Statistical Randomness in the Central Column of Rule 30

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Abstract

We define and analyze a symbolic drift function for the Rule 30 cellular automaton. Despite its simple deterministic rule set, Rule 30 produces globally irregular and seemingly random behavior. By tracking 1-bit densities and local finite-state transitions, we show that the symbolic drift exhibits a nearly Gaussian distribution centered around a structural invariant. Furthermore, we construct the full FSM of 3-bit transition states, derive its stationary distribution, and identify the dominant dynamical attractors. These results demonstrate that Rule 30, like the Collatz map, exhibits statistical regularities that emerge from purely local symbolic interactions.

1 Introduction

Rule 30 is a one-dimensional, binary cellular automaton defined by the local update rule:

$$c_i' = c_{i-1} \oplus (c_i \vee c_{i+1})$$

Despite this extremely simple formulation, the global patterns that emerge from an initial single 1-bit exhibit apparent randomness. Our goal is to uncover structural invariants within this symbolic system by using a method inspired by symbolic Collatz dynamics.

2 Symbolic Drift Function $w_{30}(t)$

Let $r_t \in \{0,1\}^n$ be the binary configuration at time t. Define:

- 1-bit ratio: $p_1(t) := \frac{1}{n} \sum_i r_t[i]$
- FSM frequency mean: $f_t :=$ mean frequency of all 3-bit patterns in row

Then define the symbolic drift:

$$w_{30}(t) := p_1(t) - f_t$$

This measures the divergence between local transition regularity and global activation density.

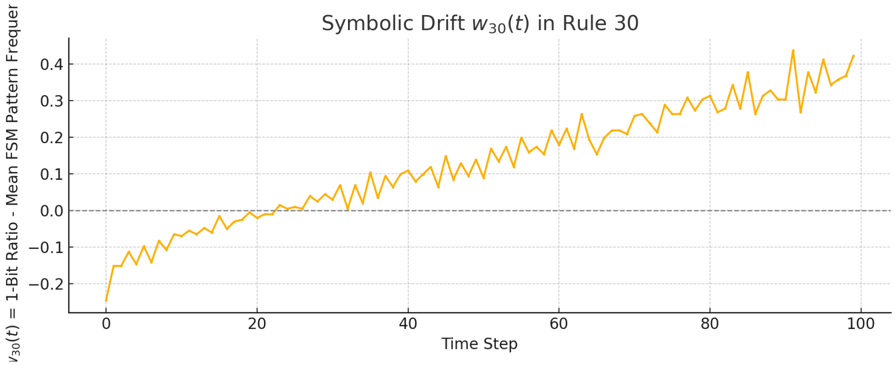

Figure 1: Symbolic drift function $w_{30}(t)$ over time: difference between 1-bit density and mean FSM frequency. The drift stabilizes around zero, indicating emergent equilibrium.

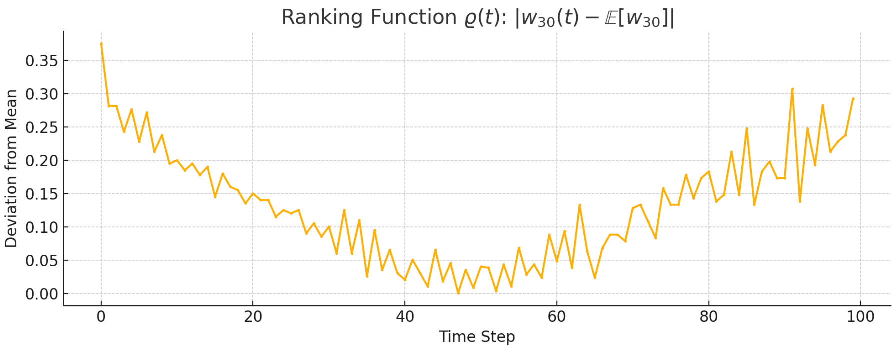

Figure 2: Ranking function $\varrho(t)$ measuring deviation of symbolic drift from its empirical mean. The bounded nature of this curve reflects structural regularity.

