (-10, -10, 5) \\ (10, 10, 5) & (10, -10, 5) \\ (10, 10, 25) & (10, -10, 25) \\ (-10, 10, 25) & (-10, -10, 25) \end{pmatrix}$

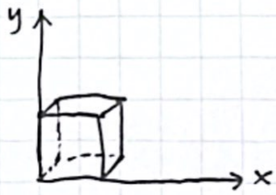
∴ $\cancel{\{a^T \sim h^T\}^T_c = ?}$ ∵ $\tilde{a}_{ic} = K \cdot \tilde{a}_{iw}$

∴ $\{a_i^T\}^T_w = \begin{pmatrix} (0 & 100 & 5) & (0 & 0 & 5) \\ (100 & 100 & 5) & (100 & 0 & 5) \\ (300 & 300 & 25) & (300 & 200 & 25) \\ (200 & 300 & 25) & (200 & 200 & 25) \end{pmatrix}$

ii). $\{a_i^T\}^T_w = \{$ x, y the same. z: 10 10 30 30, 10 10 30 30 $\}$. $K = \begin{bmatrix} 10 & 0 & 10 \\ 0 & 10 & 10 \\ 0 & 0 & 1 \end{bmatrix}$.

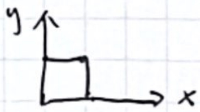
$\{a_i^T\}^T_c = \begin{Bmatrix} 0 & 200 & 10 & 0 & 0 & 10 \\ 200 & 200 & 10 & 200 & 0 & 10 \\ 400 & 400 & 30 & 400 & 200 & 30 \\ 200 & 400 & 30 & 200 & 200 & 30 \end{Bmatrix}$.

iii). $\{a_i^T\}^T_w = \{$ x, y the same. z: $\cancel{80}$ 90 90, $\cancel{80}$ 90 90, $\cancel{100}$ 110 110, 110 110 $\}$. $K = \begin{bmatrix} 90 & 0 & 10 \\ 0 & 90 & 10 \\ 0 & 0 & 1 \end{bmatrix}$



$\{a_i^T\}^T_c = \begin{Bmatrix} 0 & 1800 & 90 & 0 & 0 & 90 \\ 1800 & 1800 & 90 & 1800 & 0 & 90 \\ 2000 & 2000 & 110 & 2000 & [illegible] & 110 \\ [illegible] & 2000 & 110 & [illegible] & [illegible] & 110 \end{Bmatrix}$.

iv). $\{a_i^T\}^T_c = \{ (1\ 1\ 1)\ (1\ 1\ 1) \ \vdots \}$.



v). when z is infinitely far away.

Photometric Image Formation.

a). $\frac{1}{z_c} + \frac{1}{z_s} = \frac{1}{f}$. $z_c = \frac{1}{\frac{1}{f} - \frac{1}{z_s}} = 5100\text{mm}$.

b).

$\frac{z_s - \Delta z_s}{d} = \frac{\Delta z_s}{c}$. $c = \frac{\Delta z_s}{z_s - \Delta z_s} \cdot d \approx \frac{\Delta z_s}{z_s} \cdot d$

∴ $c = \frac{\Delta z_s \cdot f}{z_s \cdot N}$

(7). when $\Delta z_s = 0.1mm$: $\cancel{10^{-4}}$ 6.25×10^{-2}

when $\Delta z_s = \cancel{0.3mm}$ 0.03mm : $\cancel{3 \times 10^{-4}}$ 1.875×10^{-3}

when $\Delta z_s = 0.1mm$: $sp = \sqrt{\frac{64}{400^2}} = \cancel{4 \times 10^{-4}}$ $0.02 = 2 \times 10^{-2}$.

~~w~~ same for $\Delta z_s = 0.3mm$.

$\therefore$ $\cancel{2 \times 10^{-2} <}$ $1.875 \times 10^{-3} < 2 \times 10^{-2} < 6.25 \times 10^{-2}$

$\therefore$ for 0.03 mm, sharp. for 0.1mm, not enough.