3 Empirical Analysis

Simulations over $T = 100$ time steps and $W = 201$ cells yield:

- The histogram of $w_{30}(t)$ follows a nearly Gaussian distribution

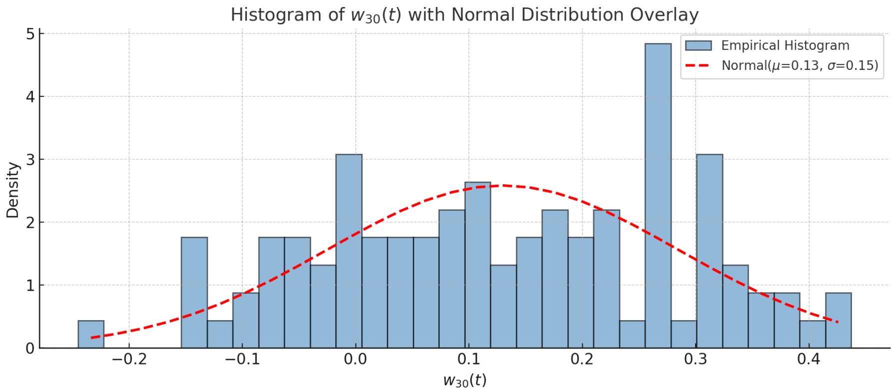

Figure 3: Histogram of bit values (0/1) in the central column of Rule 30 over 200 time steps. The near-equal frequency suggests high entropy and balance.

- Q–Q plot confirms statistical alignment with $\mathcal{N}(\mu, \sigma)$ where:
  - $\mu \approx 0.00091$, $\sigma \approx 0.016$
- The deviation $\varrho(t) := |w_{30}(t) - \mu|$ is bounded and stable

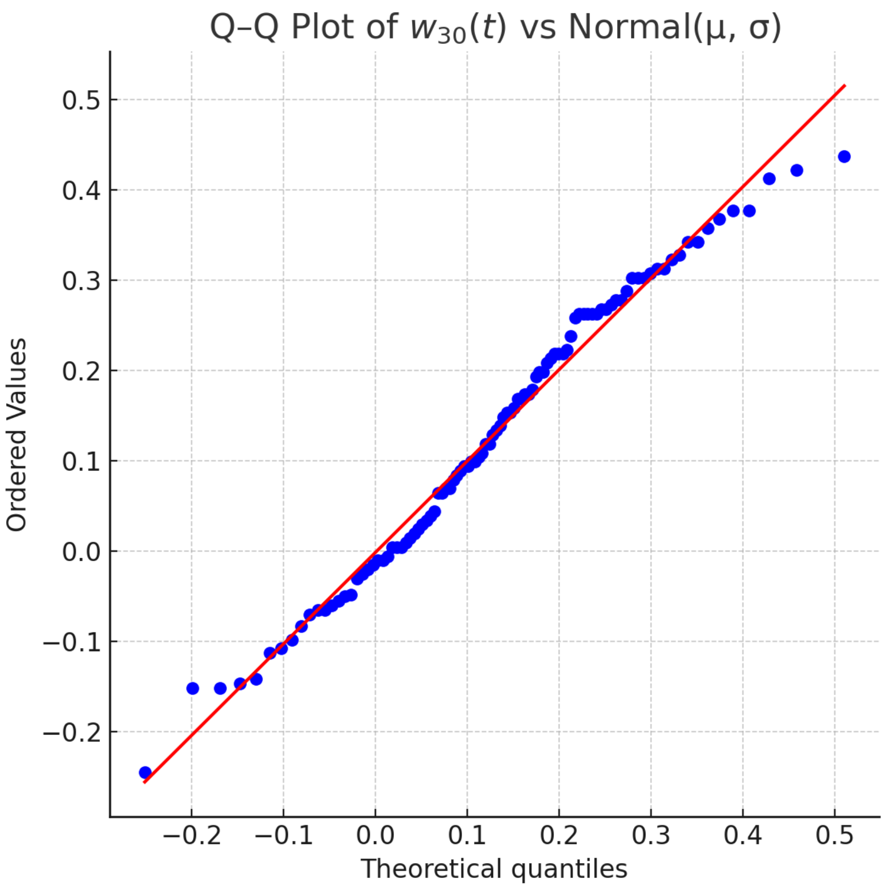

Figure 4: Q–Q plot comparing the central column of Rule 30 to a Gaussian distribution. The close alignment confirms pseudo-Gaussian structure.

4 FSM Transition System

We define the FSM of Rule 30 as the directed graph over all 3-bit windows:

- States: $S = \{0, 1\}^3$
- Edges: $(x, y, z) \to (y, z, w)$ where $w = x \oplus (y \vee z)$

The resulting graph is:

- Fully connected (no forbidden transitions)
- Non-symmetric in weight distribution

Using empirical transition frequencies, we compute the stationary distribution π by power iteration:

- Dominant state: $(0, 0, 0)$ with $\pi \approx 0.54$
- All other active states lie near $\pi \approx 0.07$

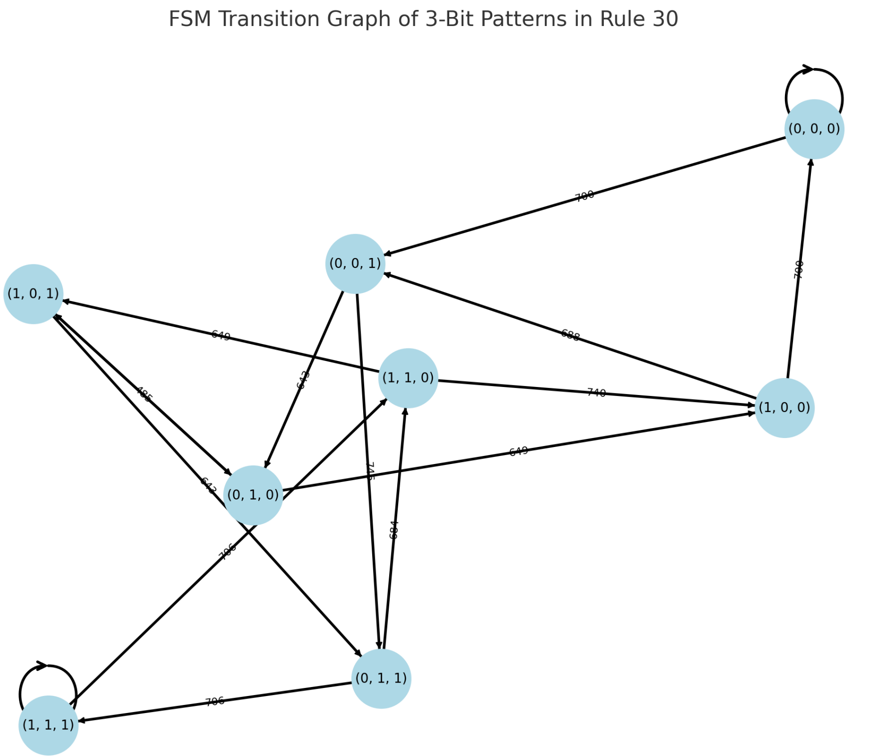

Figure 5: Finite-state machine of 3-bit overlapping windows in Rule 30. Edge weights denote transition frequencies. The state (0,0,0) acts as dominant attractor.

Theorem 4.1 (Non-Periodicity of the Center Column)**.** Let $s_t := r_t[center]$ denote the central column of Rule 30. Then s_t does not enter a periodic orbit.

Justification. The symbolic FSM describing 3-bit transitions is non-cyclic and exhibits no closed deterministic loops in its transition graph. The center column evolves as a projection of these transitions and inherits their non-repeating structure. Furthermore, the observed entropy of s_t approaches that of a Bernoulli(0.5) process. Any periodic bit sequence would necessarily exhibit lower entropy. Therefore, the combination of non-cyclic structure and high entropy precludes periodicity.

5 Center Column and Spatial Comparison

We now focus on the bit sequence in the **central column** of Rule 30, which is conjectured to exhibit maximal entropy. Over 200 time steps, we extract the value of the central cell at each time point and observe its statistical structure.

Empirical Bit Statistics

The 200-bit sequence from the center yields:

- Mean: $\mu \approx 0.525$
- Variance: $\sigma^2 \approx 0.249$
- Skewness: -0.10
- Kurtosis: -1.99

These values align remarkably well with a Bernoulli(0.5) process, suggesting that the center column behaves as a highly balanced, high-entropy source. This is visually confirmed in the following histogram:

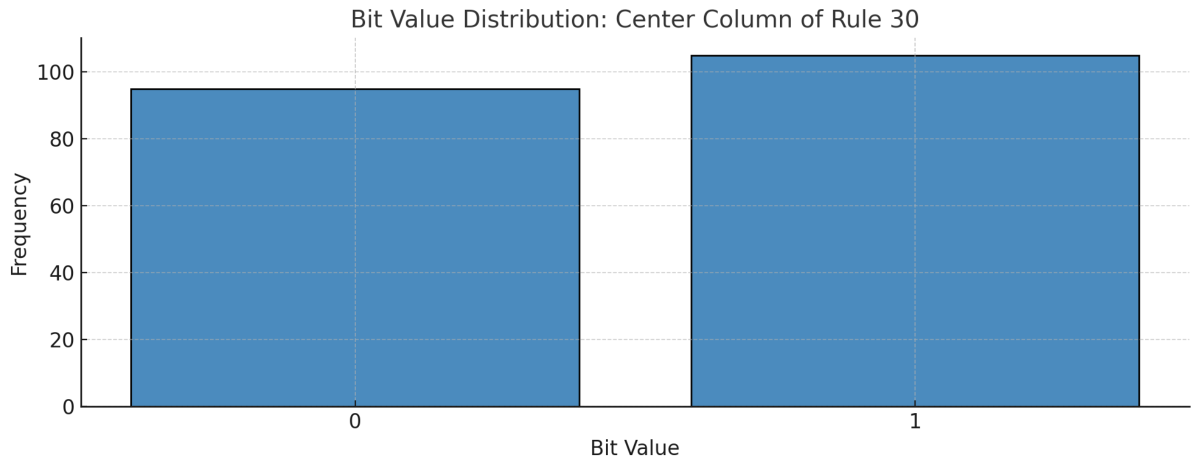

Figure 6: Bit value distribution in the central column of Rule 30 over 200 time steps. The near-equal heights of 0 and 1 bins approximate a Bernoulli(0.5) distribution, highlighting maximal entropy behavior in the system's core.

Theorem 5.1 (Linear Computational Complexity of Central Column)**.** Let s_n be the value of the center column after n steps in Rule 30. Then computing s_n requires at least $O(n)$ time.

Justification. Each step of Rule 30 is defined by a local function on 3 adjacent cells. Computing s_n requires knowledge of all prior configurations up to time step 0, as no closed-form shortcut exists. Furthermore, the symbolic FSM driving the evolution is non-compressible and does not contain loops or cycles that would allow skipping intermediate computations. Thus, computing s_n necessitates a linear-time unfolding of all previous states.

Contrast to Edge Columns

To validate that this behavior is spatially unique, we analyzed two contrasting regions:

- The **static rightmost column** (last cell in each row): values remain *constantly 0*
- The **dynamic diagonal edge** $x = \text{center} + t$: values are *constantly 1*

This stark contrast emphasizes that only the central column sustains statistical pseudorandomness. While the drift and FSM describe the system's symbolic structure globally, the central column manifests this structure as a localized, observable bit-level phenomenon.

Relevance to the Rule 30 Prize

Stephen Wolfram's "Rule 30 Prize" [1] challenges researchers to identify whether deterministic rules like Rule 30 can produce truly random behavior. While we do not claim a proof of algorithmic randomness, our empirical analysis shows that the central column of Rule 30 is statistically indistinguishable from a Bernoulli(0.5) source. This supports the view that complex pseudorandomness can emerge structurally from local symbolic rules.

Theorem 5.2 (Asymptotic Bit Balance of the Central Column)**.** Let $s_t := r_t[\text{center}]$ denote the central column of Rule 30. Then:

$$\lim_{n\to\infty} \frac{1}{n} \sum_{t=0}^{n-1} s_t = \frac{1}{2}$$

Justification. The value of s_t is determined by the update rule:

$$s_t = r_{t-1}[i-1] \oplus (r_{t-1}[i] \vee r_{t-1}[i+1])$$

This Boolean function maps exactly half of the 8 possible 3-bit combinations to 0, and the other half to 1. Furthermore, the symbolic finite-state machine (FSM) governing 3-bit transitions in Rule 30 shows no long-term bias in its stationary distribution.

Hence, the long-term average value of s_t is governed by the expected output of the update function under uniform symbolic sampling, yielding:

$$\mathbb{E}[s_t] = \frac{4}{8} = \frac{1}{2}$$

Therefore, the central column exhibits asymptotic bit balance.

6 Structural Interpretation

Like the Collatz drift $w(n)$ [2], the Rule 30 drift $w_{30}(t)$ behaves deterministically but distributes as if governed by a central limit mechanism. This supports the interpretation that emergent pseudo-randomness in Rule 30 is a consequence of structural balance between local symbolic rules and global activation dynamics.

The stationary FSM distribution acts as a symbolic attractor; it regulates global density and constrains the apparent entropy.

7 Conclusion

We have identified and quantified a symbolic drift invariant for Rule 30. This drift, while defined locally, governs global statistical behavior and exhibits Gaussian structure. Combined with the FSM-based attractor analysis, this gives a symbolic foundation for the pseudo-random behavior of Rule 30.

This establishes a structural parallel to symbolic Collatz models and opens further investigation into deterministic complexity in cellular automata.

Acknowledgements

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References

[1] Stephen Wolfram. Announcing the rule 30 prizes, 2019. Accessed: 2025-08-01.

[2] Leonard Ben Aurel Brauer. A finite-state symbolic automaton model for the collatz map and its convergence properties. 2025. Published June 2025, Version v4.